

PARTON DISTRIBUTION FUNCTIONS

AND LATTICE QCD

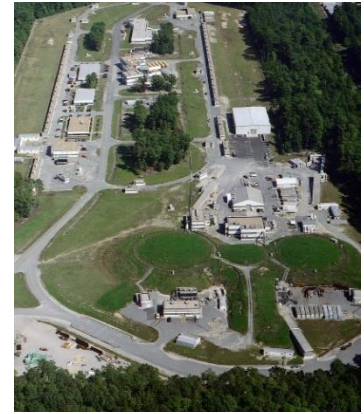
HUEY-WEN LIN



Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

∞ Many ongoing/planned experiments
(BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

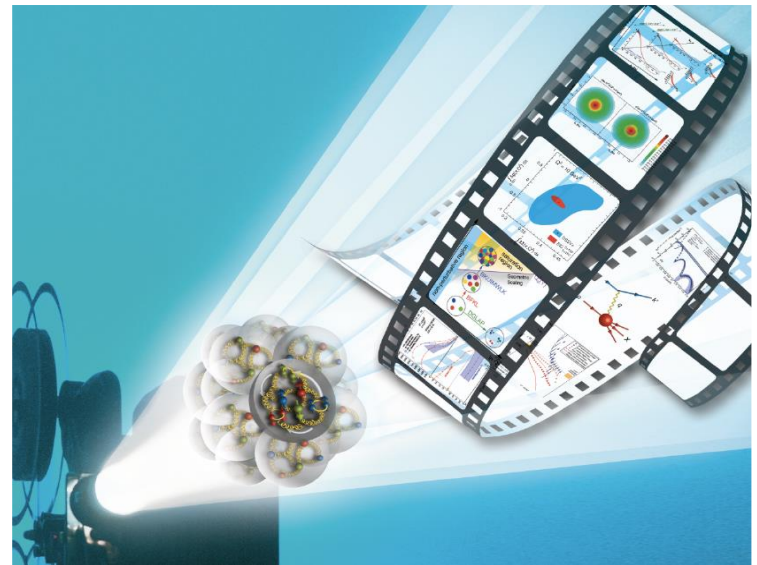


**Electron Ion Collider:
The Next QCD Frontier**

Imaging of the proton

*How are the **sea** quarks and gluons,
and their spins, distributed in space and
momentum inside the nucleon?*

EIC White Paper, 1212.1701



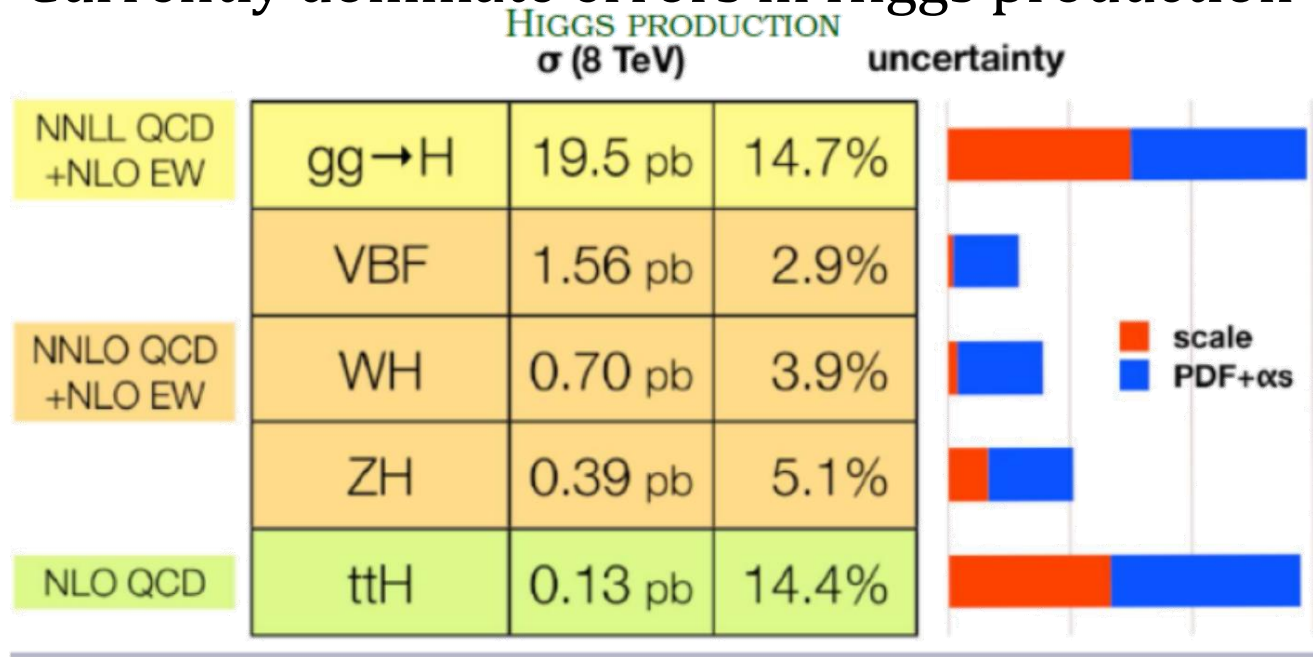
Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

↪ Many ongoing/planned experiments
(BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

§ Important inputs to discern new physics at LHC

↪ Currently dominate errors in Higgs production

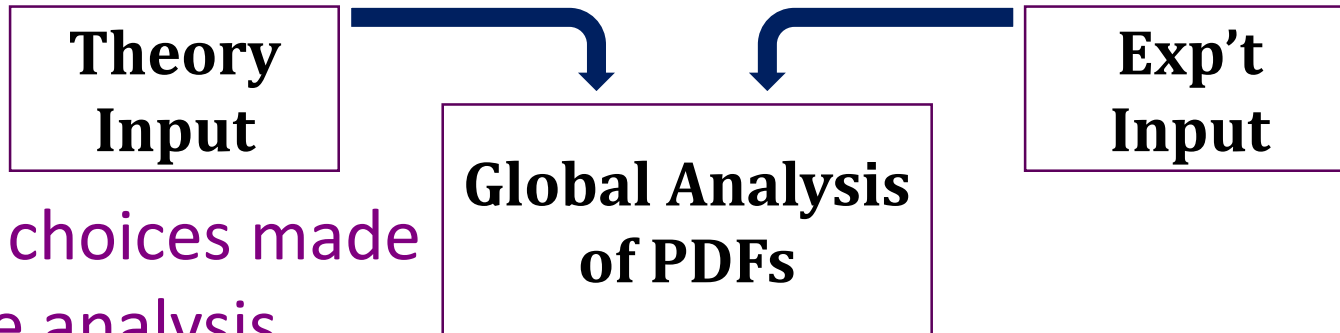


(J. Campbell, HCP2012)

Global Analysis

§ Experiments cover diverse kinematics of parton variables

⇒ Global analysis takes advantage of all data sets



§ Some choices made for the analysis

- ⇒ Choice of data sets and kinematic cuts
- ⇒ Strong coupling constant $\alpha_s(M_Z)$
- ⇒ How to parametrize the distribution

$$xf(x, \mu_0) = a_0 x^{a_1} (1 - x)^{a_2} P(x)$$

⇒ Assumptions imposed

SU(3) flavor symmetry, charge symmetry, strange and sea distributions

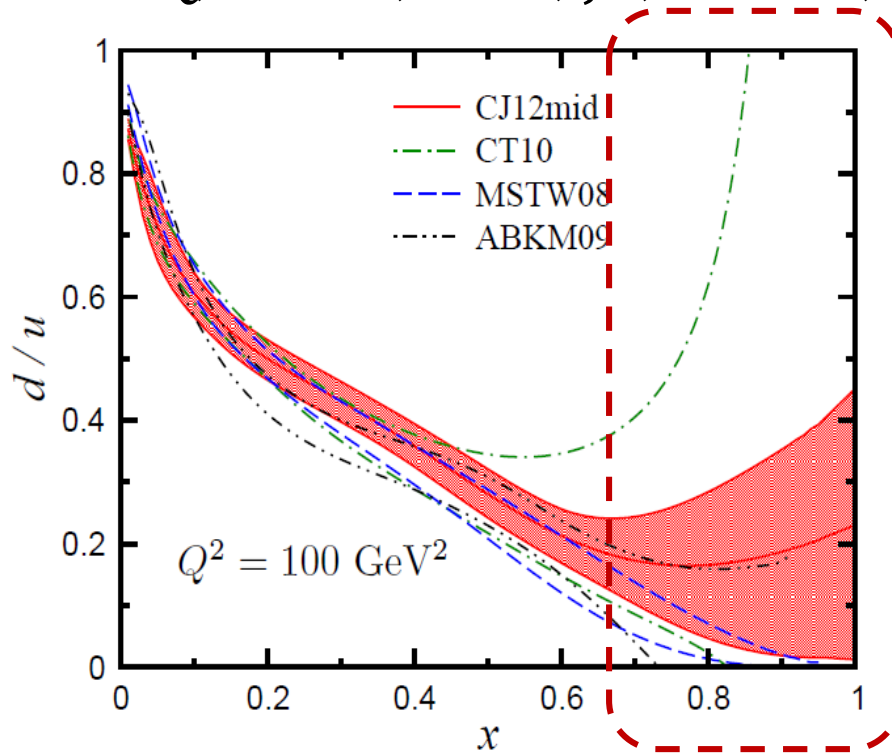
$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

Global Analysis

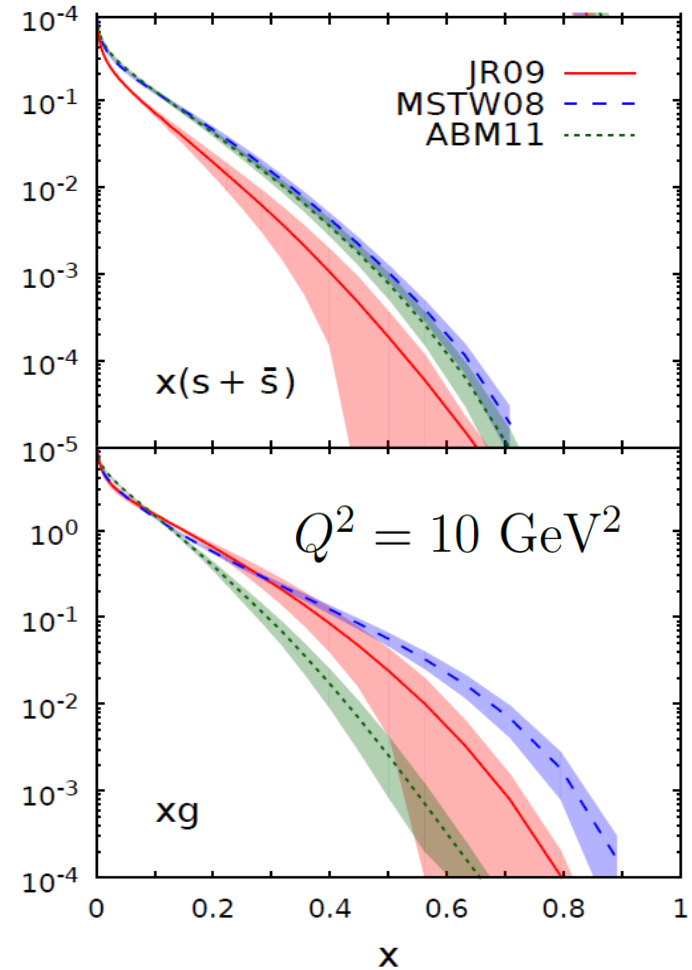
§ Discrepancies appear when data is scarce

§ Many groups have tackled the analysis

∞ CTEQ, MSTW, ABM, JR, NNPDF, etc.



Jimenez-Delgado, Melnitchouk, Owens,
J.Phys. G40 (2013) 09310



What can we do on the lattice?



Lattice QCD 101

- § Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories
- § Physical observables are calculated from the path integral

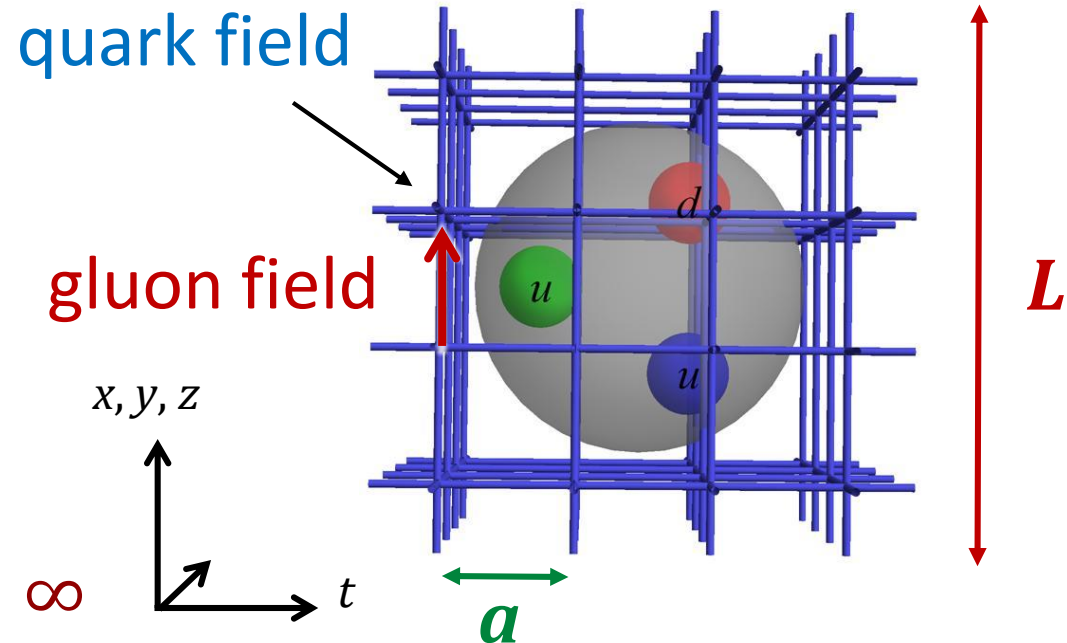
$$\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS(\bar{\psi}, \psi, A)} O(\bar{\psi}, \psi, A)$$

in **Euclidian** space

- ∞ Quark mass parameter (described by m_π)
- ∞ Impose a UV cutoff
discretize spacetime
- ∞ Impose an infrared cutoff
finite volume

