Thermodynamics for SU(2) gauge theory using gradient flow

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NFQCD2018, YITP, 5. June, 2018

arXiv:1805.07106 [hep-lat]

Introduction

Scale Setting

Thermodynamics

Summary

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Summary

SU(2) gauge theory

- SU(2) gauge theory (pure SU(2), 2-color QCD) has properties SU(3) gauge theory does.
  - Asymptotic freedom
  - Quark confinement
  - Chiral symmetry breaking
  - etc...
- Different point between SU(2) and SU(3) ...
  - the order of phase transition
    (pure SU(2): second, pure SU($N_c \geq 3$): first)
- Numerical cost of lattice simulation: SU(2) $<$ SU(3)
  $\rightarrow$ SU(2) gauge theory is a good model for
  the methodological study on SU(3) gauge theory

Gradient flow

- Yang-Mills gradient flow equation on lattice[1] ...

$$\partial_t V_t(x,\mu) = -g_0^2 \left\{ \partial_{x,\mu} S_W \right\} V_t(x,\mu),$$
$$V_t(x,\mu)\Big|_{t=0} = U(x,\mu).$$

  g_0: bare coupling, t: flow-time, $U(x,\mu)$: link variable, S_W: Wilson action.

- $V_t \rightarrow$ **renormalized field**.
- $\sqrt{t} \rightarrow$ typical energy scale of the renormalization.
- Composite operators with V_t are **UV finite in the positive flow-time ($t > 0$).**

[1] M. Luscher, JHEP **1008** (2010) 071.

Energy-momentum tensor

- Energy-momentum tensor (EMT) ...
  - Lattice regularization breaks the translational symmetry.
  - Wave function renormalization is needed.
- We can directly calculate the renormalized EMT with gradient flow[2]

$$T^R_{\mu\nu} = \lim_{t\to 0}\left\{\frac{U_{\mu\nu}}{\alpha_U} + \frac{\delta_{\mu\nu}}{4\alpha_E}\left[E - \langle E\rangle_0\right]\right\},$$

$$U_{\mu\nu} = G_{\mu\rho}G_{\nu\rho} - \frac{\delta_{\mu\nu}}{4}G_{\rho\sigma}G_{\rho\sigma},\ E = \frac{1}{4}G_{\mu\nu}G_{\mu\nu}.$$

  $G_{\mu\nu}$: the field strength consisting of V_t,
  α_U, α_E: coefficients calculated in the one-loop order of running coupling.
- EMT $\rightarrow$ entropy density, trace anomaly, etc...

[2] H. Suzuki, PTEP **2013**, 083B03 (2013)

Our motivation

1. Previous study of SU(2) gauge theory ...
   - Thermodynamic quantities: measured using integral method only[3]
     → **carry out using gradient flow in this work**
2. Calculating the shear viscosity ...
   - Measuring of correlation function of the EMT is first step.
   - A huge number of the configuration is needed (in previous work).
     → Reduce using Gradient flow method(?)
     → Prepare to calculate shear viscosity

[3] P. Giudice and S. Piemonte, Eur. Phys. J. C **77** (2017) no.12, 821

Target of this study

"Calculating thermodynamics for pure SU(2) using gradient flow method"

1. Scale setting (t_0 scale)
   $\rightarrow$ Compare with present scale settings.
2. Calculate EMT at finite temperature
   $\rightarrow$ entropy density, trace anomaly, energy density, pressure $\rightarrow$ eq. of state (EOS)
   $\rightarrow$ Compare with
   - Integral method
   - Hard-Thermal-Loop (HTL) analysis
   - SU($N_c \geq 3$) theory

Introduction

Scale Setting

Thermodynamics

Summary

Scale Setting

- Observable … "Action density"

$$E = \frac{1}{4} G_{\mu\nu} G_{\mu\nu}, \;\; t^2 \langle E(t) \rangle \propto N_c^2 - 1.$$

- Reference scale: $t^2 \left\langle E(t) \right\rangle \Big|_{t=t_0} = 0.1$

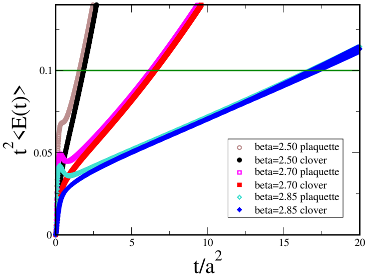
  $\rightarrow$ a natural scaling-down of the SU(3) case[4].
- Configuration generation
  - Wilson-plaquette action, $N_s = N_t = 32$
  - 1 sweep = 1 pseudo-heatbath + 20 over-relaxation
  - 100 sweep separation between measurements

β	2.42	2.50	2.60	2.70	2.80	2.85
# of Conf.	100	300	300	300	300	600

- Gradient flow
  - utilize Plaquette and Clover-leaf definition
  - $t/a^2 \in [0.00, 32.00], \;\; \Delta t/a^2 = 0.01$

[4] M. Luscher, JHEP **1008** (2010) 071.

