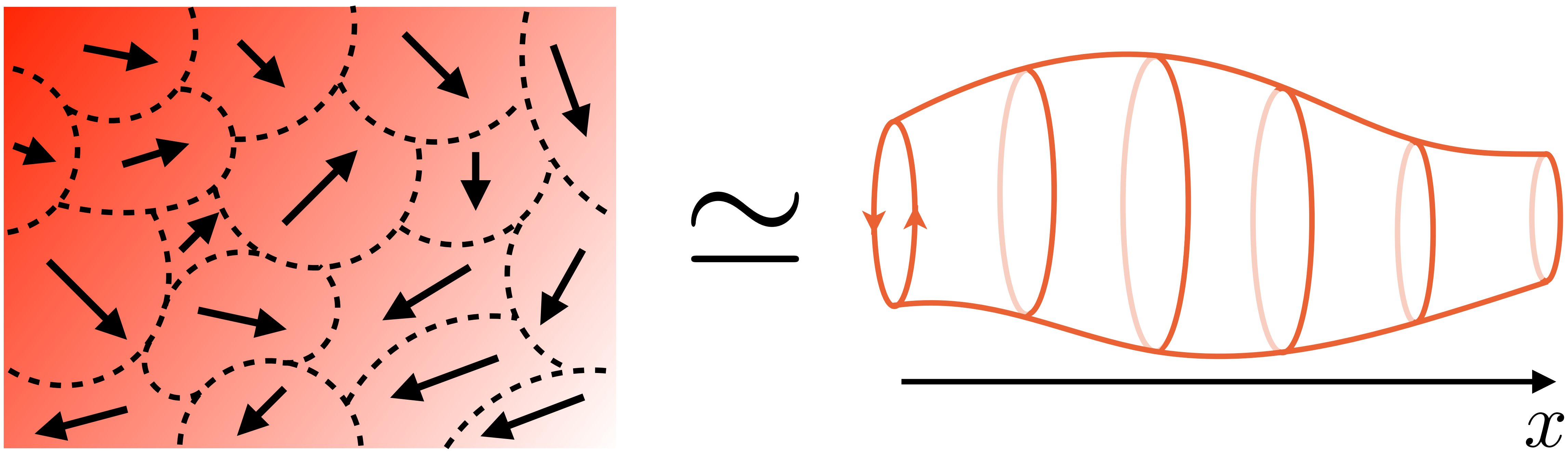
Path-integral formula for local thermal equilibrium



Masaru Hongo

RIKEN, iTHEMS program

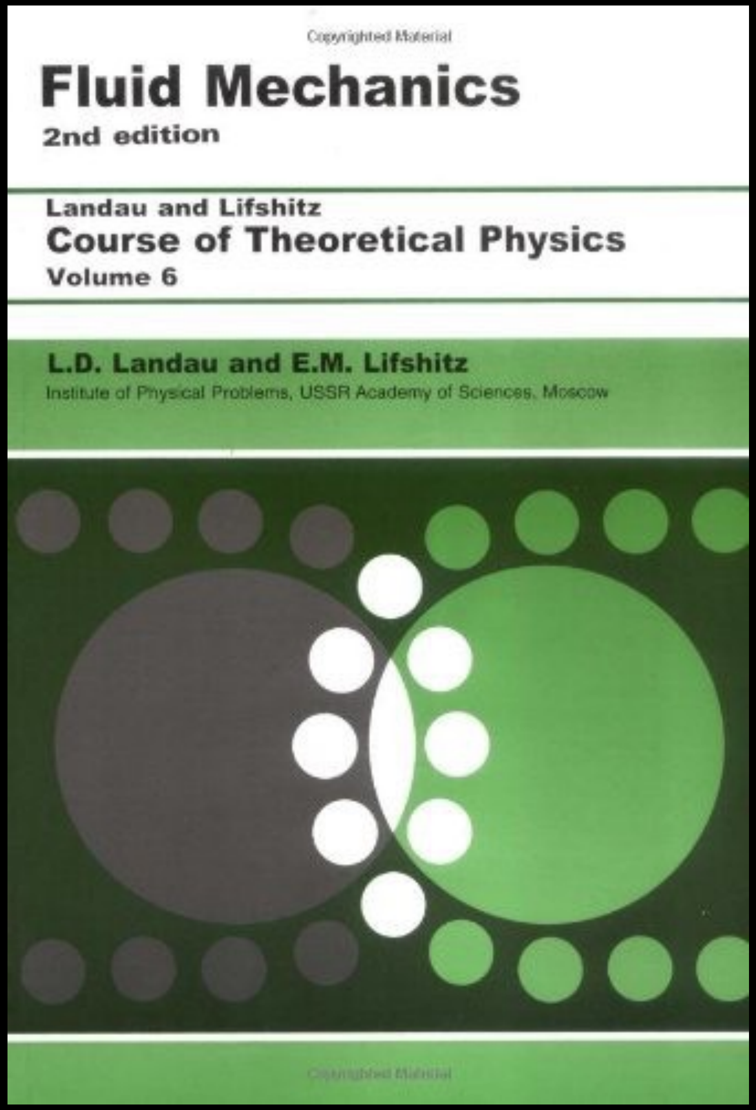
New Frontiers in QCD 2018, **2018 6/8, YITP**

Based on My Ph. D thesis
Hayata-Hidaka-MH-Noumi PRD(2015), MH Annals of Physics (2017)

Today's main Question

Q. Why $T^{\mu\nu} = (e+p)u^\mu u^\nu + pg^{\mu\nu} + \cdots$?

Answer 1.



Answer2. My talk + Challenge to audience

Outline

MOTIVATION:

Quantum field theory under
local thermal equilibrium?

APPROACH:

QFT for Local Gibbs distribution

APPLICATION:

Derivation of
Anomalous hydrodynamics

Motivation

Microscopic

$$\mathcal{L}_{\rm QCD}$$

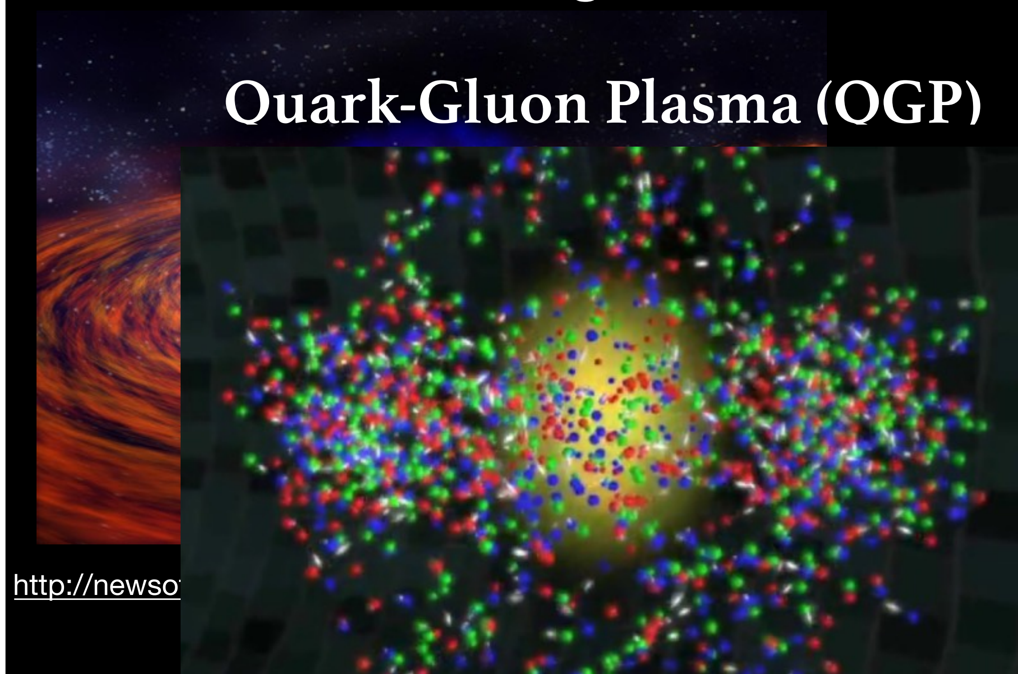
QFT

d.o.f.

Quark, Gluon

Neutron Star (Magnetar)

Quark-Gluon Plasma (QGP)



http://newso

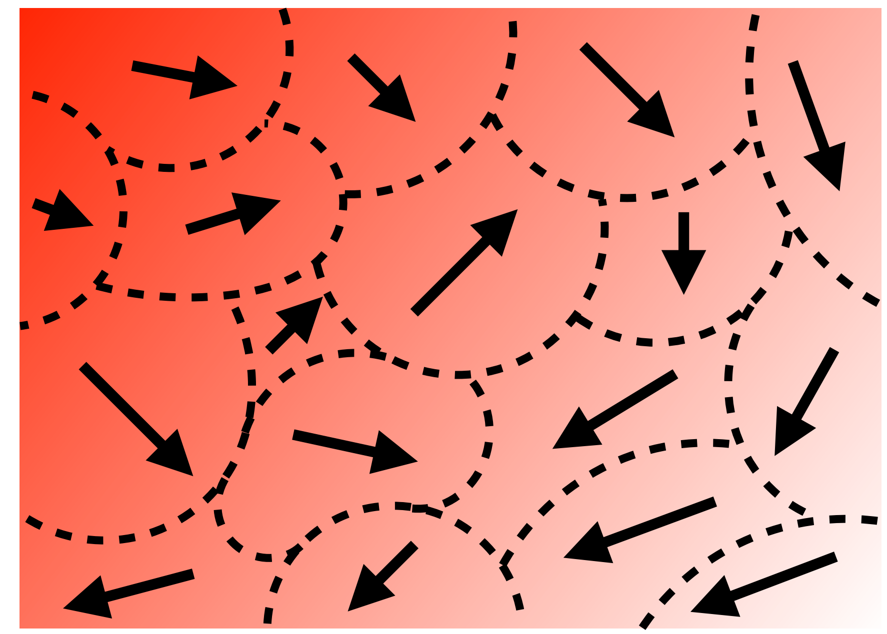
http://www.bnl.gov/rhic/news2/news.asp?a=1403&t=pr

?

Question.

How to bridge the gap between micro and macro?

Macroscopic



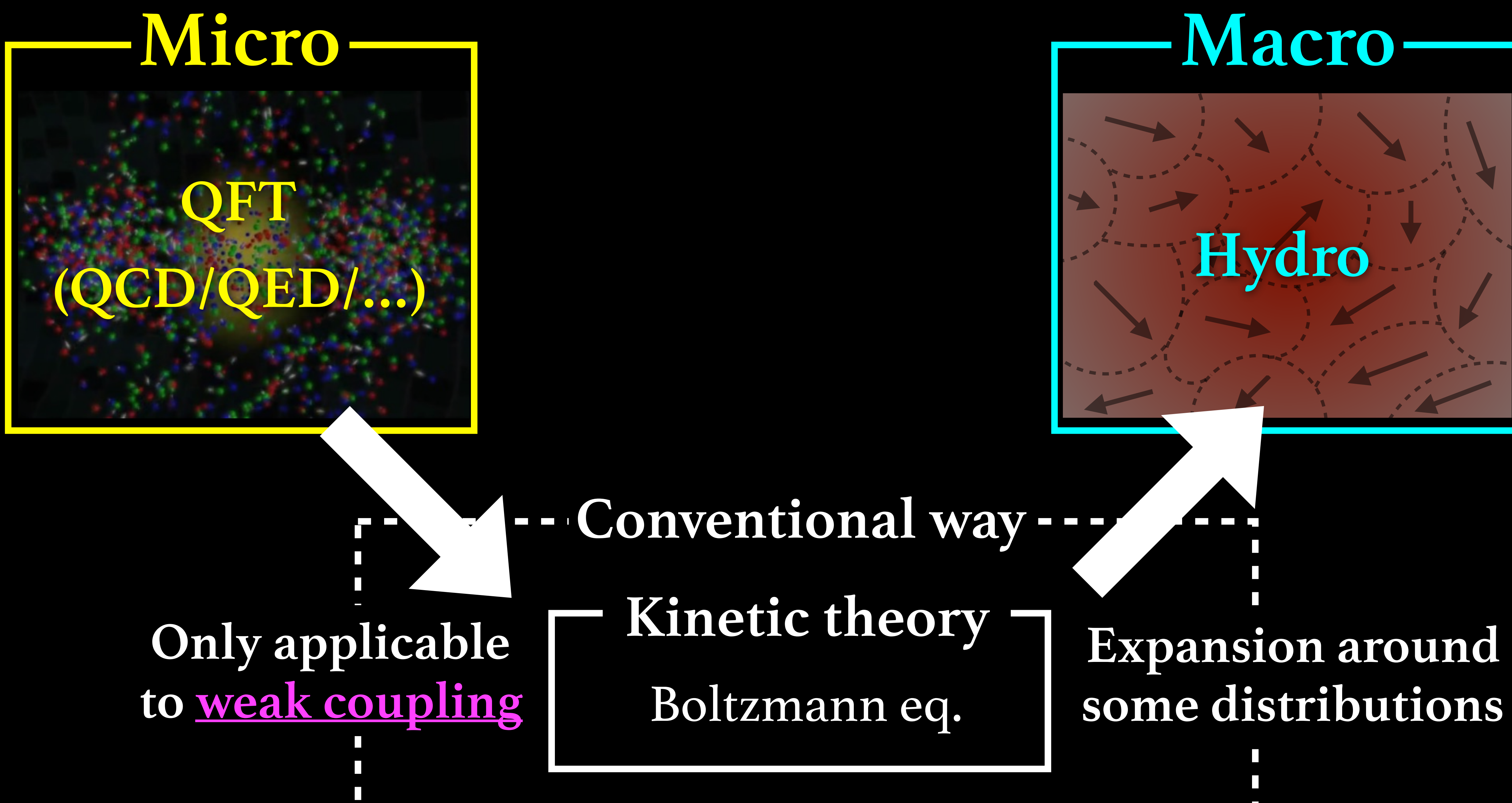
Hydrodynamics

- Haehl et al. (2015)
- Harder et al. (2015)
- Crossley et al. (2015)

