Multiplicity fluctuations of (net) charges and (net) protons from iEBE-VISHNU hybrid model

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Reference:
Jixing Li, Hao-jie Xu，Huichao Song. Phys. Rev. C97 (2018) no.1, 014902

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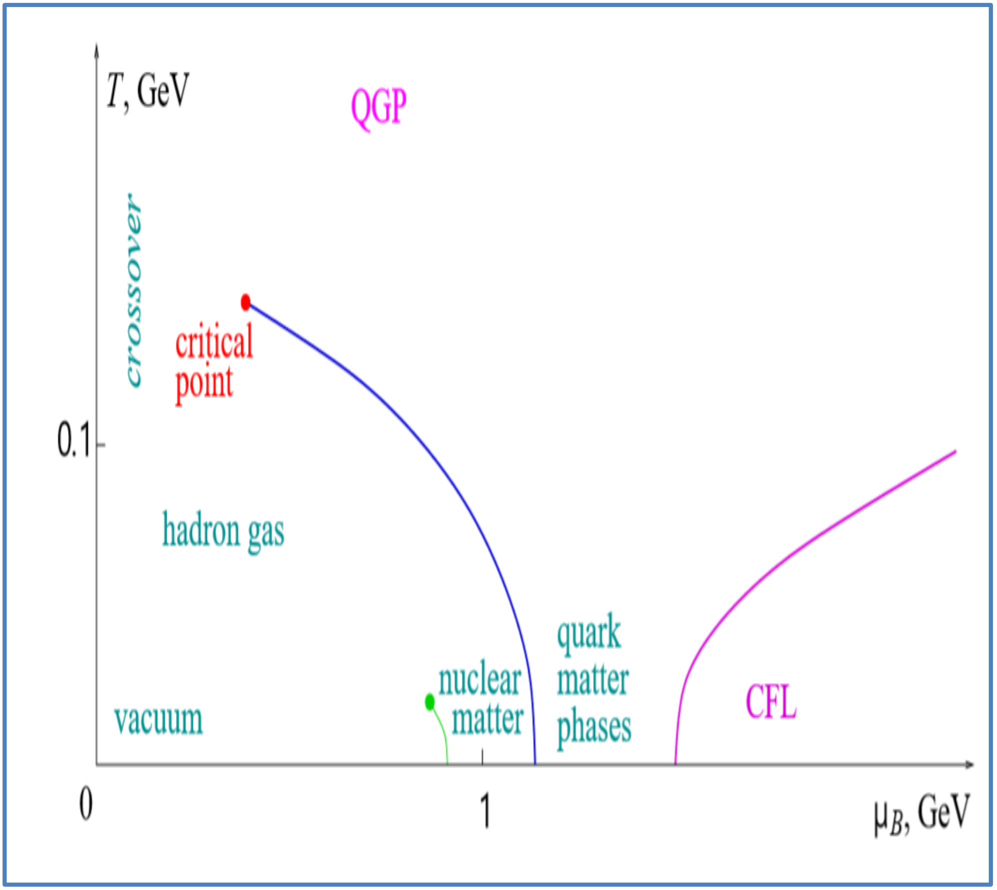
Outline

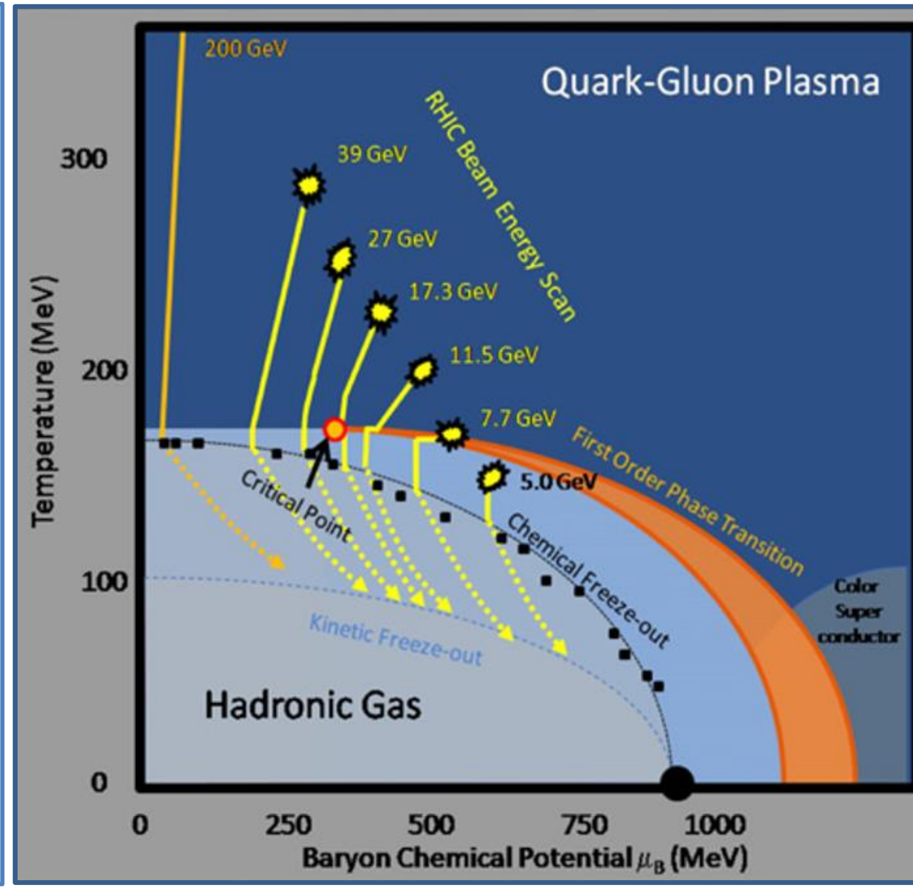
1. Introduction: QCD phase and Critical Point
2. The Revised iEBE-VISHNU Hybrid Model
3. Results and Discussion
4. Summary

1

Introduction: Phase transition and fluctuations

QCD Phase Diagram and Critical Point





- QCD phase diagram: QGP, Hadronic phase, Critical point(CP), CS…
- Critical Point（CP):The second-order critical endpoint of the first-order transition between quark-gluon plasma and hadron matter.
- In heavy ion collisions experiments, the event-by-event multiplicity fluctuations are expected to provide crucial information about the hot and dense QCD matter.

Theoretical predictions on critical fluctuations

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots\right].$$

$$\langle\sigma_0^2\rangle = \frac{T}{V}\xi^2 \qquad \langle\sigma_0^3\rangle = \frac{2\lambda_3 T}{V}\xi^6; \qquad \langle\sigma_0^4\rangle_c = \frac{6T}{V}[2(\lambda_3\xi)^2 - \lambda_4]\xi^8.$$

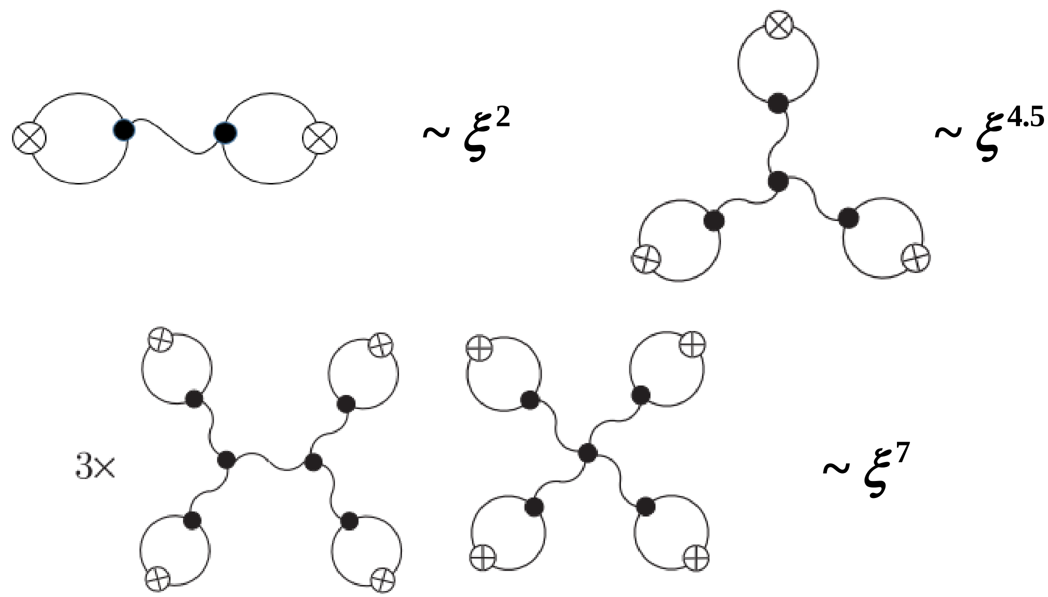
Stephanov Phys. Rev. Lett. 102 (2009) 032301

Critical Fluctuations of particles:

$$\langle(\delta N)^2\rangle \sim \xi^2$$

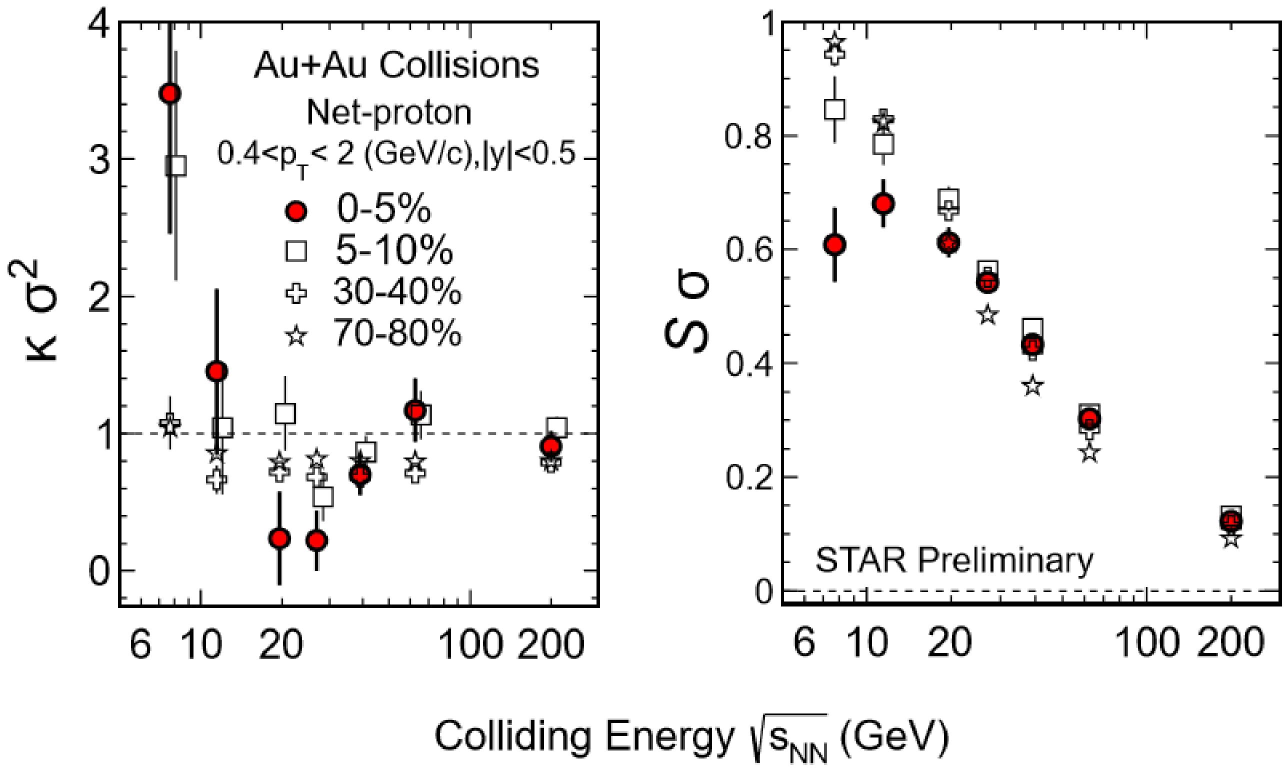
$$\langle(\delta N)^3\rangle \sim \xi^{4.5}$$

$$\langle(\delta N)^4\rangle \sim \xi^7$$



- **Higher cumulants (ratios) of conserved charges are sensitive observables to probe the QCD critical point.**

Experiment data



[X. Luo. PoS CPOD 2014 (2015) 019]

- **In experiment the net-proton high moment products show an obvious non-monotonic behavior and large deviation from the traditional Poisson baselines.**

Experiment observables: Critical + Non-Critical Fluctuation.

