Stress-Tensor Distribution in Quark-Antiquark System

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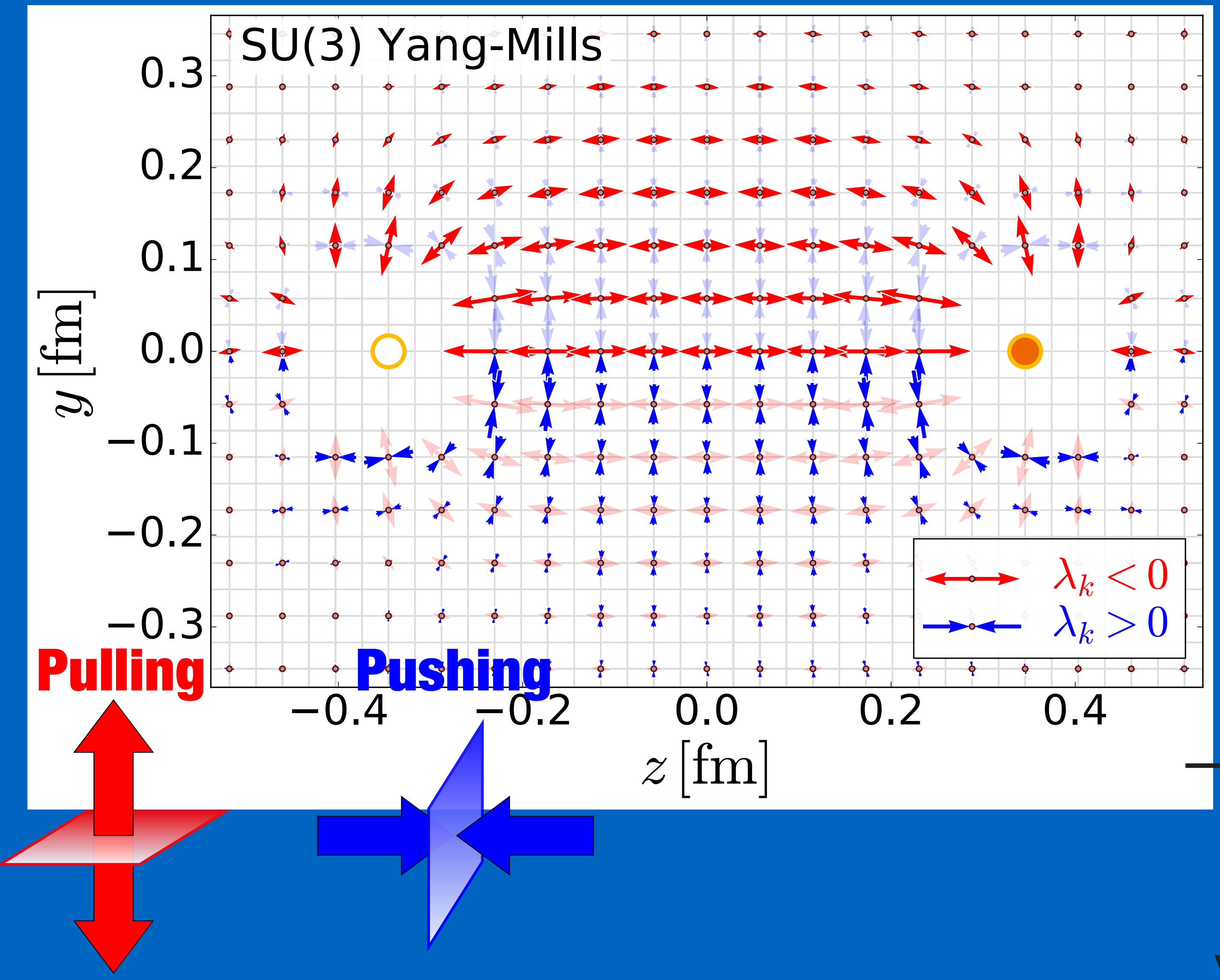
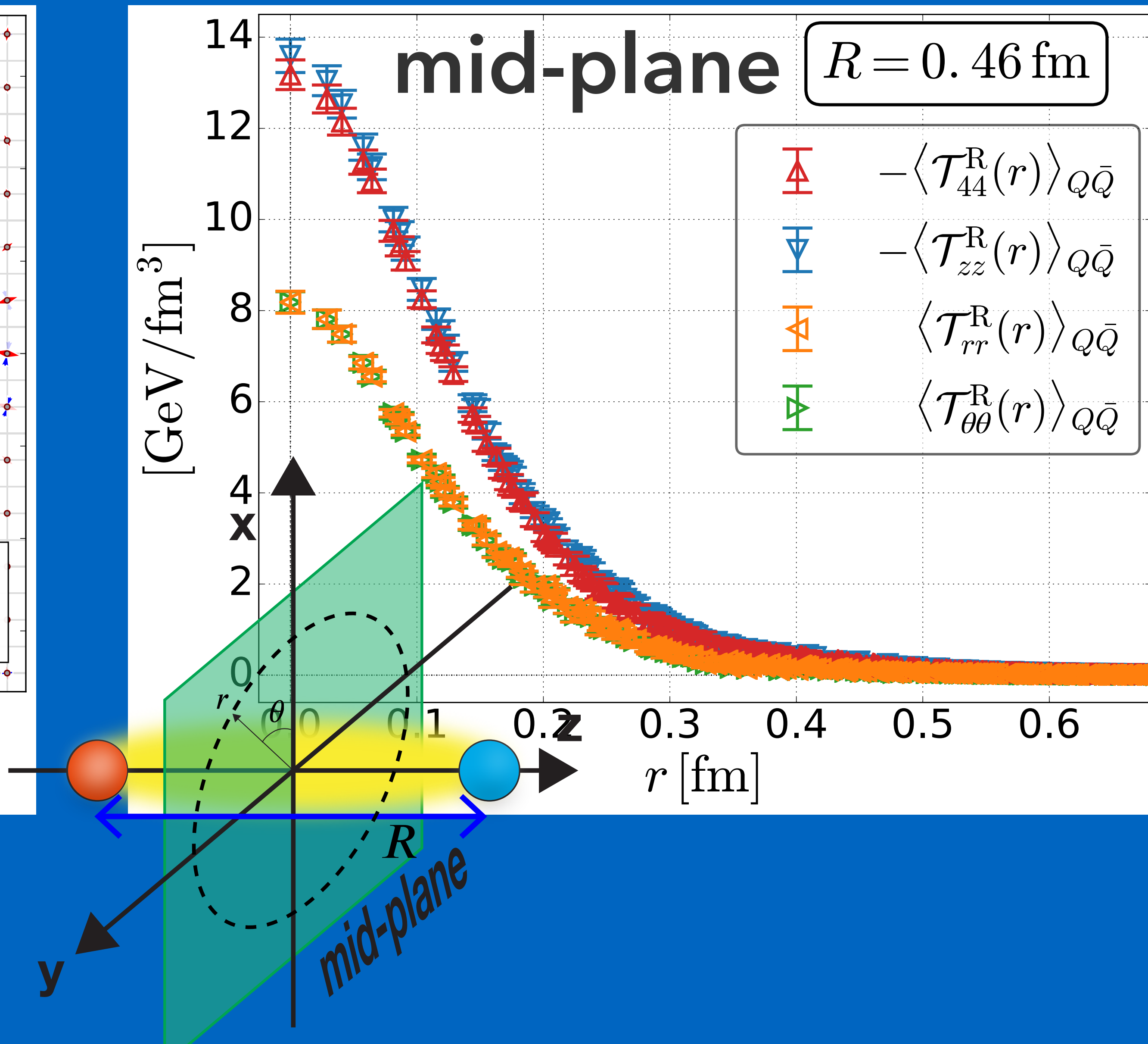
Reference arXiv:1803.05656[hep-lat]

The First Observation of the Energy-Momentum Tensor around static Quark-Antiquark





1. Introduction: Quark-Antiquark System
2. Energy-Momentum Tensor (EMT) and Gradient Flow
3. EMT around static Quark-Antiquark

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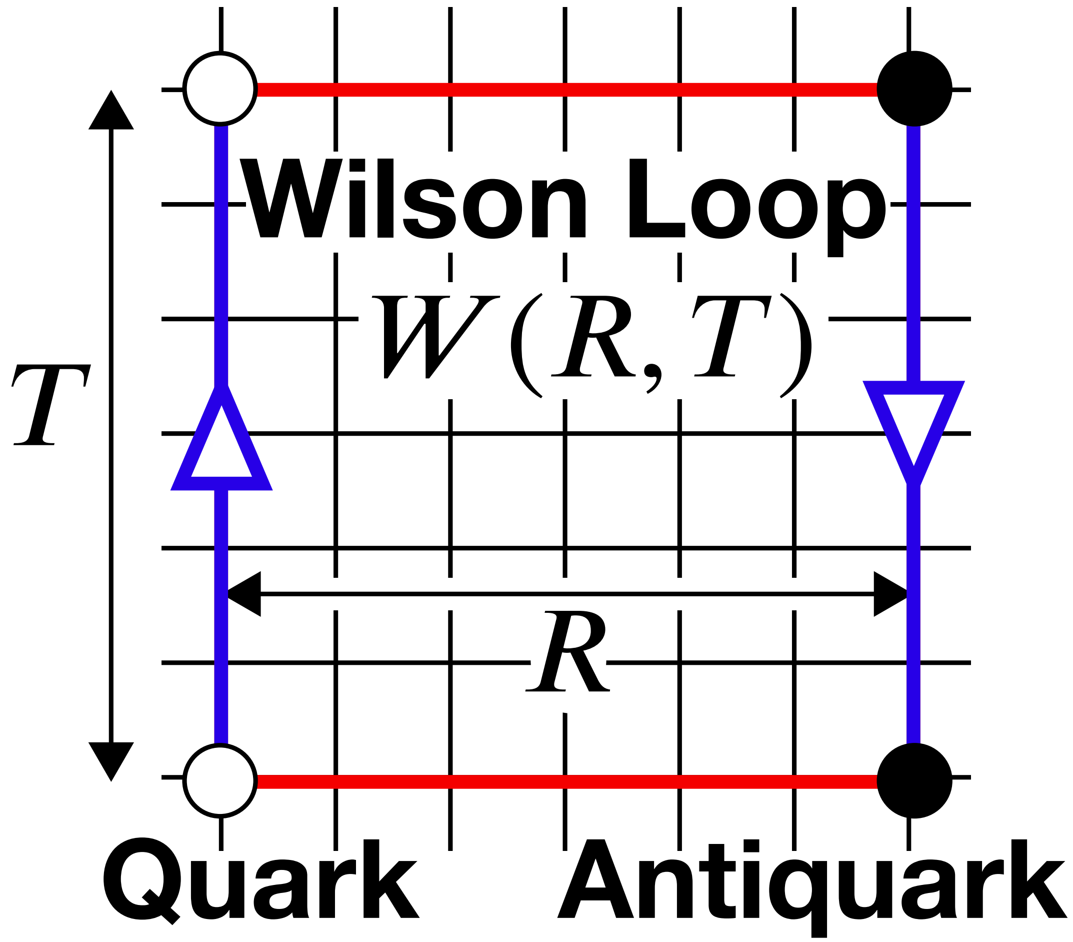
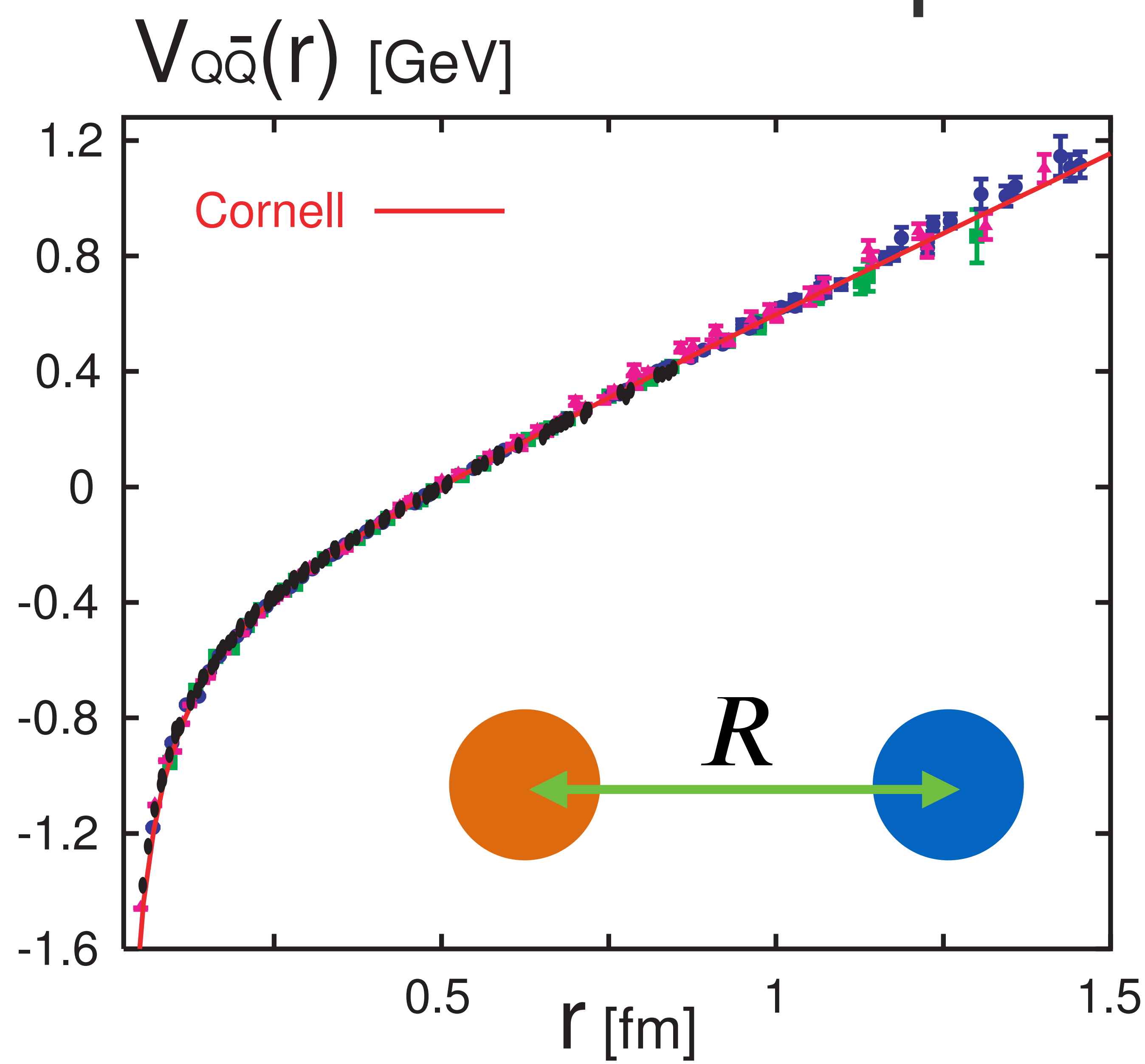
Static Quark-Antiquark System

Wilson loop: time evolution of Quark-Antiquark system

$$\langle W(R,T)\rangle \sim \langle Q\bar{Q}|e^{-HT}|Q\bar{Q}\rangle \propto \exp(-V_{Q\bar{Q}}(R)T)$$



Coulomb + Linear pot.



"Tube"-structure between Quark and Antiquark

- Origin of a linear rising potential – "**tube**" structure
- a tube-like structure is established by lattice QCD
  - *Dual-superconductor scenario?* (Nambu, 't Hooft, Mandelstam in the 1970s)

