Hadron Interactions from Lattice QCD and Applications to Exotic Hadrons

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HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

S. Aoki, T. Aoyama, T. Miyamoto, K. Sasaki (YITP, Kyoto Univ.)
T. Doi, T. M. Doi, S. Gongyo, T. Hatsuda, T. Iritani (RIKEN)
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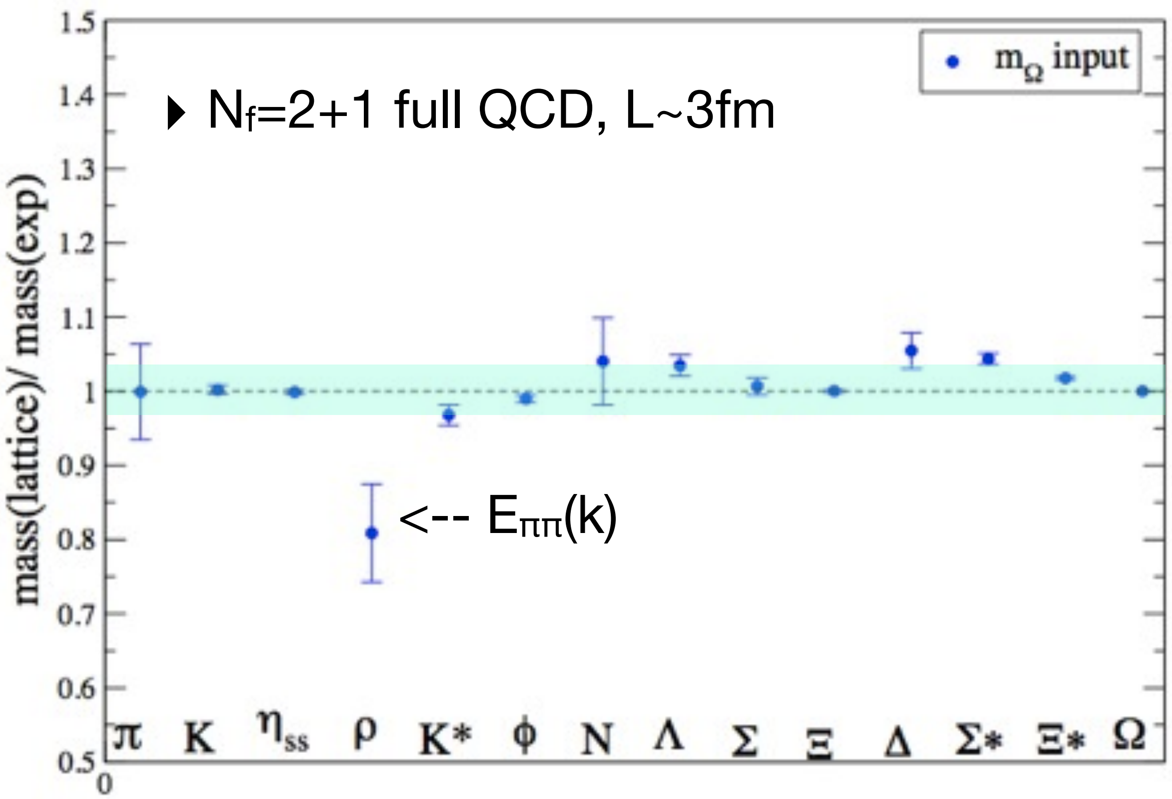
Hadrons to Atomic nuclei HAL from Lattice QCD

4th week in NFQCD2018 @ YITP, Kyoto Univ., Jun. 18-22, 2018.

Single hadron spectroscopy from LQCD

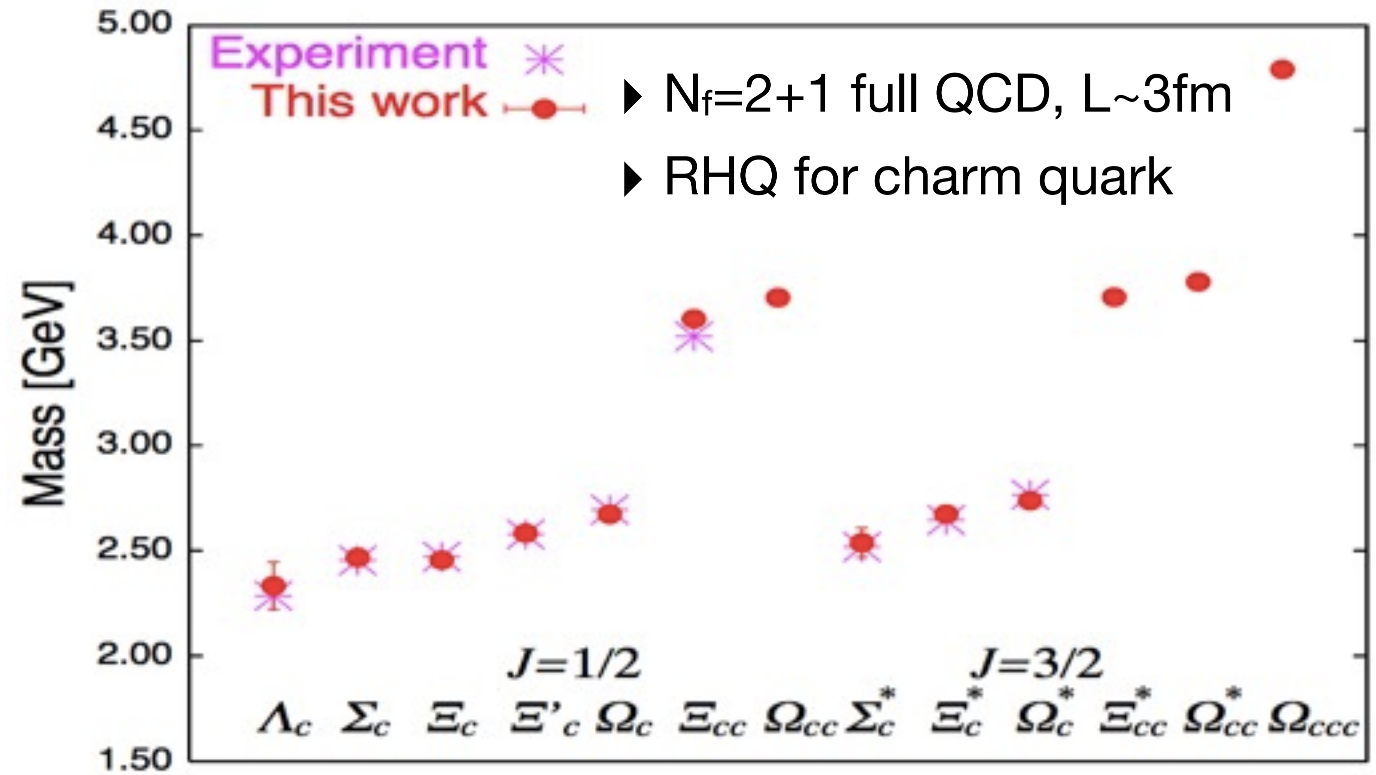
★ **Low-lying hadrons on physical point (physical m_q)**

light-quark sector

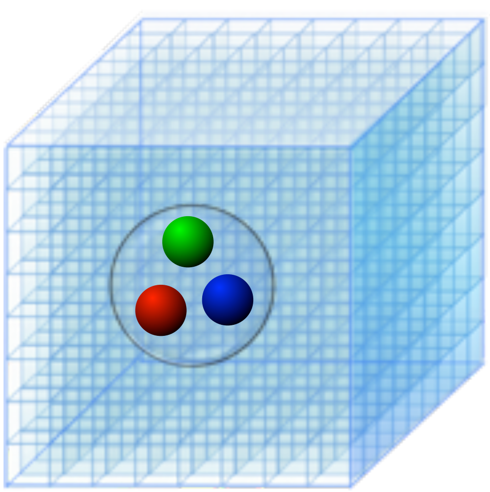


Aoki et al. (PACS-CS), PRD81 (2010).

charm baryons



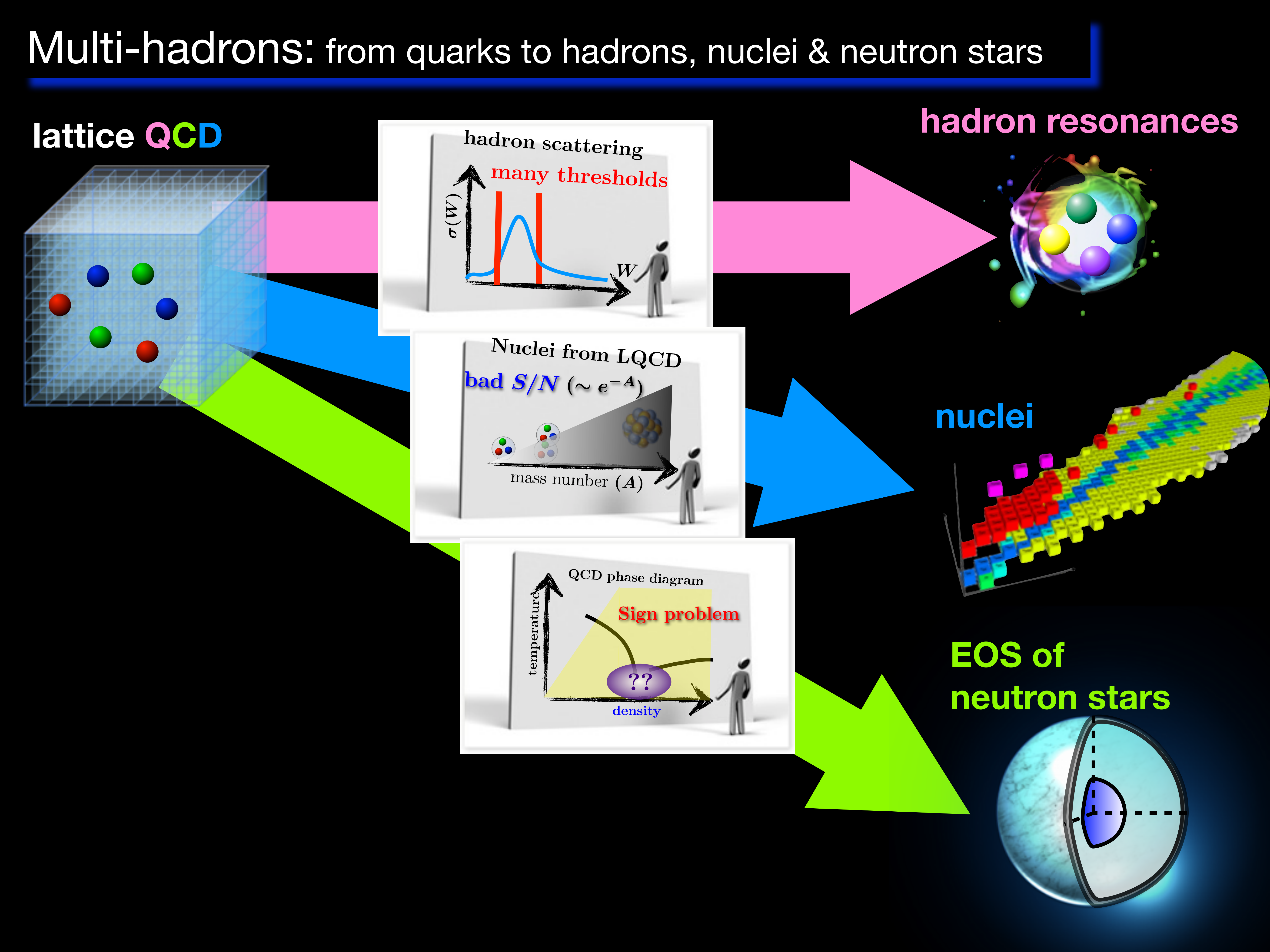
Namekawa et al. (PACS-CS), PRD84 (2011); PRD87 (2013).



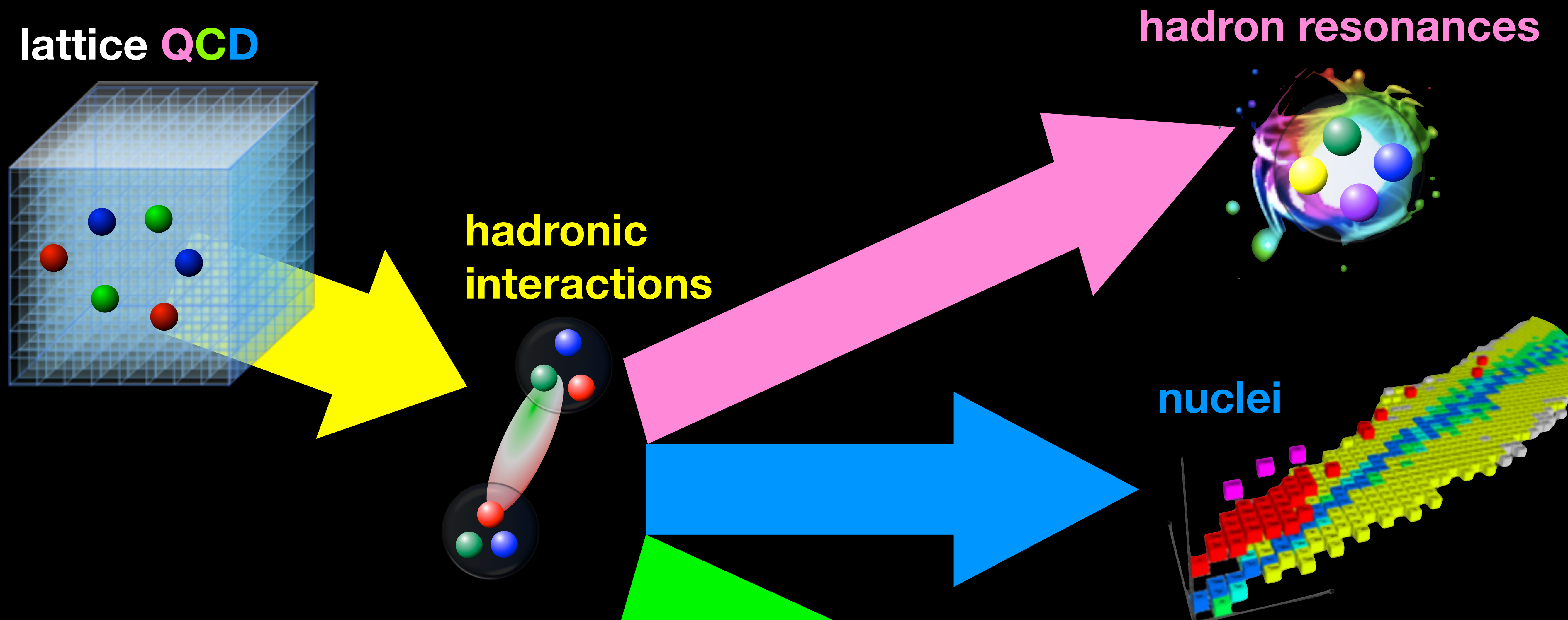
a few % accuracy already achieved for single hadrons

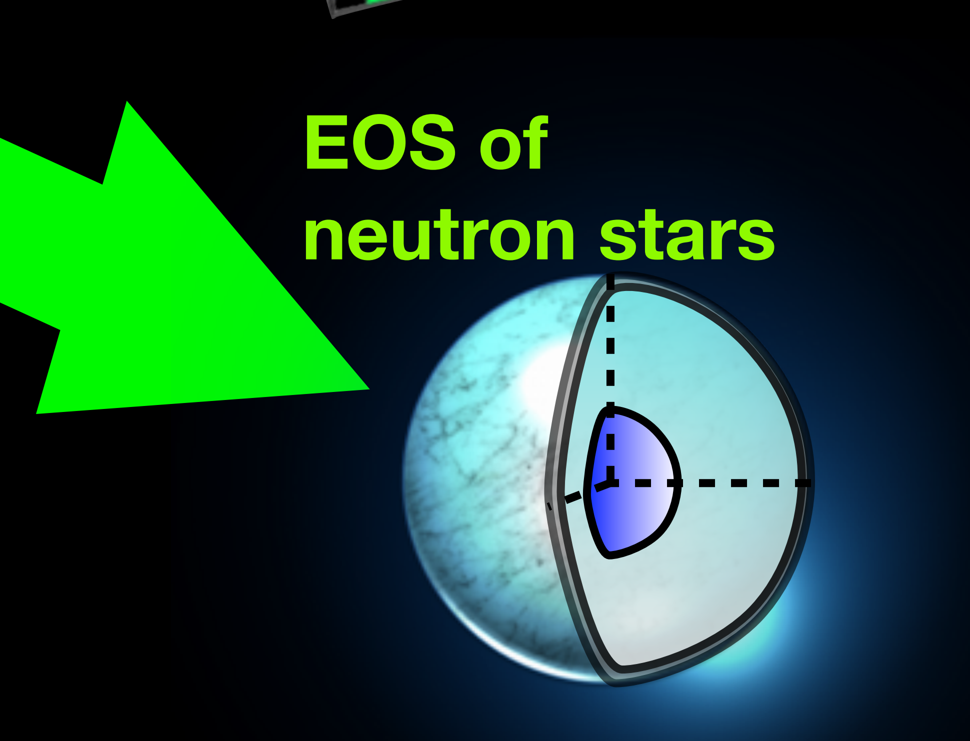
LQCD predictions of undiscovered charm hadrons (Ξ^*_{cc}, Ω_{ccc}, ...)

➡ **Next challenge : multi-hadron systems**


Multi-hadrons: from quarks to hadrons, nuclei & neutron stars
lattice QCD
hadron scattering
many thresholds
σ(W)
W
Nuclei from LQCD
bad S/N (∼ e^{−A})
mass number (A)
QCD phase diagram
Sign problem
temperature
??
density
hadron resonances
nuclei
EOS of
neutron stars

HAL QCD strategy: from quarks to hadrons, nuclei & neutron stars





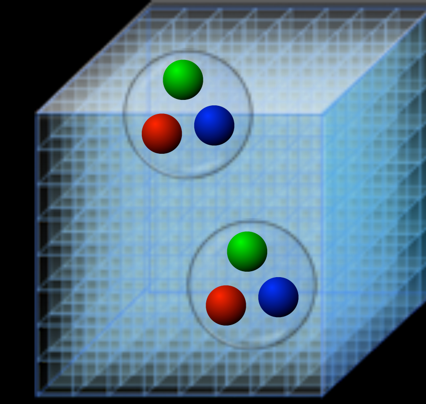
Part I: hadronic interactions

- difficulties in multi-hadron systems
- solution = HAL QCD method

Part II: applications to exotic candidates

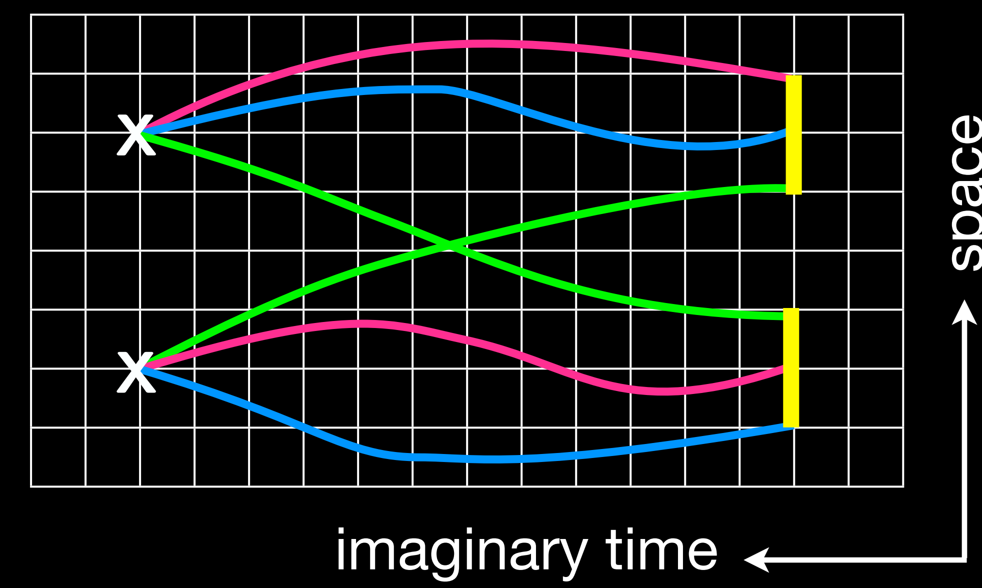
- coupled-channel scattering from LQCD
- dibaryon & tetraquark candidates

Hadronic interactions from LQCD



hadronic correlation function

$$C_{NN}(\vec{r},t) \equiv \langle 0|N_1(\vec{r},t)N_2(\vec{0},t)\mathcal{J}^\dagger(t=0)|0\rangle$$
$$= \sum_n A_n \psi_n(\vec{r}) e^{-W_n t}$$