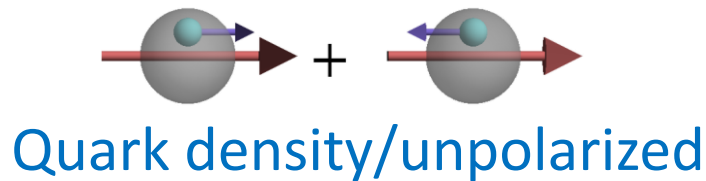
§ Recover physical limit

$$m_\pi \rightarrow m_\pi^{\text{phys}}, a \rightarrow 0, L \rightarrow \infty$$



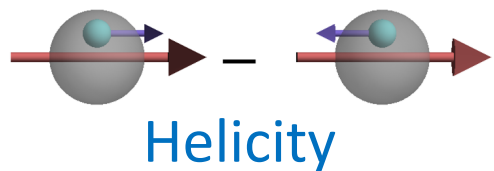
PDFs on the Lattice

§ Lattice calculations rely on operator product expansion,
only provide moments



$$\langle x^n \rangle_q = \int_{-1}^1 dx x^n q(x)$$

most well known



$$\langle x^n \rangle_{\Delta q} = \int_{-1}^1 dx x^n \Delta q(x)$$

longitudinally polarized



$$\langle x^n \rangle_{\delta q} = \int_{-1}^1 dx x^n \delta q(x)$$

transversely polarized

very poorly known



§ True distribution can only be recovered with all moments

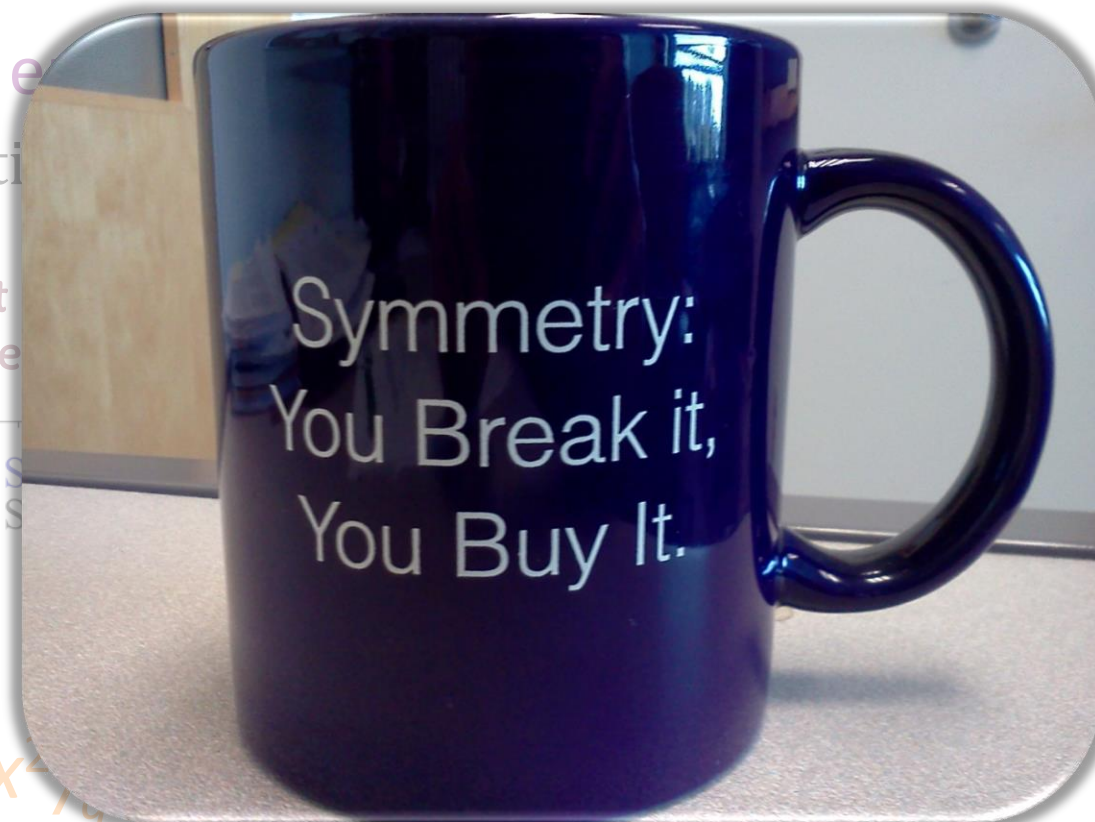
Problem with Moments

§ For higher moments, ops mix with lower-dimension ops

↪ Renormalization is difficult too

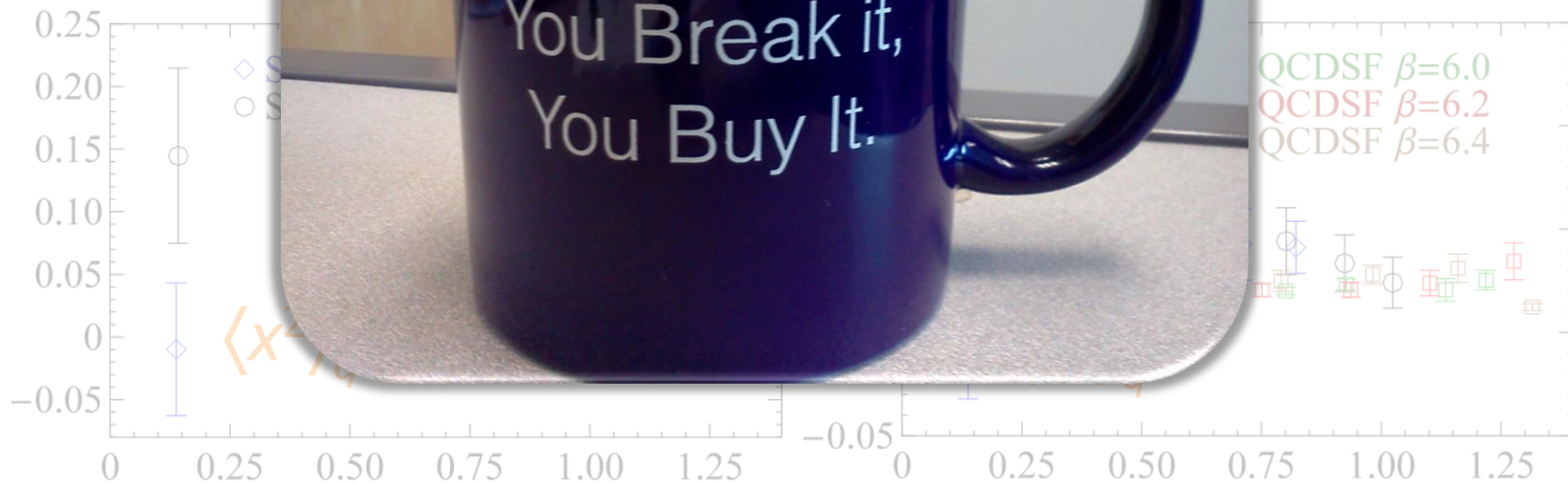
§ Relative error

↪ Calculation



(SAM):
clover

Dolgov et al
Göckeler et al



Problem with Moments

§ For higher moments, ops mix with lower-dimension ops

↪ Renormalization is difficult too

§ Relative error grows in higher moments

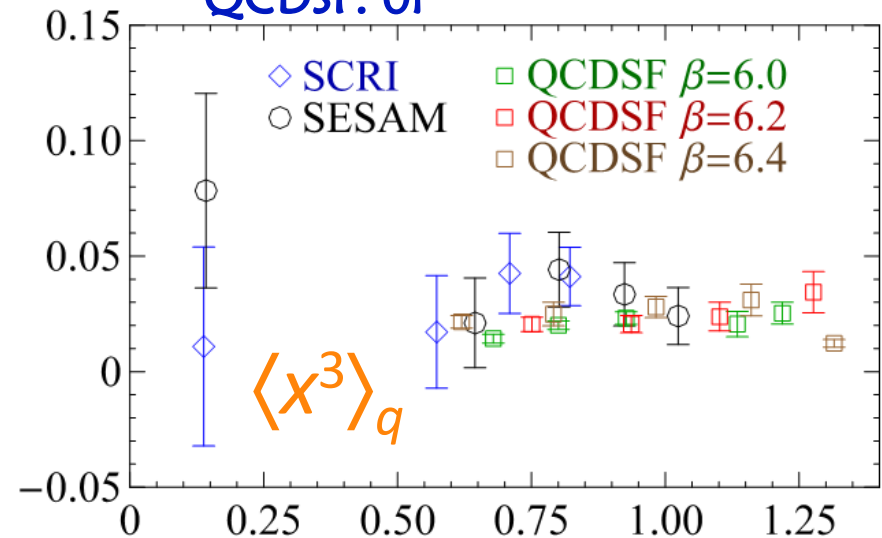
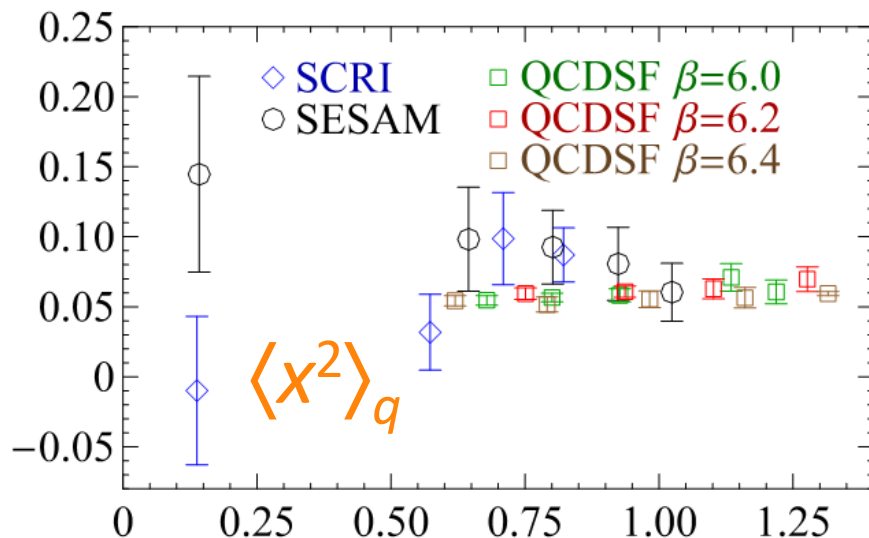
↪ Calculation would be costly and difficult

Dolgov et al. PRD66, 034506 (2002)

Göckeler et al. PRD71, 114511 (2005)

LHPC (SCRI, SESAM):
2f, Wilson and clover

QCDSF: 0f



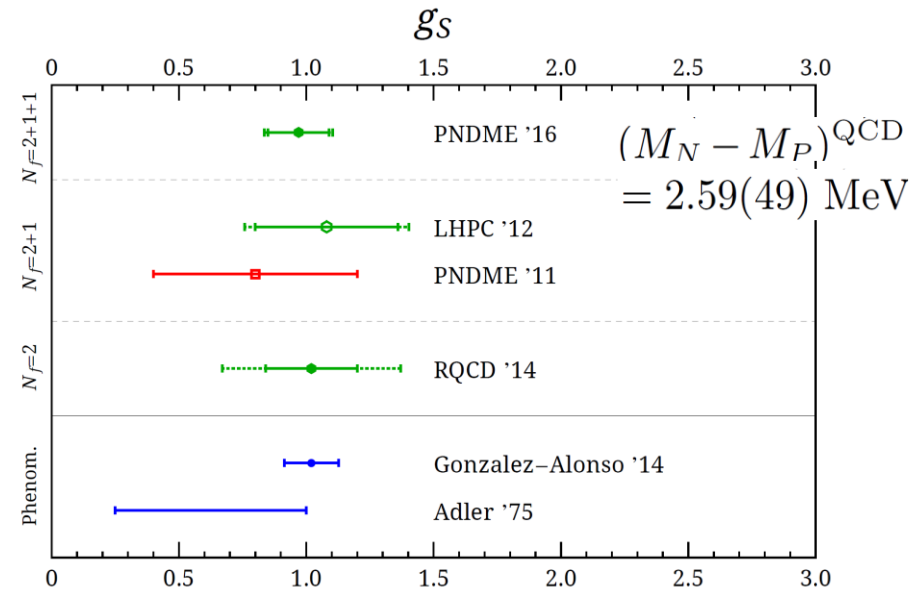
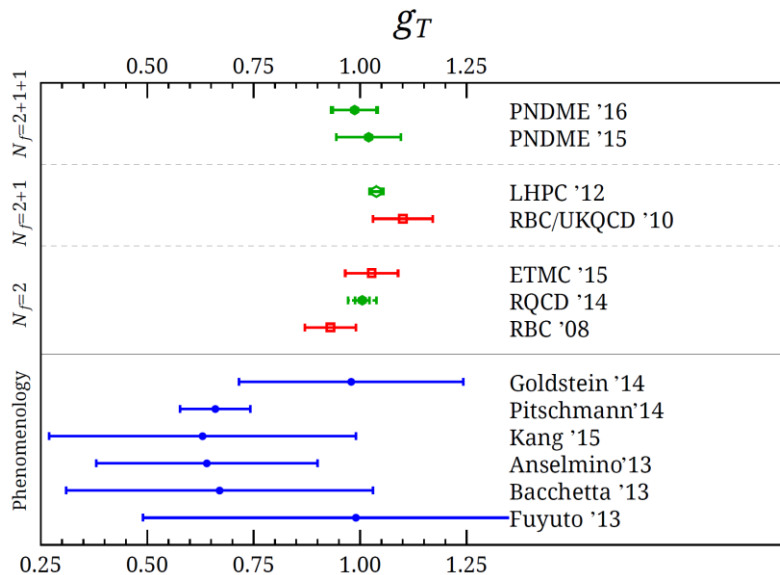
State-of-the-Art Moments

§ FLAG rating system

PNDME, 1506.06411; 1606.07049

§ New: excited-state rating

Collaboration	Ref.	publication status	N_f	chiral extrapolation	continuum extrapolation	finite volume	excited state	renormalization	g_T
PNDME'15	This work	P	2+1+1	★	★	★	★	★	1.020(76) ^a
ETMC'13	[30]	C	2+1+1	■	○	○	■	★	1.11(3) ^b
LHPC'12	[28]	A	2+1	★	○	★	○	★	1.037(20) ^c
RBC/UKQCD'10	[29]	A	2+1	○	■	★	★	★	1.10(7) ^d
RQCD'14	[31]	P	2	★	★	★	○	★	1.005(17)(29) ^e
ETMC'13	[30]	C	2	★	■	○	■	○	1.114(46) ^f
RBC'08	[32]	P	2	■	■	★	■	★	0.93(6) ^g

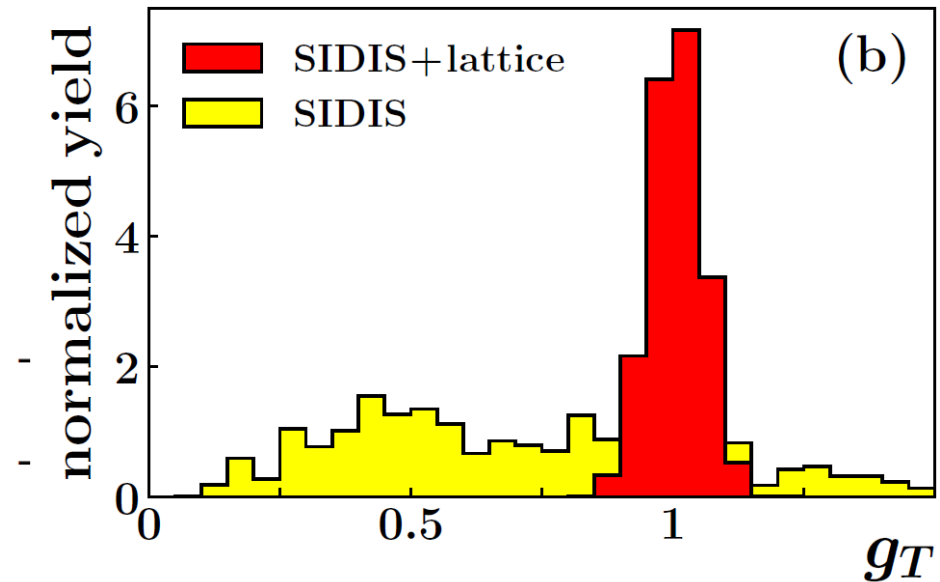
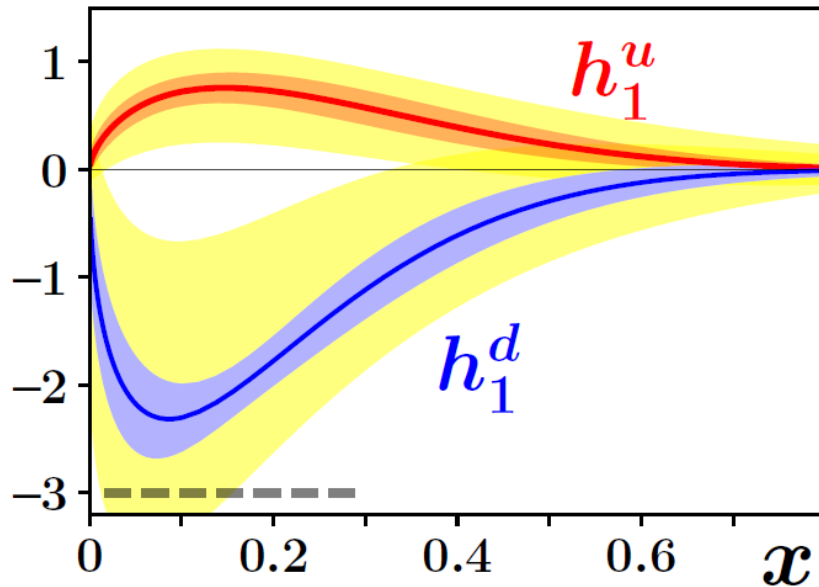


State-of-the-Art Moments

§ Improved transversity distribution with LQCD g_T

∞ Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$

∞ Use to constrain the global analysis fits SIDIS π^\pm production data from proton and deuteron targets



Lin, Melnitchouk, Prokudin, Sato, Phys. Rev. Lett. 120, 152502 (2018)

Beyond Traditional Moments?

