Hadron Interactions from Lattice QCD and Applications to Exotic Hadrons

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HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

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Single hadron spectroscopy from LQCD

★ Low-lying hadrons on physical point (physical $m_\phi$)

light-quark sector

- $N_f=2+1$ full QCD, $L\sim 3\text{fm}$

charm baryons

- $N_f=2+1$ full QCD, $L\sim 3\text{fm}$
- RHQ for charm quark

a few % accuracy already achieved for single hadrons

LQCD predictions of undiscovered charm hadrons ($\Xi^{*}_{cc}, \Omega_{ccc}, \ldots$)

⇒ Next challenge: multi-hadron systems

Aoki et al. (PACS-CS), PRD81 (2010).

Namekawa et al. (PACS-CS), PRD84 (2011); PRD87 (2013).
Multi-hadrons: from quarks to hadrons, nuclei & neutron stars

- **lattice QCD**
- **hadron resonances**
- **nuclei**
- **EOS of neutron stars**

- Hadron scattering: many thresholds
- Nuclei from LQCD: bad \( S/N \sim e^{-A} \)
- QCD phase diagram: Sign problem

- Temperature vs. density
- Mass number \( (A) \) vs. bad \( S/N \)
HAL QCD strategy: from quarks to hadrons, nuclei & neutron stars

Part I: hadronic interactions
- difficulties in multi-hadron systems
- solution = HAL QCD method

Part II: applications to exotic candidates
- coupled-channel scattering from LQCD
- dibaryon & tetraquark candidates
Hadron interactions from LQCD

hadronic correlation function

\[ C_{NN}(\vec{r}, t) \equiv \langle 0| N_1(\vec{r}, t) N_2(\vec{0}, t) \mathcal{J}^\dagger(t = 0)|0 \rangle \]

\[ = \sum_n A_n \psi_n(\vec{r}) e^{-W_n t} \]

Energy eigenvalue \( W_n \) & NBS (Nambu-Bethe-Salpeter) wave function \( \psi_n(\vec{r}) \)

Finite Volume Method

- \( W_n(L) \) \( \longrightarrow \) phase shift

Lüscher’s formula

\[ \kappa_n \cot \delta(k_n) = \frac{4\pi}{L^3} \sum_{m \in \mathbb{Z}^3} \frac{1}{p_m^2 - k_n^2} \]

\[ W_n = \sqrt{m_1^2 + k_n^2} + \sqrt{m_2^2 + k_n^2} \]

Hadronic interactions from LQCD

hadronic correlation function

\[ C_{NN}(\vec{r}, t) \equiv \langle 0|N_1(\vec{r}, t)N_2(\vec{0}, t)\mathcal{J}^\dagger(t = 0)|0\rangle = \sum_n A_n \psi_n(\vec{r}) e^{-W_n t} \]

Energy eigenvalue \( W_n \) & NBS (Nambu-Bethe-Salpeter) wave function \( \psi_n(r) \)

Finite Volume Method

- \( W_n(L) \) \( \xrightarrow{\text{---}} \) phase shift
- Lüscher’s formula

- Serious difficulty to measure \( W_n(L) \) in multi-hadron systems

HAL QCD Method

- \( \psi_n(r) \) \( \xrightarrow{\text{---}} \) 2PI kernel (\( \psi = \varphi + G_0 U \psi \))
  \( \xrightarrow{\text{---}} \) phase shift, binding energy, ...

Ishii et al. [HAL QCD], PLB 712, 437 (2012).
Fundamental difficulty in multi-hadron systems

see, Iritani, Doi et al. [HAL QCD], JHEP10 (2016) 101.

\[ C_N(t) = a_0 e^{-m_N t} + c_1 e^{-(m_N + m_\pi) t} + \cdots \rightarrow a_0 e^{-m_N t} \quad (t > t^*) \]

\[ C_{NN}(t) = b_0 e^{-W_0 t} + b_1 e^{-W_1 t} + \cdots \rightarrow b_0 e^{-W_0 t} \quad (t > t^*) \]

\[ S/N \sim \sqrt{N_{\text{conf.}}} \times 10^{-2} \]

\[ S/N \sim \sqrt{N_{\text{conf.}}} \times 10^{-30} \]

\[ t^* \sim \delta E^{-1} \sim m_N (L/2\pi)^2 \sim 10 \text{ fm} \]
Demonstration of plateau method by mock-up data

“Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD”
Iritani, Doi et al. [HAL QCD], JHEP10 (2016) 101.