Scale Setting



β	2.42	2.50	2.60
t_0/a^2	1.083(2)	1.839(3)	3.522(10)
β	2.70	2.80	2.85
t_0/a^2	6.628(36)	11.96(12)	16.95(17)

- Best fit function (t_0/a^2 .vs. β)

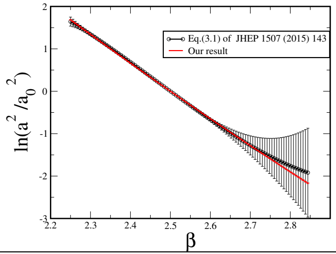
$$\ln(t_0/a^2) = 1.258 + 6.409(\beta - 2.600) - 0.7411(\beta - 2.600)^2.$$

Compare with other scale setting

- with "r_0 scale[5] (r_c scale[6])" : define via $q\bar{q}$ force

$$\rightarrow \frac{\sqrt{8t_0}}{r_0} = 0.6020(86)(40), \ \frac{\sqrt{8t_0}}{r_c} = 1.126(7)(7),$$
$$\rightarrow \sqrt{8t_0} = 0.3010(43)(20)[\text{fm}].$$
$$r_0 = 0.5[\text{fm}], \ r_c = 0.26[\text{fm}].$$

- with scale setting using the string tension[7] ($a^2\sigma$)



- take reference lattice spacing a at $\beta = 2.50$
- $2.42 \leq \beta \leq 2.60$ ... both functions are available
- 1–σ consistent
- we cover $T \leq 2T_c$

[5] R. Sommer, Nucl. Phys. B 411, 839 (1994).
[6] S. Necco and R. Sommer, Nucl. Phys. B 622, 328 (2002).
[7] M. Caselle, A. Nada and M. Panero, JHEP **1507** (2015) 143.

Introduction

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Procedure and Simulation setup

- Steps to calculate renormalized EMT[8]
  1. Generate configuration at $t = 0$ on $N_s^3 \times N_\tau$
  2. Solve gradient flow eq. in $a \ll \sqrt{8t} \ll R$
  3. Construct renormalized EMT at each t
  4. Carry out an extrapolation, first $a \to 0$, next $t \to 0$
- Simulation setup
  - Wilson-plaquette action
  - $N_s/N_\tau = 4, \ N_\tau = 6, 8, 10, 12$
  - # of Conf. for each parameter: 200
  - 1 sweep = 1 pseudo-heatbath+N_t over-relaxation
  - 100 sweep separation between measurements
- Gradient flow
  - utilize Clover-leaf definition for action density
  - $t/a^2 \in [0.00, 5.00], \ \Delta t/a^2 = 0.01$

[8] M. Asakawa *et al.*, Phys. Rev. D **90**, no. 1, 011501 (2014)

Observables

- β .vs. T/T_c for each N_t [9]

T/T_c	$N_\tau = 6$	$N_\tau = 8$	$N_\tau = 10$	$N_\tau = 12$
0.95	—	2.50	2.57	2.62
0.98	2.42	2.51	2.58	2.63
1.01	2.43	2.52	2.59	2.64
1.04	2.44	2.53	2.60	2.65
1.08	2.45	2.54	2.61	2.66
1.12	2.46	2.55	2.62	2.67
1.28	2.50	2.59	2.66	2.72
1.50	2.55	2.64	2.71	2.77
1.76	2.60	2.69	2.76	2.82
2.07	2.65	2.74	2.81	—

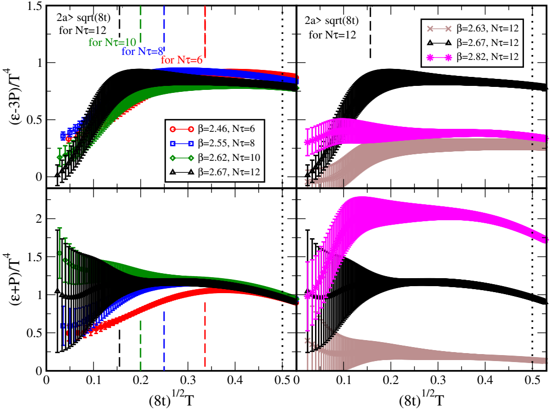
- entropy density (s), trace anomaly (Δ)

$$sT = \varepsilon + P = T^R_{11} - T^R_{44}, \quad \Delta = \varepsilon - 3P = -\sum_{\mu=1}^{4} T^R_{\mu\mu}.$$

- ε: energy density, P: pressure
  $\rightarrow$ Compute ε and P from s and Δ.