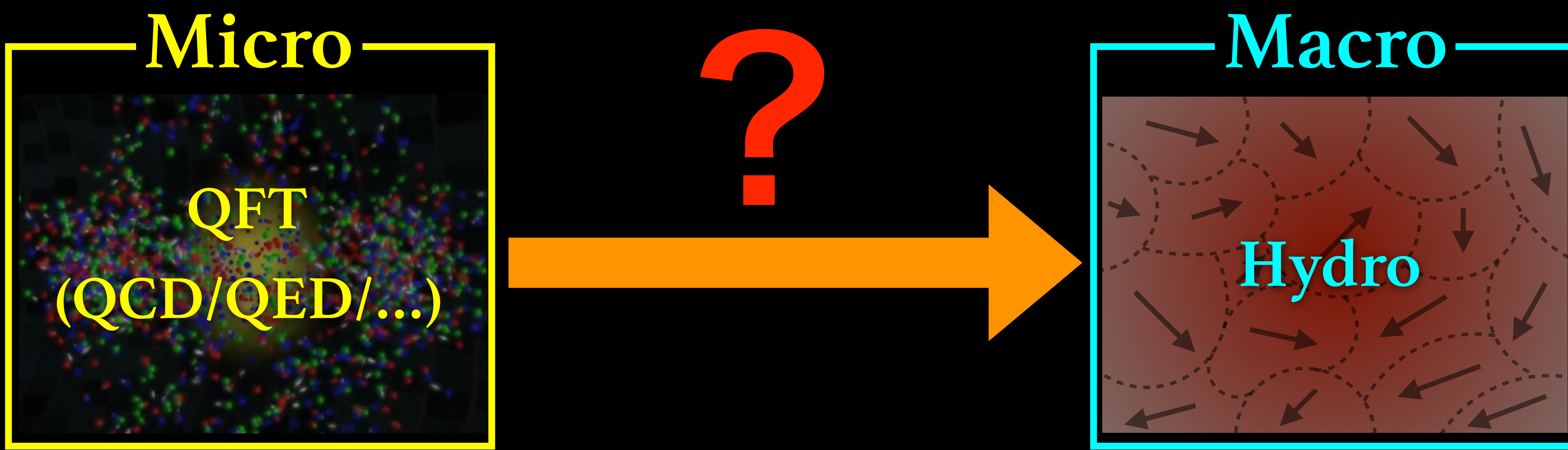
d.o.f.

$T(x),\ \vec{v}(x),\ \mu(x)$

How to construct hydrodynamics



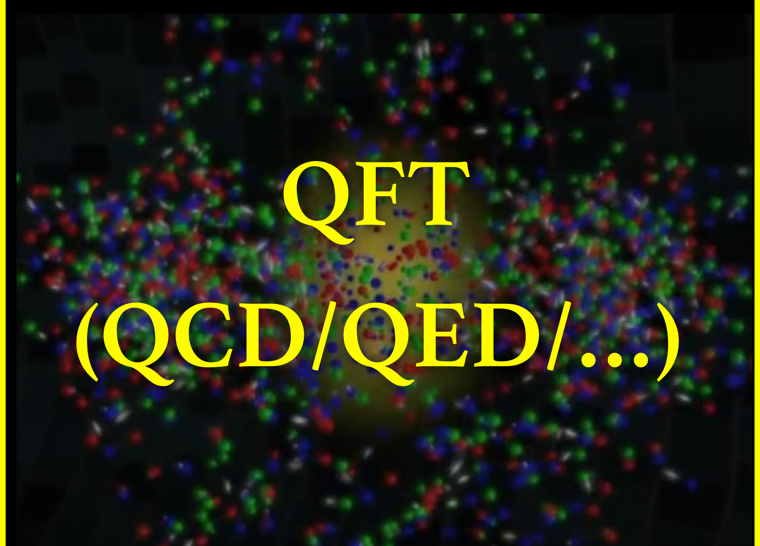
How to construct hydrodynamics



How to construct hydrodynamics

Nakajima (1957), Mori (1958), McLennan (1960)
Zubarev et al. (1979), Becattini et al. (2015)
Hayata-Hidaka-MH-Noumi (2015)

Micro

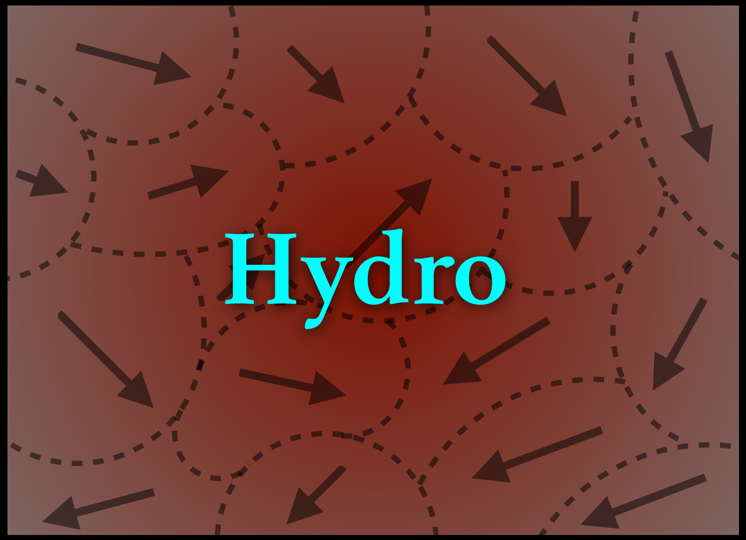


Local Thermal equil.
+ Small deviation

Also applicable to
strong coupling

Controllable
EOS, Kubo formula, ...

Macro



Thermal QFT in a Nutshell

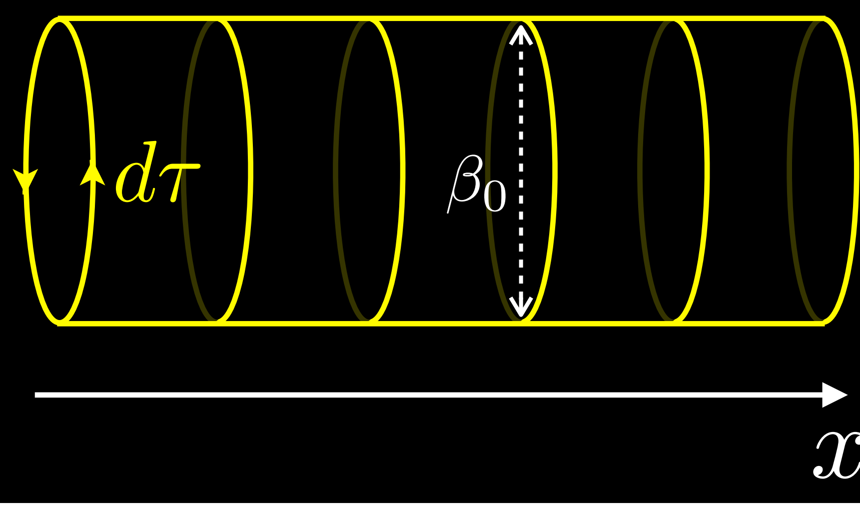
Global equil. β_0

$$T = \text{const.}$$

Path int.

Thermal QFT (Matsubara formalism)

[Matsubara, 1955]



QFT in the flat spacetime with size β_0

Gibbs dist.: $\hat{\rho}_G = \dfrac{e^{-\beta(\hat{H}-\mu\hat{N})}}{Z} = e^{-\beta(\hat{H}-\mu\hat{N})-\Psi[\beta,\nu]}$

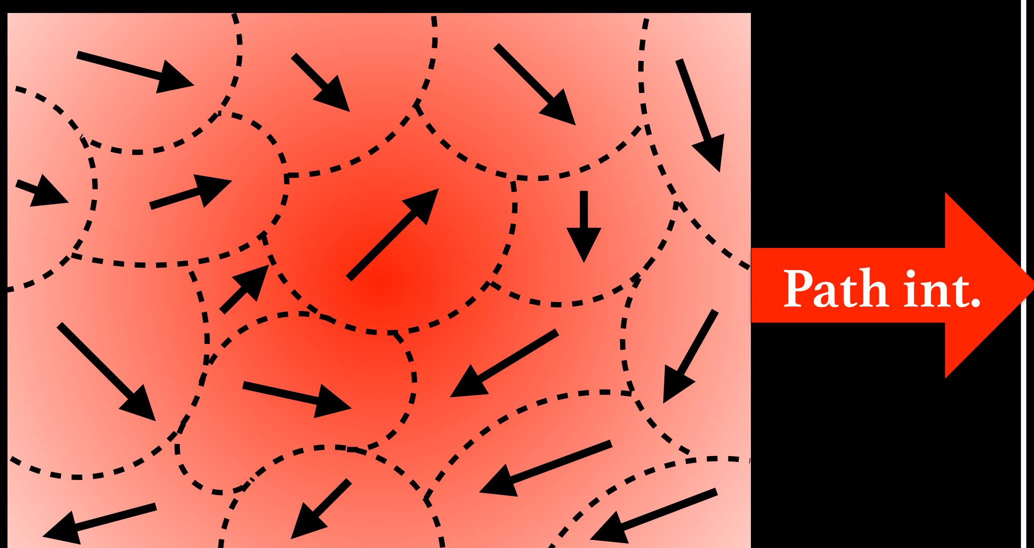
Thermodynamic potential with Euclidean action

$$\Psi[\beta,\nu] = \log \operatorname{Tr} e^{-\beta(\hat{H}-\mu\hat{N})} = \log \int d\varphi \langle \pm\varphi | e^{-\beta(\hat{H}-\mu\hat{N})} | \varphi \rangle$$

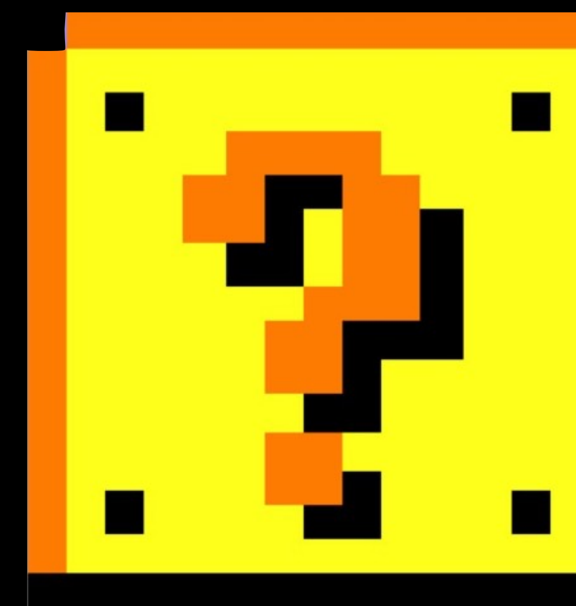
$$= \log \int_{\varphi(\beta)=\pm\varphi(0)} \mathcal{D}\varphi \, e^{+S_E[\varphi]}, \quad S_E[\varphi] = \int_0^\beta d\tau \int d^3x \, \mathcal{L}_E(\varphi, \partial_\mu \varphi)$$

QFT for local thermal equilibrium?

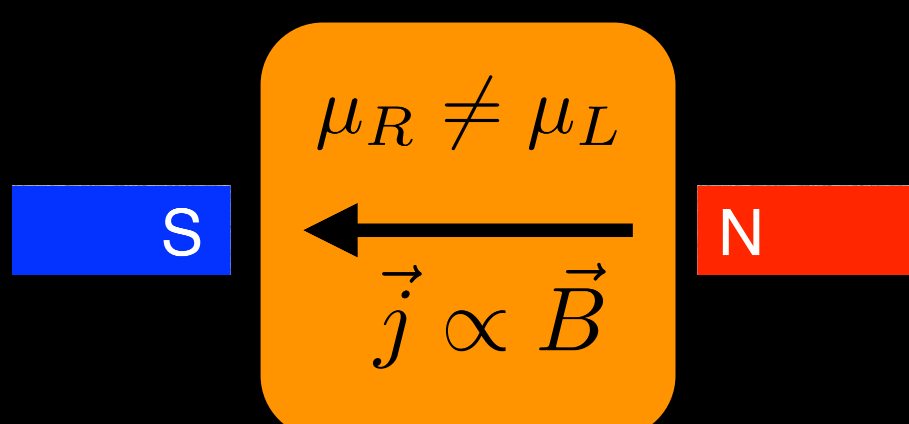
Local equil. $\{\beta(x),\ \vec{v}(x)\}$



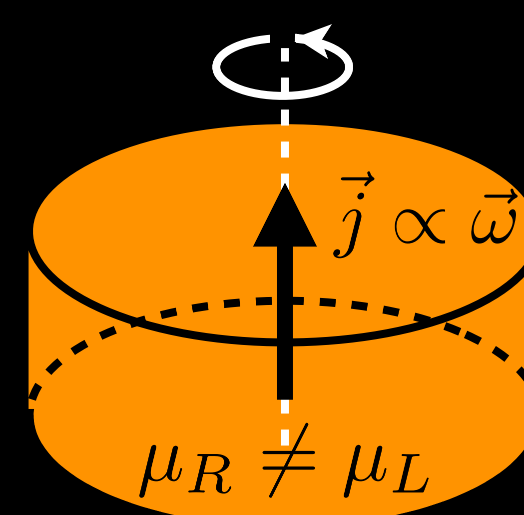
Local Thermal QFT



Local thermal QFT can describe anomaly-induced transport



Chiral Magnetic Effect

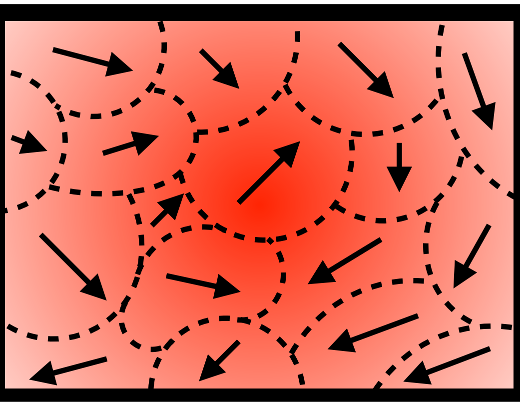


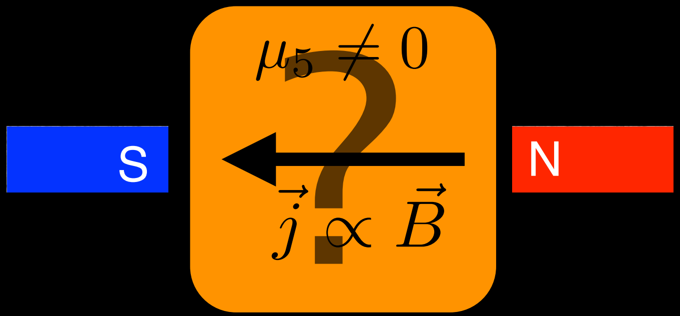
Chiral Vortical Effect

Outline

MOTIVATION:

Quantum field theory under
local thermal equilibrium?





