Driven-dissipative chiral condensate

Masaru Hongo (RIKEN iTHEMS)

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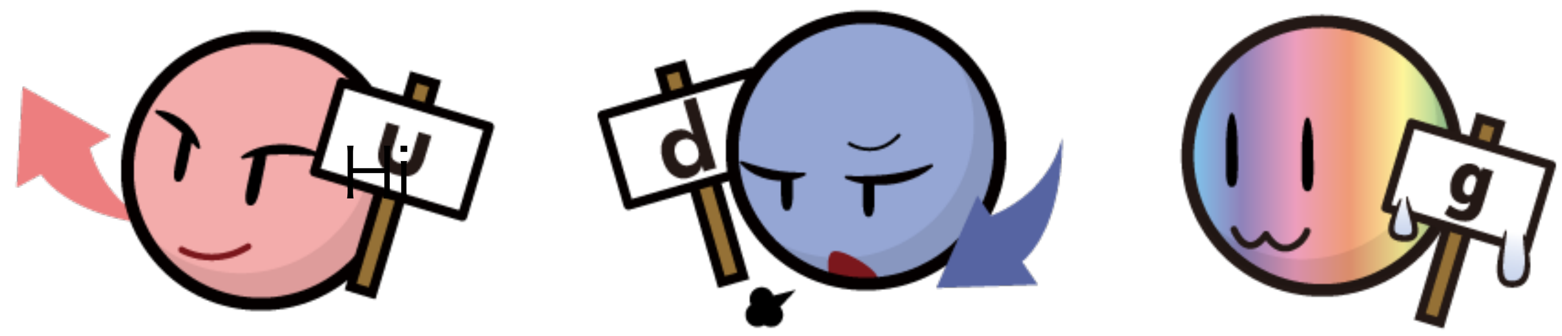
Based on ongoing collaboration with Yoshimasa Hidaka,
and **MH**, Kim, Noumi, Ota, arXiv:1805.06240 [hep-th]

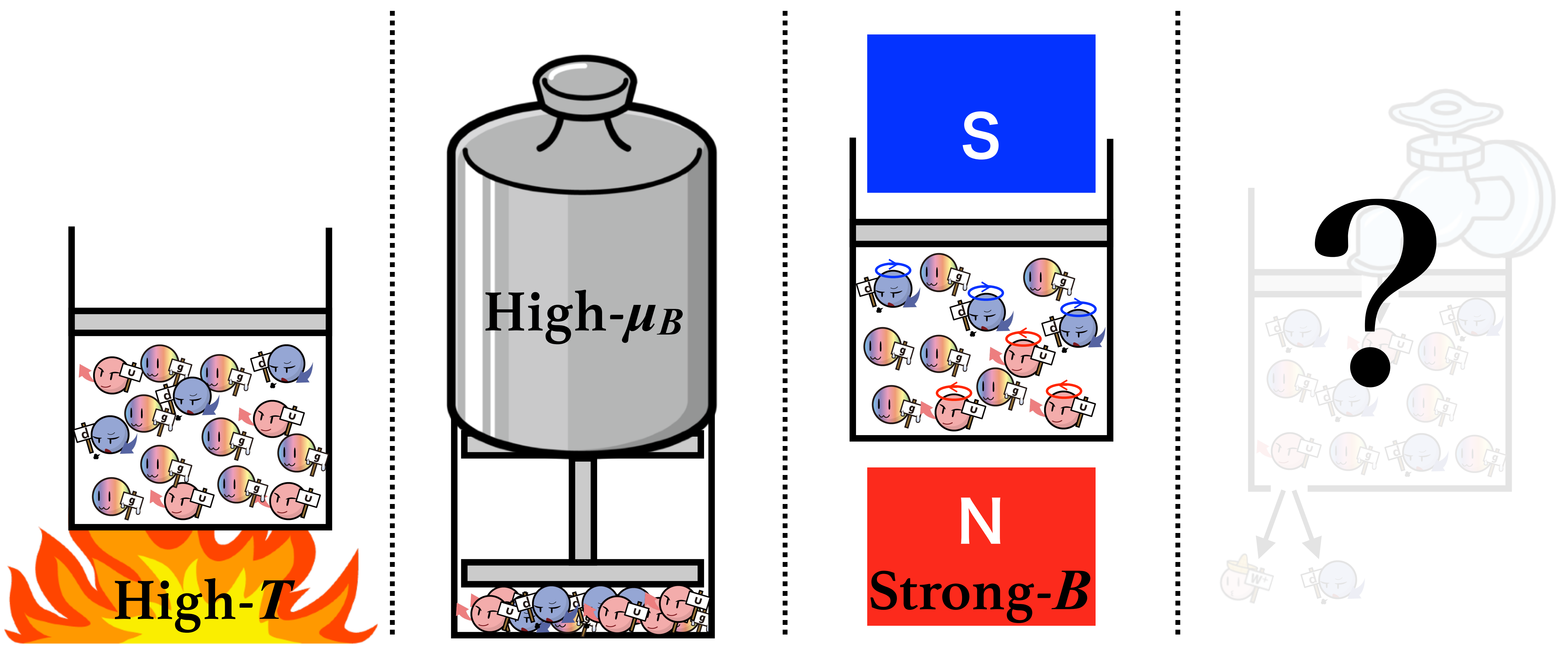
QCD matter in extreme situations

Main characters in QCD matter



[http://higgstan.com/]

◆ QCD matter in several extreme situations



Outline

MOTIVATION:

Time crystalline behavior and NG mode for chiral order parameter?

APPROACH:

Schwinger-Keldysh formalism for nonequilibrium open system

RESULT:

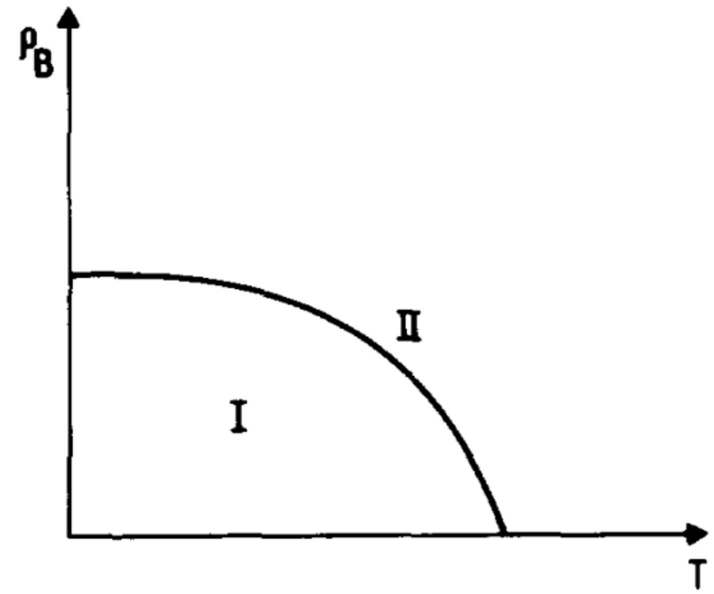
Noneq. Ginzburg-Landau expansion

Schwinger-Keldysh based EFT

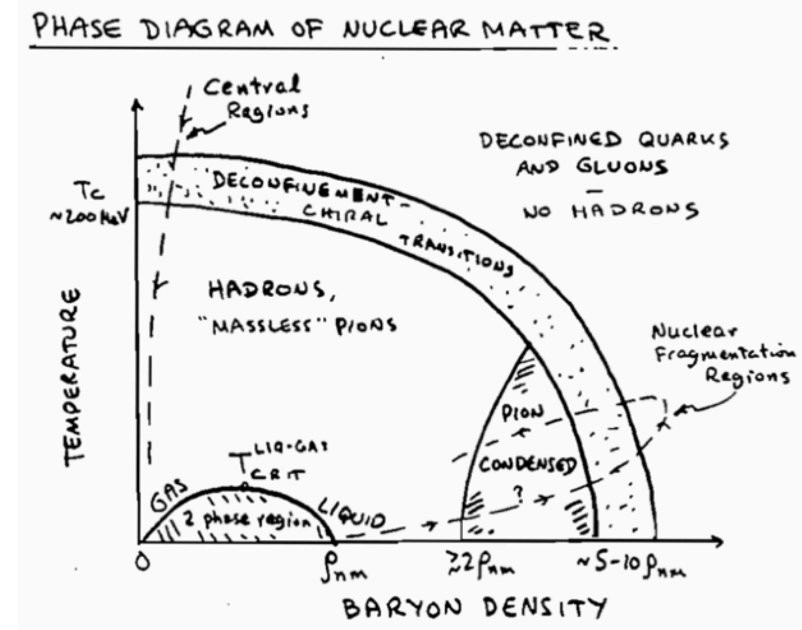
Time-evolution of QCD phase diagram

◆ Initial condition [Cabbibo-Parisi (1975)]