➢A brief review of Critical Fluctuation

Critical Fluctuations

- **Static (equilibrium) Critical fluctuation**
  - Misha Stephanov , Phys. Rev. Lett. 102 (2009) 032301
  - Christiana Athanasiou, Krishna Rajagopal , Misha Stephanov. PRD82 (2010) 074008
  - Masayuki Asakawa, Shinji Ejiri, Masakiyo Kitazawa. PRL103 (2009) 262301
  - Lijia Jiang, Pengfei Li , Huichao Song. PRC94 (2016) no.2, 024918.
  - ...

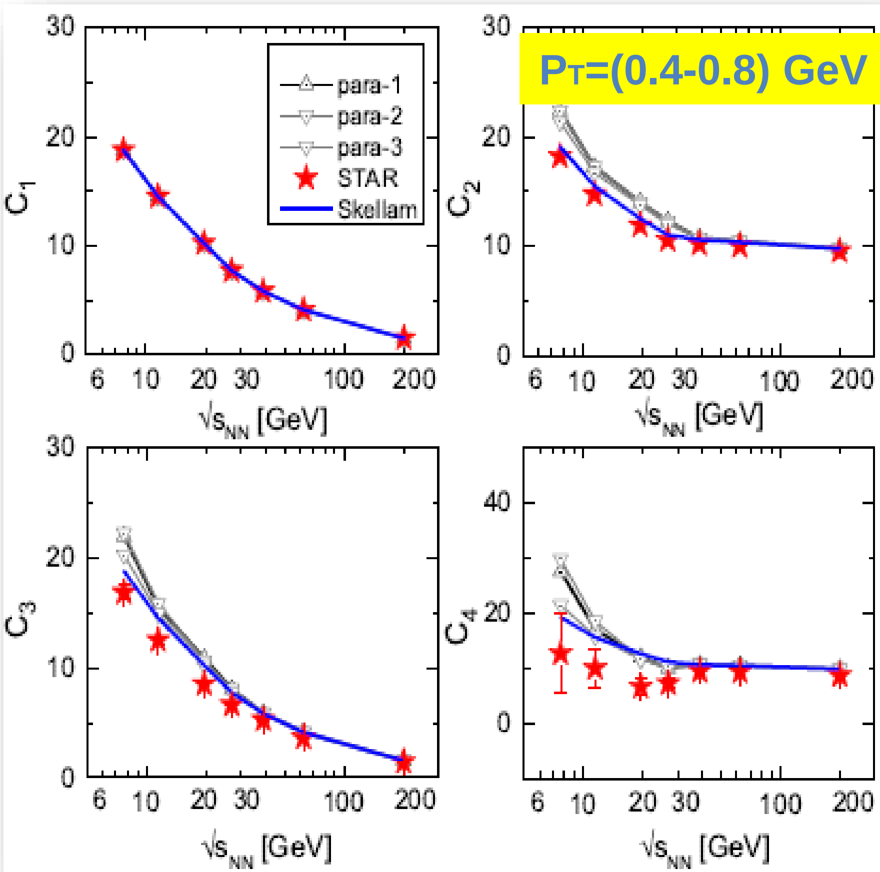
- **Dynamic (non-equilibrium) Critical fluctuation**
  - Masakiyo Kitazawa, Masayuki Asakawa, Hirosato Ono. PLB728 (2014) 386-392
  - S. Mukherjee, R. Venugopalan, Y. Yin, PRC92 (2015)
  - C. Herold, Marlene Nahrgang,Y. Yan, C. Kobdaj.PRC93 (2016) no.2, 021902
  - Lijia Jiang, Shanjin Wu, Huichao Song. Nucl.Phys. A967 (2017) 441-444
  - ...

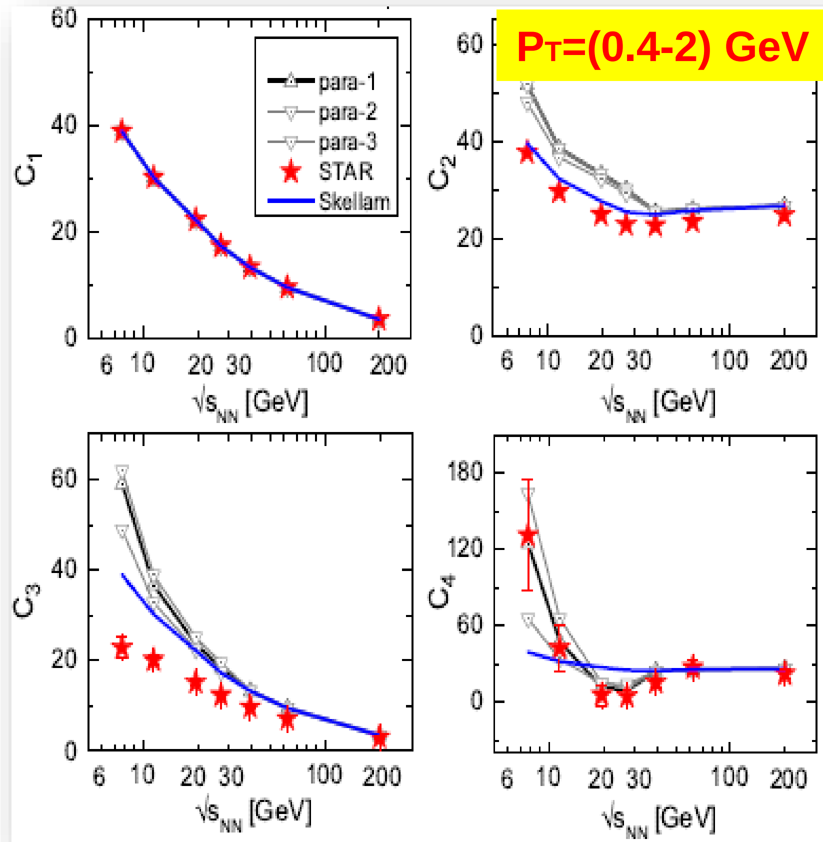
◆ **We expected the critical fluctuations of conserved charges could give us more information about the critical point or the phase transition.**

Cumulants of net protons

Net Protons 0-5%

Lijia Jiang, Pengfei Li , Huichao Song. PRC94 (2016) no.2, 024918





- The correlated fluctuations of net protons on the hydro. freeze-out surface for different P_T range has been numerically calculated with a modified distribution function that satisfied specific static fluctuations.
- Static critical fluctuations give positive contribution to C_2 , C_3; well above the Poisson baselines; can NOT explain/describe the C_2 , C_3 experiment data.

Effects from dynamical evolutions

$$\partial_\tau P(\sigma;\tau) = \frac{1}{m_\sigma^2 \tau_{eff}} \left[\partial_\sigma \left[\partial_\sigma \Omega_0(\sigma) + V_4^{-1} \partial_\sigma\right] P(\sigma;\tau)\right]$$

-Model A

S. Mukherjee, R. Venugopalan, Y. Yin, PRC92 (2015)

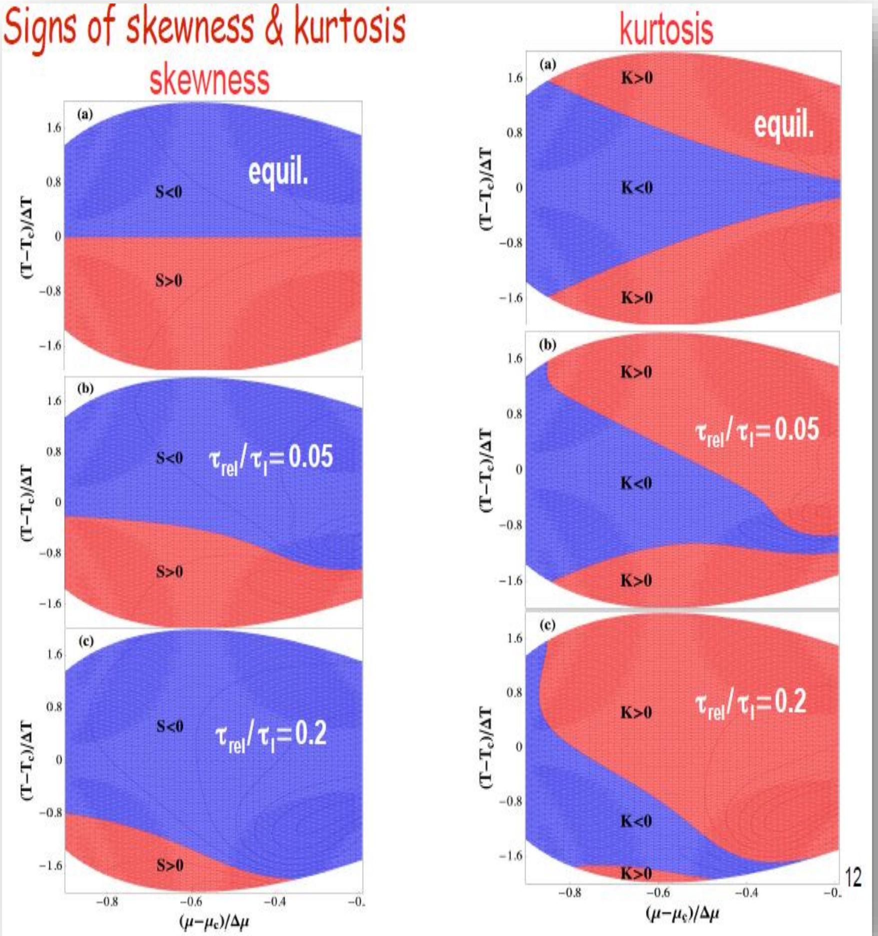
near-equilibrium limit: $\delta\kappa_n = \kappa_n - \kappa_n^{eq}$

$$\partial_\tau \kappa_2 = -2\tau_{eff}^{-1} a_2 \delta\kappa_2$$

$$\partial_\tau \kappa_3 = -3\tau_{eff}^{-1}\left[a_2 \delta\kappa_2 + a_3 \delta\kappa_3\right]$$

$$\partial_\tau \kappa_4 = -4\tau_{eff}^{-1}\left[a_2 \delta\kappa_2 + a_3 \delta\kappa_3 + a_4 \delta\kappa_4\right]$$

- **Sign of non-Gaussian cumulants can be different from equilibrium case due to the dynamic evolution memory effects.**