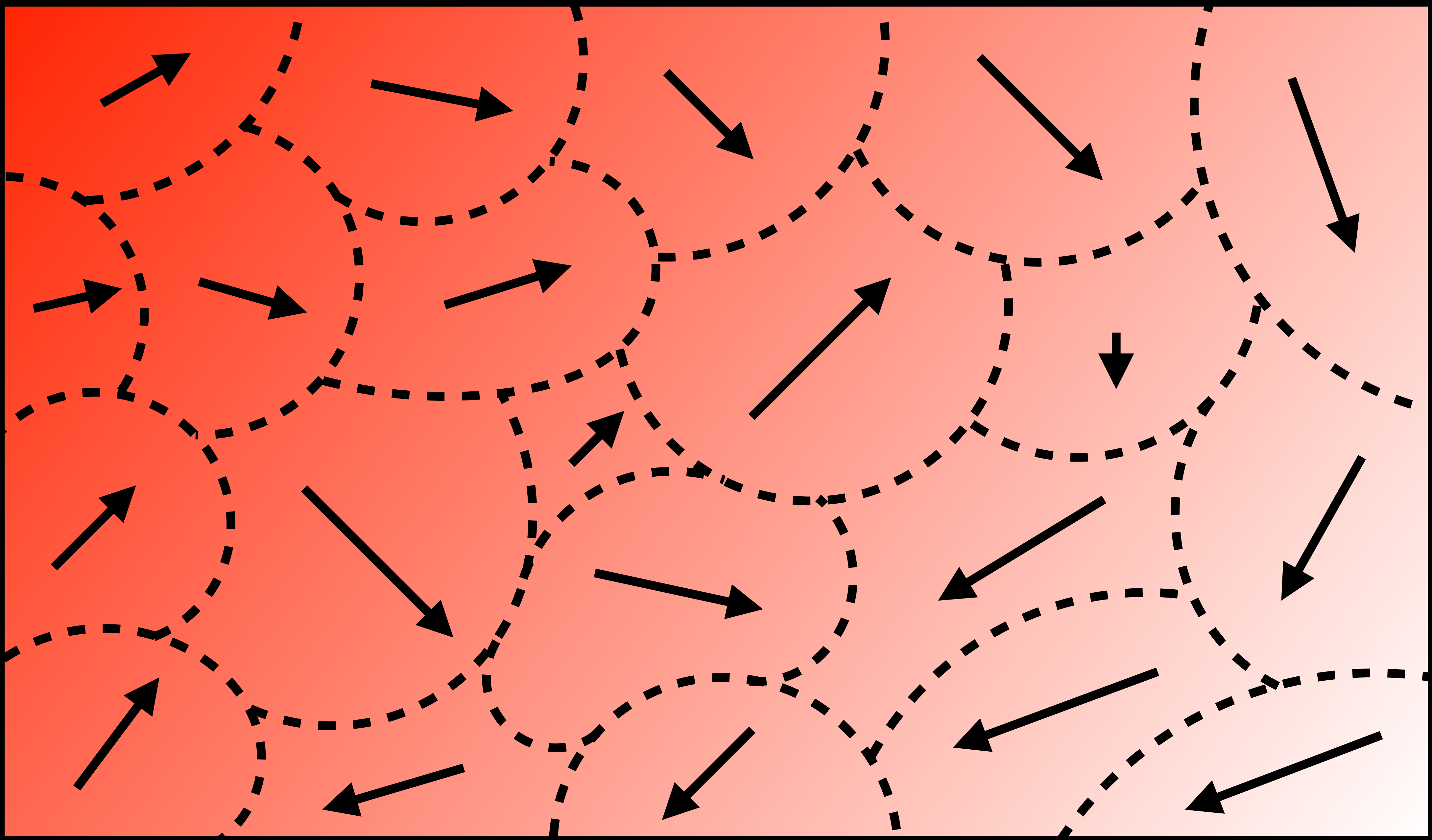
APPROACH:

QFT for Local Gibbs distribution

APPLICATION:

Derivation of
Anomalous hydrodynamics

Local thermal equilibrium



Determined only by **local temperature, local velocity…** at that time

How to describe local thermal equil.

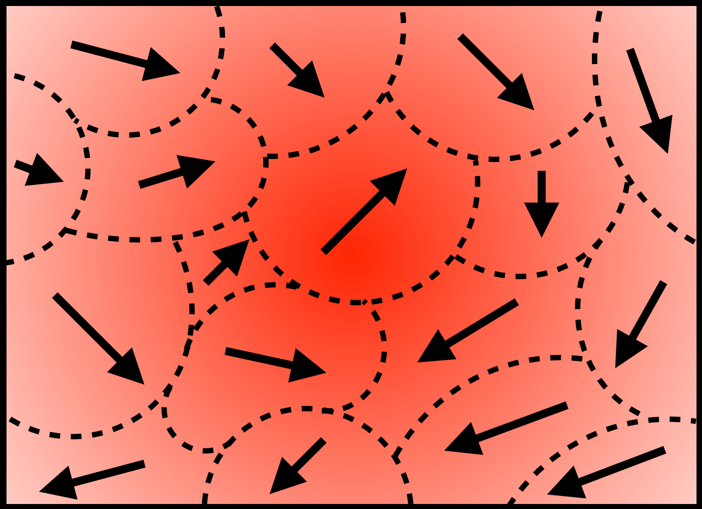
$T = \text{const.}$

Global thermal equilibrium:

Gibbs distribution:

$$\hat{\rho}_G = e^{-\beta \hat{H} - \Psi[\beta]}, \quad \Psi[\beta] \equiv \log \mathrm{Tr} e^{-\beta \hat{H}}$$

Localize



$\{\beta(x),\ \vec{v}(x)\}$

Local thermal equilibrium:

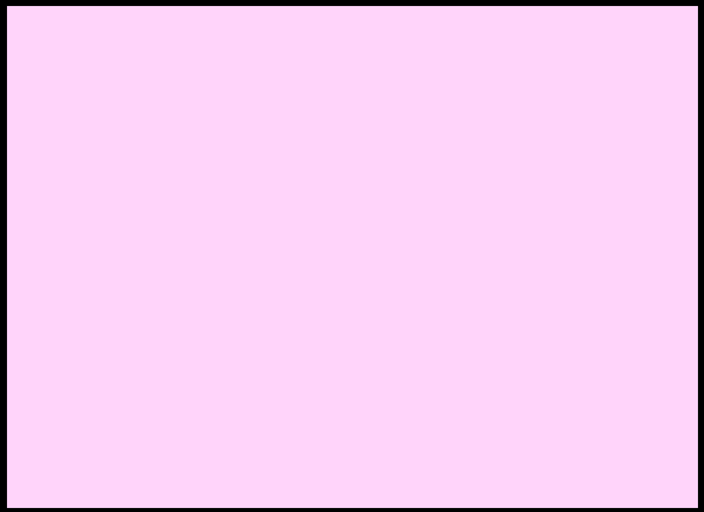
Local Gibbs (LG) distribution:

$$\hat{\rho}_{LG} = e^{-\hat{K} - \Psi[\beta^\mu(x), \nu(x)]}$$

$$\hat{K} = -\int d^3x \left(\beta^\mu(\boldsymbol{x}) \hat{T}^0{}_\mu(\boldsymbol{x}) + \nu(\boldsymbol{x}) \hat{J}^0(\boldsymbol{x}) \right)$$

What is Local Gibbs distribution?

Gibbs distribution



What is the state with maximizing information entropy: $S(\hat{\rho}) = -\mathrm{Tr}\hat{\rho}\log\hat{\rho}$

under constraints:

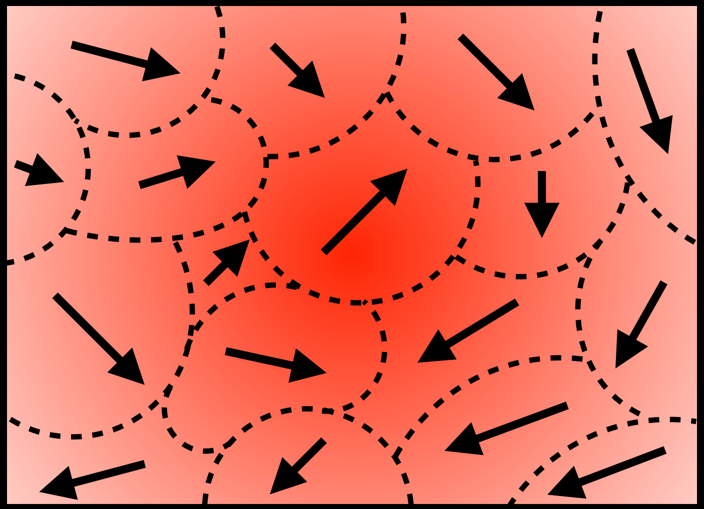
$\langle\hat{H}\rangle = E = \mathrm{const.},\ \langle\hat{N}\rangle = N = \mathrm{const.}$

Answer:

$$\hat{\rho}_{\mathrm{G}} = e^{-\beta\hat{H}-\nu\hat{N}-\Psi[\beta,\nu]}$$

Lagrange multipliers: $\Lambda^a = \{\beta, \nu = \beta\mu\}$

Local Gibbs distribution



What is the state with maximizing information entropy: $S(\hat{\rho}) = -\mathrm{Tr}\hat{\rho}\log\hat{\rho}$

under constraints:

$\langle\hat{T}^0{}_\mu(x)\rangle = p_\mu(x),\ \langle\hat{J}^0(x)\rangle = n(x)$

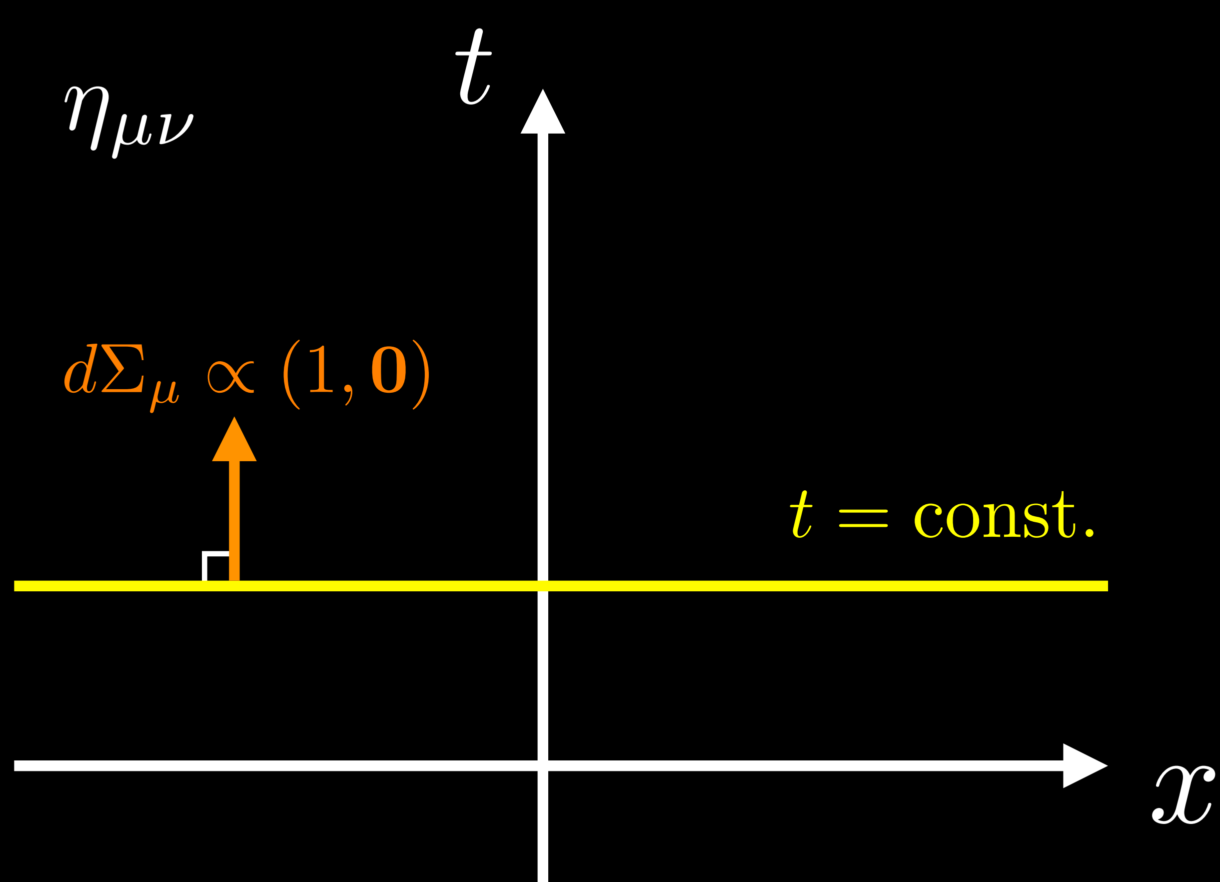
Answer:

$$\hat{\rho}_{\mathrm{LG}} = e^{-\int d^{d-1}x(\beta^\mu\hat{T}^0{}_\mu+\nu\hat{J}^0)-\Psi[\beta^\mu,\nu]}$$