Earliest QCD
Phase diagram

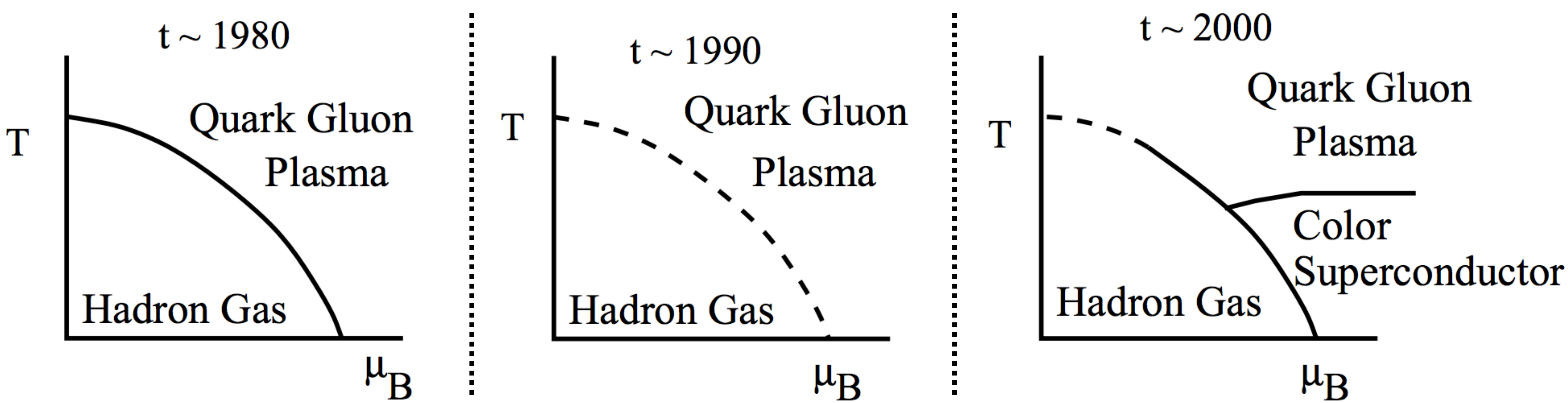


[Cabibbo-Parisi (1975)]

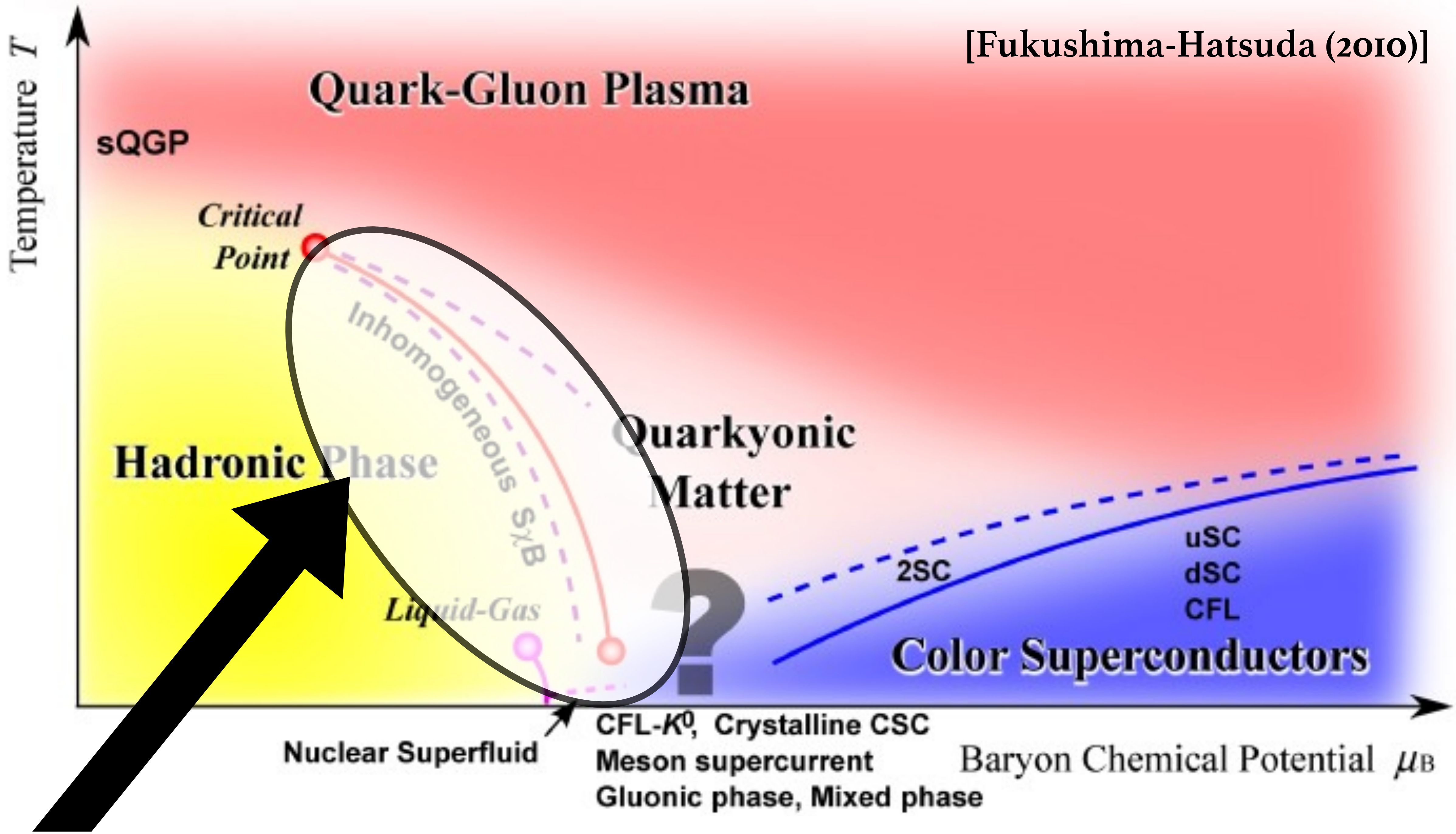


[Baym et al. "Long range plan" (1983)]

◆ Time evolution [McLerran (2003)]



QCD phase diagram in 2010's



Possibility for inhomogeneous chiral order parameter!

[Nakano-Tatsumi (2005), Nickel (2009), Kojo et al.(2010), Hidaka et al. (2015), ...]

Time crystal & No-Go theorem

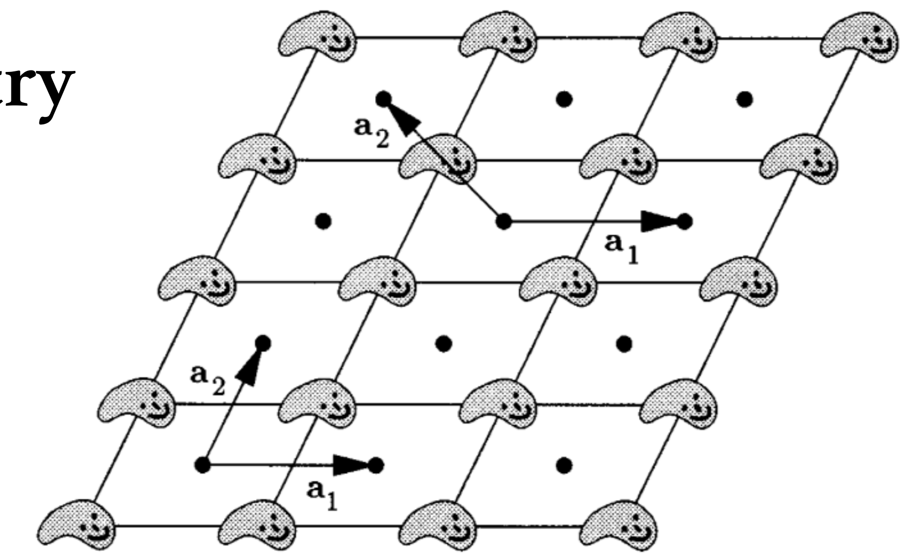
◆ Inhomogeneous phase = SSB of spatial translation symmetry

Lagrangian: Continuous translational symmetry

SSB → Crystalline state

Only discrete translational symmetry

(Classified by the space group!)