Dynamical critical fluctuations of the sigma field

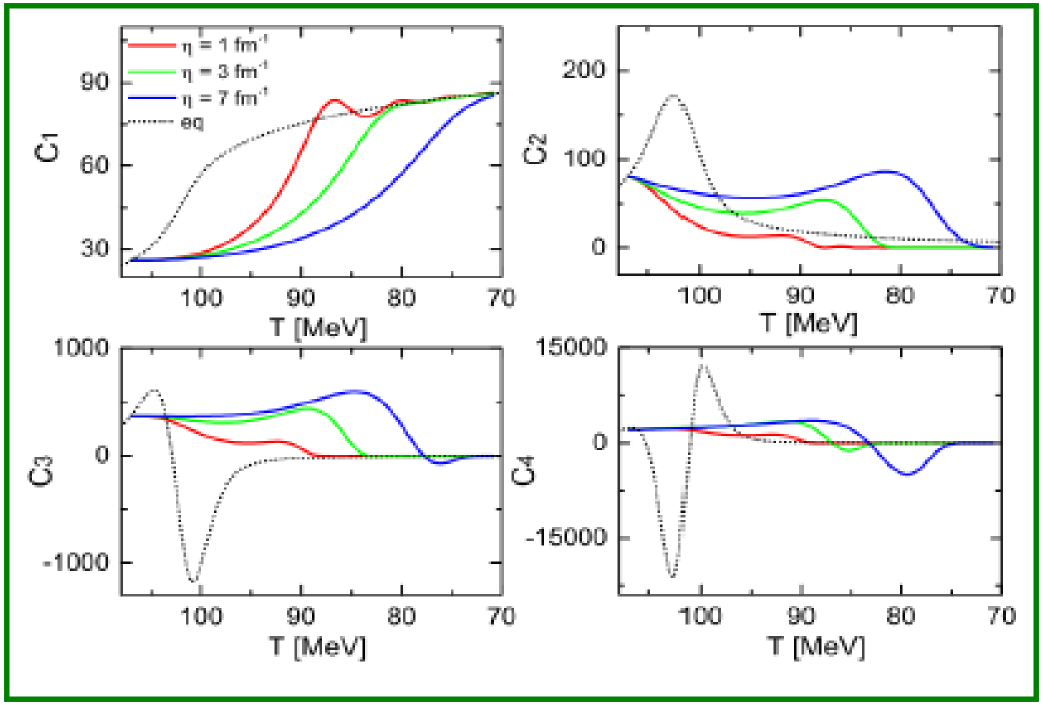
Langevin dynamics: $\partial^{\mu}\partial_{\mu}\sigma(t,x)+\eta\partial_t\sigma(t,x)+V'_{eff}(\sigma)=\xi(t,x)$ **-Model A**

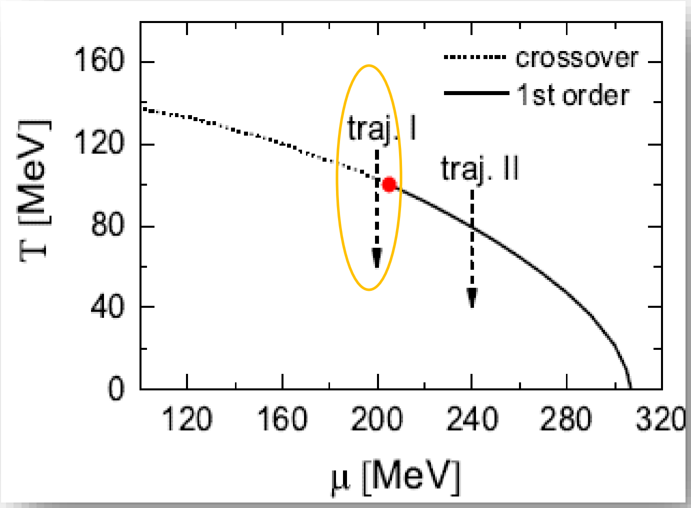
with effective potential from linear sigma model with constituent quarks

$$V_{eff}(\sigma)=U(\sigma)+\Omega_{\bar{q}q}(T,\sigma)=\frac{\lambda^2}{4}\left(\sigma^2-\nu^2\right)^2-h_q\sigma-U_0-2d_qT\int\frac{d^3p}{(2\pi)^3}\ln\left(1+\exp\left(-\frac{E}{T}\right)\right)$$

Lijia Jiang, Shanjin Wu, Huichao Song. Nucl.Phys. A967 (2017) 441-444

On the crossover side





- **The signs of C_3 & C_4 are different from the equilibrium. ones due to memory effects.**

➢Non-Critical Fluctuation

Non-Critical (Thermal) Fluctuations

➢ Detection and analysis technology

The efficiency corrections and acceptance of the detector

Bzdak A, Holzmann R, Koch V. PRC94 (2016) no.6, 064907 …

Bin width effect and centrality dependence

McDonald D, STAR Collaboration. Nuclear Physics A, 2013, 904: 907c-910c…

Auto-correlation effects(ACE)

Luo X, Xu J, Mohanty B, et al. JPG, 2013, 40(10): 105104…

Acceptance dependence of fluctuation

Ling B, Stephanov M A. arXiv preprint arXiv:1512.09125, 2015; Bzdak A, Koch V. Phys. Rev. C, 2012, 86(4): 044904; Masayuki Asakawa and Masakiyo Kitazawa. arXiV:1512.0038…

…

➢ Physical effect

Conservations law for charges and baryons

Bzdak A, Koch V, Skokov V. PRC, 2013, 87(1): 014901…

Volume fluctuations

Xu H..arXiv:1602.07089, 2016; Xu H. arXiv:1602.06378, 2016; S. Jeon, hep-ph/0304012; M. I. Gorenstein, Phys.Rev. C 78, 041902; V. Skokov, Phys.Rev. C 88, 034911…

Hadronic evolution & rescattering

X.Luo,J. Xu, B. Mohanty,and N. Xu, J.P.G 40,105104(2013); Xu, Ji; Yu, Shili; Liu, Feng; Luo, Xiaofeng arXiv:1606.03900

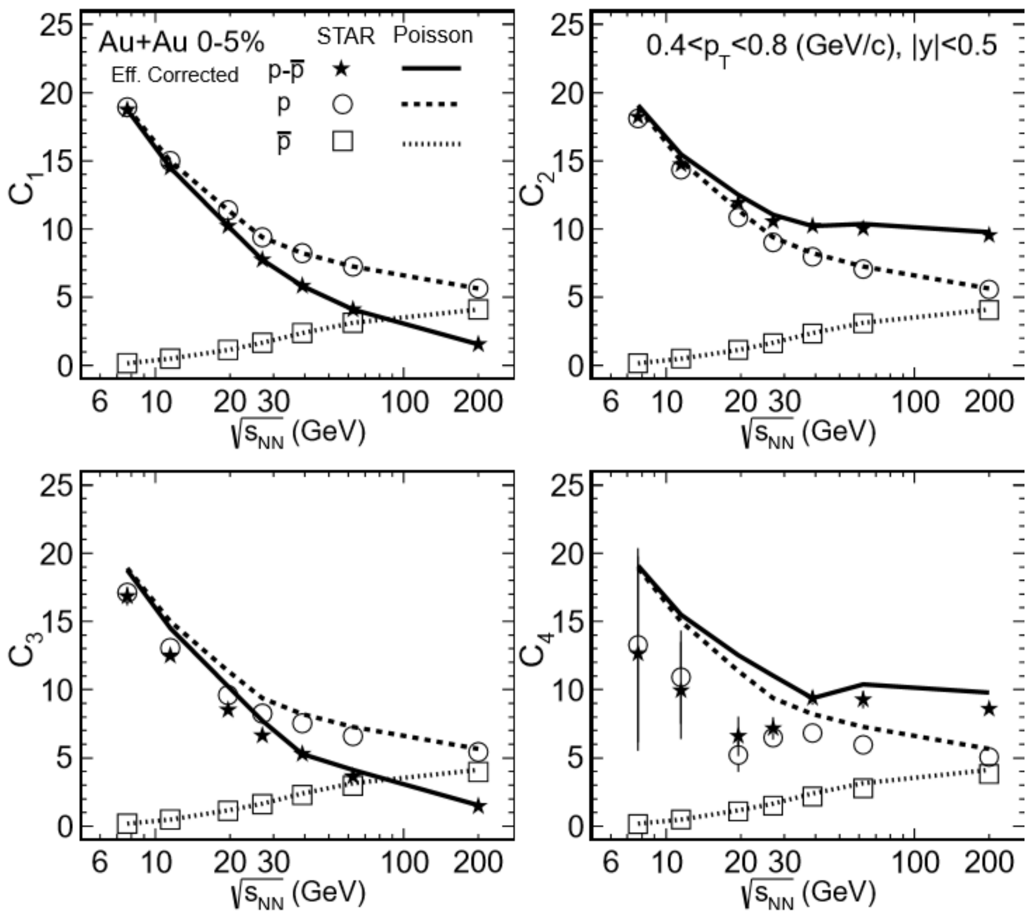
Resonance decay

Garg P, Mishra D K, et al. Phys. Lett. B, 2013, 726(4): 691-696; Andronic A, Braun-Munzinger P, Stachel J. Nucl. Phys. A , 2006, 772(3): 167-199; Andronic A, Braun-Munzinger P. Phys. Lett. B, 2009, 673(2): 142-145; Cleymans J, Kämpfer B, Kaneta M, et al.. Phys. Rev. C, 2005, 71(5): 054901…

…

Traditional Poisson Baselines

[X.Luo,Nucl.Phys. A931 (2014) 808-813]



➢ **In experiment, Poisson fluctuations are generally served as the thermal fluctuation baselines, especially for the multiplicity fluct. of (anti)protons**

- **Where does the Poisson baselines come from?**
- **How various factors influence / destroy Poisson distributions?**
  (volume fluctuations, hadronic evolution, resonance decays … …)

The origin of the Poisson Baseline

- In the grand canonical ensemble(GCE) of the thermal system:

$$\ln Z(T,\mu_B,\mu_Q,\mu_S) = \sum_{i\in\text{mesons}} \ln Z_i^{+}(T,\mu_Q,\mu_S) + \sum_{i\in\text{baryons}} \ln Z_i^{-}(T,\mu_B,\mu_Q,\mu_S)$$

- With Boltzmann approximation:

$$\ln Z_i^{\pm}(T,V,\vec{\mu}) = \frac{VTg_i m_i^2}{2\pi^2} K_2(m_i/T)\exp(\vec{\mu}/T)$$

- Thermodynamic pressure and susceptibilities of thermal qualities:

$$P/T^4 = \lim_{V\to\infty}\frac{1}{VT^3}\ln Z(T,V,\vec{\mu}) \qquad \chi_q^{(n)}(T,\mu_B,\mu_S,\mu_Q) = \left.\frac{\partial^n (P/T^4)}{\partial(\mu_q/T)^n}\right|_T$$

- The susceptibilities of those quantities satisfy:

$$\chi_q^{(n)}(T,\mu_B,\mu_S,\mu_Q)=\chi_q^{(1)}(T,\mu_B,\mu_S,\mu_Q)$$

- Cumulants and susceptibilities have the relation:

$$C_{n,q} = VT^3\chi_q^{(n)}(T,\mu_B) \quad \Rightarrow \quad C_{n,q} = C_{1,q} = \text{Mean}_q$$

⇨ Cumulants satisfied the character of Poisson distribution's cumulant where $C_n = C_1$

Cleymans J, Kämpfer B, Kaneta M, et al.. Phys. Rev. C, 2005, 71(5): 054901.

