Diffusive Nambu-Goldstone modes in quantum time crystals

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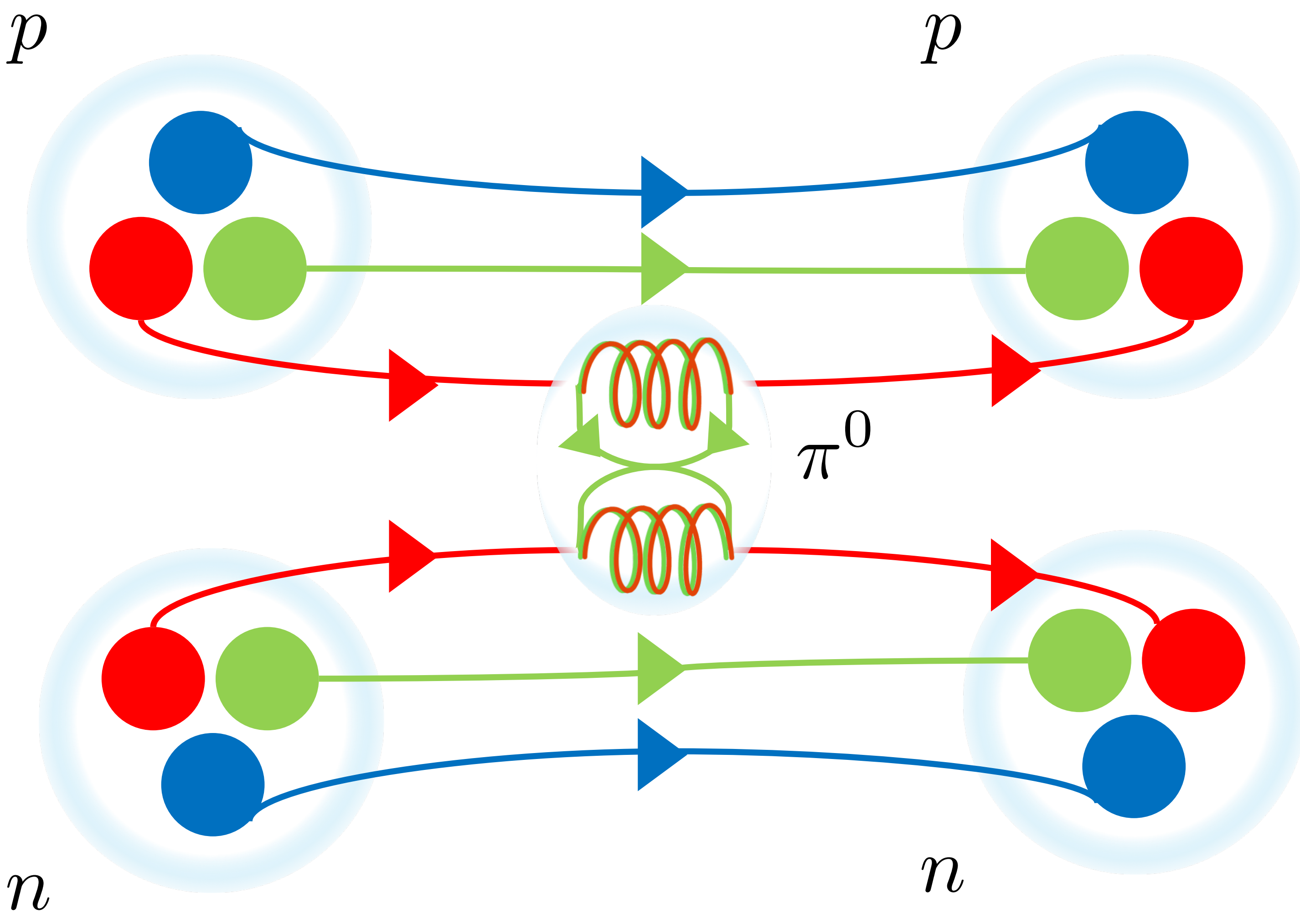
TH, Y. Hidaka (RIKEN), in preparation

Nambu-Goldstone (NG) modes

○ Gapless modes associated with spontaneous symmetry breaking



Nambu-Goldstone (NG) modes

	Patterns	N_{BS}	$\omega = p$	$\omega = p^2$
Pions	$\mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{SU}(2)_{\mathrm{L}} \rightarrow \mathrm{SU}(2)_{\mathrm{V}}$	3	3	0
K-CFL phase	$\mathrm{SU}(2)_{\mathrm{I}} \times U(1)_{\mathrm{Y}} \rightarrow U(1)_{\mathrm{em}}$	3	1	1
Magnons	$\mathrm{SO}(3) \rightarrow \mathrm{SO}(2)$	2	0	1