Energy eigenvalue W_n & NBS (Nambu-Bethe-Salpeter) wave function $\psi_n(r)$

Finite Volume Method

- $W_n(L)$ --------> phase shift

↑ **Lüscher's formula**

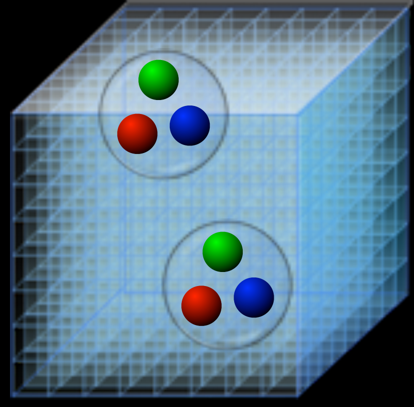
Lüscher's finite volume formula

Lüscher, Nucl. Phys. B354, 531 (1991).

$$k_n \cot\delta(k_n) = \frac{4\pi}{L^3} \sum_{m\in\mathbb{Z}^3} \frac{1}{\vec{p}_m^{\,2} - k_n{}^2}$$

$$W_n = \sqrt{m_1^2 + k_n^2} + \sqrt{m_2^2 + k_n^2}$$

Hadronic interactions from LQCD



hadronic correlation function

$$C_{NN}(\vec{r},t) \equiv \langle 0|N_1(\vec{r},t)N_2(\vec{0},t)\mathcal{J}^\dagger(t=0)|0\rangle$$
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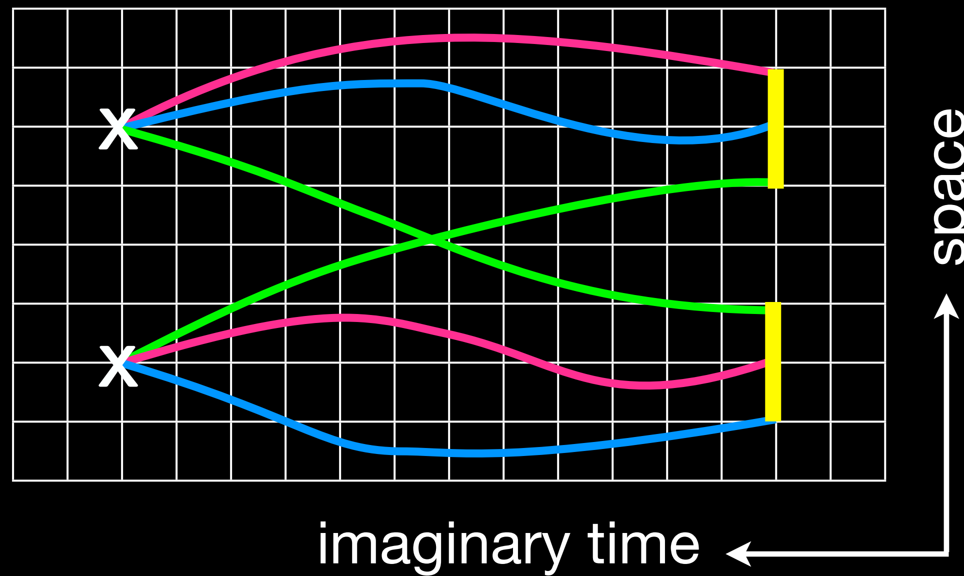


Energy eigenvalue W_n & NBS (Nambu-Bethe-Salpeter) wave function $\psi_n(r)$

Finite Volume Method

- $W_n(L)$ --------> phase shift

  ↑ **Lüscher's formula**

  Lüscher, Nucl. Phys. B354, 531 (1991).

➡ Serious difficulty to measure $W_n(L)$ in multi-hadron systems

HAL QCD Method

- $\psi_n(r)$ --> **2PI kernel** ($\psi = \varphi + G_0 U \psi$)

  --> phase shift, binding energy, ...

  Ishii, Aoki, Hatsuda, PRL 99, 022001 (2007).

  Ishii et al. [HAL QCD], PLB 712, 437 (2012).

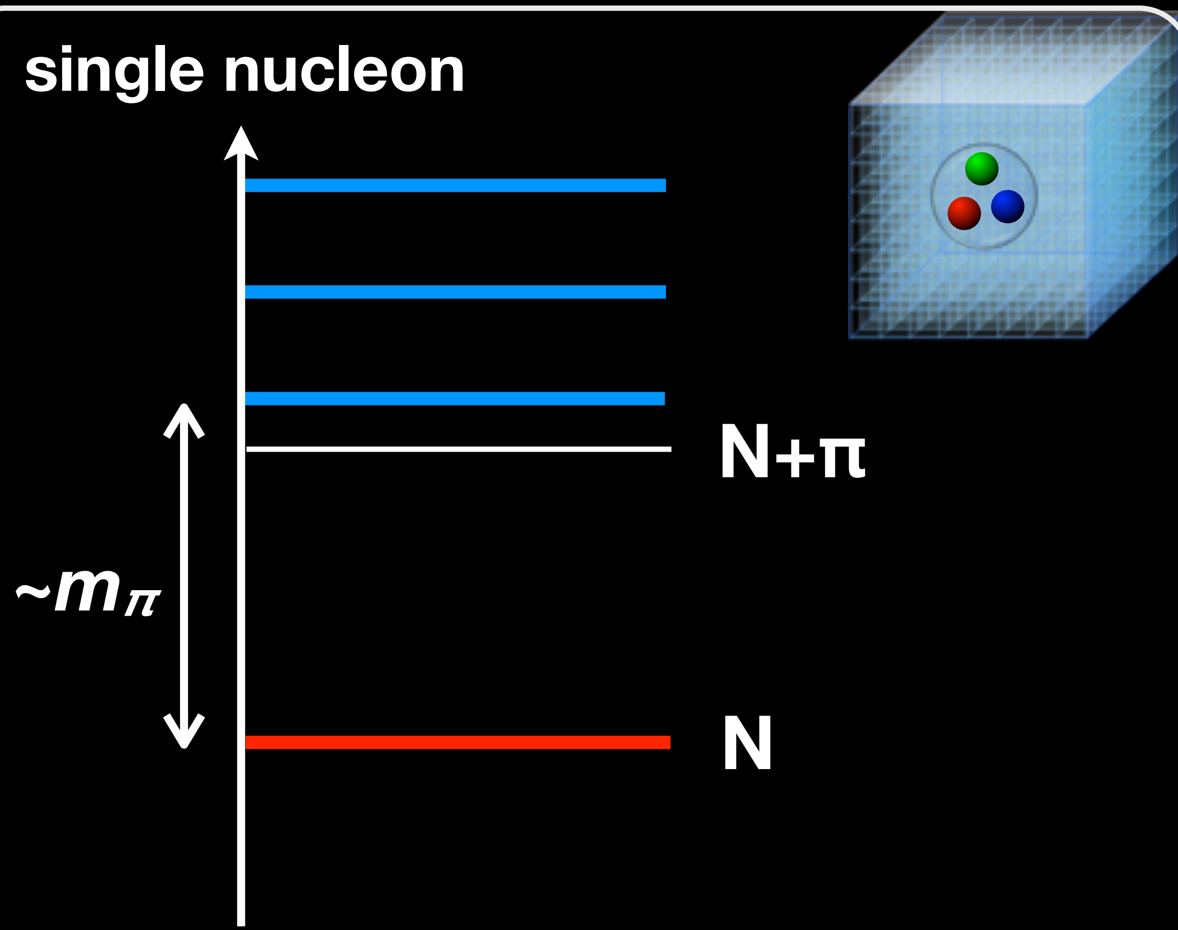
Fundamental difficulty in multi-hadron systems

see, Iritani, Doi et al. [HAL QCD], JHEP10 (2016) 101.

$$C_N(t) = a_0 e^{-m_N t} + c_1 e^{-(m_N + m_\pi)t} + \cdots$$
$$\longrightarrow a_0 e^{-m_N t} \quad (t > t^*)$$

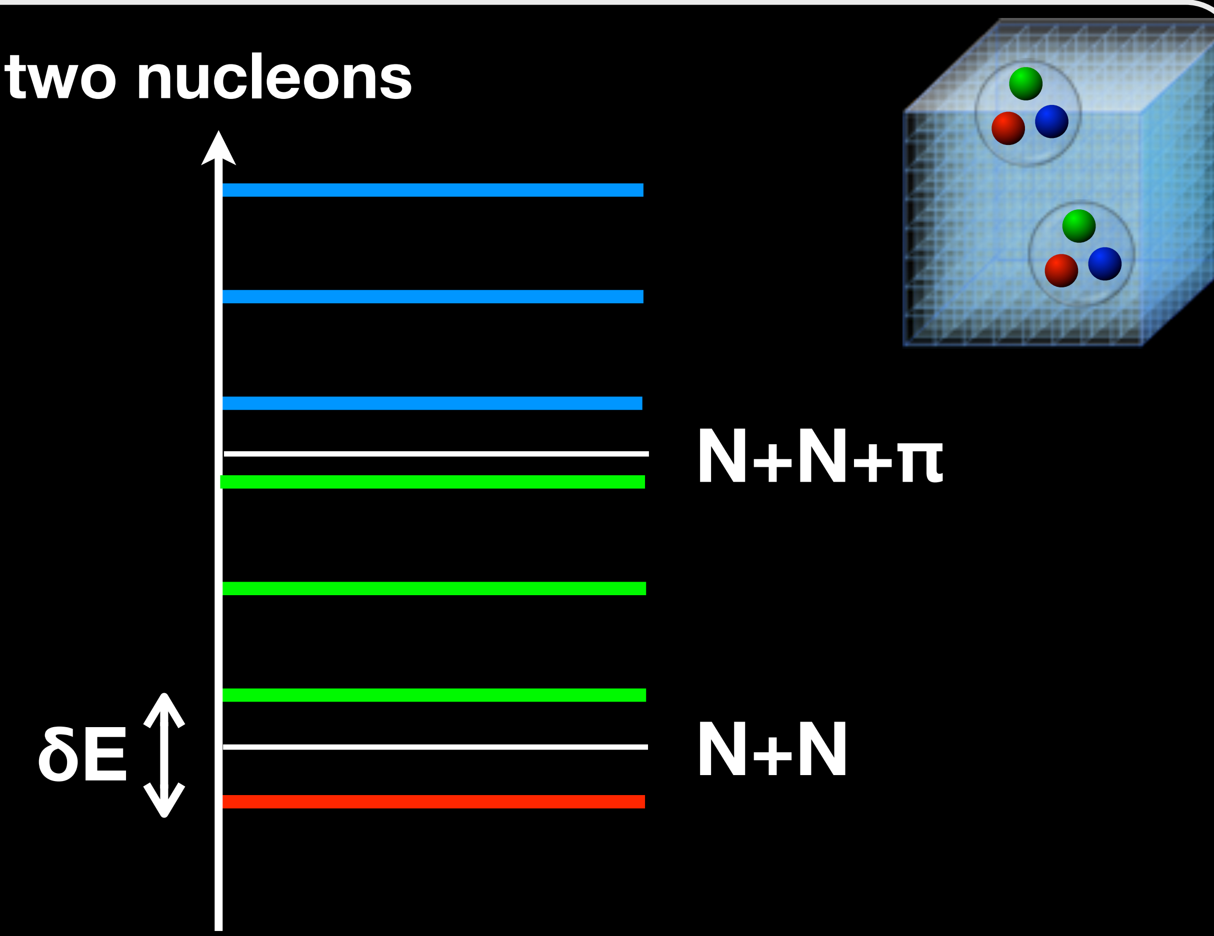
$$C_{NN}(t) = b_0 e^{-W_0 t} + b_1 e^{-W_1 t} + \cdots$$
$$\longrightarrow b_0 e^{-W_0 t} \quad (t > t^*)$$

single nucleon



$t^* \sim m_\pi^{-1} \sim 1$ fm

two nucleons



$t^* \sim \delta E^{-1} \sim m_N (L/2\pi)^2 \sim 10$ fm

$$\mathcal{S}/\mathcal{N} \sim \sqrt{N_{\rm conf.}} \times 10^{-2}$$

$$\mathcal{S}/\mathcal{N} \sim \sqrt{N_{\rm conf.}} \times 10^{-30}$$

➡ S/N becomes far worse in multi-hadron systems

Demonstration of plateau method by mock-up data

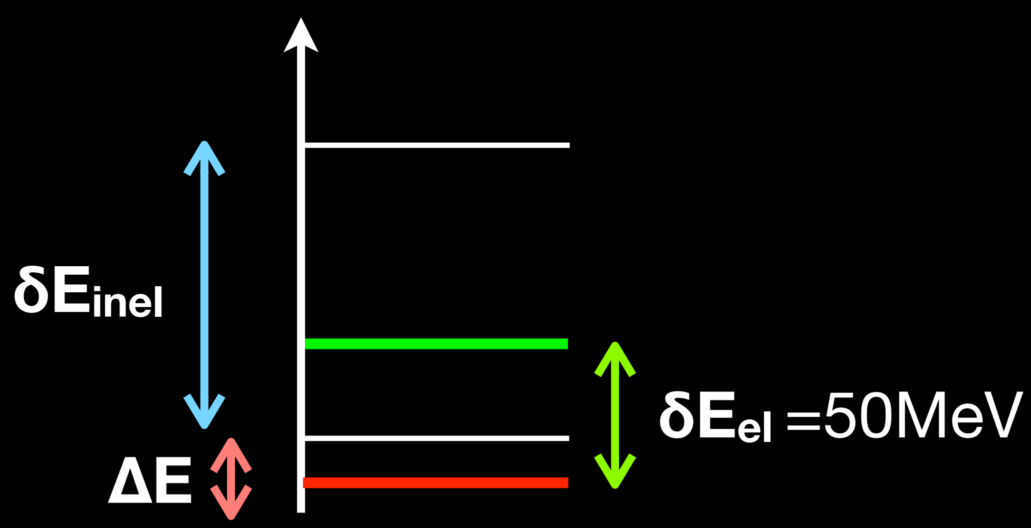
"Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD"

Iritani, Doi et al. [HAL QCD], JHEP10 (2016) 101.

- **Normalized correlation func. *R(t)*** for two baryons in mock-up data

$$R(t) = \frac{C_{BB}(t)}{C_B(t)^2} = b_1 e^{-\Delta E t} + b_2 e^{-\delta E_{\rm el} t} + c_1 e^{-\delta E_{\rm inel} t}$$

$$\Delta E^{\rm eff}(t) = \log\left[\frac{R(t)}{R(t+1)}\right] \xrightarrow[t > t^*]{} \Delta E$$



- Ground state energy $\Delta E = W_{BB} - 2m_B$

  ~1 MeV precision necessary (nuclear physics scale)

- Elastic scattering states $\delta E_{\rm el}$

  $\delta E_{\rm el}$ = 50MeV, $b_2/b_1 = \pm 0.1, 0$ (10% contamination)

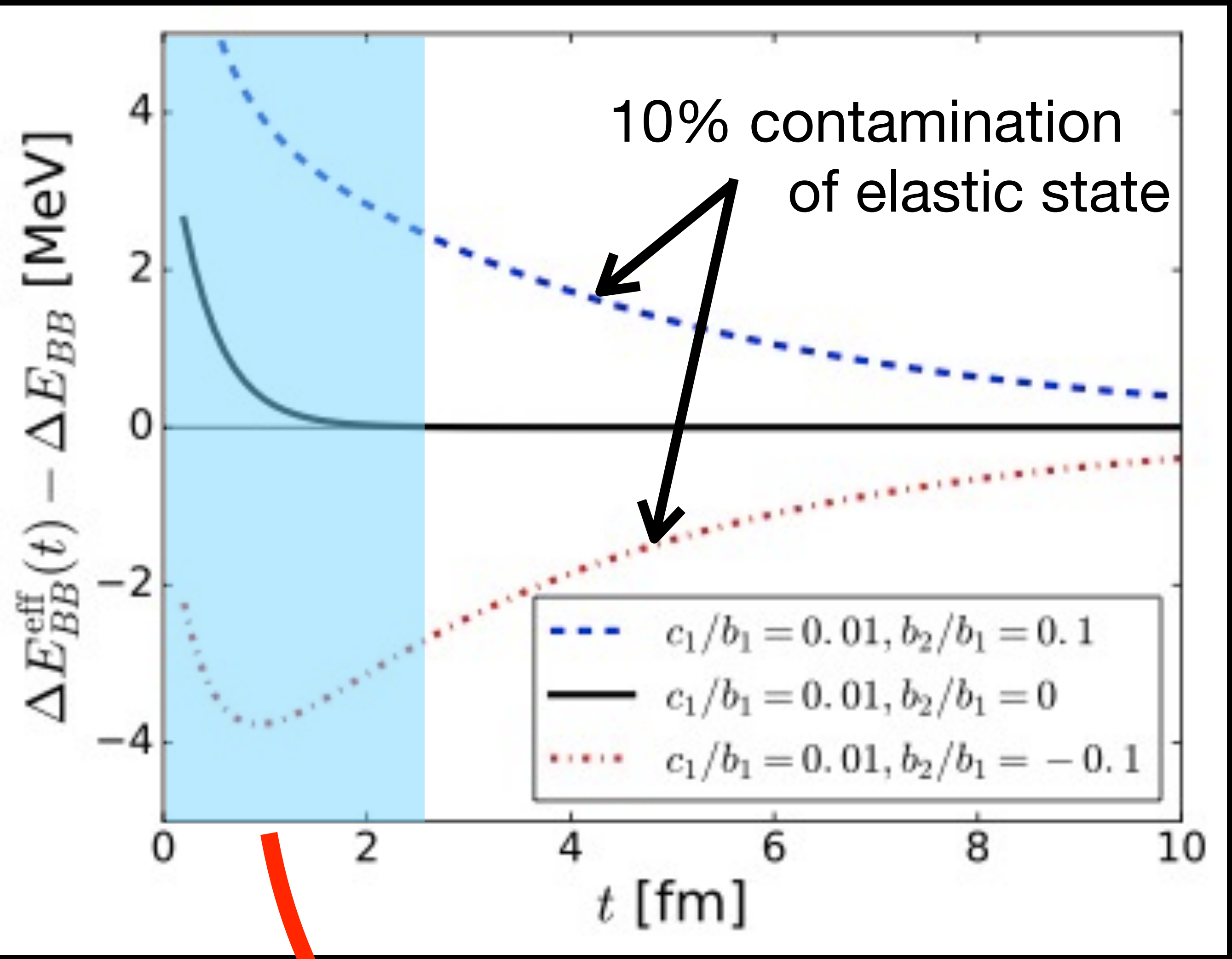
- Inelastic threshold $\delta E_{\rm inel}$

  $\delta E_{\rm inel}$ = 500MeV, $c_1/b_1 = 0.01$ (1% contamination)

Demonstration of plateau method by mock-up data

"Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD"

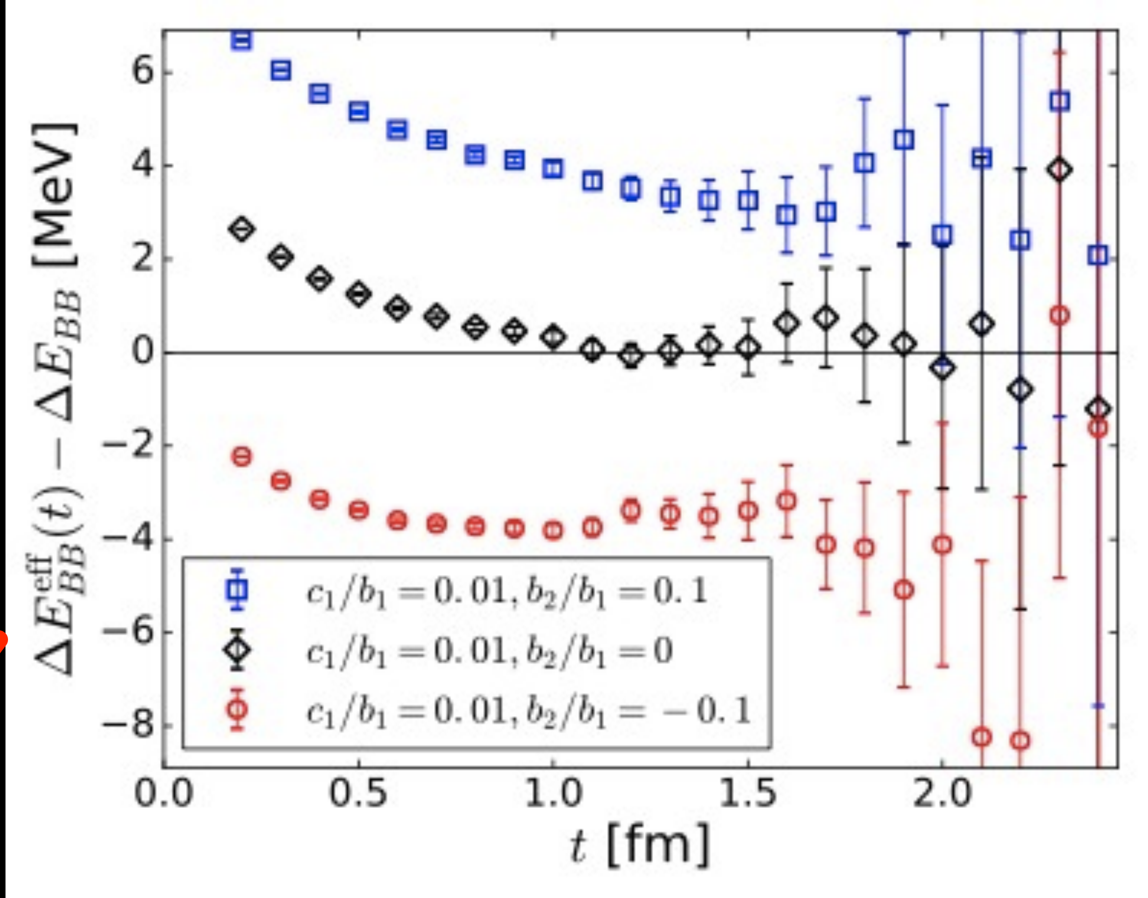
Iritani, Doi et al. [HAL QCD], JHEP10 (2016) 101.