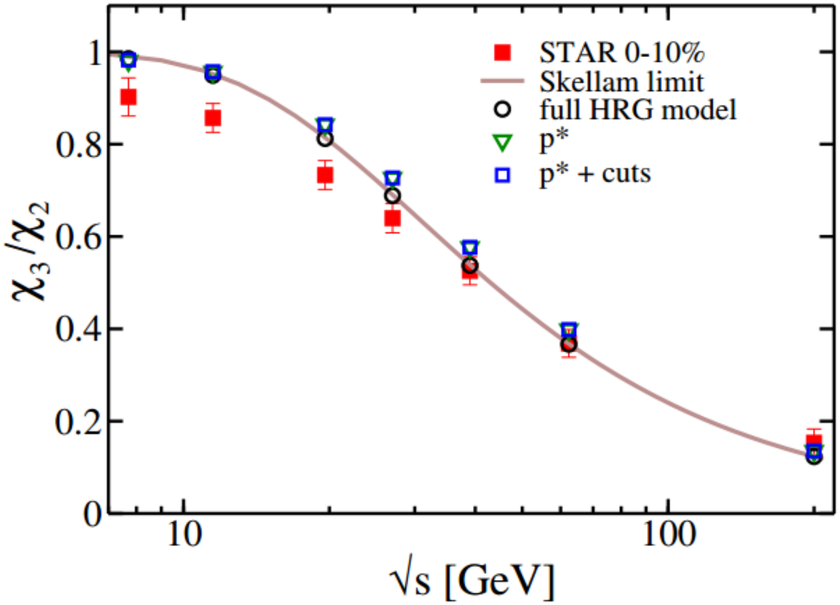
Garg P, Mishra D K, Netrakanti P K, et al. Phys. Lett. B, 2013, 726(4): 691-696.

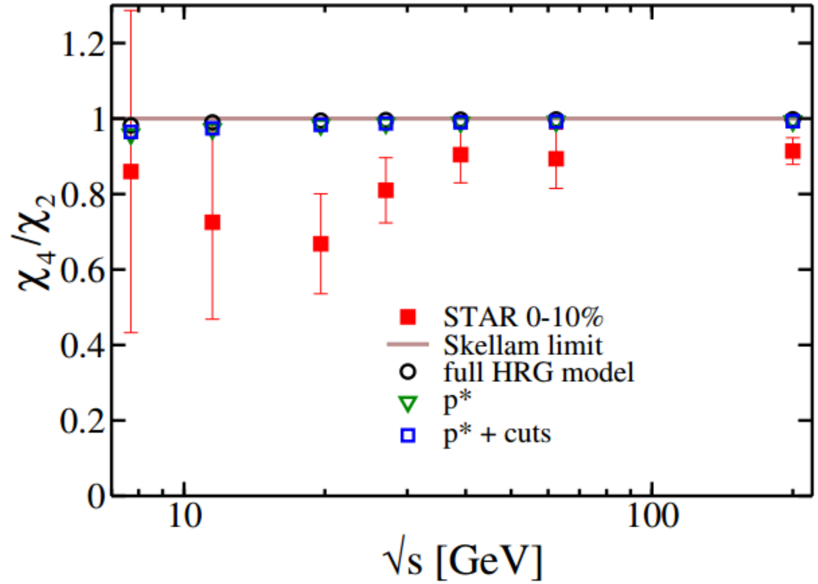
Improvements form the Hadron Resonance Gas (HRG) Model

- Kinematic acceptance in y and p_T

$$n_k(T,\mu_k) = \frac{d_k}{4\pi^2}\int_{-y_{\mathrm{MAX}}}^{y_{\mathrm{MAX}}} \mathrm{d}y \int_{p_T^{\mathrm{MIN}}}^{p_T^{\mathrm{MAX}}} \mathrm{d}p_T \frac{p_T\sqrt{p_T^2+m_k^2}\,\mathrm{Cosh}[y]}{(-1)^{B_k+1}+\exp((\mathrm{Cosh}[y]\sqrt{p_T^2+m_k^2}-\mu_k)/T)}$$

- Resonance decays
  - The cumulants of multiplicity : $VT^3 \left.\frac{\partial(P/T^4)}{\partial(\mu_h/T)}\right|_T = \langle N_h\rangle + \sum_R \langle N_R\rangle\langle n_h\rangle_R$





[Nahrgang, et al. EPJC75 (2015) no.12, 573]

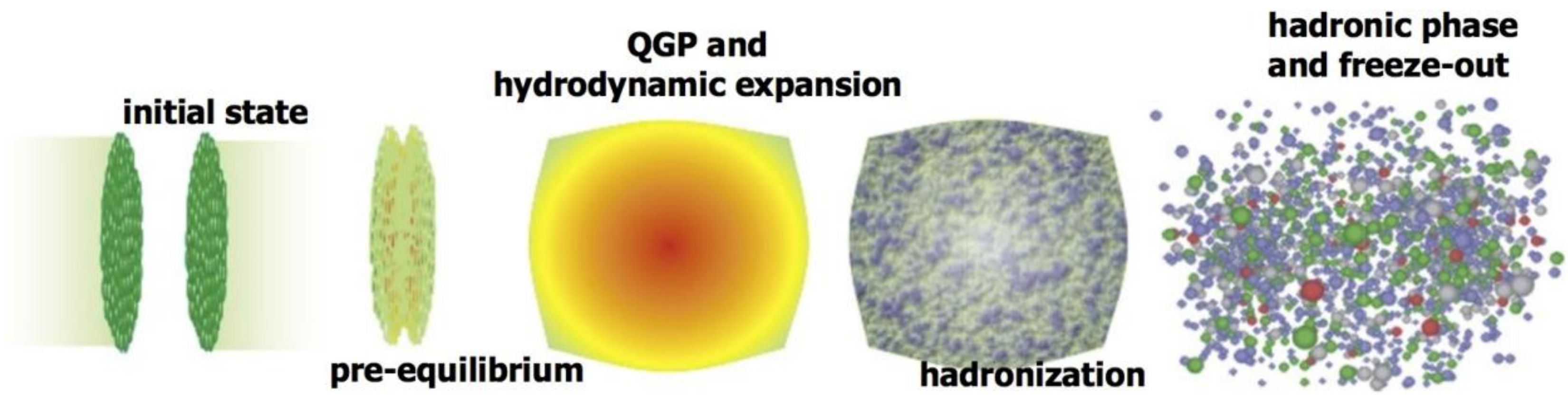
- The decay processes do not significantly modify the cumulants ratios of primordial net-protons (without resonance decays) .
- It is also important to investigate the non-critical effects with more realistic dynamical models.

2 The Revised iEBE-VISHNU Hybrid Model

- Poisson fluctuation assumptions
  - ---Resonance decay
  - ---Hadronic scattering evolution
  - ---Volume fluctuation effects

Jixing Li, Hao-jie Xu，Huichao Song. Phys. Rev. C97 (2018) no.1, 014902

The revised iEBE-VISHNU Hybrid Model



Super MC Vis. Hydro. iSS URQMD Weakdecay

- Cooper-Frye formula: $E\frac{d^3N_i}{d^3p}(x) = \frac{g_i}{(2\pi)^3} p^\mu d^3\sigma_\mu(x) f(x,p)$ where $f(x,p) = f_0 + \delta f$
- We assume our system is a traditional grand canonical ensemble where f_0:

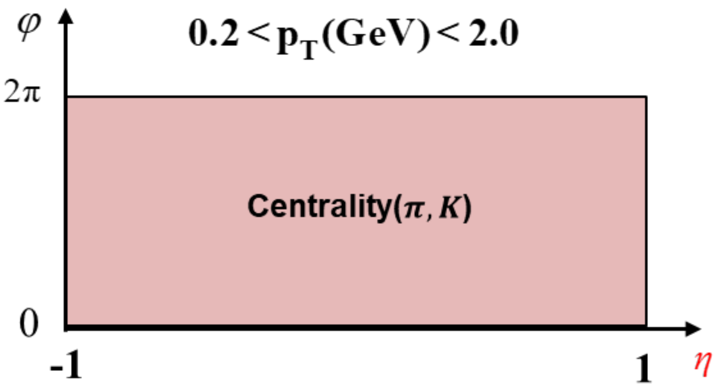
$$f_0 = \frac{1}{\gamma_s^{-|S_i|} e^{(p^\nu \cdot u_\nu - \vec{c}_i \cdot \vec{\mu}_i)/T} \pm 1}$$

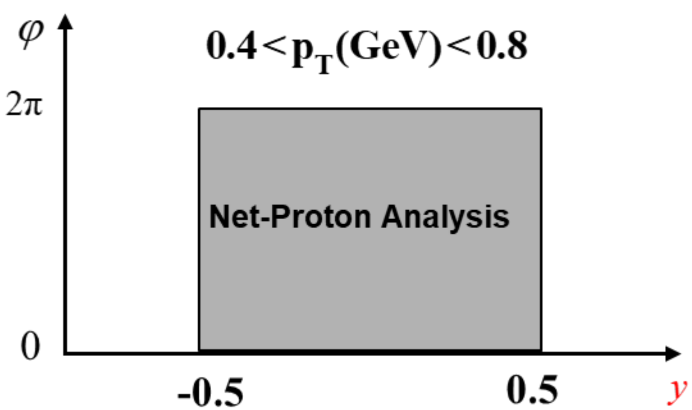
Cooper-Frye formula specifically written as: $E\frac{d^3N_i}{d^3p}(x) \propto p^\mu d^3\sigma_\mu(x) \gamma_s^{|S_i|} e^{-p^\nu \cdot u_\nu / T} e^{\vec{c}_i \cdot \vec{\mu}_i / T}$

- Poisson fluctuations： $P_i(k) = \frac{\lambda_i^k e^{-\lambda_i}}{k!}$, with $\lambda_i = N_i$ $N_i = w_y * dN^i/dy$

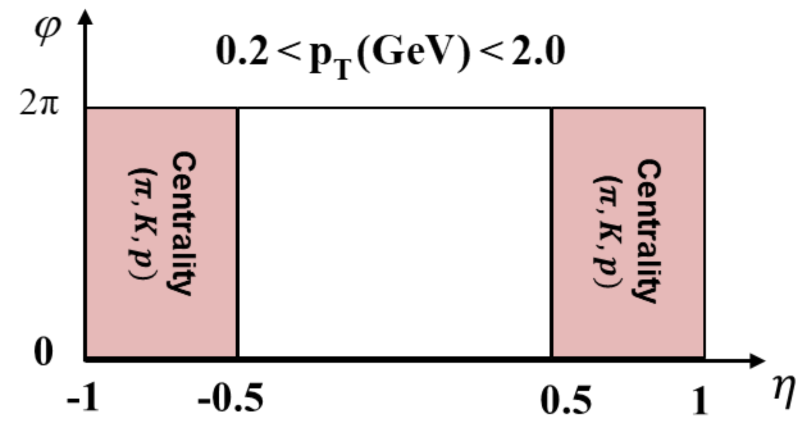
Data Analyze

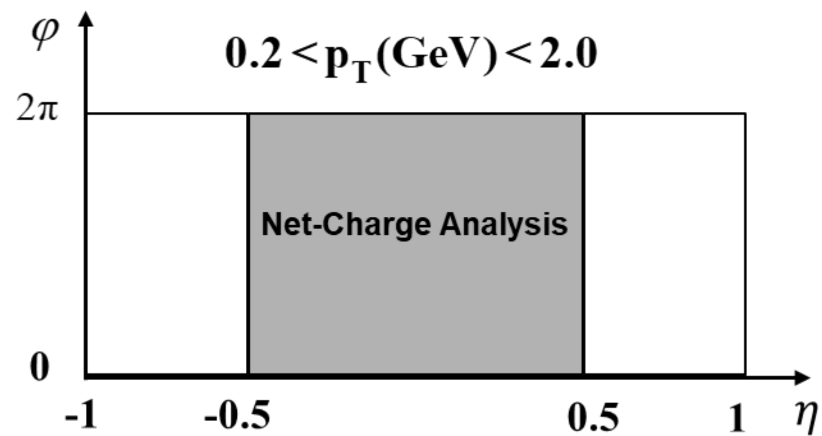
Net-Proton





Net-Charge





Aggarwal M M, et al. arXiv preprint arXiv:1007.2613, 2010.

Adamczyk L, et al. Phys. Rev. Lett, 2014, 112(3): 032302.