Counting rules

Hidaka, PRL 110, 091601 (2013)
Watanabe-Murayama, PRL 108, 251602 (2013)

$$N_{\mathrm{NG}} = N_{\mathrm{A}} + N_{\mathrm{B}}$$

$$N_{\mathrm{A}} = N_{\mathrm{BS}} - 2N_{\mathrm{B}}$$

$$\boxed{N_{\mathrm{B}} = \frac{1}{2}\mathrm{rank}\langle[i\hat{Q}_a, \hat{Q}_b]\rangle}$$

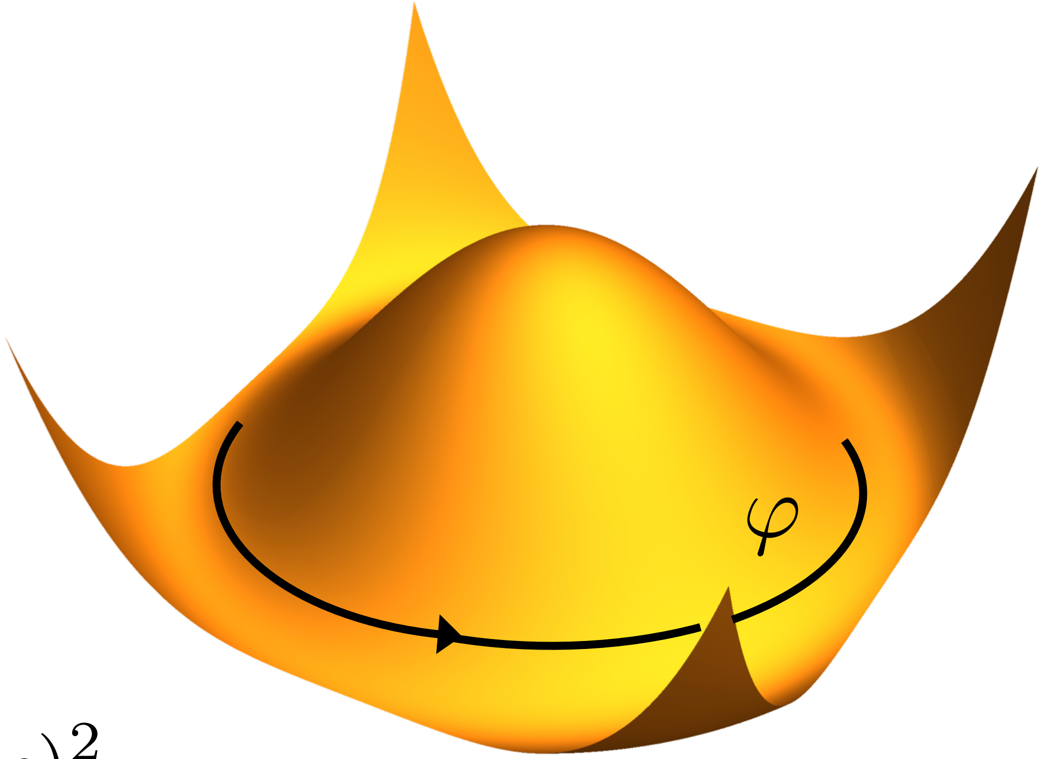
- Type A $\quad \omega = \pm|\boldsymbol{p}|$
- Type B $\quad \omega = \pm\boldsymbol{p}^2$