➡ **True ground state for t > 8 fm with 10% contamination**

Fake plateaux or "Mirage" appear at t ~ 1 fm

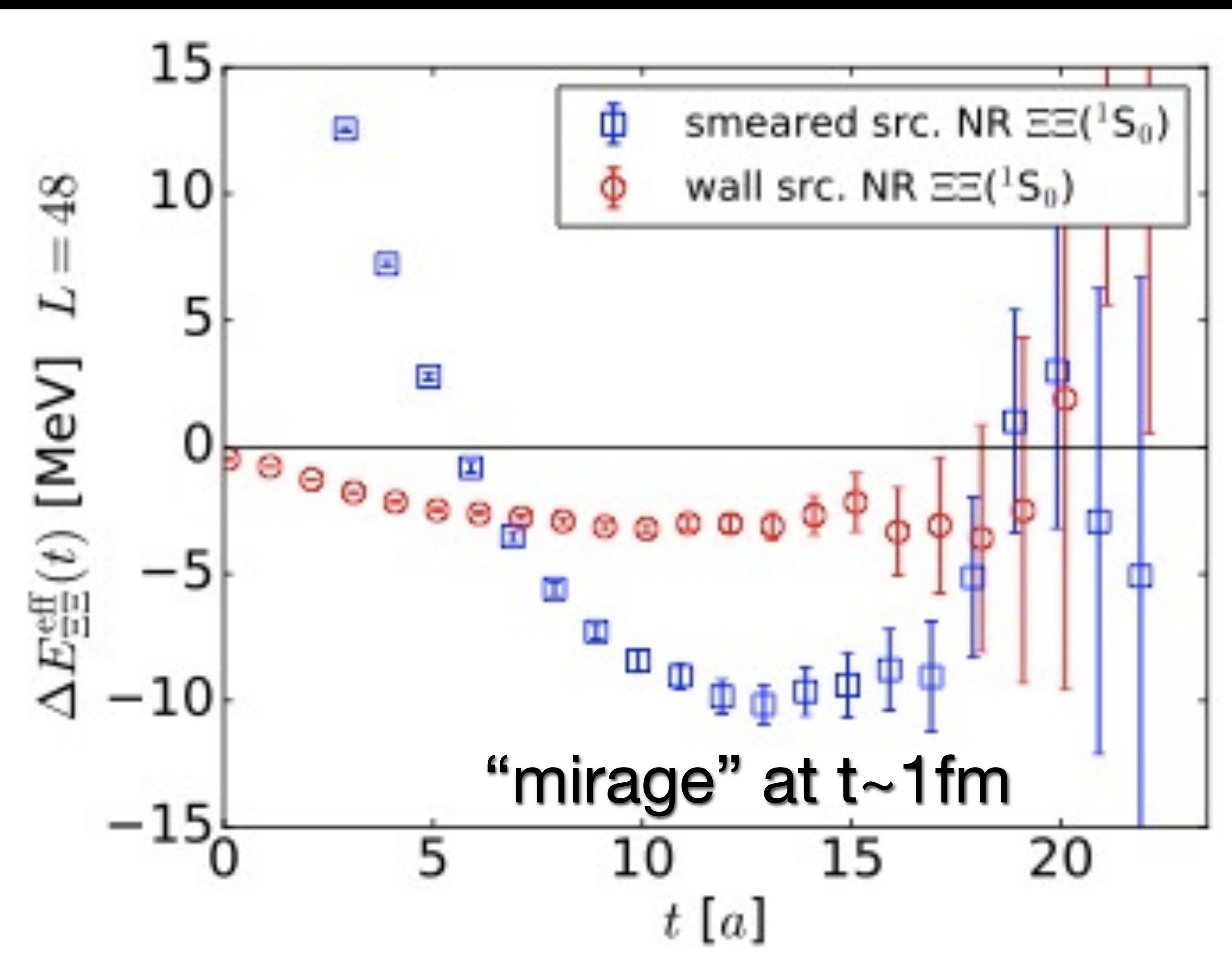
Zoom + typical stat. error



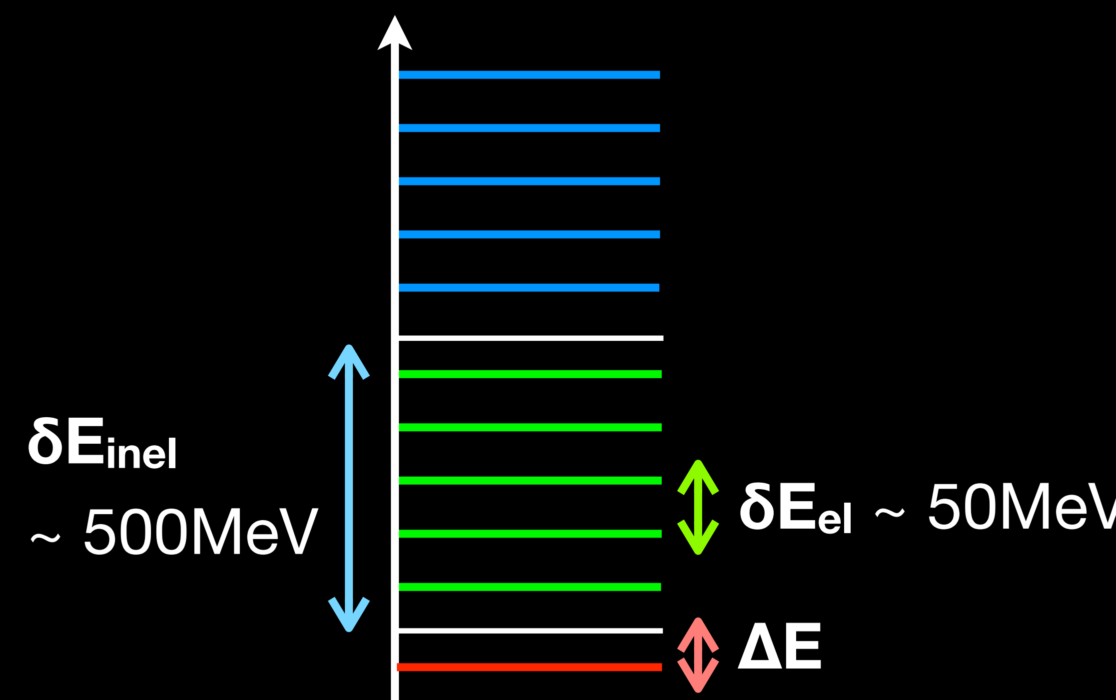
Actual data for ΞΞ ($^1S_0$) @m_π=0.51GeV, L=4.3fm, a=0.09fm

Source-operator dependence in plateau method

$$R(t) = \sum_{\vec{x},\vec{y}} \langle 0 | B_1(\vec{x},t) B_2(\vec{y},t) \mathcal{J}^\dagger(t=0) | 0 \rangle / C_B(t)^2$$



$$\Delta E^{\rm eff}(t) = \log\left[\frac{R(t)}{R(t+1)}\right] \xrightarrow[t > t^*]{} \Delta E$$



➡ **True ground state is to appear at t > t* (t*~8 fm)**

- At least one of "plateaux" is fake! (Data at t~1fm is too early to identify plateau.)
- Naive plateau method does NOT work --> variational method (next talk)

A solution: HAL QCD method -- potential as a representation of S-matrix --

- The scattering states do exist, and we should tame the scattering states

➡ **HAL QCD method** Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al. [HAL QCD], PLB 712, 437 (2012).

✓ define energy-independent potential *U(r,r')*

$$\int d\vec{r}' U(\vec{r},\vec{r}')\psi_n(\vec{r}') = (E_n - H_0)\,\psi_n(\vec{r})$$

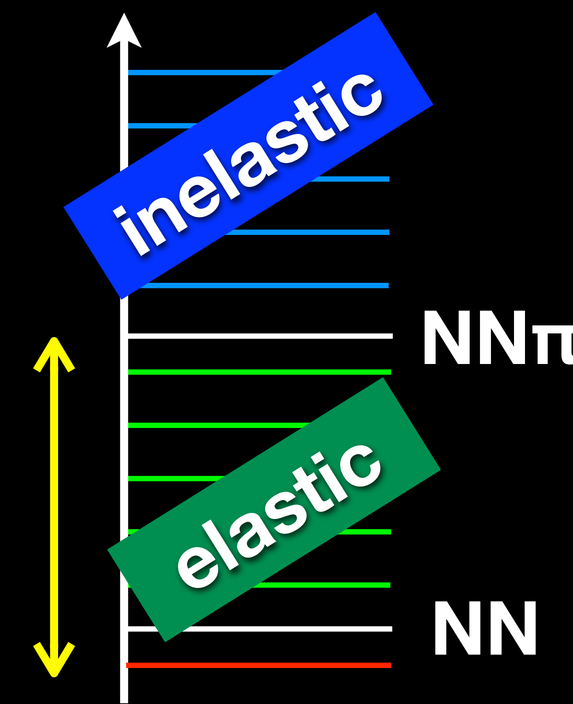
$$U(\vec{r},\vec{r}') \equiv \sum_{n=0}^{n_{\rm th}} (E_n - H_0)\,\psi_n(\vec{r})\bar{\psi}_n(\vec{r}')$$

➡ All elastic states share the same potential U(r,r')

$U\,\psi_0 = (E_0 - H_0)\,\psi_0$

$U\,\psi_1 = (E_1 - H_0)\,\psi_1$

⋮



✓ derive *U(r,r')* from time-dependent Schrödinger-type eq.

$$\int d\vec{r}' U(\vec{r},\vec{r}') R(\vec{r}',t) = \left(-\frac{\partial}{\partial t} + \frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - H_0\right) R(\vec{r},t)$$

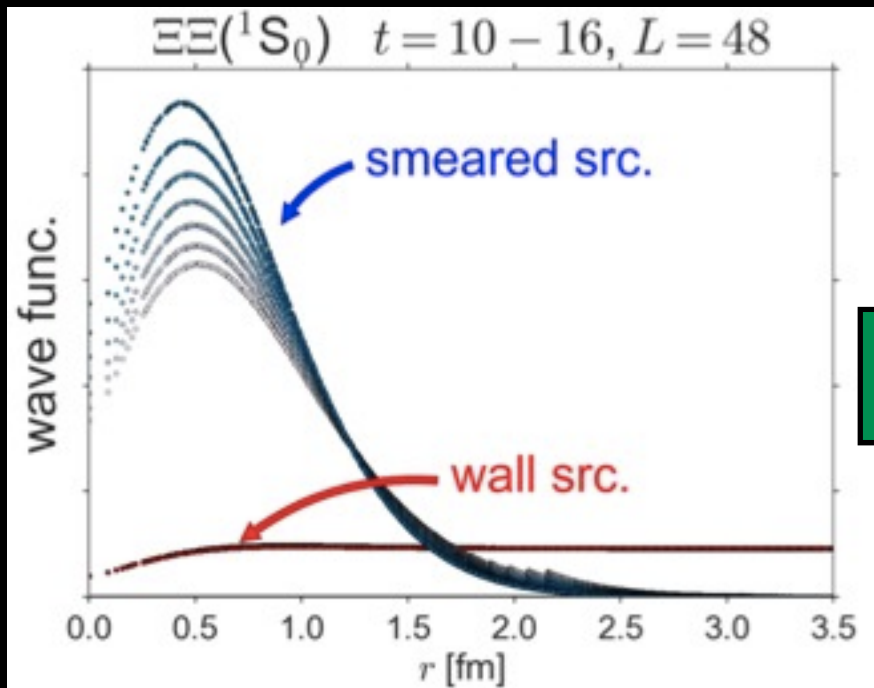
$$R(\vec{r},t) = e^{2m_B t} C_{BB}(\vec{r},t)$$

➡ **Elastic scat. states are no more contamination than signal** (***t*~1fm***)

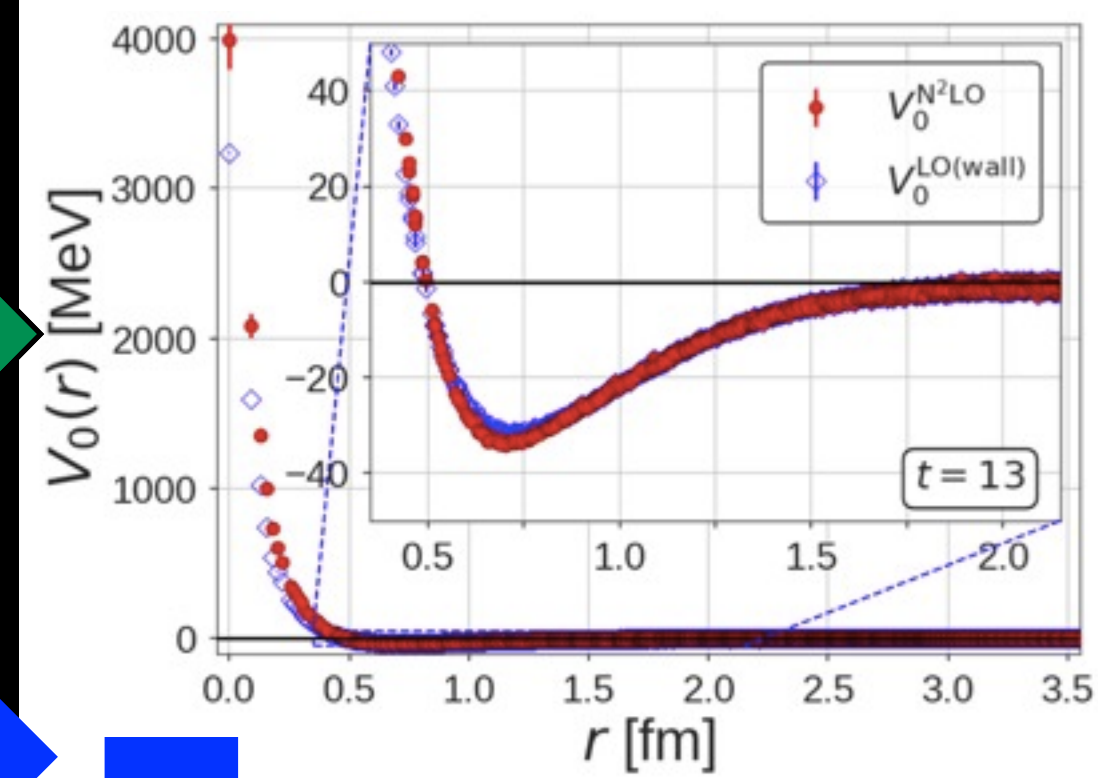
ΞΞ ($^1S_0$) in HAL QCD method @m_π=0.51GeV, L=4.3fm, a=0.09fm

Iritani et al. [HAL QCD], arXiv:1805.02365 [hep-lat].

source dependence of R(r,t)

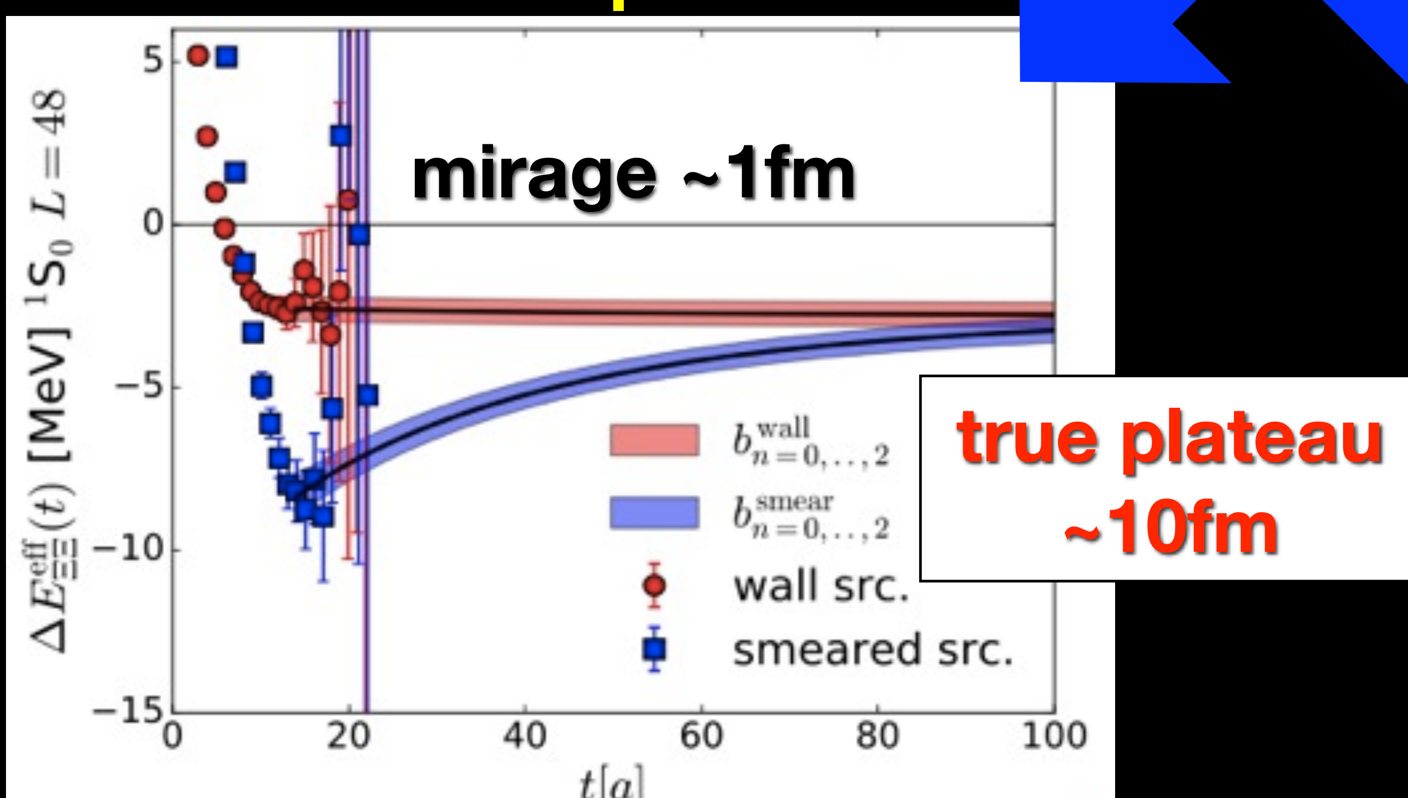


$$U(\vec{r}, \vec{r}') = [V_0(\vec{r}) + V_2(\vec{r})\nabla^2 \cdots]\, \delta(\vec{r} - \vec{r}')$$

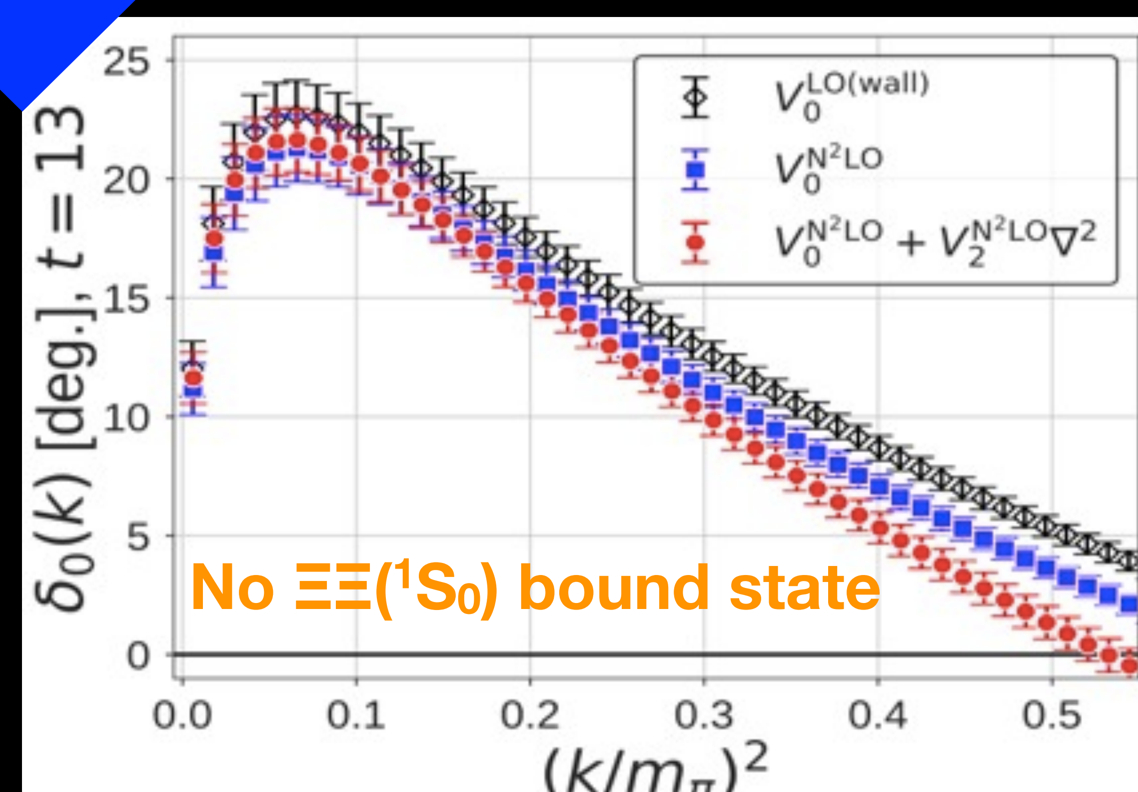


finite V calc.