• **Normalized correlation func.** \(R(t)\) for two baryons in mock-up data

\[
R(t) = \frac{C_{BB}(t)}{C_B(t)^2} = b_1 e^{-\Delta E t} + b_2 e^{-\delta E_{el} t} + c_1 e^{-\delta E_{inel} t}
\]

\[\Delta E^{eff}(t) = \log \left[ \frac{R(t)}{R(t+1)} \right] \rightarrow \Delta E \quad t > t^*\]

• Ground state energy \(\Delta E = W_{BB} - 2m_B\)
  ~1 MeV precision necessary (nuclear physics scale)

• Elastic scattering states \(\delta E_{el}\)
  \(\delta E_{el} = 50\text{MeV},\ b_2/b_1 = \pm 0.1, 0\) (10% contamination)

• Inelastic threshold \(\delta E_{inel}\)
  \(\delta E_{inel} = 500\text{MeV},\ c_1/b_1 = 0.01\) (1% contamination)
Demonstration of plateau method by mock-up data

“Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD”

Iritani, Doi et al. [HAL QCD], JHEP10 (2016) 101.
Actual data for $\Xi\Xi\ (^1S_0) \quad @ m_\pi=0.51\text{GeV}, \quad L=4.3\text{fm}, \quad a=0.09\text{fm}$

**Source-operator dependence in plateau method**

$$R(t) = \sum_{\vec{x},\vec{y}} \langle 0| B_1(\vec{x}, t) B_2(\vec{y}, t) \mathcal{J}^\dagger(t = 0)|0\rangle / C_B(t)^2$$

$$\Delta E_{\text{eff}}(t) = \log \left[ \frac{R(t)}{R(t + 1)} \right] \quad \rightarrow \quad \Delta E$$

- **At least one of “plateaux” is fake!** (Data at $t\sim1\text{fm}$ is too early to identify plateau.)
- **Naive plateau method does NOT work --> variational method (next talk)**

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$\delta E_{\text{inel}} \sim 500\text{MeV}$

$\delta E_{\text{el}} \sim 50\text{MeV}$

$\Rightarrow$ **True ground state is to appear at $t > t^\ast$ \ (t*~8 fm)**
A solution: HAL QCD method -- potential as a representation of S-matrix --

- The scattering states do exist, and we should tame the scattering states

**HAL QCD method**

- define energy-independent potential $U(r,r')$

\[
\int d\vec{r}' U(\vec{r}, \vec{r}') \psi_n(\vec{r}') = (E_n - H_0) \psi_n(\vec{r})
\]

$U(\vec{r}, \vec{r}') \equiv \sum_{n=0}^{n_{th}} (E_n - H_0) \psi_n(\vec{r}) \overline{\psi}_n(\vec{r}')$

- All elastic states share the same potential $U(r,r')$

\[
U \psi_0 = (E_0 - H_0) \psi_0
\]

\[
U \psi_1 = (E_1 - H_0) \psi_1
\]

\[
\vdots
\]

- derive $U(r,r')$ from time-dependent Schrödinger-type eq.

\[
\int d\vec{r}' U(\vec{r}, \vec{r}') R(\vec{r}', t) = \left( -\frac{\partial}{\partial t} + \frac{1}{4m_B} \frac{\partial^2}{\partial t^2} - H_0 \right) R(\vec{r}, t)
\]

\[
R(\vec{r}, t) = e^{2m_B t} C_{BB}(\vec{r}, t)
\]

**Elastic scat. states** are no more contamination than signal ($t^* \sim 1 fm$)
$\Xi(^{1}S_{0})$ in HAL QCD method $@m_{\pi}=0.51\text{GeV}, L=4.3\text{fm}, a=0.09\text{fm}$

source dependence of $R(r,t)$

$Iritani et al. [HAL QCD], arXiv:1805.02365 [hep-lat]$. 

$U(\vec{r}, \vec{r}') = \left[ V_{0}(\vec{r}) + V_{2}(\vec{r}) \nabla^{2} \cdots \right] \delta(\vec{r} - \vec{r}')$

finite $V$ calc.  ➞ Fate of fake plateaux

infinite $V$ calc.

$\Xi(1S_{0})$ bound state

mirage $\sim 1\text{fm}$

true plateau $\sim 10\text{fm}$

No $\Xi(1S_{0})$ bound state
Multi-hadron spectroscopy from LQCD

lattice QCD

HAL QCD method

hadron scattering

many thresholds

• Resonances are embedded into coupled-channel scattering states
How can we find resonances?