Type-A NG modes

○ Harmonic oscillators

$$m\frac{d^2}{dt^2}x = -\frac{mg}{l}x$$

- Elastic variable (EV) φ



$$V = \frac{mg}{2l}x^2 \rightarrow \frac{f_\pi^2}{2}(\nabla\varphi)^2$$

○ Linear dispersion relations

$$m \rightarrow f_\pi^2 \quad g/l \rightarrow \boldsymbol{p}^2$$

$$\omega = \pm|\boldsymbol{p}|$$

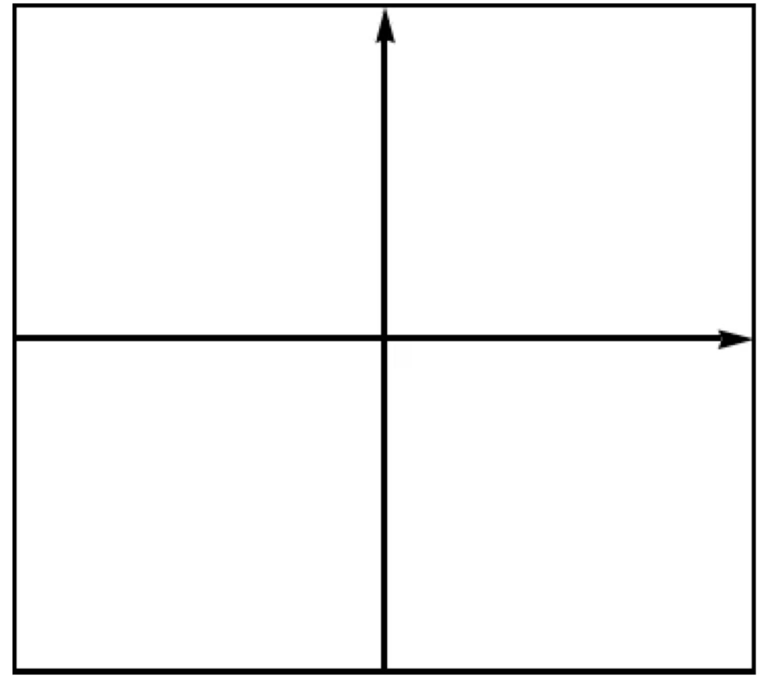
Type-B NG modes

○ Coupled harmonic oscillators

$$m\frac{d^2}{dt^2}x = -\frac{mg}{l}x + 2m\Omega\frac{d}{dt}y$$

Coriolis force

$$m\frac{d^2}{dt^2}y = -\frac{mg}{l}y - 2m\Omega\frac{d}{dt}x$$



- Foucault's pendulum

$$\Omega = \langle[\hat{x}, \hat{y}]\rangle$$

○ Quadratic dispersion relations

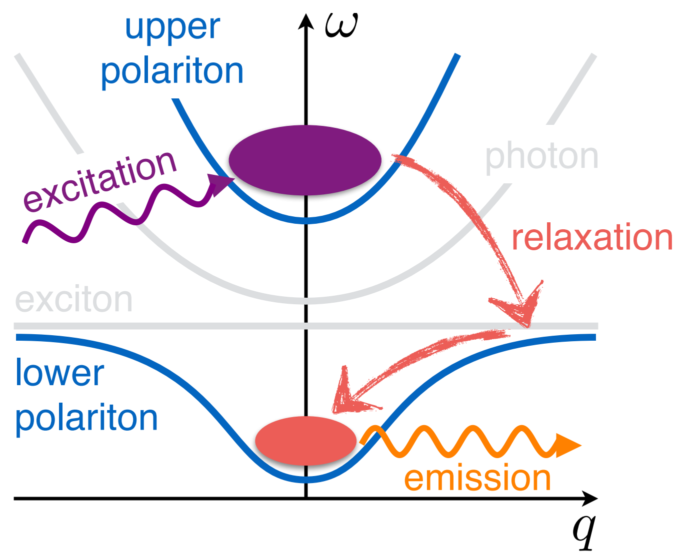
$$\omega = \pm\frac{\boldsymbol{p}^2}{2\Omega}$$

Dissipative quantum systems

○ Driven-dissipative BEC

Sieberer1-Buchhold-Diehl, Rep. Prog. Phys. 79 096001 (2016)

$$S = \int d^4x\, \Phi_A^* \left(i\partial_t + \frac{\nabla^2}{2m} - \underline{u^* - \lambda^* |\Phi_R|^2} \right) \Phi_R + \Phi_R^* \left(i\partial_t + \frac{\nabla^2}{2m} - \underline{u - \lambda |\Phi_R|^2} \right) \Phi_A + iA|\Phi_A|^2$$



○ Dissipative Higgs model

$$\Phi^a_{R,A} = (\Phi^1_{R,A}, \Phi^2_{R,A})$$

$$S = \int d^4x\, \Phi_A^{a*} \left(-(\partial_t + i\mu)^2 + \nabla^2 \underline{+ \gamma\partial_t} - 2\lambda |\Phi_R^b|^2 \right) \Phi_R^a + \Phi_R^{a*} \left(-(\partial_t + i\mu)^2 + \nabla^2 \underline{- \gamma\partial_t} - 2\lambda |\Phi_R^b|^2 \right) \Phi_A^a + iA|\Phi_A^a|^2$$

Minami-Hidaka, PRE 97, 012130 (2018)