§ Longstanding obstacle!

§ Holy grail of structure calculations

§ Applies to many structure quantities:

∞ Generalized parton distributions (GPDs)

∞ Transverse-momentum distributions (TMD)

∞ Meson distribution amplitudes...

∞ Wigner distribution



"Marvelously
zany humor."
— NEWSWEEK

Beyond Traditional Moments?

§ Reaching for higher moments

↪ Fictitious heavy quarks (Detmold and Lin, hep-lat/0507007)

↪ Smearred lat. ops (Davoudi et al. 1204.4146)

§ Direct calculation of x dependence

↪ Hadronic tensor currents

(Liu et al., hep-ph/9806491, ... 1603.07352)

↪ Inversion method/OPE without OPE

(QCDSF, hep-lat/9809171, ...1703.01153)

↪ Euclidean correlation functions (RQCD, 1709.04325)

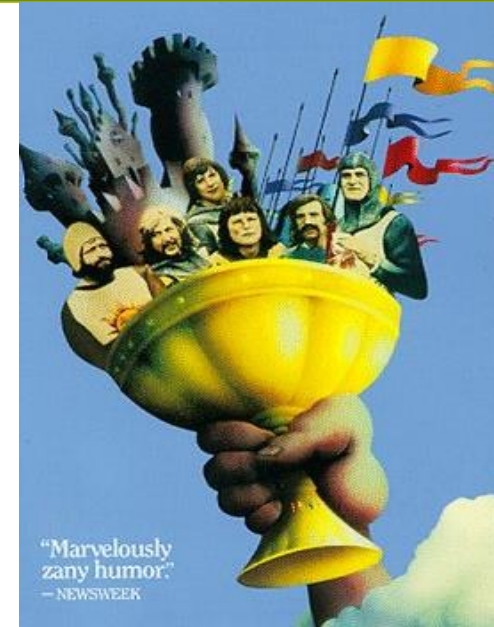
↪ Lattice cross-section method (Y.-Q. Ma and J. Qiu, 2014, 2017)

↪ Large-momentum effective theory (LaMET) and variations

↪ Original LaMET (“quasi-PDF” method) **This talk**

↪ Pseudo-PDF method: differs in FT (A. Radyushkin, 2017)

↪ Smearred quasi-PDF (C. Monahan and K. Orginos, 2017)



A Promising “New” Direction

Large-Momentum Effective Theory (LaMET)



Lattice Parton Physics Project (LP³)

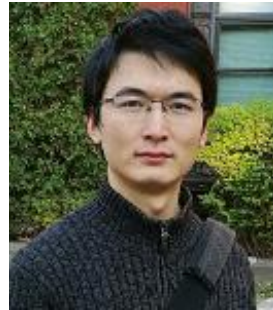
<https://www.pa.msu.edu/~hwlin/LP3/>



HWL
(MSU)



Xiangdong Ji
(UMD)



Luchang Jin
(Conn)



Ruizi Li
(MSU*)



Yi-Bo Yang
(MSU)



Yong Zhao
(MIT)

International collaborators



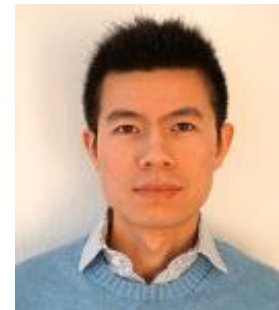
Jiunn-Wei Chen
(NTU)



Yu-Sheng Liu
(SJTU)



Andreas Schäfer
(Regensburg)



Jian-Hui Zhang
(Regensburg)

A New Direction

Large-Momentum Effective Theory (LaMET) X. Ji, PRL. 111, 262002 (2013)

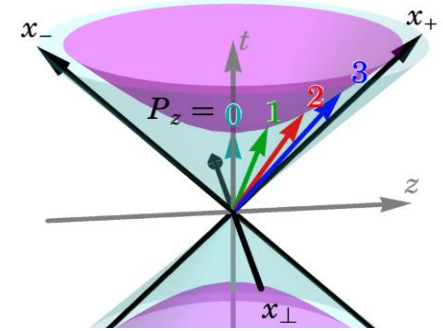
§ Calculate the parton distributions through the infinite-momentum frame Feynman, Phys. Rev. Lett. 23, 1415 (1969)

§ Finite-momentum quark distribution (quasi-distribution)

∞ Suggested operator:

$$\tilde{Q}(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \langle P | \bar{\psi}(z) \gamma_z \exp\left(-ig \int_0^z dz' A_z(z')\right) \psi(0) | P \rangle$$

$x = k_z/P_z$ (blue arrow pointing to x)
 Lattice z coordinate (green arrow pointing to z)
 hadron momentum $P_\mu = \{P_0, 0, 0, P_z\}$ (red arrow pointing to P)
 Product of lattice gauge links (purple arrow pointing to $\exp(-ig \int_0^z dz' A_z(z'))$)



§ Take the infinite- P_z limit to recover lightcone functions

∞ Just another limit to take, like taking $a \rightarrow 0$ or $V \rightarrow \infty$

Progress in the theoretical development of LaMET

- **Renormalization:**

Ji and Zhang, 2015; Ishikawa et al., 2016, 2017; Chen, Ji and Zhang, 2016;

Xiong, Luu and Meißner, 2017; Constantinou and Panagopoulos, 2017; Ji, Zhang, and Y.Z., 2017; J. Green et al., 2017; Ishikawa et al. (LP3), 2017; Wang, Zhao and Zhu, 2017; Spanouides and Panagopoulos, 2018.

- **Factorization:**

Ma and Qiu, 2014, 2015, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **One-loop matching:**

Xiong, Ji, Zhang and Y.Z., 2014; Ji, Schaefer, Xiong and Zhang, 2015; Xiong and Zhang, 2015; Constantinou and Panagopoulos, 2017; I. Stewart and Y. Z., 2017; Wang, Zhao and Zhu, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **Power corrections:**

J.-W. Chen et al., 2016; A. Radyushkin, 2017.

- **Transvers momentum dependent parton distribution function:**

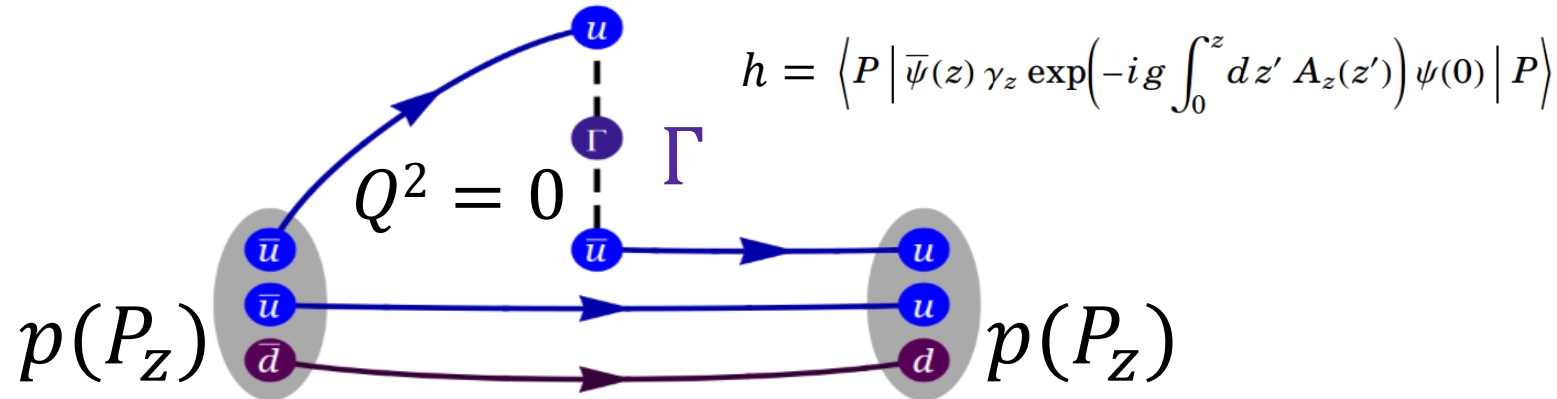
Ji, Xiong, Sun, Yuan, 2015; Ji, Jin, Yuan, Zhang and Y.Z., 2018; Ebert, Stewart and Y.Z., in progress.

Slide credit: Yong Zhao, CIPANP 2018 Plenary talk

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs X. Ji, PRL. 111, 262002 (2013)

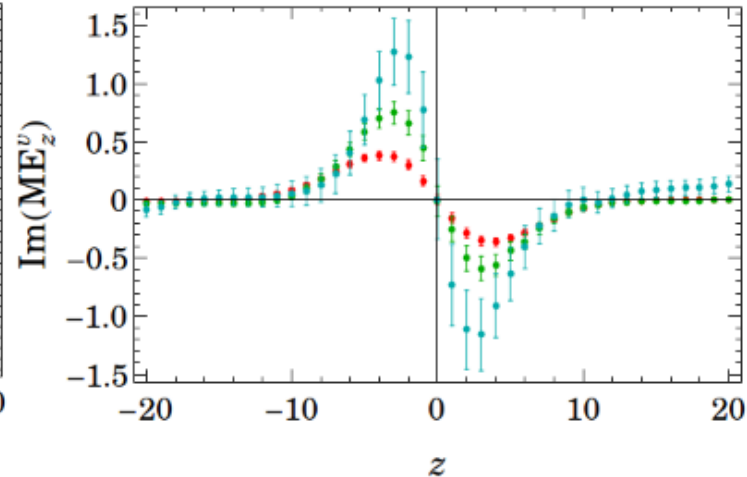
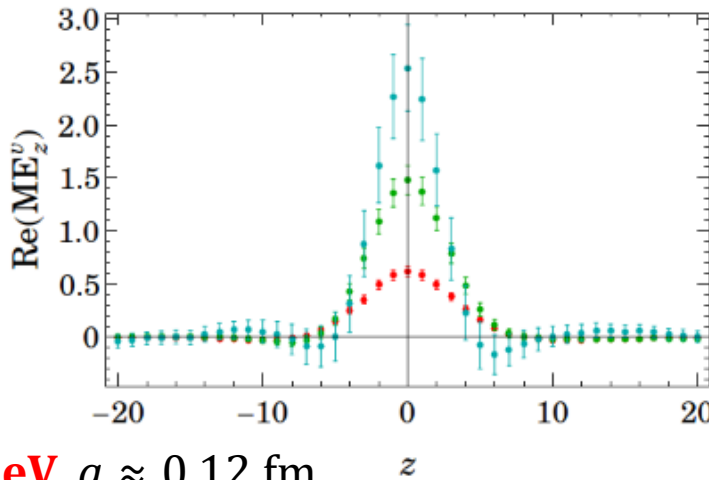
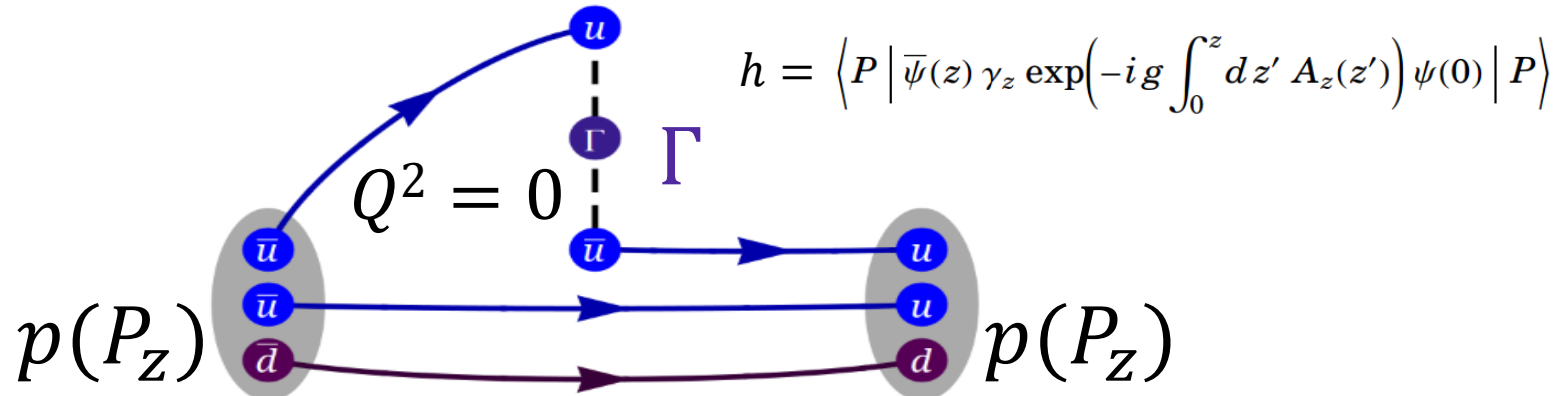
1) Calculate nucleon matrix elements on the lattice



LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs X. Ji, PRL. 111, 262002 (2013)

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$M_\pi \approx 310 \text{ MeV}$, $a \approx 0.12 \text{ fm}$

HWL et al. 1402.1462

$P_z \in \{0.43, 0.86, 1.29\} \text{ GeV}$

LaMET: Step-by-Step

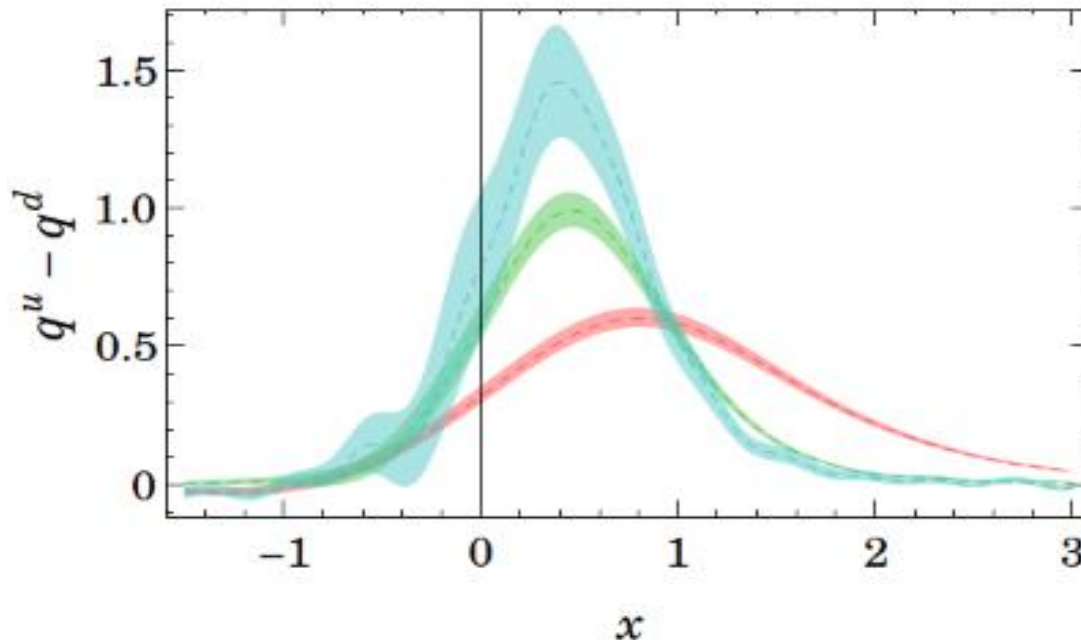
Large-Momentum Effective Theory for PDFs X. Ji, PRL. 111, 262002 (2013)

2) Compute “quasi-distribution” via

$$\tilde{q}(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-i x z P_z} h(z, \mu, P_z)$$

$M_\pi \approx 310$ MeV, $a \approx 0.12$ fm
HWL et al. 1402.1462

$P_z \in \{0.43, 0.86, 1.29\}$ GeV



LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs X. Ji, PRL. 111, 262002 (2013)

3) Recover true distribution (take $P_Z \rightarrow \infty$ limit)

$$\tilde{q}(x, \mu, P_Z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_Z}\right) q(y, \mu) + \mathcal{O}(M_N^2/P_Z^2) + \mathcal{O}(\Lambda_{\text{QCD}}^2/P_Z^2)$$

Finite $P_Z \leftrightarrow \infty$ perturbative matching

$$Z(x, \mu/P_Z) = C\delta(x-1) - \frac{\alpha_s}{2\pi} Z^{(1)}(x, \mu/P_Z)$$

Non-singlet case only

X. Xiong, X. Ji, J. Zhang, Y. Zhao, 1310.7471;

Ma and Qiu, 1404.6860

LaMET: Step-by-Step

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Dominant correction
(for nucleon);
known scaling form

HWL et al. 1402.1462

J.-W. Chen et al, 1603.06664

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs X. Ji, PRL. 111, 262002 (2013)

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complicated higher-twist operator;
smaller P_z correction for nucleon
J.-W. Chen et al, 1603.06664 and reference within
(extrapolate it away)

§ Some similarity in more broadly-studied HQET...

