Nucleon quasi-parton distributions within the chiral quark soliton model

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(CRC110 Symmetries and the emergence of Structure in QCD)

Parton distribution functions

- **Momentum distribution of the partons inside hadron**
  ~probability interpretation

- **Factorisation theorem: DIS, HH →Jets, etc…**

- **Parton distributions at a low renormalisation point**
  ~ initial conditions of the QCD evolution:
  model calculation, lattice simulation?

Quasi-parton distribution functions

- **Lattice simulation of QCD:**

  Direct calculation of the parton distribution functions(PDFs) is difficult, but the moments.

- **Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013).**

  Quasi-parton distribution functions(qPDFs):

  Nucleon is (slightly) off the light-cone

  PDFs can be obtained in the limit $P \to \infty$

Nucleon PDFs from Chiral quark soliton model

- **At low energy, a successful description of the PDFs was made: Positivity, sum rules, …**

  [D. Diakonov, V. Y. Petrov, P. V. Pobylitsa, M. Polyakov, and C. Weiss, Nuclear Physics B 480, 341 (1996)]

- **Quasi-parton distribution function**

  Nucleon matrix element in Euclidean separation

  Lorentz boost → PDFs

- **Properties of qPDFs:**

  Sum-rules, positivity

  evolution in P

Outline

- ***Chiral quark-soliton model***
- ***Nucleon qPDFs in the model: isoscalar unpolarised***
- ***Properties of the qPDF***
- ***Numerical result & discussions***

Chiral quark-soliton model

$$\exp(iS_{\text{eff}}[\pi(x)]) = \int D\psi D\bar{\psi} \exp\left(i \int d^4x \bar{\psi}(i\partial\!\!\!/ - MU^{\gamma_5})\psi\right)$$

$$U^{\gamma_5} = \exp(i\pi^a(x)\tau^a\gamma_5)$$

From ***QCD*** by the ***instanton*** approach, Λ = 1/ρ ~ 600 MeV

Fully field-theoretic: Static & dynamic observables of the baryons

~ mass splitting, couplings, form factors, and

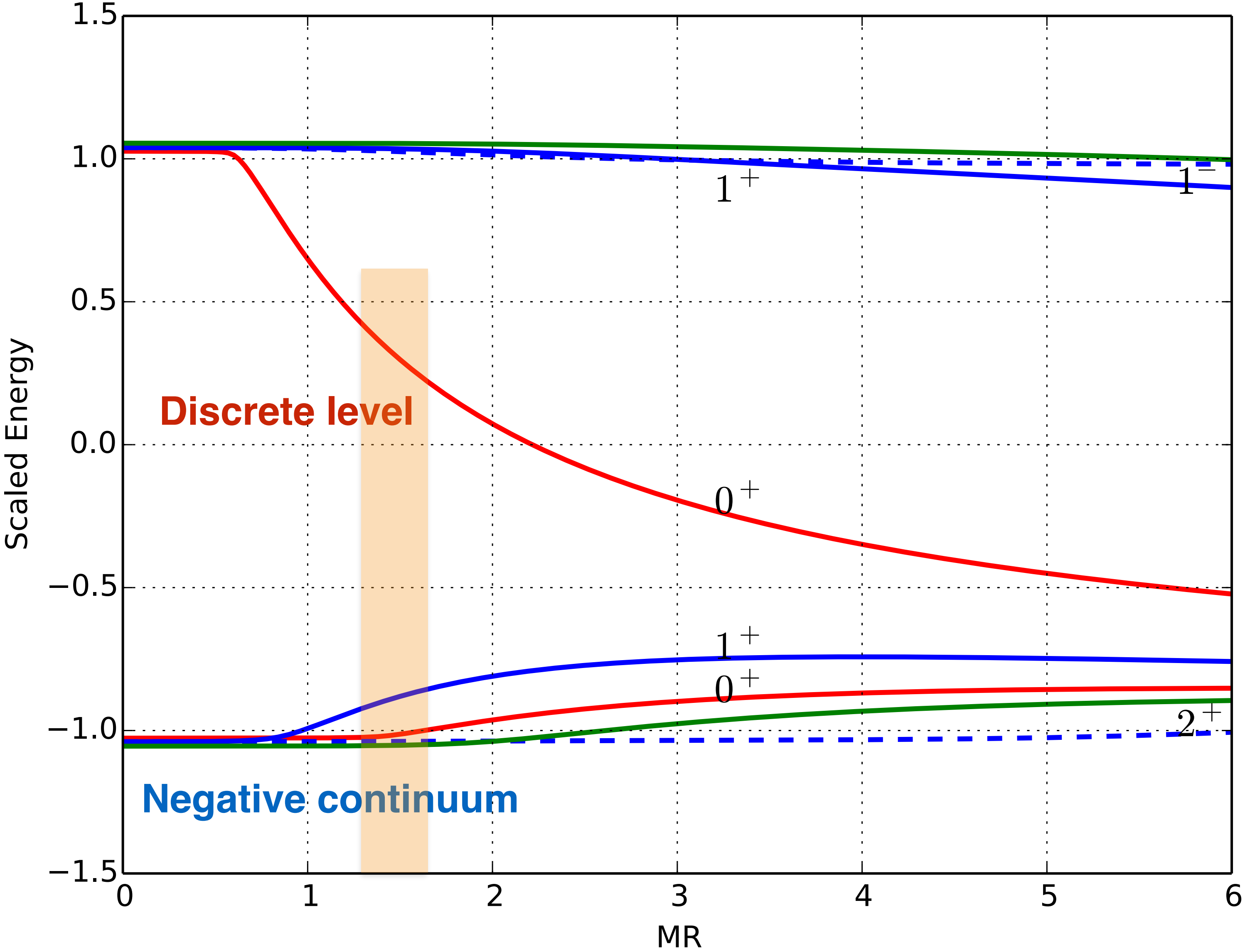
parton distribution functions

Bridges the quark model and solitonic pictures

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

Chiral quark-soliton model

Hedgehog Ansatz: $U_{\mathrm{SU}(2)} = \exp\left[i\gamma_5 \mathbf{n}\cdot\boldsymbol{\tau} P(r)\right]$