Lagrange multipliers: $\lambda^a(x) = \{\beta^\mu(x), \nu(x)\}$

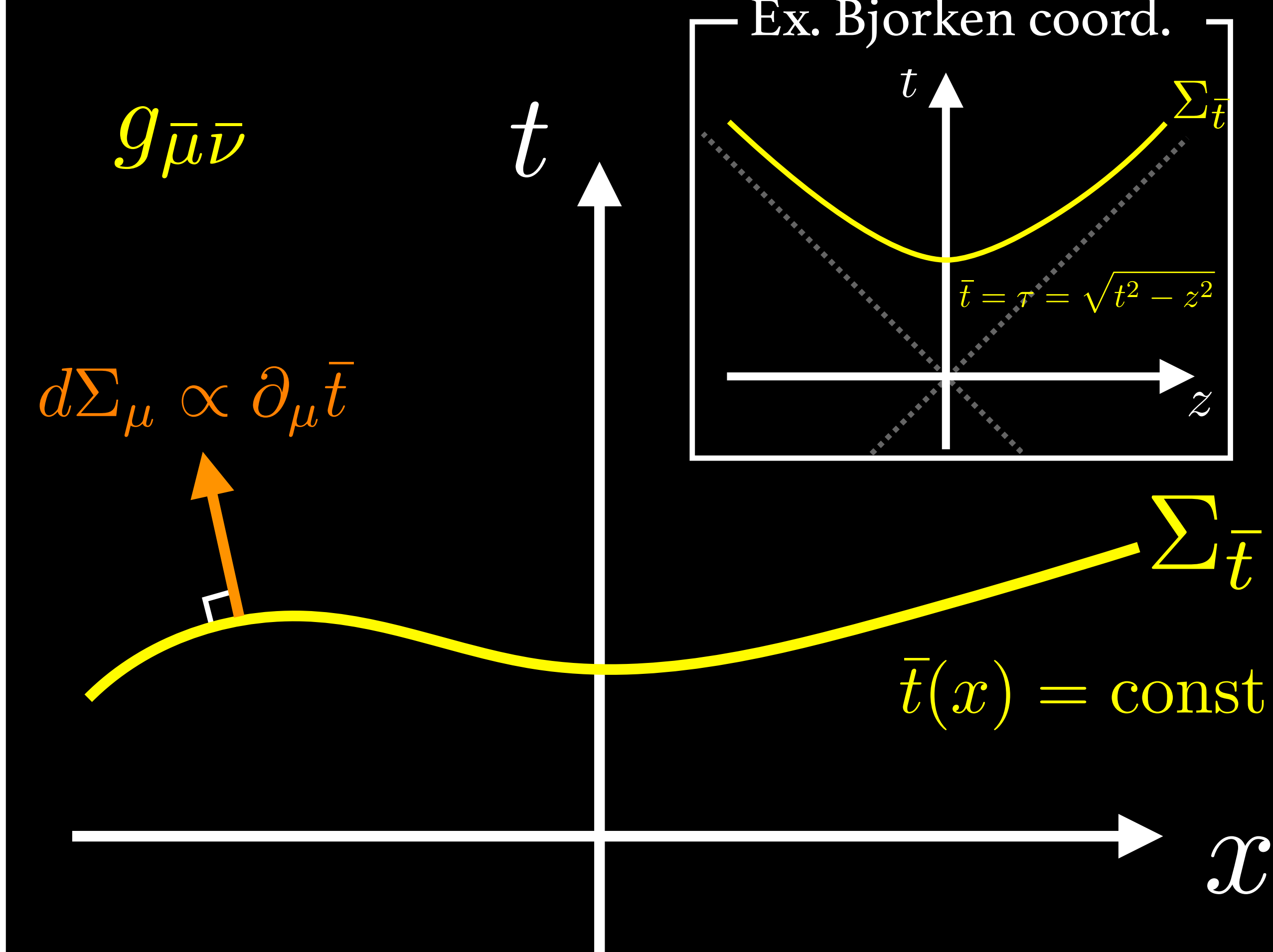
Introducing background metric

Flat spacetime



$$\hat{K} = -\int d^3x \left(\beta^\mu(\boldsymbol{x}) \hat{T}^0{}_\mu(\boldsymbol{x}) + \nu(\boldsymbol{x}) \hat{J}^0(\boldsymbol{x}) \right)$$

Curved spacetime

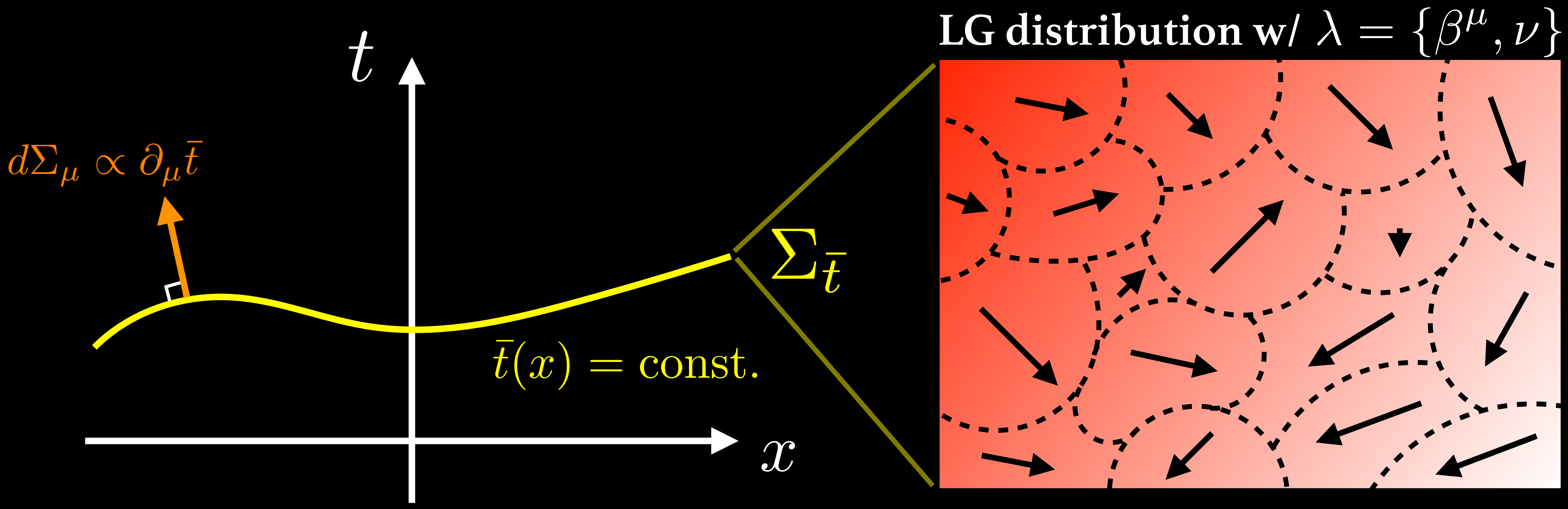


$$\hat{K} = -\int d\Sigma_{\bar{t}\nu} \left(\beta^\mu(x) \hat{T}^\nu{}_\mu(x) + \nu(x) \hat{J}^\nu(x) \right)$$

→ ① Formulation becomes manifestly covariant

② Background metric plays a role as external field coupled to $T^{\mu\nu}$

(Local) Thermodynamic Potential



Masseiu-Planck functional

$$\Psi[\bar{t};\lambda] \equiv \log \mathrm{Tr} \exp \left[\int d\Sigma_{\bar{t}\nu} \left(\beta^\mu(x) \hat{T}^\nu{}_\mu(x) + \nu(x) \hat{J}^\nu(x) \right) \right]$$

$$= \log \mathrm{Tr} \exp \left[- \int d^3\bar{x} \sqrt{-g} \left(\beta^{\bar{\mu}}(\bar{x}) \hat{T}^{\bar{0}}{}_{\bar{\mu}}(\bar{x}) + \nu(\bar{x}) \hat{J}^{\bar{0}}(\bar{x}) \right) \right]$$

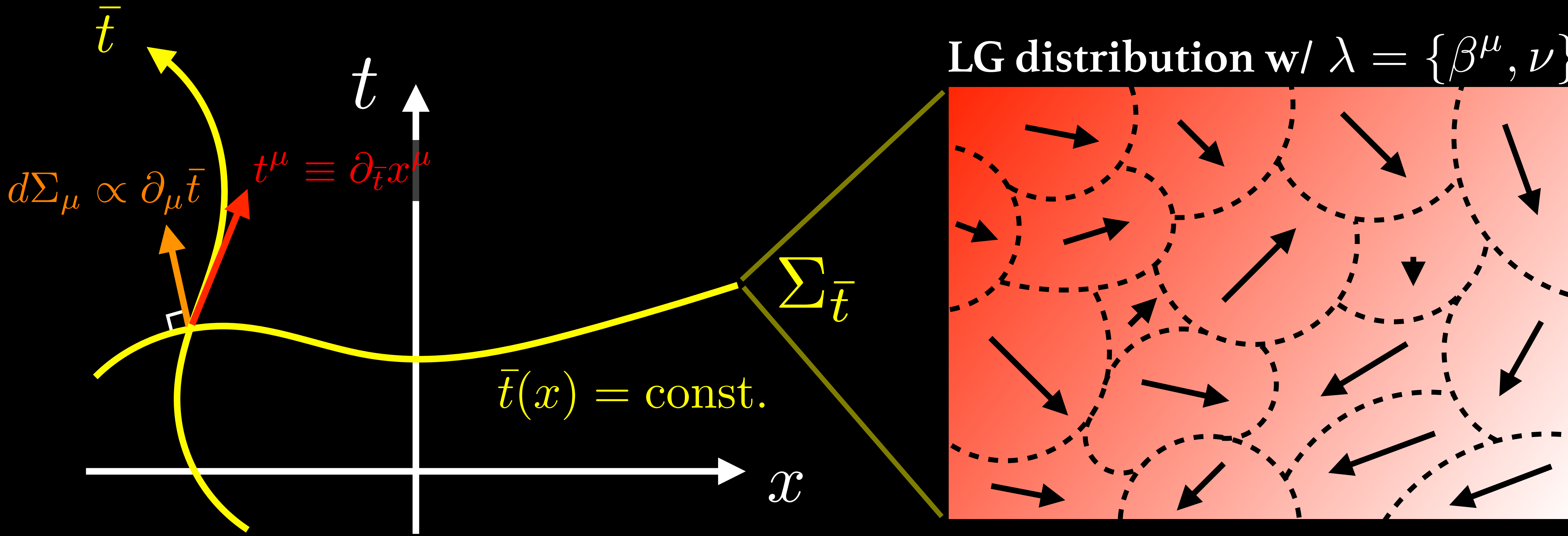
Variation formula for local equil.

[Banerjee et al.(2012), Jensen et al.(2012) , Haehl et al. (2015), MH(2016)]

Variation formula in "hydrostatic gauge"

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}(x)} \Psi[\bar{t};\lambda], \quad \langle \hat{J}^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{1}{\sqrt{-g}} \frac{\delta}{\delta A_{\mu}(x)} \Psi[\bar{t};\lambda]$$

(Local) Thermodynamic Potential

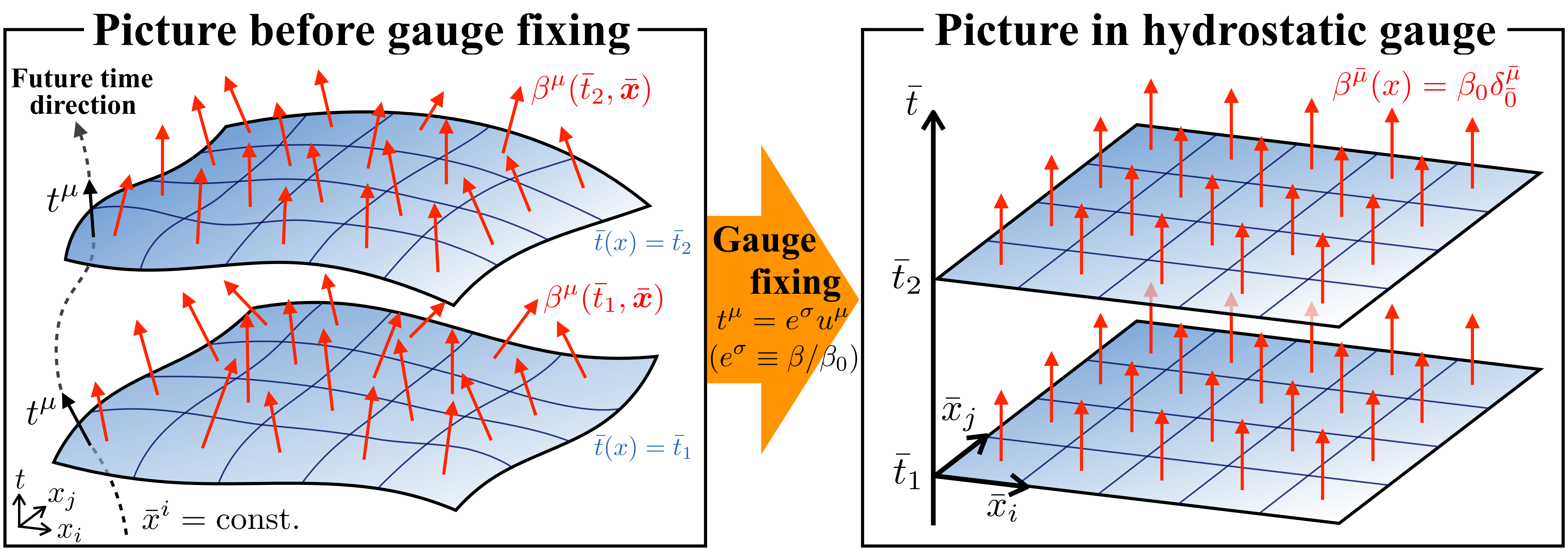


Masseiu-Planck functional

$$\Psi[\bar{t};\lambda] \equiv \log \mathrm{Tr} \exp \left[\int d\Sigma_{\bar{t}\nu} \left(\beta^\mu(x) \hat{T}^\nu{}_\mu(x) + \nu(x) \hat{J}^\nu(x) \right) \right]$$

$$= \log \mathrm{Tr} \exp \left[- \int d^3\bar{x} \sqrt{-g} \left(\beta^{\bar{\mu}}(\bar{x}) \hat{T}^{\bar{0}}{}_{\bar{\mu}}(\bar{x}) + \nu(\bar{x}) \hat{J}^{\bar{0}}(\bar{x}) \right) \right]$$

Hydrostatic gauge fixing



We can choose the time direction vector $t^\mu(x) \equiv \partial_{\bar{t}} x^\mu$

Hydrostatic gauge fixing

Let us choose $t^\mu(x) = \beta^\mu(x)/\beta_0, \ A_{\bar{0}}(x) = \nu(x)$

Variation formula for local equil.