3

Results and Discussion

- **Charge moments and moments products**

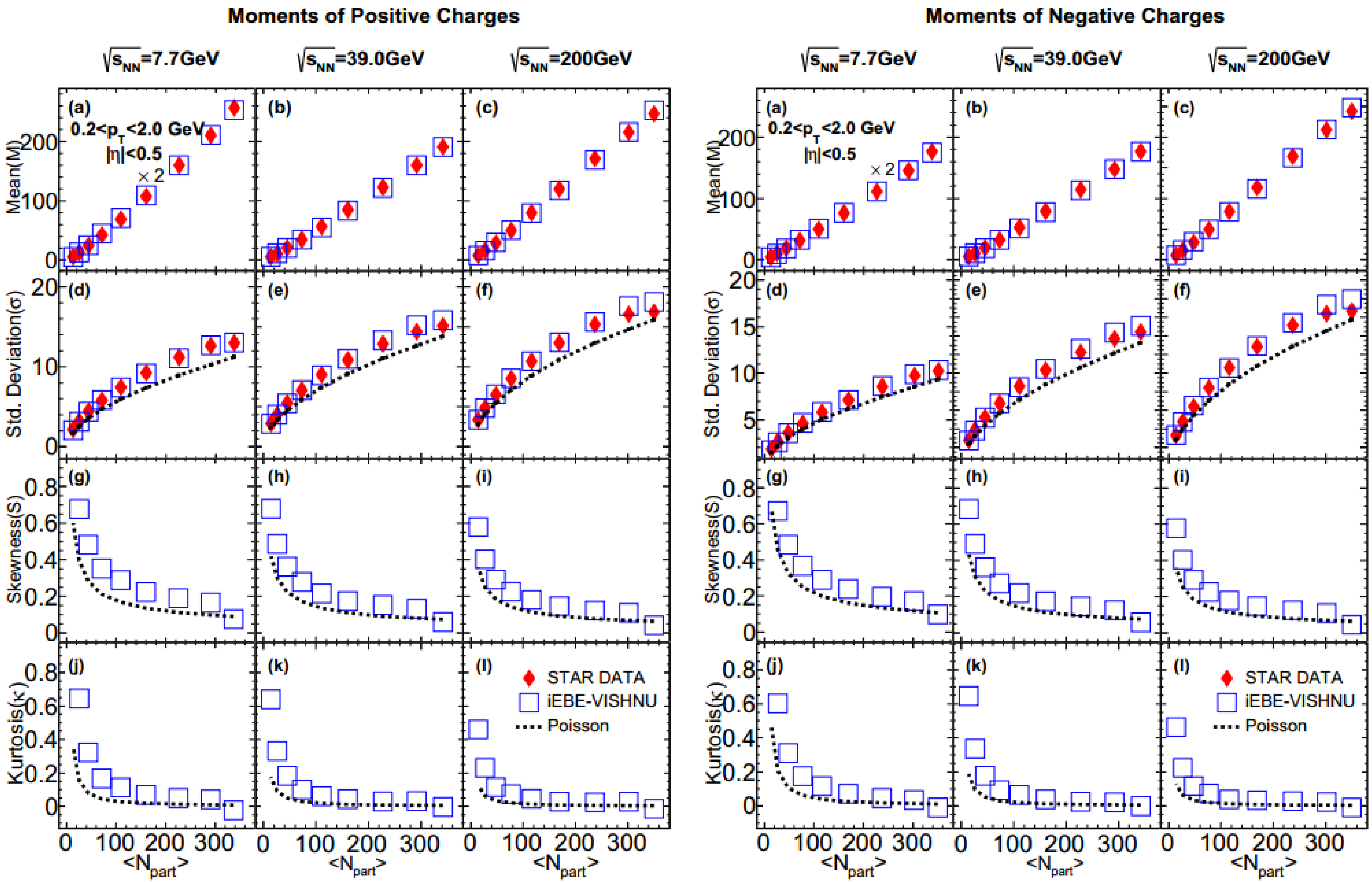
- **Proton moments and Moments products**

- **The effects from volume corrections, resonance decays and hadronic evolution**

- **Non-Critical fluctuations**

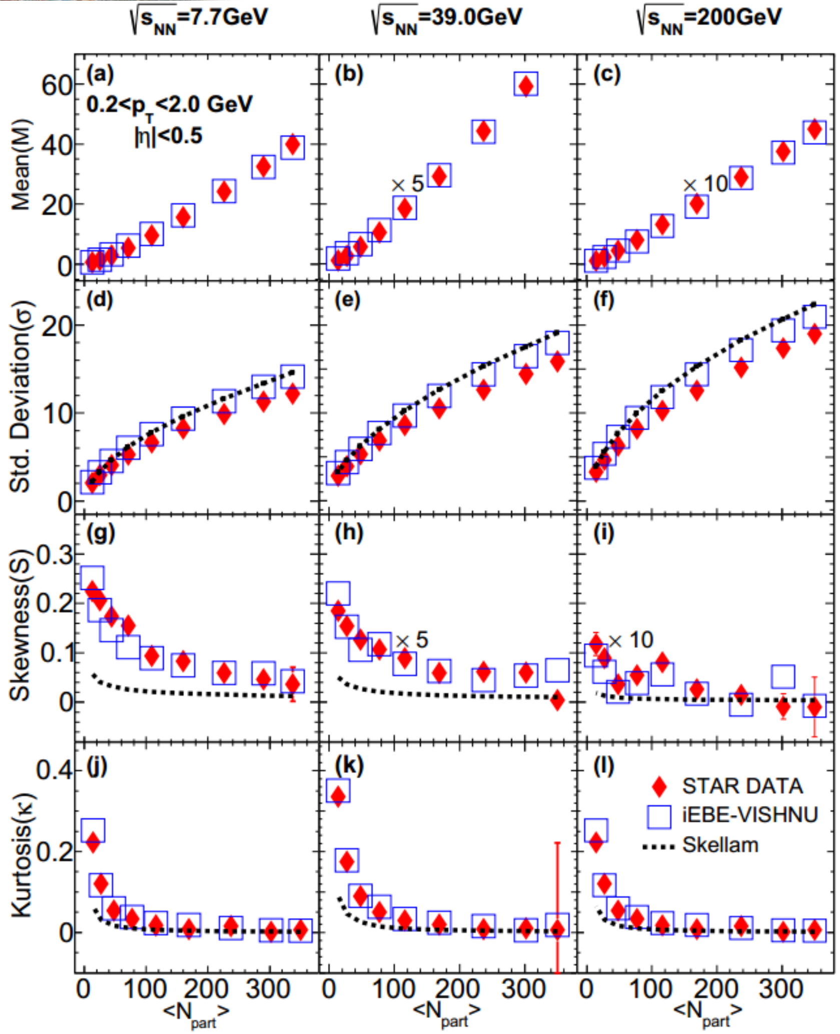
  ---iEBE-VISHNU vs. UrQMD

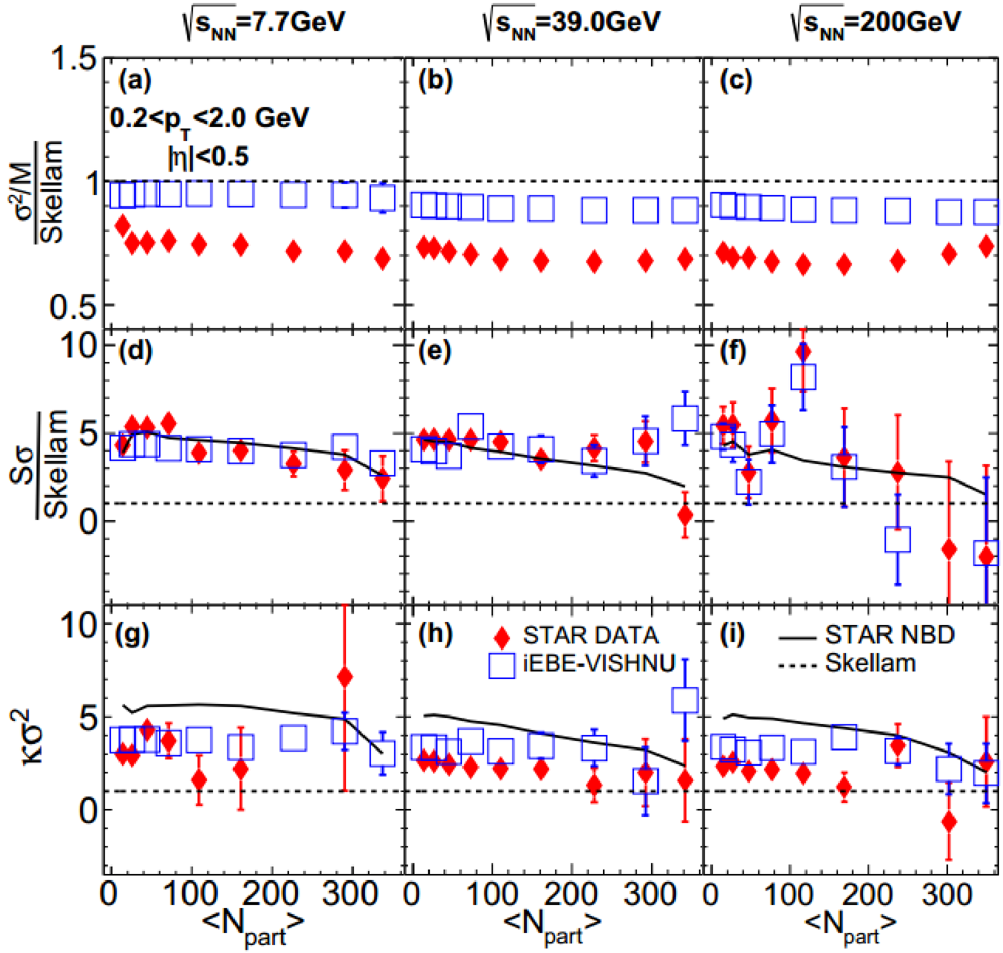
Moments of Positive/Negative Charges



- iEBE-VISHNU model give a very good description for the mean and standard deviation of positive/negative charge distributions .
- Due to the contribution from volume corrections, the iEBE-VISHNU predictions are larger than the Poisson baselines. (Xu.PLB.765(2016))
- The results of the negative charge are similar to ones of the positive charge.

Moments and Moment products of net-charges





- The model works well for the mean, Skewness, Kurtosis of net-charge distributions.
- Volume corrections play different role for the moments of net charges.
- Other correlations of net charge, e.g. the related conservation law, should be contained
- Model calculations qualitatively describe the data of $S\sigma$ and $\kappa\sigma^2$ better than the NBD.