Quantum Numbers:

G = J + τ

P = $(-1)^{G,G+1}$

Quarks are bound by the pion mean-field

Quasi parton distribution function

Quasi-parton distribution

$$q(x,\mu,P^z) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P|\bar{\psi}(0)\gamma^z \exp\left[-ig\int_0^z dz' A^z(z')\right]\psi(z)|P\rangle + \mathcal{O}(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2})$$

$$g(x,\mu^2,P_z) = \int \frac{dz}{4\pi k^z} e^{izk^z} \langle P|F^{3\mu}(z)\exp\left[-ig\int_0^z dz'\ A^z(z')\right] F^3_\mu(0)|P\rangle$$

Large Momentum Effective Theory

Spacelike separation→ Lattice simulation can calculate this!

Approaches to PDFs in the limit v → 1

[Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013).]

Nucleon qPDF in the χQSM

- **In the large Nc limit,**

Isosinglet unpolarised	$u(x) + d(x)$	$\sim \; N_c^2 \, \rho(N_c x)$
Isovector polarised	$\Delta u(x) - \Delta d(x)$	
Isosinglet polarised	$u(x) - d(x)$	$\sim \; N_c \, \rho(N_c x)$
Isovector unpolarised	$\Delta u(x) + \Delta d(x)$	

Nucleon qPDF in the χQSM

qPDF (isosinglet unpolarised) in χQSM

$$\sum_f q_f(x,v)_{occ} = N_c M_N v \sum_{occ} \langle n|\delta(P^3 + vE_n - vM_N x)(1+v\gamma_0\gamma^3)|n\rangle$$
$$-(U\to 1)$$

Low-energy effective Λ ~ 600 MeV

Constituent quarks as effective degrees of freedom

Recovers the PDF as v →1

$\sum_{occ}$	: sum over the occupied levels $H\|n\rangle = E_n\|n\rangle$ *valence + negative continuum*
$(U\to 1)$	:vacuum subtraction
v	: boost velocity
Γ	: γ^0

Properties of the PDF

Baryon number

$$\int_0^1 dx\; q(x) - \bar{q}(x) = N_C B$$

Momentum

$$\int_0^1 dx\; x(q(x) + \bar{q}(x)) = 1$$

Positivity: Probability interpretation of the PDFs

What we know: *continuum contribution is essential!*

Baryon Number Sum rule

$$\sum_f q_f(x,v)_{occ} = N_c M_N v \sum_{occ} \langle n|\delta(P^3 + vE_n - vM_N x)(1 + v\gamma_0\gamma^3)|n\rangle$$
$$-(U \to 1)$$

Equivalent representation

$$\sum_f q_f(x,v) = N_c M_N v \int_{-\infty}^{E_{\rm level}} dw \ {\rm Tr}\left[\delta(w-H)\delta(wv - xvM_N + P^3)(1 + v\gamma^0\gamma^3)\right]$$
$$-(U \to 1)$$

Nucleon mass

$$M_N = N_c {\rm Tr}[H\Theta(E_{\rm level} - H)] - (U \to 1)$$

$\Rightarrow$

$$\int_{-\infty}^{\infty} \sum_f q(x,v) dx = \int_{-\infty}^{\infty} dx N_c M_N v \int_{-\infty}^{E_{\rm level}} dw {\rm Tr}[\delta(w-H)$$
$$\delta(wv - xvM_N + P^3)(1 + v\gamma^0\gamma^3)] - (U \to 1) = N_c B$$

Momentum Sum rule

$$\sum_f q_f(x,v)_{occ} = N_c M_N v \sum_{occ} \langle n|\delta(P^3 + vE_n - vM_N x)(1 + v\gamma_0\gamma^3)|n\rangle$$

$$-(U \to 1)$$

For Γ = γ^0,

$$\int_{-\infty}^{\infty} \sum_f q_f(x,v)_{occ}\, x\, dx = \frac{N_c}{vM_N}\mathrm{Tr}[\Theta(E_{\mathrm{level}} - H)(H + P^3\gamma_0\gamma^3)v] = 1$$

For Γ = γ^3,

$$\int_{-\infty}^{\infty} \sum_f q_f(x,v)_{occ}\, x\, dx = v$$

Momentum sum rule is satisfied only with the self-consistent pion field

Divergences & derivative expansion

Expansion in small pion-momentum

$$\sum_f q_f(x,v) = \sum_{n=0}^{\infty}\sum_f q_f^{(n)}(x,v)$$

$$n^0 = 1,\ n^3 = -\frac{1}{v},\ n_\perp = 0$$

$$\bar{n}^0 = 1,\ \bar{n}^3 = -v,\ \bar{n}_\perp = 0$$

$$\sum_f q_f^{(n)}(x,v) = \frac{N_c M_N}{T}\mathrm{ImTr}\left[(i\gamma^\mu\partial_\mu + MU^{-\gamma_5})(-1)^n((-\partial^2 - M^2 + i\epsilon)^{-1} iM(\partial\!\!\!/ U^{-\gamma_5}))^n \right.$$
$$\left. (-\partial^2 - M^2 + i\epsilon)^{-1}\delta(in^\mu\partial_\mu - xM_N)(\bar{n}\cdot\partial)\right] - (U\to 1)$$

1. To understand divergence
2. Simple calculation for the sea contribution

Divergences & derivative expansion

Expansion in small pion-momentum

$$\sum_f q_f^{(0)}(x,v) = 0$$

$$\sum_f q_f^{(1)}(x,v) = -\,4N_C M_N \mathrm{Im}\, M^2 \int \frac{d^3k}{(2\pi)^3}\frac{d^4p}{(2\pi)^4}[(p+k)^2 - M^2 + i\epsilon]^{-1}[p^2 - M^2 + i\epsilon]^{-1}$$
$$\times\, \delta(n^\mu p_\mu - xM_N)(\bar{n}\cdot k)\mathrm{tr}\left[\tilde{U}(\vec{k})^\dagger \tilde{U}(\vec{k})\right]$$