[Banerjee et al.(2012), Jensen et al.(2012) , Haehl et al. (2015), MH(2016)]

Variation formula in "hydrostatic gauge"

$$\langle \hat{T}^{\mu\nu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g_{\mu\nu}(x)}\Psi[\bar{t};\lambda], \quad \langle \hat{J}^{\mu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = \frac{1}{\sqrt{-g}}\frac{\delta}{\delta A_{\mu}(x)}\Psi[\bar{t};\lambda]$$

Proof. Consider time derivative of $\Psi[\lambda]$

$$\partial_{\bar{t}}\Psi[\bar{t};\lambda] = \int d^{d-1}\bar{x}\sqrt{-g}\left(\nabla_\mu\beta_\nu\langle\hat{T}^{\mu\nu}\rangle_{\bar{t}}^{\mathrm{LG}} + (\nabla_\mu\nu + F_{\nu\mu}\beta^\nu)\langle\hat{J}^\mu\rangle_{\bar{t}}\right)$$

$$= \int d^{d-1}\bar{x}\sqrt{-g}\left(\frac{1}{2}(\nabla_\mu\beta_\nu + \nabla_\nu\beta_\mu)\langle\hat{T}^{\mu\nu}\rangle_{\bar{t}}^{\mathrm{LG}} + (\beta^\nu\nabla_\nu A_\mu + A_\nu\nabla_\mu\beta^\nu)\langle\hat{J}^\mu\rangle_{\bar{t}}\right)$$

$$= \int d^{d-1}\bar{x}\sqrt{-g}\left(\frac{1}{2}\pounds_\beta g_{\mu\nu}\langle\hat{T}^{\mu\nu}\rangle_{\bar{t}}^{\mathrm{LG}} + \pounds_\beta A_\mu\langle\hat{J}^\mu\rangle_{\bar{t}}\right)$$

On the other hand, since $t^\mu = \beta^\mu$, we can express the LHS as

$$\partial_{\bar{t}}\Psi[\bar{t};\lambda] = \int d^{d-1}\bar{x}\left(\pounds_\beta g_{\mu\nu}\frac{\delta\Psi}{\delta g_{\mu\nu}} + \pounds_\beta A_\mu\frac{\delta\Psi}{\delta A_\mu}\right)$$

Matching them gives the above variation formula! □

Q. How can we calculate $\Psi \equiv \log Z$?

Case study 1: Scalar field

$$\mathcal{L} = -\frac{g^{\bar{\mu}\bar{\nu}}}{2}\partial_{\bar{\mu}}\phi\partial_{\bar{\nu}}\phi - V(\phi)$$

$$\Rightarrow \quad \hat{T}^{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}} = \partial^\mu\hat{\phi}\partial^\nu\hat{\phi} + g^{\mu\nu}\mathcal{L}(\hat{\phi}, \partial_\rho\hat{\phi})$$

$$\begin{aligned}\Psi[\bar{t};\lambda] &= \log \mathrm{Tr}\exp\left[-\int d^{d-1}\bar{x}\sqrt{-g}\beta^\mu(x)\hat{T}^{\bar{0}}{}_\mu(x)\right] \\ &= \log\int \mathcal{D}\phi\exp\left(S_E[\phi,\beta^\mu]\right) = \log\int\mathcal{D}\phi\exp\left(S_E[\phi,\tilde{g}]\right)\end{aligned}$$

$$S[\phi,\beta^\mu] = \int_0^{\beta_0} d\tau\int d^3\bar{x}\sqrt{-g}e^\sigma u^{\bar{0}}\left[-\frac{e^{-2\sigma}}{2u^{\bar{0}}u_{\bar{0}}}(i\dot{\phi})^2 - \frac{-e^{-\sigma}u^{\bar{i}}}{u^{\bar{0}}u_{\bar{0}}}(i\dot{\phi})\partial_{\bar{i}}\phi - \frac{1}{2}\left(\gamma^{\bar{i}\bar{j}} + \frac{u^{\bar{i}}u^{\bar{j}}}{u^{\bar{0}}u_{\bar{0}}}\right)\partial_{\bar{i}}\phi\partial_{\bar{j}}\phi - V(\phi)\right]$$

$$= \int_0^{\beta_0} d\tau\int d^3\bar{x}\sqrt{-\tilde{g}}\left[-\frac{\tilde{g}^{\bar{\mu}\bar{\nu}}}{2}\partial_{\bar{\mu}}\phi\partial_{\bar{\nu}}\phi - V(\phi)\right] \qquad \left(e^{\sigma(\bar{x})} \equiv \beta(\bar{x})/\beta_0\right)$$

ψ in terms of thermal metric

$$\Psi[\bar{t};\lambda] = \log \int \mathcal{D}\phi \exp\left(S_E[\phi, ; \tilde{g}]\right)$$

Thermal metric

$$\tilde{g}_{\bar{\mu}\bar{\nu}} = \begin{pmatrix} -e^{2\sigma} & e^{\sigma} u_{\bar{j}} \\ e^{\sigma} u_{\bar{i}} & \gamma_{\bar{i}\bar{j}} \end{pmatrix}$$

$$(e^{\sigma(\bar{x})} \equiv \beta(\bar{x})/\beta_0)$$

Inverse thermal metric

$$\tilde{g}^{\bar{\mu}\bar{\nu}} = \begin{pmatrix} \dfrac{e^{-2\sigma}}{u^{\bar{0}} u_{\bar{0}}} & -\dfrac{e^{-\sigma} u^{\bar{j}}}{u^{\bar{0}} u_{\bar{0}}} \\ -\dfrac{e^{-\sigma} u^{\bar{i}}}{u^{\bar{0}} u_{\bar{0}}} & \gamma^{\bar{i}\bar{j}} + \dfrac{u^{\bar{i}} u^{\bar{j}}}{u^{\bar{0}} u_{\bar{0}}} \end{pmatrix}$$

◆ Interpretation of above result

$\Psi[\bar{t};\lambda]$ **is described by QFT in "curved spacetime" s. t.**

$$d\tilde{s}^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}} dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}} dx^{\bar{i}} dx^{\bar{j}}$$

$$(a_{\bar{i}} \equiv e^{-\sigma} u_{\bar{i}}, \quad \gamma'_{\bar{i}\bar{j}} \equiv \gamma_{\bar{i}\bar{j}} + u_{\bar{i}} u_{\bar{j}}, \quad d\tilde{t} = -i d\tau)$$

Case study 2: Dirac field

$$\mathcal{L} = -\frac{1}{2}\bar{\psi}\left(\gamma^a e_a{}^{\bar{\mu}} \overrightarrow{D}_{\bar{\mu}} - \overleftarrow{D}_{\bar{\mu}} \gamma^a e_a{}^{\bar{\mu}}\right)\psi - m\bar{\psi}\psi$$

Symmetric energy-momentum tensor

$$T^{\bar{\mu}}{}_{\bar{\nu}} = -\delta^{\bar{\mu}}_{\bar{\nu}}\mathcal{L} - \frac{1}{4}\bar{\psi}(\gamma^{\bar{\mu}}\overrightarrow{D}_{\bar{\nu}} + \gamma_{\bar{\nu}}\overrightarrow{D}^{\bar{\mu}} - \overleftarrow{D}_{\bar{\nu}}\gamma^{\bar{\mu}} - \overleftarrow{D}^{\bar{\mu}}\gamma_{\bar{\nu}})\psi$$

◆ **Result of path integral**

$$\Psi[\bar{t};\lambda] \equiv \log \mathrm{Tr} \exp\left[\int d\Sigma_{\bar{t}\nu}\left(\beta^\mu(x)\hat{T}^\nu{}_\mu(x) + \nu(x)\hat{J}^\nu(x)\right)\right]$$

$$= \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(S_E[\psi,\bar{\psi};\tilde{e}]\right)$$

ψ in terms of thermal vielbein

$$\Psi[\bar{t};\lambda] = \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(S_E[\psi,\bar{\psi};\tilde{e}]\right)$$

◆ **Euclidean action with thermal vielbein**

$$S_E[\psi,\bar{\psi};\tilde{e}] = \int_0^{\beta_0} d\tau \int d^3\bar{x}\tilde{e}\left[-\frac{1}{2}\bar{\psi}\left(\gamma^a \tilde{e}_a{}^{\bar{\mu}}\overrightarrow{D}_{\bar{\mu}} - \overleftarrow{D}_{\bar{\mu}}\gamma^a\tilde{e}_a{}^{\bar{\mu}}\right)\psi - m\bar{\psi}\psi\right]$$

Thermal vielbein : $\tilde{e}_{\bar{0}}{}^a = e^\sigma u^a, \ \tilde{e}_{\bar{i}}{}^a = e_{\bar{i}}{}^a \quad (e^\sigma \equiv \beta(x)/\beta_0)$

◆ **Interpretation of above result**

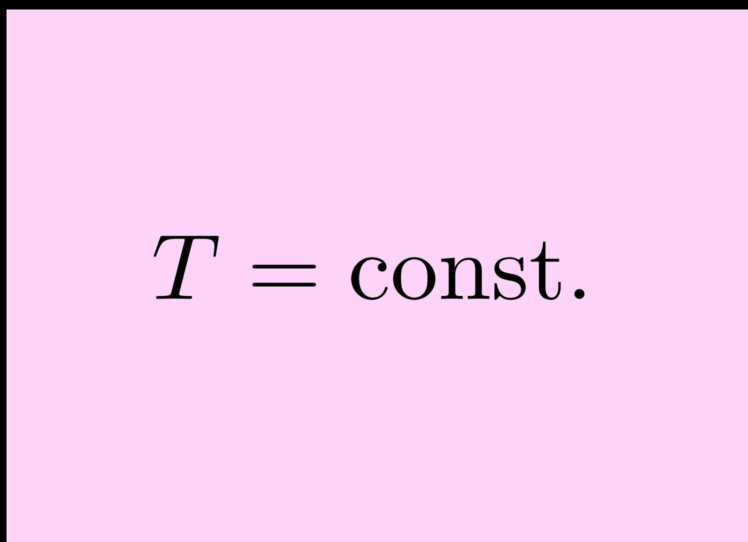
$\Psi[\bar{t};\lambda]$ **is described by QFT in "curved spacetime" s. t.**

$$d\tilde{s}^2 = \tilde{e}_{\bar{\mu}}{}^a\tilde{e}_{\bar{\nu}}{}^b\eta_{ab}dx^{\bar{\mu}}dx^{\bar{\nu}} = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}}dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}}dx^{\bar{i}}dx^{\bar{j}}$$

$$(a_{\bar{i}} \equiv e^{-\sigma}u_{\bar{i}}, \quad \gamma'_{\bar{i}\bar{j}} \equiv \gamma_{\bar{i}\bar{j}} + u_{\bar{i}}u_{\bar{j}}, \quad d\tilde{t} = -id\tau)$$

Local Thermal QFT

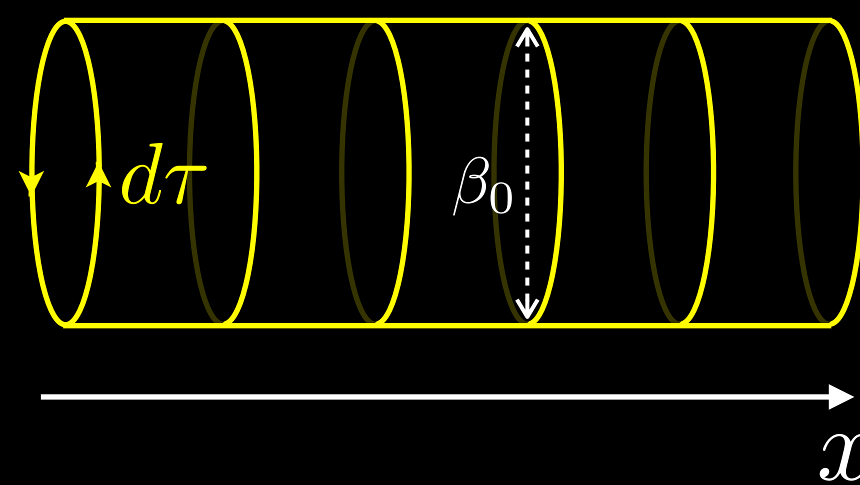
Global equil. β_0



Path int.