Type-A NG modes

○ Damped Harmonic oscillators

$$m\frac{d^2}{dt^2}x = -\frac{mg}{l}x - 2m\gamma\frac{dx}{dt}$$

Friction

○ Diffusive dispersion relations

$$m \to f_\pi^2 \quad g/l \to \boldsymbol{p}^2$$

$$\omega = -\frac{i}{\gamma}\boldsymbol{p}^2$$

- Overdamped

Type-B NG modes

○ Coupled damped harmonic oscillators

$$m\frac{d^2}{dt^2}x = -\frac{mg}{l}x + 2m\Omega\frac{d}{dt}y - \underline{2m\gamma\frac{dx}{dt}}$$

Friction

$$m\frac{d^2}{dt^2}y = -\frac{mg}{l}y - 2m\Omega\frac{d}{dt}x - \underline{2m\gamma\frac{dy}{dt}}$$

○ Quadratic dispersion relations

$$\omega = \frac{1}{4\Omega^2 + \gamma^2}(\pm 2\Omega - i\gamma)\boldsymbol{p}^2$$

- Damping oscillation

Nambu-Goldstone (NG) modes

Minami-Hidaka, PRE 97, 012130 (2018)

	Patterns	Hamiltonian	Dissipative
Type A	$\langle [iQ, \phi] \rangle \neq 0$	$\omega = \pm \lvert \boldsymbol{p} \rvert$	$\omega = -i \lvert \boldsymbol{p} \rvert$
Type B	$\langle [iQ_a, Q_b] \rangle \neq 0$	$\omega = \pm \boldsymbol{p}^2$	$\omega = \pm \boldsymbol{p}^2 - i \boldsymbol{p}^2$

Static condensate
➔
Dynamic condensate

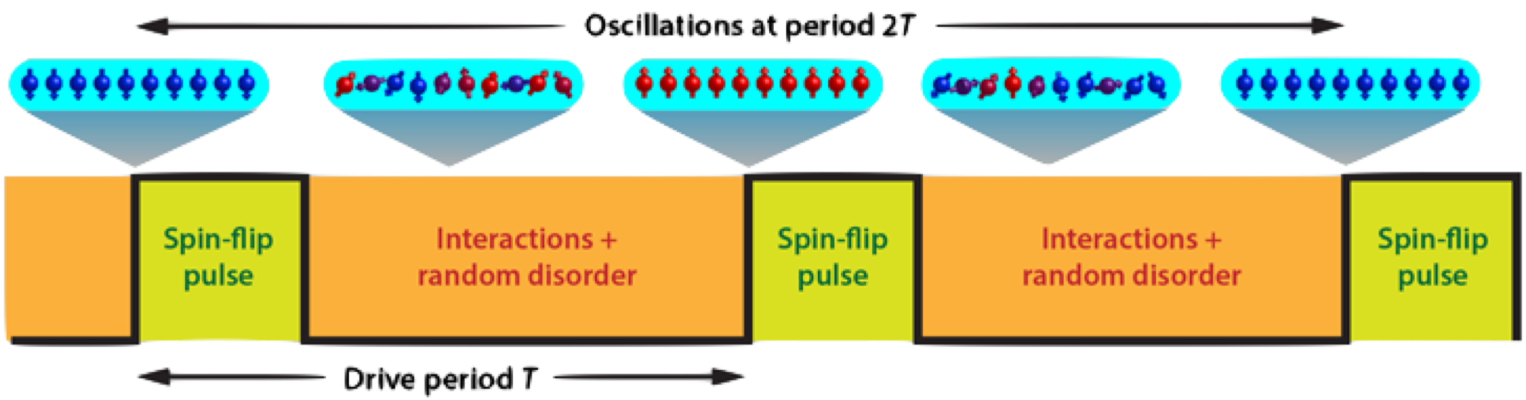
Quantum time-crystals

Wilczek, PRL109, 160401 (2012)

- No-Go theorem in Hamiltonian systems Watanabe- Oshikawa, PRL114, 251603 (2015)

○ Floquet (discrete) time-crystal [Ising chain]

Yao-Potter-Potirniche-Vishwanath, PRL 118, 030401 (2017)



○ Continuous time-crystal (Spin precision) in excited states

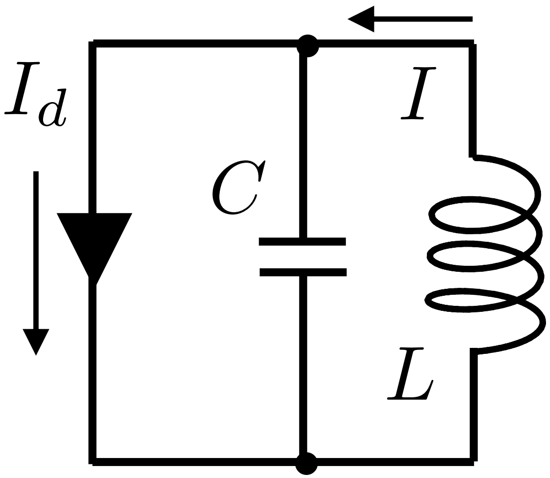
Autti-Eltsov-Volovik, PRL120, 215301 (2018)