Coupled-channel scatterings

\[ S^{(\ell)} (W) \]

Partial wave analysis of expt. data

- Cross sections \( (d\sigma/d\Omega) \)
- Spin polarization observables
- Etc.

Analyticity of S-matrix is \textit{uniquely} determined

Identical theorem + dispersion theory

Bound state (1st sheet)
- Pole position \( \rightarrow \) binding energy
- Residue \( \rightarrow \) coupling to scattering state

Resonance (2nd sheet)
- Analytic continuation onto 2nd sheet
- Pole position \( \rightarrow \) resonance energy
- Residue \( \rightarrow \) coupling to scat. state, partial decay

Reaction plane
Strategy to search for complex poles on the lattice

Coupled-channel scatterings from lattice QCD

\[ S^{(\ell)}(W) = \langle 0 | \phi_1(\vec{r}, t) \phi_2(\vec{0}, t) \mathcal{J}^\dagger(t = 0) | 0 \rangle = \sum_n A_n \psi_n(\vec{r}) e^{-W_n t} \]

- coupled-channel Lüscher’s method
  \[ W_n(L) \rightarrow \delta^1(W_n), \delta^2(W_n), \eta(W_n) \]

- (coupled-channel scattering difficult)
  \[ \delta^1(W_n), \delta^2(W_n), \eta(W_n) \leftarrow W_n(L_1) = W_n(L_2) = W_n(L_3) \]
**Coupled-channel HAL QCD method**

- **measure relevant NBS wave function** --> channel is defined

\[
\langle 0 | \phi^a_1(\vec{x} + \vec{r}, t) \phi^a_2(\vec{0}, t) J^\dagger(0) | 0 \rangle = \sqrt{Z^a_1 Z^a_2} \sum_n A_n \psi^a_n(\vec{r}) e^{-W_n t}
\]

- **Nambu-Bethe-Salpeter (NBS) wave function in each channel**
  - derive 2PI kernel (potential) as a representation of S-matrix

\[
\left( \nabla^2 + \left( k^a_n \right)^2 \right) \psi^a_n(\vec{r}) = 2 \mu^a \sum_b \int d\vec{r}' \ U^{ab}(\vec{r}, \vec{r}') \psi^b_n(\vec{r}')
\]

- **coupled-channel potential** \( U^{ab}(r, r') \):
  - \( U^{ab}(r, r') \) is faithful to coupled-channel S-matrix
  - \( U^{ab}(r, r') \) is energy independent (until new threshold opens)
  - Non-relativistic approximation is not necessary
  - \( U^{ab}(r, r') \) contains all 2PI contributions


**Full details, Aoki et al. (HAL QCD), PRD87, 034512 (2013); Proc. Jpn. Acad., Ser. B, 87 (2011).**
Octet BB forces & H-dibaryon

\[ n(udd) \quad p(uud) \]
\[ \Sigma^0(uds) \quad \Lambda(uds) \quad \Sigma^+(uus) \]
\[ \Xi^-(dss) \quad \Xi^0(uss) \]

\[ n(udd) \quad p(uud) \]
\[ \Sigma^0(uds) \quad \Lambda(uds) \quad \Sigma^+(uus) \]
\[ \Xi^-(dss) \quad \Xi^0(uss) \]

\[ = \left( 27 \oplus 8 \oplus 1 \right)_{\text{sym.}} \oplus \left( 10^* \oplus 10 \oplus 8 \right)_{\text{anti-sym.}} \]

NN \( (^1S_0) \)

H-dibaryon \( (\Lambda\Lambda-\Xi N-\Sigma \Sigma) \)?