Logarithmic divergence of $q_f^{(1)}$

$$\sum_f q_f^{(2)}(x,v) = -\,4N_C M_N \mathrm{Im}\, M^2 \int \frac{d^3k}{(2\pi)^3}\frac{d^4p}{(2\pi)^4}[(p+k)^2 - M^2 + i\epsilon]^{-1}[p^2 - M^2 + i\epsilon]^{-2}$$
$$\times\, \delta(n^\mu p_\mu - xM_N)(\bar{n}\cdot p)k^2\mathrm{tr}\left[\tilde{U}(\vec{k})^\dagger \tilde{U}(\vec{k})\right]$$

Numerical calculation

Regularisation: Pauli-Villars

$$q(x,v)^{PV} = q(x,v)^{\text{level}} + q(x,v)_{occ} - \frac{M^2}{M_{PV}^2} q(x,v)_{occ}(M \to M_{PV})$$

$$F_\pi^2 = \frac{N_c M^2}{4\pi^2} \log(M_{PV}^2/M^2)$$

$$M = 350 \text{ MeV}$$
$$M_{PV} = 557 \text{ MeV}$$

Pion profile

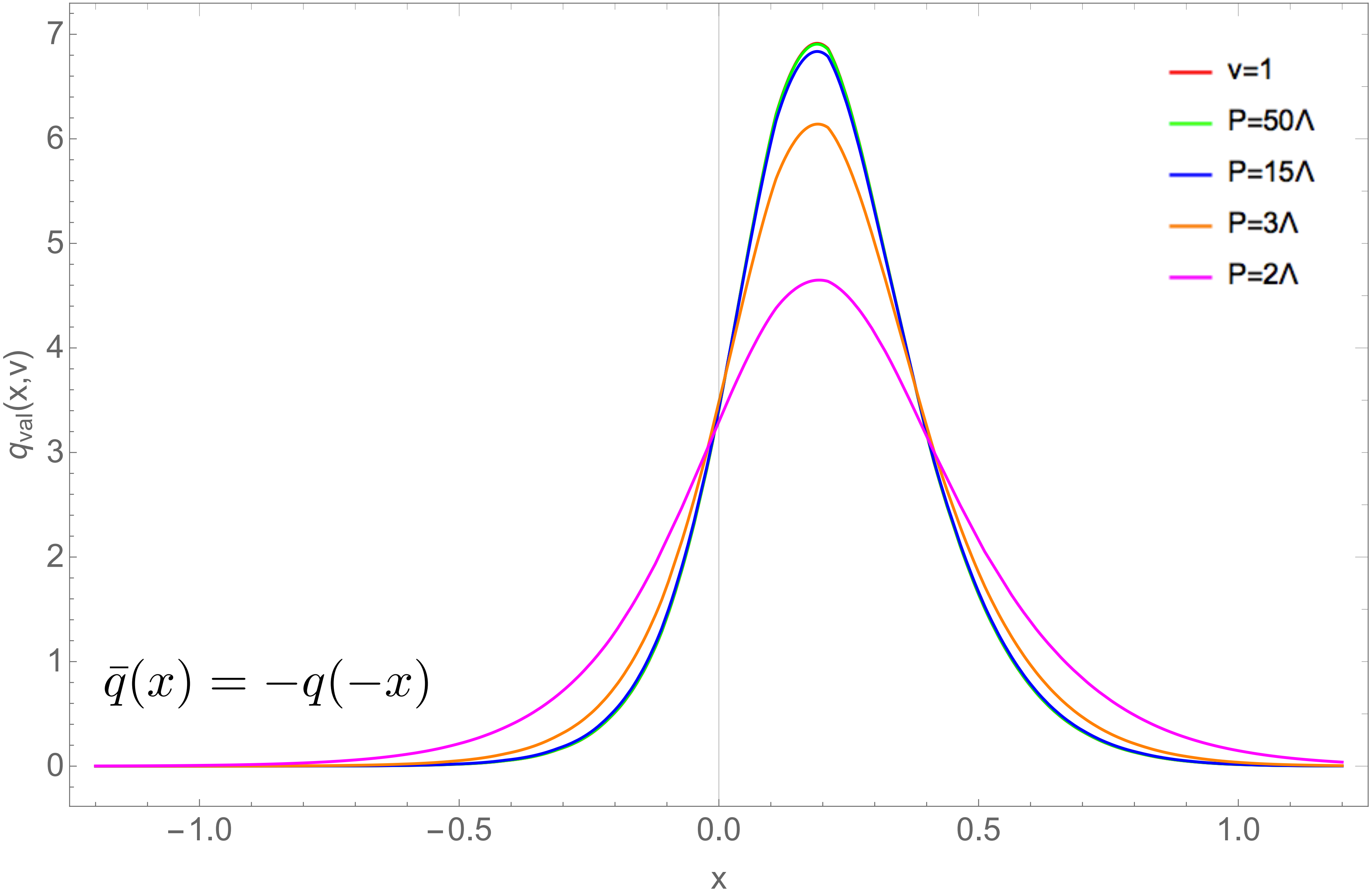
~10% from the self-consistent solution

$$P(r) = 2 \text{ Arctan}\left(\frac{r_0^2}{r^2}\right) \quad r_0 \approx 1/M$$

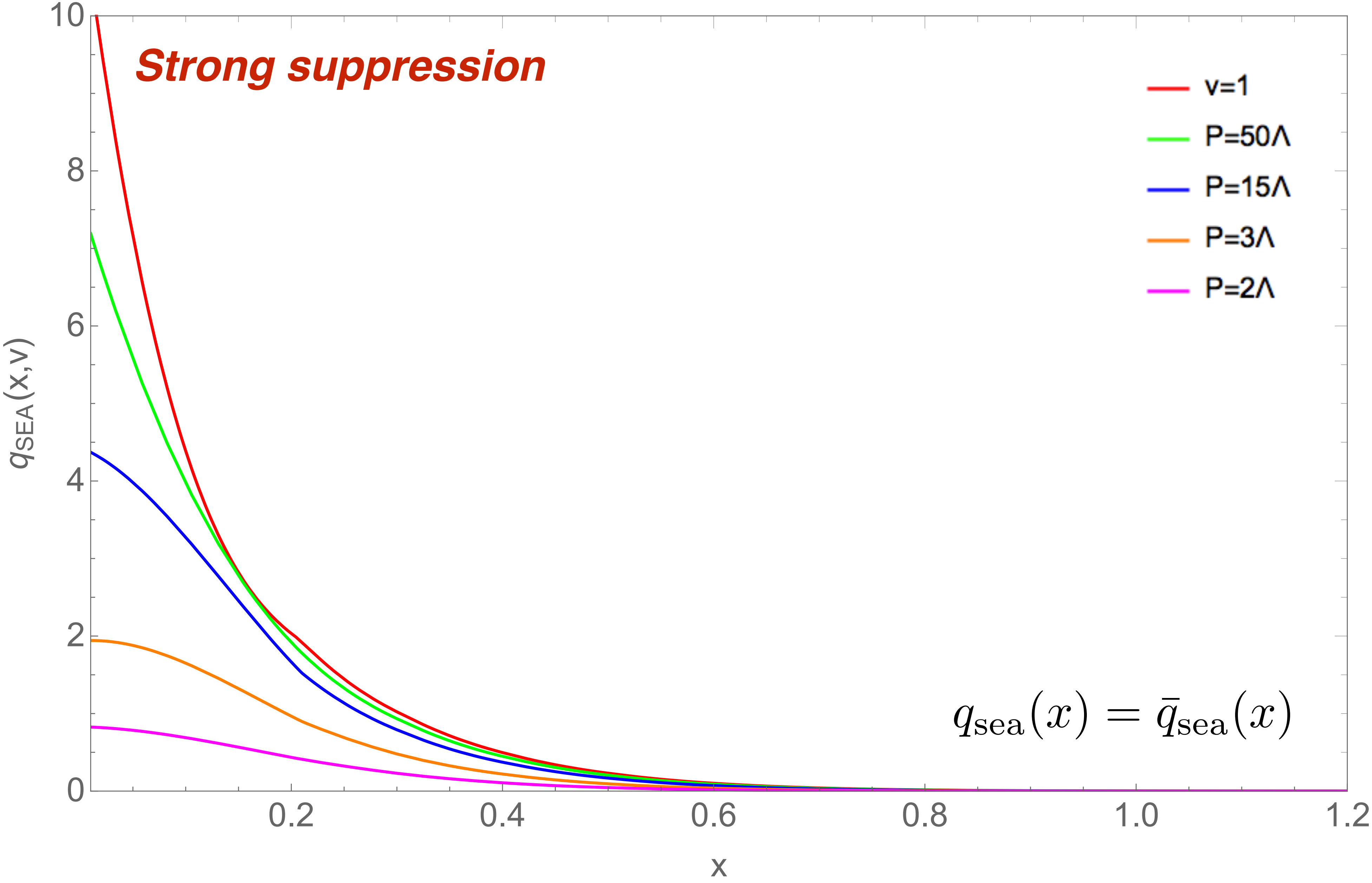
Nucleon mass (calculated)