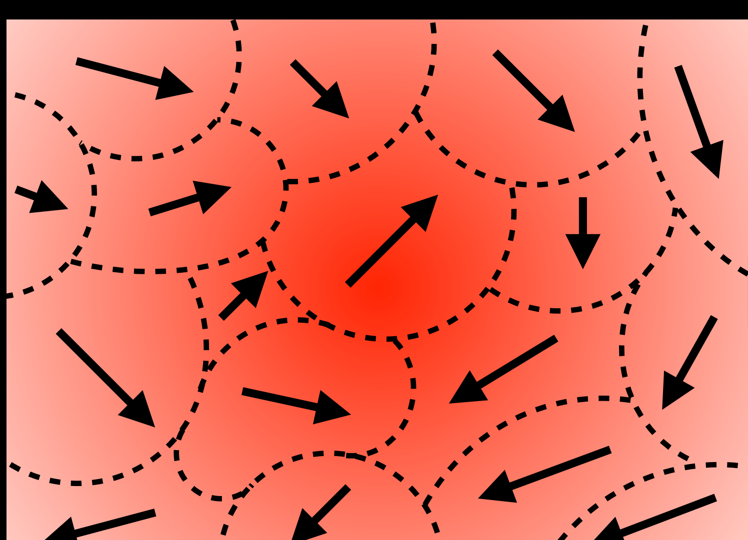
Thermal QFT (Matsubara formalism)

[Matsubara, 1955]



QFT in the flat spacetime with size β_0

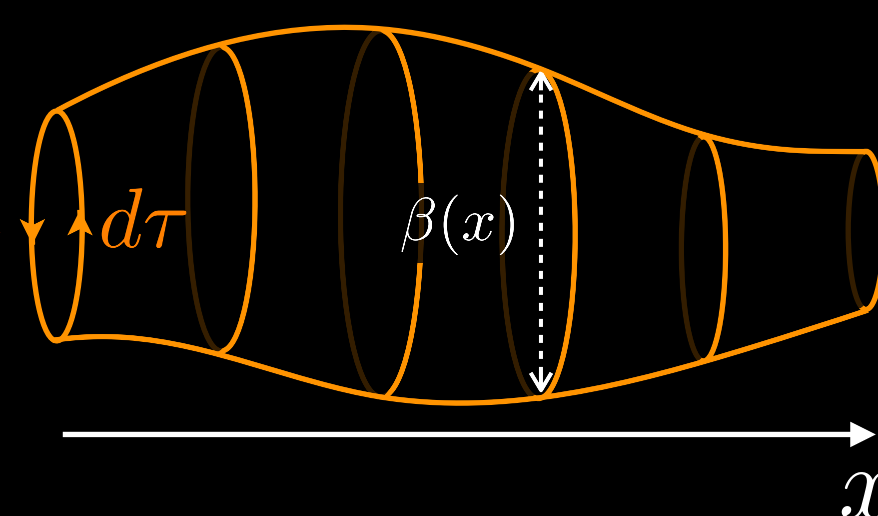
Local equil. $\{\beta(x),\ \vec{v}(x)\}$



Path int.

Local Thermal QFT

[Hayata-Hidaka-MH-Noumi PRD(2015)]
[MH (2017)]



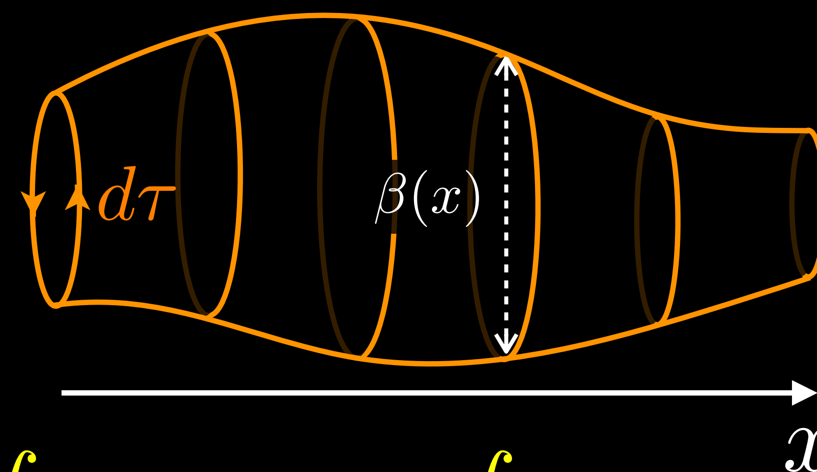
QFT in the "curved spacetime" with "line element"

$$d\tilde{s}^2 = d\tilde{s}^2(\beta, \vec{v})$$

Two ways to construct $\Psi \equiv \log Z$

1. Use diffeo & gauge invariance!

- Ψ is expressed by $\{\tilde{g}_{\mu\nu}, \tilde{A}_\mu\}$
- Ψ is diffeo & gauge invariant!



→ Ψ is expressed in terms of $\beta = \oint d\tilde{s},\ \beta\mu = \oint \tilde{A},\ \tilde{R},\ \tilde{F}_{\mu\nu}$

2. Use symmetry from imaginary-time nature!

- Ψ is spatial diffeomorphism invariant
- Ψ is Kaluza-Klein gauge invariant!

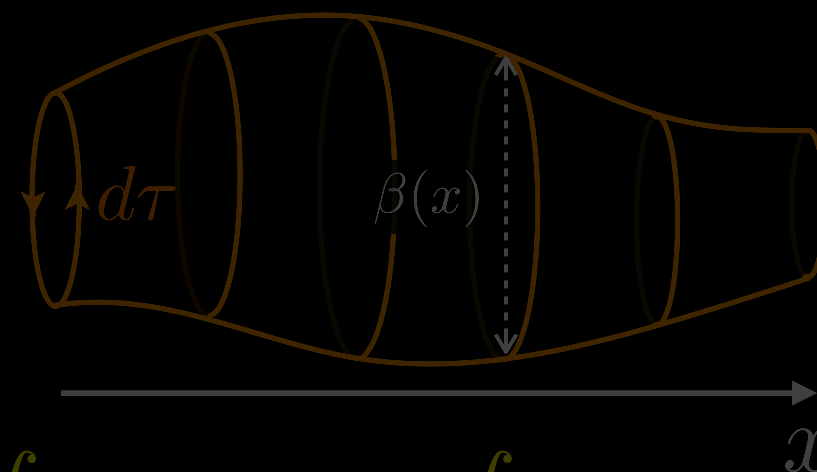
→ $\Psi \equiv \log Z$ should respect these two symmetries!!

[cf. Hydrostatic partition function method Banerjee et al.(2012), Jensen et al.(2012)]

Two ways to construct $\Psi \equiv \log Z$

1. Use diffeo & gauge invariance!

- Ψ is expressed by $\{\tilde{g}_{\mu\nu}, \tilde{A}_\mu\}$
- Ψ is diffeo & gauge invariant!



→ Ψ is expressed in terms of $\beta = \oint d\tilde{s},\ \beta\mu = \oint \tilde{A},\ \tilde{R},\ \tilde{F}_{\mu\nu}$

2. Use symmetry from imaginary-time nature!

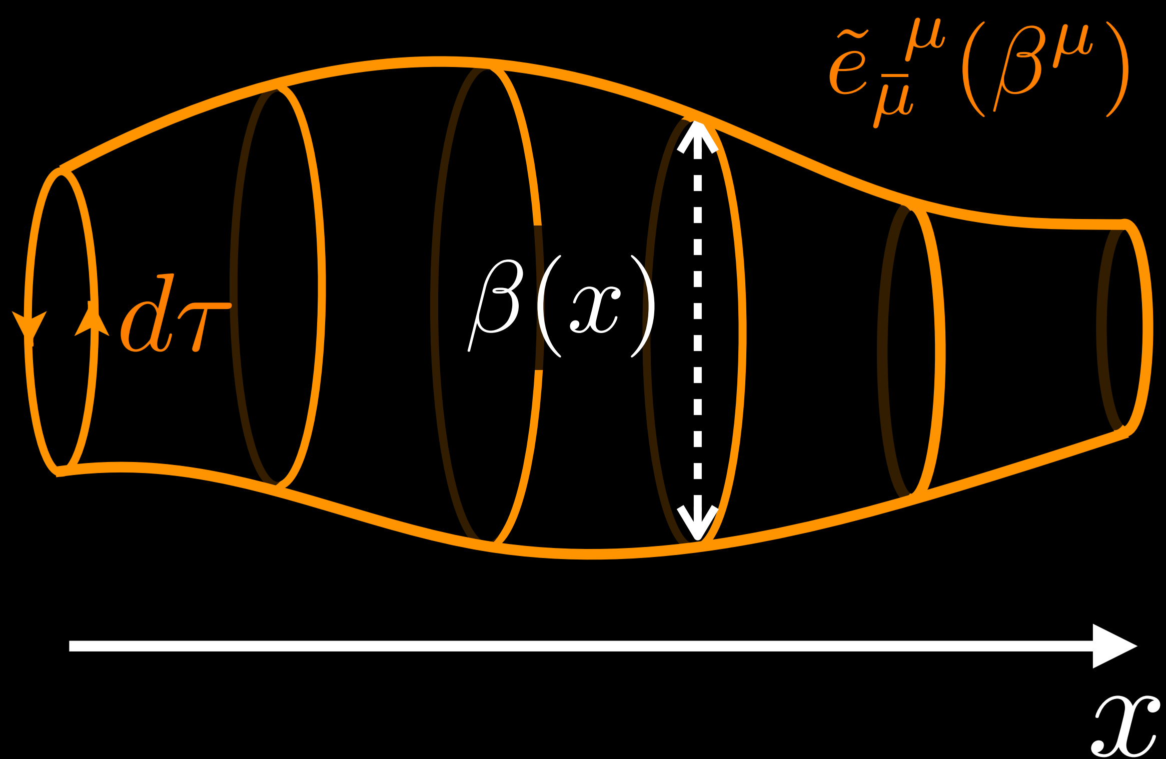
- Ψ is spatial diffeomorphism invariant
- Ψ is Kaluza-Klein gauge invariant!

→ $\Psi \equiv \log Z$ should respect these two symmetries!!

[cf. Hydrostatic partition function method Banerjee et al.(2012), Jensen et al.(2012)]

Kaluza-Klein gauge symmetry

$$d\tilde{s}^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}}dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}}dx^{\bar{i}}dx^{\bar{j}} \;\; (d\tilde{t} = -id\tau)$$



Parameters λ don't depend on imaginary time τ.

"Kaluza-Klein" gauge tr.

$$\begin{cases} \tilde{t} \to \tilde{t} + \chi(\bar{\boldsymbol{x}}) \\ \bar{\boldsymbol{x}} \to \bar{\boldsymbol{x}} \\ a_{\bar{i}}(\bar{\boldsymbol{x}}) \to a_{\bar{i}}(\bar{\boldsymbol{x}}) - \partial_{\bar{i}}\chi(\bar{\boldsymbol{x}}) \end{cases}$$

$$\Psi[\lambda] = \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{S[\psi,\bar{\psi},\tilde{e}]} \ni$$

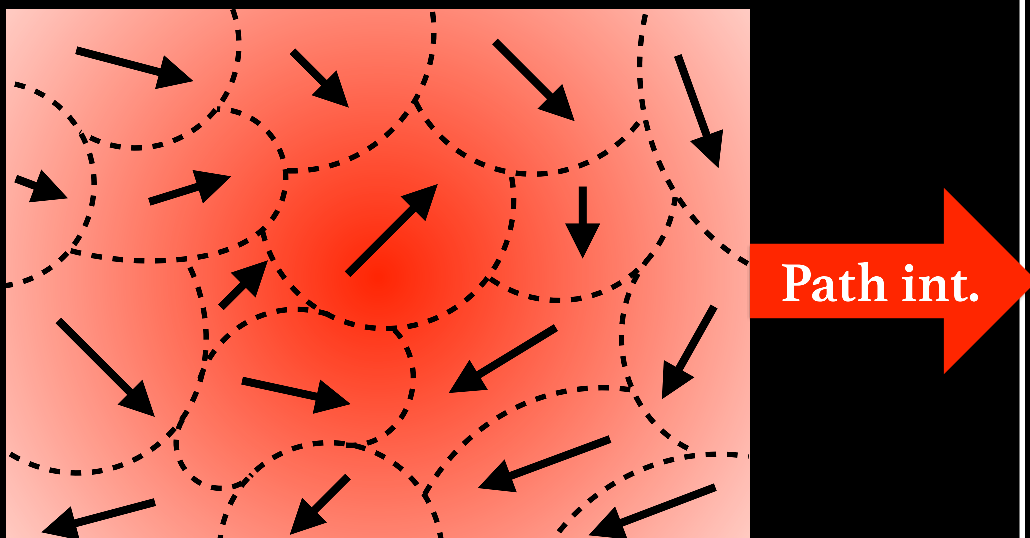
$$(f_{\bar{i}\bar{j}} \equiv \partial_{\bar{i}}a_{\bar{j}} - \partial_{\bar{j}}a_{\bar{i}})$$

 $f^{\bar{i}\bar{j}}f_{\bar{i}\bar{j}}, \cdots$

 $a_{\bar{i}},\;\; a_{\bar{i}}a^{\bar{i}}, \cdots$

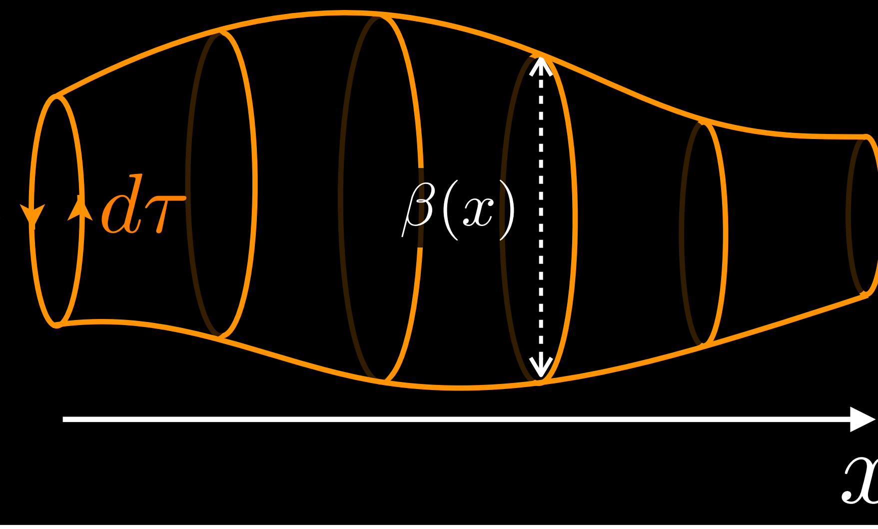
Short Summary: Local Thermal QFT

Local equil. $\{\beta(x),\ \vec{v}(x)\}$



Local Thermal QFT

[Hayata-Hidaka-MH-Noumi PRD(2015)]
[MH (2017)]



QFT in the "curved spacetime" with "line element"

$$d\tilde{s}^2 = d\tilde{s}^2(\beta, \vec{v})$$

$$\Psi[\bar{t};\lambda] \equiv \log \mathrm{Tr} \exp\left[\int d\Sigma_{\bar{t}\nu}\left(\beta^\mu(x)\hat{T}^\nu{}_\mu(x) + \nu(x)\hat{J}^\nu(x)\right)\right]$$