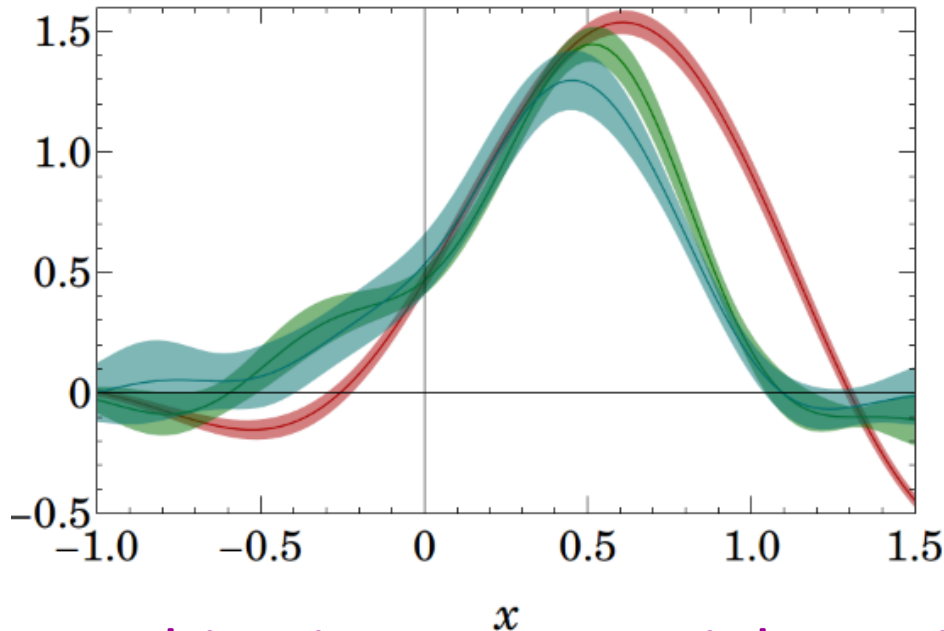
$$\mathcal{O}\left(\frac{m_b}{\Lambda}\right) = Z\left(\frac{m_b}{\Lambda}, \frac{\Lambda}{\mu}\right) o(\mu) + \mathcal{O}\left(\frac{1}{m_b}\right) + \dots$$

LaMET: Step-by-Step

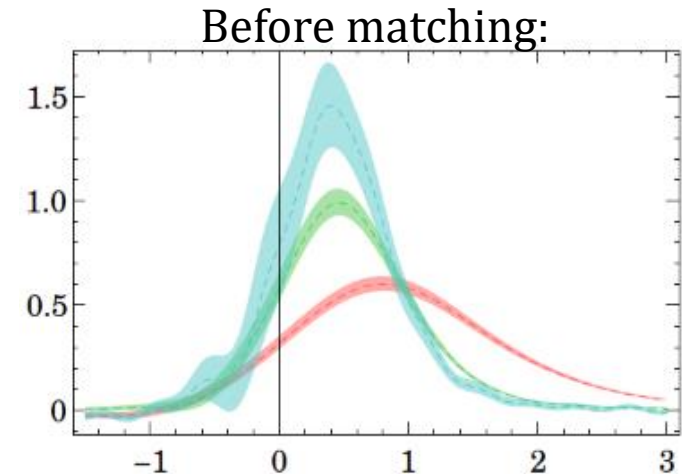
Large-Momentum Effective Theory for PDFs X. Ji, PRL. 111, 262002 (2013)

3) Recover true distribution (take $P_Z \rightarrow \infty$ limit)

$$\tilde{q}(x, \mu, P_Z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_Z}\right) \mathbf{q}(y, \mu) + \mathcal{O}(M_N^2/P_Z^2) + (\Lambda_{\text{QCD}}^2/P_Z^2)$$



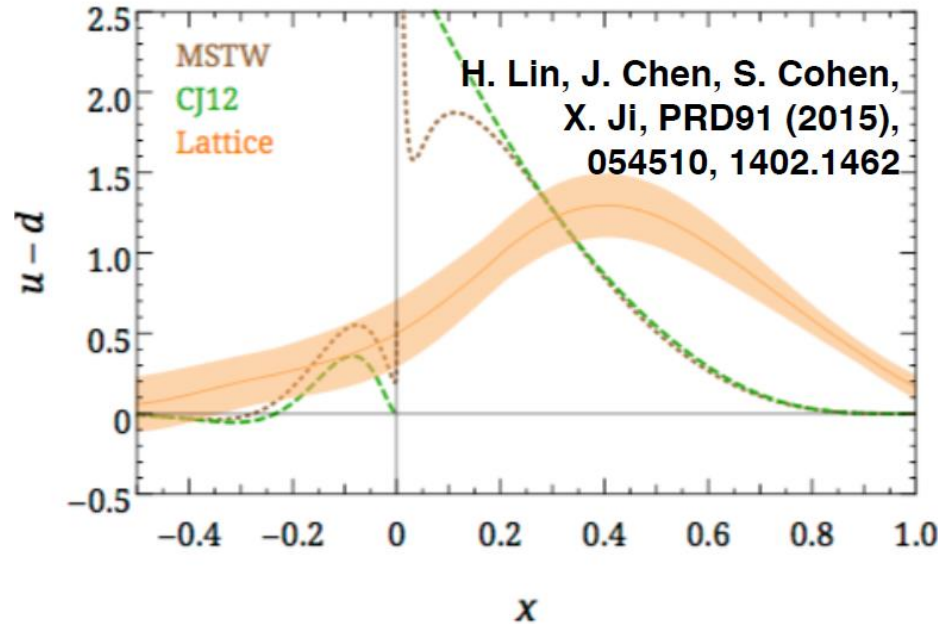
HWL et al. 1402.1462



§ Matching is a very crucial step in recovering the true lightcone distribution

Nucleon Unpolarized PDF

§ From 2014 to 2018



§ First result in 2014

- ↻ $M_\pi \approx 310 \text{ MeV}$, $a \approx 0.12 \text{ fm}$
($M_\pi L \approx 4.5$)
- ↻ Largest $P_Z \approx 1.3 \text{ GeV}$
- ↻ 1-loop $\overline{\text{MS}}$ matching + target-mass correction

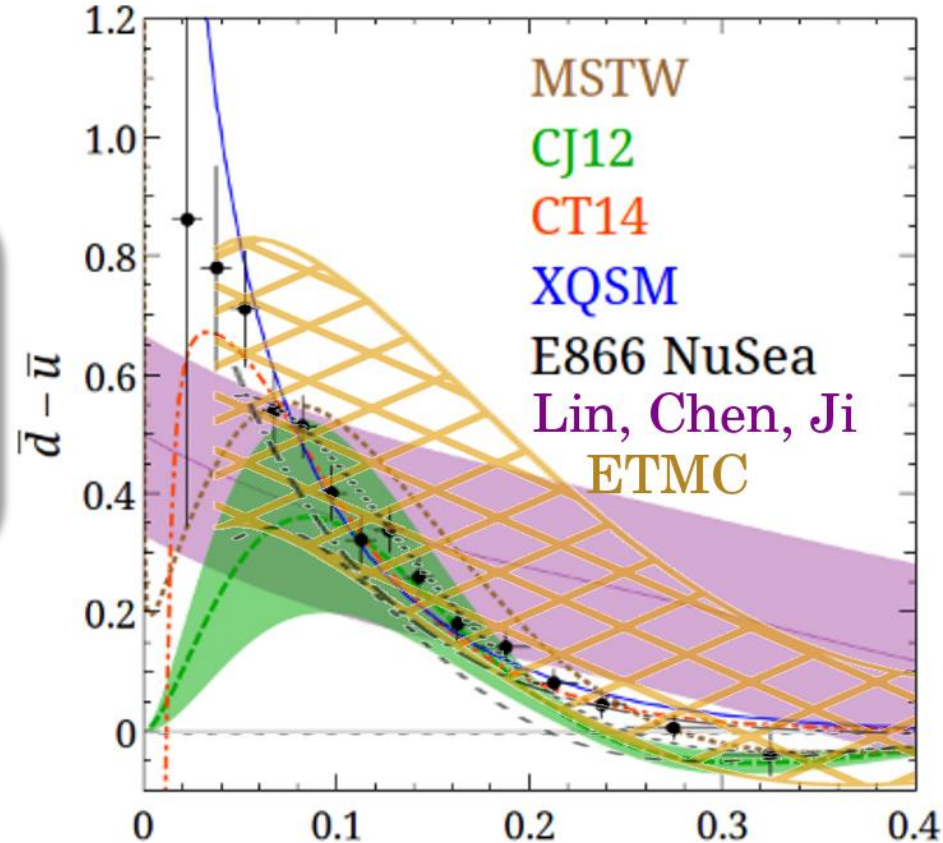
§ Updated results in 2017/18

- ↻ Improved quasi-distribution definition
- ↻ RI/MOM nonperturbative renormalization and corresponding matching to lightcone distribution

Nucleon Unpolarized PDF

§ From 2014 to 2018

Similar results repeated by
ETMC,
at $M_\pi \approx 373$ MeV, $a \approx 0.8$ fm
ETMC, 1504.07455



§ Updated results in 2017/18

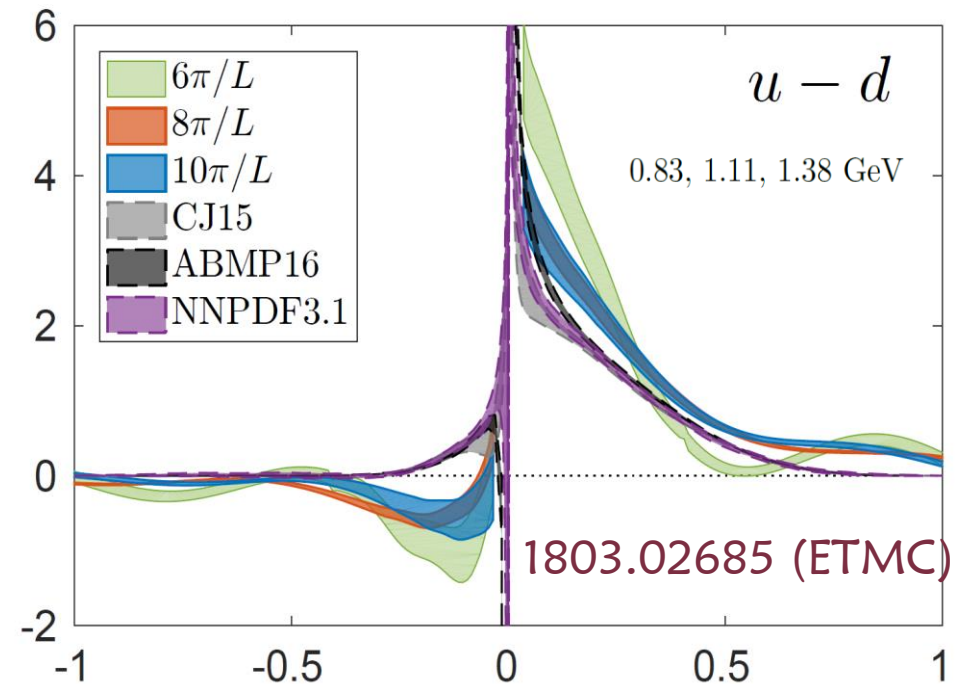
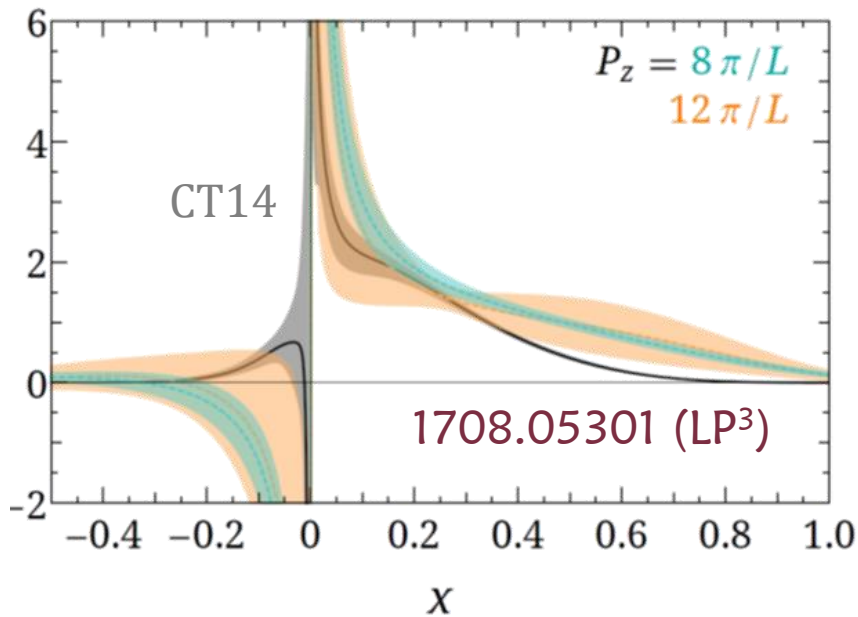
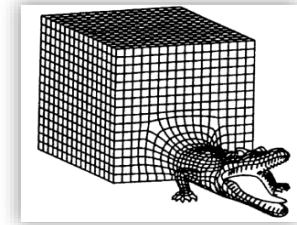
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Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