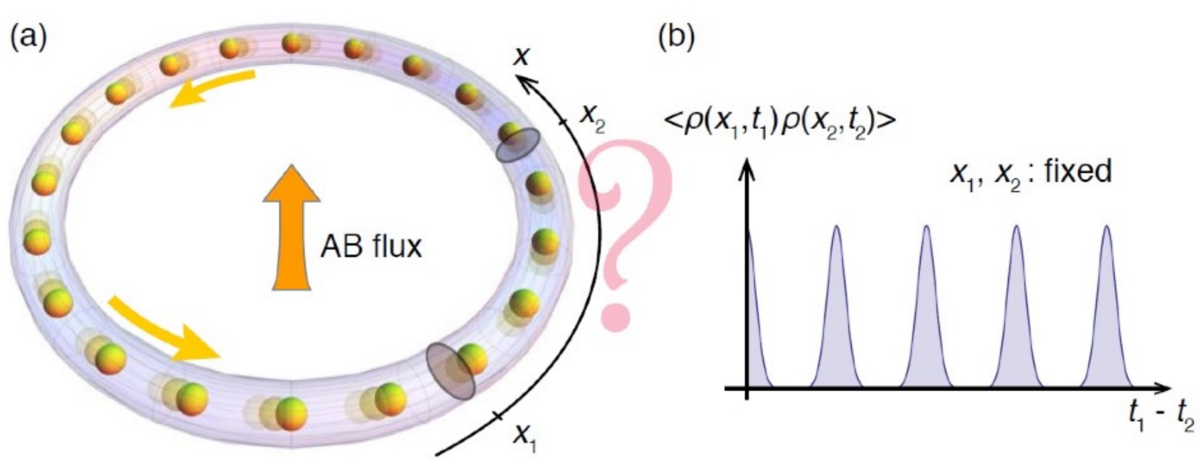
◆ Possibility of time crystal? [Wilczek (2012)]

Lagrangian: Continuous time-translational symmetry

SSB? → ~~Time crystal?~~

~~Only discrete symmetry~~

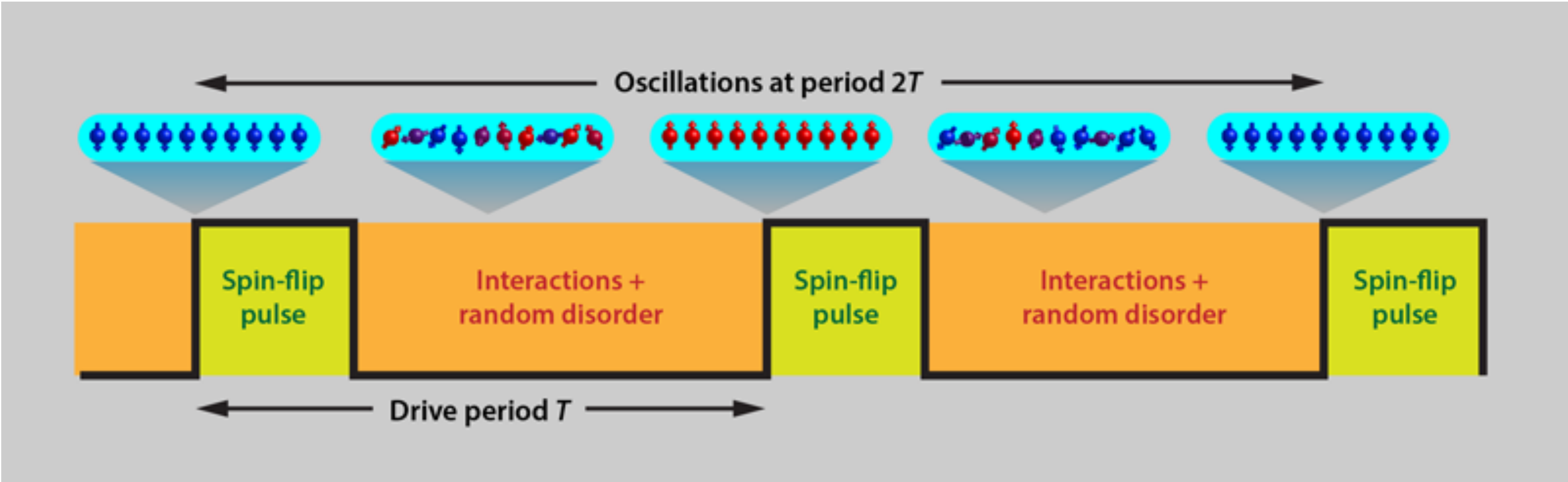
No-Go theorem for equilibrium state



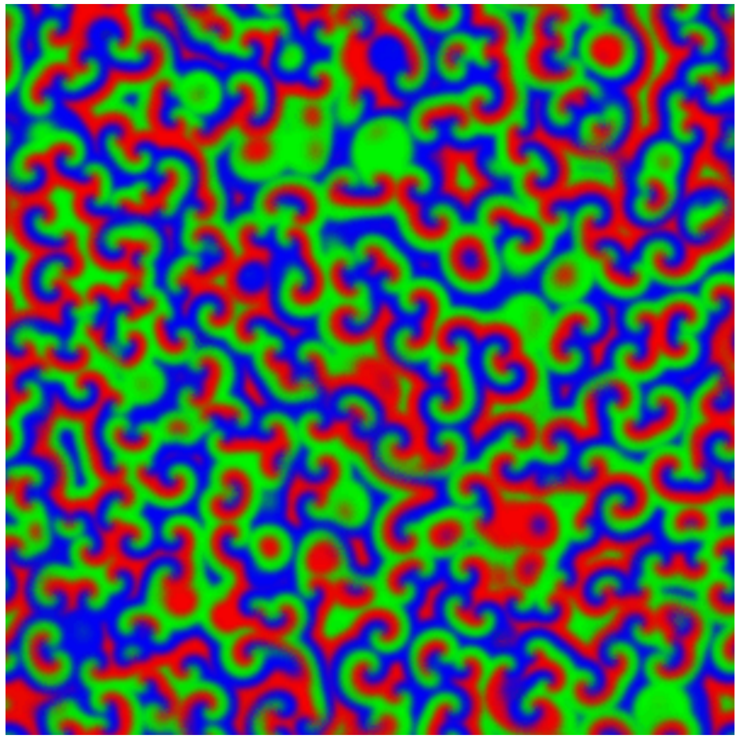
[Watanabe-Oshikawa (2015)], ...

Possibility in nonequilibrium system

◆ Discrete time crystal in periodically driven system [Yao et al. (2017),...]

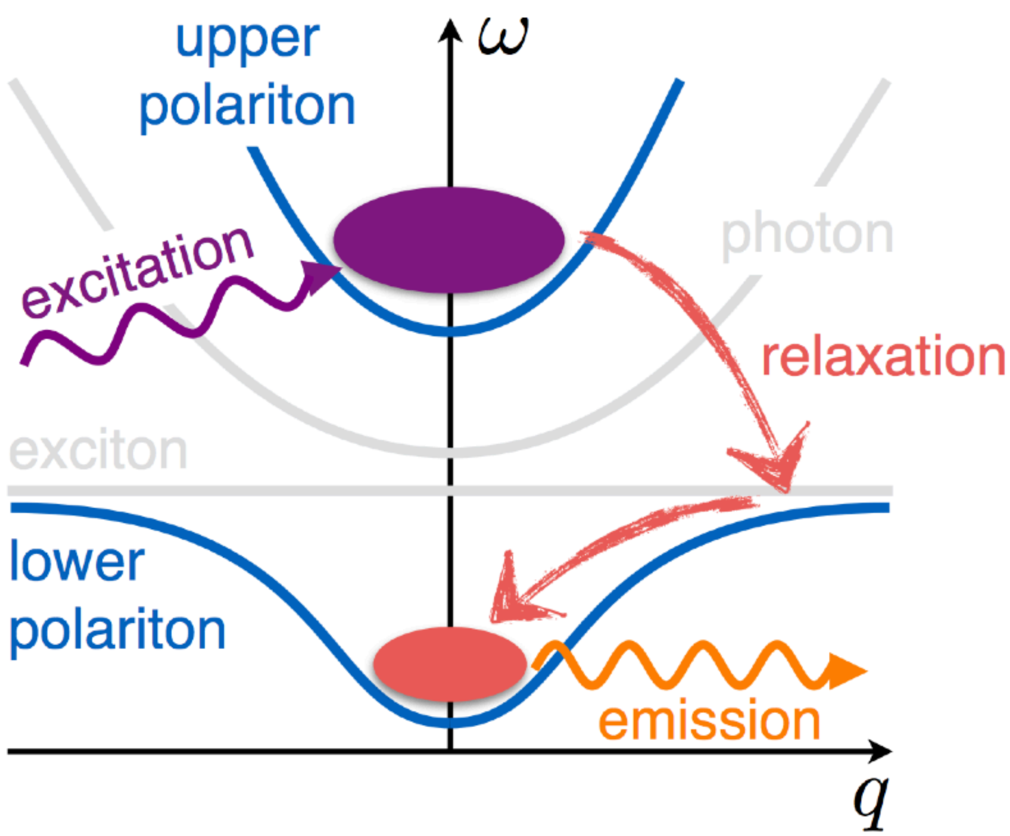


◆ Continuous time crystal in chemical reaction (BZ reaction)