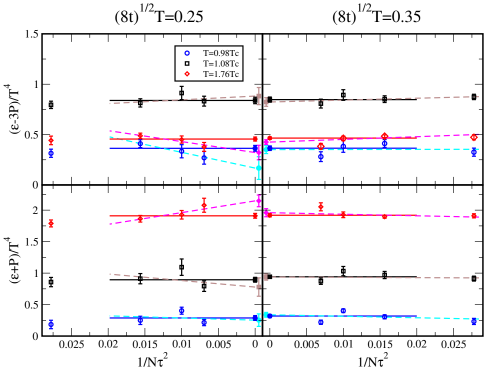
[9] the critical β on $N_t = 6$ from [J. Engels, J. Fingberg and D. E. Miller, Nucl. Phys. B **387** (1992) 501.]

Flow-time dependence of s and Δ



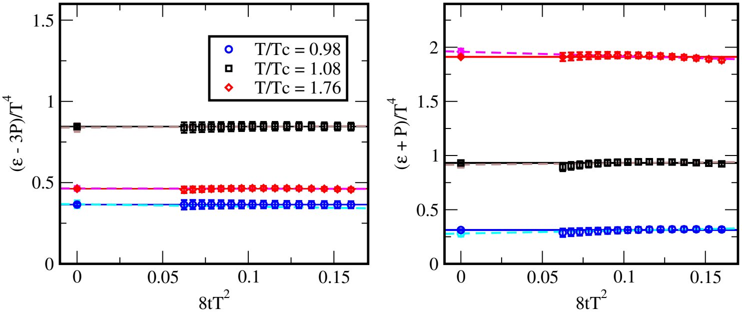
- left panel: $T = 1.12\,T_c$
- right panel: $N_t = 12$
- Fiducial window: $2a \leq \sqrt{8t} \leq N_t a/2$
- lower: Using clover-leaf operator
- upper: Considering the effect of smearing by gradient flow

$a \to 0$ limit



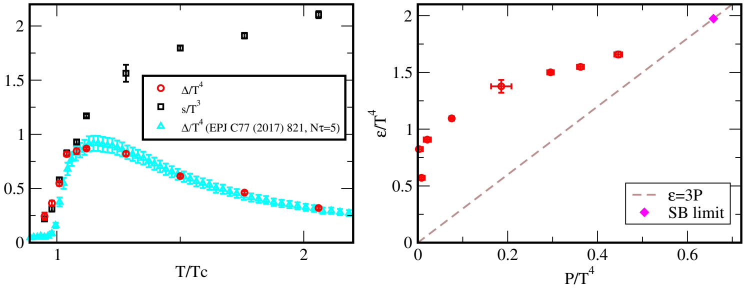
- $\sqrt{8t}T \in [0.25, 0.40]$, $\delta(\sqrt{8t}T) = 0.01$
- Each data is adopted closest to the fixed $\sqrt{8t}T$
- Constant extrapolation: to calculate the central value
- Linear extrapolation: to estimate the systematic error with constant extrapolation
- Constant .vs. Linear ... 2–σ consistent

$t \longrightarrow 0$ limit



- $\sqrt{8t}T \in [0.25, 0.40]$, $\delta(\sqrt{8t}T) = 0.01$
- Carry out both constant- and linear-extrapolation
- We take the central result which is the better χ^2/d.o.f
- Constant .vs. Linear ... 2–σ consistent

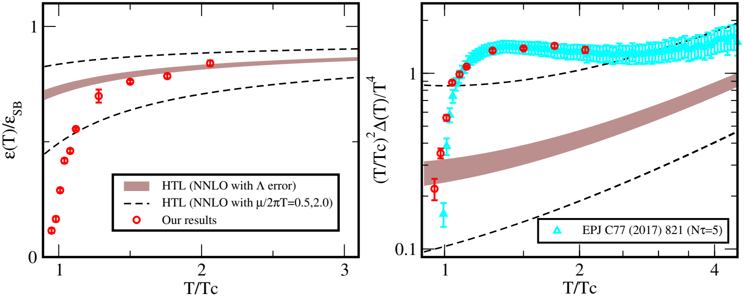
Result



- Left panel: s/T^4 (black symbol), Δ/T^4 (red symbol)
  - Δ/T^4 ... consistent with the integral method[10] in $T > T_c$
- Right panel: ε/T^4 .vs. P/T^4 (EOS) in $T > T_c$
  - Toward to SB limit $(P/T^4, \varepsilon/T^4) = (\pi^2/15, \pi^2/5)$
  - $70 \sim 80\%$ of SB limit for $T \sim 2T_c$
    $\rightarrow$ NOT describe two-color QGP around $T \leq 2T_c$