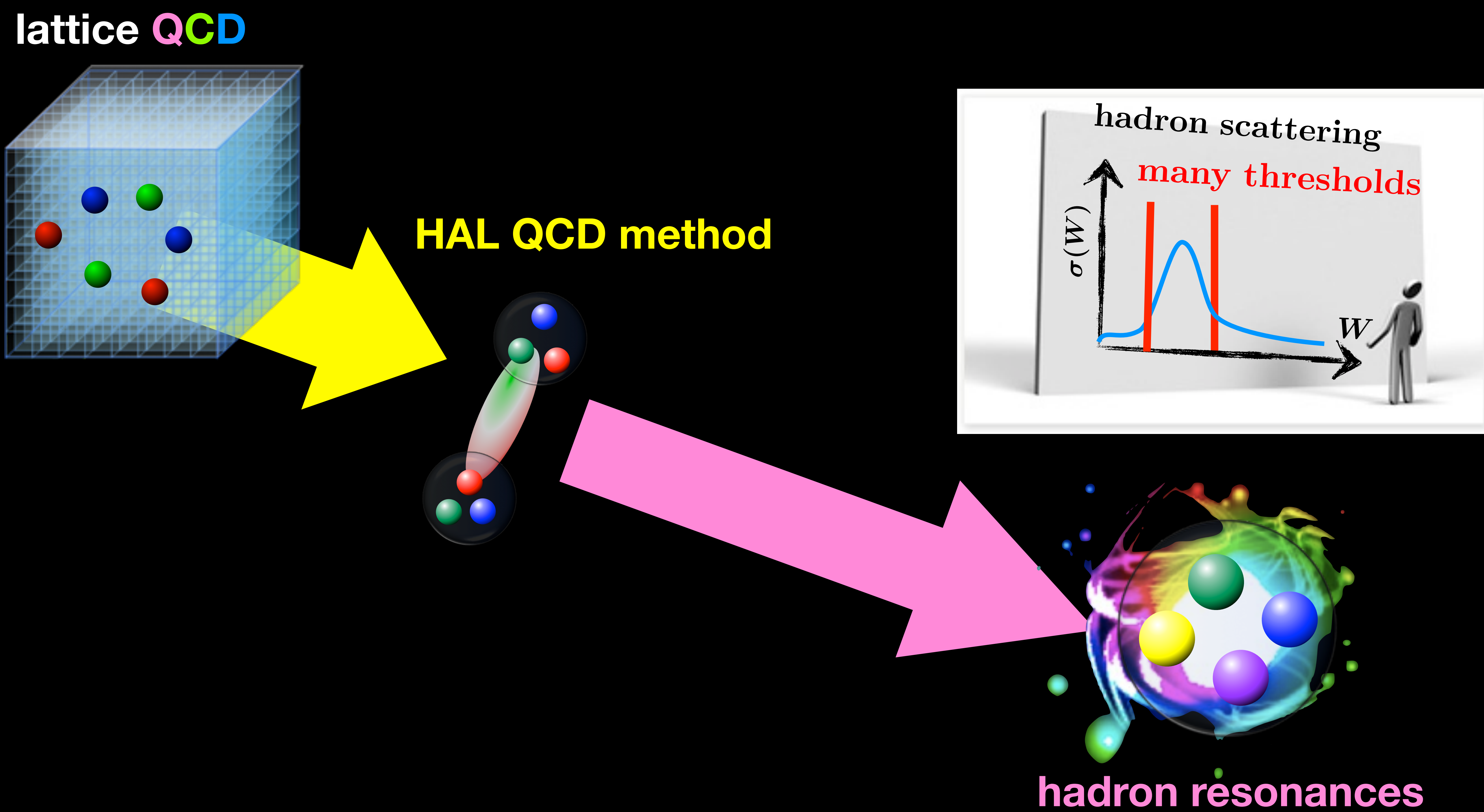
➡ Fate of fake plateaux



infinite V calc.

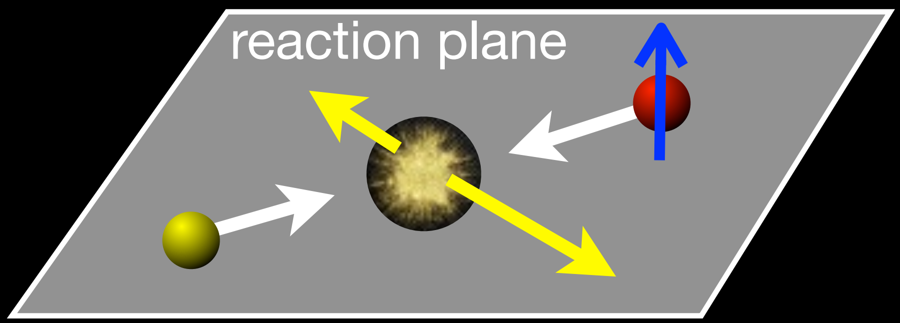


Multi-hadron spectroscopy from LQCD



- Resonances are embedded into coupled-channel scattering states

How can we find resonances?



Coupled-channel scatterings

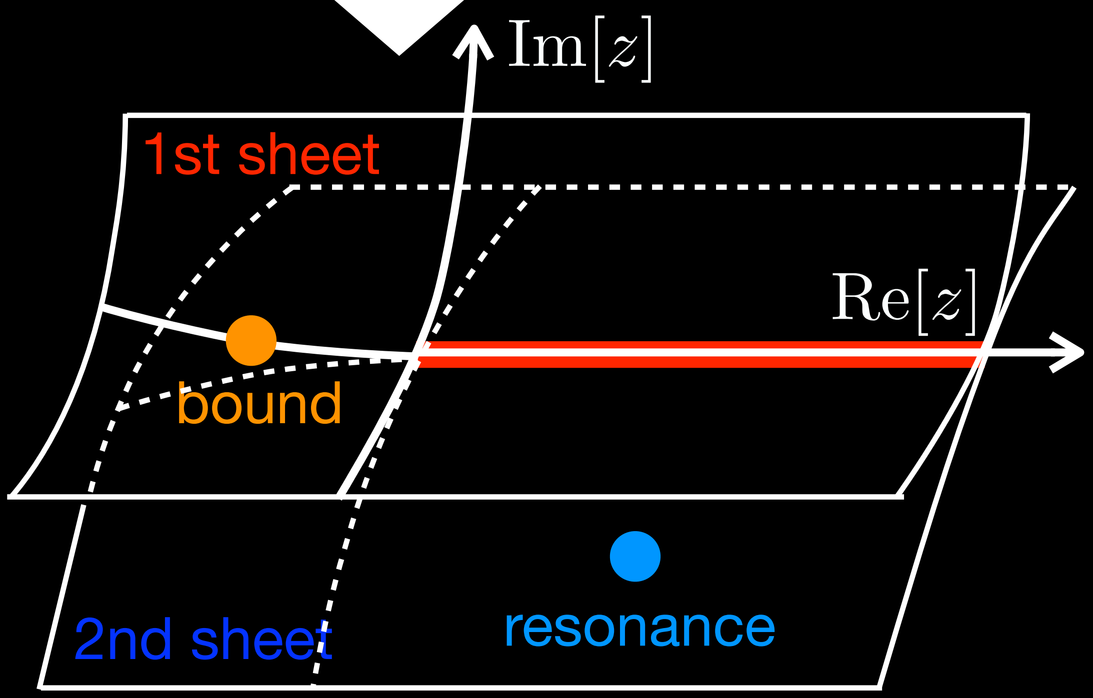
$S^{(\ell)}(W)$ ⇐ **partial wave analysis of expt. data**

- cross sections (dσ/dΩ)
- spin polarization observables
- etc.

⇓

Analyticity of S-matrix is **uniquely** determined ⇐ **identical theorem + dispersion theory**

⇓



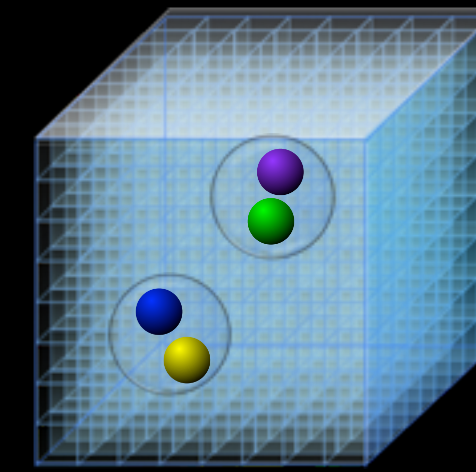
bound state (1st sheet)

- pole position --> binding energy
- residue --> coupling to scattering state

resonance (2nd sheet)

- analytic continuation onto 2nd sheet
- pole position --> resonance energy
- residue --> coupling to scat. state, partial decay

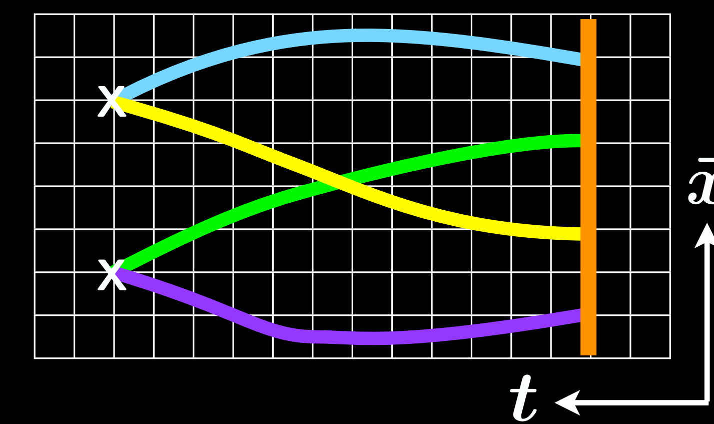
Strategy to search for complex poles on the lattice

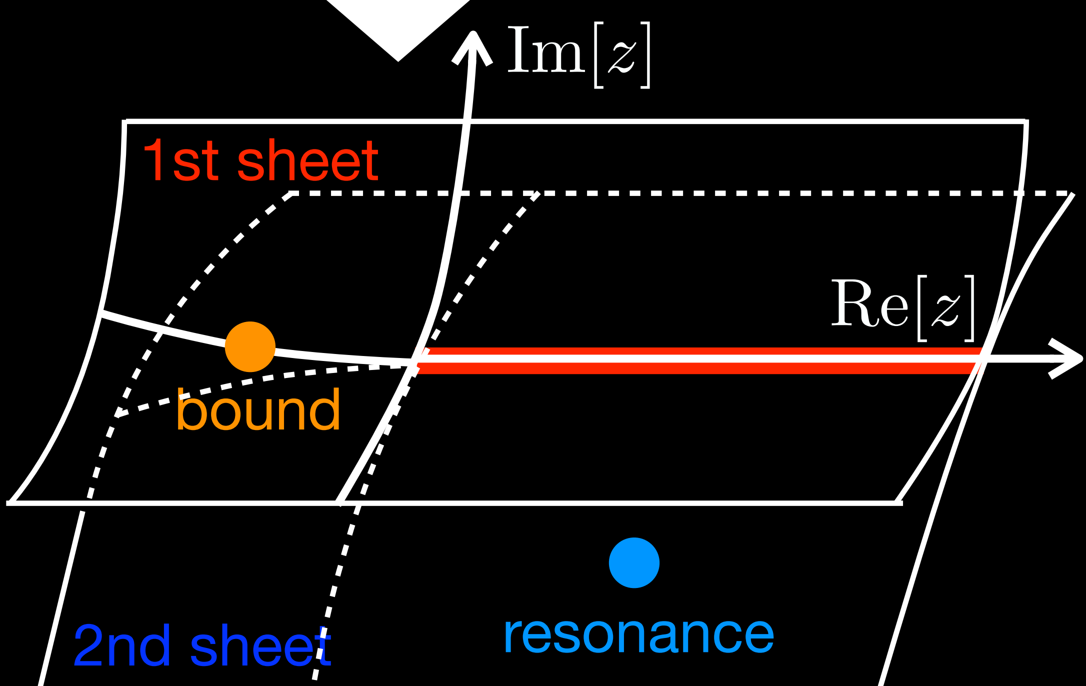


Coupled-channel scatterings from lattice QCD

$S^{(\ell)}(W)$

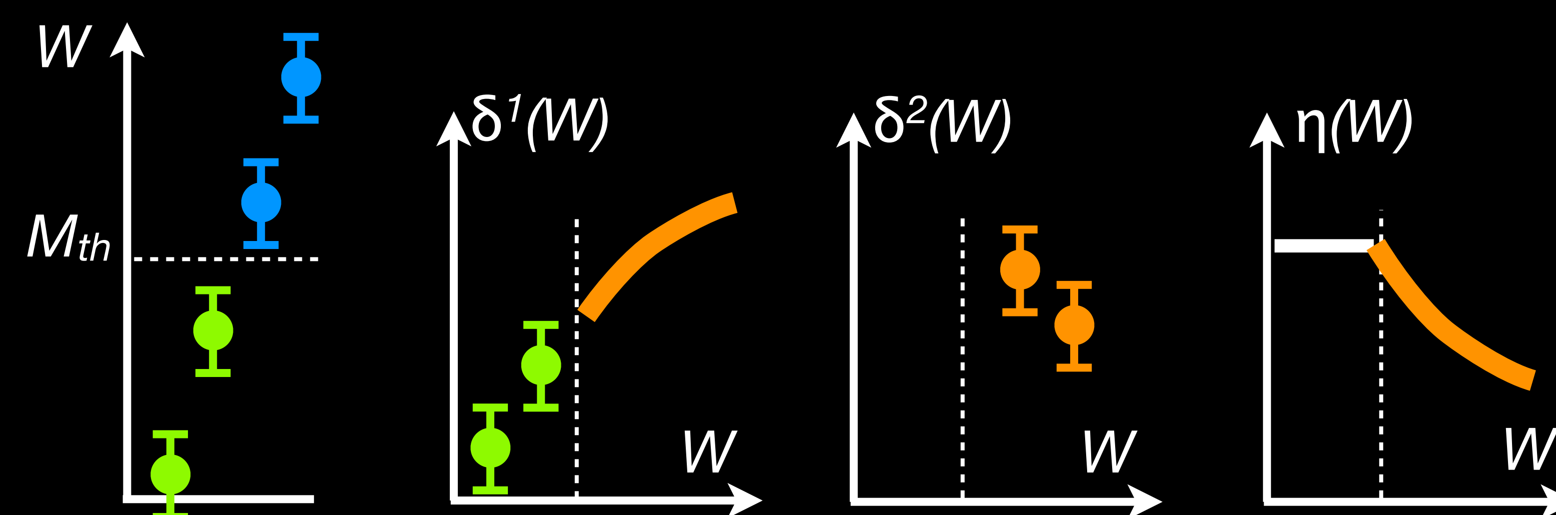
$$\langle 0|\phi_1(\vec{r},t)\phi_2(\vec{0},t)\mathcal{J}^\dagger(t=0)|0\rangle = \sum_n A_n \psi_n(\vec{r}) e^{-W_n t}$$





❖ **coupled-channel Lüscher's method**

➡ $W_n(L)$ --> $\delta^1(W_n)$, $\delta^2(W_n)$, $\eta(W_n)$

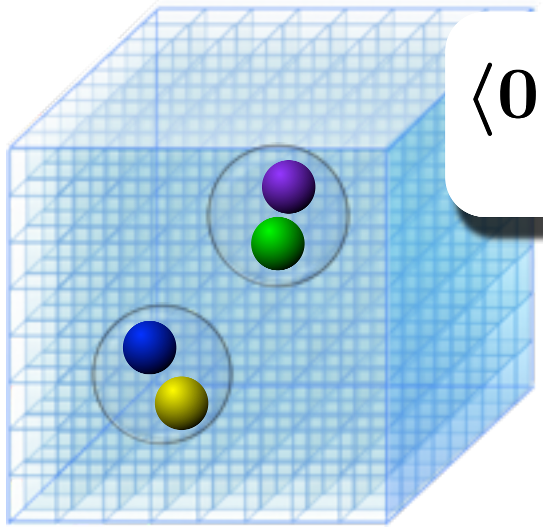


(**coupled-channel scattering difficult**)

‣ $\delta^1(W_n)$, $\delta^2(W_n)$, $\eta(W_n)$ <-- $W_n(L_1) = W_n(L_2) = W_n(L_3)$

Coupled-channel HAL QCD method

✦ **measure relevant NBS wave function --> channel is defined**



$$\langle 0|\phi_1^a(\vec{x}+\vec{r},t)\phi_2^a(\vec{0},t)\mathcal{J}^\dagger(0)|0\rangle = \sqrt{Z_1^a Z_2^a}\sum_n A_n \psi_n^a(\vec{r})e^{-W_n t}$$

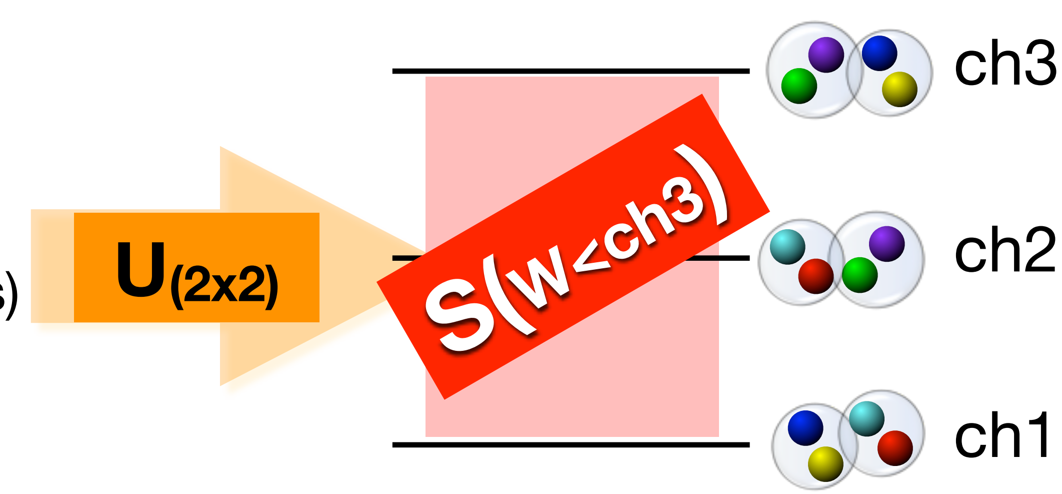
Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al. (HAL QCD), PLB712, 437(2012).