∞ Boost momenta $P_z \leq 1.4$ GeV

∞ Study of systematics still needed

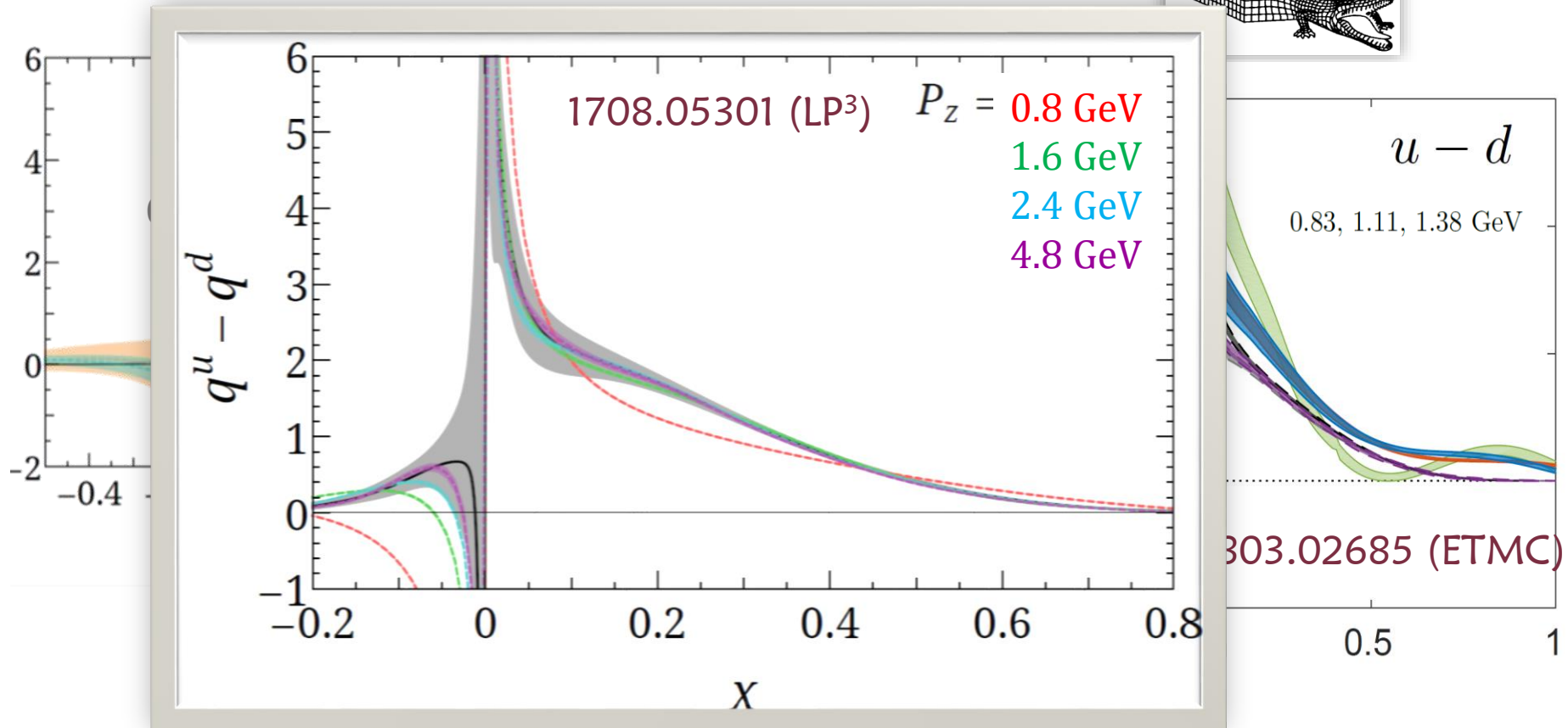
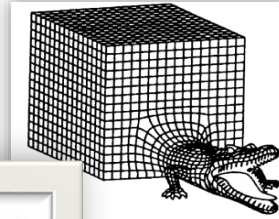


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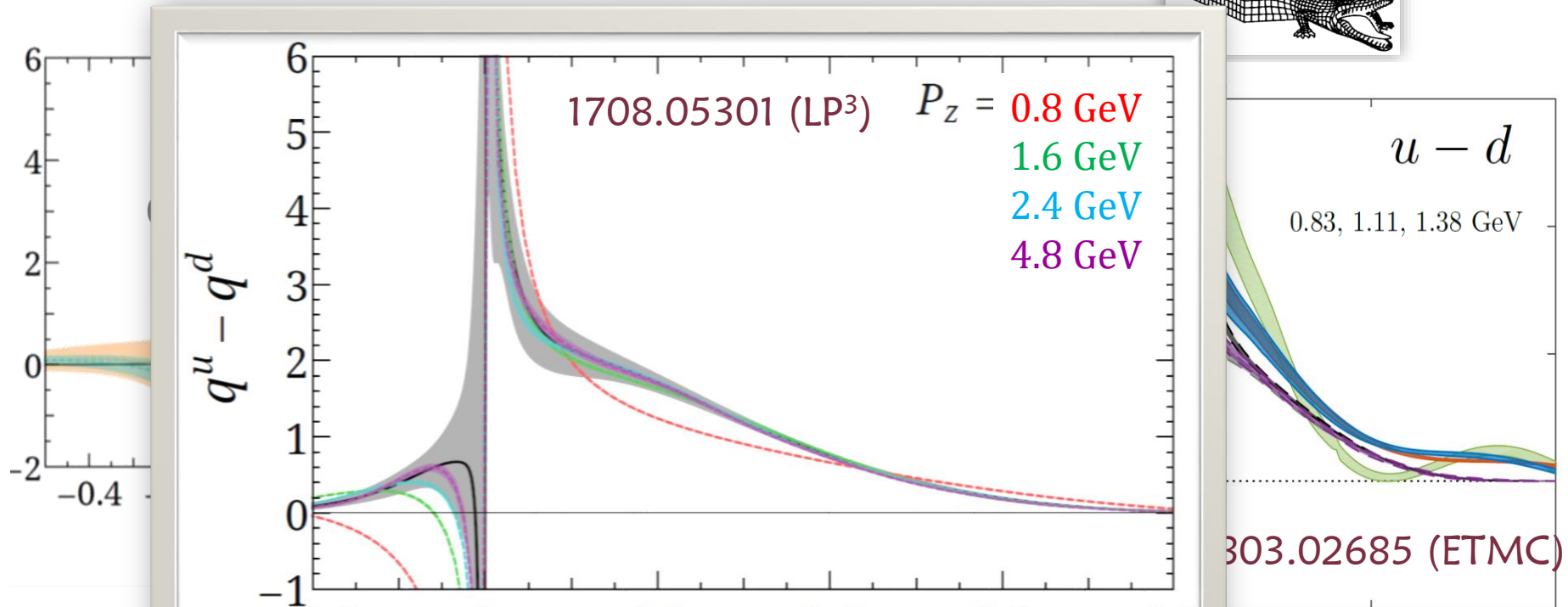
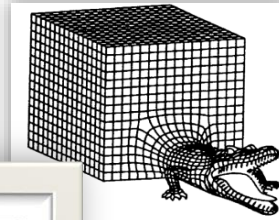


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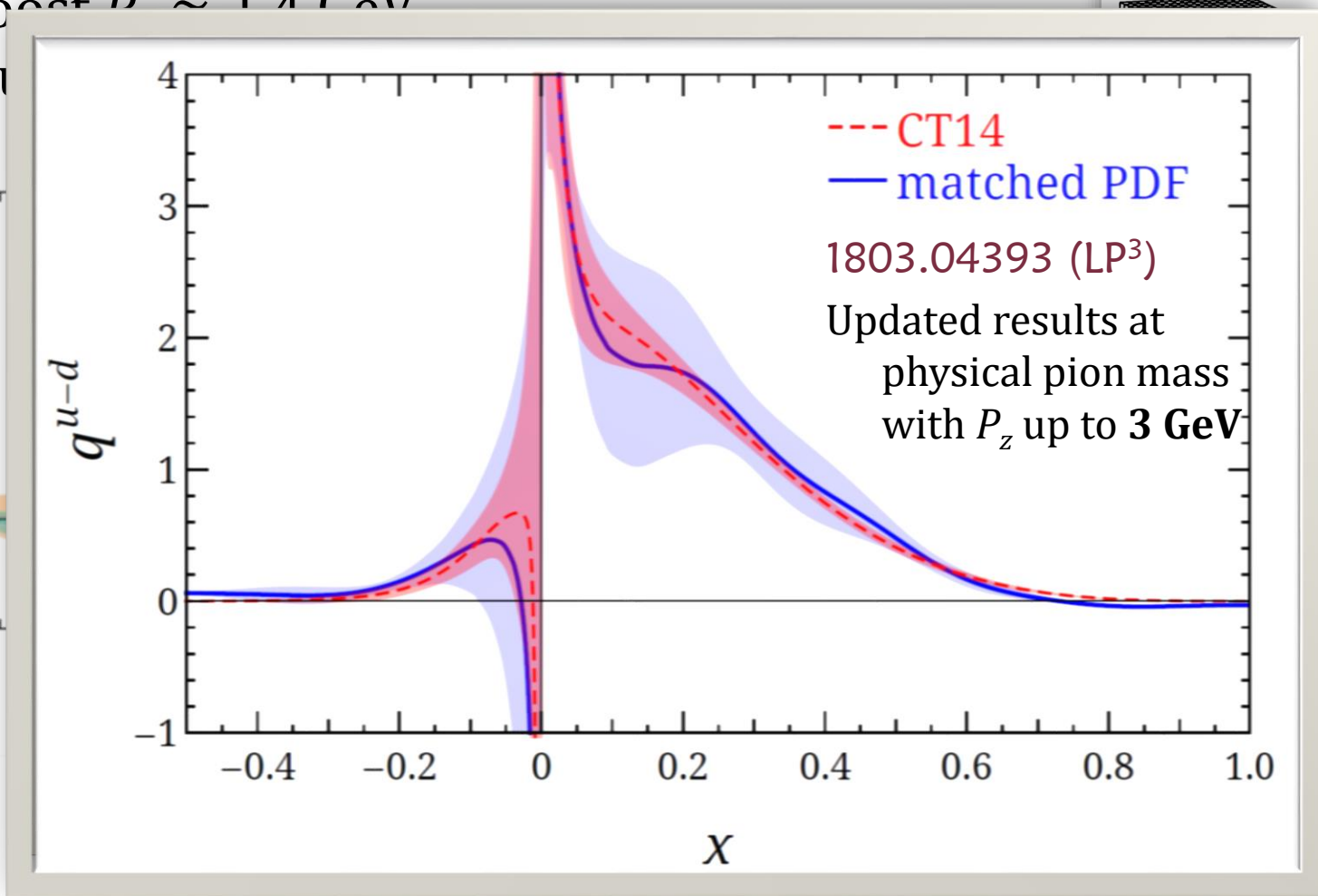
Reaching $x < 0.1$ for antiquark remains challenging especially without relying on an assumed parametrization

Physical Pion Mass Results

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∞ Boost $P_z \sim 1.4 \text{ GeV}$