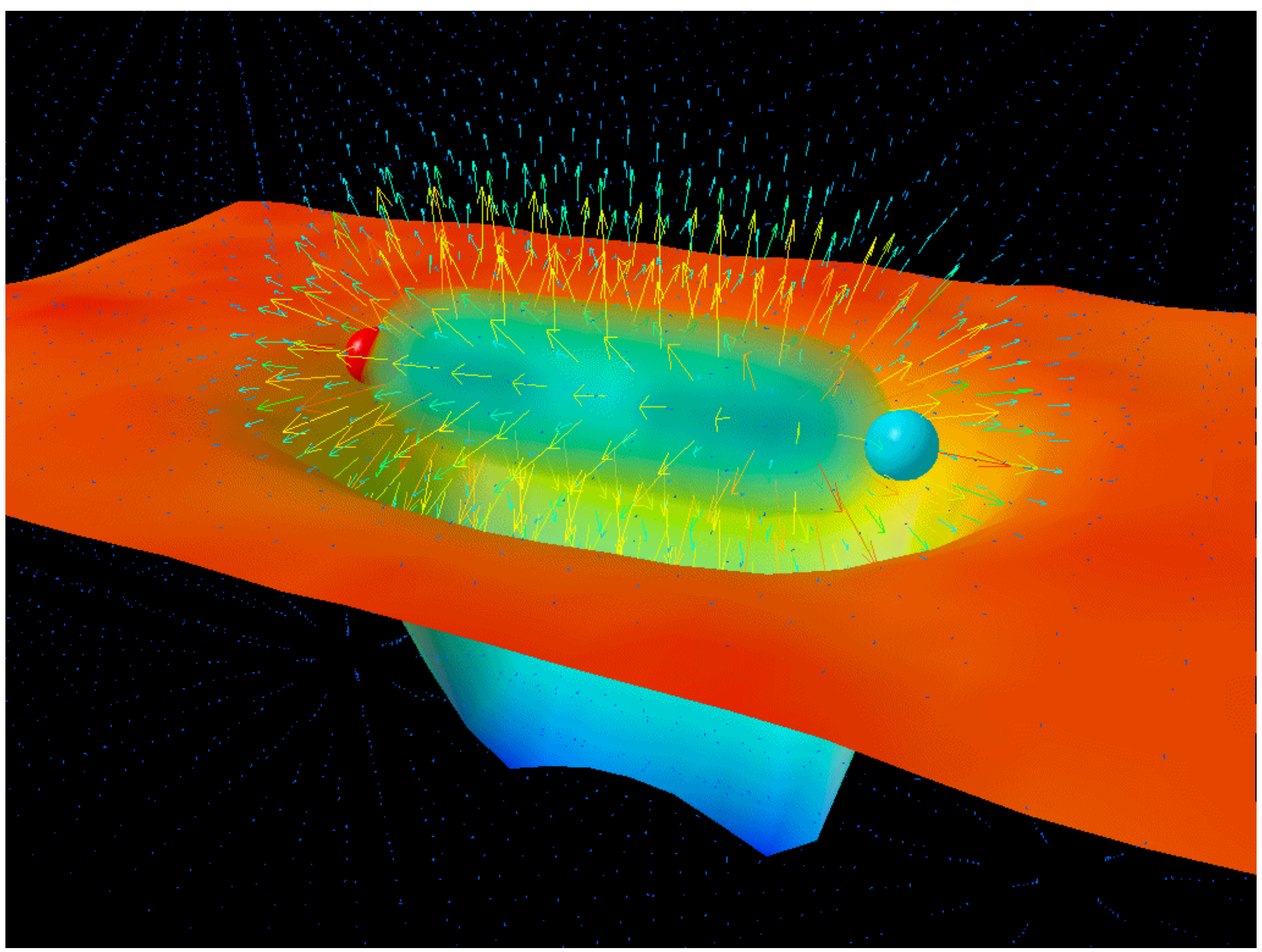
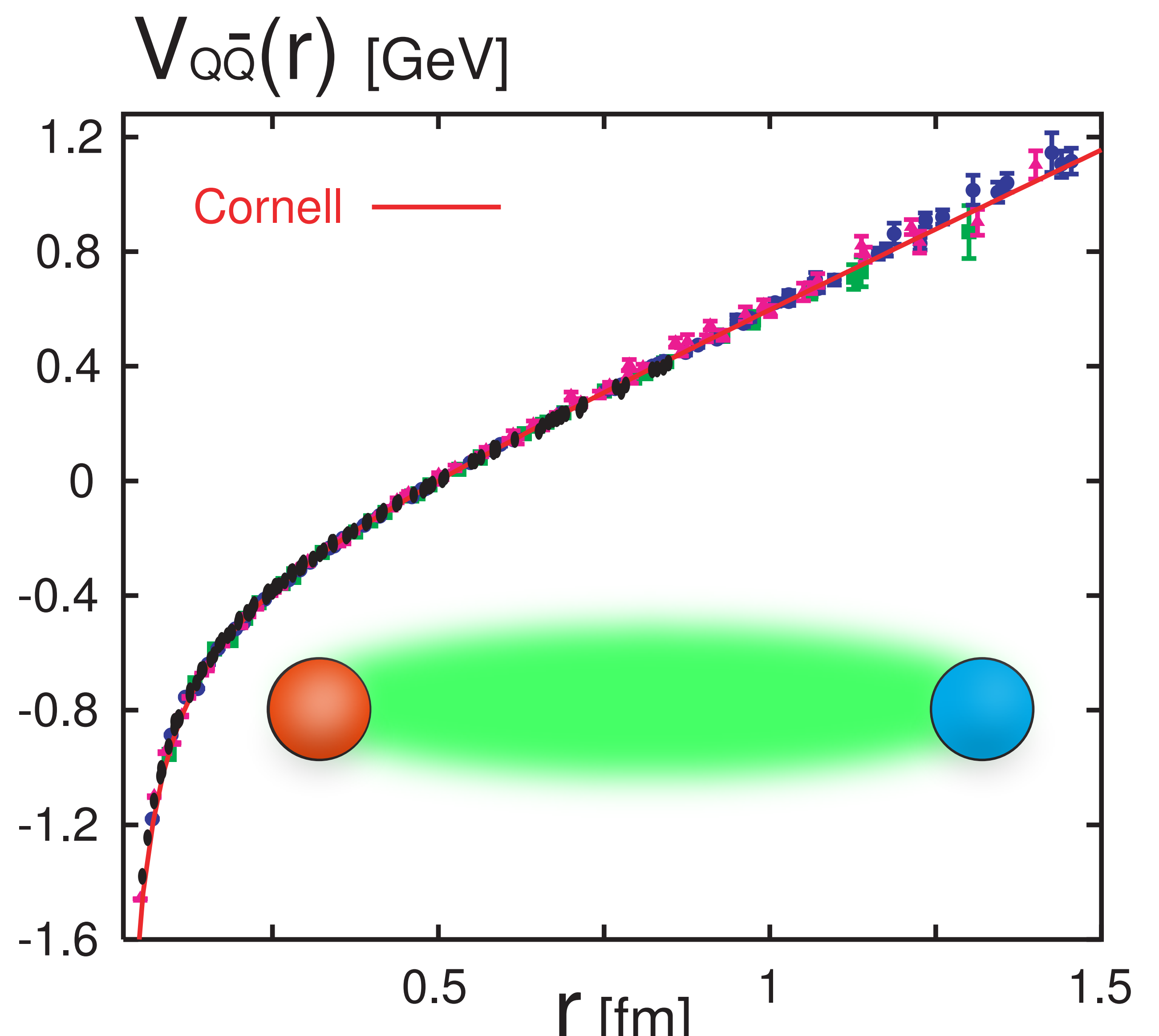


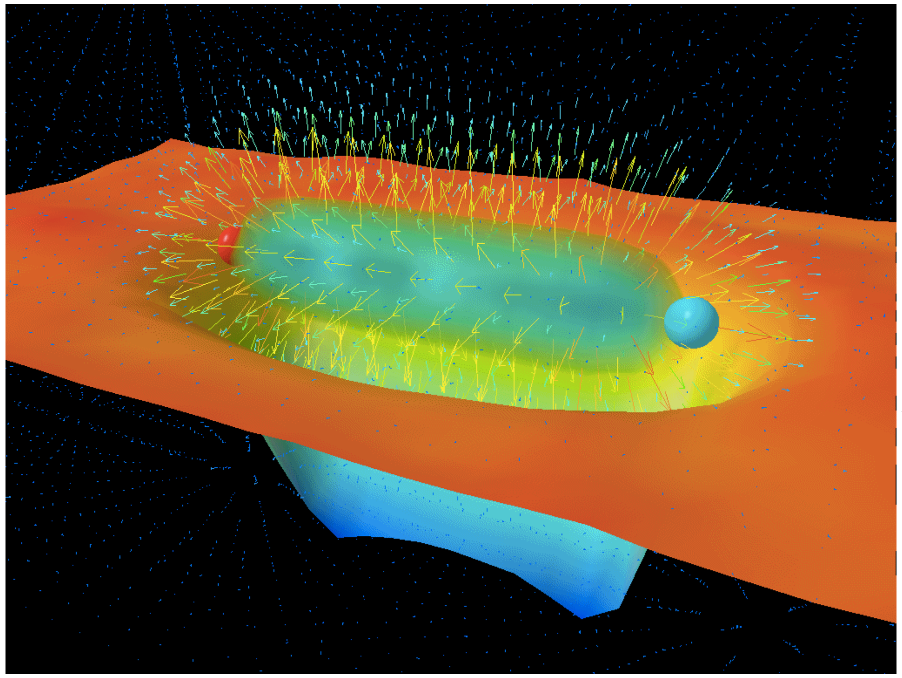
Fig. Lattice QCD calc. by Leinweber

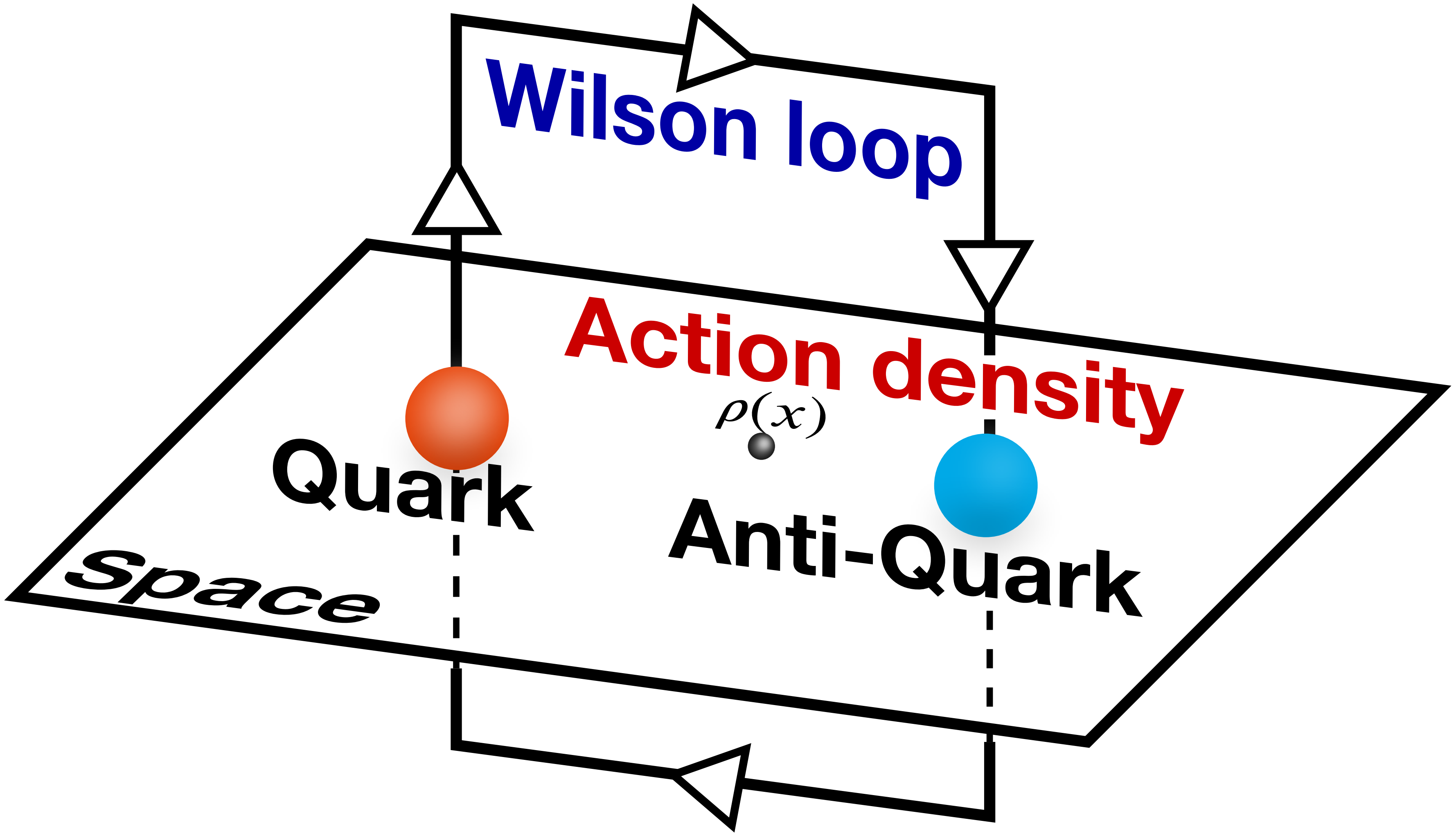


How to Observe the Tube from Lattice QCD?

- Spatial correlation of "**action density**" & **Wilson loop**

$$\langle \rho(x) \rangle_W = \frac{\langle \rho(x) W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \rho \rangle$$





Probing into QQ^{bar} System with Various Operators

longitudinal chromoelectric fields are dominant components

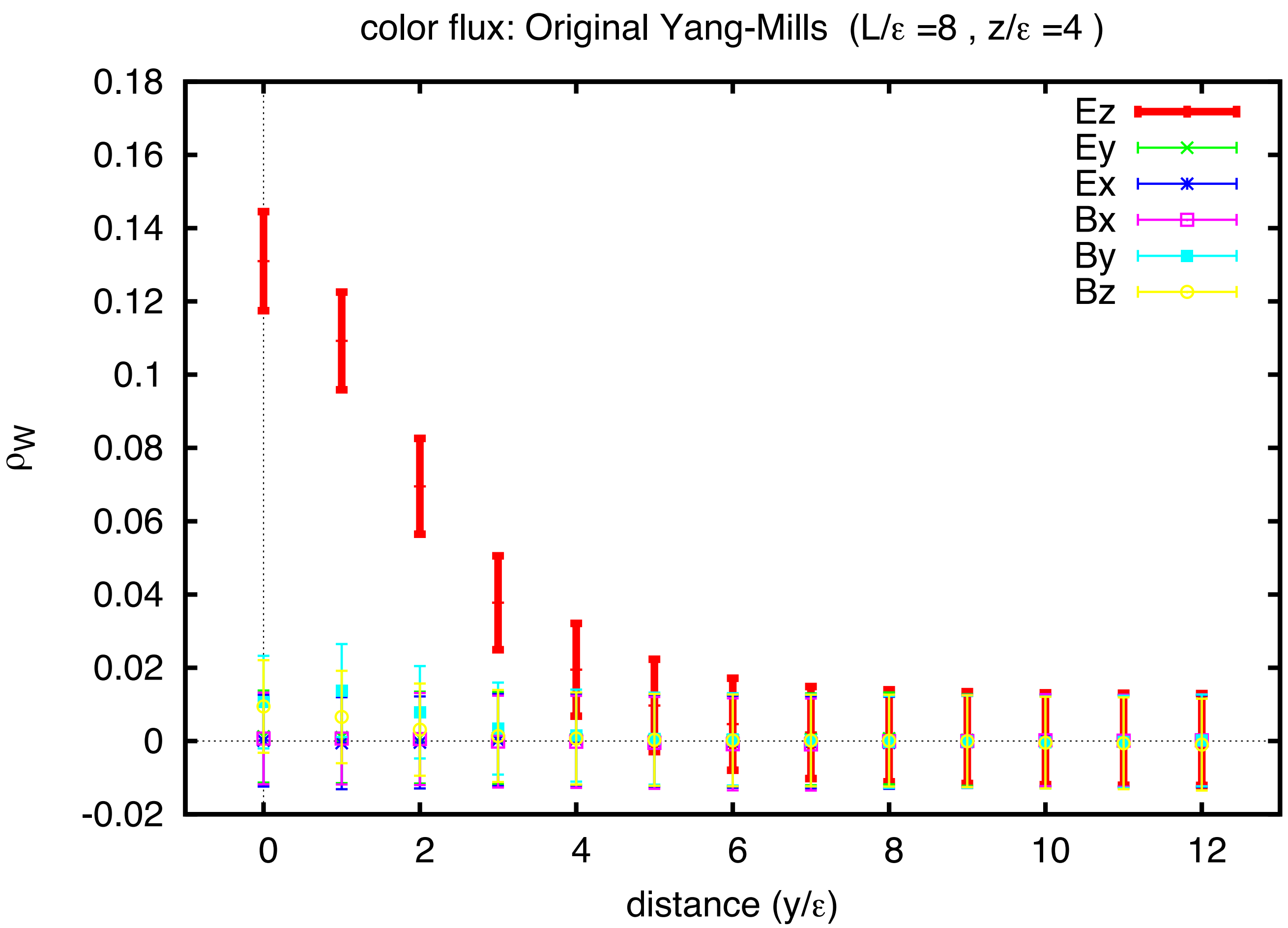
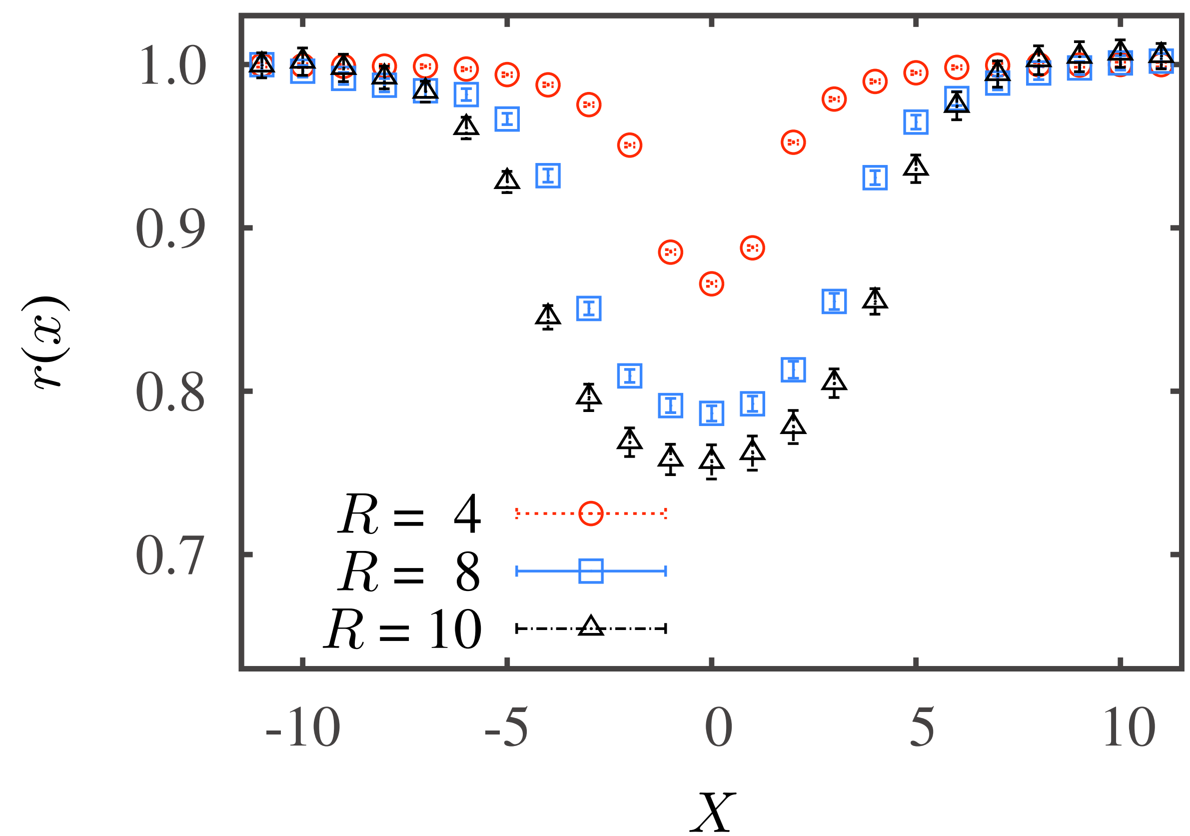


Fig. Shibata-Kondo-Kato-Shinohara '12

See also Cea *et al.*

chiral condensate is effectively **reduced**

$$r(x) = \langle \bar{q}q(x) \rangle / \langle \bar{q}q \rangle_{\mathrm{vac.}}$$