$$M_N \approx 1200 \text{ MeV}$$

Valence

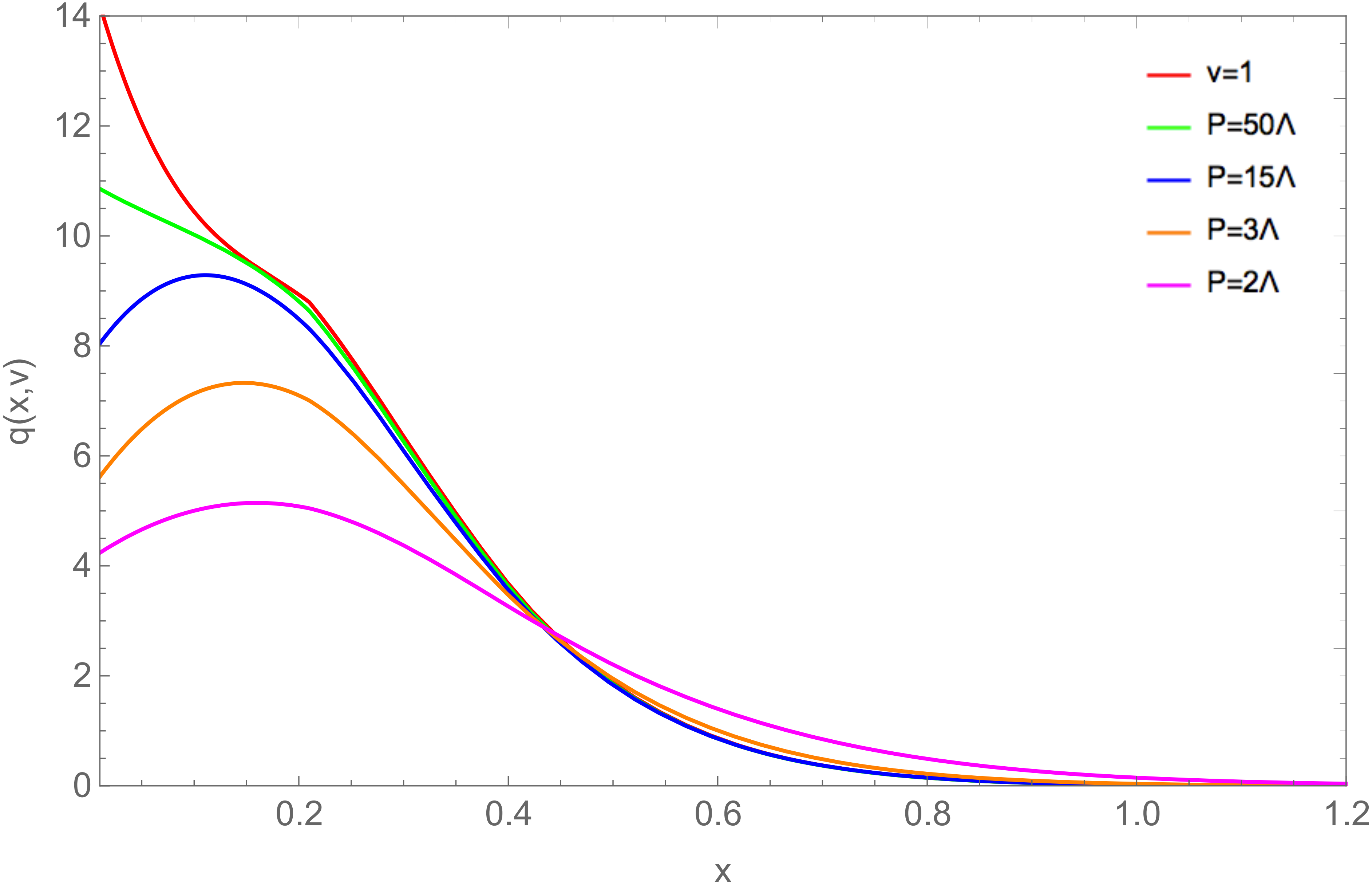


Sea

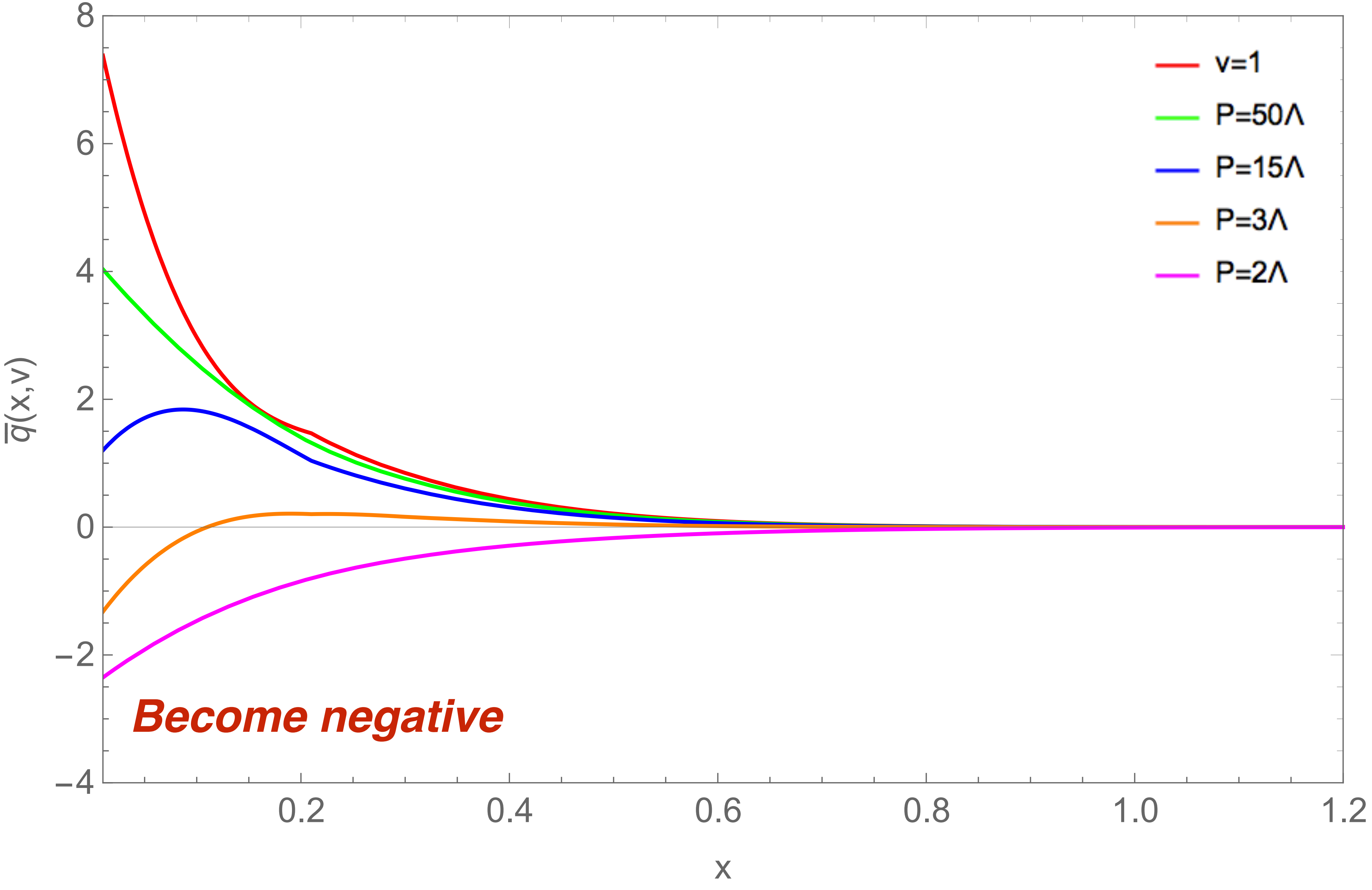


Strong suppression
v=1
P=50Λ
P=15Λ
P=3Λ
P=2Λ
$q_{\rm SEA}(x,v)$
x
0
2
4
6
8
10
0.2
0.4
0.6
0.8
1.0
1.2
$q_{\rm sea}(x) = \bar{q}_{\rm sea}(x)$

q(x,v)