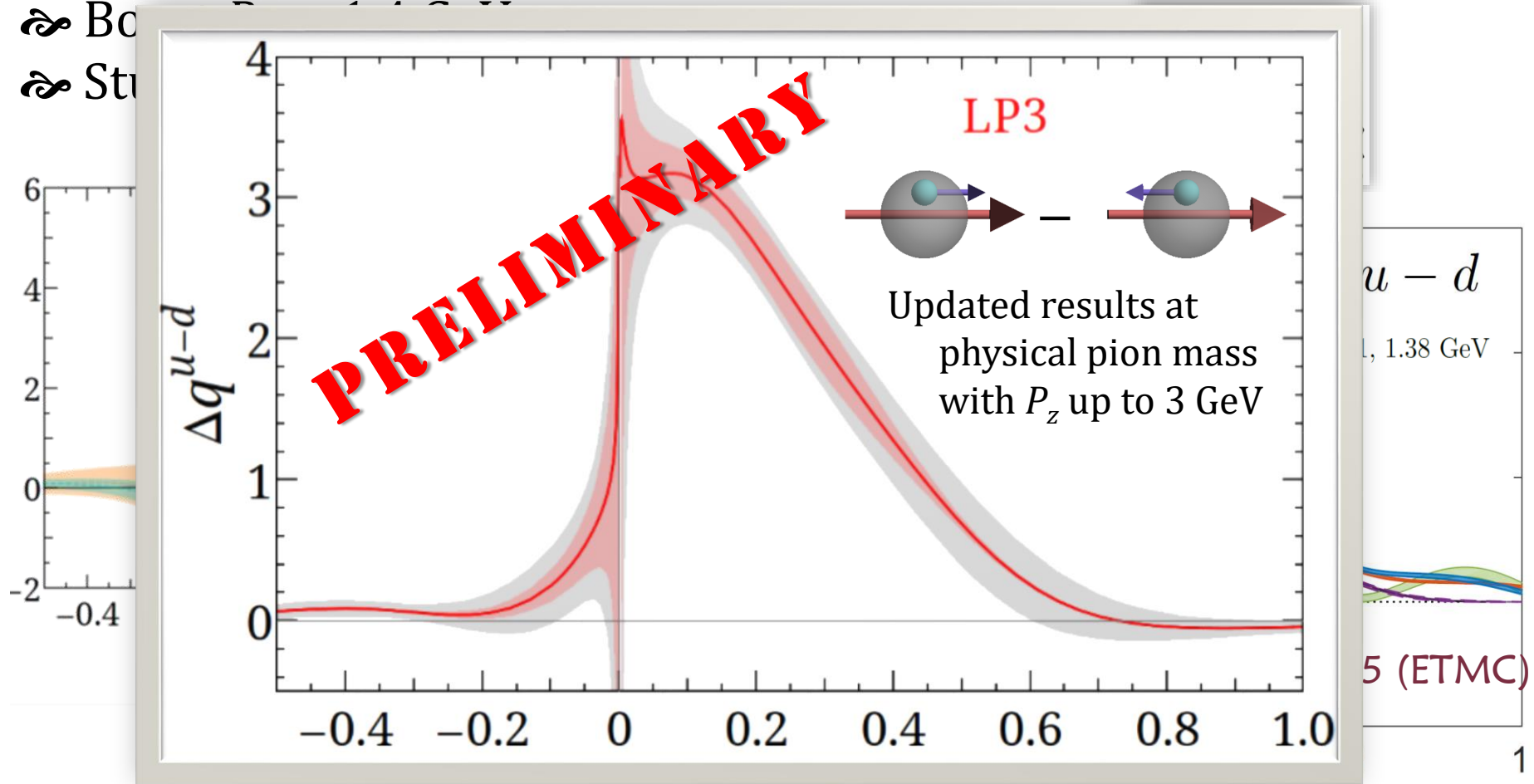
∞ St



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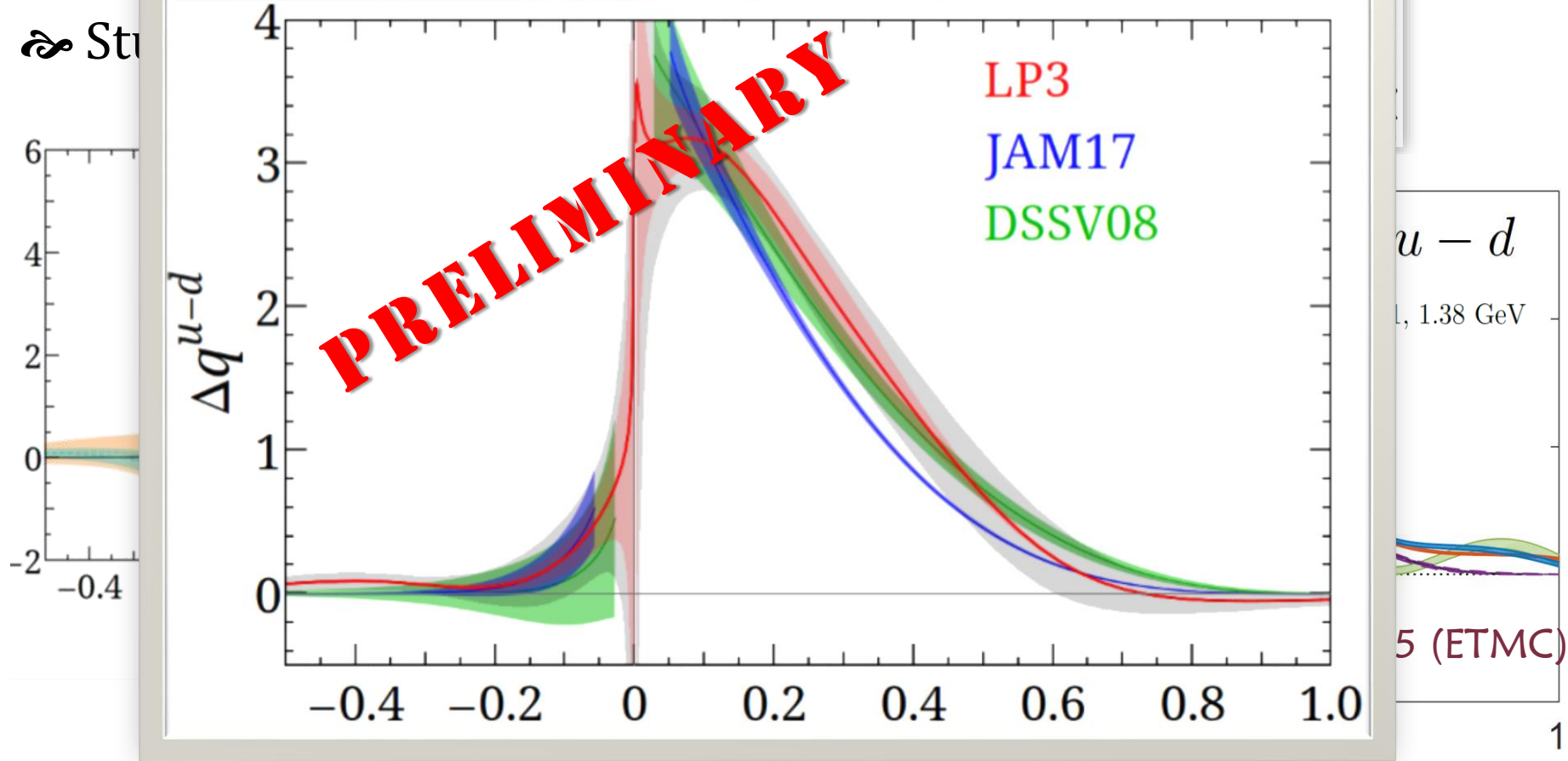
∞ Bo
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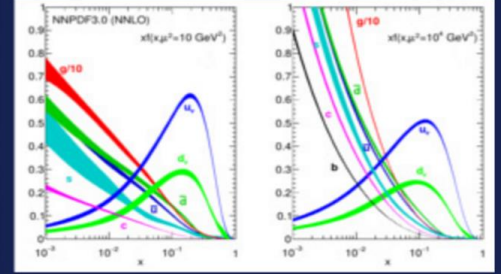
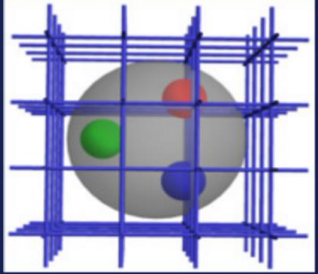


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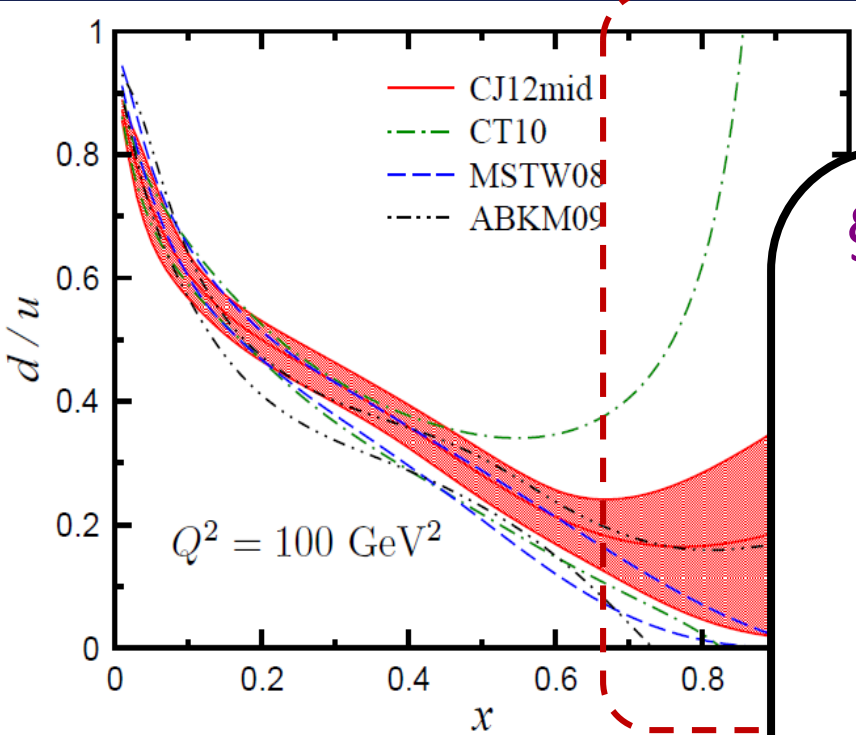
∞ Bo
∞ St





Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

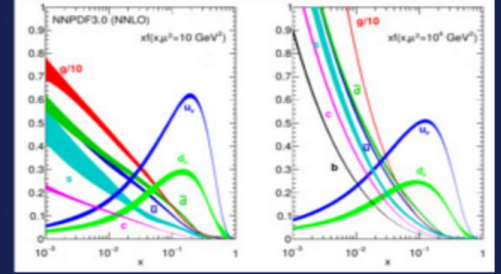
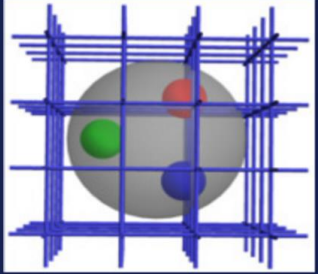
22-24 March 2017, Oxford, UK



§ A first joint workshop with global-fitting community to address key LQCD inputs

- ⌘ <http://www.physics.ox.ac.uk/confs/PDFlattice2017>
- ⌘ Whitepaper study the needed precision of lattice PDFs in the large- x region

Jimenez-Delgado, Melnitchouk, O...
J.Phys. G40 (2013) 09310



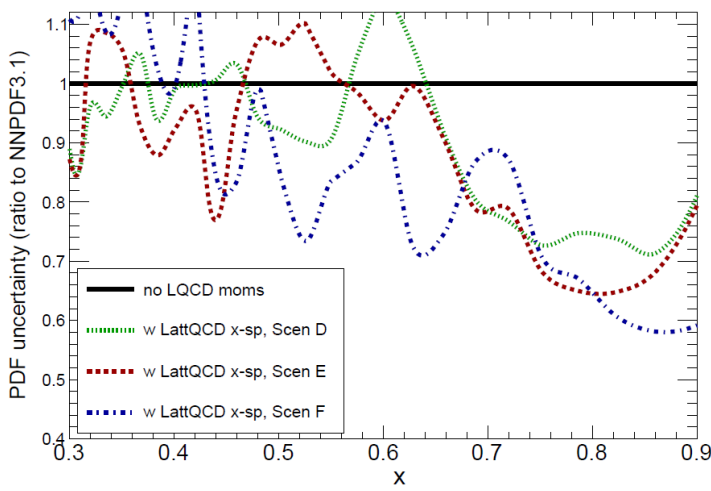
Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

22-24 March 2017, Oxford, UK

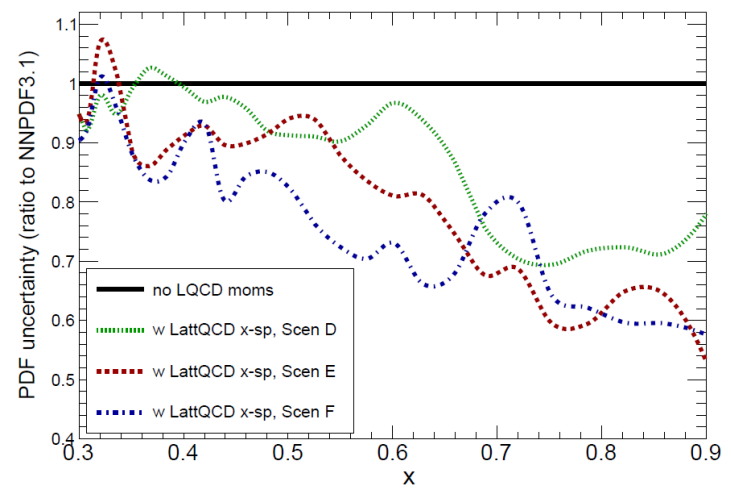
§ Implementing the pseudo-data from LQCD with $x=0.7-0.9$

$$u(x_i, Q^2) - d(x_i, Q^2) \text{ and } \bar{u}(x_i, Q^2) - \bar{d}(x_i, Q^2)$$

$\delta(\bar{u}) @ Q^2=4 \text{ GeV}^2, \text{ NNPDF3.1}$

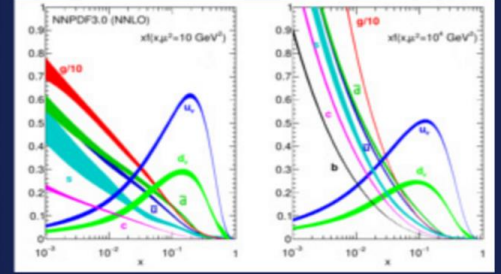
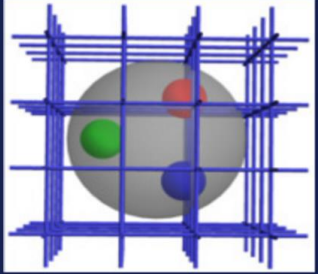


$\delta(\bar{d}) @ Q^2=4 \text{ GeV}^2, \text{ NNPDF3.1}$



D: 12%
E: 6%
F: 3%

Lin et al, *Progress in Particle and Nuclear Physics* 100, 106 (2018)



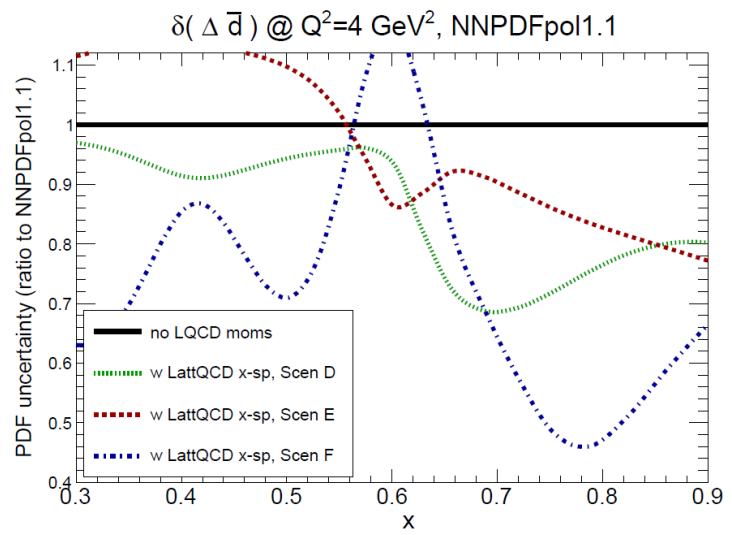
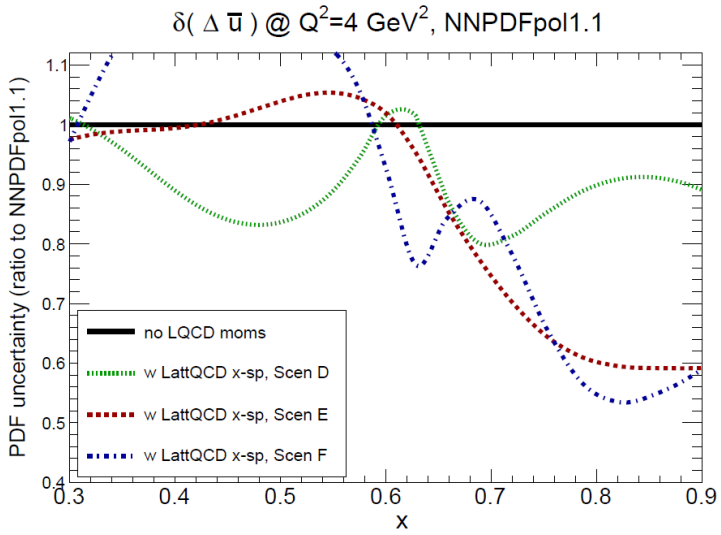
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E: 6%
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Lin et al, *Progress in Particle and Nuclear Physics* 100, 106 (2018)

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion-Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

Summary & Outlook

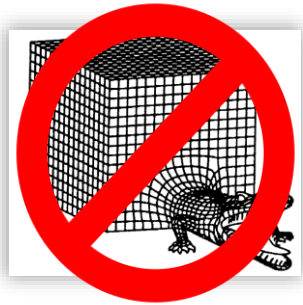
Exciting time for studying structure on the lattice

§ Overcoming longstanding obstacle to full x -distribution

- ↪ Most importantly, this can be done with today's computers
- ↪ Progress made in meson distribution amplitudes (1702.0008, 1712.10025) and first lattice pion PDF (1804.01483)

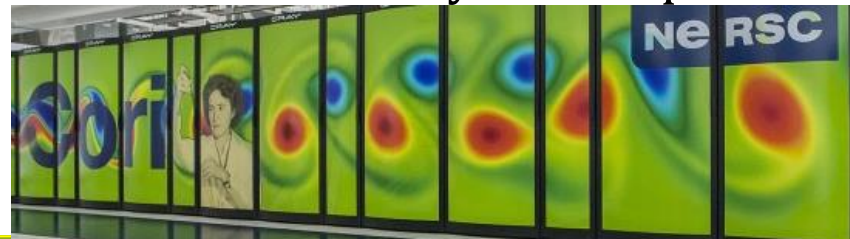
§ Moving on to remove the systematics of earlier study

- ↪ Progress on **renormalization**, further work on larger momentum boost with finer lattice spacing, higher statistics, etc.
- ↪ Long-term future for lattice hadronic physics



§ LQCD impacts for current PDFs in the next few years

- ↪ Large- x isovector PDFs
- ↪ Combined analysis with precision moments to access small- x regions



Backup Slides



Pseudo-PDF

§ A variation of LaMET: A. Radyushki, 1705.01488

$$\ni \mathcal{P}(x, z^2, \mu, \epsilon) = \int dz (p_z/2\pi) e^{ix \cdot v} h(v, z^2, \mu, \epsilon)$$