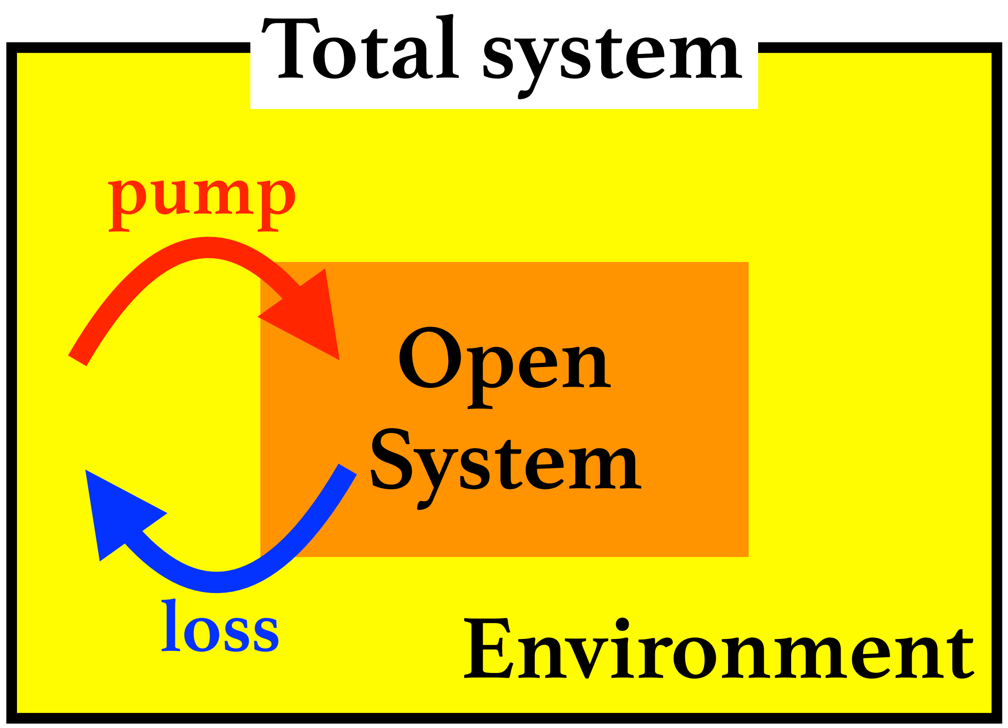
Driven-dissipative condensate

◆ **Driven-dissipative environment in exciton-polariton system**



[From Sieberer et al. (2016)]

$\simeq$



➡ Time-oscillating condensate (≒ time crystal) can be realized!

➡ Q. Do we have something special for the open system?

NG mode in open system?

◆ Spontaneous symmetry breaking & Nambu-Goldstone mode

For some conserved charge $\hat{Q}_a$

$$\exists\,\hat{\Phi}(x) \text{ such that } \langle[i\hat{Q}_a,\hat{\Phi}_i(x)]\rangle \equiv \mathrm{Tr}\big(\hat{\rho}[i\hat{Q}_a,\hat{\Phi}_i(x)]\big) \neq 0$$

Spontaneous Symmetry Breaking (SSB)

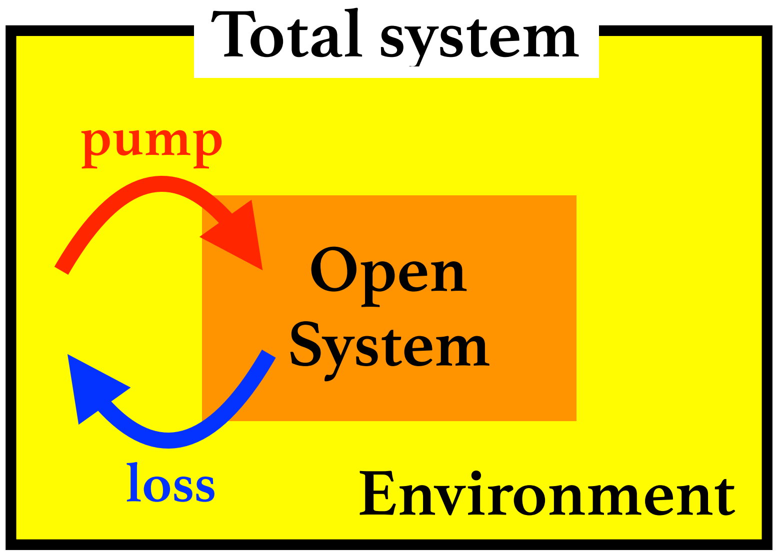
➡ **Massless mode = Nambu-Goldstone (NG) mode appears!**

※ In open system, charge can diffused to environment!

Charge is no longer conserved!

➡ { - What is "symmetry and its breaking"?
- Is there Nambu-Goldstone mode?

[Minami-Hidaka (2018)]



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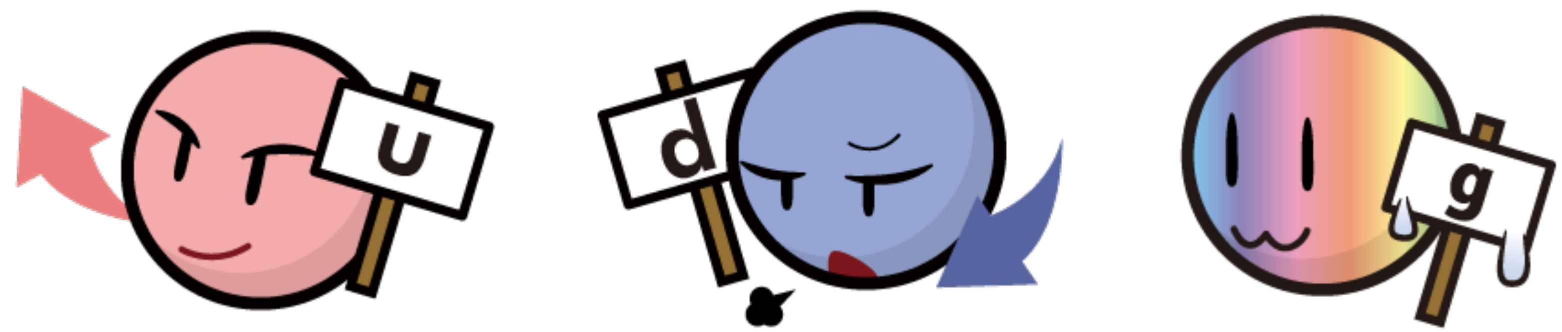
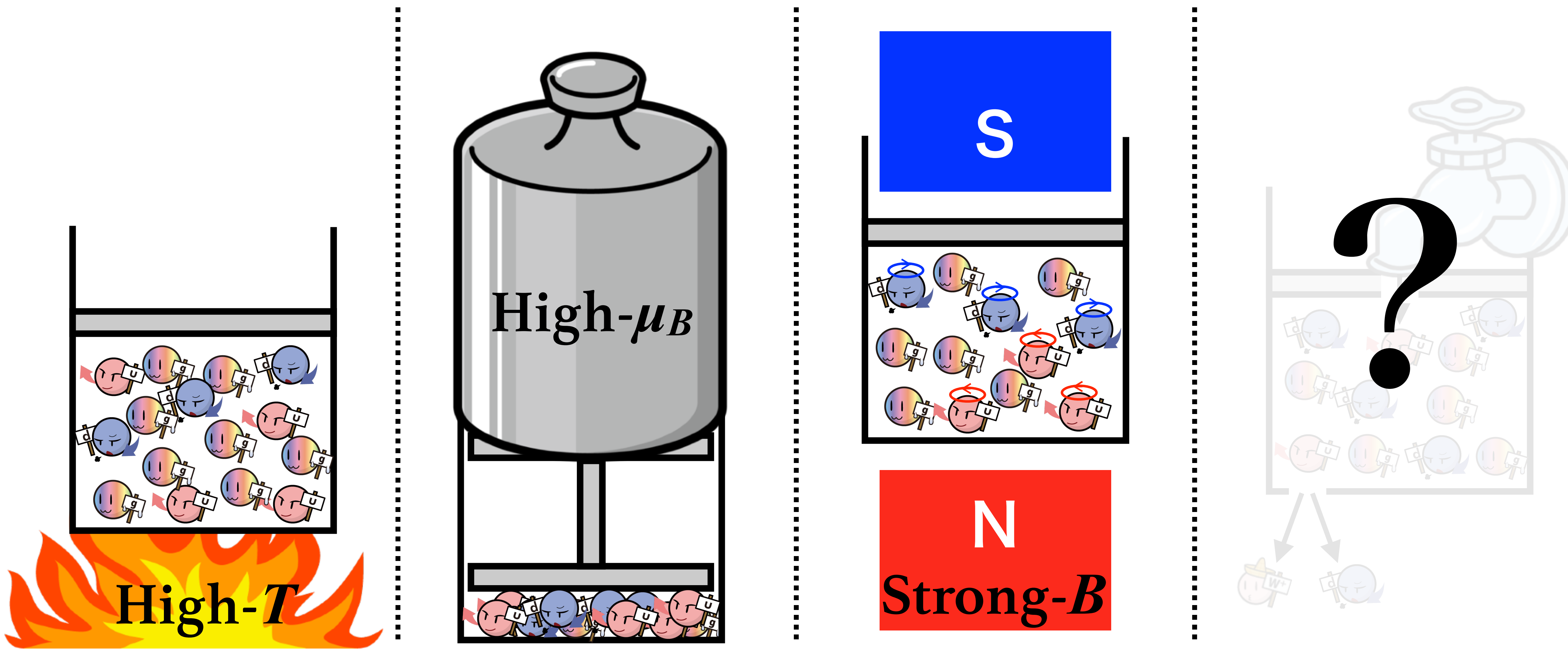