① $\Psi[\lambda]$ plays a role as the generating functional: $\langle \hat{T}^{\mu\nu}(x)\rangle^{\mathrm{LG}} = \dfrac{2}{\sqrt{-g}}\dfrac{\delta}{\delta g_{\mu\nu}(x)}\Psi[\lambda]$

② $\Psi[\lambda]$ is written in terms of QFT in curved spacetime

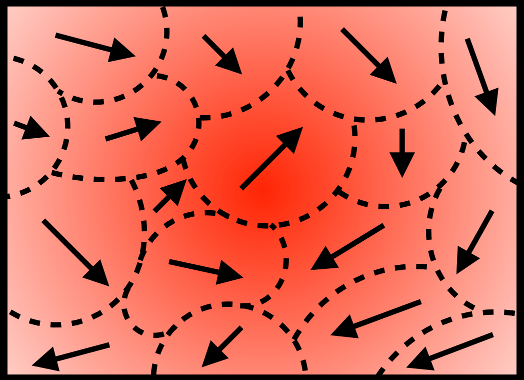
$$d\tilde{s}^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}}dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}}dx^{\bar{i}}dx^{\bar{j}}$$

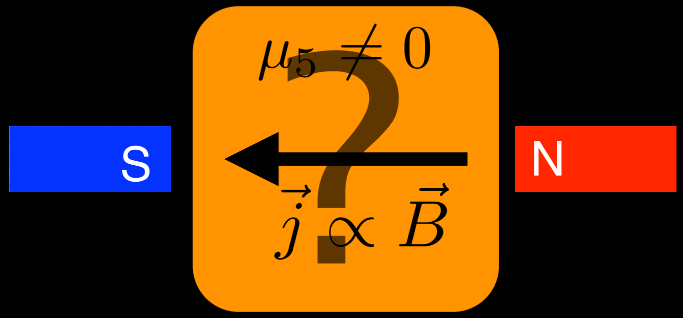
Symmetry = Spatial diffeomorphism + Kaluza-Klein gauge

Outline

MOTIVATION:

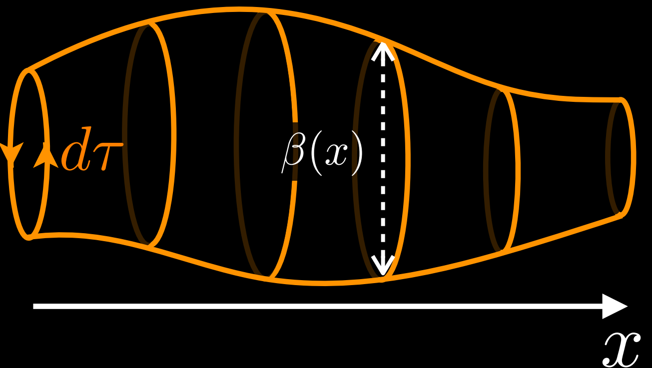
Quantum field theory under
local thermal equilibrium?





APPROACH:

QFT for **Local Gibbs distribution**



① Variation formula: $\langle \hat{T}^{\mu\nu}(x) \rangle^{\mathrm{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}(x)} \Psi[\lambda]$

② $\Psi[\lambda]$ is written in terms of QFT in "curved spacetime"

$$ds^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}})dx^{\bar{i}} + \gamma'_{\bar{i}\bar{j}} dx^{\bar{i}} dx^{\bar{j}}$$

Symmetry = Spatial diffeomorphism + Kaluza-Klein gauge

APPLICATION:

Derivation of
Anomalous hydrodynamics

Parity-even case

$$\mu_R = \mu_L$$

Derivative expansion of Ψ

Derivative expansion of ψ

$$\Psi[\beta^\mu,\nu] = \Psi^{(0)}[\beta^\mu,\nu] + \Psi^{(1)}[\beta^\mu,\nu,\partial] + \mathcal{O}(\partial^2) + \cdots$$

$\Psi^{(0)}[\beta^\mu,\nu] \simeq \beta p$ — Symmetry property

$\Psi^{(1)}[\beta^\mu,\nu,\partial] = 0$ — **Parity-even system**

Non-dissipative constitutive relation

$$\langle \hat{T}^{\mu\nu}(x)\rangle^{\mathrm{LG}}_{\bar{t}} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g_{\mu\nu}(x)}\Psi[\bar{t};\lambda] = T^{\mu\nu}_{(0)}[\lambda(x)] + T^{\mu\nu}_{(1)}[\lambda(x),\nabla\lambda(x)] + \cdots$$

$$\langle \hat{J}^{\mu}(x)\rangle^{\mathrm{LG}}_{\bar{t}} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta A_{\mu}(x)}\Psi[\bar{t};\lambda] = J^{\mu}_{(0)}[\lambda(x)] + J^{\mu}_{(1)}[\lambda(x),\nabla\lambda(x)] + \cdots$$

$T^{\mu\nu}_{(1)}[\lambda(x),\nabla\lambda(x)]$, $J^{\mu}_{(1)}[\lambda(x),\nabla\lambda(x)]$ $= 0$

Recipe for Masseiu-Planck fcn.

[Banerjee et al.(2012), Jensen et al.(2012)]

Masseiu-Planck functional

$$\Psi[\lambda] = \log \int \mathcal{D}\phi e^{S[\phi,\tilde{g}]} = \underset{\mathcal{O}(p^0)}{\Psi^{(0)}[\lambda]} + \underset{\mathcal{O}(p^1)}{\Psi^{(1)}[\lambda, \partial]} + \mathcal{O}(\partial^2)$$

- **Building blocks**：$\lambda = \{e^\sigma, \ a_{\bar{i}}, \ \mu, \ A_{\bar{i}}\}$

- **Symmetry**：Spatial diffeo, Kaluza-Klein, Gauge

  $A_{\bar{i}}$ ：not Kaluza-Klein inv. ➡ $\bar{A}_{\bar{i}} \equiv A_{\bar{i}} - \mu a_{\bar{i}}$

- **Power counting scheme**：$\lambda = \mathcal{O}(p^0)$

  $f_{\bar{i}\bar{j}} \equiv \partial_{\bar{i}} a_{\bar{j}} - \partial_{\bar{j}} a_{\bar{i}} = \mathcal{O}(p^1)$ ➡ $ff = \mathcal{O}(p^2)$

$\Psi^{(0)}$: Order $\mathcal{O}(p^0)$

Masseiu-Planck functional

$$\Psi[\lambda] = \log \int \mathcal{D}\phi e^{S[\phi,\tilde{g}]} = \Psi^{(0)}[\lambda] + \Psi^{(1)}[\lambda, \partial] + \mathcal{O}(\partial^2)$$

$\mathcal{O}(p^0)$ $\mathcal{O}(p^1)$

- **Building blocks** : $\lambda = \{e^\sigma, \cancel{a_{\bar{i}}}, \mu, \cancel{\bar{A}_{\bar{i}}}\}$

$$\Psi^{(0)}[\lambda] = \int_0^{\beta_0} d\tau \int d^3\bar{x} \sqrt{\gamma'} e^\sigma p(\beta, \mu)$$

Perfect fluid

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = (e+p)u^\mu u^\nu + p\eta^{\mu\nu}$$

$$\langle \hat{J}^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = n u^\mu$$

Parity-odd case

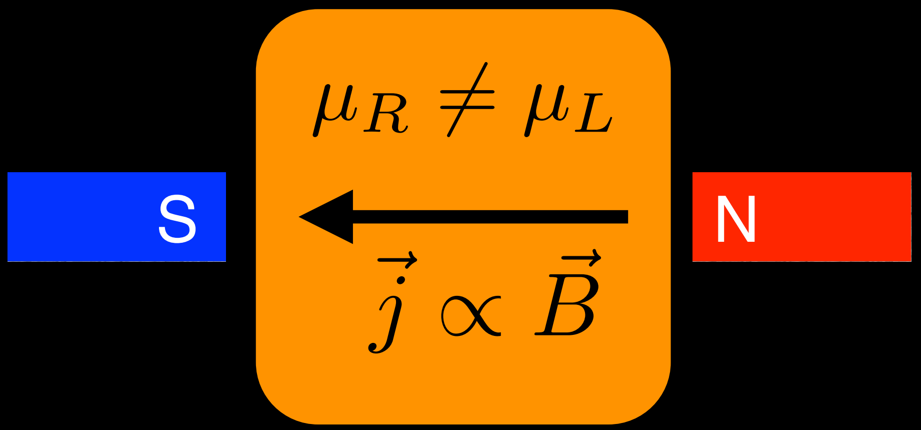
$$\mu_R \neq \mu_L$$

Anomaly-induced transport

- **Chiral Magnetic Effect (CME)**

[Fukushima et al.2008, Vilenkin 1980]

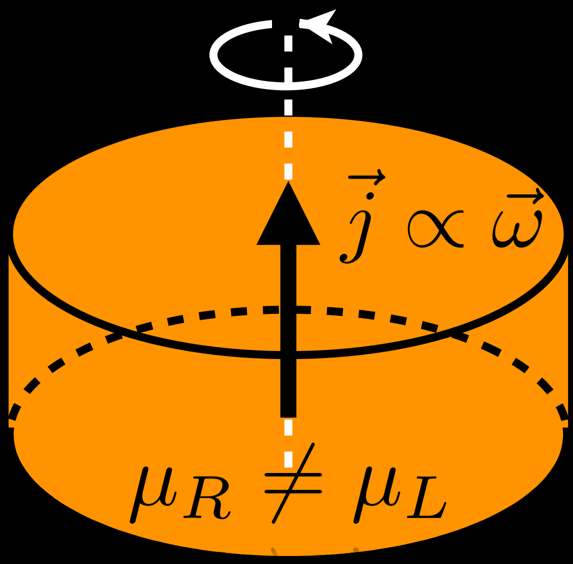
$$\vec{j} = \frac{e\mu_5}{2\pi^2}\vec{B}$$



- **Chiral Vortical Effect (CVE)**

[Erdmenger et al. 2008, Son-Surowka 2009]

$$\vec{j} = \frac{\mu\mu_5}{2\pi^2}\vec{\omega}$$



Derivative expansion of Ψ

Derivative expansion of ψ

$$\Psi[\beta^\mu,\nu] = \Psi^{(0)}[\beta^\mu,\nu] + \Psi^{(1)}[\beta^\mu,\nu,\partial] + \mathcal{O}(\partial^2) + \cdots$$

$\Psi^{(0)}[\beta^\mu,\nu] \simeq \beta p$

$\Psi^{(1)}[\beta^\mu,\nu,\partial] = 0$ **Parity-even system**

Symmetry property $\neq 0$ **Parity-odd system**

Non-dissipative constitutive relation

$$\langle \hat{T}^{\mu\nu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g_{\mu\nu}(x)}\Psi[\bar{t};\lambda] = T^{\mu\nu}_{(0)}[\lambda(x)] + T^{\mu\nu}_{(1)}[\lambda(x),\nabla\lambda(x)] + \cdots$$

$$\langle \hat{J}^{\mu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta A_{\mu}(x)}\Psi[\bar{t};\lambda] = J^{\mu}_{(0)}[\lambda(x)] + J^{\mu}_{(1)}[\lambda(x),\nabla\lambda(x)] + \cdots$$

$= 0$ $\neq 0$

Recipe for Masseiu-Planck fcn.

Weyl fermion : $\mathcal{L} = \frac{i}{2}\xi^\dagger \left(e_m^{\ \mu} \sigma^m \overrightarrow{D}_\mu - \overleftarrow{D}_\mu \sigma^m e_m^{\ \mu} \right) \xi$

$$\Psi[\lambda] = \log \int \mathcal{D}\xi^\dagger \mathcal{D}\xi e^{S[\xi,\xi^\dagger,A,\tilde{e}]} = \underbrace{\Psi^{(0)}[\lambda]}_{\mathcal{O}(p^0)} + \underbrace{\Psi^{(1)}[\lambda,\partial]}_{\mathcal{O}(p^1)} + \mathcal{O}(\partial^2)$$

- **Building blocks** : $\lambda = \{e^\sigma,\ a_{\bar{i}},\ \mu_R,\ \bar{A}_{\bar{i}}\}$

- **Symmetry** : Spatial diffeo, Kaluza-Klein, Gauge

  $A_{\bar{i}}$: not Kaluza-Klein inv. → $\bar{A}_{\bar{i}} \equiv A_{\bar{i}} - \mu_R a_{\bar{i}}$

- **Power counting scheme** : $\lambda = \mathcal{O}(p^0)$

  $f_{\bar{i}\bar{j}} \equiv \partial_{\bar{i}} a_{\bar{j}} - \partial_{\bar{j}} a_{\bar{i}} = \mathcal{O}(p^1)$ → $ff = \mathcal{O}(p^2)$

$\Psi^{(0)}$: Order $\mathcal{O}(p^0)$

Weyl fermion : $\mathcal{L} = \frac{i}{2}\xi^\dagger \left(e_m^{\ \mu} \sigma^m \overrightarrow{D}_\mu - \overleftarrow{D}_\mu \sigma^m e_m^{\ \mu} \right) \xi$

$$\Psi[\lambda] = \log \int \mathcal{D}\xi^\dagger \mathcal{D}\xi e^{S[\xi, \xi^\dagger, A, \tilde{e}]} = \underbrace{\Psi^{(0)}[\lambda]}_{\mathcal{O}(p^0)} + \underbrace{\Psi^{(1)}[\lambda, \partial]}_{\mathcal{O}(p^1)} + \mathcal{O}(\partial^2)$$

- **Building blocks** : $\lambda = \{e^\sigma, \cancel{a_{\bar{i}}}, \mu_R, \cancel{\bar{A}_{\bar{i}}}\}$

$$\Psi^{(0)}[\lambda] = \int_0^{\beta_0} d\tau \int d^3\bar{x} \sqrt{\gamma'} e^\sigma p(\beta, \mu_R)$$