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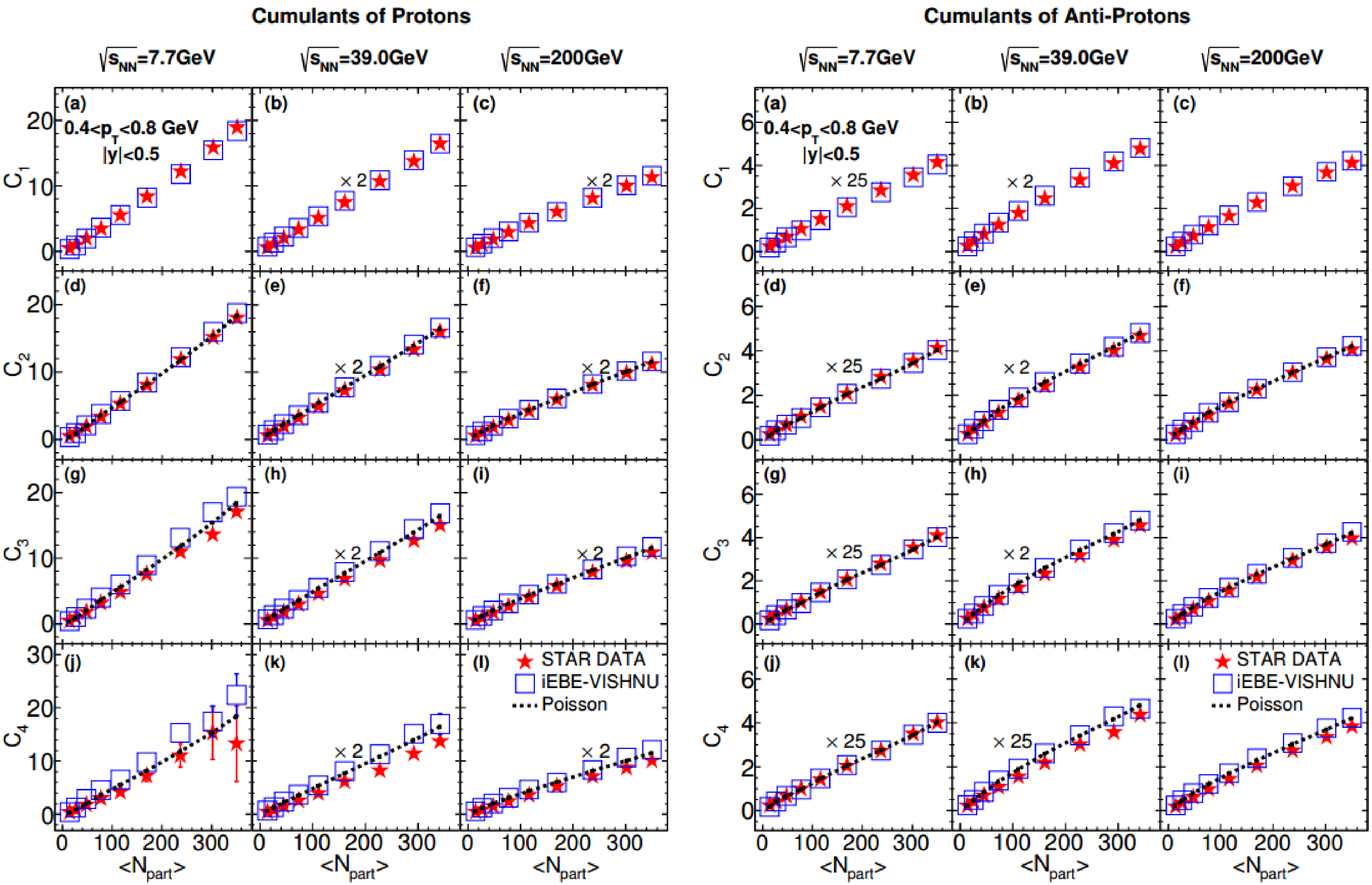
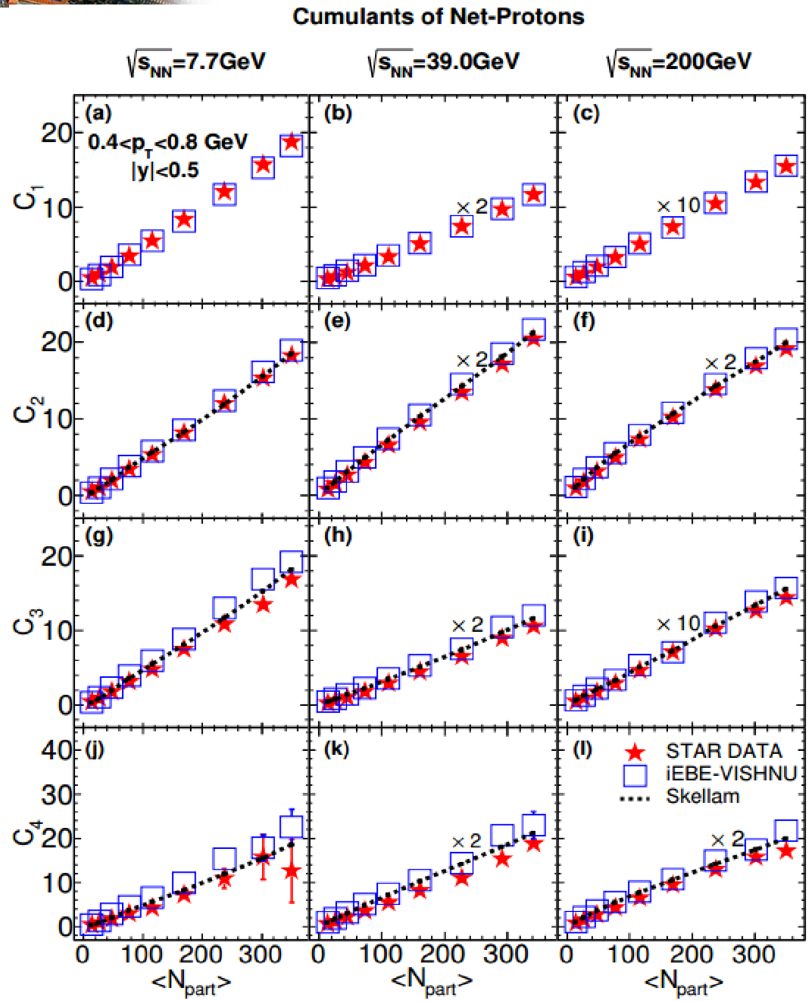
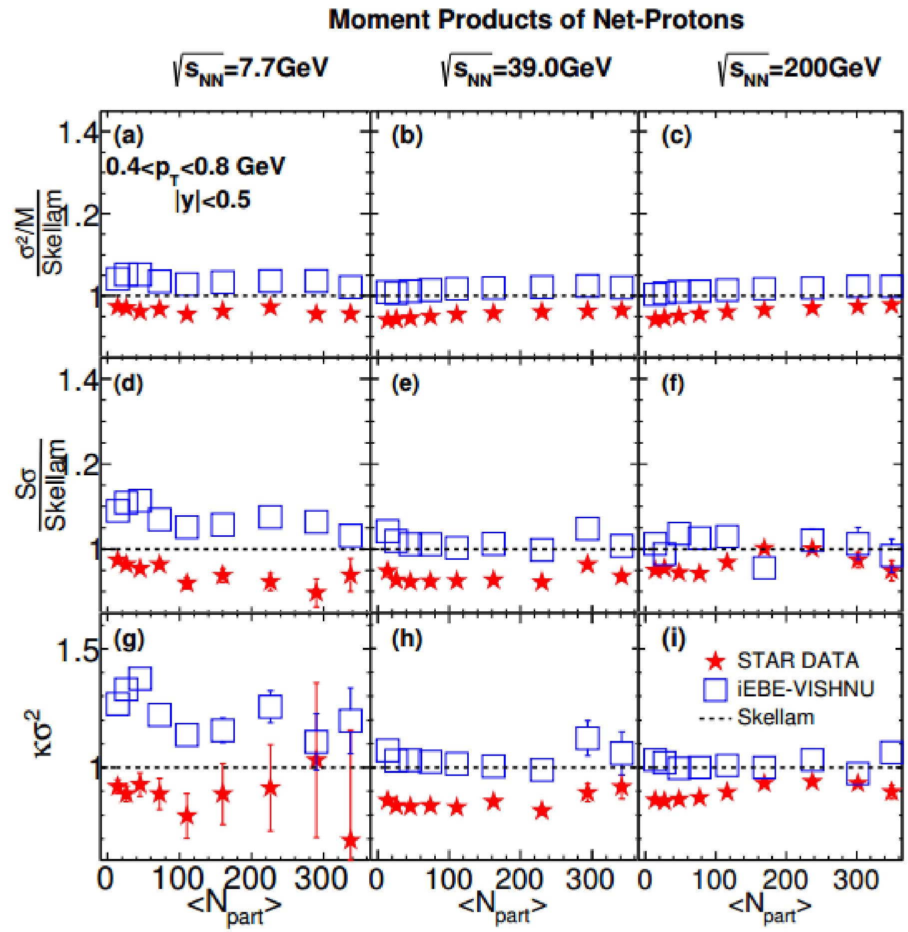
Cumulants of Proton/Anti-proton



- The difference between our model calculations and the Poisson baselines are pretty small.
- The results of anti-proton are similar to the ones of proton.

Cumulants and Moment Products of Net-proton





- The distribution of Net-proton is close to the Skellam distribution:

$$P(N) = \left(\frac{M_p}{M_{\bar{p}}}\right)^{N/2} I_N(2\sqrt{M_p M_{\bar{p}}}) \exp[-(M_p + M_{\bar{p}})],$$

- Our model overpredicts the data for $S\sigma$ and $\kappa\sigma^2$.
- More critical/non-critical physical effects should be considered to give a quantitative description of the data.

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- **The effects from volume corrections, resonance decays and hadronic evolution**
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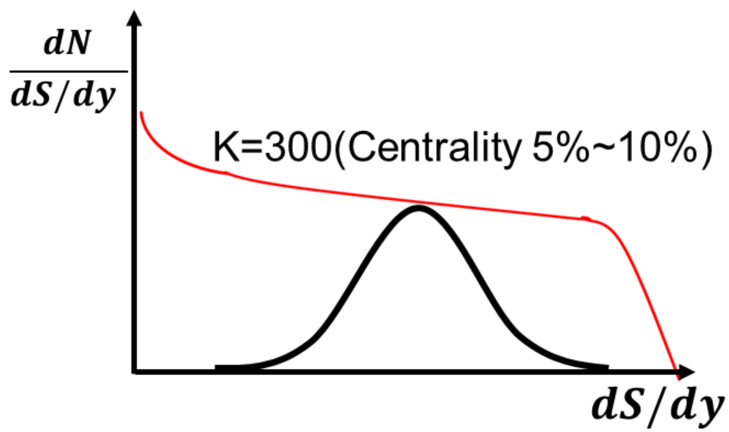
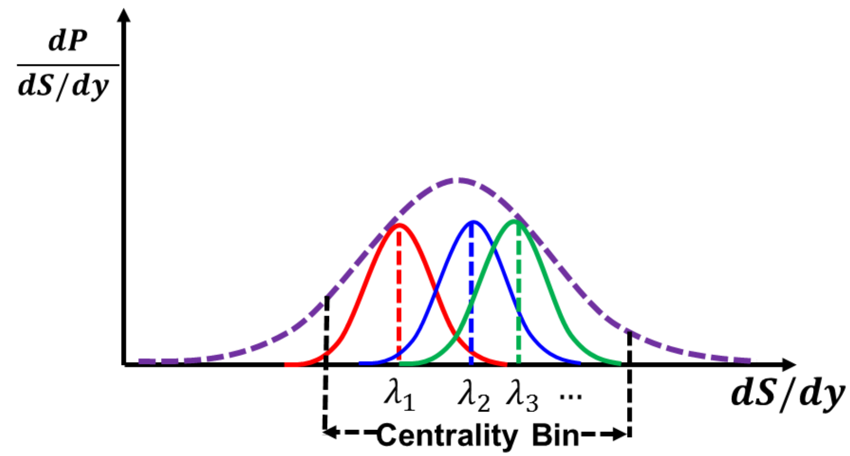
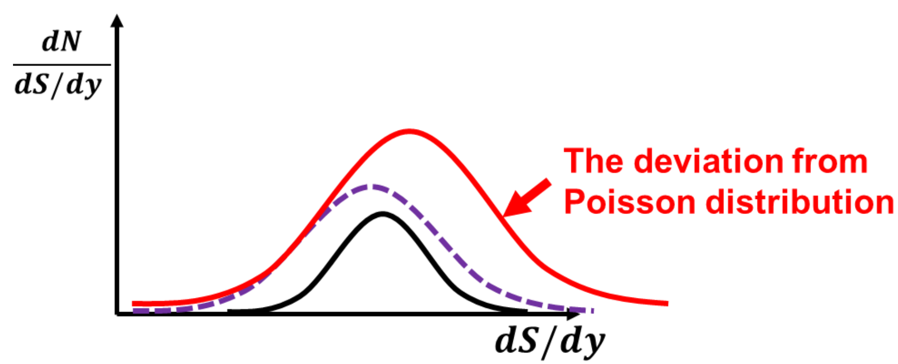
---iEBE-VISHNU vs. UrQMD

Volume Corrections

- Volume variation within a finite centrality bin size [Luo,et al. JPGN.40(2013)105104]
- The initial volume fluctuations;
  - Particle multiplicity and the impact parameter does not correspond one-to-one.
  - There are different initial collision geometry resolutions (centrality resolutions) for different centrality definitions with particle multiplicity.