$$\langle S_x + iS_y \rangle = S_0 e^{i2\Omega t}$$

○ Van der Pol time-crystal in a nonequilibrium steady state

This work

Van der Pol oscillator



$$I_d = \alpha\left(-V + V^3/3\right)$$

$$\frac{d^2V}{dt^2} - \gamma\left(1 - V^2\right)\frac{dV}{dt} + m^2 V = 0$$

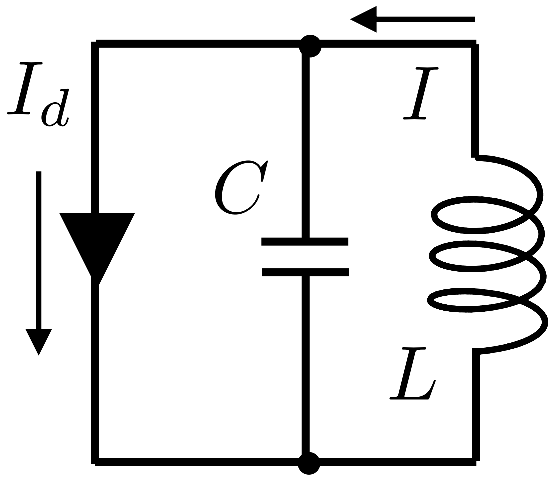
$$\gamma = \alpha/L \quad m^2 = 1/LC$$

- Nonlinear friction

$V^2 < 1 \rightarrow$ Negative friction $V \sim e^{\gamma t}$

$V^2 > 1 \rightarrow$ Positive friction $V \sim e^{-\gamma^* t}$

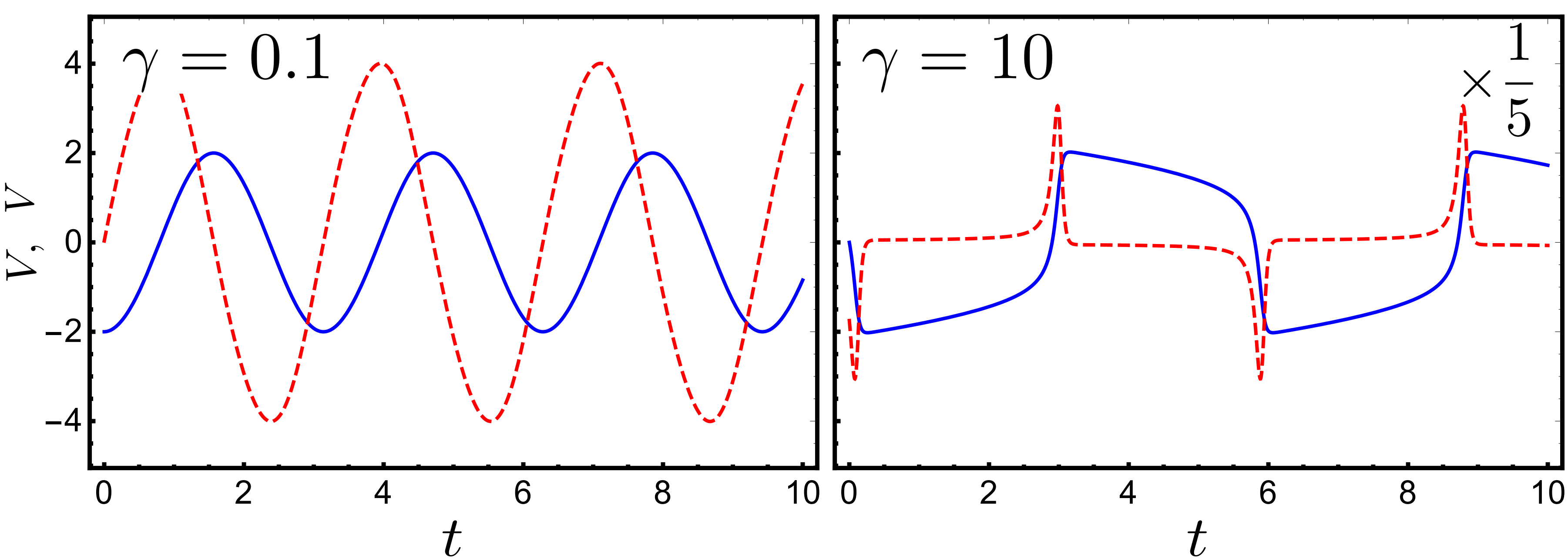
Van der Pol oscillator



$$I_d = \alpha\left(-V + V^3/3\right)$$

$$\frac{d^2V}{dt^2} - \gamma\left(1 - V^2\right)\frac{dV}{dt} + m^2V = 0$$

- Spontaneous oscillation (Limit cycle)