◆ QCD matter in several extreme situations



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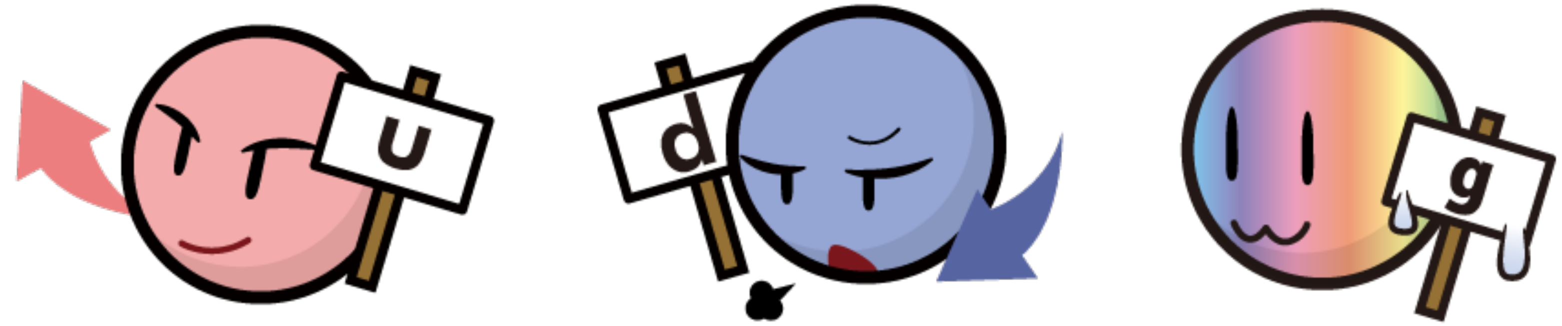

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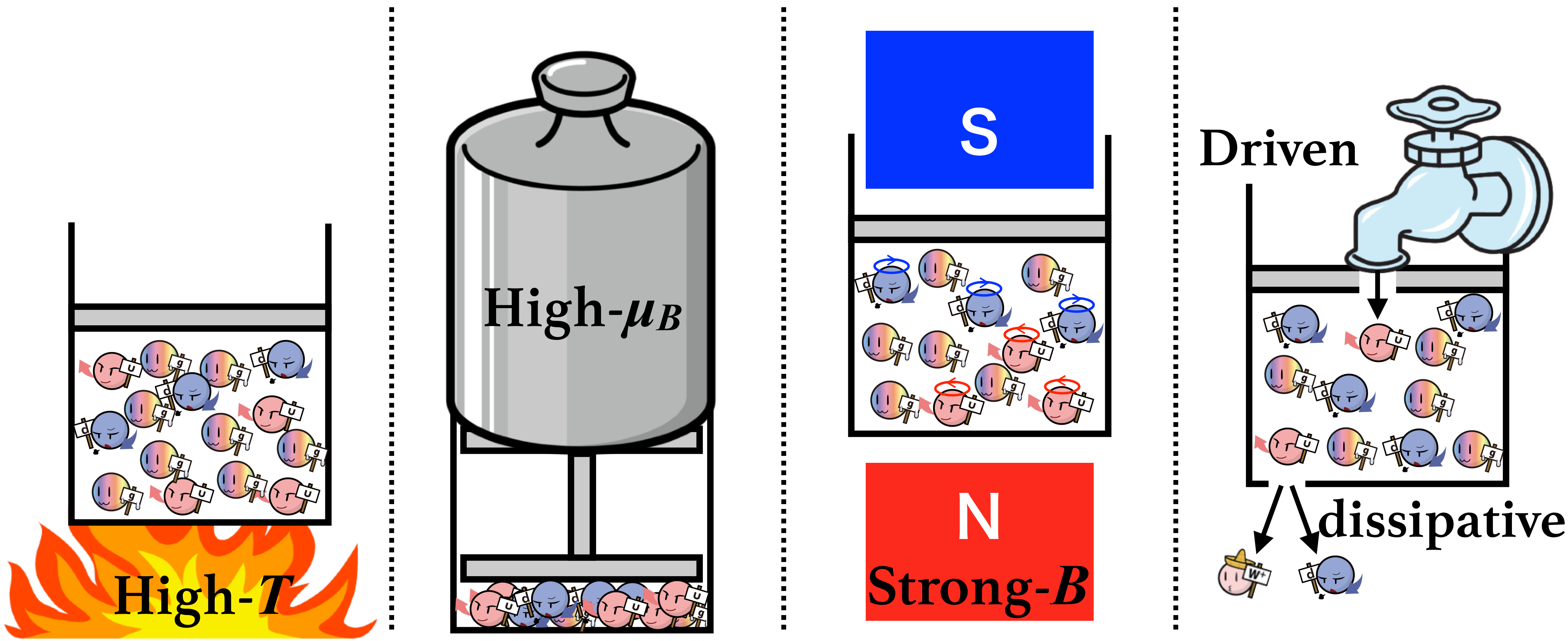

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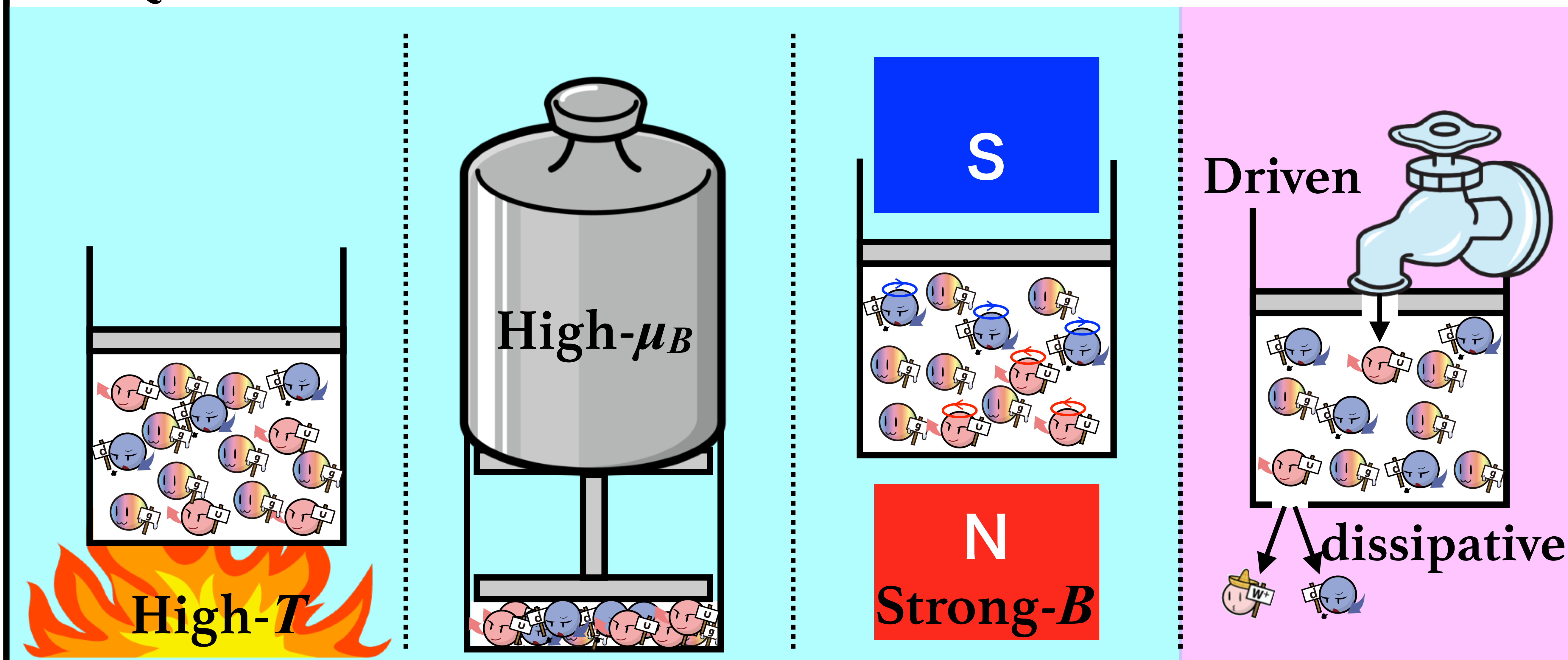
Need for real-time formalism