§ Versus quasi-PDF Ji, Zhang, Zhao 1706.07416

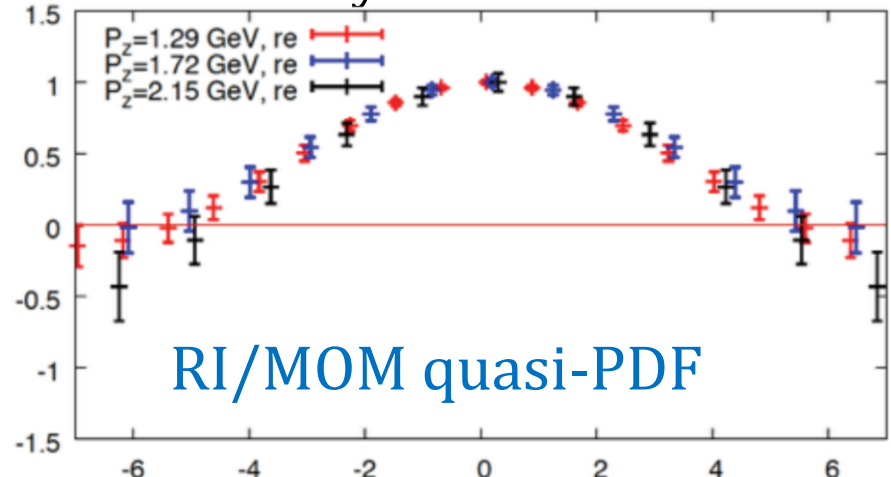
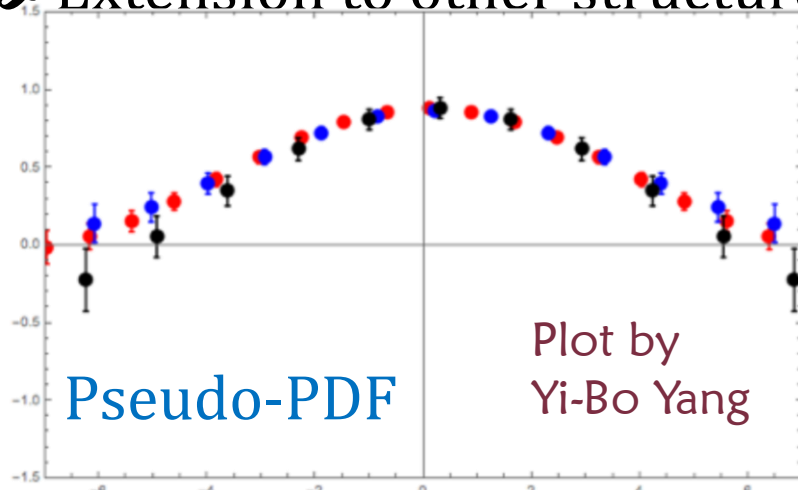
$$\ni \tilde{q}(x, p_z, \mu, \epsilon) = \int (dz/2\pi) e^{ix \cdot z} p_z h(z p_z, z^2, \mu, \epsilon)$$

§ Similarity and issues: $h(v, z^2)/h(0, z^2) = M(v, z^2)$

§ One of the numerical attractions

\ni Similar matrix elements; same problems we have

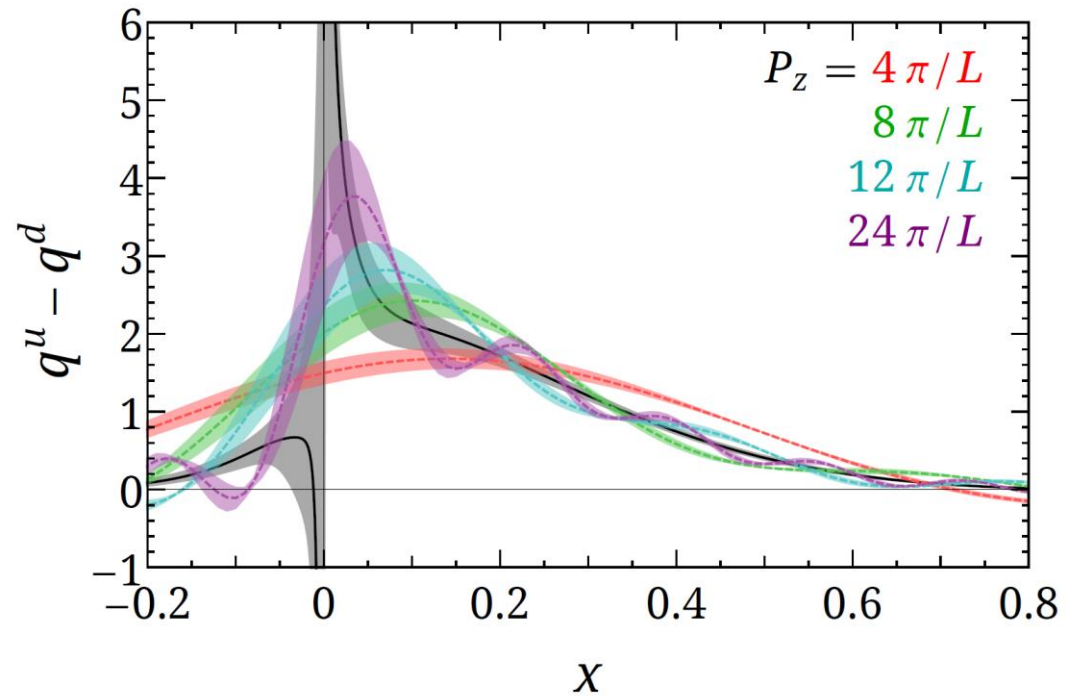
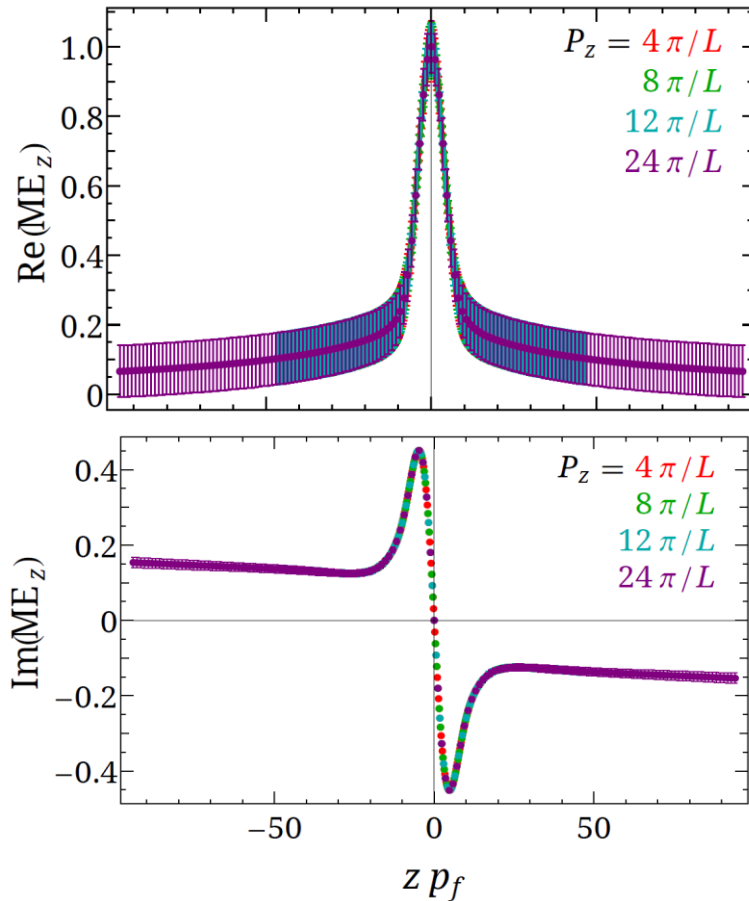
\ni Extension to other structures is not clear yet



Physical Pion Mass

§ Not a lattice problem but Fourier transform issue

§ Simple exercise with CT14 PDF 1506.07443



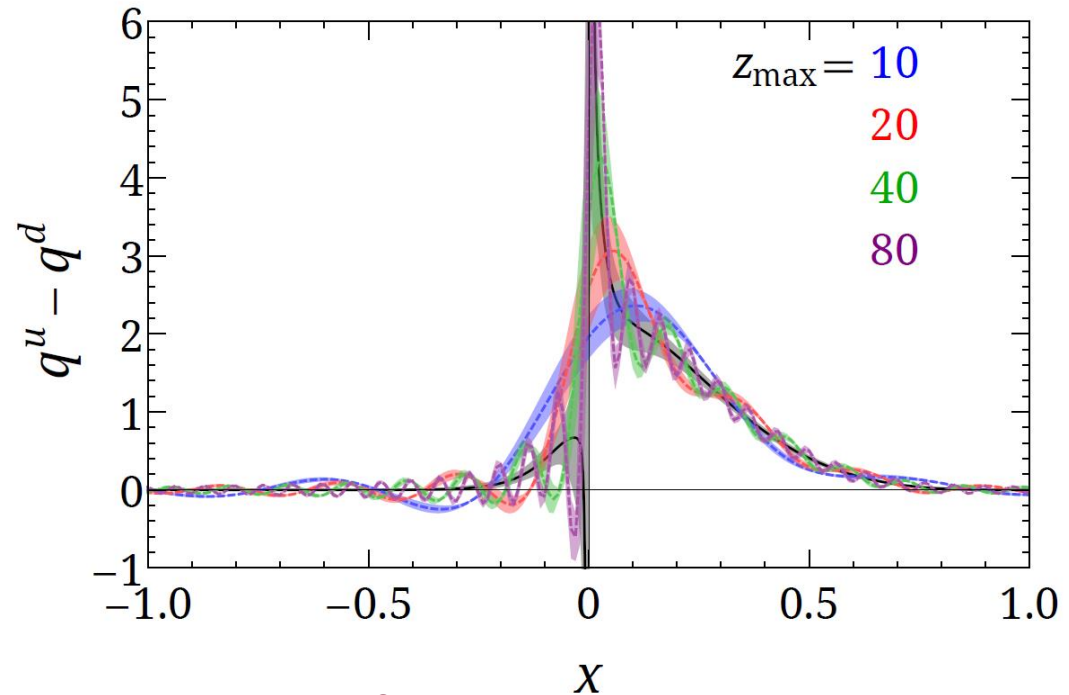
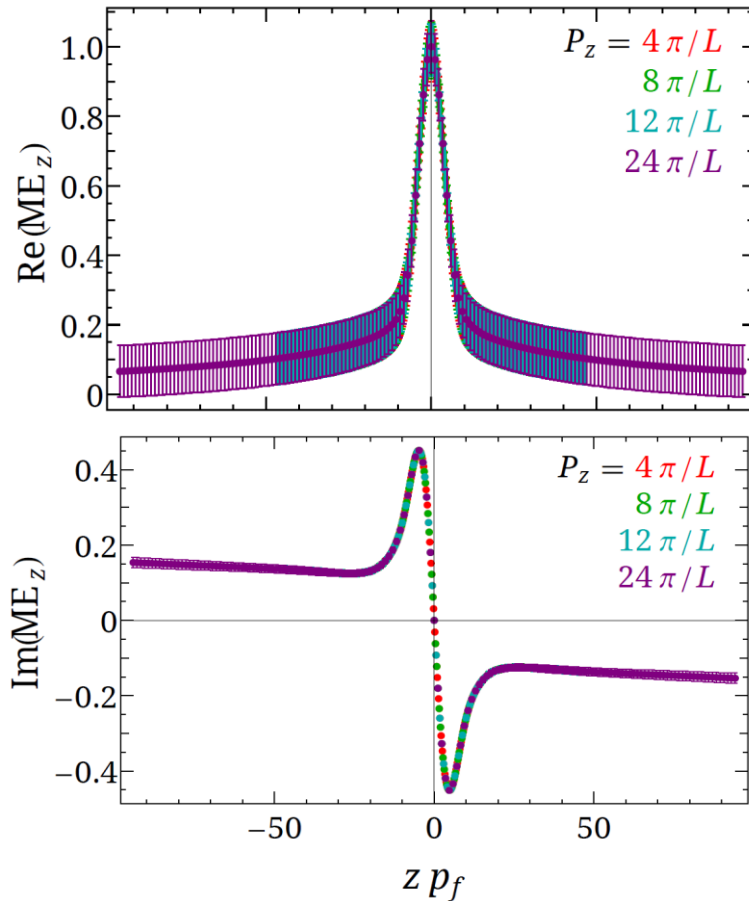
1708.05301 (LP3)

Fixed $L_z = 32$

Physical Pion Mass

§ Not a lattice problem but Fourier transform issue

§ Simple exercise with CT14 PDF 1506.07443



1708.05301 (LP³)

Fixed $P_z = 24\pi/L$

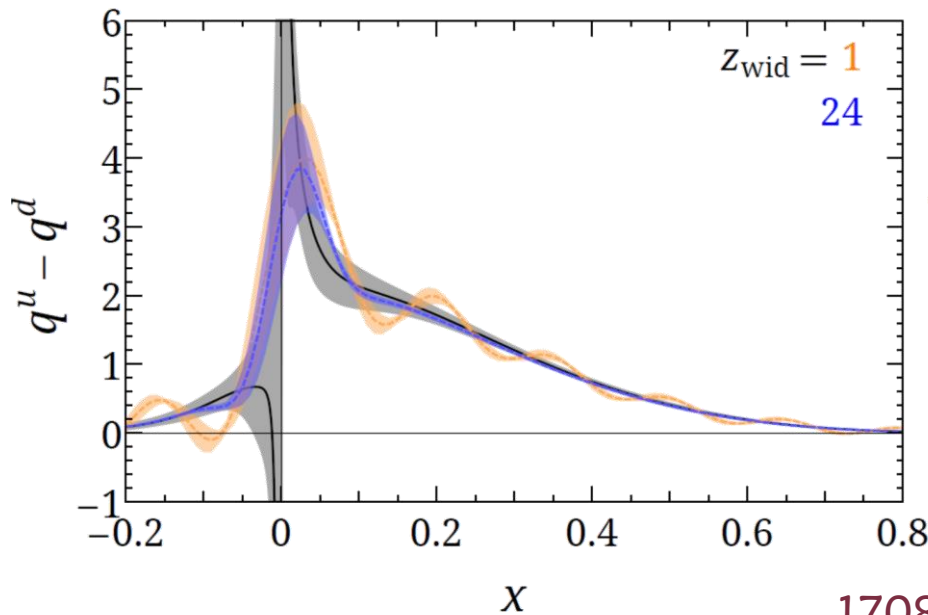
Physical Pion Mass

§ Not a lattice problem but Fourier transform issue

§ Two possible solutions proposed (likely more)

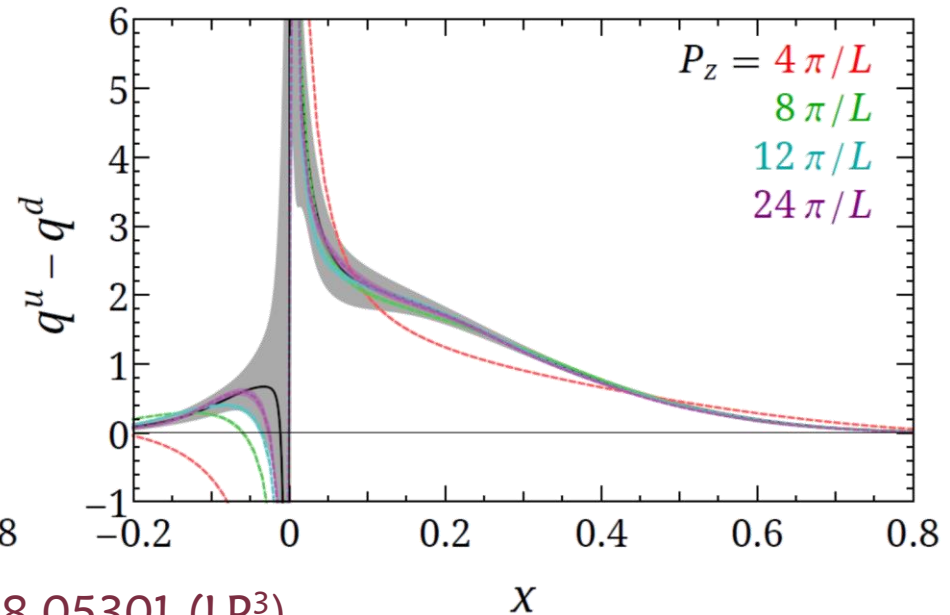
Filter approach

$$F(z_{\text{lim}}, z_{\text{wid}}) = \frac{1 + \operatorname{erf}\left(\frac{z + z_{\text{lim}}}{z_{\text{wid}}}\right)}{2} \frac{1 - \operatorname{erf}\left(\frac{z - z_{\text{lim}}}{z_{\text{wid}}}\right)}{2}$$