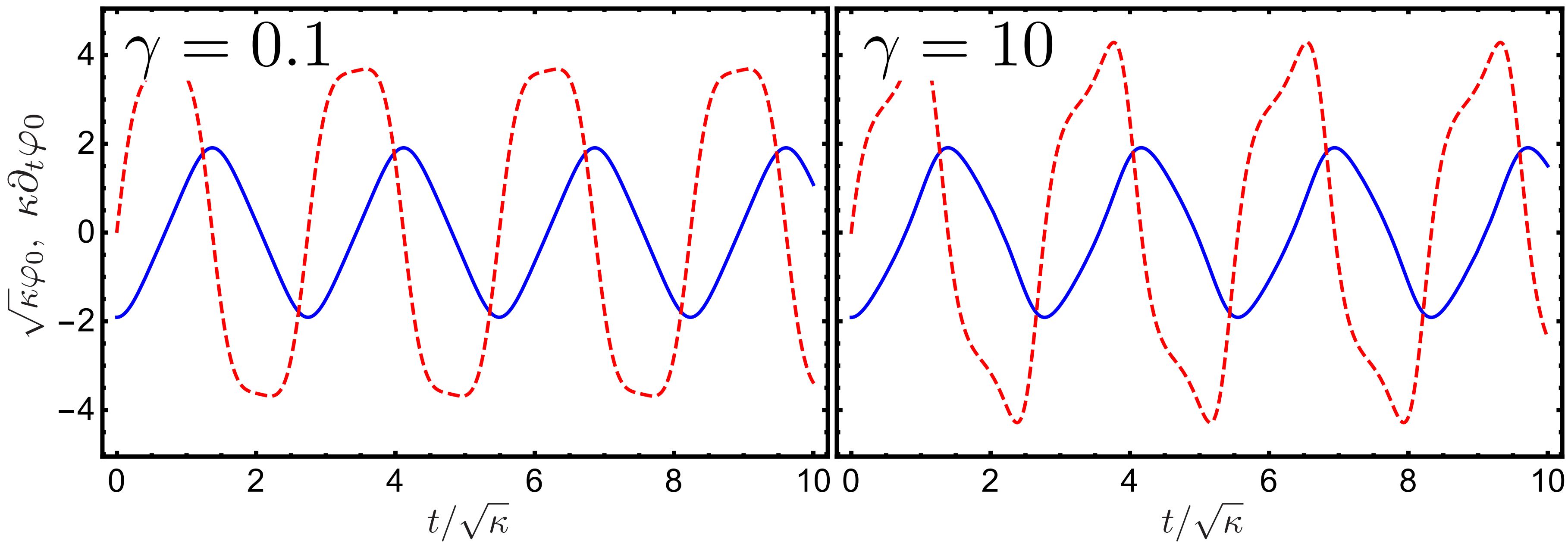
Schwinger-Keldysh action

Real-scalar fields $\varphi_{R,A}$

$$S = \int d^4x \varphi_A \left(-\partial_t^2 + \nabla^2 + \gamma\left(1 - \kappa\varphi_R^2\right)\partial_t - 2\lambda\varphi_R^2\right)\varphi_R + iA(\varphi_A)^2$$

- Linearly unstable, but nonlinearly stable

○ Mean-field approximation $\varphi_0 = \langle\varphi_{\mathrm{R}}\rangle$



- Spontaneous breaking of the time-translation symmetry

Type-A NG mode in quantum time-crystals

○ Elastic variable $\varphi_0(t) \to \varphi_0(t+\chi_{\mathrm{R}})$

$$-\partial_t^2\varphi_0 + \gamma\left(1-\kappa\varphi_0^2\right)\partial_t\varphi_0 - 2\lambda\varphi_0^3 = 0$$

○ EOM of the EV

$$\left(\dot{\varphi}_0\left(-\partial_t^2+\nabla^2\right) - 2\partial_t\dot{\varphi}_0\partial_t + \dot{\varphi}_0\gamma\left(1-\kappa\varphi_0^2\right)\partial_t\right)\chi_R = 0$$

- Gapless mode necessarily appears!