★ **Nambu-Bethe-Salpeter (NBS) wave function in each channel**
➡ **derive 2PI kernel (potential) as a representation of S-matrix**

$$\left(\nabla^2 + (\vec{k}_n^a)^2\right)\psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}\,' U^{ab}(\vec{r},\vec{r}\,')\psi_n^b(\vec{r}\,')$$

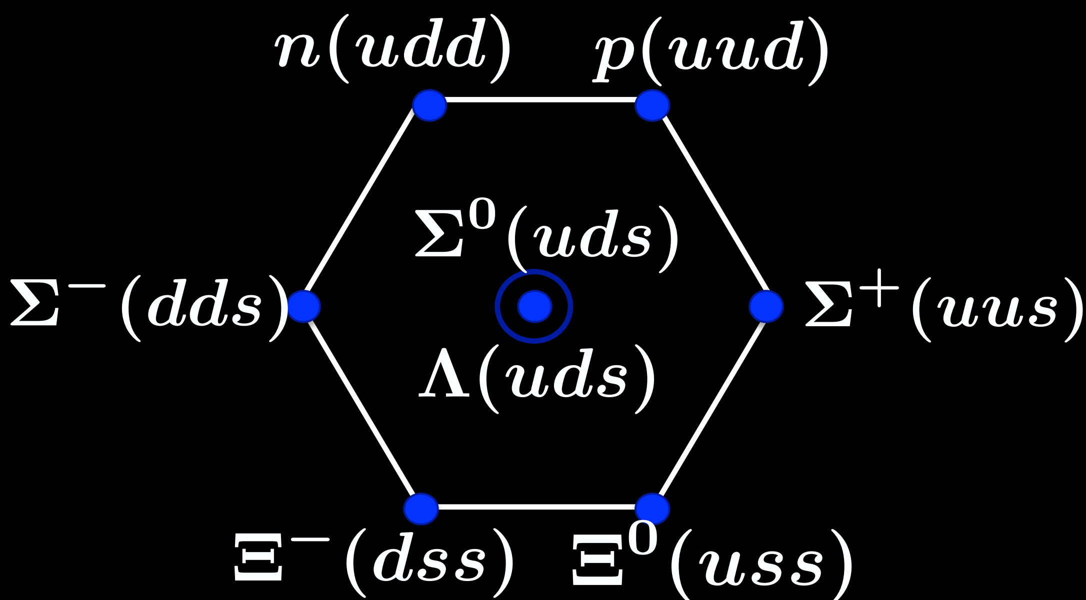
★ **coupled-channel potential U^{ab}(r,r'):**

- U^{ab}(r,r') is faithful to **coupled-channel S-matrix**
- U^{ab}(r,r') is **energy independent** (until new threshold opens)
- Non-relativistic approximation is not necessary
- U^{ab}(r,r') contains all 2PI contributions

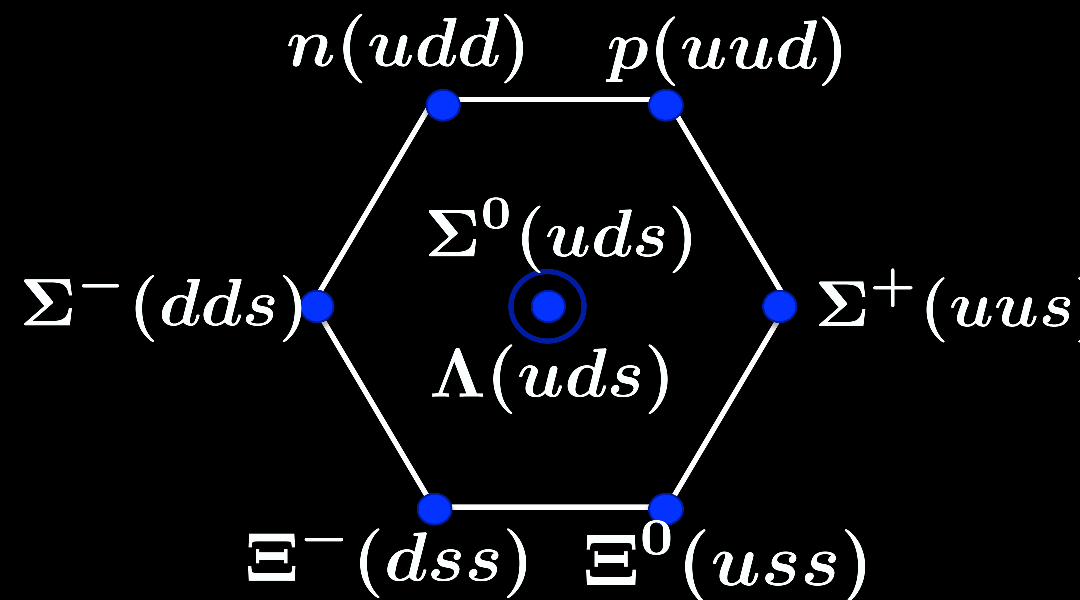


Full details, Aoki et al. (HAL QCD), PRD87, 034512 (2013); Proc. Jpn. Acad., Ser. B, 87 (2011).

Octet BB forces & H-dibaryon



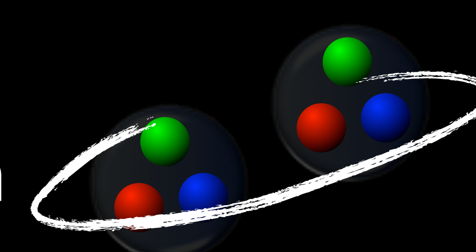
X



$$= (27 \oplus 8 \oplus 1)_{\text{sym.}} \oplus (10^* \oplus 10 \oplus 8)_{\text{anti-sym.}}$$

↑ *NN* ($^1S_0$)

↑ *NN* ($^1S_0$) : deuteron



↑ H-dibaryon (ΛΛ-ΞN-ΣΣ)? **Jaffe (1977)**

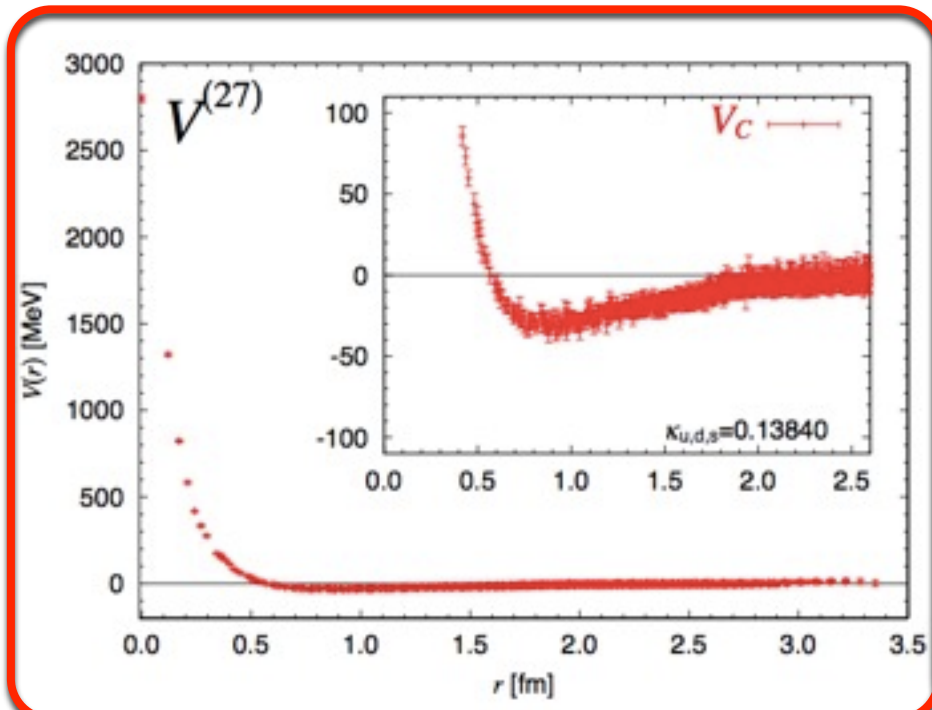
Generalized BB forces in flavor SU(3) limit

- Full QCD in $SU(3)_F$ limit : m_π~0.47GeV, L=3.9 fm

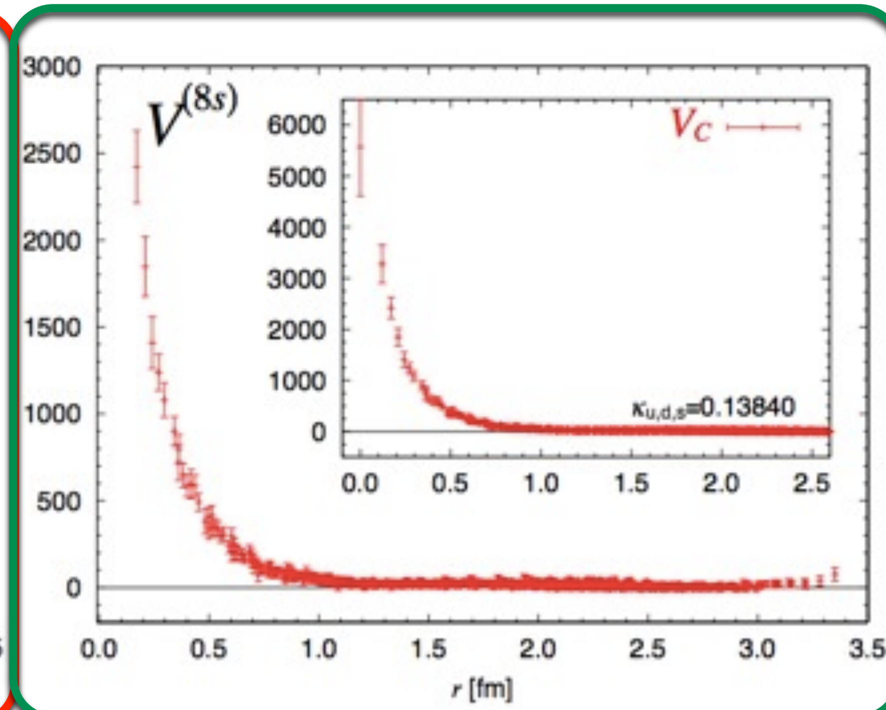
Inoue et al. (HAL QCD), PRL106 (2011), NPA881 (2012).

★ **potentials in flavor symmetric channels --> 27 + 8_s + 1**

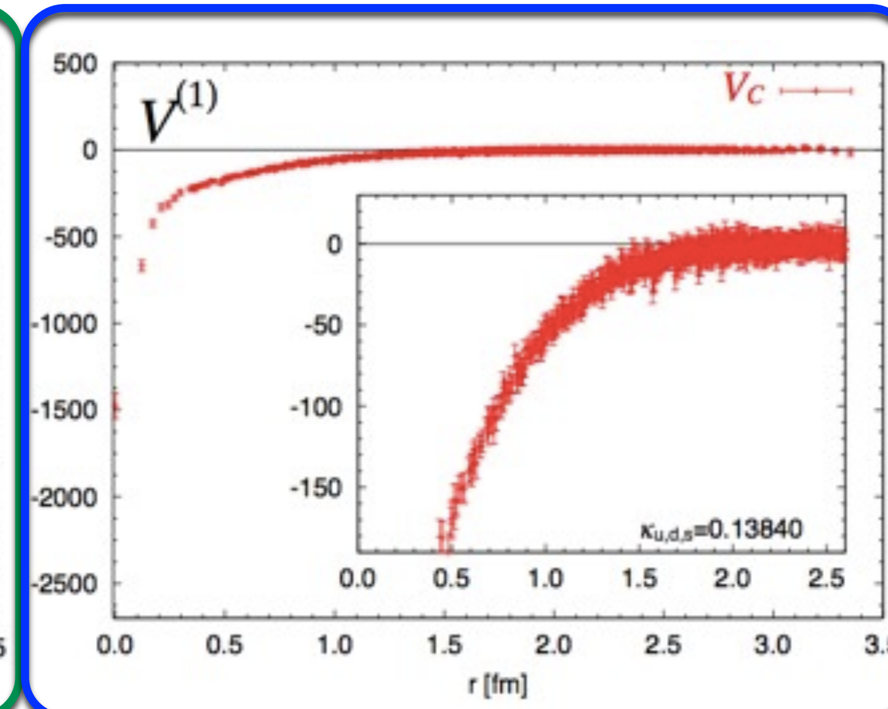
NN $^1S_0$ channels
(partially Pauli blocked)



8_s channel
(Pauli forbidden)



H-dibaryon channel
(Pauli allowed)



bound H-dibaryon?

✦ **origin of repulsive core <--> Pauli principle**

(+ magnetic gluon coupling)

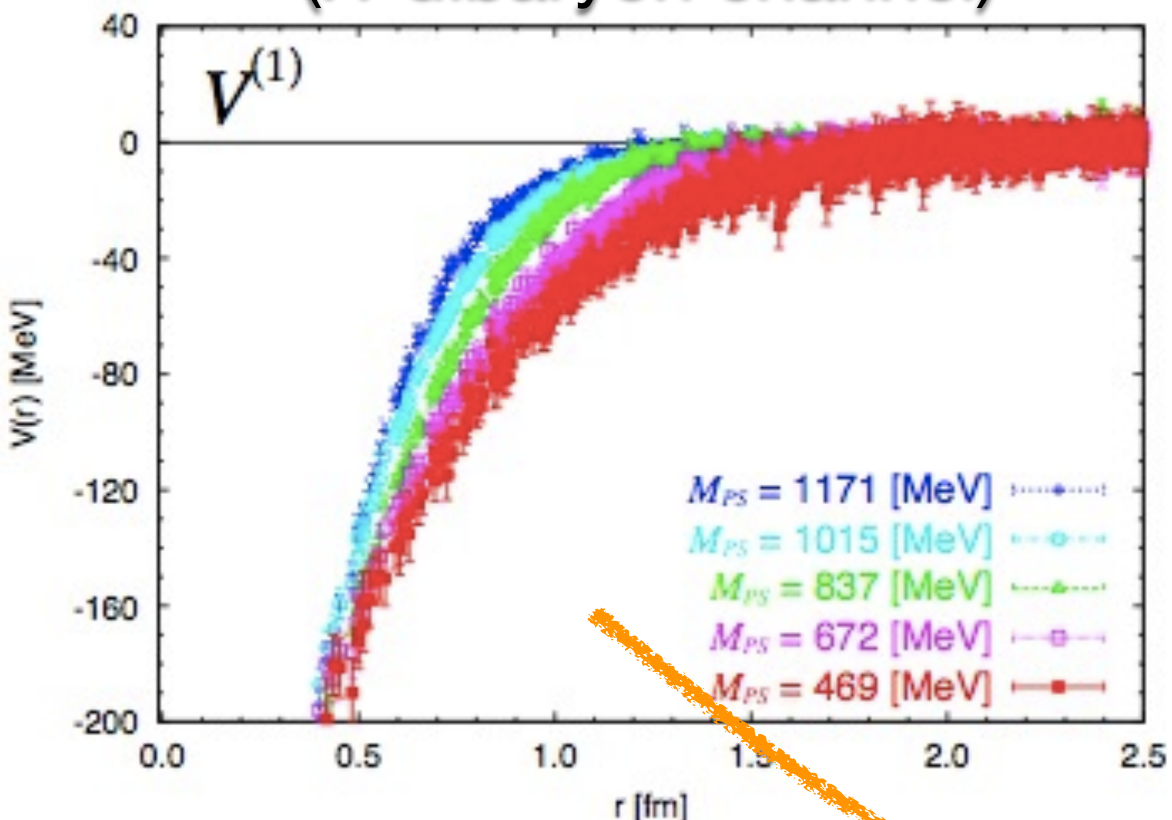
see, Oka & Yazaki, NPA464 (1987)

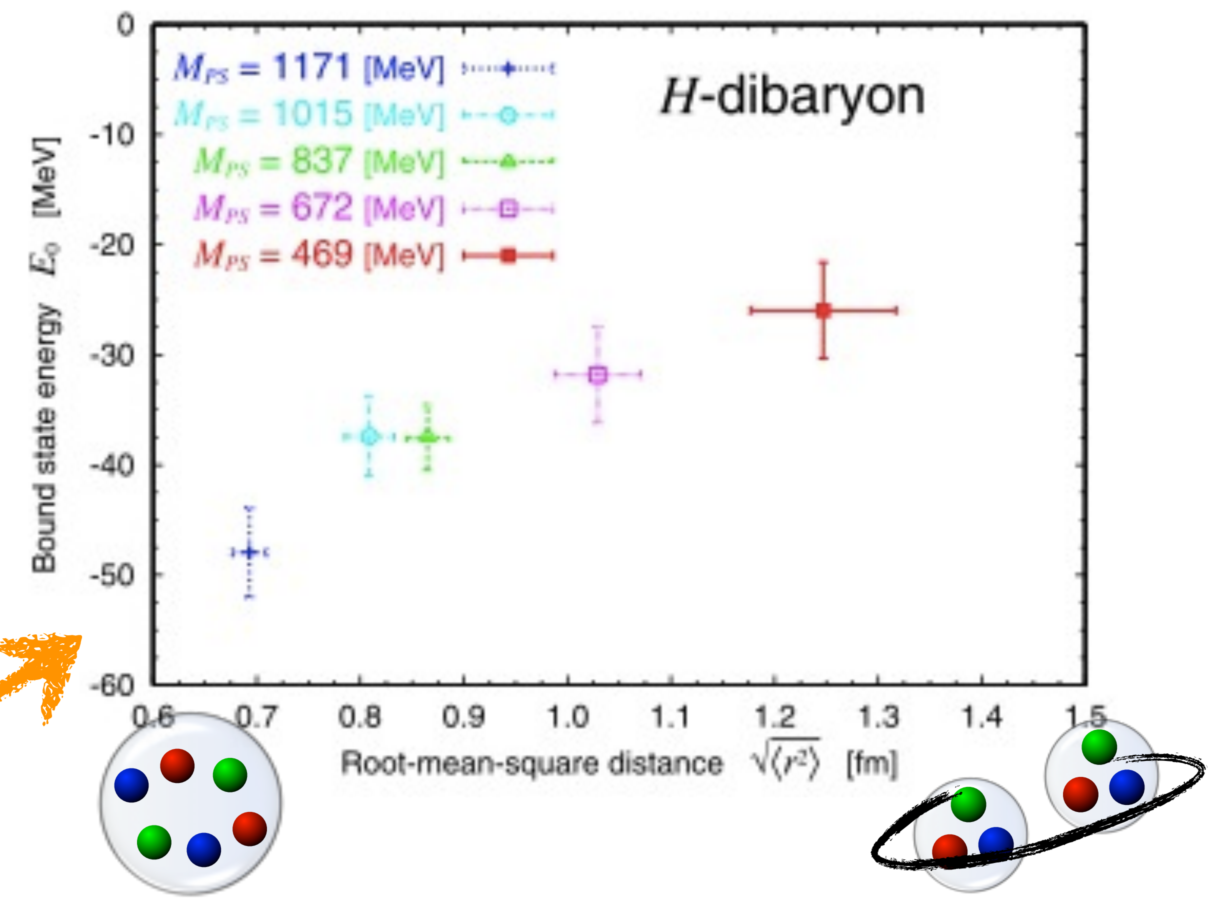
Structure of H-dibaryon in flavor SU(3) limit

★flavor singlet potential $V^{(1)}$
(H-dibaryon channel)

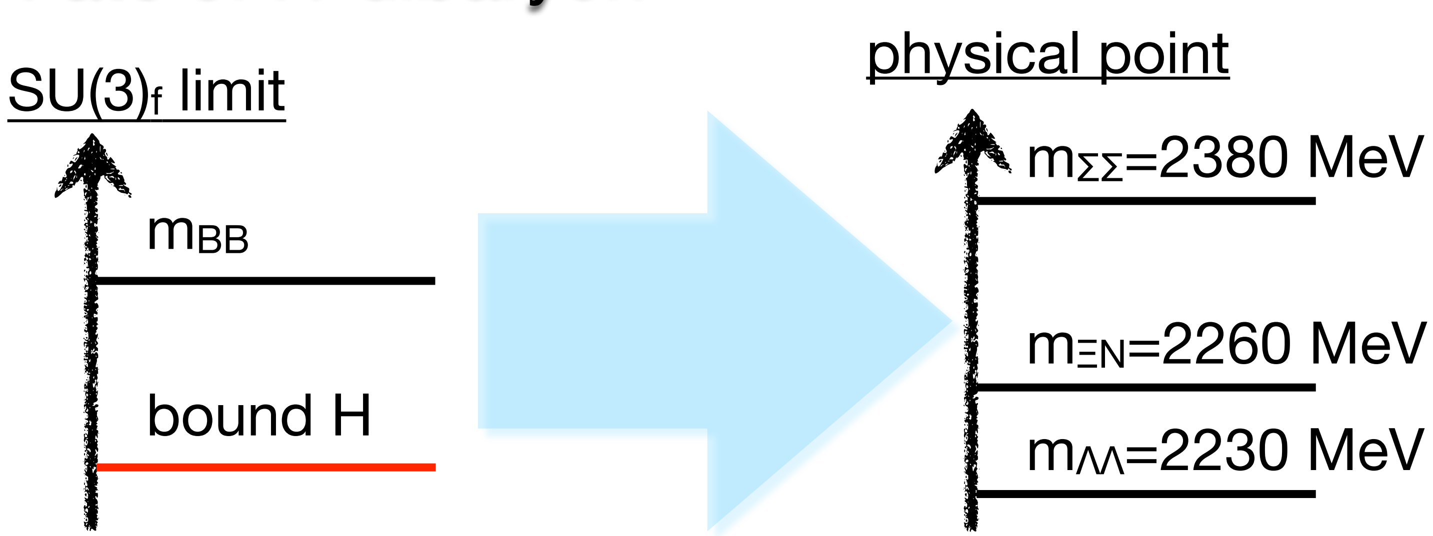
❖ N_f=3 full QCD : m_π~0.47-1.17GeV, L=3.9 fm

Inoue et al. (HAL QCD), PRL106 (2011), NPA881 (2012).