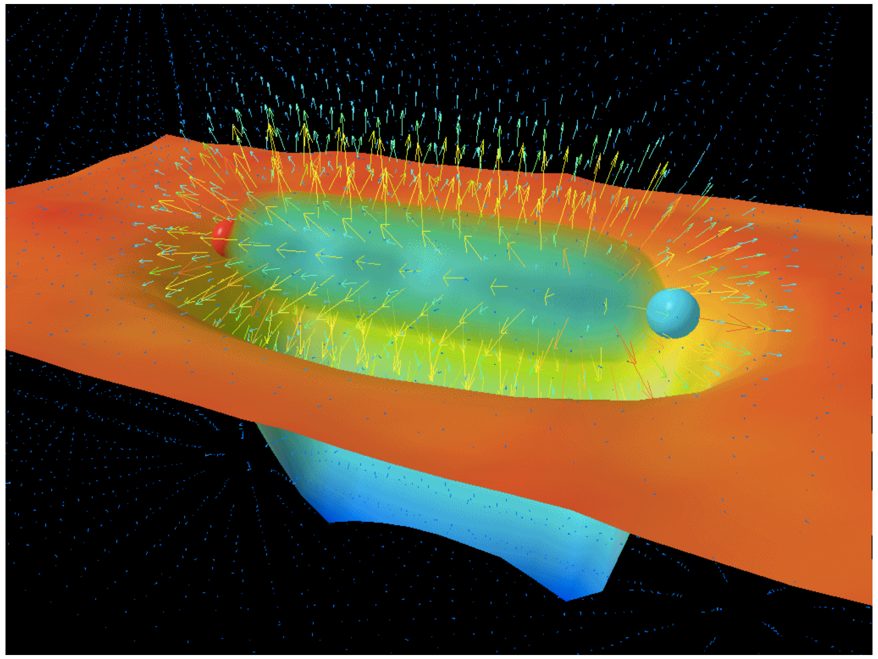
TI-Cossu-Hashimoto '13

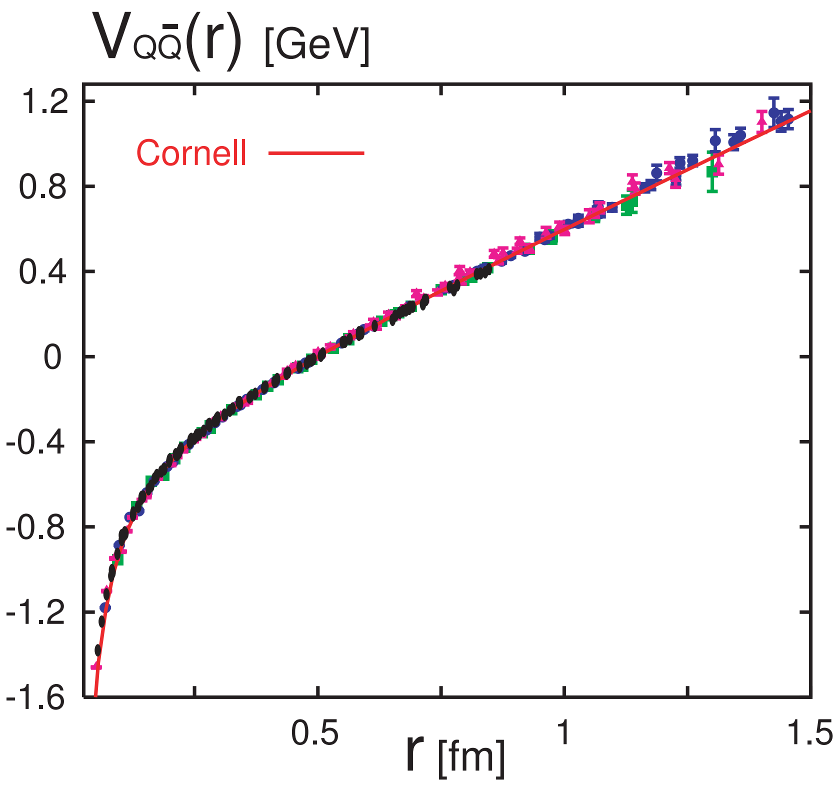
How to Extract Quantitative Information?

- **Tube** structures are observed from lattice QCD, but
  - **Action density or Chromo-Electric/Magnetic Fields**

    renormalization ? continuum limit ?

    what about the relation between interquark force ?





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Energy-Momentum Tensor (EMT)

- $T_{\mu\nu}$: generator of **Poincaré group**

energy momentum

$$T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

stress

pressure

Energy-Momentum Tensor (EMT)

- $T_{\mu\nu}$: generator of **Poincaré group**
  - **conservation law** $\partial^\mu T_{\mu\nu} = 0$
  - **trace anomaly** $T^\mu_\mu = \frac{\beta}{2g} G_{\mu\nu} G^{\mu\nu}$

- **energy density** $\varepsilon(x) = -T_{44}(x)$
- **stress tensor** $\sigma_{ij}(x) = -T_{ij}(x) \longrightarrow \mathcal{F}_i = \sigma_{ij} n_j$ **force per area**

At finite temperature

$\langle T_{\mu\nu} \rangle$

$\langle (T_{\mu\nu})^2 \rangle$

$\langle T_{\mu\nu} T_{\rho\sigma} \rangle$

bulk thermodynamics

$\varepsilon(T), P(T), s(T)$

fluctuations

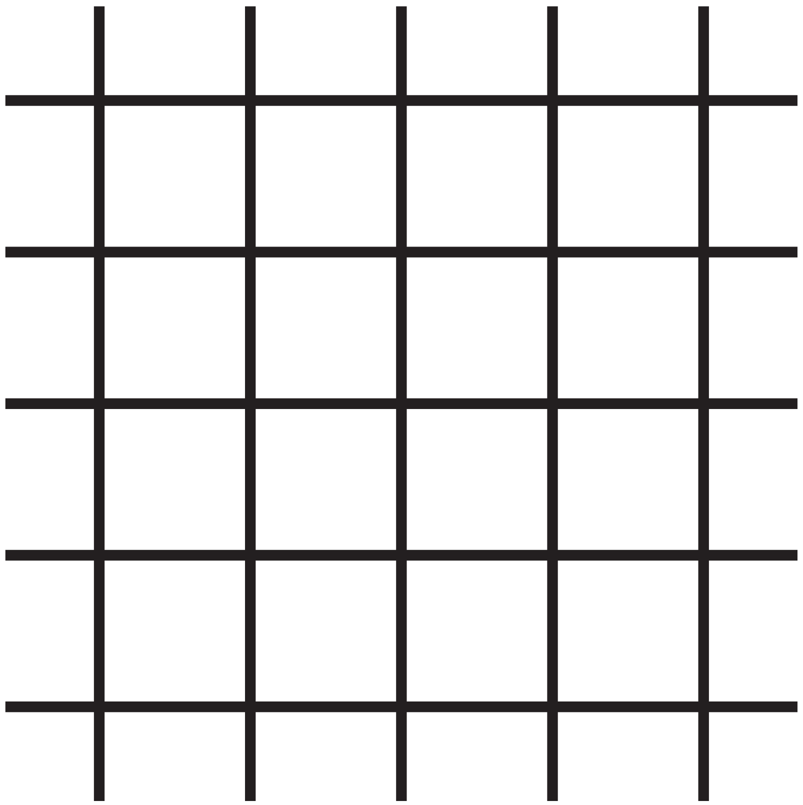
$C_V(T)$

transport coeffs.

$\eta(T), \zeta(T)$

Problems of EMT in Lattice Gauge Theory

- **Lorentz sym.** is **broken**: definition is non-trivial



$$T_{\mu\nu}^{\rm lat} = F_{\mu\rho}^{\rm lat} F_{\nu\rho}^{\rm lat} - \frac{1}{4}\delta_{\mu\nu} F^{\rm lat} F^{\rm lat}$$

$$\lim_{a\to 0} T_{\mu\nu}^{\rm lat} \neq T_{\mu\nu}$$

- dim. 4 operator – **UV fluctuation** is difficult to control

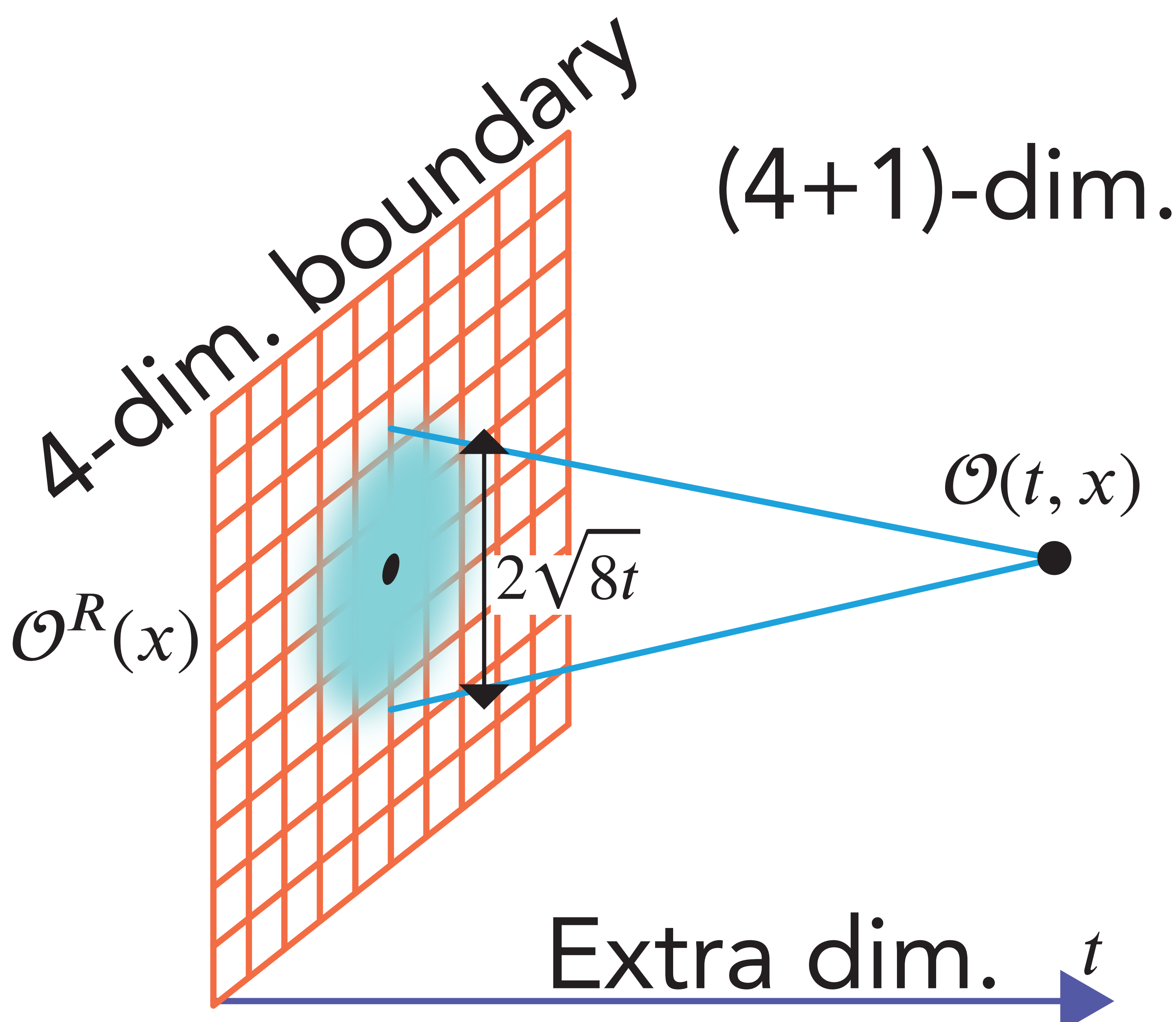
several strategies:
ex. matching bulk thermodynamic quantities, shifted boundary conditions, …

Gradient Flow

Diffusion Eq. to "Extra-dim."

Lüscher & Weisz '11

$$\frac{\partial B_\mu(t,x)}{\partial t} = -g_0^2 \frac{\delta S[B]}{\delta B_\mu(t,x)}, \qquad B_\mu(0,x) = A_\mu(x)$$



flowed ops. **UV finite**

$\mathcal{O}(t,x)$

small-t expansion

$$\mathcal{O}(t,x) \overset{t\to 0}{\longrightarrow} \sum c_i(t)\mathcal{O}_i^R(x)$$

renormalized ops. of

original theory

$\mathcal{O}^R(x)$

Renormalized EMT by Gradient Flow

H. Suzuki '13

$$T^R_{\mu\nu}(x) = \lim_{t\to 0} T_{\mu\nu}(t,x)$$

flow time = renormalization scale

$$T_{\mu\nu}(t,x) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} \left[E(t,x) - \langle E(t,x)\rangle_0 \right]$$

dim. 4 ops.

$$U_{\mu\nu}(t,x) = G^a_{\mu\rho}(t,x)G^a_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G^a_{\rho\sigma}(t,x)G^a_{\rho\sigma}(t,x)$$

$$E(t,x) = \frac{1}{4}G^a_{\mu\nu}(t,x)G^a_{\mu\nu}(t,x)$$

coefficients

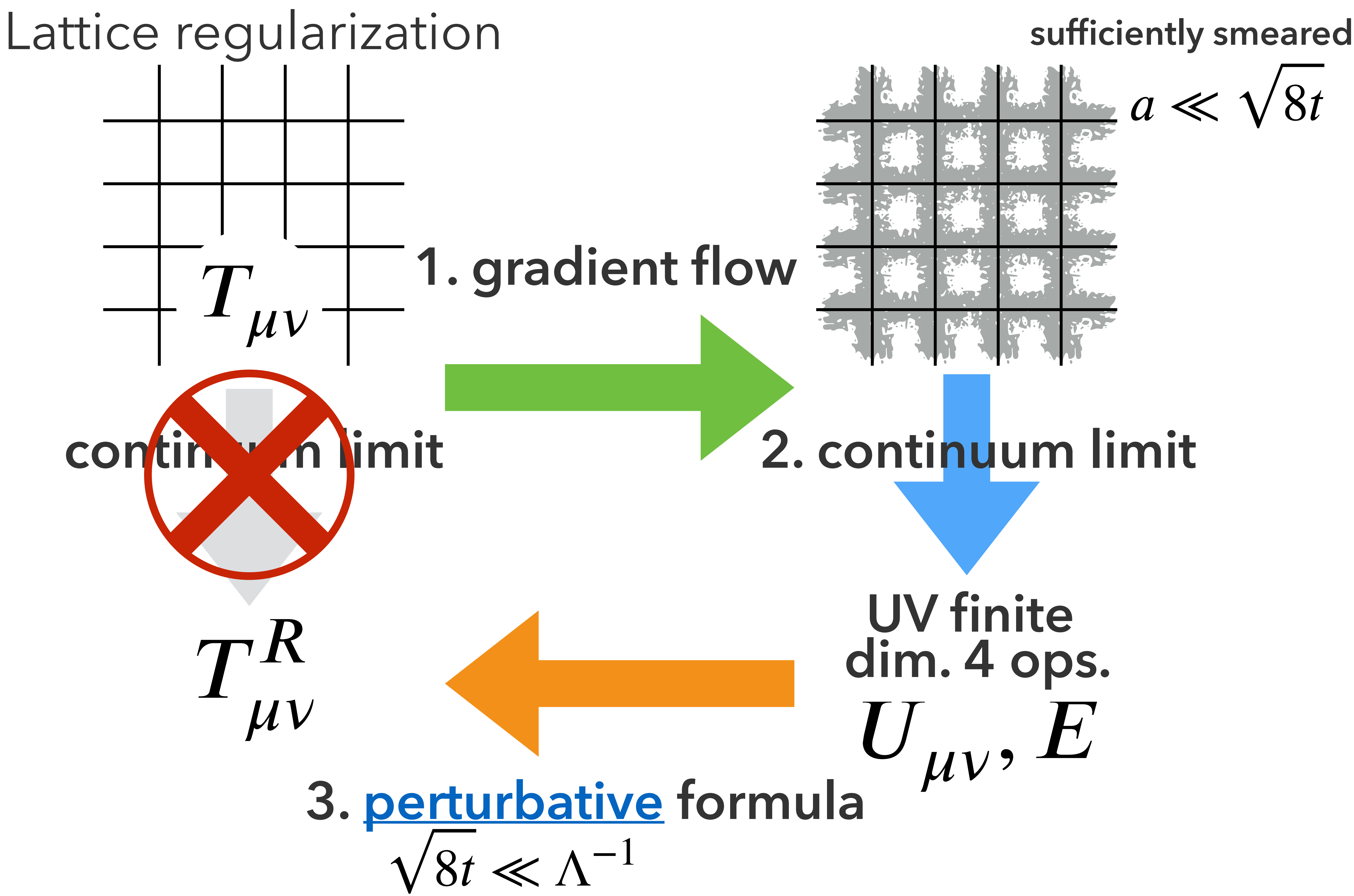
$$\alpha_U(t) = \bar{g}^2\left[1 + 2b_0\bar{s}_1\bar{g}^2 + \mathcal{O}(\bar{g}^4)\right], \quad \alpha_E(t) = \frac{1}{2b_0}\left[1 + 2b_0\bar{s}_2\bar{g}^2 + \mathcal{O}(\bar{g}^4)\right]$$

$\bar{g} \equiv \bar{g}(1/\sqrt{8t})$: running coupling in MS-bar scheme

$$b_0 = \frac{1}{(4\pi)^2}\frac{11}{3}N_c, \quad \bar{s}_1 \simeq -0.008635\cdots, \quad \bar{s}_2 \simeq 0.055785\cdots$$

with fermion H. Makino & H. Suzuki '14

EMT using Gradient Flow



Ex. Thermodynamics of SU(3) YM from EMT

- Trace anomaly

$$\Delta = \varepsilon - 3p = -\langle T_{\mu\mu}(x) \rangle$$

- Entropy density

$$sT = \varepsilon + p = -\langle T_{44}(x) \rangle + \frac{1}{3} \sum_{i=1}^{3} \langle T_{ii}(x) \rangle$$

1. Calc. EMT from "flowed fields"

$$T_{\mu\nu}^{\mathrm{lat.}}(t, x)$$

2. Continuum limit

$$T_{\mu\nu}(t, x) = \lim_{a \to 0} \left[T_{\mu\nu}^{\mathrm{lat.}}(t, x) + O(a^2) \right]$$