Jaffe (1977)
Generalized BB forces in flavor SU(3) limit

- Full QCD in SU(3)$_F$ limit: $m_\pi \approx 0.47$ GeV, $L = 3.9$ fm


- Potentials in flavor symmetric channels $\rightarrow 27 + 8_s + 1$

  - NN $^1S_0$ channels (partially Pauli blocked)
  - $8_s$ channel (Pauli forbidden)
  - H-dibaryon channel (Pauli allowed)

- Origin of repulsive core $\leftrightarrow$ Pauli principle
  (+ magnetic gluon coupling)

  see, Oka & Yazaki, NPA464 (1987)
Structure of H-dibaryon in flavor SU(3) limit

★ Flavor singlet potential $V^{(1)}$ (H-dibaryon channel)

✓ Fate of H-dibaryon

- $m_{\pi} \sim 0.47\text{-}1.17\text{GeV}$, $L=3.9\text{ fm}$
- $N_f=3$ full QCD:
- $m_{\Sigma \Sigma} = 2380\text{ MeV}$
- $m_{\Xi N} = 2260\text{ MeV}$
- $m_{\Lambda \Lambda} = 2230\text{ MeV}$

Coupled-channel analysis on physical point

Fate of H-dibaryon @ almost physical point

- \( N_f=2+1 \) full QCD, \( m_\pi \approx 0.146 \text{GeV} \) (almost physical), \( L \approx 8.1 \text{fm} \) (large volume)

\[
\begin{pmatrix}
|\Sigma\Sigma\rangle \\
|N\Xi\rangle \\
|\Lambda\Lambda\rangle
\end{pmatrix}
= \frac{1}{\sqrt{40}}
\begin{pmatrix}
-1 & -\sqrt{24} & \sqrt{15} \\
\sqrt{12} & \sqrt{8} & \sqrt{20} \\
\sqrt{27} & -\sqrt{8} & -\sqrt{5}
\end{pmatrix}
\begin{pmatrix}
|27\rangle \\
|8s\rangle \\
|1\rangle
\end{pmatrix}
\]

\( N\Xi - N\Xi \) - repulsive \((8s)\)

\( N\Xi - \Lambda\Lambda \) - attractive \((1)\)

\( \Lambda\Lambda - \Lambda\Lambda \) - attractive \((27)\)

Sasaki et al. [HAL QCD], in preparation.
Fate of H-dibaryon @ almost physical point

★ ΛΛ and ΞN phase shifts

\[ S(k) = \begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1 - \eta^2} e^{i(\delta_1 + \delta_2)} \\ i\sqrt{1 - \eta^2} e^{i(\delta_1 + \delta_2)} & \eta e^{2i\delta_2} \end{pmatrix} \]

Original prediction of H-dibaryon

Jaffe (1977) based on quark model, “Perhaps a Stable Dihyperon”

Answer from QCD for H-dibaryon

“Perhaps near threshold Dihyperon”
Decuplet BB forces & $\Omega\Omega$-dibaryon

$\Delta^- \Delta^0 \Delta^+ \Delta^{++}$

$\Sigma^{*-} \Sigma^{*0} \Sigma^{*+}$

$\Xi^{-} \Xi^{*0}$

$\Omega^{-}$

$\times$

$\Delta^- \Delta^0 \Delta^+ \Delta^{++}$

$\Sigma^{*-} \Sigma^{*0} \Sigma^{*+}$

$\Xi^{-} \Xi^{*0}$

$\Omega^{-}$

$= (28 \bigoplus 27)_{\text{sym.}} \bigoplus (35 \bigoplus 10^*)_{\text{anti-sym.}}$

$\Omega\Omega (J=0): the\ most\ strange\ dibaryon?$

Dyson & Young, PRL14 (1965).

Most strange dibaryon @ almost physical point

★ ΩΩ system in $^1S_0$

Gongyo, Sasaki et al. [HAL QCD], PRL 120, 212001 (2018).

• repulsive core + attractive pocket

\[ \mathcal{H}_{\text{int.}} = V_{\Omega\Omega}^{\text{LQCD}}(r) + \alpha/r \]

• ΩΩ is bound against strong interaction
• ΩΩ is close to unitary region together with Coulomb force
  ➞ 2-particle correlation func. in future HIC

see talks by Hatsuda & Morita

See also, Yamada [HAL QCD], PTEP2015 (2015)., for $m_\pi=700$ MeV, $L\sim3$ fm
Charmed tetra-quark candidate $Z_c(3900)$

★ $Z_c(3900)$ in experiments

$e^+$

$e^-$

$Y(4260)$

$Z_c(3900)$

$\pi$

$\pi$

$M_{\pi J/\psi}$

$J/\psi$

$Z_c(3900)$ found in $\pi^{+/−} J/\psi$ (cc$^\text{bar}$ud$^\text{bar}$)

BESIII (2013).

Belle (2013).