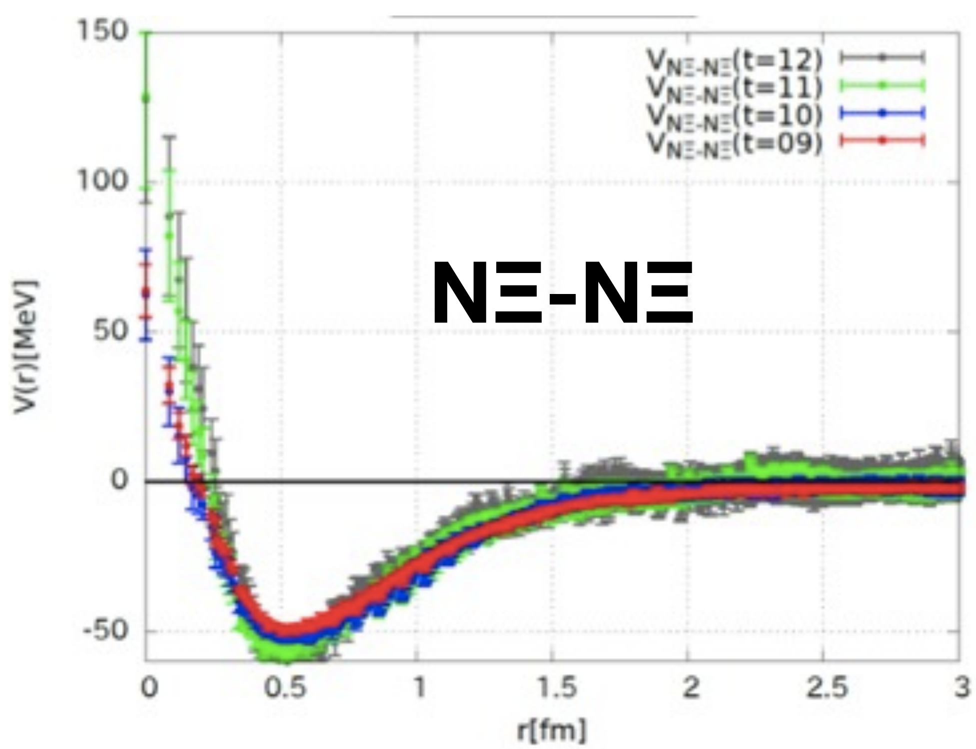
✓ Fate of H-dibaryon

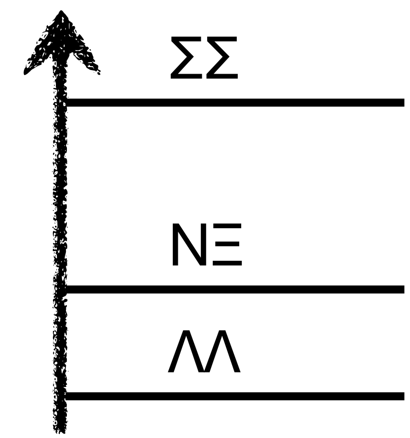


‣ Coupled-channel analysis
on physical point

Fate of H-dibaryon @ almost physical point

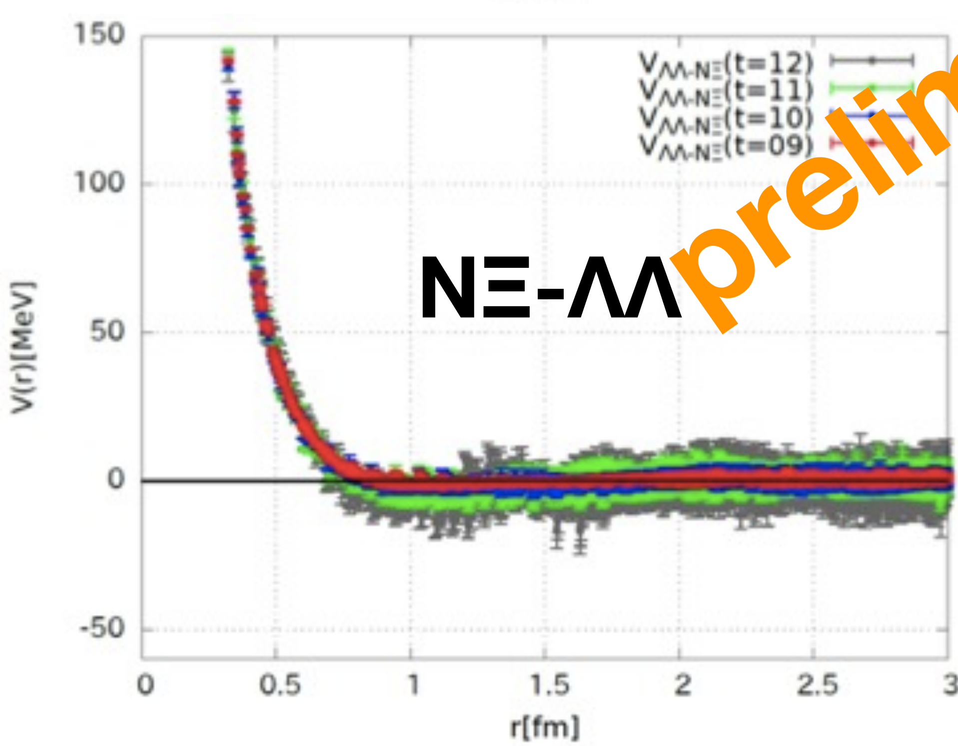
- N_f=2+1 full QCD, m_π~0.146GeV (almost physical), L~8.1fm (large volume)

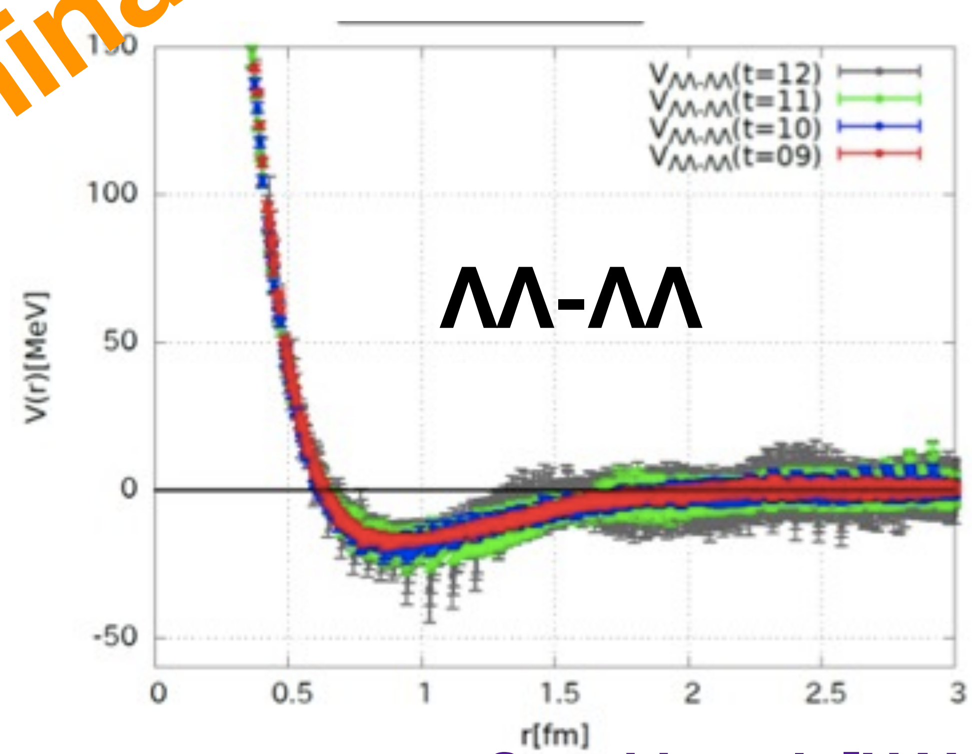




$$\begin{pmatrix}|\Sigma\Sigma\rangle\\|N\Xi\rangle\\|\Lambda\Lambda\rangle\end{pmatrix} = \frac{1}{\sqrt{40}}\begin{pmatrix}-1 & -\sqrt{24} & \sqrt{15}\\ \sqrt{12} & \sqrt{8} & \sqrt{20}\\ \sqrt{27} & -\sqrt{8} & -\sqrt{5}\end{pmatrix}\begin{pmatrix}|27\rangle\\|8_s\rangle\\|1\rangle\end{pmatrix}$$

preliminary





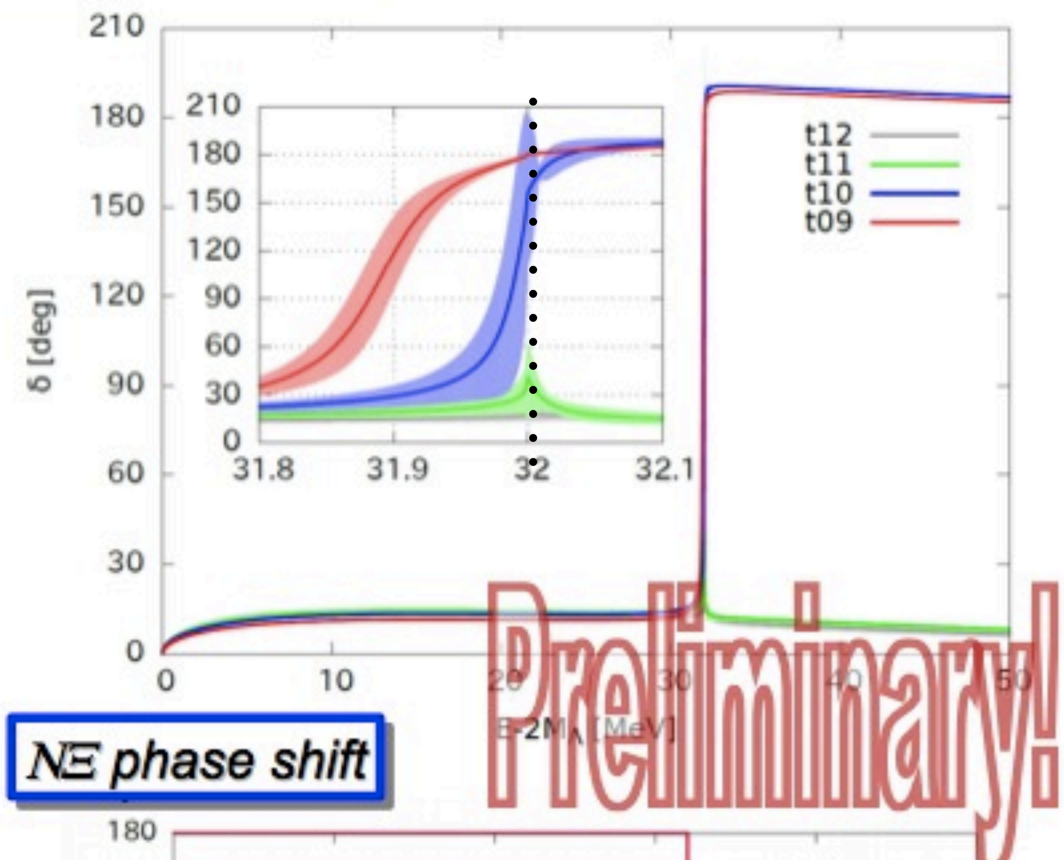
Sasaki et al. [HAL QCD], in preparation.

Fate of H-dibaryon @ almost physical point

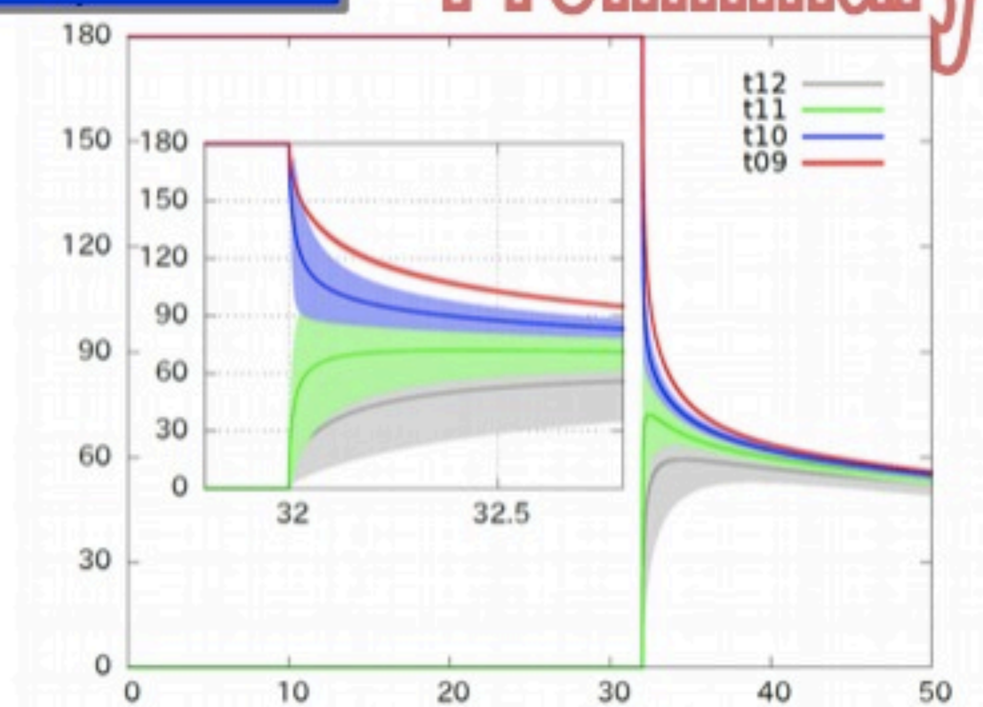
★ ΛΛ and ΞN phase shifts

$$S(k) = \begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$

ΛΛ phase shift



NΞ phase shift

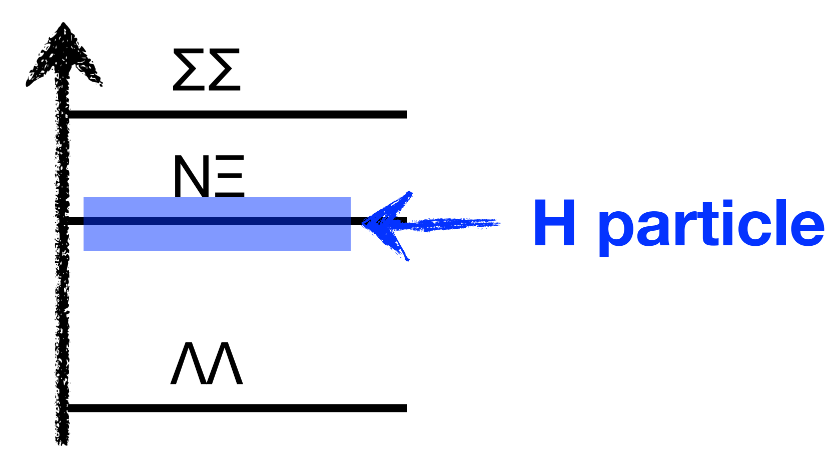


Original prediction of H-dibaryon

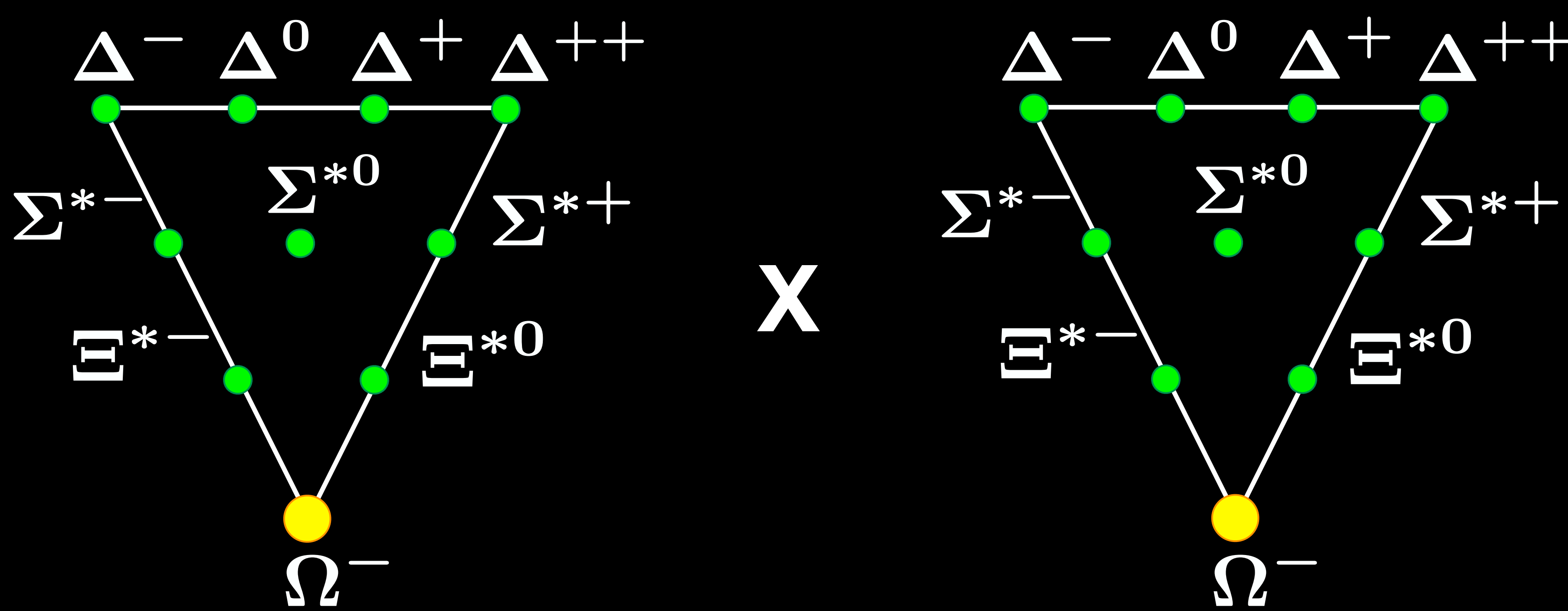
Jaffe (1977) based on quark model,
<u>“Perhaps a Stable Dihyperon”</u>

Answer from QCD for H-dibaryon

<u>“Perhaps near threshold Dihyperon”</u>



Decuplet BB forces & ΩΩ-dibaryon



$$= (\mathbf{28} \oplus \mathbf{27})_{\text{sym.}} \oplus (\mathbf{35} \oplus \mathbf{10}^*)_{\text{anti-sym.}}$$

↑

ΩΩ (J=0) : the most strange dibaryon?

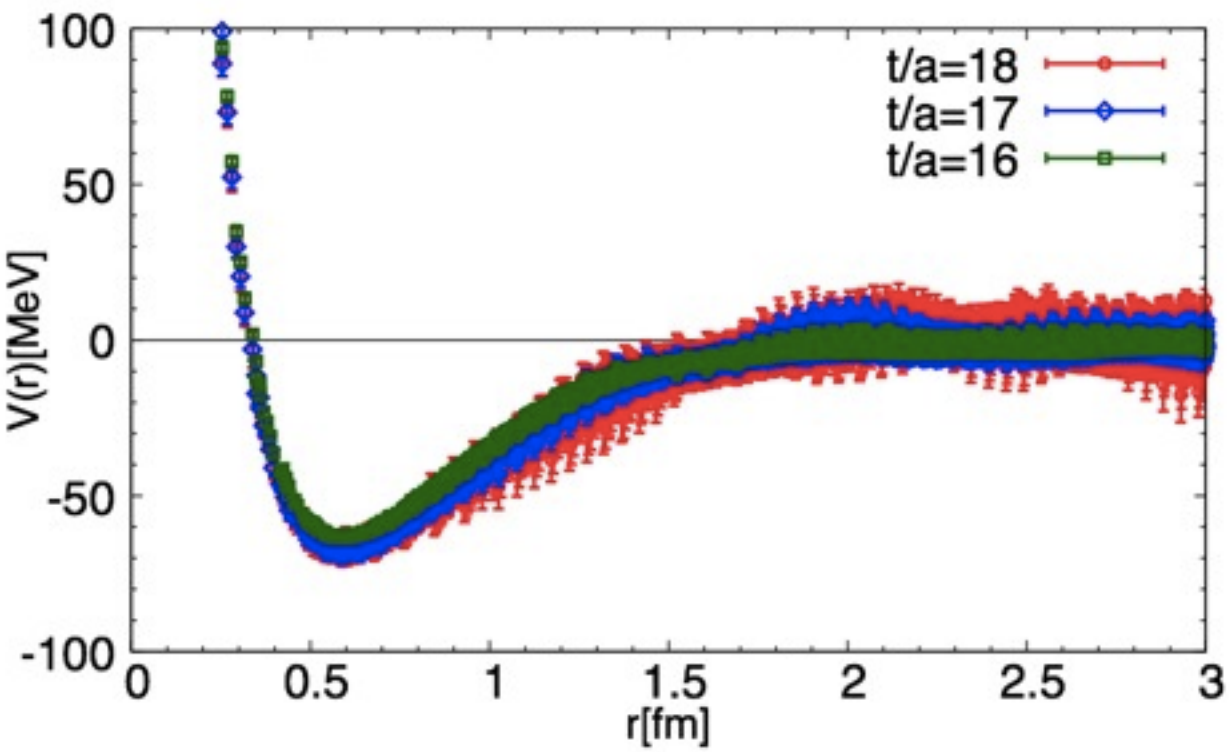
Dyson & Xoung, PRL14 (1965).