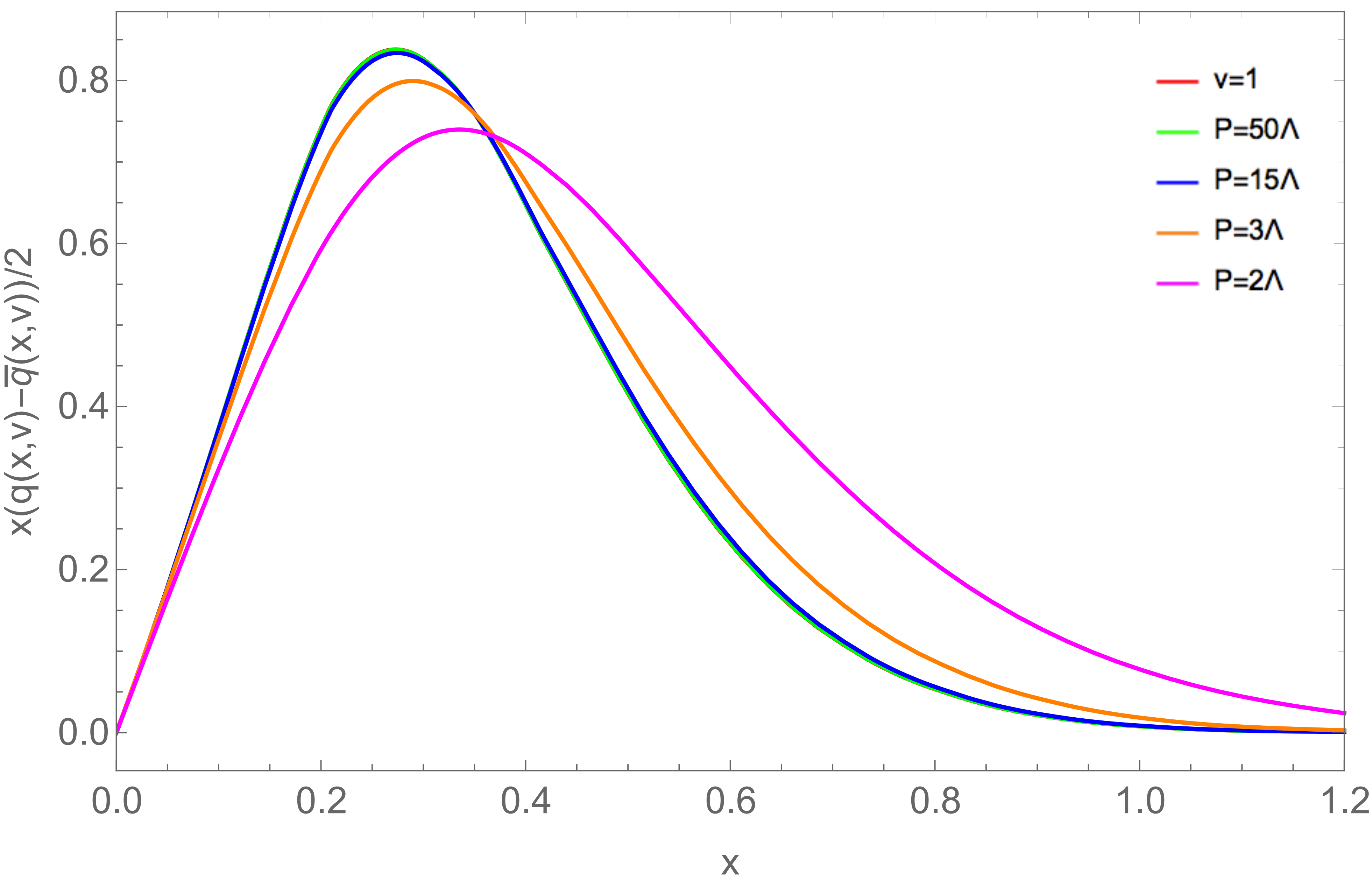


$\bar{q}(x,v)$



8
6
4
2
0
−2
−4
0.2
0.4
0.6
0.8
1.0
1.2
x
q̄(x,v)
v=1
P=50Λ
P=15Λ
P=3Λ
P=2Λ
Become negative

$x\,(\,q(x,v) - \bar{q}(x,v)\,)\,/2$



Perspectives

- Isosinglet unpolarized qPDF & sum rules

  positivity: no clear proof → full calc.