◆ QCD matter in several extreme situations



Equilibrium state: Imaginary-time formalism!

Nonequilibrium (steady) state

➡ We need to use the real-time formalism of quantum theory!

Real-time formalism of

Quantum Field Theory in a Nutshell

Schwinger-Keldysh in a Nutshell

QFT for T=0

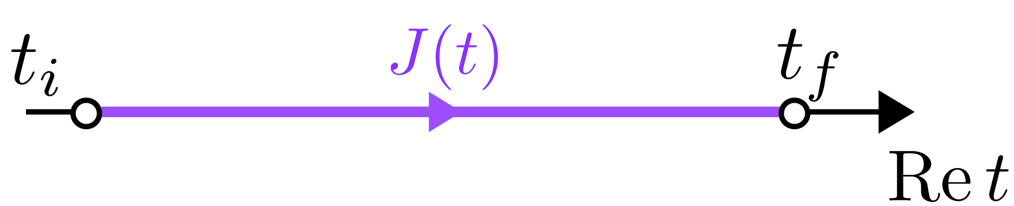
- Observable : Transition amp.

$$\langle \mathrm{out}|\hat{\mathcal{O}}|\mathrm{in}\rangle$$

Ex. $S_{\beta\alpha} = \langle\beta|\hat{S}|\alpha\rangle$

- Generating functional

$$Z[J] = \langle \mathrm{vac}|\hat{U}_J|\mathrm{vac}\rangle$$



Non-equilibrium QFT

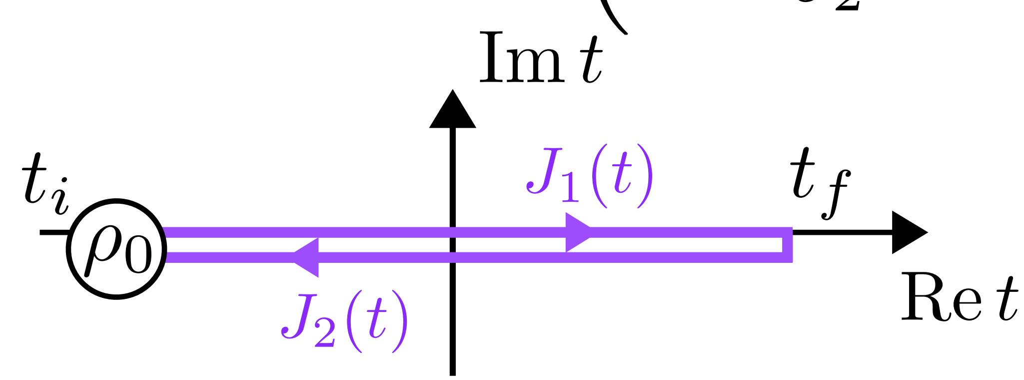
- Observable : Average value

$$\sum_{\mathrm{in}} \langle \mathrm{in}|\hat{\mathcal{O}}|\mathrm{in}\rangle$$

Ex. $\langle\hat{\varphi}\rangle = \mathrm{Tr}\left(\hat{\rho}_0\hat{\varphi}\right)$

- CTP generating functional

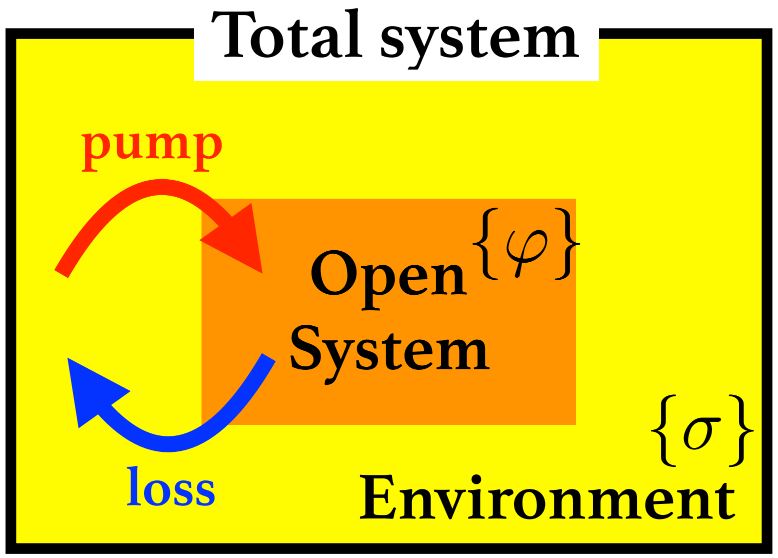
$$Z[J_1, J_2] = \mathrm{Tr}\left(\hat{\rho}_0 \hat{U}^\dagger_{J_2}\hat{U}_{J_1}\right)$$



Doubled symmetry structure

◆ Dynamical degrees of freedom

Total system (***G*-inv.**) = System $\{\varphi\}$ + Environment $\{\sigma\}$



$$S_{\rm tot}[\varphi,\sigma] = S_{\rm sys}[\varphi] + S_{\rm env}[\sigma] + S_{\rm int}[\varphi,\sigma]$$

◆ Doubled symmetry structure in total & open systems

$$Z = \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2 \mathcal{D}\sigma_1 \mathcal{D}\sigma_2 e^{i(S_{\rm tot}[\varphi_1,\sigma_1]-S_{\rm tot}[\varphi_2,\sigma_2])}\rho_0(\varphi,\sigma)$$

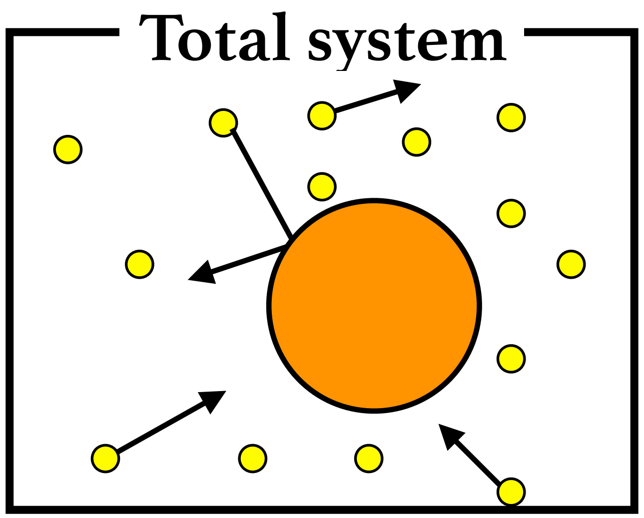
$$= N\int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2 e^{i(S_{\rm sys}[\varphi_1]-S_{\rm sys}[\varphi_2])+i\Gamma[\varphi_1,\varphi_2]}\rho_0(\varphi)$$

$S_{\rm tot}[\varphi_1,\sigma_1] - S_{\rm tot}[\varphi_2,\sigma_2]$ **is invariant under** $G_1 \times G_2 = G_r \times G_a$

$S_{\rm sys}[\varphi_1] - S_{\rm sys}[\varphi_2] + \Gamma[\varphi_1,\varphi_2]$ **can be only invariant under** G_r