Zhang (1992), Wang(1992)

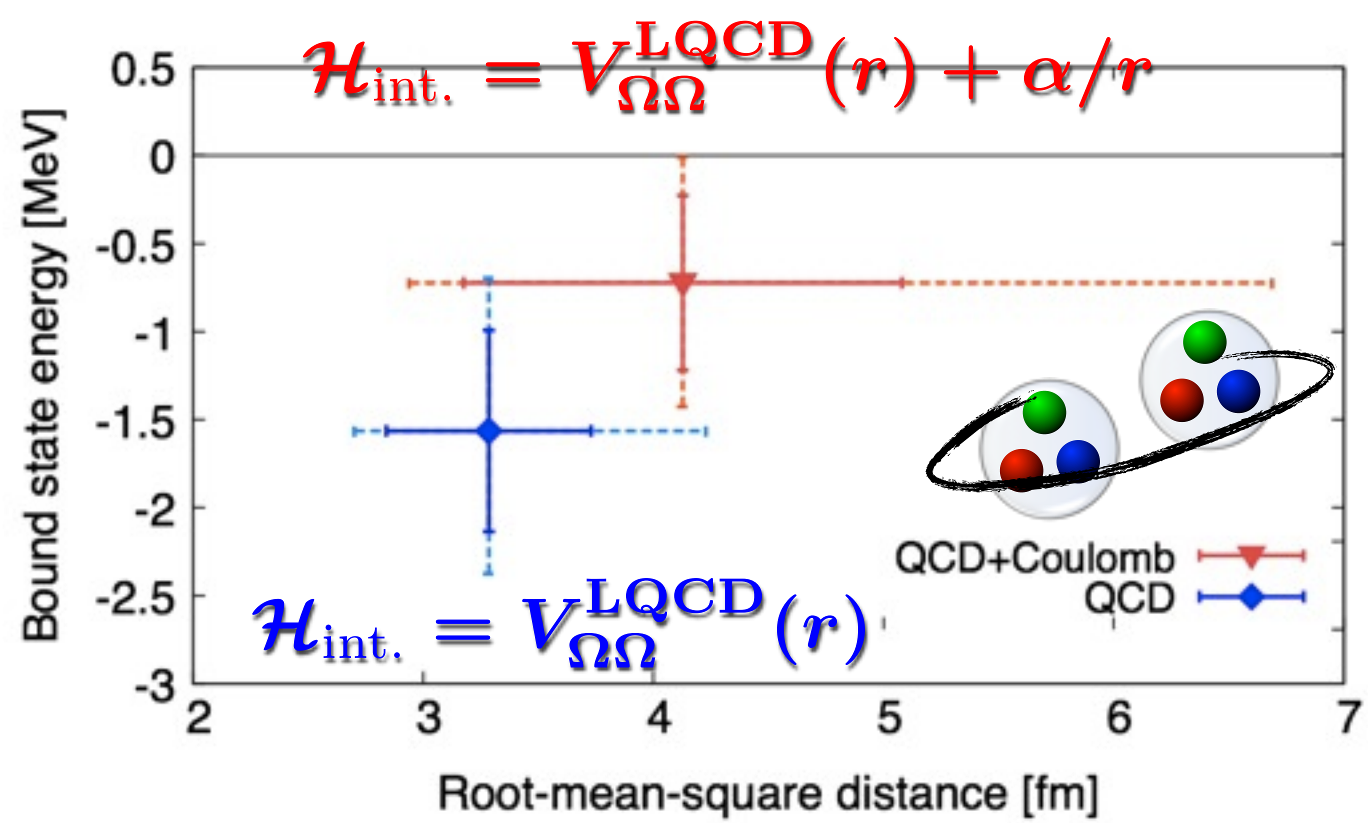
Most strange dibaryon @ almost physical point

★ ΩΩ system in $^1S_0$

Gongyo, Sasaki et al. [HAL QCD], PRL 120, 212001 (2018).



- repulsive core + attractive pocket



- ΩΩ is bound against strong interaction
- ΩΩ is close to unitary region together with Coulomb force

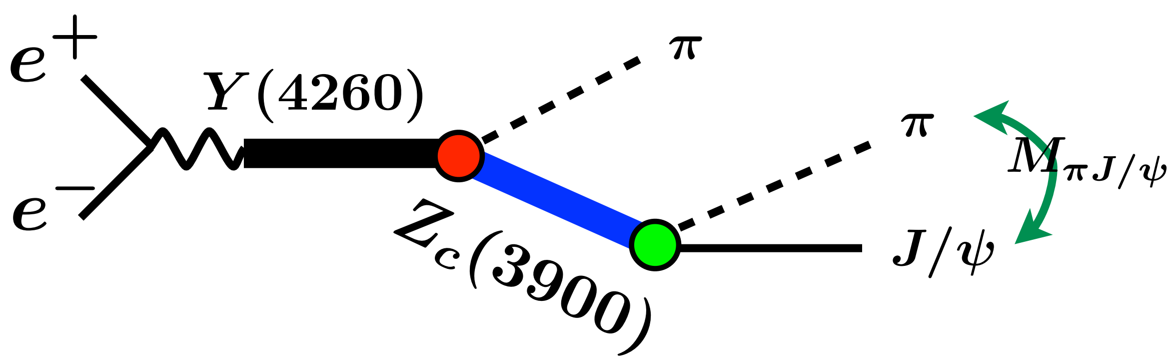
➡ 2-particle correlation func. in future HIC

see talks by Hatsuda & Morita

see also, Yamada [HAL QCD], PTEP2015 (2015)., for m_π=700 MeV, L~3fm

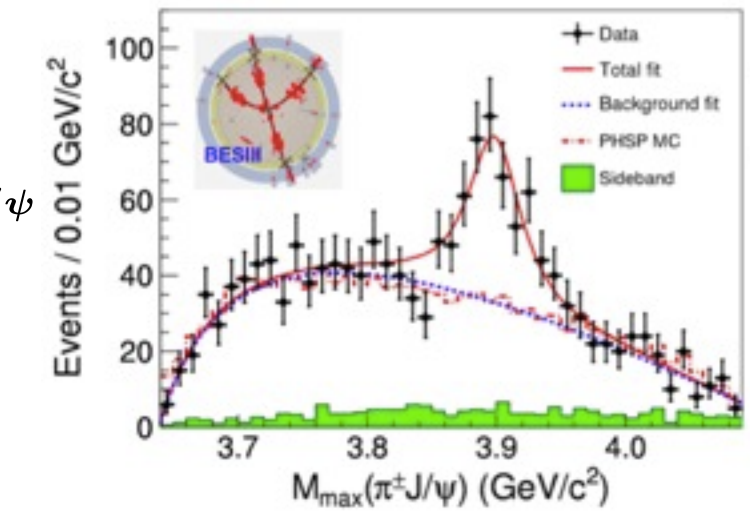
Charmed tetra-quark candidate $Z_c(3900)$

★ $Z_c(3900)$ in experiments

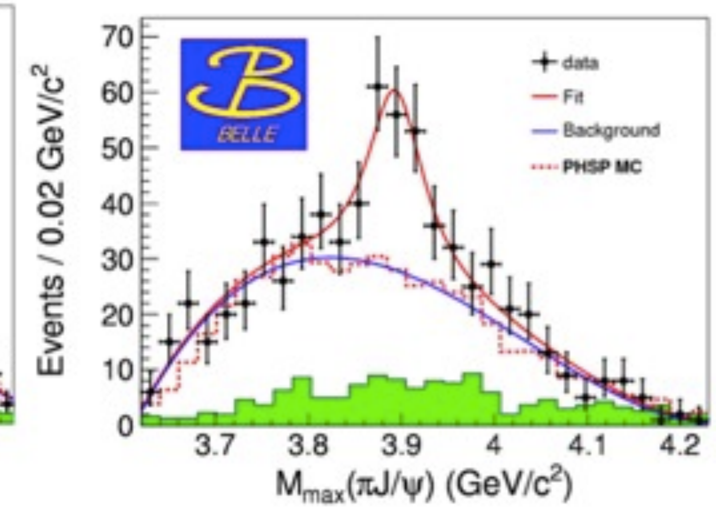


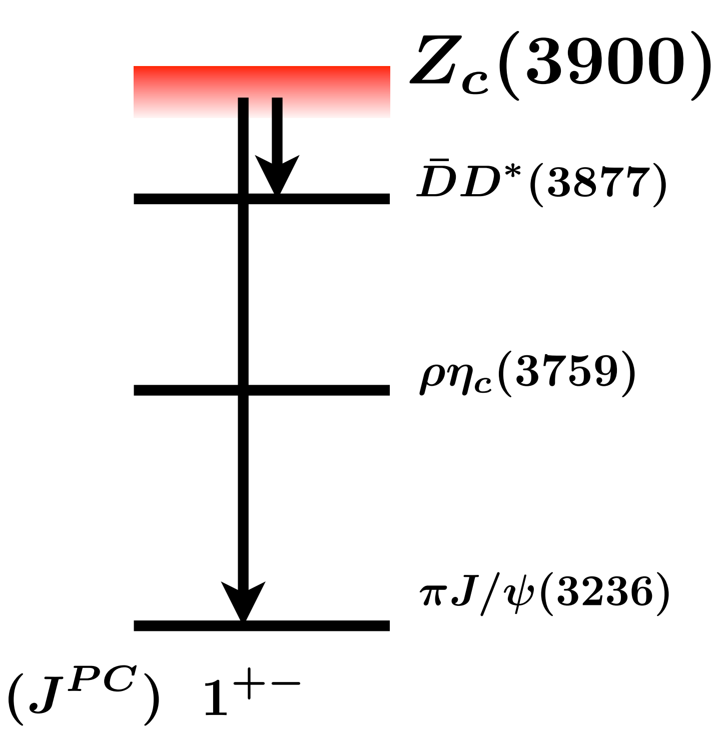
▸$Z_c(3900)$ found in $\pi^{+/-}J/\psi$ ($cc^{bar}ud^{bar}$)

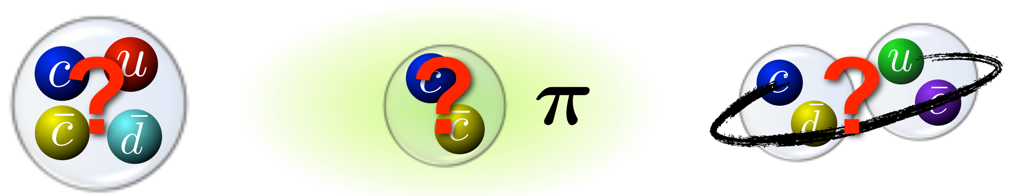
BESIII (2013).



Belle (2013).







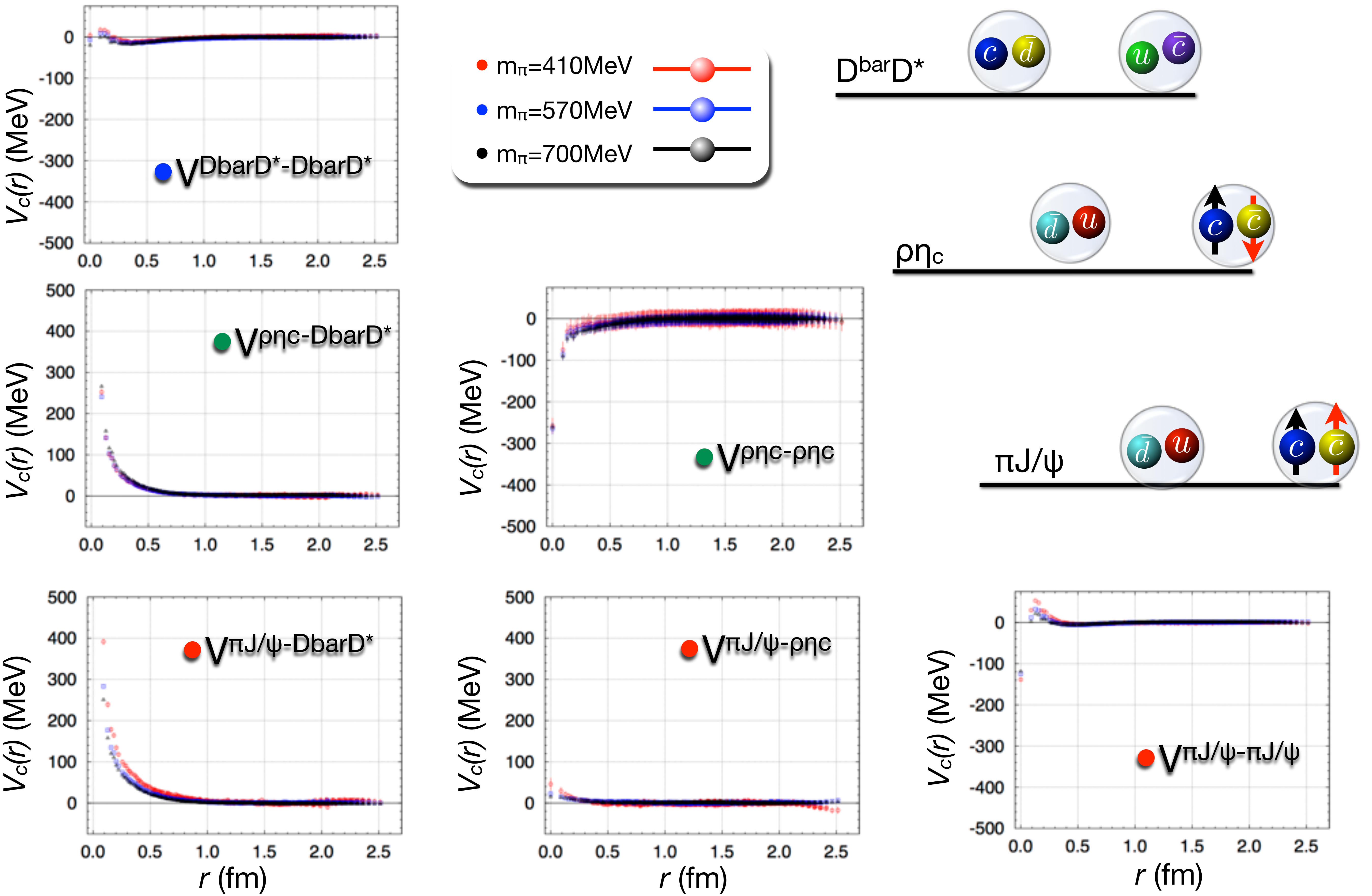
★ $Z_c(3900)$ from lattice QCD

➡coupled-channel HAL QCD approach

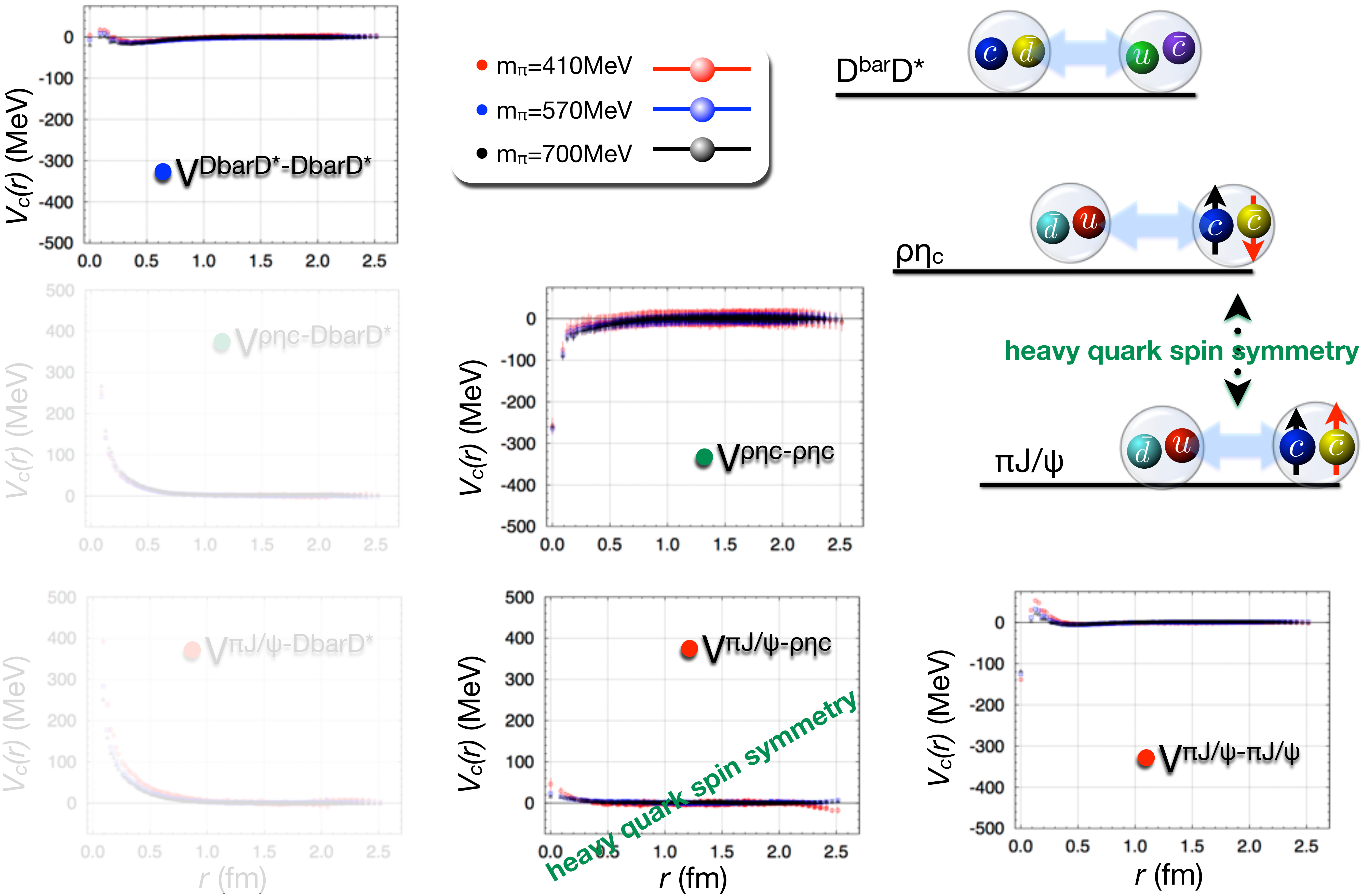
- coupled-channel $\pi J/\psi$-$\rho\eta_c$-$D^{bar}D^*$ potentials
- understand the nature of $Z_c(3900)$