- Momentum sum rule: self-consistent profile is necessary

- **Δu-Δd is coming soon!** $\Delta\bar{u} - \Delta\bar{d}$

- Polarised & transverse distributions

- Comparison to lattice calculations & other models

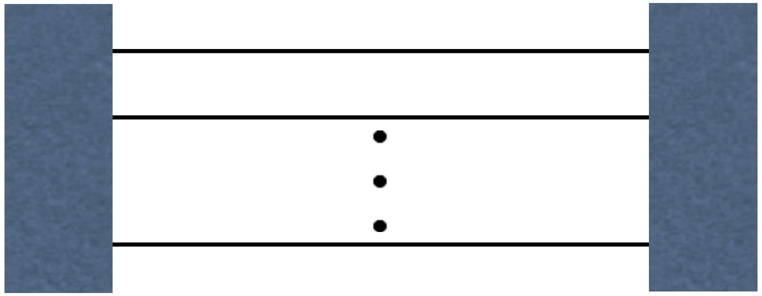
- Pseudo-parton distributions: ~ TMDs

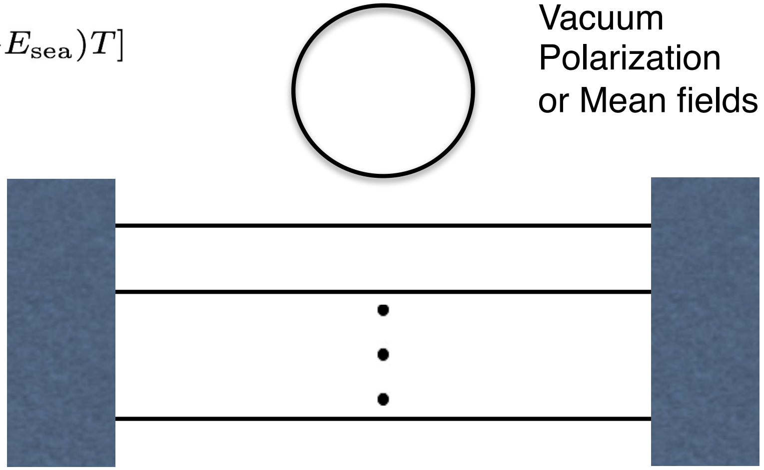
  [A.V. Radyushkin ,Phys.Rev. D96 (2017) no.3, 034025]

Thank you very much!

Classical Soliton

$$\langle J_N(\vec{x},T) J_N^\dagger(\vec{y},-T)\rangle_0 \sim \Pi_N(T) \sim e^{-[(N_c E_{\rm val}+E_{\rm sea})T]}$$

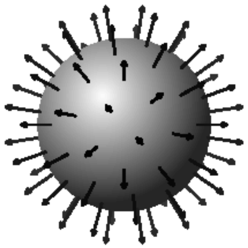




$$\frac{\delta}{\delta U}(N_c E_{\rm val} + E_{\rm sea}) = 0 \;\rightarrow\; M_{\rm cl} = N_c E_{\rm val}(U_c) + E_{\rm sea}(U_c)$$

Hedgehog Ansatz:

$$U_{\rm SU(2)} = \exp\left[i\gamma_5 \mathbf{n}\cdot\boldsymbol{\tau} P(r)\right]$$



[C. V. Christov, A. Blotz, H.-Ch. Kim, P. V. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola, and K. Goeke, PPNP 37, 91 (1996),
Figures from H.-Ch. Kim's talk, Orsay Workshop on Nucleon and Resonance Structure with Hard Exclusive Processes, 29-31 May 2017

Description of qPDFs

Quark distribution

$$q(x,\mu) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle P|\bar{\psi}(0)\gamma^+ \mathrm{P}\exp\left[-ig\int_0^z dz'^\alpha A_\alpha(z')\right]\psi(z)|P\rangle\Big|_{z^+=0,z_\perp=0,\mu}$$

At low energy, Λ ~ 600 MeV

Well approximated in large Nc that 'constituent quark' is the only ingredient. The effect of the non-perturbative gluons are in the quark mass

$$q(x) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle P|\bar{\psi}(0)\gamma^+\psi(z)|P\rangle\Big|_{z^+=0,z_\perp=0}$$

Nucleon qPDF in the χQSM

Low-energy effective theory

$$D_i(x,v) = \int \frac{d^3k}{(2\pi)^3}\delta\left(x - \frac{k^3}{P_N}\right)\int d^3x_1 d^3x_2\, e^{-i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}\langle N_v|\bar{\psi}(\vec{x}_2,t)\Gamma_i\psi(\vec{x}_1,t)|N_v\rangle,$$

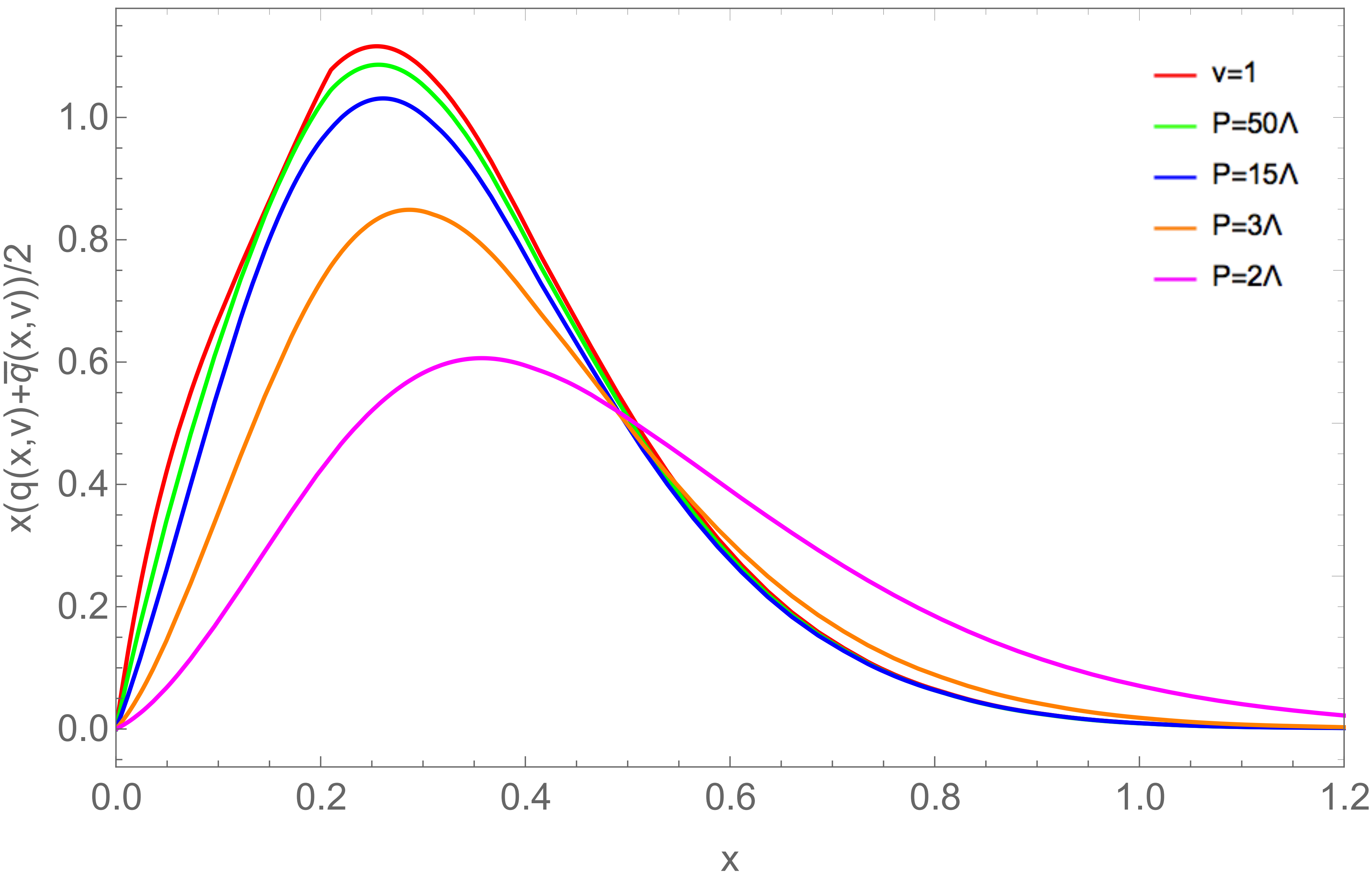
$$\bar{D}_i(x,v) = \int \frac{d^3k}{(2\pi)^3}\delta\left(x - \frac{k^3}{P_N}\right)\int d^3x_1 d^3x_2\, e^{-i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}\langle N_v|\mathrm{Tr}\left[\Gamma_i\psi(\vec{x}_2,t)\bar{\psi}(\vec{x}_1,t)\right]|N_v\rangle.$$