Derivative approach

$$q(x) = \int_{-z_{\text{max}}}^{+z_{\text{max}}} dz \frac{-P_z e^{ixP_z z}}{2\pi i P_z x} h'(z)$$



1708.05301 (LP³)

Physical Pion Mass

§ Not a lattice problem but Fourier transform issue

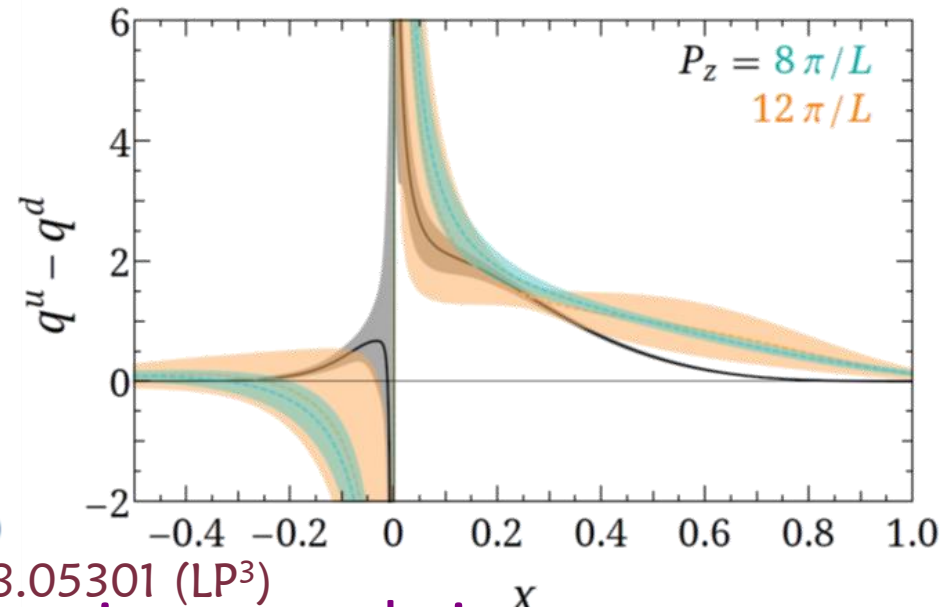
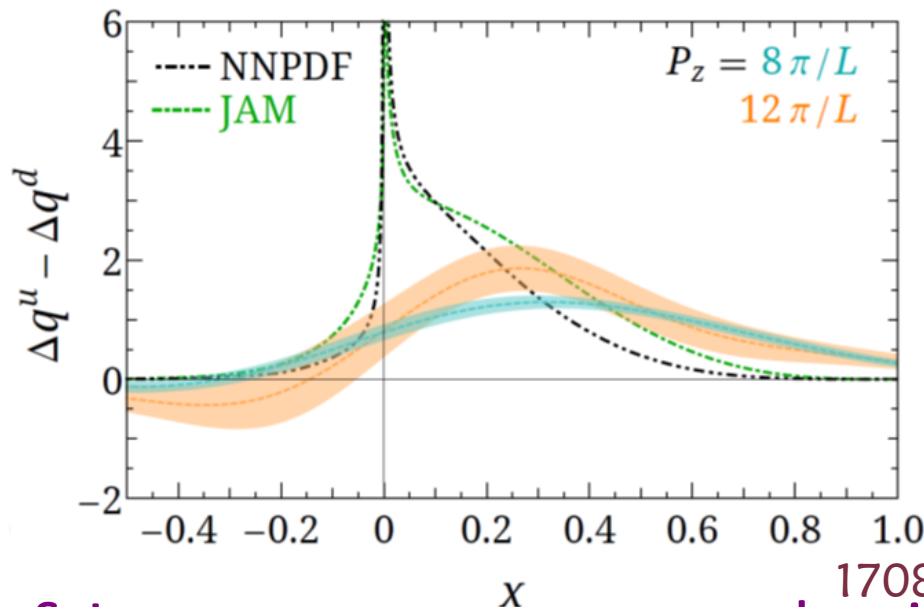
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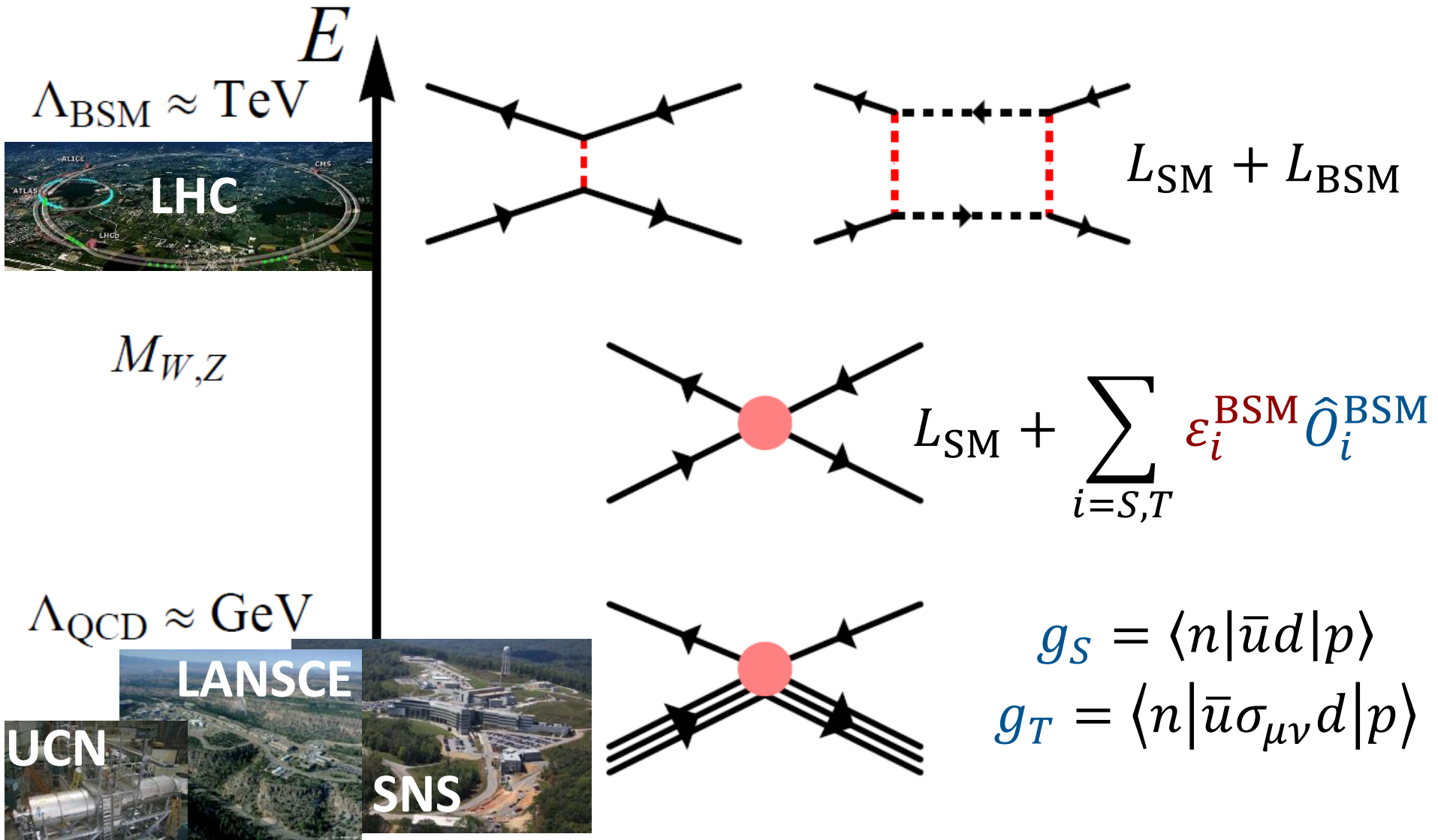
Derivative approach

$$q(x) = \int_{-z_{\text{max}}}^{+z_{\text{max}}} dz \frac{-P_z}{2\pi} \frac{e^{ixP_z z}}{iP_z x} h'(z)$$



§ Larger momentum production is currently in progress

New Physics in TeV Scale



Beta Decays & BSM

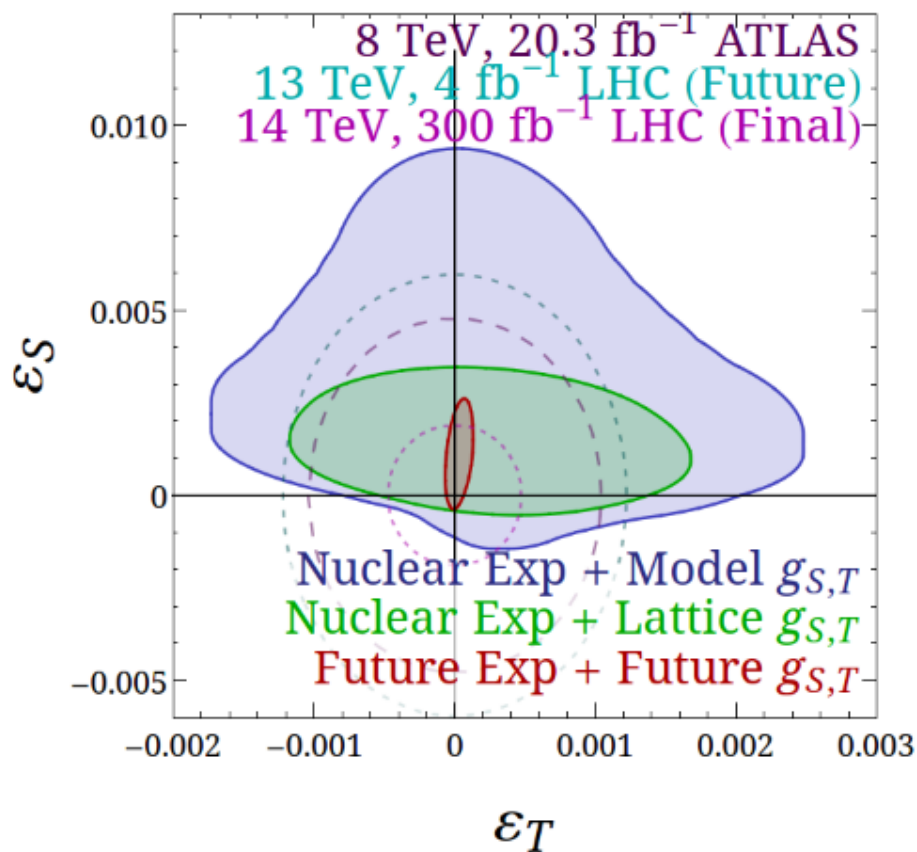
§ Given precision $g_{S,T}$ and O_{BSM} , predict new-physics scales

Low-Energy

Expt

$$O_{\text{BSM}} = f_O(\epsilon_{S,T} g_{S,T})$$

Precision LQCD input
($m_\pi \rightarrow 140$ MeV, $a \rightarrow 0$)



$$\epsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$

Upcoming precision

low-energy experiments

LANL/ ORNL UCN neutron
decay exp't

$$|B_1 - b|_{\text{BSM}} < 10^{-3}$$

$$|b|_{\text{BSM}} < 10^{-3}$$

CENPA: ${}^6\text{He}(b_{\text{GT}})$ at 10^{-3}

PNDME, PRD85 054512 (2012);

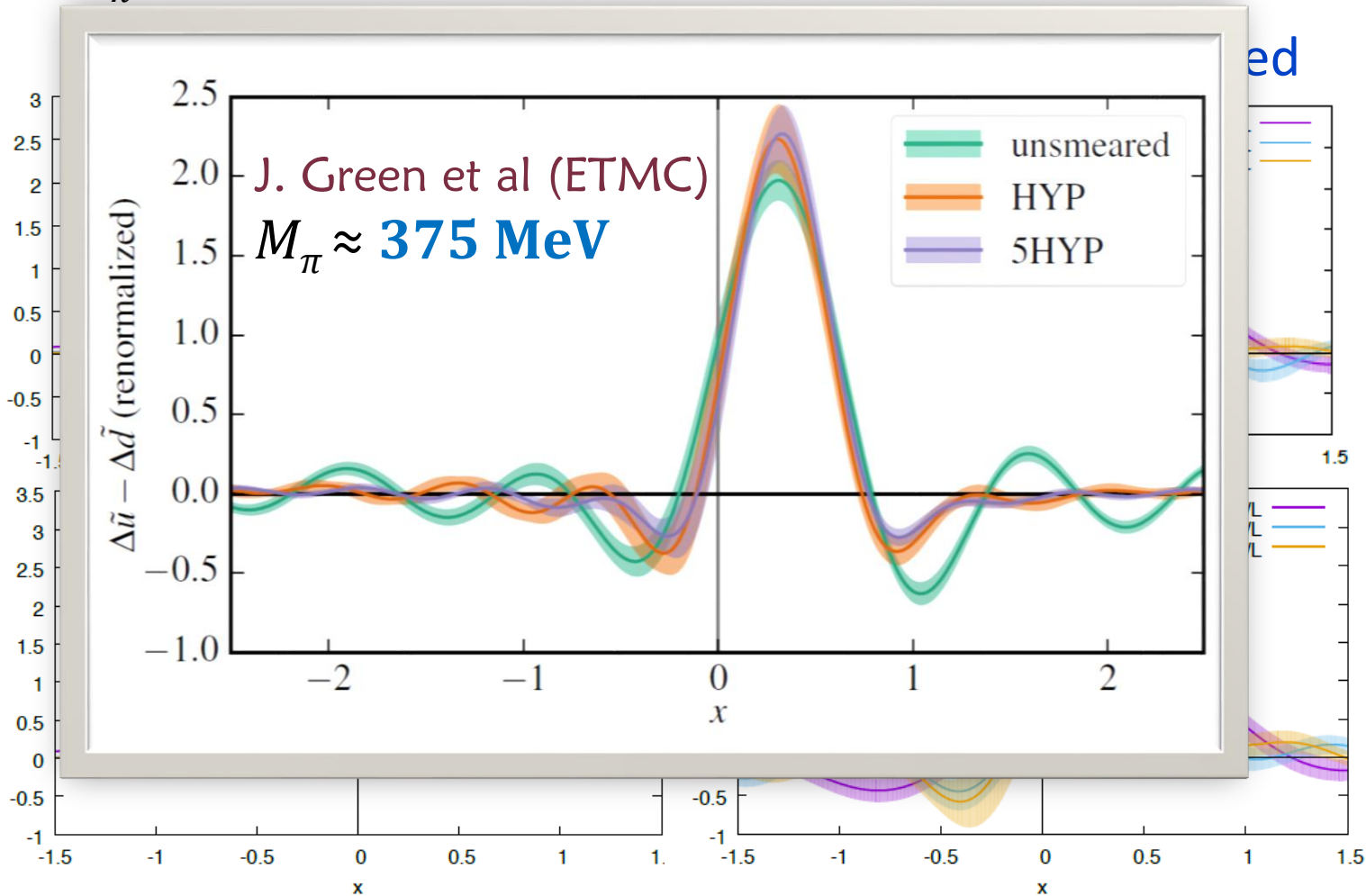
$$1306.5435; 1606.07049 \quad \Lambda_S > 7 \text{ TeV}$$

$$\Lambda_T > 13 \text{ TeV}$$

Physical Pion Mass

§ The problem persists/worsens at physical pion mass

$$M_\pi \approx 135 \text{ MeV}, a \approx 0.09 \text{ fm}, L \approx 5.6 \text{ fm}$$



Yi-Bo Yang
(MSU)