★ $Z_c(3900)$ from lattice QCD

$\Rightarrow$ coupled-channel HAL QCD approach

- coupled-channel $\pi J/\psi$-$\rho \eta_c$-$D^\text{bar}D^*$ potentials
- understand the nature of $Z_c(3900)$

Y. Ikeda, et al. [HAL QCD], PRL117, 242001 (2016).
3x3 potential matrix (\(\pi J/\psi - \rho \eta_c - D^{\text{bar}}D^*\))

\(V_{\text{c}}(r)\) (MeV)

- \(m_\pi=410\text{MeV}\)
- \(m_\pi=570\text{MeV}\)
- \(m_\pi=700\text{MeV}\)

\(V_D^{\text{bar}}D^* - D^{\text{bar}}D^*\)
\(V_\rho \eta_c - D^{\text{bar}}D^*\)
\(V_\pi J/\psi - D^{\text{bar}}D^*\)
\(V_\pi J/\psi - \rho \eta_c\)
\(V_\pi J/\psi - \pi J/\psi\)

Diagram showing different potential interactions and their corresponding energy levels.
3x3 potential matrix \((\pi J/\psi - \rho \eta_c - D^{\text{bar}D^*})\)

- \(V_{\text{DbarD}^*-\text{DbarD}^*}(r)\)
- \(V_{\pi J/\psi-\rho \eta_c}(r)\)
- \(V_{\rho \eta_c-\rho \eta_c}(r)\)
- \(V_{\rho \eta_c-D^{\text{bar}D^*}}(r)\)
- \(V_{\pi J/\psi-D^{\text{bar}D^*}}(r)\)
- \(V_{\pi J/\psi-\pi J/\psi}(r)\)

- \(m_\pi=410\text{MeV}\)
- \(m_\pi=570\text{MeV}\)
- \(m_\pi=700\text{MeV}\)

heavy quark spin symmetry
3x3 potential matrix ($\pi J/\psi - \rho \eta_c - D^{\ast}\bar{D}$)

- $m_\pi = 410$ MeV
- $m_\pi = 570$ MeV
- $m_\pi = 700$ MeV

Strong $V_{\pi J/\psi}, D^{\ast}\bar{D}$ and $V_{\rho \eta_c}, D^{\ast}\bar{D}$

Charm quark exchange process
Mass spectra of $\pi J/\psi$ (2-body scattering)

★ 2-body scattering (the most ideal to understand $Z_c(3900)$)

$\pi$ $\pi$ $\pi$

$J/\psi$ $J/\psi$

$\pi J/\psi - \rho \eta_c - D_{bar}D^*$

$Z_c(3900)$ in expt.

$Y(4260)$ $\pi$

$Z_c(3900)$ $J/\psi$

$M_{\pi J/\psi}$

✓ Enhancement just above $D_{bar}D^*$ threshold

⇒ effect of strong $V^{\pi J/\psi, D_{bar}D^*}$ (black $\rightarrow V^{\pi J/\psi, D_{bar}D^*}=0$)

• line shape not Breit-Wigner

✓ Is $Z_c(3900)$ a conventional resonance? $\rightarrow$ pole of S-matrix
Pole of S-matrix \( (\pi J/\psi : 2\text{nd}, \rho \eta_c : 2\text{nd}, D^{\text{bar}}D^*: 2\text{nd}) \)

- Pole corresponding to "virtual state"
- Pole contribution to scat. observables is small (far from scat. axis)
- \( Z_c(3900) \) is not a resonance but "threshold cusp" induced by strong \( V_{\pi J/\psi, D^{\text{bar}}D^*} \)
Summary

**HAL QCD method**

- NBS wave function $\psi(r)$ --> 2PI kernel ($\psi = \phi + G_0 U \psi$)
- Crucial for multi-hadron & coupled-channel scatterings

  Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
  Ishii et al. [HAL QCD], PLB 712, 437 (2012).
  Aoki et al. (HAL QCD), PRD87, 034512 (2013).

**Exotic candidates, H, ΩΩ, Zc(3900)**

- H particle is very close to $N\Xi$ threshold --> J-PARC?
  Sasaki et al [HAL QCD], in preparation.

- $ΩΩ$ is very close to unitary region --> HIC?
  Gongyo, Sasaki et al [HAL QCD], PRL120, 212001 (2018).

- $Z_c(3900)$ is threshold cusp induced by strong $V^{D\bar{D}^*}, \pi J/\psi$
  Ikeda et al. [HAL QCD], PRL117, 242001 (2016).

**Future: many hadron resonances & nuclear structures at physical point**
Thank you for your attention!