Perfect fluid

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = (e + p) u^\mu u^\nu + p \eta^{\mu\nu}$$

$$\langle \hat{J}_R^\mu(x) \rangle_{\bar{t}}^{\mathrm{LG}} = n_R u^\mu$$

$\Psi^{(1)}$: Order $\mathcal{O}(p)$

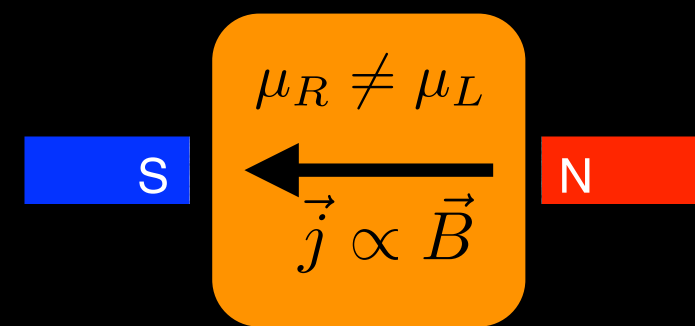
Weyl fermion : $\mathcal{L} = \frac{i}{2}\xi^\dagger \left(e_m^{\ \mu}\sigma^m \overrightarrow{D}_\mu - \overleftarrow{D}_\mu \sigma^m e_m^{\ \mu} \right)\xi$

$$\Psi[\lambda] = \log \int \mathcal{D}\xi^\dagger \mathcal{D}\xi e^{S[\xi,\xi^\dagger,A,\tilde{e}]} = \Psi^{(0)}[\lambda] + \Psi^{(1)}[\lambda,\partial] + \mathcal{O}(\partial^2)$$

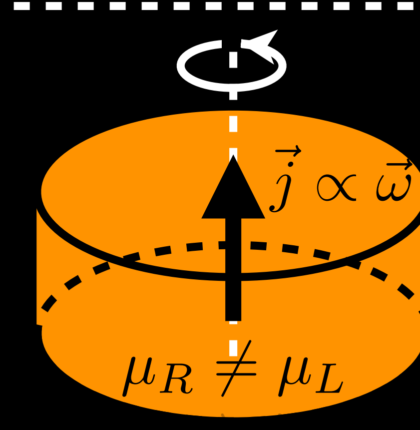
$\mathcal{O}(p^0)$ $\mathcal{O}(p^1)$

- Building blocks : $\lambda = \{e^\sigma,\ a_{\bar{i}},\ \mu_R,\ \bar{A}_{\bar{i}}\}$

$$\int d^3\bar{x}\sqrt{\gamma'}C_1(\beta,\mu_R)\epsilon^{\bar{i}\bar{j}\bar{k}}\bar{A}_{\bar{i}}\partial_{\bar{j}}\bar{A}_{\bar{k}}$$



$$\int d^3\bar{x}\sqrt{\gamma'}C_2(\beta,\mu_R)\epsilon^{\bar{i}\bar{j}\bar{k}}\bar{A}_{\bar{i}}\partial_{\bar{j}}a_{\bar{k}}$$



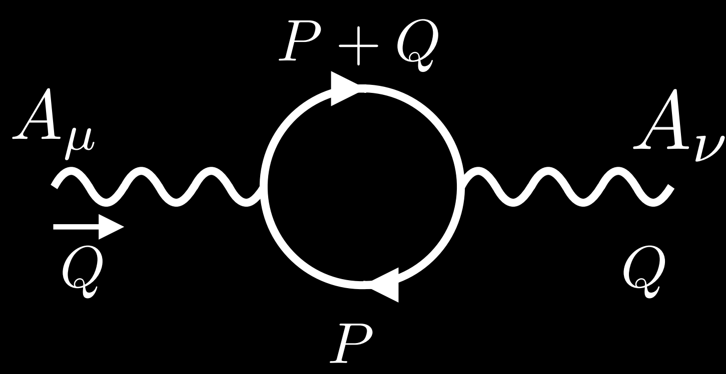
Anomalous transport coefficients

① **Non-perturbative way (WZ consistency condition …)**

[Banerjee et al.(2012), Jensen et al.(2012) , Haehl et al. (2015)]

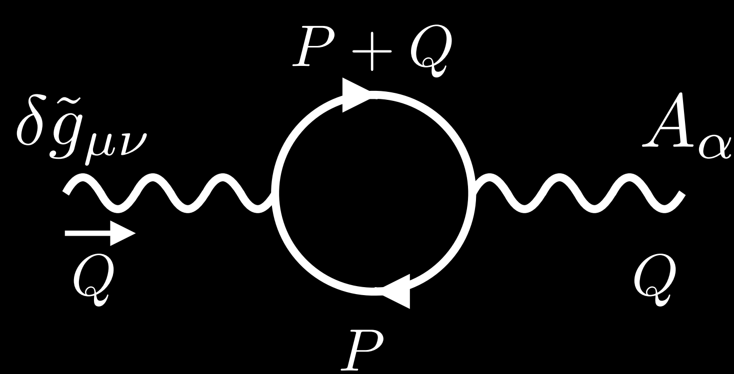
② Perturbative evaluation of ψ in external field

$$\frac{\delta^2\Psi}{\delta A_\mu \delta A_\nu} =$$



$$\simeq -i\varepsilon^{0\mu\rho\nu}\tilde{Q}_\rho \frac{\mu_R}{4\pi^2}$$

$$\frac{\delta^2\Psi}{\delta \tilde{g}_{\mu\nu} \delta A_\alpha} =$$



$$\simeq i\tilde{Q}_\rho \underbrace{C}_{= \frac{\mu_R^2}{8\pi^2} + \frac{T^2}{24}} (\eta^{\nu 0}\varepsilon^{\rho\mu 0\alpha} + \delta_{ij}\eta^{\nu i}\epsilon^{\rho\mu j\alpha})$$

$$\Rightarrow \Psi^{(1)}[\lambda] = \int d^3x \varepsilon^{0ijk}\left[\frac{\nu_R}{8\pi^2}A_i\partial_j A_k + \left(\frac{\nu_R\mu_R}{8\pi^2} + \frac{T}{24}\right)A_i\partial_j\tilde{g}_{0k}\right]$$

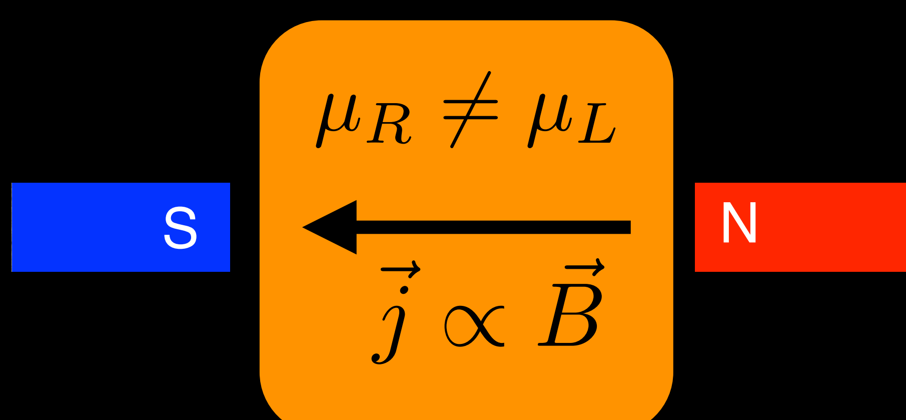
Derivation of CME/CVE

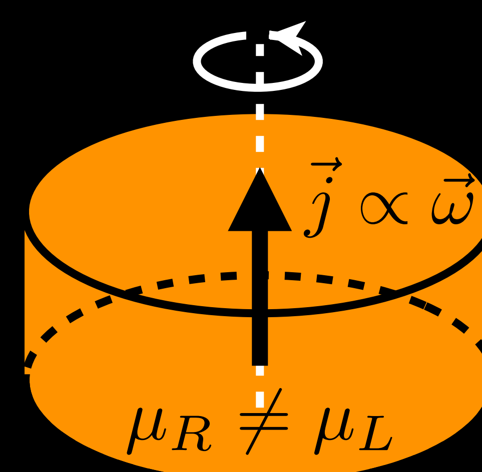
$$\Psi^{(1)}[\lambda] = \int d^3x \varepsilon^{0ijk} \left[\frac{\nu_R}{8\pi^2} A_i \partial_j A_k + \left(\frac{\nu_R \mu_R}{8\pi^2} + \frac{T}{24} \right) A_i \partial_j \tilde{g}_{0k} \right]$$

$$\Rightarrow \langle \hat{J}^i_R(x) \rangle^{\mathrm{LG}}_{(0,1)} = \frac{1}{\sqrt{-g}} \frac{\delta \Psi^{(1)}}{\delta A_i(x)} = \frac{\mu_R}{4\pi^2} B^i + \left(\frac{\mu_R^2}{8\pi^2} + \frac{T^2}{24} \right) \omega^i$$

$$\langle \hat{J}^i_V(x) \rangle^{\mathrm{LG}}_{(0,1)} = \frac{\mu_5}{2\pi^2} B^i + \frac{\mu \mu_5}{2\pi^2} \omega^i$$

$$\langle \hat{J}^i_A(x) \rangle^{\mathrm{LG}}_{(0,1)} = \frac{\mu}{2\pi^2} B^i + \left(\frac{\mu^2 + \mu_5^2}{4\pi^2} + \frac{T^2}{12} \right) \omega^i$$

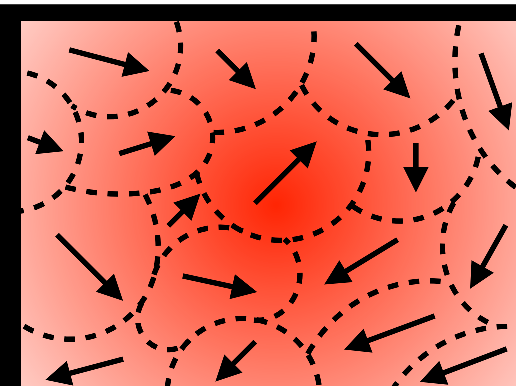


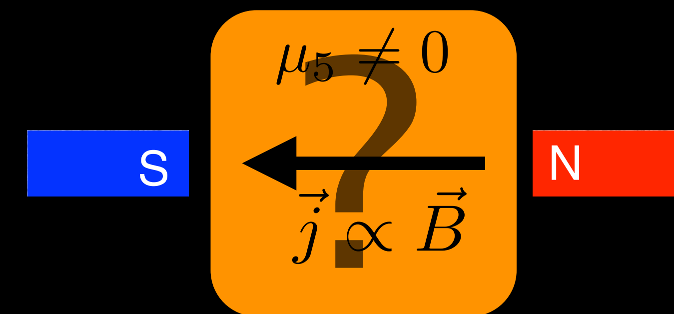


Summary

MOTIVATION:

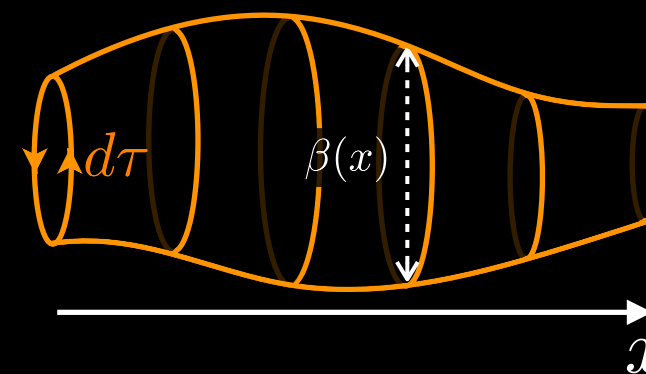
Quantum field theory under
local thermal equilibrium?





APPROACH:

QFT for **Local Gibbs distribution**



① Variation formula: $\langle \hat{T}^{\mu\nu}(x) \rangle^{\mathrm{LG}} = \dfrac{2}{\sqrt{-g}} \dfrac{\delta}{\delta g_{\mu\nu}(x)} \Psi[\lambda]$

② $\Psi[\lambda]$ is written in terms of QFT in “curved spacetime”

$$ds^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}})dx^{\bar{i}} + \gamma'_{\bar{i}\bar{j}} dx^{\bar{i}} dx^{\bar{j}}$$

Symmetry = Spatial diffeomorphism + Kaluza-Klein gauge

APPLICATION:

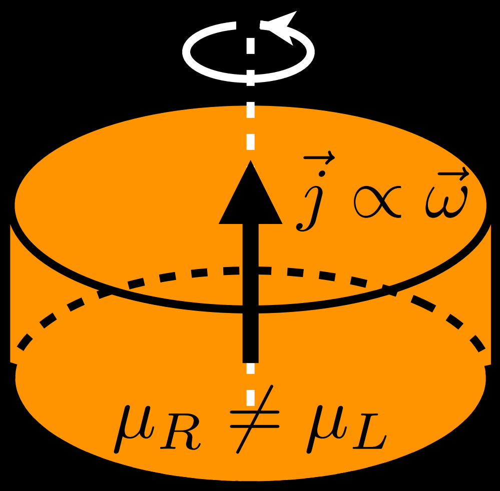
Derivation of
Anomalous hydrodynamics

$\Psi^{(1)} \to \vec{j} = \dfrac{e\mu_5}{2\pi^2} \vec{B}$

Outlook: Challenge for audience

CHALLENGE FOR LATTICIAN:

Q1. CVE coefficient from Lattice QCD [Braguta et al. (2014)]



$$\langle \hat{J}^i_A(x) \rangle^{\mathrm{LG}}_{(0,1)} = \frac{\mu}{2\pi^2} B^i + \left(\frac{\mu^2 + \mu_5^2}{4\pi^2} + \frac{T^2}{12} \right) \omega^i$$

→ Gradient flow method to evaluate $\sigma^5_\omega \sim \langle T^{0i} J^i_5 \rangle$

Q2. Thermodynamic properties of rotating/inhomogeneous QGP

→ Usual Monte-Carlo? Complex Langevin??

CHALLENGE FOR NON-LATTICIAN:

Q1. (Non-)Perturbative calculation with strong inhomogeneity

Q2. Find some New physics captured by local thermal equilibrium!!

Backup