3. Flow time zero limit

$$T_{\mu\nu}^{R}(x) = \lim_{t \to 0} T_{\mu\nu}(t, x)$$

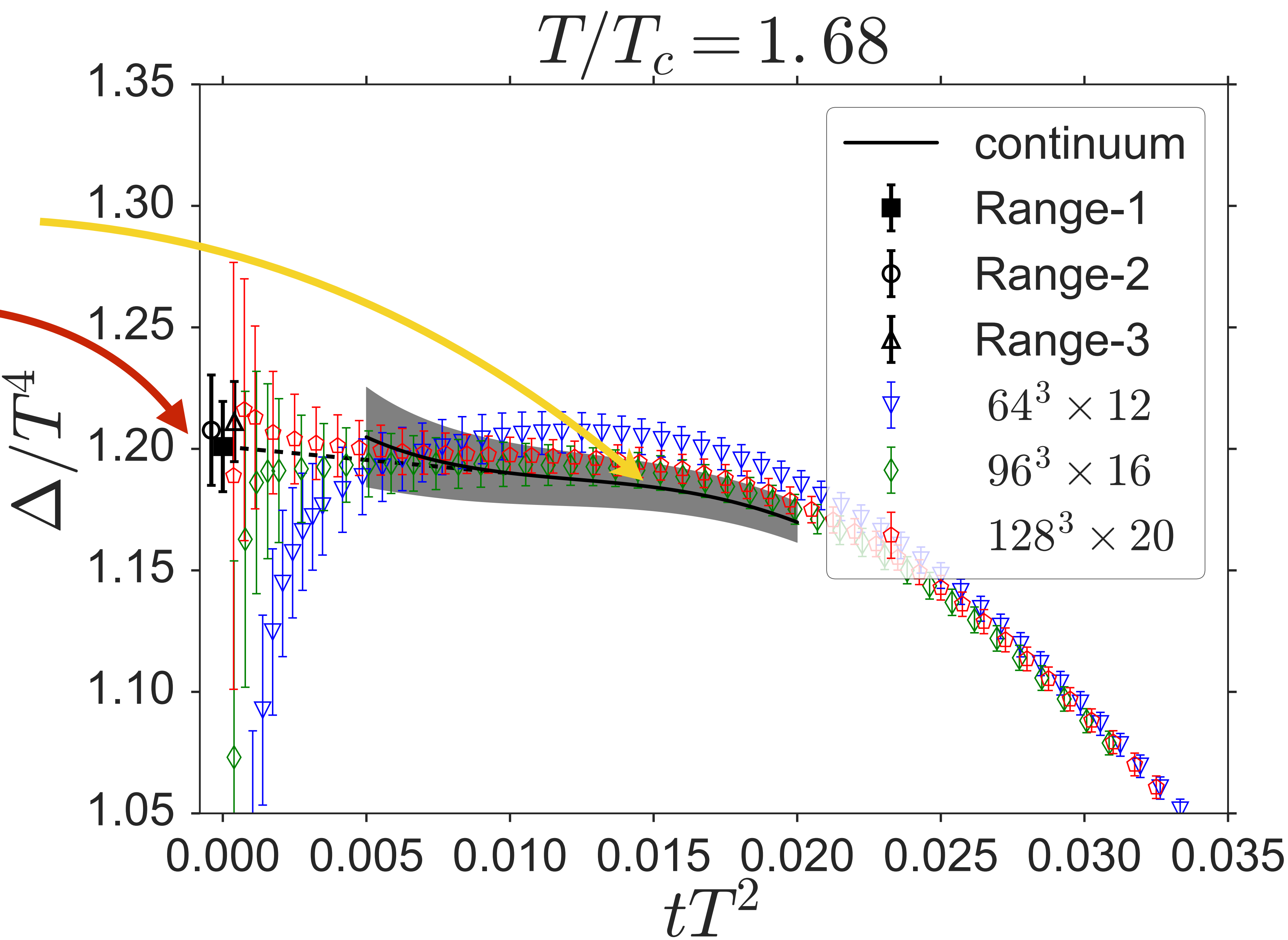
Ex. Double Limit Analysis

- **Smearing length**: $\sqrt{8t}$
- **Fitting window**: smeared & perturbative region

$$a \ll \sqrt{8t} \ll \Lambda^{-1}$$

fine lattice is mandatory

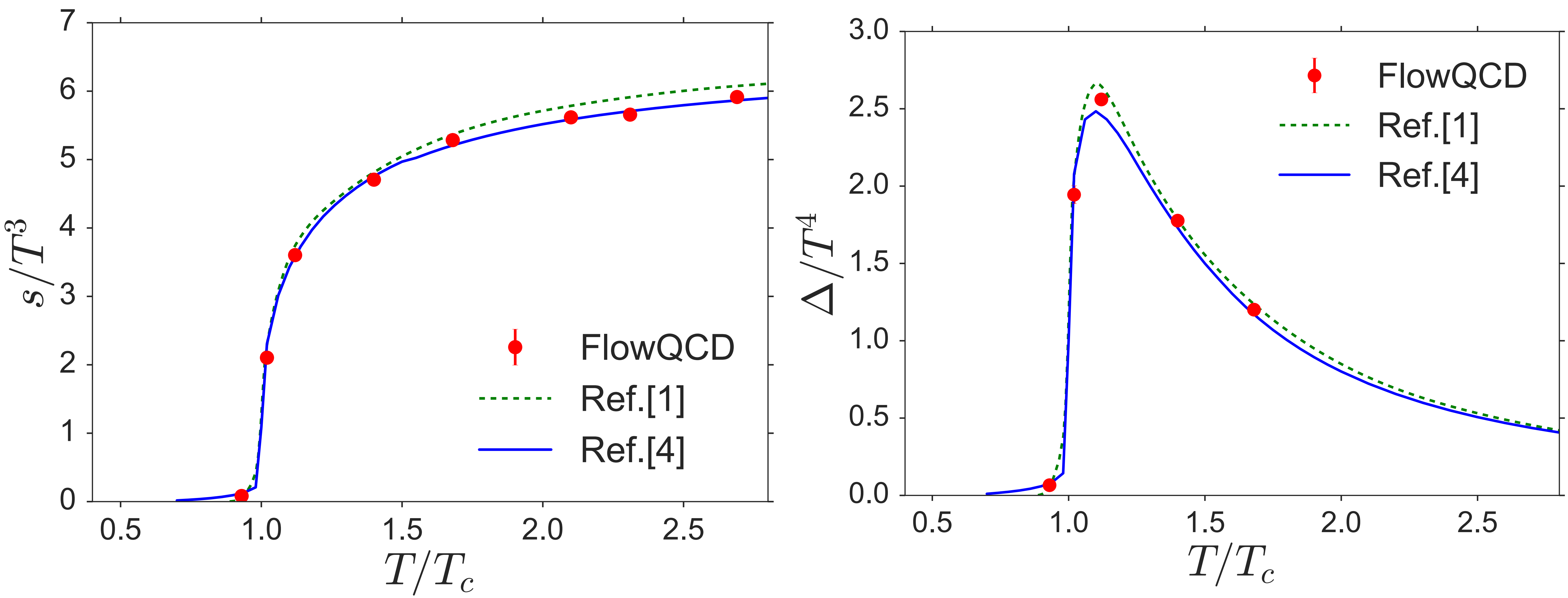
1. **Continuum limit**
2. **Flow zero limit**



Ex. Entropy Density and Trace Anomaly in SU(3)

- Consistent with integral method (w/o using EMT).
- EMT from Gradient Flow works well!

FlowQCD '16



Ref. [1] Boyd et al. '96, Ref. [2] Borsanyi et al. '12

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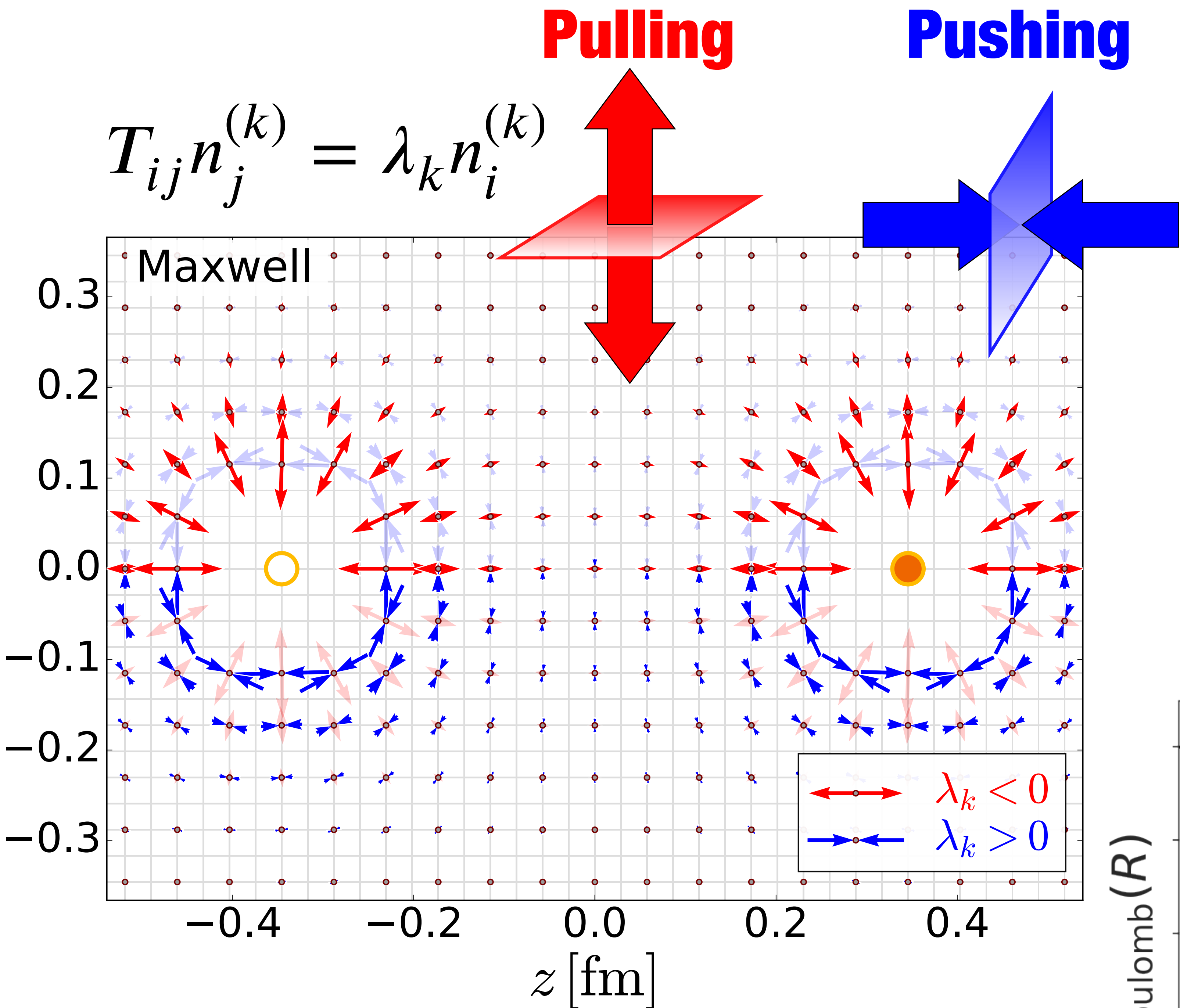
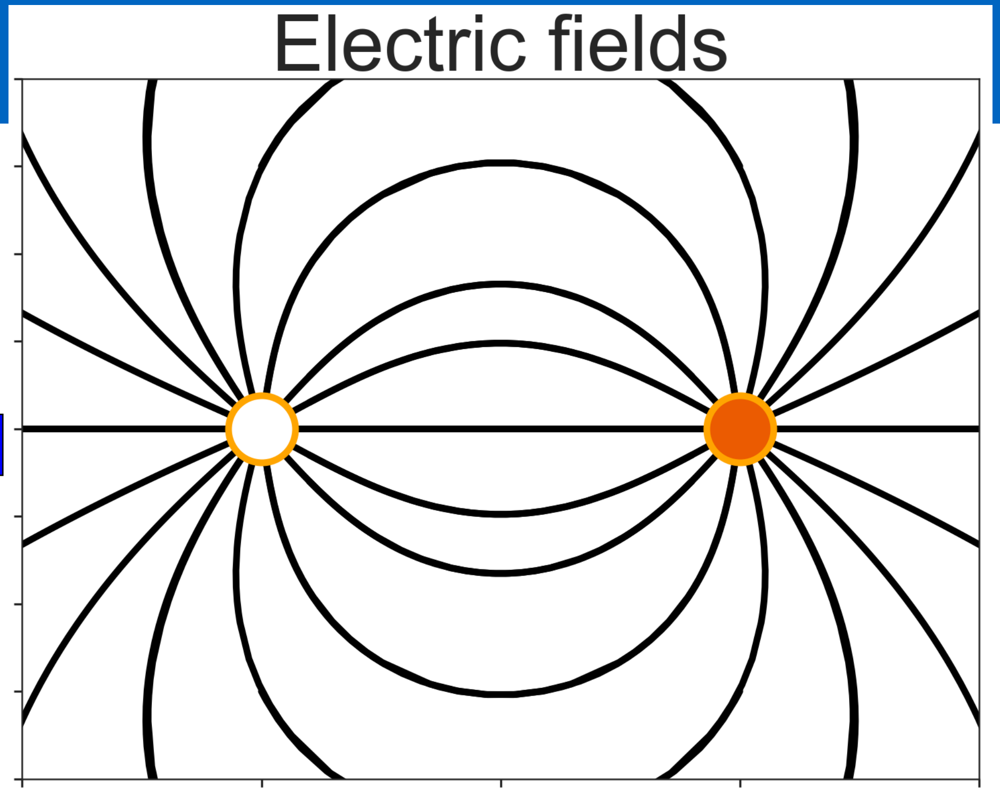
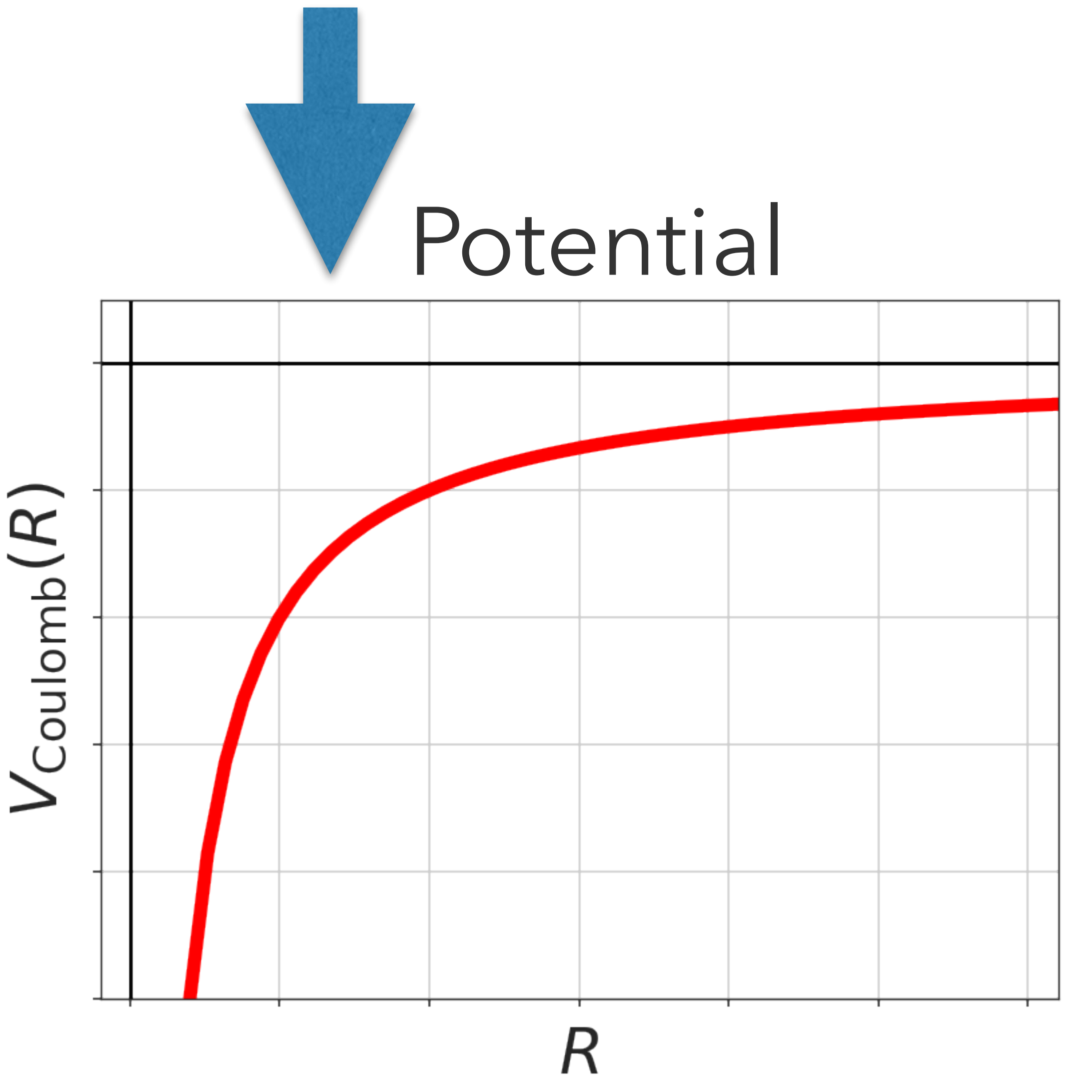
Ex. Maxwell Stress Tensor

$$T_{ij} n_j^{(k)} = \lambda_k n_i^{(k)}$$



the length of the arrow $\propto |\lambda_k|^{1/2}$





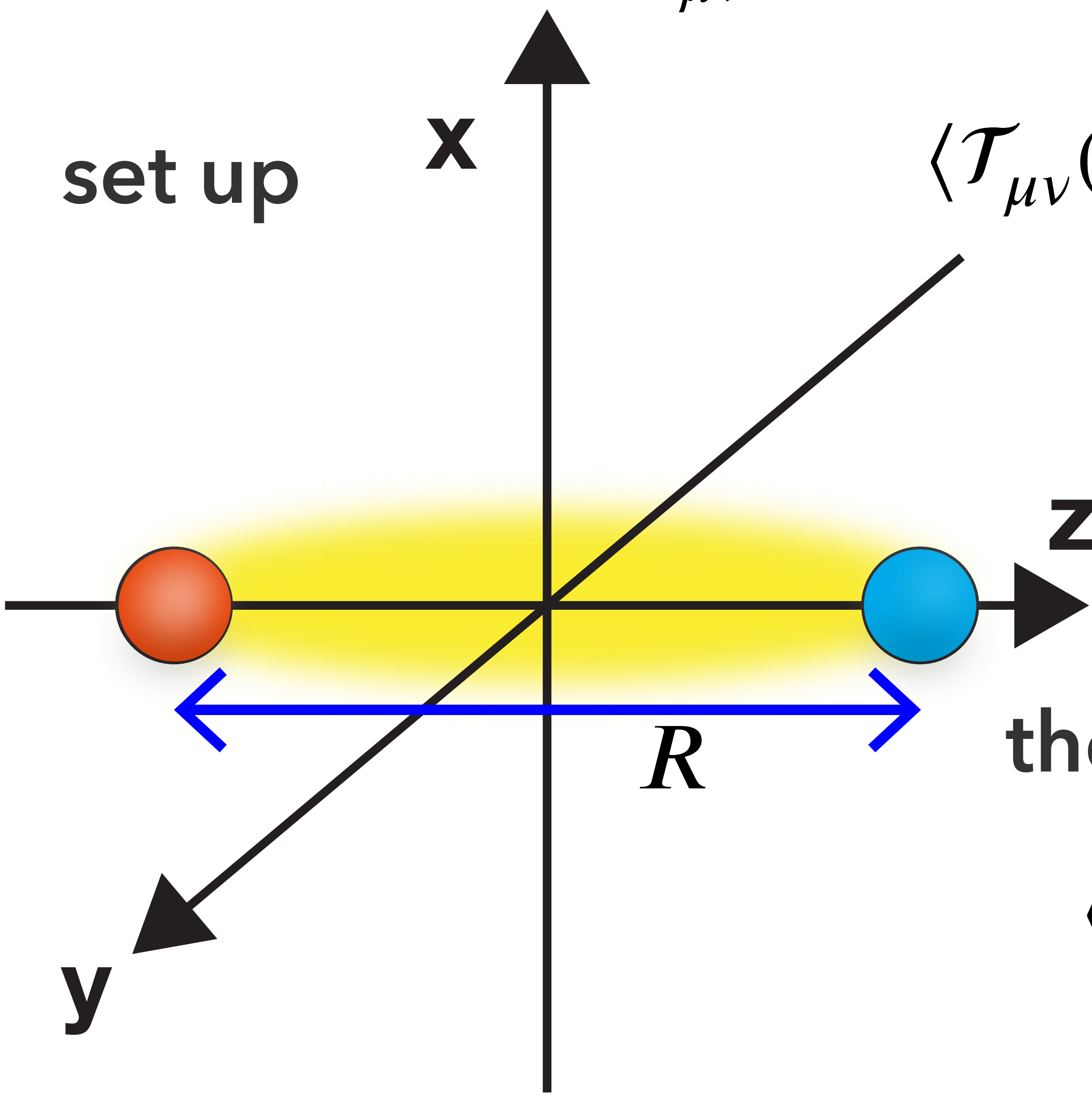
EMT around static Quark-Antiquark

Setup: SU(3) Yang-Mills (quenched QCD)