Y. Ikeda, et al. [HAL QCD], PRL117, 242001 (2016).
Reviewed in J. Phys. G45, 024002 (2018).

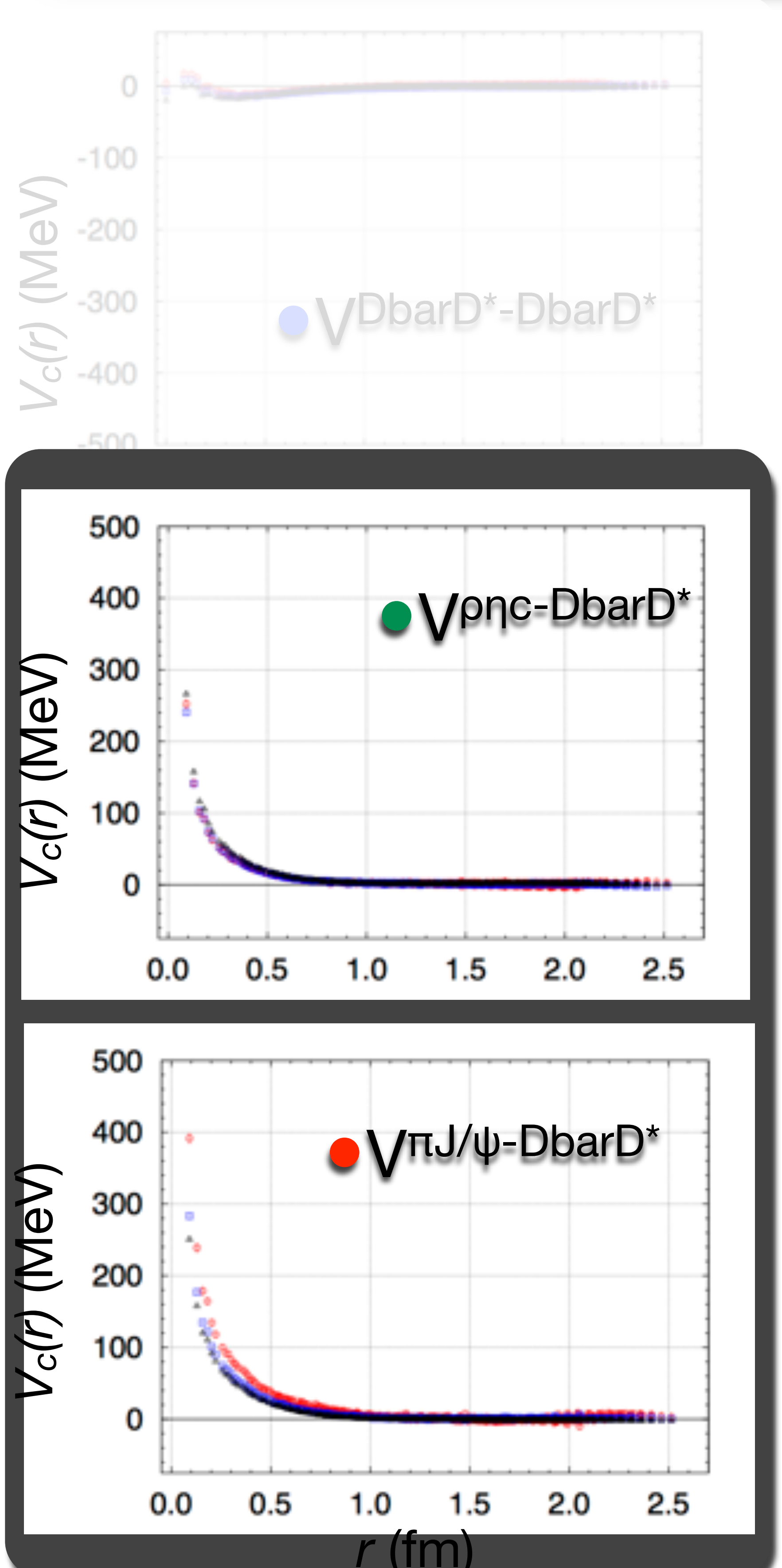
3x3 potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$)

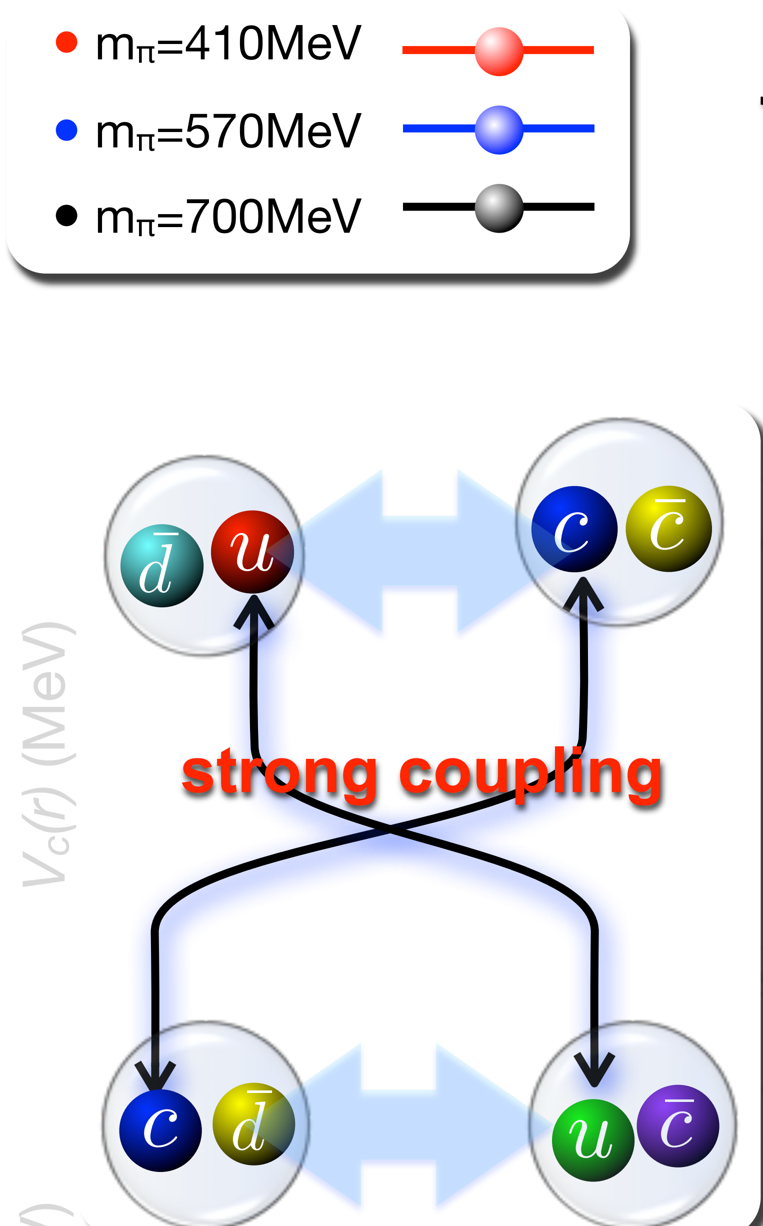


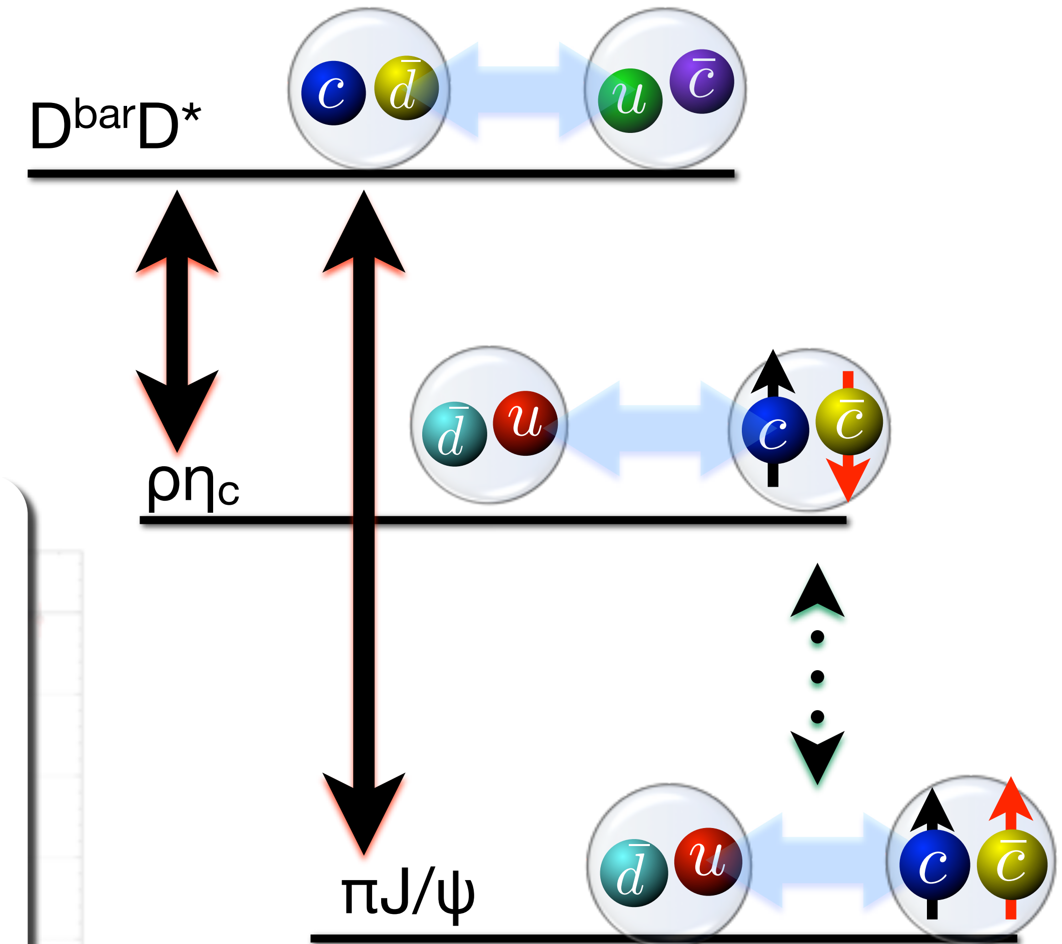
3x3 potential matrix (πJ/ψ - $\rho\eta_c$ - $D^{bar}D^*$)



3x3 potential matrix (πJ/ψ - $\rho\eta_c$ - $D^{bar}D^*$)





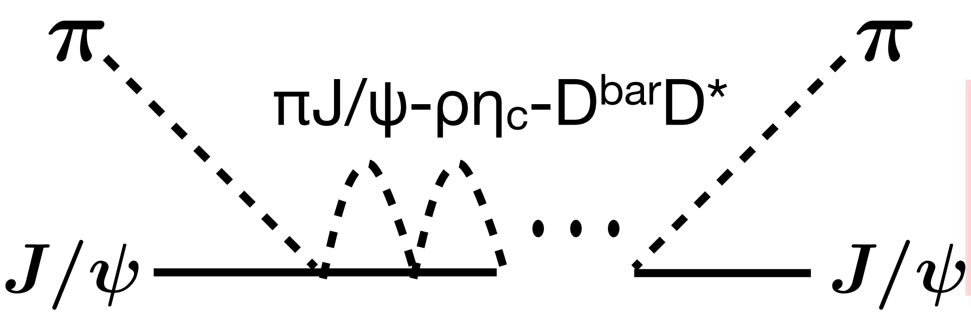


- strong $V^{\pi J/\psi,DbarD^*}$ & $V^{\rho\eta c,DbarD^*}$

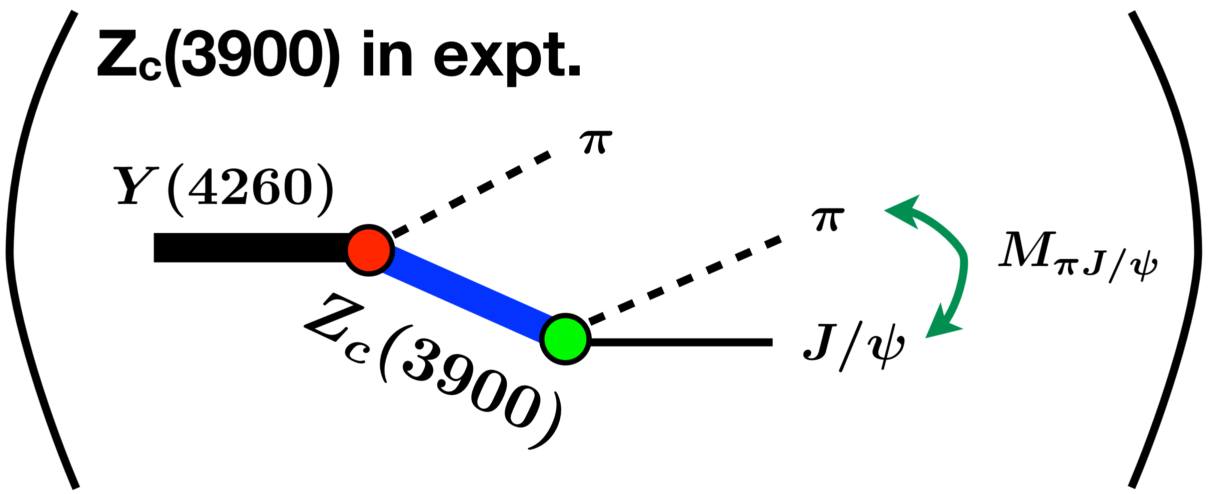
➡ charm quark exchange process

Mass spectra of πJ/ψ (2-body scattering)

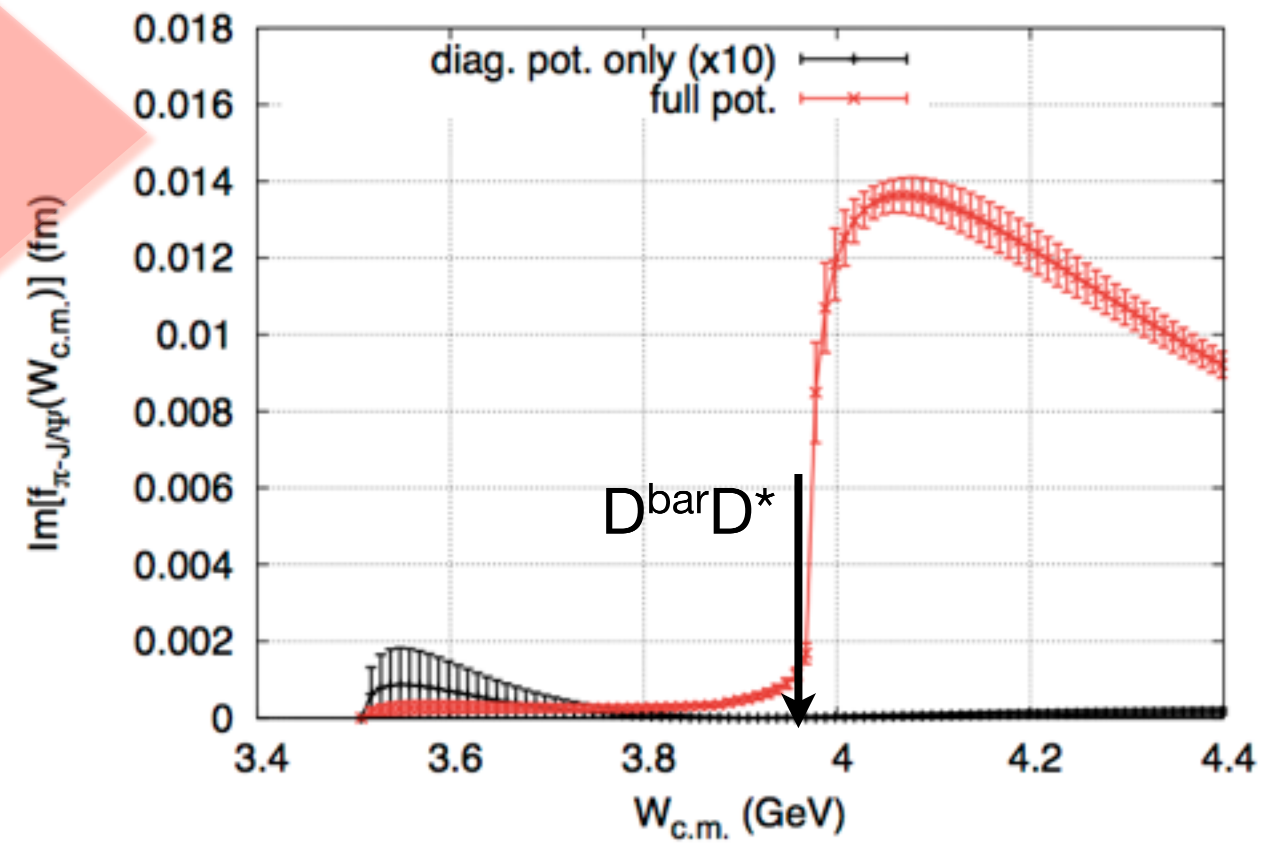
★ 2-body scattering (the most ideal to understand $Z_c(3900)$)



$Z_c(3900)$ in expt.



● πJ/ψ invariant mass (m_π=410MeV)



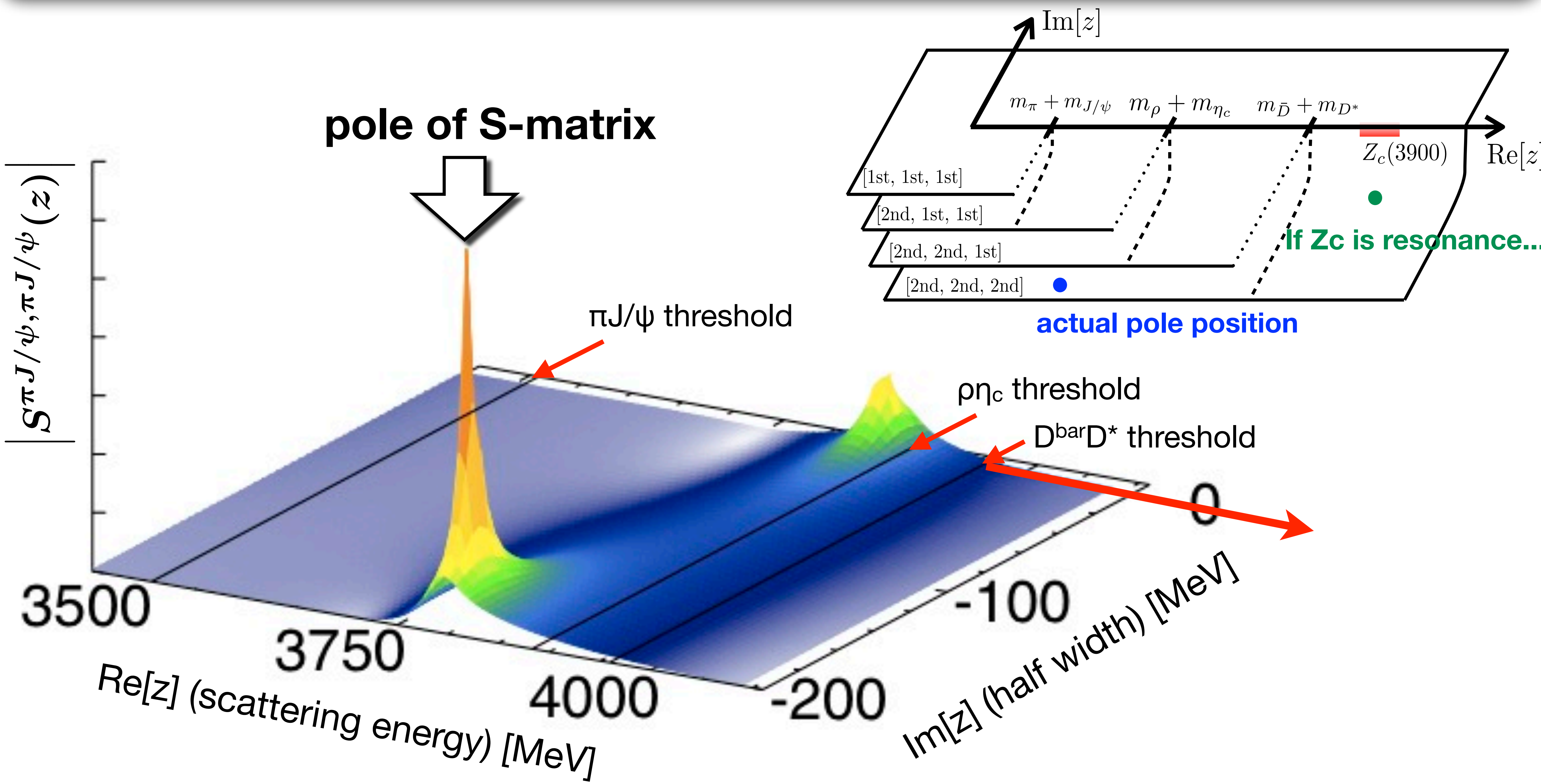
✓ Enhancement just above D^{bar}D* threshold

➡ effect of strong $V^{\pi J/\psi,\ DbarD^*}$ (black --> $V^{\pi J/\psi,\ DbarD^*}$=0)

● line shape not Breit-Wigner

✓ Is $Z_c(3900)$ a conventional resonance? --> pole of S-matrix

Pole of S-matrix ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{bar}D^*$:2nd)



- Pole corresponding to "**virtual state**"
- Pole contribution to scat. observables is small (far from scat. axis)
- **Z_c(3900) is not a resonance but "threshold cusp" induced by strong $V^{\pi J/\psi, DbarD^*}$**

Summary

✤ HAL QCD method

- NBS wave function $\psi(r)$ --> 2PI kernel ($\psi = \phi + G_0 U \psi$)
- Crucial for multi-hadron & coupled-channel scatterings

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al. [HAL QCD], PLB 712, 437 (2012).
Aoki et al. (HAL QCD), PRD87, 034512 (2013).

✤ Exotic candidates, H, ΩΩ, $Z_c(3900)$

- H particle is very close to NΞ threshold --> J-PARC?

Sasaki et al [HAL QCD], in preparation.

- ΩΩ is very close to unitary region --> HIC?

Gongyo, Sasaki et al [HAL QCD], PRL120, 212001 (2018).

- $Z_c(3900)$ is threshold cusp induced by strong $V^{DbarD^*, \pi J/\psi}$

Ikeda et al. [HAL QCD], PRL117, 242001 (2016).
Ikeda [HAL QCD], J. Phys. G45, 024002 (2018).

✤ Future: many hadron resonances & nuclear structures at physical point

Thank you for your attention!