○ Type-A NG mode

$$\omega = -iC\boldsymbol{p}^2$$

Schwinger-Keldysh action

Complex-scalar fields $\Phi_{R,A}$

$$S =$$

$$\int d^4x\, \Phi_A^* \Big(-(\partial_t + i\mu)^2 + \nabla^2$$

$$+\gamma\left(1-\kappa|\Phi_R|^2\right)\partial_t - m^2 - 2\lambda|\partial_t\Phi_R|^2\Big)\Phi_R$$

$$+\Phi_R^* \Big(-(\partial_t + i\mu)^2 + \nabla^2$$

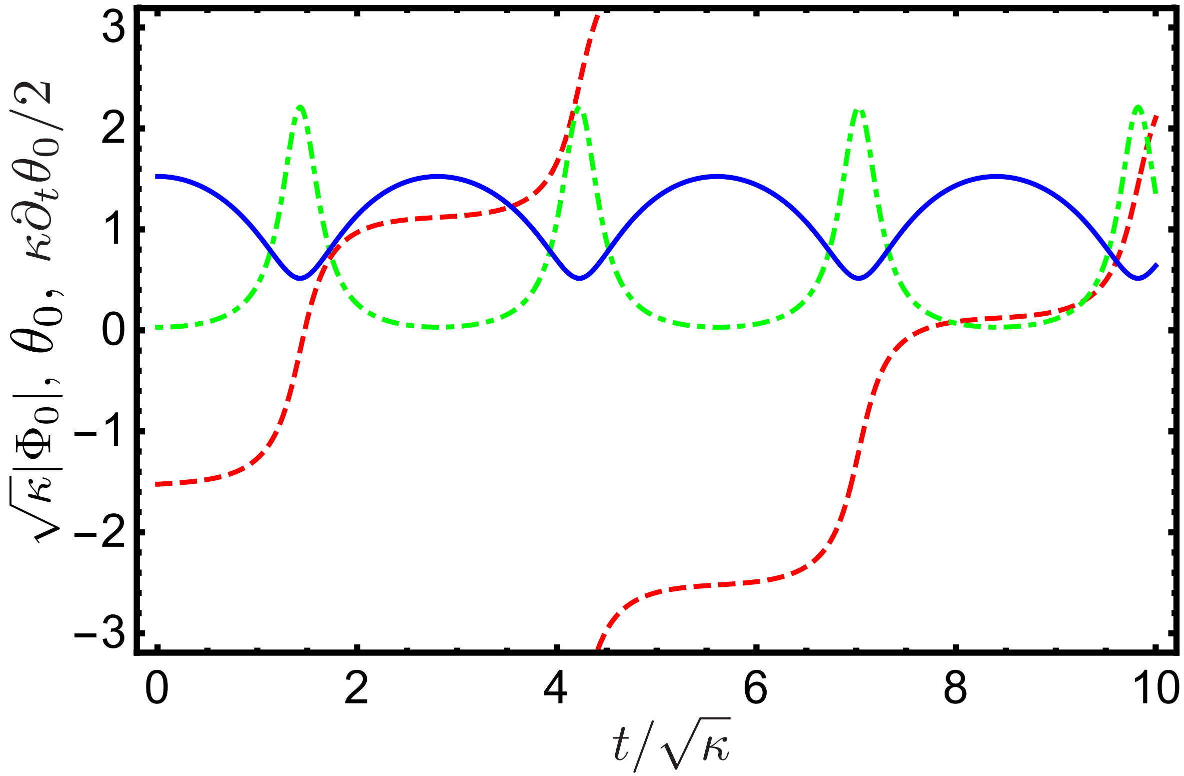
$$-\partial_t\gamma\left(1-\kappa|\Phi_R|^2\right) - m^2 - 2\lambda|\partial_t\Phi_R|^2\Big)\Phi_A + iA|\Phi_A|^2$$

- Effective theory of unstable superfluids

Superfluid kink-time crystal

○ Mean-field solution $\Phi_0 = \langle \Phi_{\rm R} \rangle$

$$-\left(\partial_t + i\mu\right)^2 \Phi_0 + \gamma\left(1 - \kappa|\Phi_0|^2\right)\partial_t \Phi_0 - (m^2 + 2\lambda|\partial_t \Phi_0|^2)\Phi_0 = 0$$