Nucleon matrix element in large Nc

$$-i\langle N_v|\mathrm{T}\{\psi(\vec{x}_1,t_1)\bar{\psi}(\vec{x}_2,t_2)\}|N_v\rangle = G_F(x_1,x_2)$$

$$-iG_F = S[\vec{v}]\Bigg[\Theta(t_2-t_1)\sum_{occ}\Phi_n(\vec{x}_1')\Phi_n^\dagger(\vec{x}_2')\gamma_0\exp(-iE_n(t_1'-t_2')) - \Theta(t_1-t_2)\sum_{nonocc}\Phi_n(\vec{x}_1')\Phi_n^\dagger(\vec{x}_2')\gamma_0\exp(-iE_n(t_1'-t_2'))\Bigg]S^{-1}[\vec{v}].$$

$x\ (\ q(x,v) + \bar{q}(x,v)\ )\ /2$



Isosinglet unpolarised

[C. Weiss, K. Goeke, hep-ph/9712447]

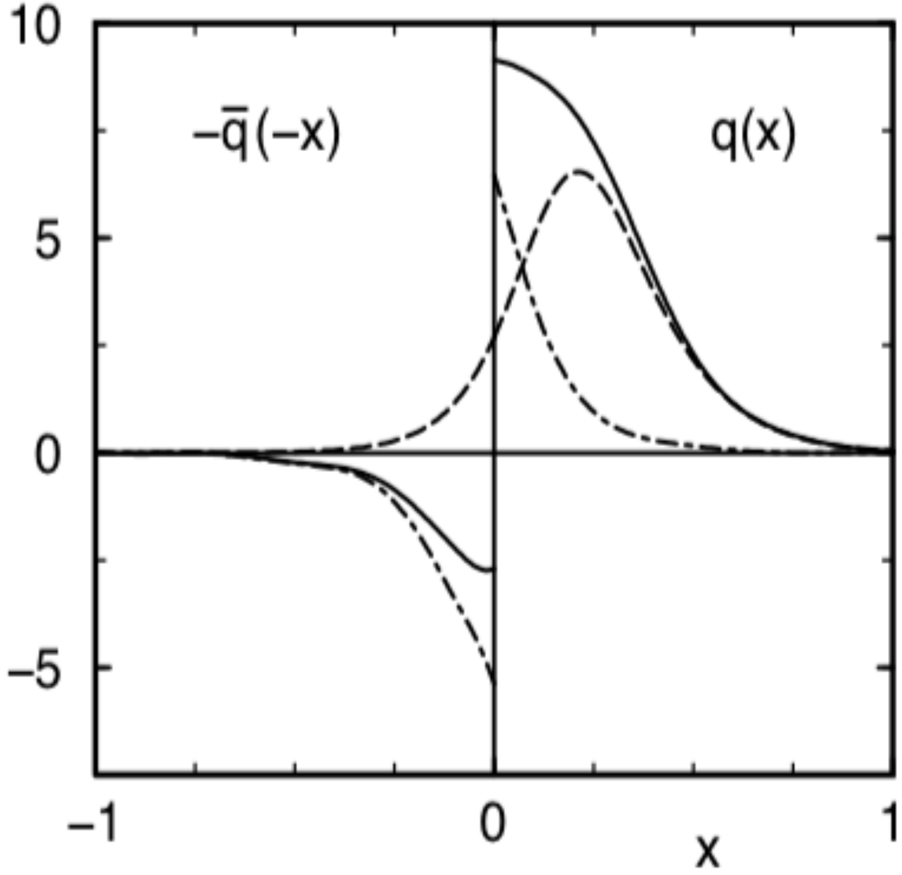


Figure 1: The isosinglet distribution function, Eq.([1]), corresponding to $u(x) + d(x)$ for $x > 0$ and $-\bar{u}(-x) - \bar{d}(-x)$ for $x < 0$, for the self-consistent soliton with $M =$ 350 MeV. *Dashed line:* contribution of the bound-state level, $f_{lev}(x)$, giving a negative contribution to the antiquark distribution. *Dot-dashed line:* contribution of the Pauli–Villars regularized Dirac continuum. *Solid line:* total result (bound-state level plus Dirac continuum). The total antiquark distribution is positive.