Simple example: Brownian motion

◆ Dynamical degrees of freedom



Total system (G-inv.) = Brownian particle $\{X\}$ + Surrounding particles $\{x_n\}$

$$S_{\rm tot}[X, x_n] = S_{\rm sys}[X] + S_{\rm env}[x_n] + S_{\rm int}[X, x_n]$$

◆ Key point

- **Generating functional:** $Z = N \int \mathcal{D}X_1 \mathcal{D}X_2 e^{iS_{\rm eff}[X_1, X_2]} \rho_0(X)$
- **Effective action (in a simplified case):**

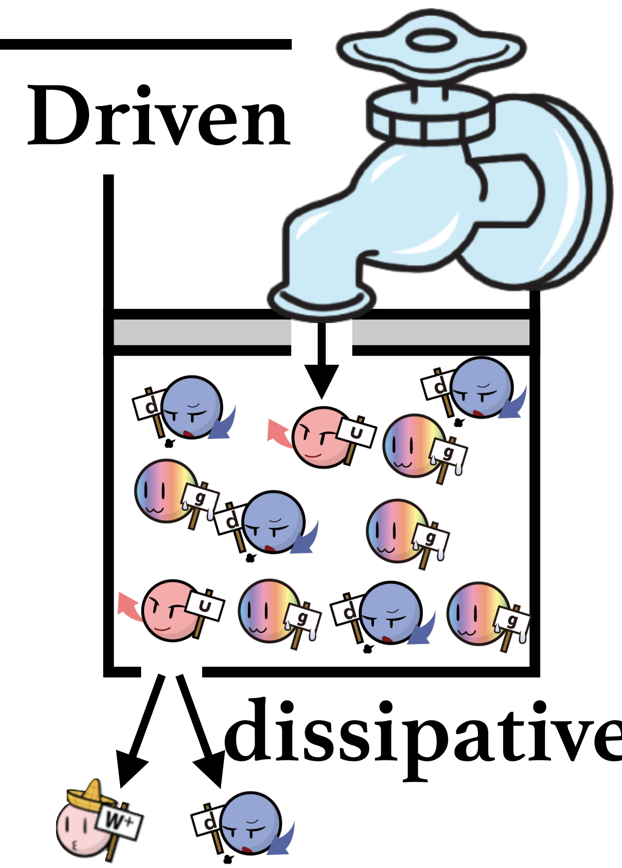
$$iS_{\rm eff}[X_1, X_2] = iS_{\rm sys}[X_1] - iS_{\rm sys}[X_2] - \frac{1}{2}\int dt \left[i\gamma(X_1\dot{X}_2 - X_2\dot{X}_1) + 2\gamma T(X_1 - X_2)^2\right]$$

- $S_{\rm eff}[X_1, X_2]$ **is only invariant under** $\begin{cases} X_1(t) \to X_1'(t) = X_1(t+\epsilon_R) \\ X_2(t) \to X_2'(t) = X_2(t+\epsilon_R) \end{cases}$

Energy of Brownian particle is not conserved, but we still have symmetry!

Towards driven-dissipative QCD

◆ Lesson from the simple example



- Doubled dynamical degrees of freedom: $\{\varphi_1, \varphi_2\}$
- Only one (vector-like) symmetry: $G_r \times$ ~~G_a~~
- Driven-dissipative coupling:

  $(\gamma < 0)$ $(\gamma > 0)$ for the Brownian particle

  (In general, coupling could be complex!)

➡ **We can discuss SSB of surviving G_r-symmetry!!** [Minami-Hidaka (2018)]

◆ Driven-dissipative 2-flavor QCD in chiral limit

- **Symmetry:** $(SU(2)_R \times SU(2)_L)_r \times$ ~~$(SU(2)_R \times SU(2)_L)_a$~~ $= O(4)_r \times$ ~~$O(4)_a$~~

  $\rightarrow$ **Possible SSB for** $(SU(2)_R \times SU(2)_L)_r = O(4)_r$

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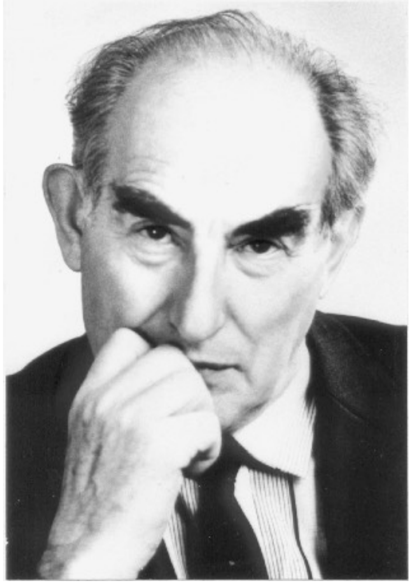
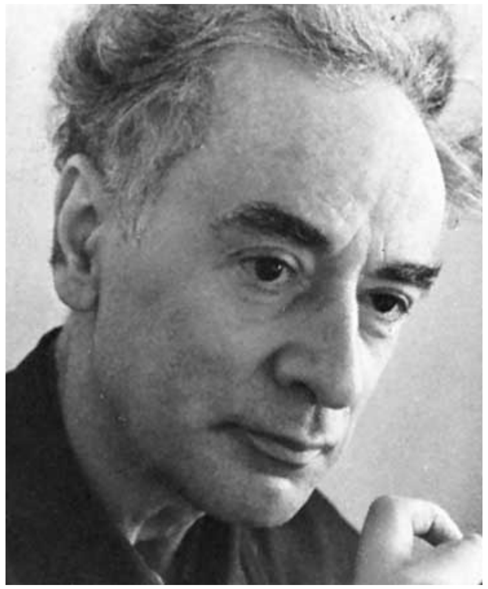
Two (theoretically) simple situations

① Near (2nd-order) phase transition point

Ginzburg-Landau theory

(Linear sigma model)





② Deep inside the ordered-phase

Effective field theory/Lagrangian

(Nonlinear sigma model)

Towards driven-dissipative TDGL

◆ **Driven-dissipative 2-flavor QCD in chiral limit**

- **Symmetry:** $(SU(2)_R \times SU(2)_L)_r \times \cancel{(SU(2)_R \times SU(2)_L)_a} = O(4)_r \times \cancel{O(4)_a}$

→ **Possible SSB for** $(SU(2)_R \times SU(2)_L)_r = O(4)_r$

- **Building blocks:** $\Phi_1 = \begin{pmatrix} \sigma_1 \\ \boldsymbol{\pi}_1 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \sigma_2 \\ \boldsymbol{\pi}_2 \end{pmatrix}$

- **Symmetry property 1:** $\begin{cases} \Phi_1 \to \Phi_1' = R_1 \Phi_1 \\ \Phi_2 \to \Phi_2' = R_2 \Phi_2 \end{cases}$ with $R_1 = R_2 \in O(4)_r$

- **Symmetry property 2:** **NO** **Lorentz invariance for mixing terms**

- **Invariants:** $\begin{cases} \text{LO} \ni \Phi_1^t \Phi_1, \ \Phi_2^t \Phi_2, \ \Phi_1^t \Phi_2, \ \Phi_2^t \Phi_1 \\ \text{NLO} \ni \partial_\mu \Phi_1^t \partial^\mu \Phi_1, \ \partial_\mu \Phi_2^t \partial^\mu \Phi_2, \ (\Phi_1^t \Phi_1)^2, \ (\Phi_2^t \Phi_2)^2, \cdots \\ \qquad \Phi_1^t \partial_0 \Phi_2, \ \Phi_2^t \partial_0 \Phi_1, \ (\Phi_2^t \Phi_1)^2, \ (\Phi_1^t \Phi_2)^2, \cdots \end{cases}$

Driven-dissipative Time-Dep. GL

[Talk by Hayata last week]

◆ Driven-dissipative TDGL action in *ra*-basis

$$S_{\rm eff} = \int d^4x \Phi_a^t \left(-\partial_0^2 + \nabla^2 - \gamma(1-\kappa\Phi_r^t\Phi_r)\partial_t - m^2 - g^2\Phi_r^t\Phi_r\right)\Phi_r + \cdots$$

with $\Phi_r(x) \equiv \dfrac{\Phi_1(x)+\Phi_2(x)}{2}, \quad \Phi_a(x) \equiv \Phi_1(x) - \Phi_2(x)$

➡ $\left(\partial_0^2 + \gamma(1-\kappa\sigma^2)\partial_0 + m^2 + g^2\sigma^2\right)\sigma(t) = 0$ with $\langle \Phi_r(x) \rangle = \begin{pmatrix} \sigma(t) \\ \mathbf{0} \end{pmatrix}$