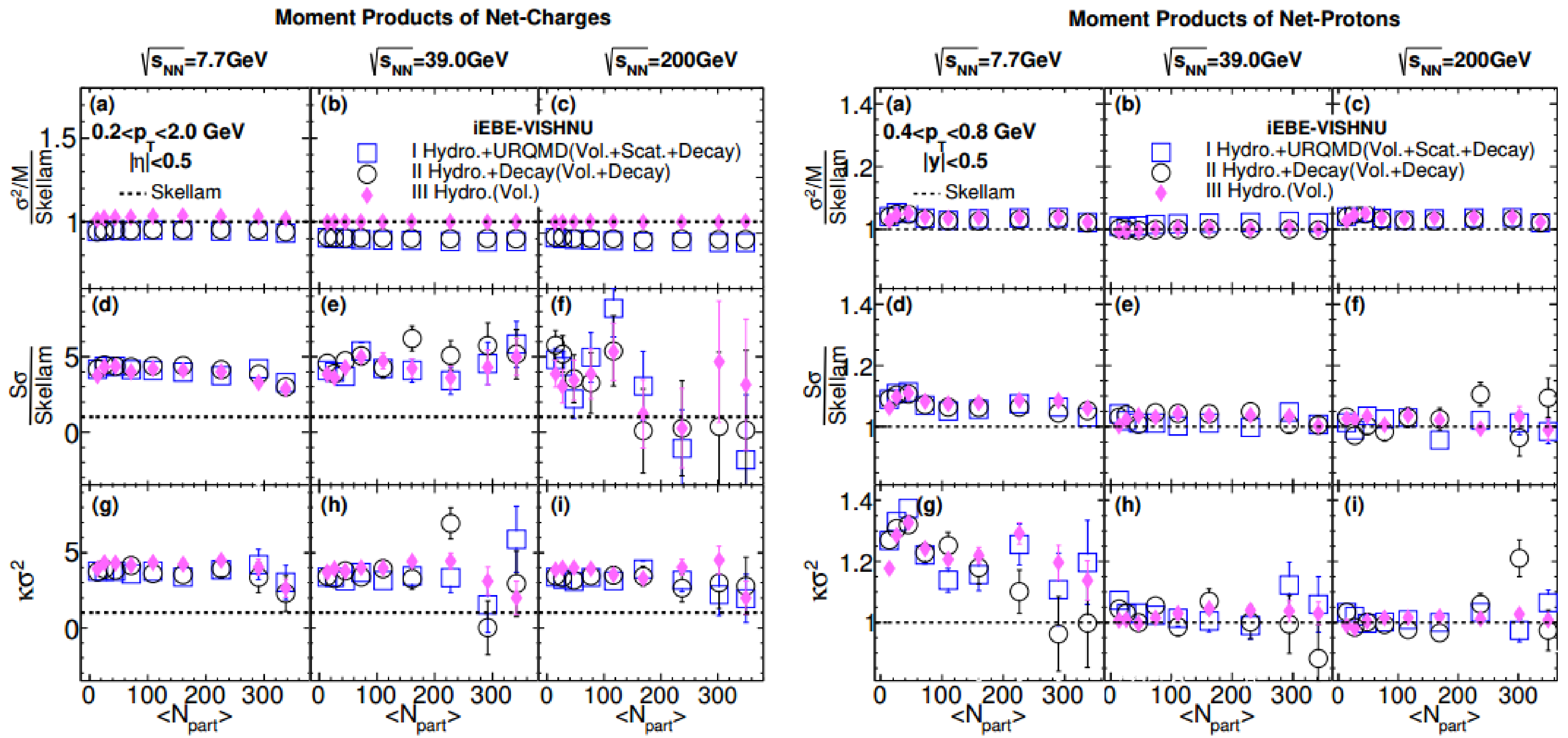






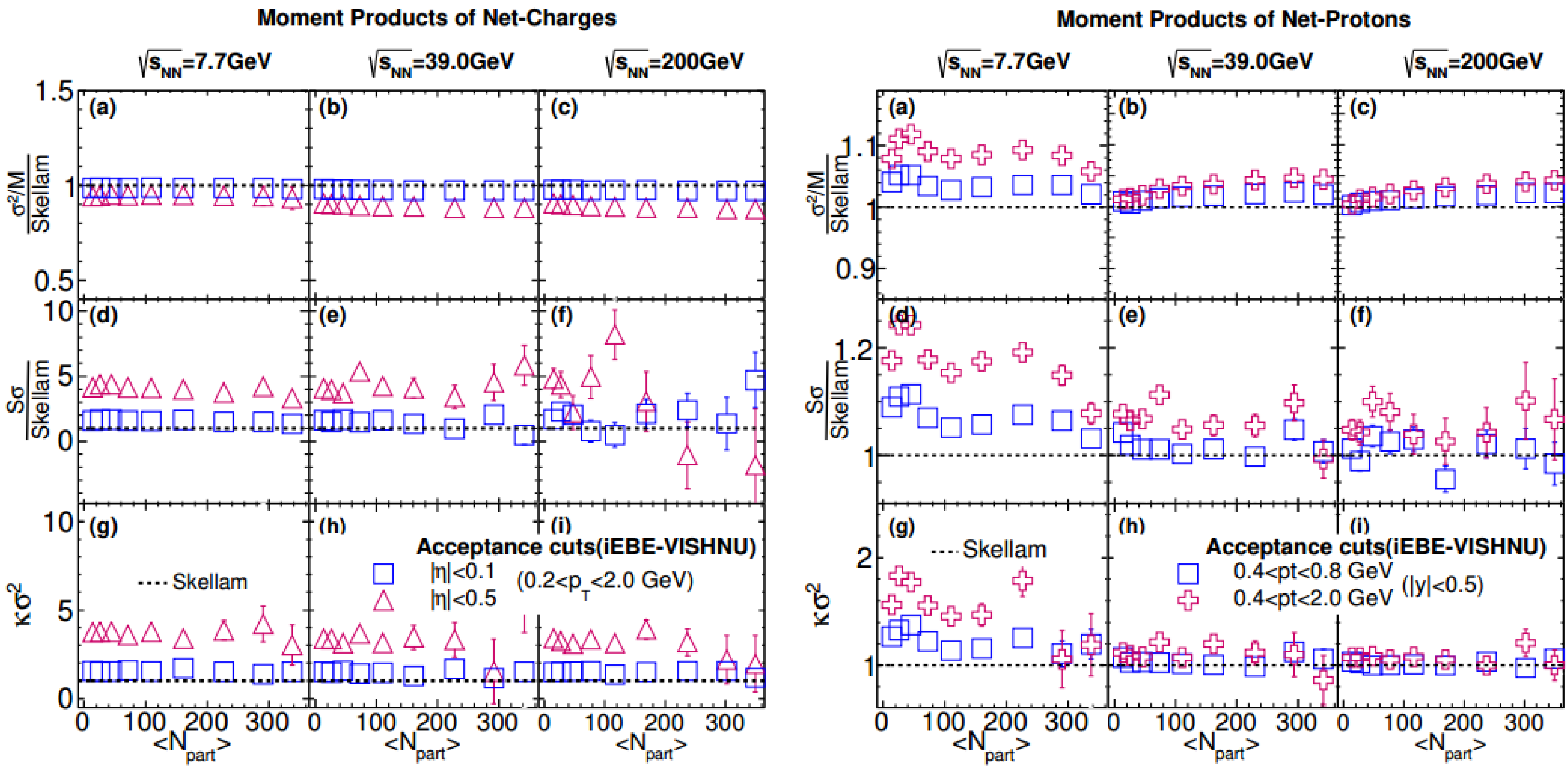
[Xu.PLB 765 (2017) 188–192]

Volume corrections, resonance decays and hadronic evolution



- The volume corrections for σ^2/M of net charges are pretty small or even negligible, while the volume corrections for the corresponding $S\sigma$ and $\kappa\sigma^2$ are very large.
- For net protons, the volume corrections for σ^2/M , $S\sigma$ and $\kappa\sigma^2$ are all pretty small, but also gradually increase with the decrease of collision energy.
- Hadronic evolution and resonance decay progress make no significant contributions to the moment products for both net-protons and net-charges.

Acceptance dependence of the moments products



- The volume correction is the main factor to influence the multiplicity fluctuations of net charges, especially for moments products of Skewness and Kurtosis.
- The measured acceptance dependence for the moments products of net protons present different behaviors, when compared with our calculations.

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- **Non-Critical fluctuations**

  ---iEBE-VISHNU vs. UrQMD

Non-Critical fluctuations

---iEBE-VISHNU vs. UrQMD

iEBE-VISHNU

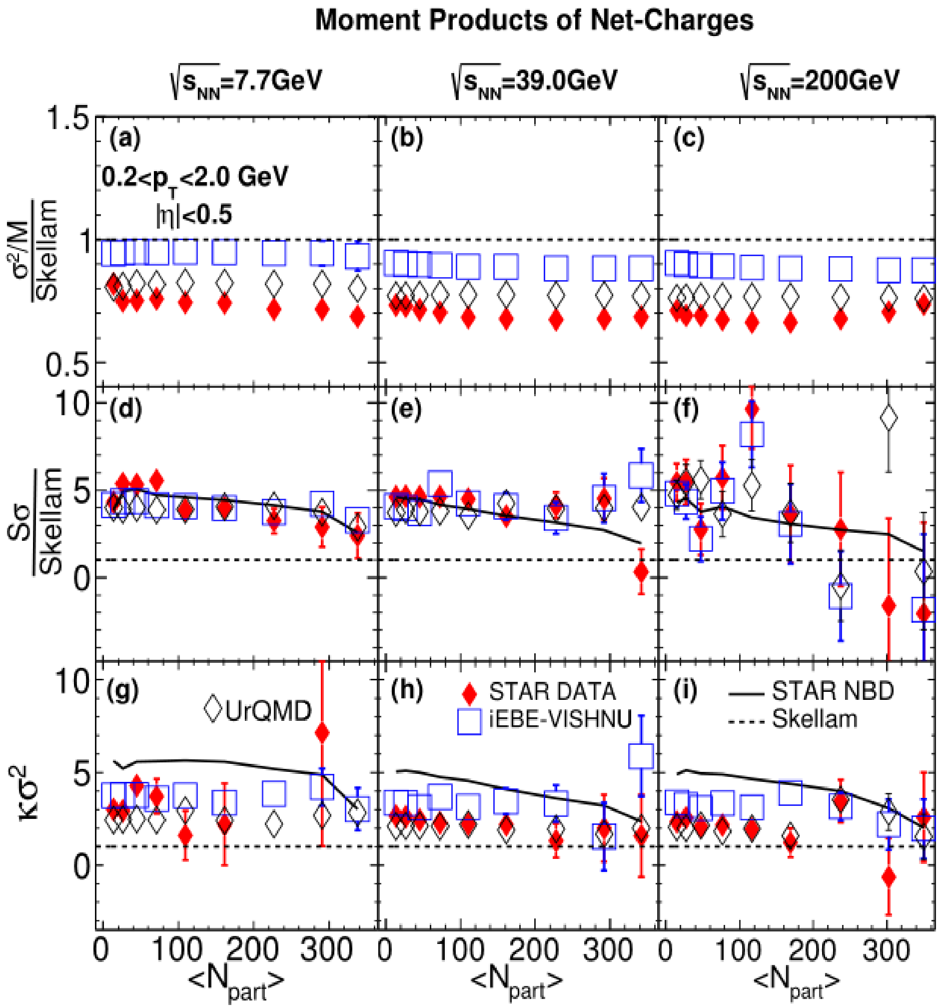
<u>Initialization of iEBE-VISHNU: </u>statistical hadronization, independent particle production, Poisson fluctuations