Operators: $W(R,T)$: APE smearing for spatial link variables & multi-hit procedure for temporal link

$\mathcal{T}_{\mu\nu}(t,x)$: flowed link variables

set up



$$\langle \mathcal{T}_{\mu\nu}(t,x) \rangle^{\text{lat.}}_{Q\bar{Q}} = \lim_{T\to\infty} \frac{\langle \mathcal{T}_{\mu\nu}(t,x) W(R,T) \rangle_0}{\langle W(R,T) \rangle_0}$$

double limit $a \to 0$ and $t \to 0$

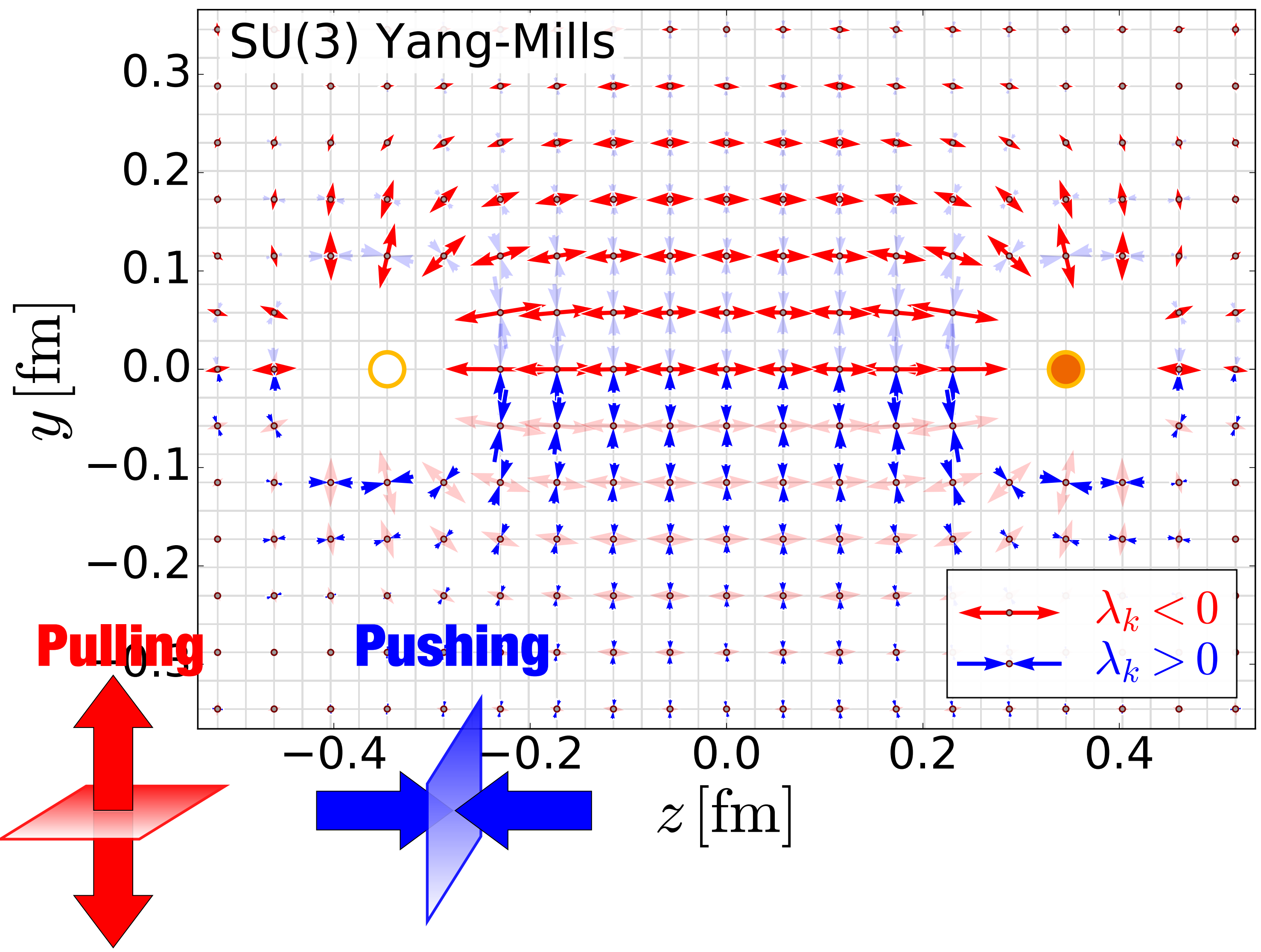
the renormalized EMT distribution

$$\langle \mathcal{T}^{R}_{\mu\nu}(x) \rangle_{Q\bar{Q}} = \lim_{t\to 0} \lim_{a\to 0} \langle \mathcal{T}_{\mu\nu}(t,x) \rangle^{\text{lat.}}_{Q\bar{Q}}$$

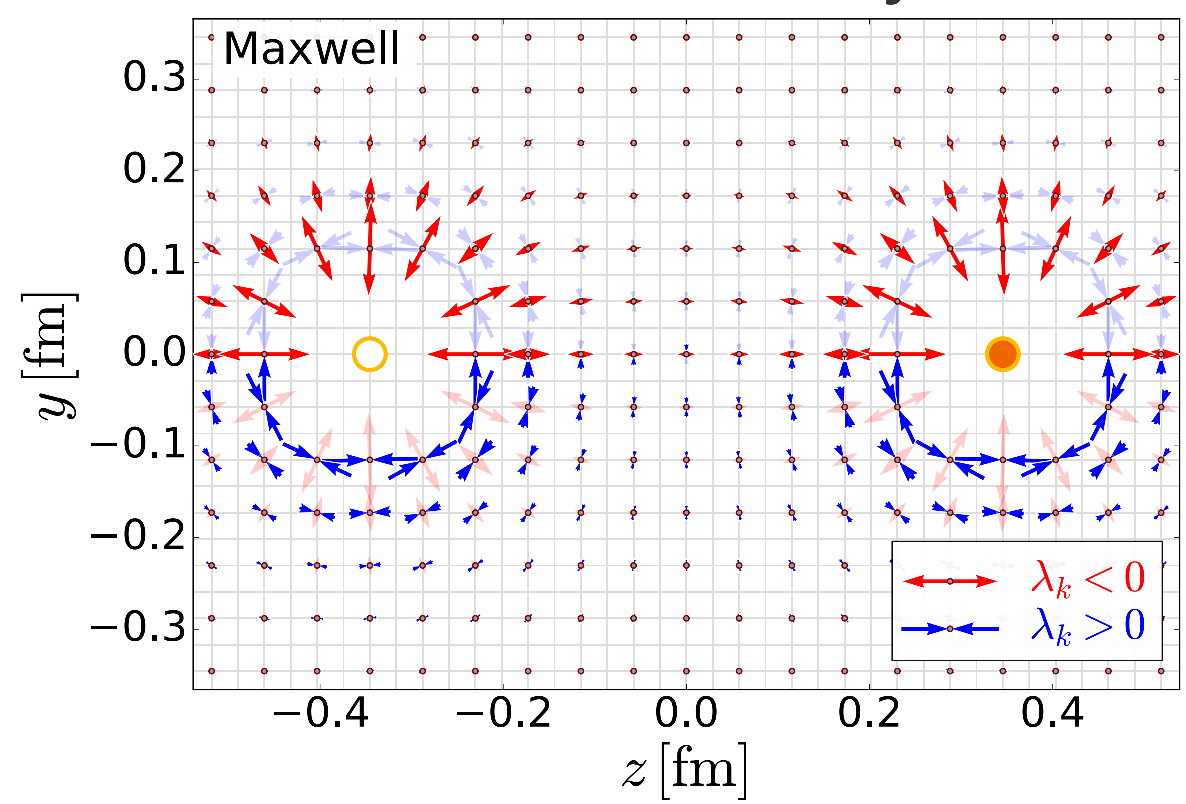
EMT around Quark-Antiquark

Distribution of the principal axes $T_{ij}n_j^{(k)} = \lambda_k n_i^{(k)}$

First observation of "tube" structure by **the EMT!**



cf. classical electrodynamics



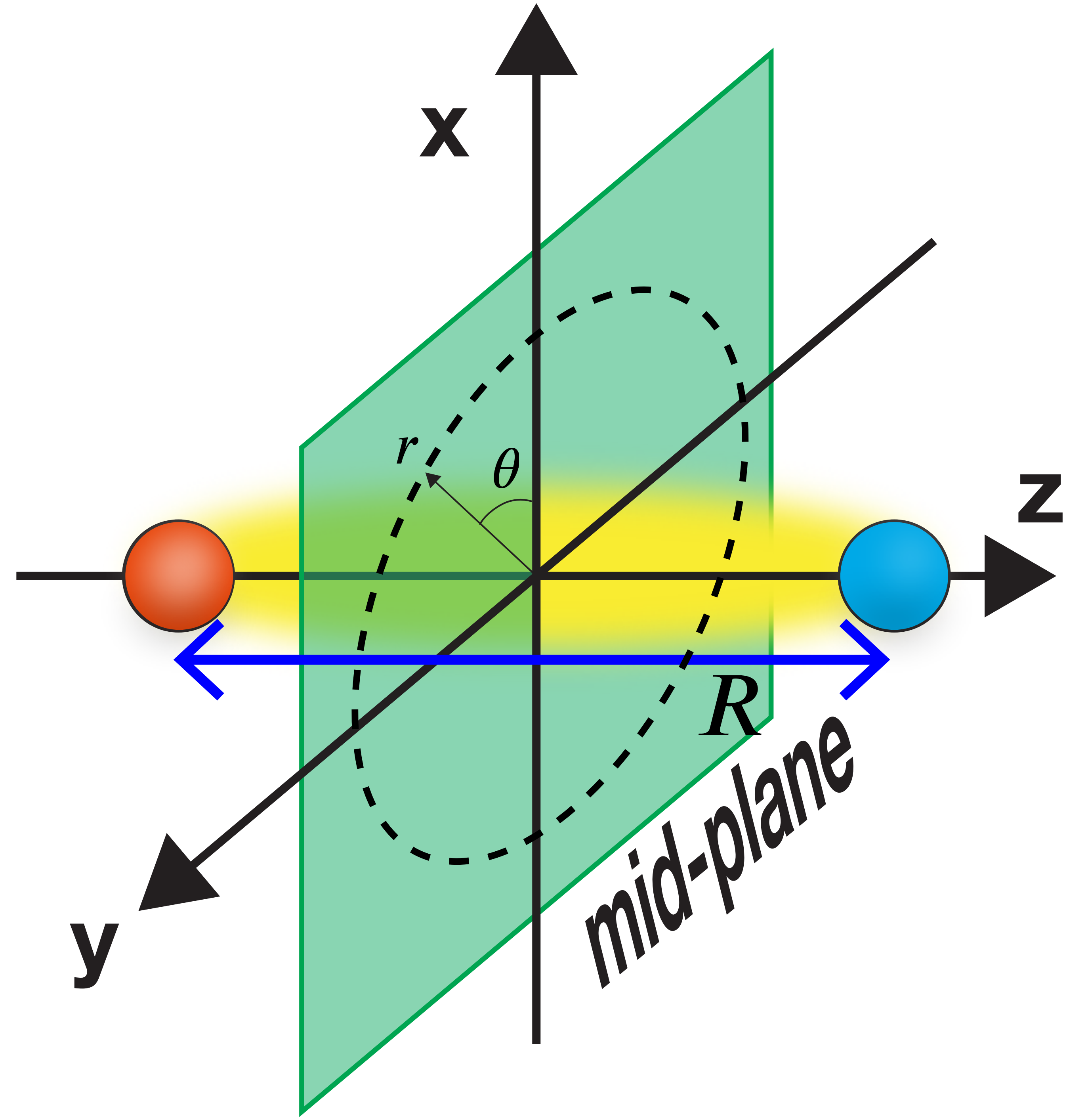
orange circles: Quark, Antiquark

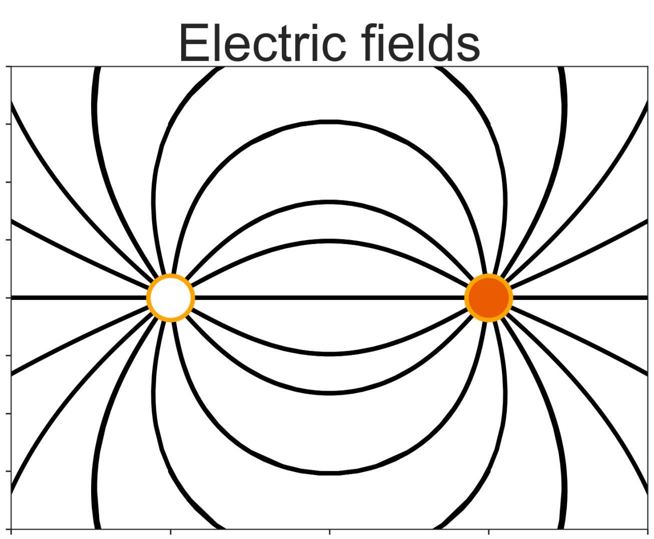
the length of the arrow $\propto |\lambda_k|^{1/2}$, finite flow time w/o cont. limit

Mid-plane

we focus on **the mid-plane**
in **cylindrical coordinates**

$$\mathcal{T}_{cc'} = \begin{pmatrix} \mathcal{T}_{rr} & & & \\ & \mathcal{T}_{\theta\theta} & & \\ & & \mathcal{T}_{zz} & \\ & & & \mathcal{T}_{44} \end{pmatrix}$$





degeneration in Maxwell

$$\mathcal{T}_{44}(r) = \mathcal{T}_{zz}(r) = -\mathcal{T}_{rr}(r) = -\mathcal{T}_{\theta\theta}(r)$$

Renormalized Stress-Tensor in Mid-plane (1) R = 0.46 fm

- **non-trivial degeneracies**

$$\langle \mathcal{T}_{44}^{R}(r)\rangle_{Q\bar{Q}} \simeq \langle \mathcal{T}_{zz}^{R}(r)\rangle_{Q\bar{Q}} < 0$$

$$\langle \mathcal{T}_{rr}^{R}(r)\rangle_{Q\bar{Q}} \simeq \langle \mathcal{T}_{\theta\theta}^{R}(r)\rangle_{Q\bar{Q}} > 0$$

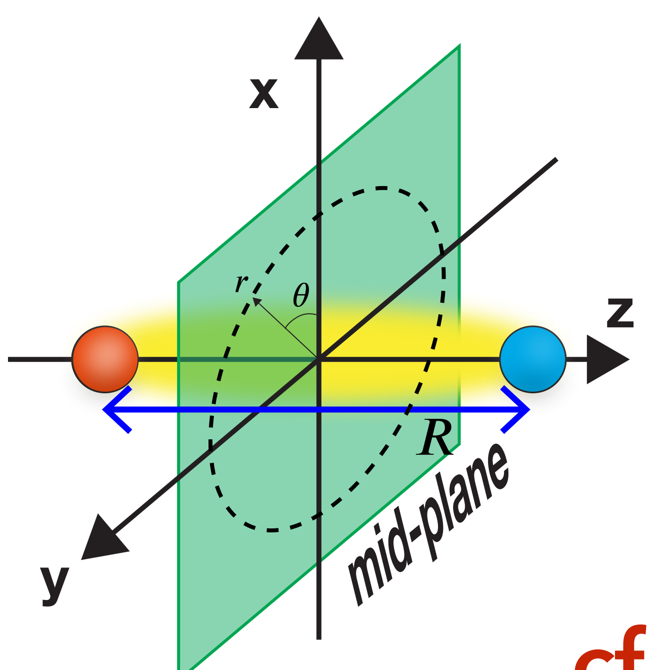
- partially restoration of **trace anomaly**