[10] P. Giudice and S. Piemonte, Eur. Phys. J. C **77** (2017) no.12, 821

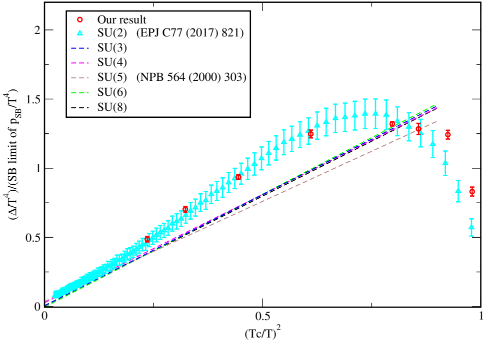
Compare with HTL analysis



- Hard-Thermal-Loop (HTL) analysis[11]
  ... 2-color case in NNLO
- Left panel: $\varepsilon/\varepsilon_{\mathrm{SB}}$
  Our result is consistent with HTL in $T > T_c$
- Right panel: $(T/T_c)^2\Delta/T^4$
  plateau and approaches to HTL result in $1.5T_c \leq T$

[11] J. O. Andersen *et al.*, JHEP **1008** (2010) 113.

Compare with $SU(N_c \geq 3)$ theory



- Dashed lines: trace anomaly for SU($N_c \geq 3$) theory[12]
- In high T: leading correction of Δ/T^4 is $1/T^2$
- SU($N_c \geq 3$): linear behavior until $(T_c/T)^2 \leq 0.9$
- SU(2): **Different behavior**

  $\leftarrow$ from the difference of order of phase transition

[12] M. Panero, Phys. Rev. Lett. **103** (2009) 232001

Introduction

Scale Setting

Thermodynamics

Summary

Summary and Future works

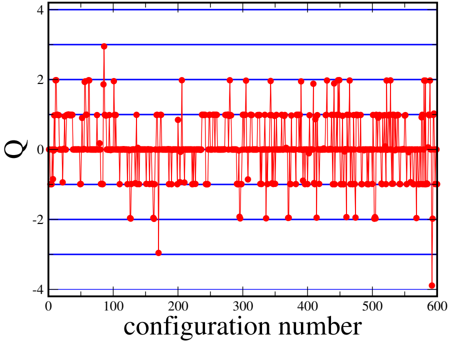
Summary

- We investigate the thermodynamics of the pure SU(2) gauge theory
  1. Scale setting
     $\rightarrow t^2 \langle E(t) \rangle \Big|_{t=t_0} = 0.1$ for SU(2)
  2. Obtaining $s/T^3,\ \Delta/T^4$, EOS
     $\rightarrow$ Consistent with integral method and HTL analysis

Future works

- Calculation of shear viscosities for pure SU(2)
  $\rightarrow$ systematic study on T- and N_c-dependence
- Constructing effective model of two-color QCD
  $\rightarrow$ analyze the physics in 2-color QCD
  @finite chemical potential

Back Up: Topological Charge using gradient flow



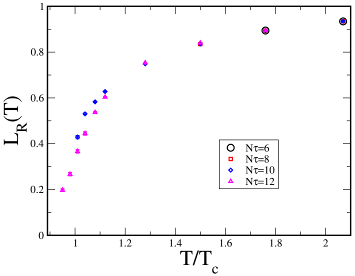
- Figure: Result at $\beta = 2.85, t/a^2 = 32$
- Topological charge Q

$$Q = \frac{1}{32\pi^2} \int d^4x \; \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr} G_{\mu\nu} G_{\rho\sigma}$$

- Q takes an almost integer-value
  $\rightarrow$ autocorrelation can be negligible in our data sets

Back Up: Renormalized Polyakov loop

- It is believed that universality class of pure SU(2) is same as that of 3-D Ising model
- Renormalization condition[13] $L_R(T = 1.76\,Tc) = 0.894$



- Critical exponent (0.3265(3) in 3-D Ising model)
  - $N_\tau = 10$: 0.159(3)
  - $N_\tau = 12$: 0.242(3)

[13] S. Borsanyi *et al.*, Phys. Lett. B **713** (2012) 342.