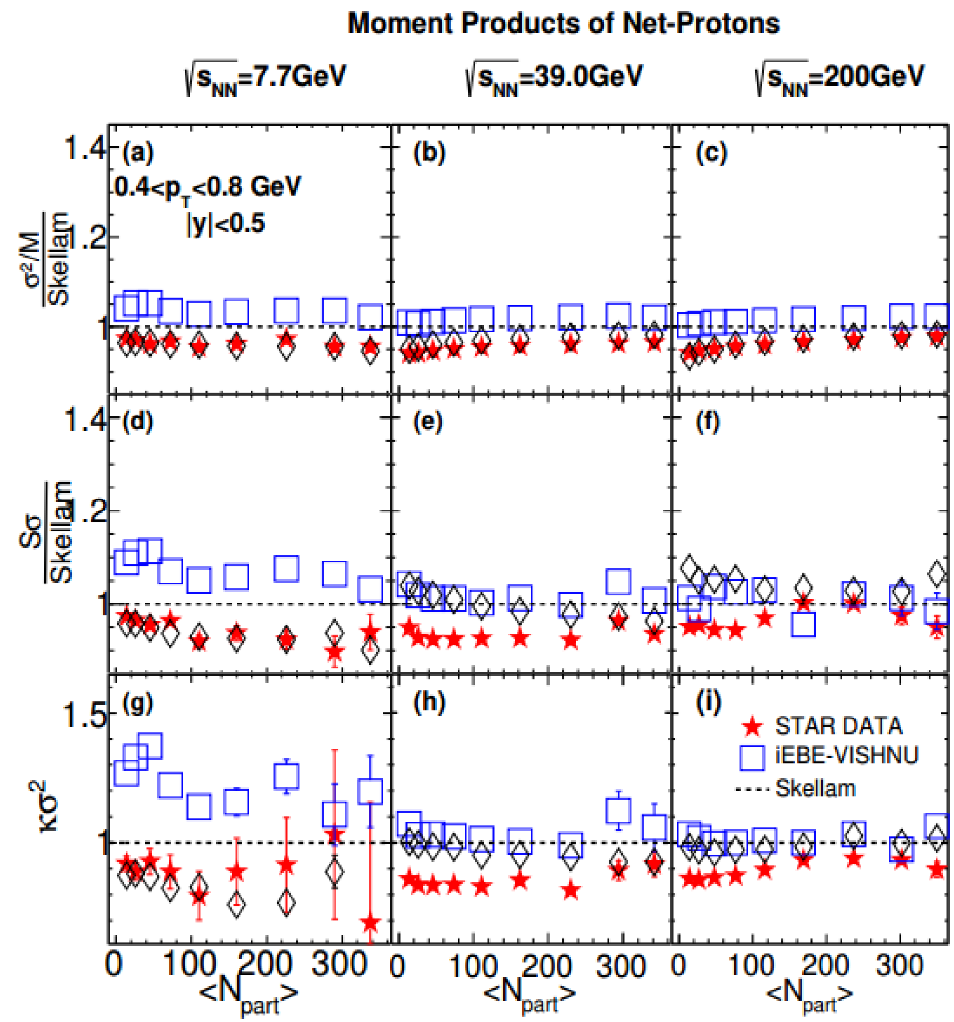
<u>UrQMD</u>

<u>Initialization of UrQMD: projectile & target nuclei</u>, charge conservation

iEBE-VISHNU vs UrQMD

-Li, Luo, Xu & Song, unpublished notes





-Volume fluctuations is the main factors to influence the multiplicity fluct. of (net) charges.

-Charge/baryon conservation should be further included in iEBE-VISHNU.

3

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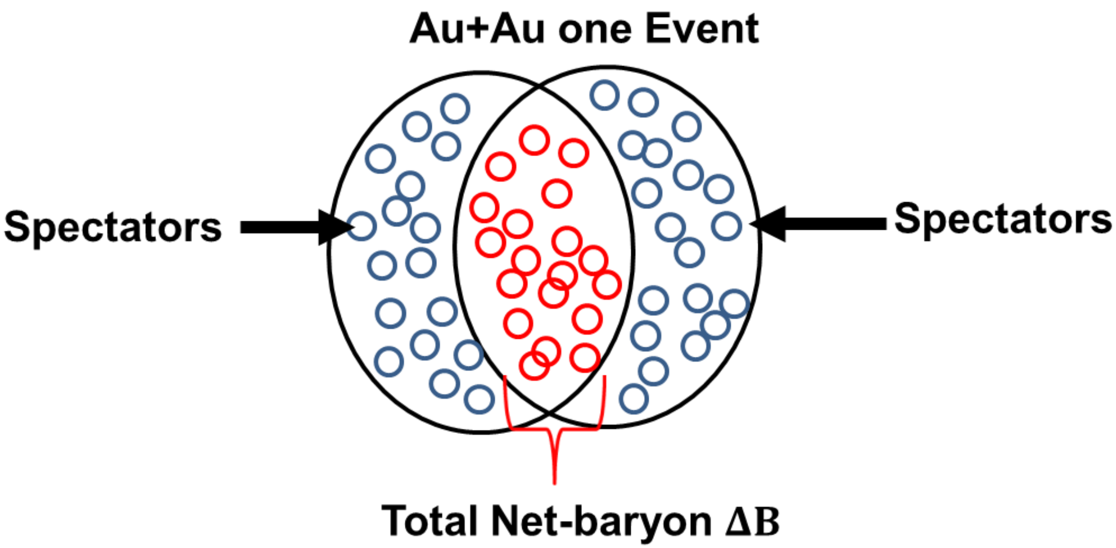
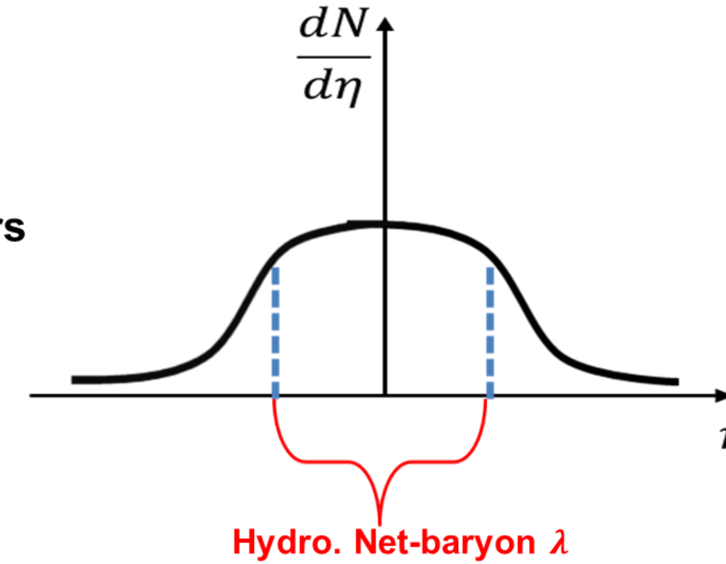
- ---iEBE-VISHNU vs. UrQMD
- **The effects from global net-baryon conservation**

Model set up and assumption





- For each event, we assumed that the thermalized net-baryon number q within a selected midrapidity slice satisfies a binomial distribution:

$$B(q|\Delta B) = \frac{\Delta B!}{q!(\Delta B - q)!} p^q (1-p)^{\Delta B - q}, \qquad p = \frac{\lambda}{\Delta B}$$

- After sampling the net-baryon number q from binomial distribution, we implement the SER algorithm to sample the number of baron (B) and antibaryon ($\bar{B}$) from the Poisson distribution with the restriction $B - \bar{B} = q$ for each event.

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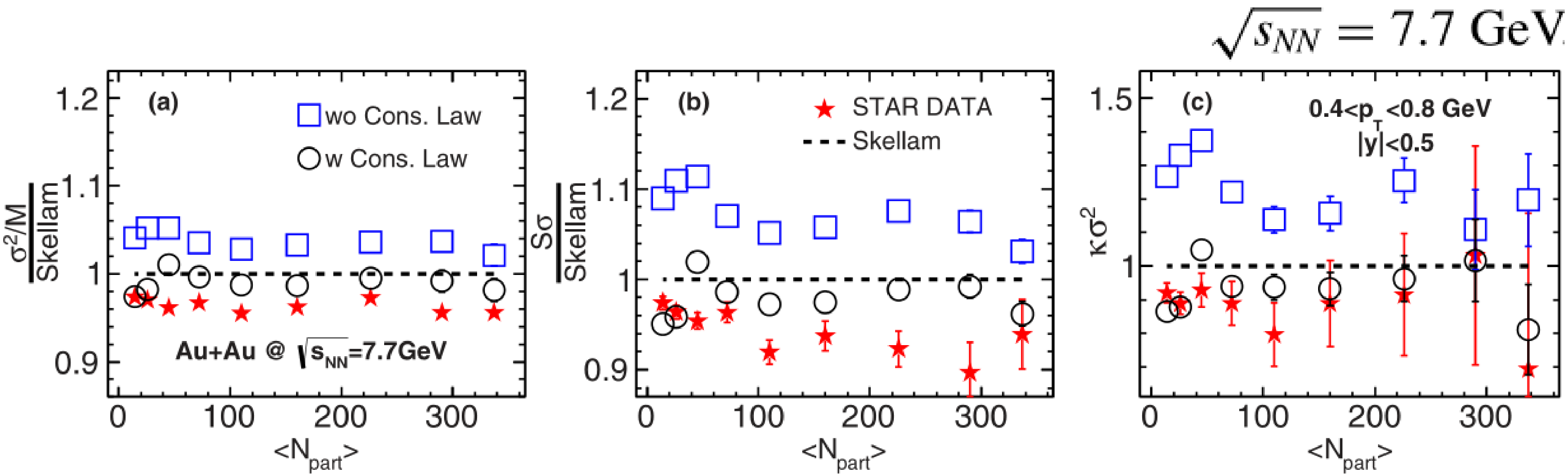
SER :L.-G. Pang, etc. J. Phys. G: Nucl. Part. Phys. 45, 015001 (2018).

The effects form global charge conservation law



- The effect of the global conservation law can largely reduce the cumulant ratios of (net) protons, which makes our model prediction more close to the experimental data for all these three ratios.

- There still exist gaps between the data and our results, which indicates other effects should be further investigated in the near future.

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4

Summary

Summary

- We have studied the no-critical multiplicity distribution fluctuations of the net-proton and net-charge from the revised iEBE-VISHNU Hybrid Model, which gave a more realistic baseline compared to the traditional Poisson baseline.

- Various effects, such as volume corrections hadronic scatterings and evolution, resonance decays, as well as realistic centrality cuts and acceptance cuts have been embed in our model calculations.

- With properly tuned parameters, iEBE-VISHNU roughly describe the centrality dependent moments and cumulants of (net) charges and (net) protons measured in experiments.

- Further comparison simulations show that the volume correlations significantly influence the higher moments of (net) charges, but do not obviously affect the multiplicity fluctuations of (net) protons

- For quantitative description for the data, more correlation effects should be considered, e.g. both baryon and charge conservation law.

Thank you!