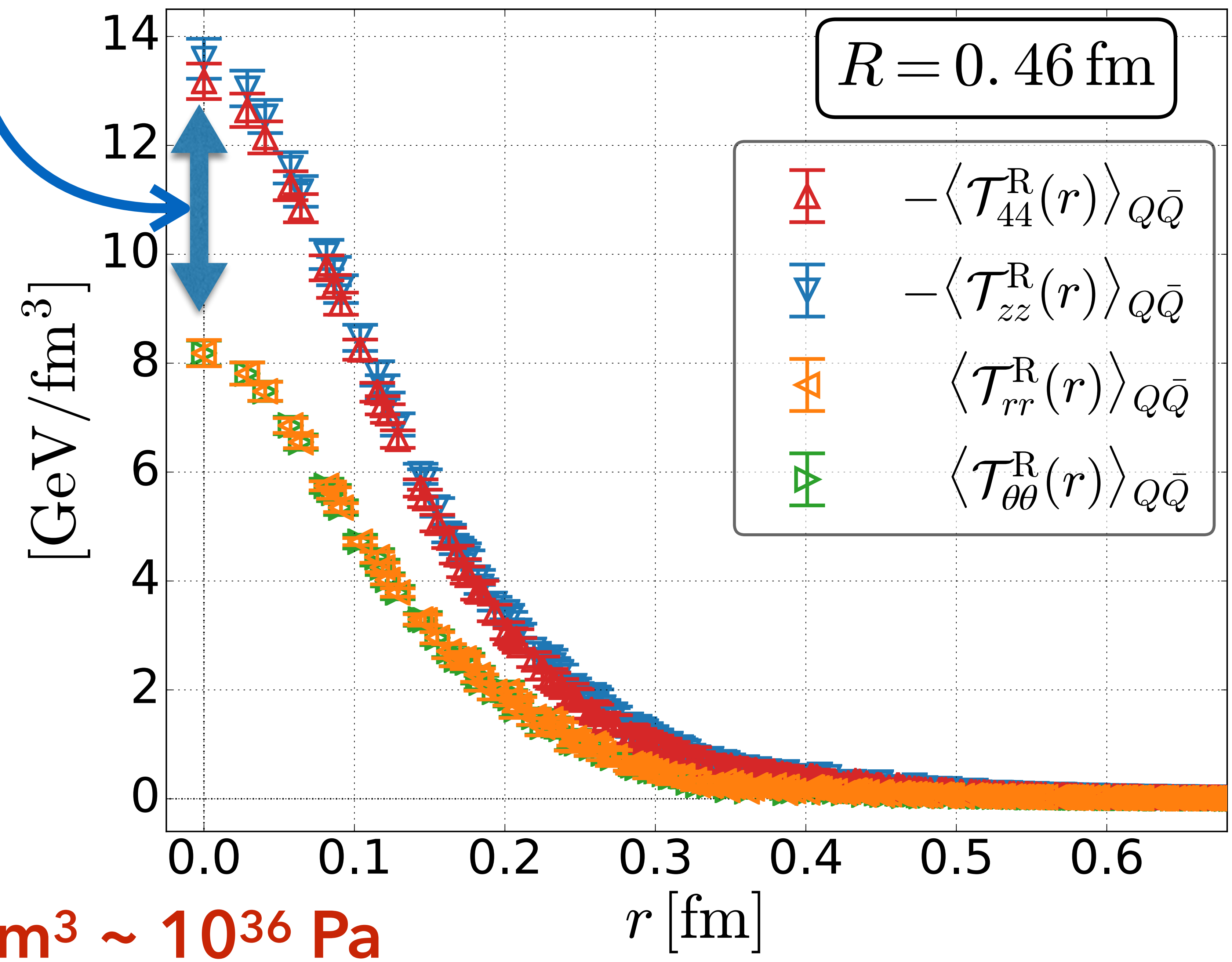
$$\langle \mathcal{T}_{\mu\mu}^{R}(r)\rangle_{Q\bar{Q}} < 0$$

cf. classical electrodynamics

$$\mathcal{T}_{44}(r) = \mathcal{T}_{zz}(r) = -\mathcal{T}_{rr}(r) = -\mathcal{T}_{\theta\theta}(r)$$

$$\mathcal{T}_{\mu\mu}(r) = 0$$





cf. 6 GeV/fm^3 ~ 10^{36} Pa

Renormalized Stress-Tensor in Mid-plane (2) R-dep.

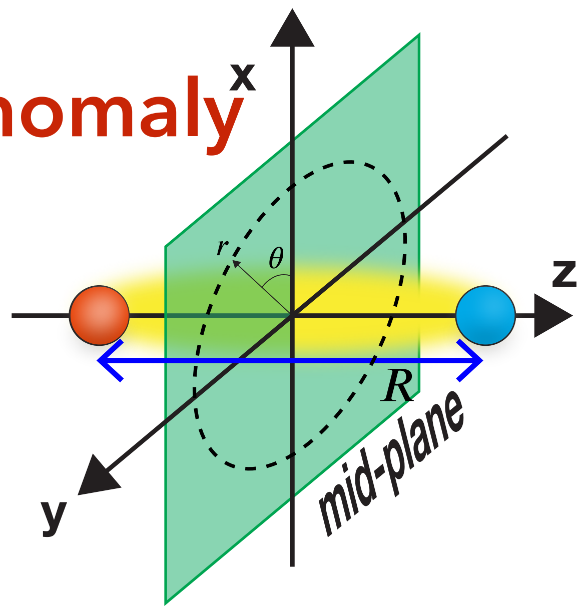
non-trivial degeneracies

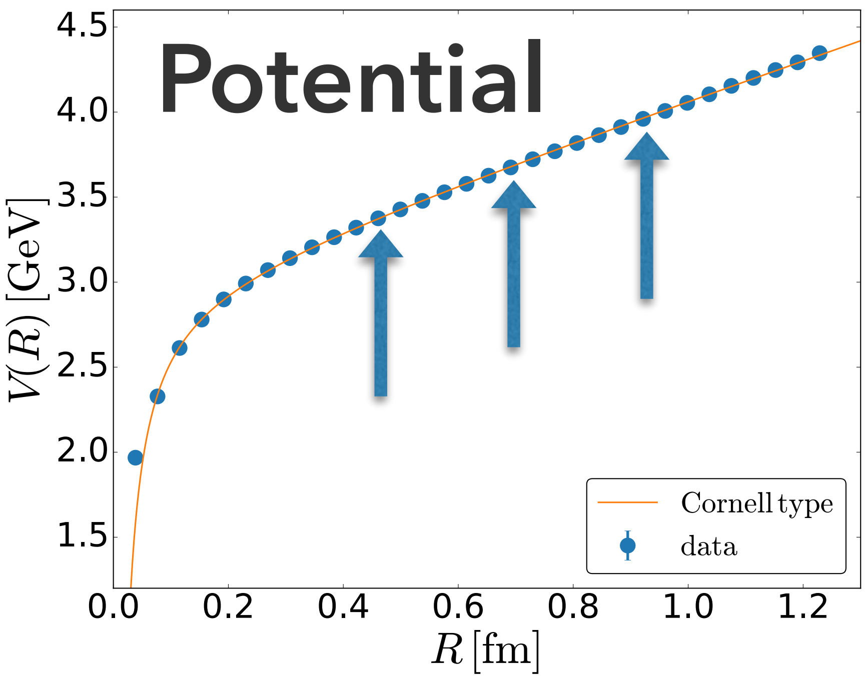
$$\langle \mathcal{T}_{44}^{R}(r)\rangle_{Q\bar{Q}} \simeq \langle \mathcal{T}_{zz}^{R}(r)\rangle_{Q\bar{Q}} < 0$$

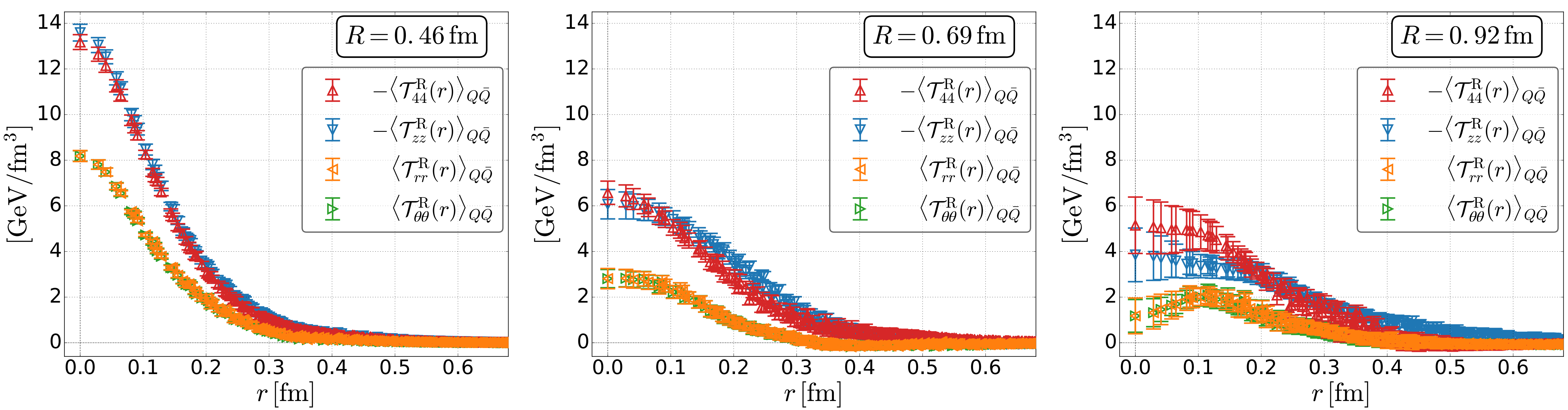
$$\langle \mathcal{T}_{rr}^{R}(r)\rangle_{Q\bar{Q}} \simeq \langle \mathcal{T}_{\theta\theta}^{R}(r)\rangle_{Q\bar{Q}} > 0$$

partially restoration of **trace anomaly**

$$\langle \mathcal{T}_{\mu\mu}^{R}(r)\rangle_{Q\bar{Q}} < 0$$

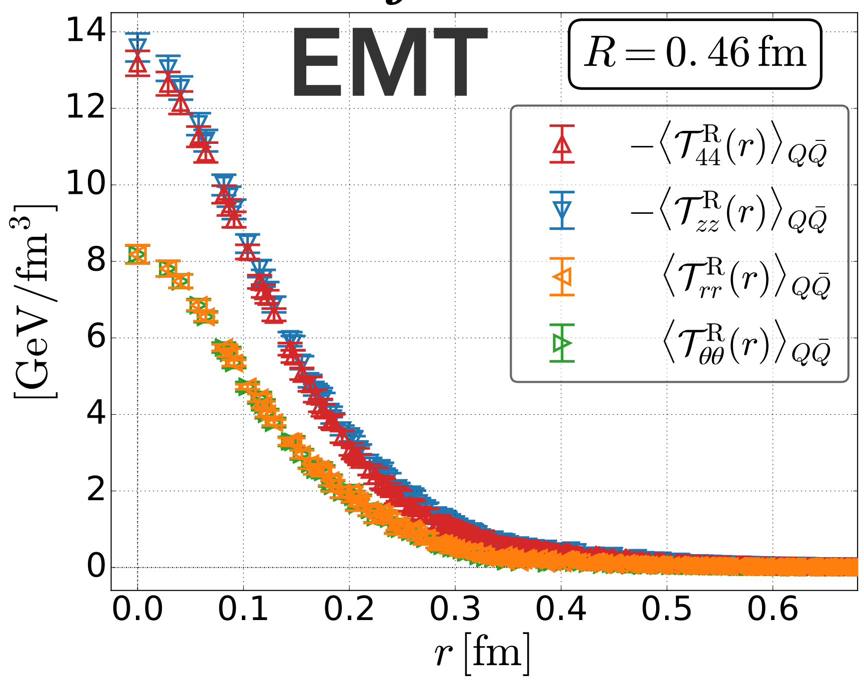




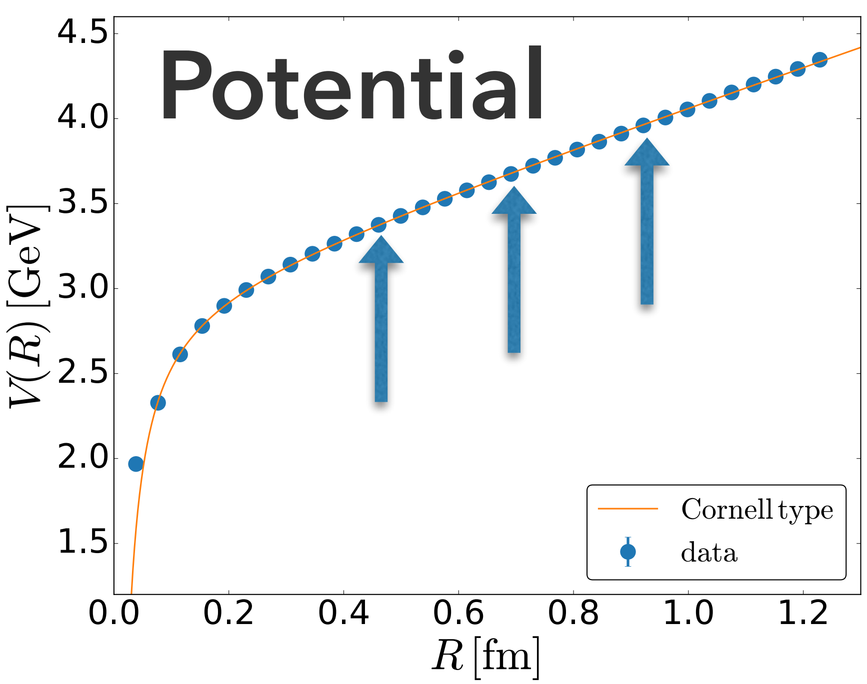


Interquark Force from the Stress-Tensor

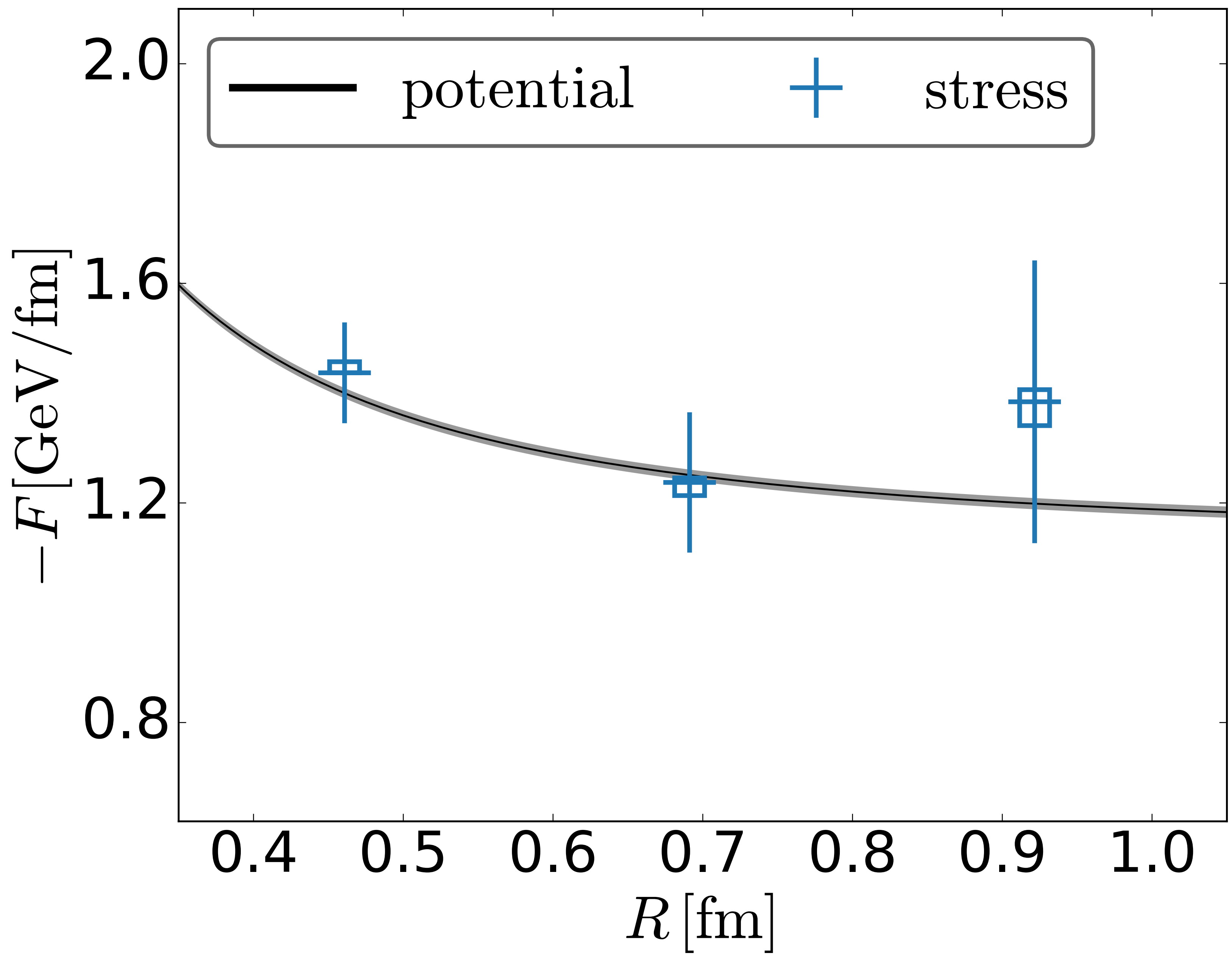
$$F_{\rm stress} = -\int \langle \mathcal{T}^R_{zj}(x) \rangle_{Q\bar{Q}} dS_j$$



$$F_{\rm pot.} = -\frac{dV(R)}{dR}$$



non-trivial consistency check

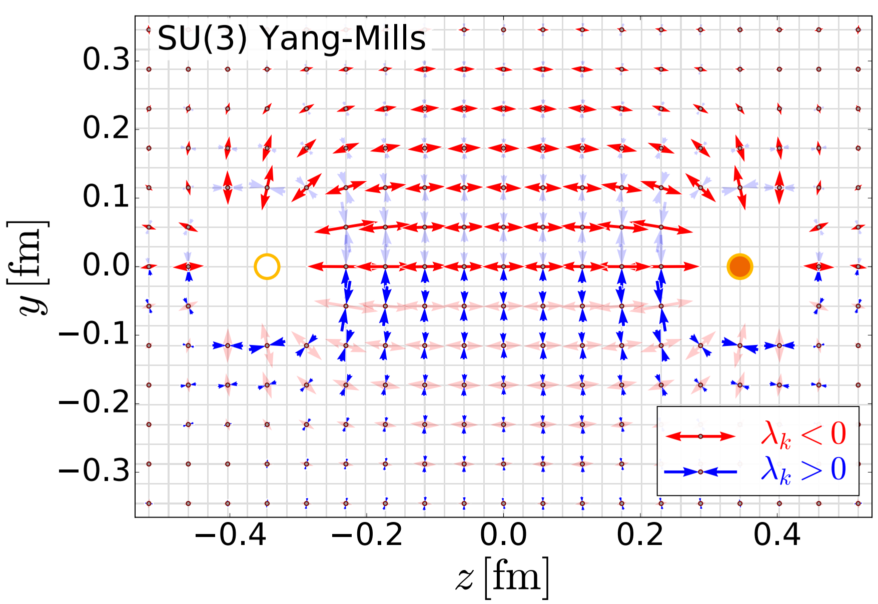


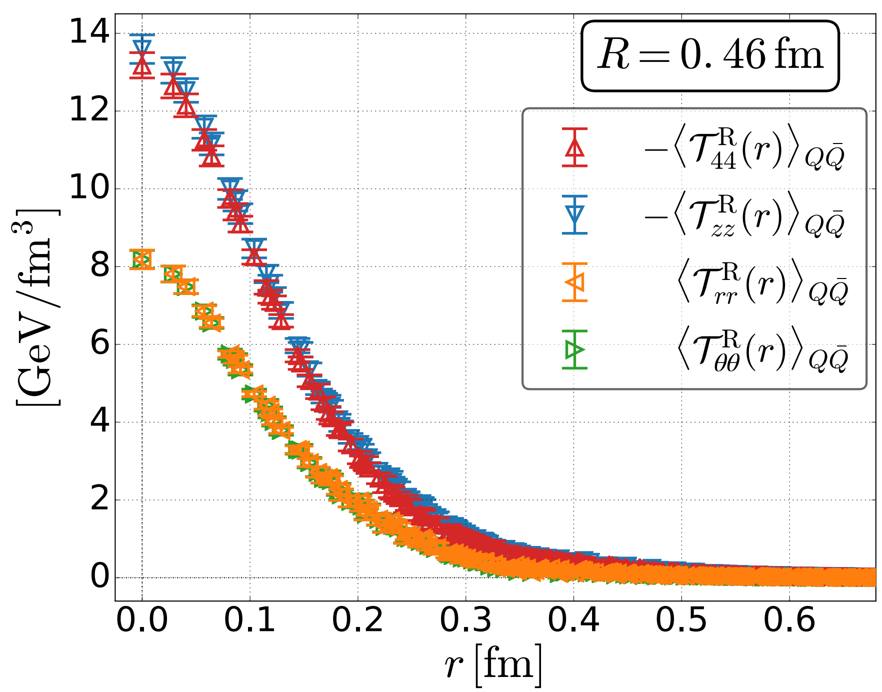
the “**action-at-a-distance**” interquark force is derived from “local” **stress tensor**