◆ Linear analysis in the trivial phase: $\sigma(t) = 0 + \delta\sigma(t)$

Solution: $\delta\sigma(t) = c_1 e^{-\frac{1}{2}(\gamma+\sqrt{\gamma^2-4m^2})t} + c_2 e^{-\frac{1}{2}(\gamma-\sqrt{\gamma^2-4m^2})t}$

➡ If $\gamma < 0$ (driven nature), trivial phase is **linearly unstable!!**

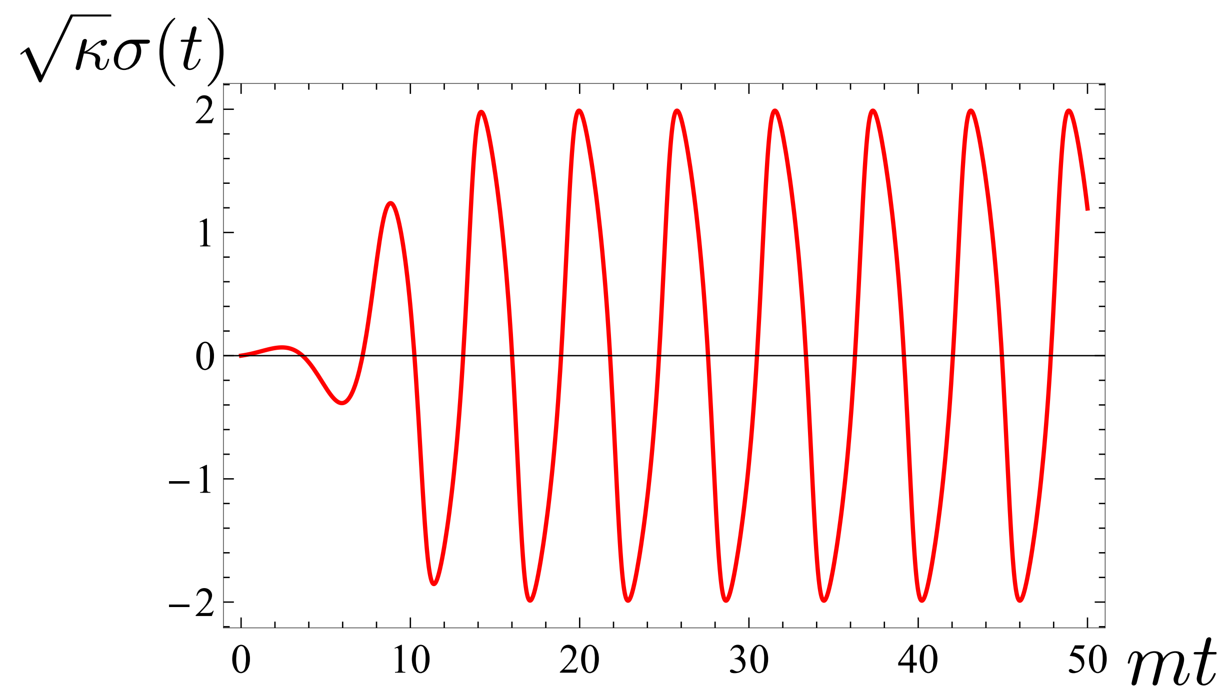
➡ **Non-linear terms will be important!**

SSB of time-translational symmetry

$\gamma/m = 0.2 > 0,\ g^2/(m^2\kappa) = 0.1$

Trivial phase
due to the dissipation

$\gamma/m = -1 > 0,\ g^2/(m^2\kappa) = 0.1$



Oscillating χ condensate
due to drive+dissipation

◆ NG mode $\pi(x)$ associated with SSB of time translation

$$\Phi(x) = \begin{pmatrix} \sigma(t + \pi(t, \boldsymbol{x})) \\ \boldsymbol{0} \end{pmatrix}$$

EOM ⇒

Dispersion relation: $\omega = -iD\boldsymbol{k}^2$

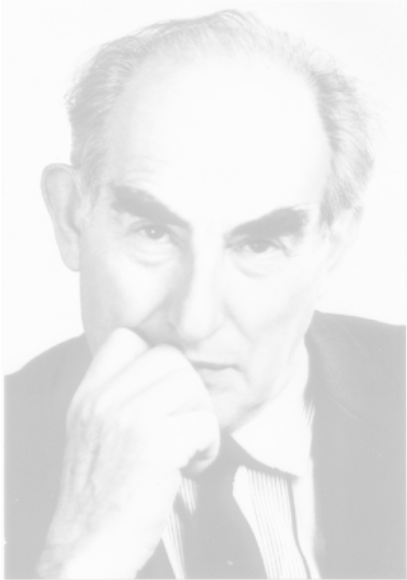
Diffusive NG mode!!

Two (theoretically) simple situations

① Near (2nd-order) phase transition point

Ginzburg-Landau theory

(Linear sigma model)



② Deep inside the ordered-phase

Effective field theory/Lagrangian

(Nonlinear sigma model)

EFT for time-dependent condensate

◆ **Time-translationally breaking & Nambu-Goldstone field**

$$\exists \phi(t, \boldsymbol{x}) \quad \text{such that} \quad \langle \phi(t, \boldsymbol{x}) \rangle = \bar{\phi}(t) \quad \text{with} \quad \dot{\bar{\phi}}(t) \neq 0$$

➡ **Introducing** $\pi(t, \boldsymbol{x})$ **as embedding:** $\phi(t, \boldsymbol{x}) = \bar{\phi}\big(t + \pi(t, \boldsymbol{x})\big)$

Ex. Driven-dissipative condensate, Dynamics of inflaton, Synchronization

- Symmetry properties: Time-translation = Nonlinearly realized

$$\pi(t, \boldsymbol{x}) \to \pi'(t, \boldsymbol{x}) = \pi(t + \epsilon^0, \boldsymbol{x} + \boldsymbol{\epsilon}) + \epsilon^0$$

- Invariants: $t + \pi(t, \boldsymbol{x})$ and its derivatives e.g. $P_\mu \equiv \partial_\mu \big(t + \pi(t, \boldsymbol{x})\big)$

- General effective Lagrangian for in-out formalism:

$$\mathcal{L}_{\text{eff}}(\pi, \partial_\mu \pi) = \alpha_0(t + \pi) + \alpha_1(t + \pi) P_\mu P^\mu + \sum_{n \geq 2} \alpha_n(t + \pi)(P_\mu P^\mu + 1)^n$$

Noneq. EFT and diffusive NG mode

◆ *ra*-basis NG fields and transformation properties

$$\pi_r(t,\boldsymbol{x}) \equiv \frac{\pi_1(t,\boldsymbol{x})+\pi_2(t,\boldsymbol{x})}{2}, \quad \pi_a(t,\boldsymbol{x}) \equiv \pi_1(t,\boldsymbol{x})-\pi_2(t,\boldsymbol{x})$$

which transforms as
$$\begin{cases} \pi_r(t,\boldsymbol{x}) \to \pi'_r(t,\boldsymbol{x}) = \pi_r(t+\epsilon^0_r, \boldsymbol{x}+\boldsymbol{\epsilon}_r) + \epsilon^0_r \\ \pi_a(t,\boldsymbol{x}) \to \pi'_a(t,\boldsymbol{x}) = \pi_a(t+\epsilon^0_r, \boldsymbol{x}+\boldsymbol{\epsilon}_r) \end{cases}$$

◆ Part of effective Lagrangian in slow-roll limit ($\dot{\bar{\phi}}(t) = \text{const.}$)

$$\mathcal{L}^{(2)}_{\text{eff}} = (\alpha_1 - 2\alpha_2)\dot{\pi}_a\dot{\pi}_r - \alpha_1\partial_i\pi_a\partial_i\pi_r - 2\gamma_1\pi_a\dot{\pi}_r$$
$$+i\left[\beta_1\pi_a^2 - (\beta_2-\beta_4)\dot{\pi}_a^2 + \beta_2(\partial_i\pi_a)^2\right]$$

➡ **In the low-energy limit,** $c_s k \ll \gamma$

- **One gapped mode**
- **One diffusive NG mode:** $\omega \simeq -i\dfrac{c_s^2}{\gamma^2}\boldsymbol{k}^2$

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→ Diffusive NG mode!

$$\omega = -iD\boldsymbol{k}^2$$

Outlook

DERIVATION OF DRIVEN-DISSIPATIVE MODEL :

Integrating out environment (cf. Lindbrad equation for heavy quarks)

MODEL ANALYSIS OF NJL-MEAN FIELD AND BEYOND :

Solving time-dependent gap equation: DCDW vs Kink in real-time

APPLICATION TO DENSE SYSTEM :

Equation of state, transport properties, quantum anomaly ...