- Periodic array of temporal-domain-walls

Type-B NG mode in quantum time-crystals

○ Two elastic variables

Time-translation: $\Phi_0(t) \to \Phi_0(t + \chi_{1\mathrm{R}})$

U(1): $\Phi_0(t) \to e^{i\chi_{2\mathrm{R}}}\Phi_0(t)$

○ EOM of the EVs

$$\Big(\partial_t\Phi_0\left(-\partial_t^2 + \nabla^2\right) - 2\partial_t^2\Phi_0\partial_t - 2i\mu\partial_t\Phi_0\partial_t + \partial_t\Phi_0\gamma\left(1 - \kappa|\Phi_0|^2\right)\partial_t - 2\lambda\Phi_0|\partial_t\Phi_0|^2\partial_t\Big)\chi_{1R} = 0$$

$$\Big(\Phi_0^*\left(-\partial_t^2 + \nabla^2\right) - 2\partial_t\Phi_0^*\partial_t + 2i\mu\Phi_0^*\partial_t + \Phi_0^*\gamma\left(1 - \kappa|\Phi_0|^2\right)\partial_t - 2\lambda\Phi_0\mathrm{Im}\left[\Phi_0^*\partial_t\Phi_0\right]\partial_t\Big)\chi_{2R} = 0$$

- Two gapless directions

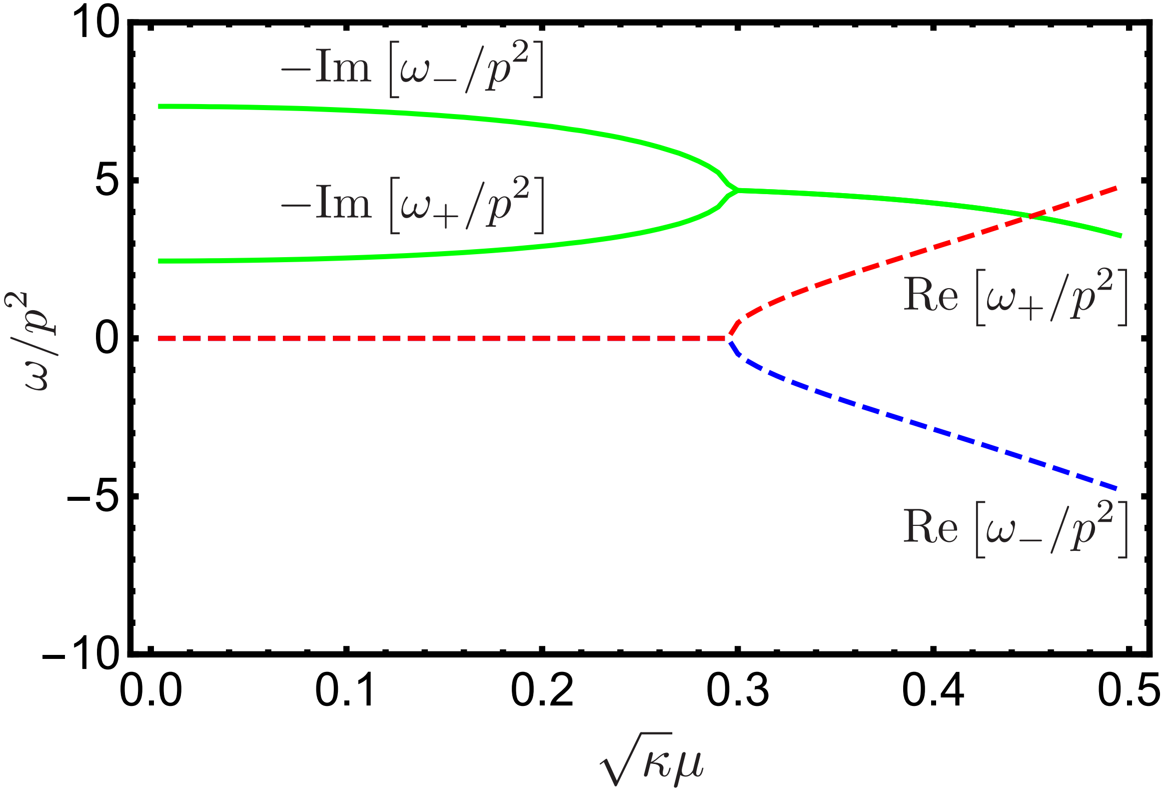
○ One type-B NG mode

$$\omega_\pm = \left(\pm C_1 - iC_2\right)\boldsymbol{p}^2$$

Discussion

○ Noncommutativity btw broken charges

$$\langle [iQ_A^t, Q_R^{\mathrm{U}(1)}] \rangle \neq -\langle [iQ_A^{\mathrm{U}(1)}, Q_R^t] \rangle \neq 0$$



- Overdamped type-B NG mode

Summary

○ NG modes in quantum time-crystals

- Type-A NG mode in the Van der Pol oscillator
- Type-B NG mode in the superfluid kink time-crystal

○ Future prospects

- Phenomenological applications
- Transport properties of quantum time-crystals