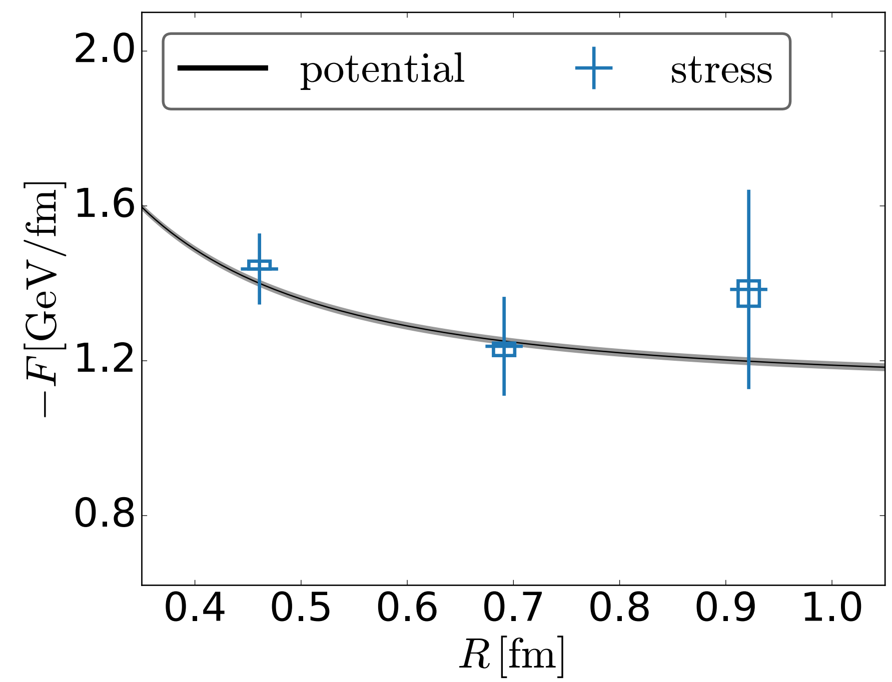
Summary

EMT around static Quark-Antiquark

- "**First direct measurement**" of **the stress-tensor** distribution around **the static Quark-Antiquark**.
- the "**action-at-a-distance**" force is derived from the local properties from **the stress-tensor**
- Deeper & Correct understandings of the Quark-Antiquark system from **local properties**.







*"**Gradient Flow**" is useful approach to **the EMT***

***Outlooks**: Quark-Antiquark system at finite temperature, Stringy excitations of Quark-Antiquark system, Three Quark system, Full QCD, …*

Backup

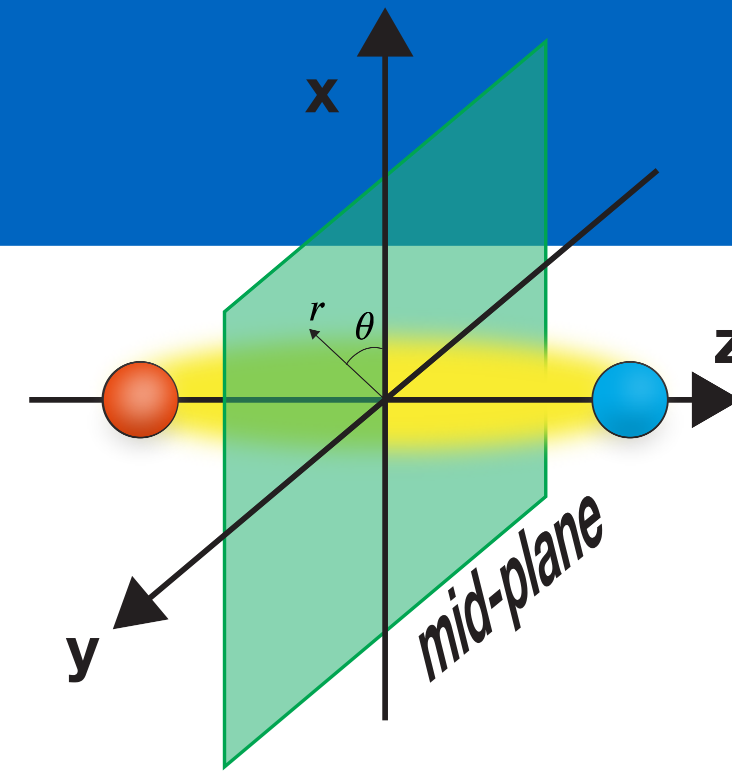
Setups

SU(3) Wilson gauge action

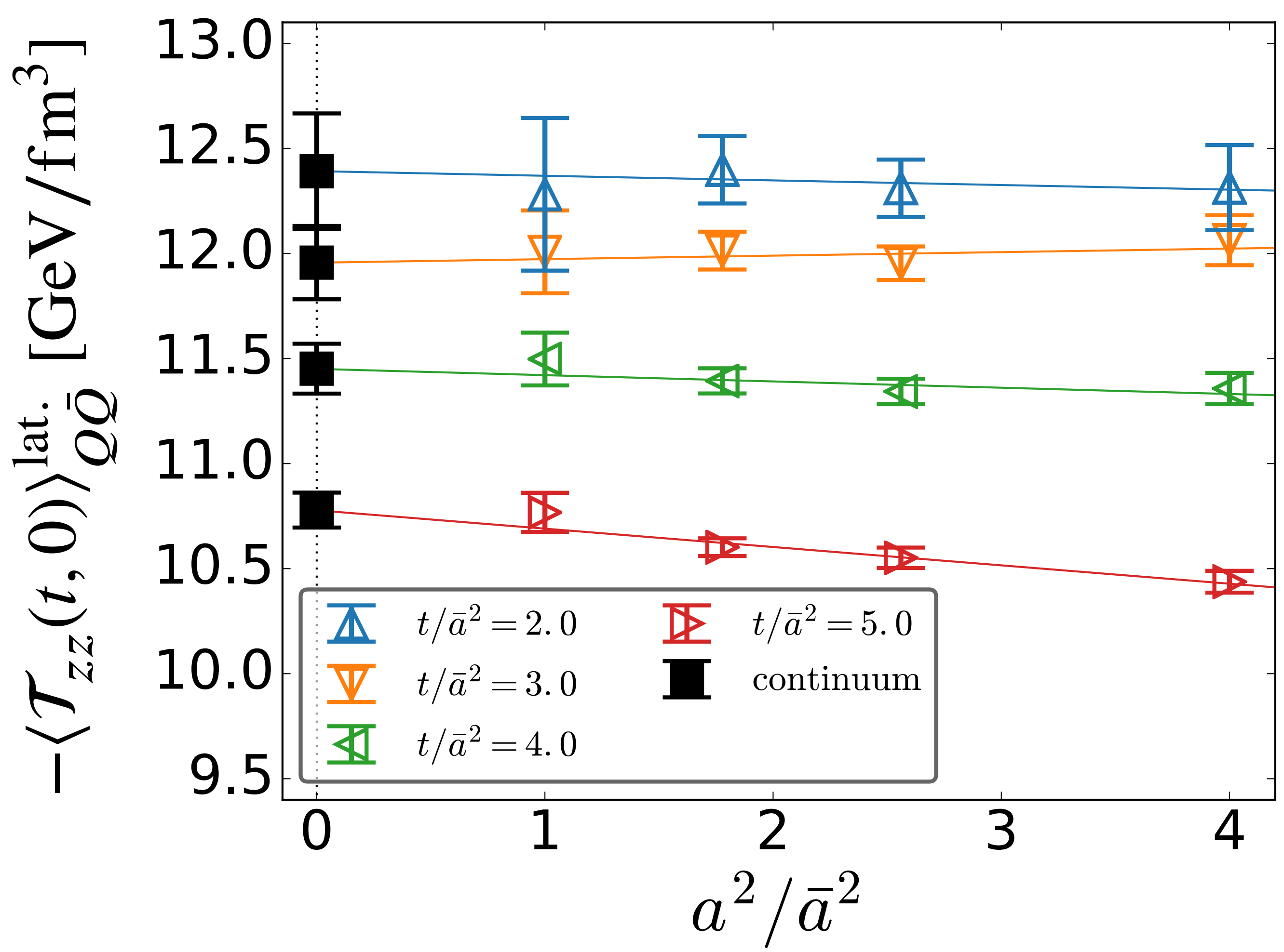
beta	a [fm]	Ns	Nconf	R_1/a	R_2/a	R_3/a	T/a
6.304	0.058	48	140	8	12	16	8
6.465	0.046	48	440	10		20	10
6.513	0.043	48	600		16		10
6.600	0.038	48	1500	12	18	24	12
6.819	0.029	64	1000	16	24	32	16
	w_0-scaling			0.46 fm	0.69 fm	0.92 fm	

Wilson loop: APE smearing for the spatial links,
& multi-hit procedure for the temporal link

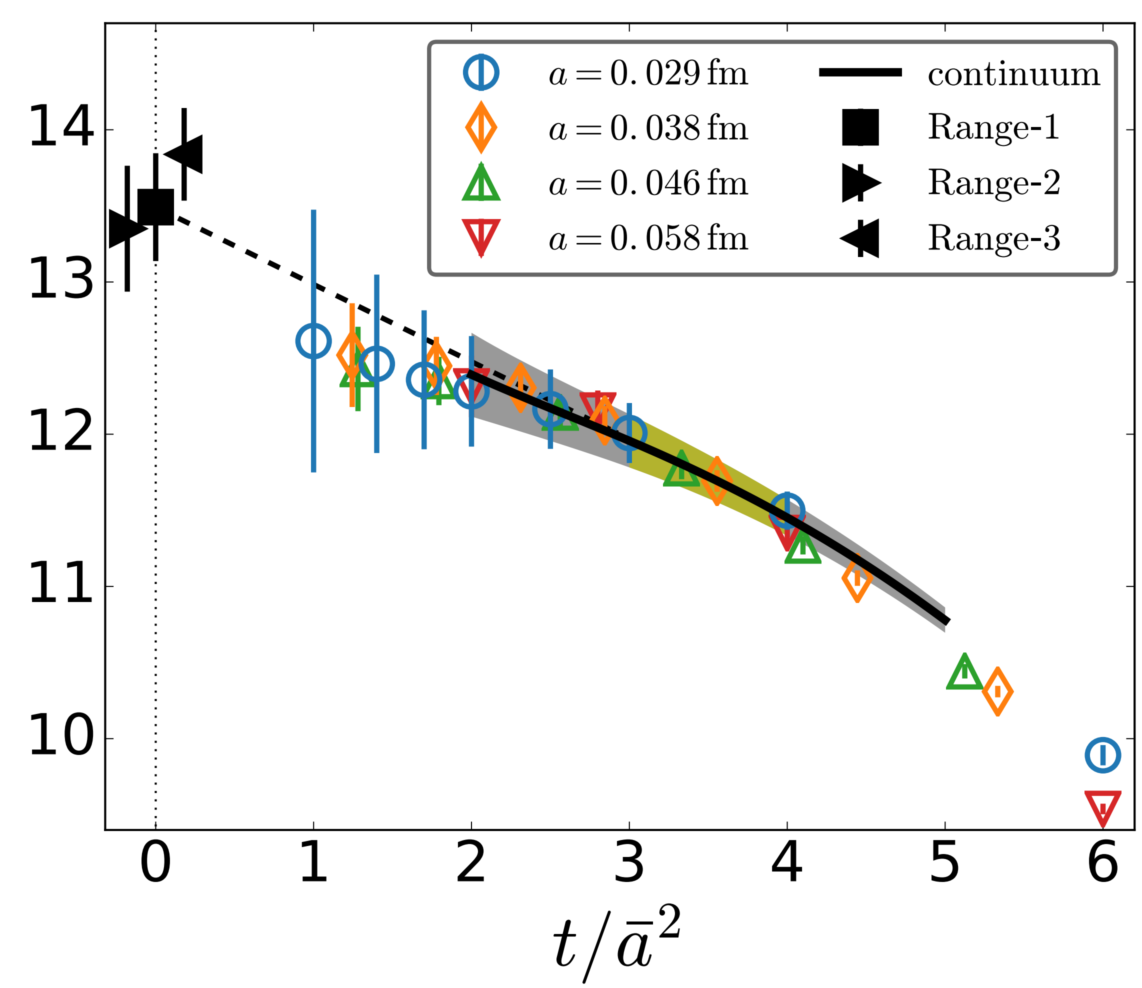
Double Limit at the Center of Mass



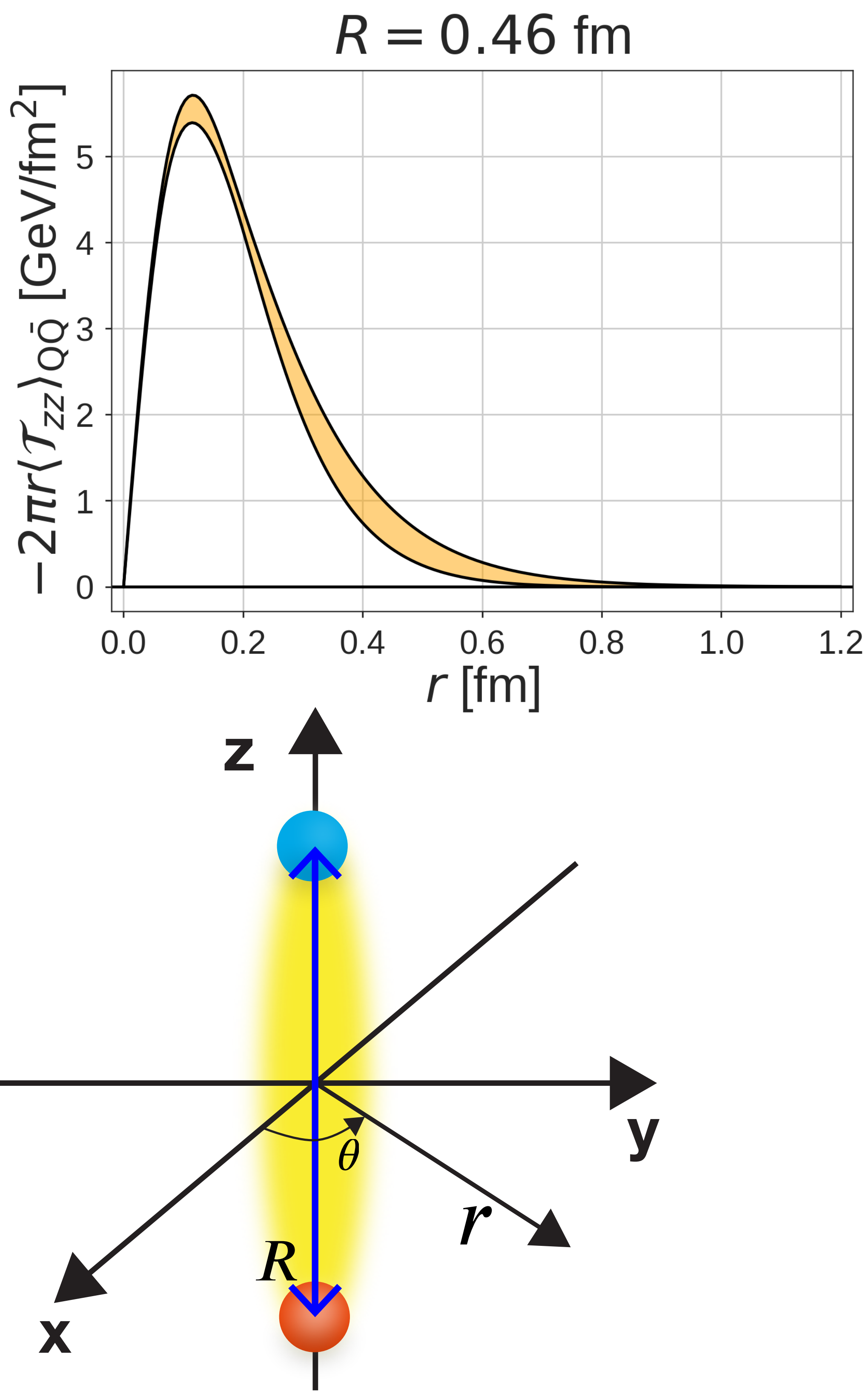
(1) continuum limit



(2) flow time zero limit



Pressure distribution



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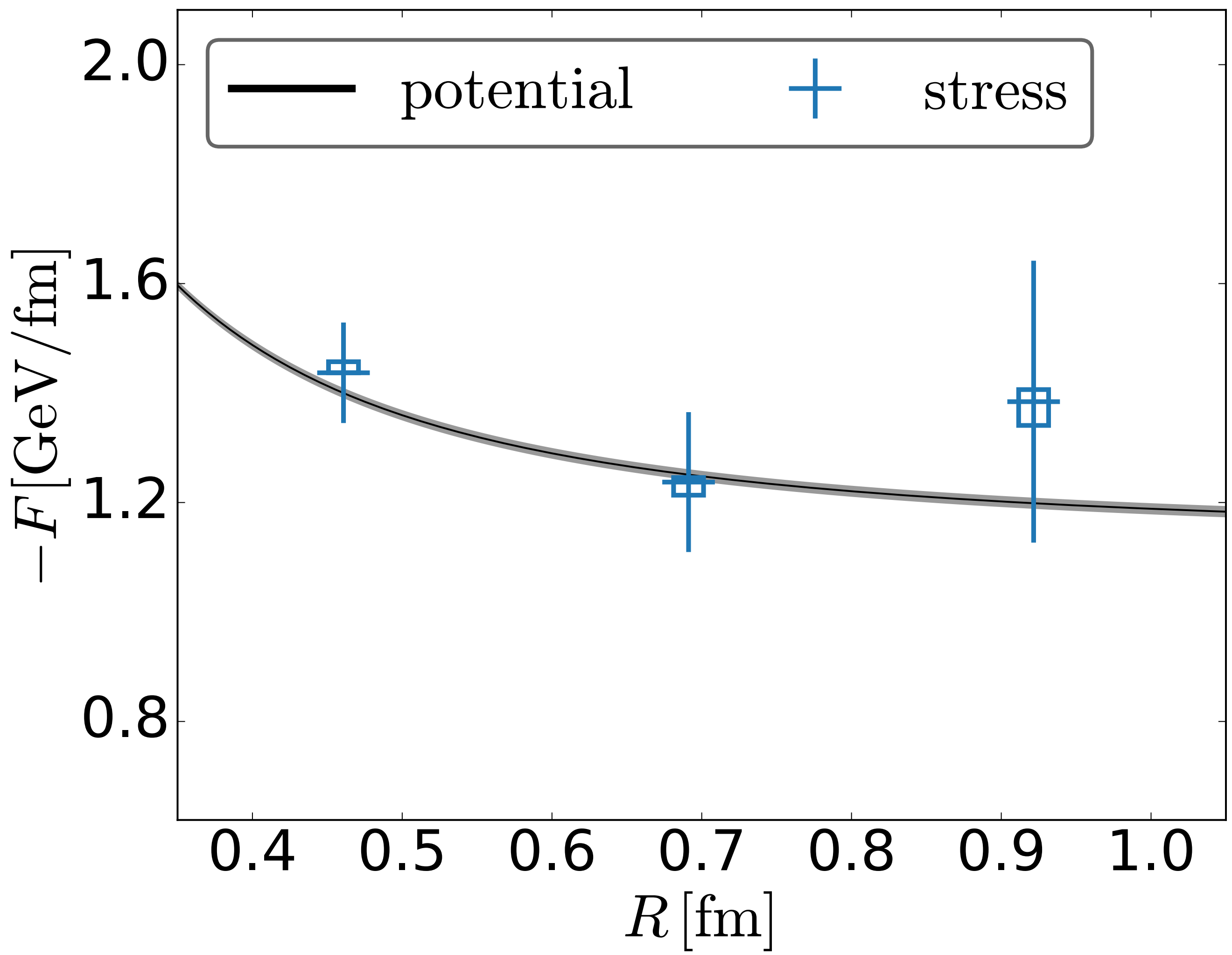
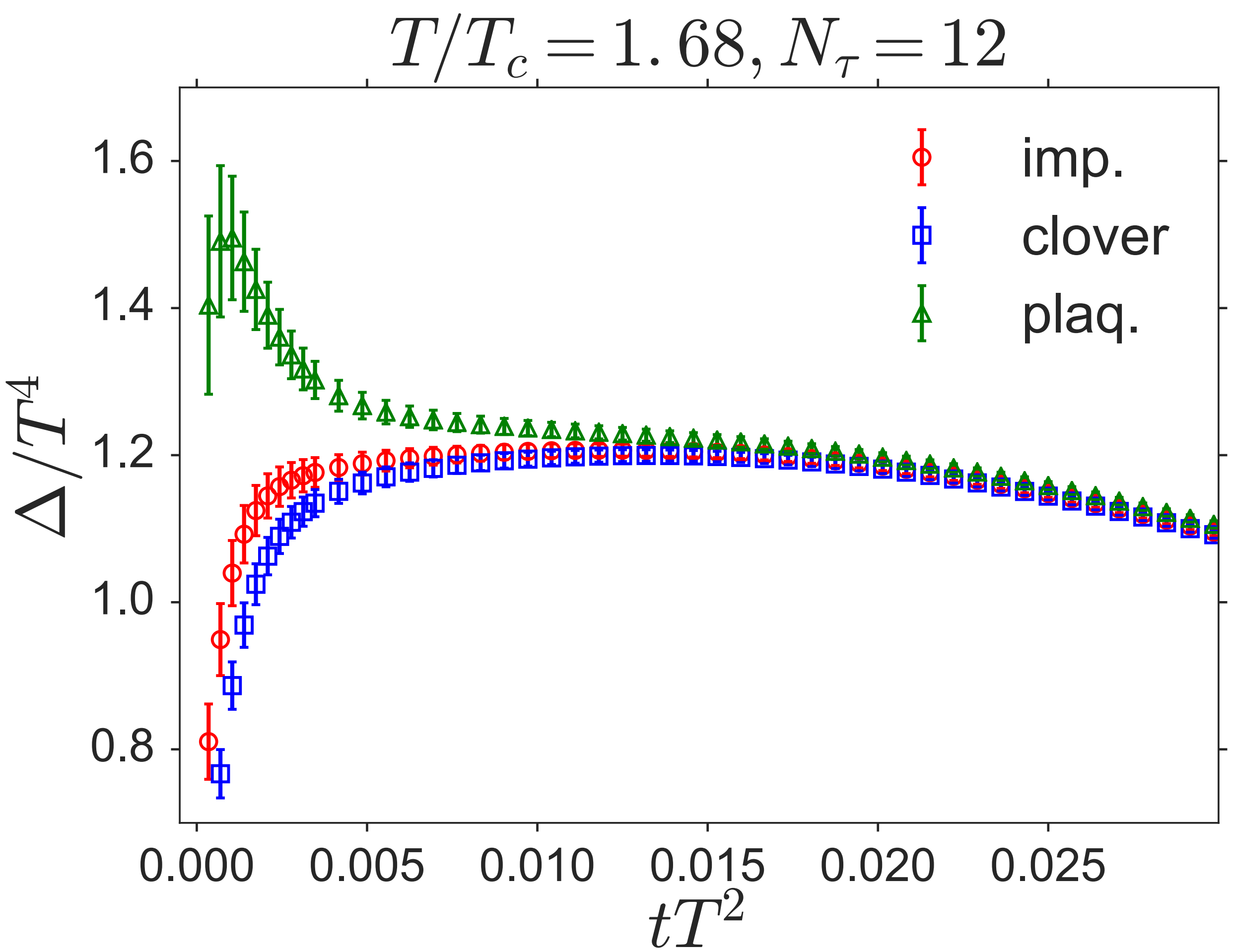
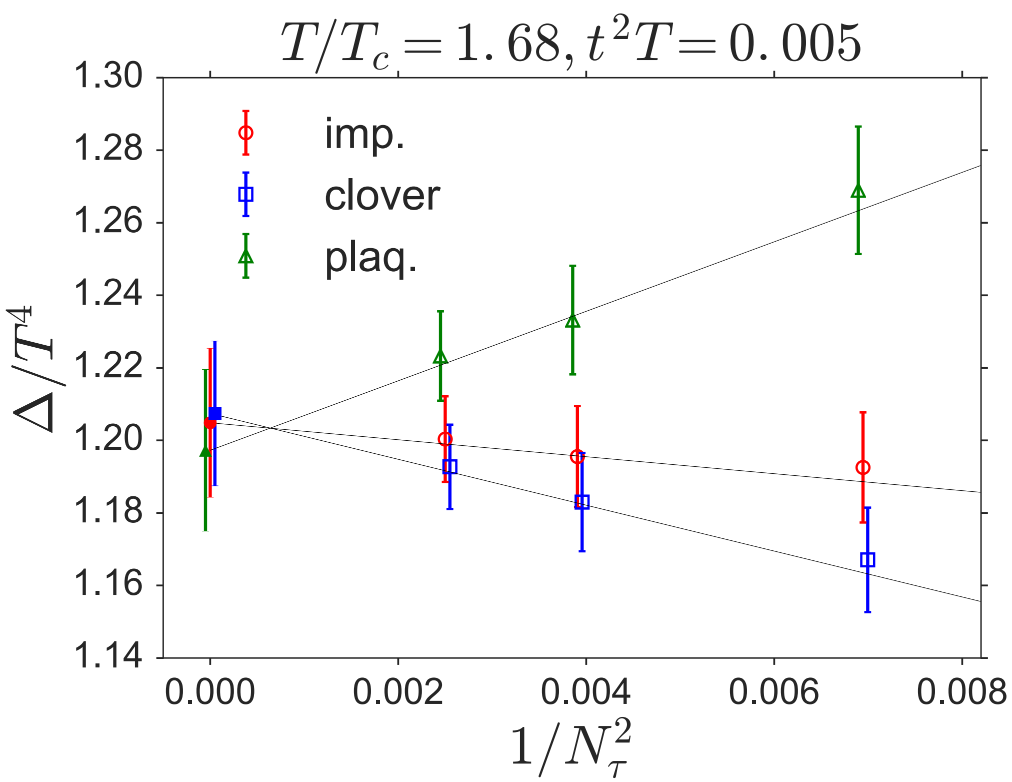
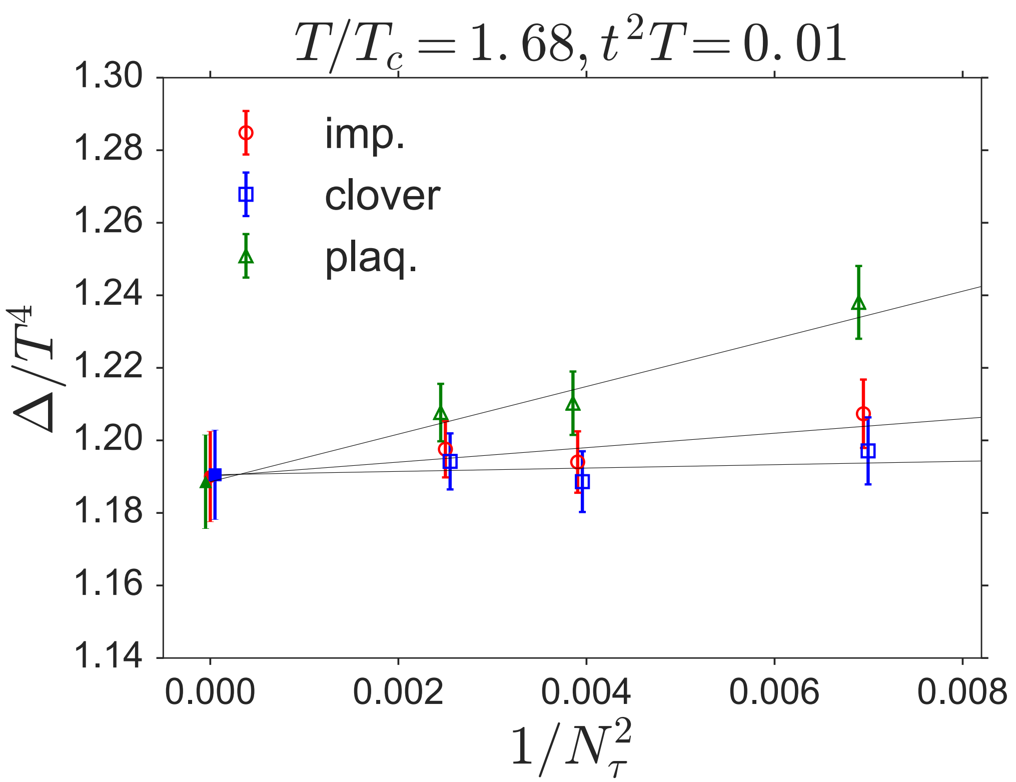


Double Limit of EoS (1) Continuum Limit

- different discretization
  - $O(a^2)$ systematic errors
- **continuum limit**
  - converge





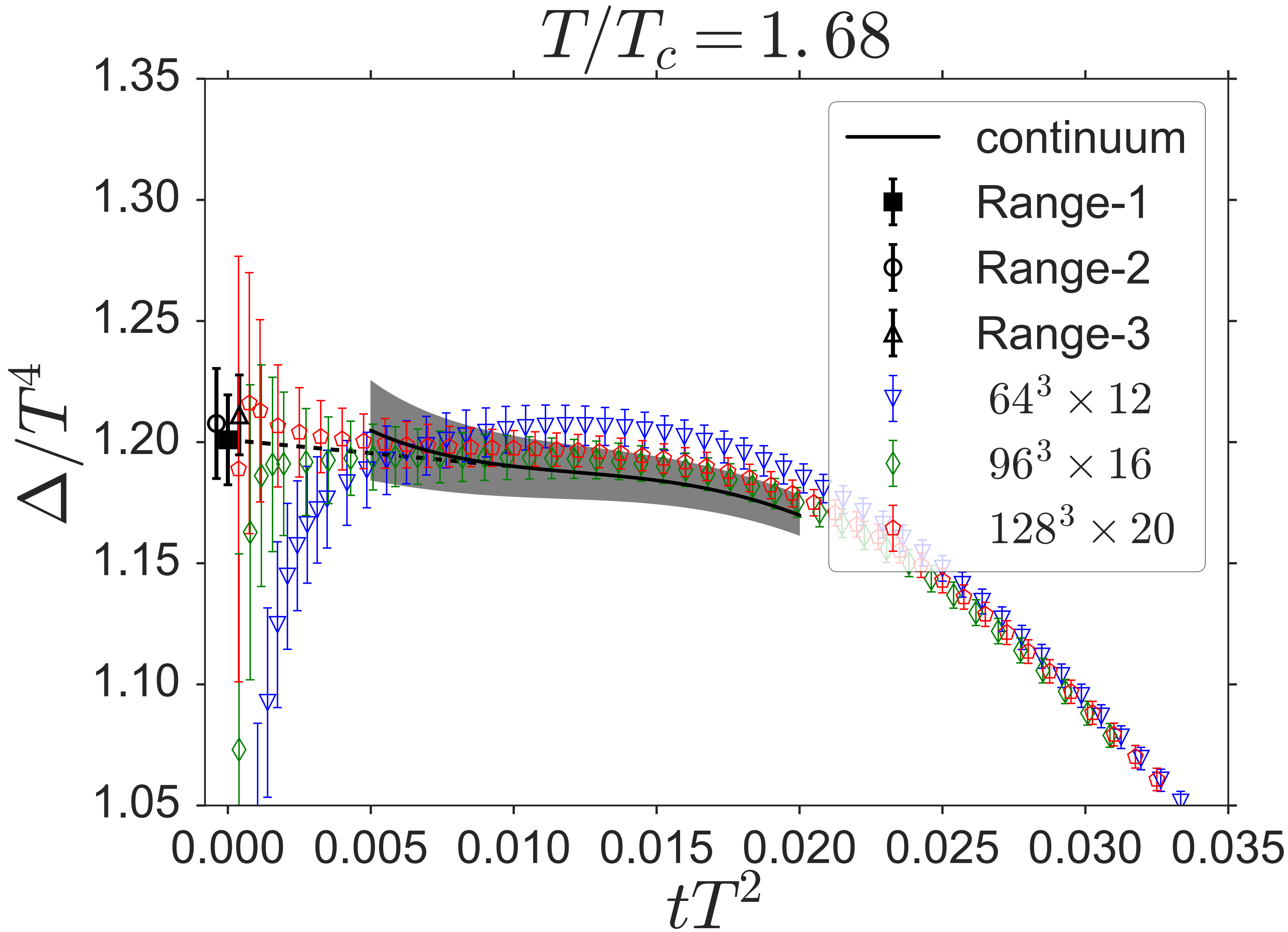


Double Limit of EoS (2) Flow Time Zero Limit

- **Smearing length**: $\sqrt{8t}$
- **Fitting window**: smeared & perturbative region

$$a \ll \sqrt{8t} \ll \Lambda^{-1}$$

linear fit
Range-1: [0.01:0.015]
Range-2: [0.005:0.015]
Range-3: [0.01:0.02]



Integral Method

J. Engels et al. '90

action:

$$S = -\beta \left[c_0 \sum_{n,\mu<\nu} \mathrm{Re\,Tr}\, U_{\mu\nu}^{1\times1}(n) + c_1 \sum_{n,\mu\neq\nu} \mathrm{Re\,Tr}\, U_{\mu\nu}^{2\times1}(n) \right]$$

exp. value of plaq.

$$\frac{I}{T^4} \equiv \frac{\varepsilon - 3p}{T^4} = \left(\frac{N_t^3}{N_s^3} \right) a \frac{d\beta}{da} \left(\langle \frac{dS}{d\beta} \rangle - \langle \frac{dS}{d\beta} \rangle_{\mathrm{vac}} \right)$$

$a(\beta) = \cdots$ scaling func.

ex. Edwards-Heller-Klassen '98

$$T = 1/(N_t a),\ V = (N_s a)^3$$

integration:

$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} \frac{I(T')}{T'^5} dT'$$

$$\varepsilon = I + 3p, \quad s = \frac{\varepsilon + p}{T}$$

Cross Section along z-axis

fixed flow time w/o cont. limit

The flux-tube at mid-plane (z=0) at R = R1 is affected by the peak structure around **quark and anti-quark**.

dot-dashed vertical line: location of the sources