Spectroscopy of H dibaryon from Lattice QCD

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New Frontiers in QCD 2018

Yukawa Institute of Theoretical Physics, Kyoto University

Dibaryon project at Mainz

- People involved in the project :
  - Hartmut Wittig
  - Andrew Hanlon (current postdoc)
  - Former postdocs : A Francis, T.Rae, J. Green, *PMJ,* C. Miao
- Works presented at conferences :
  - C. Miao et al., PoS **LATTICE2013,** 440 [1311.3933]
  - J. Green et al., PoS **LATTICE2014,** 107 [1411.1643]
  - *PMJ* et al., PoS **LATTICE2015,** 082, PoS **CD2015,** 079 [1511.01849]
  - *PMJ* et al., Talk at Confinement XII (2016)
- Full results : **arXiv:1805.03966**

Outline

- ✤ History of the H dibaryon
- ✤ Experimental status
- ✤ Review of lattice calculation of H dibaryon
- ✤ Calculation with hexaquark and two-baryon operators.
- ✤ Calculation with distillation
- ✤ Finite volume analysis
- ✤ Conclusions and future directions.

The H dibaryon

Predicted by Jaffe in 1977 using MIT bag model as a deeply bound state:

$$J = I = 0, S = -2$$

$$m_H < 2m_\Lambda \sim -80 \text{ MeV}$$

A flavor singlet in SU(3),

$$BB^{\mathbf{1}} = -\sqrt{\tfrac{1}{8}}(\Lambda\Lambda) + \sqrt{\tfrac{3}{8}}(\Sigma\Sigma) + \sqrt{\tfrac{4}{8}}(N\Xi)_+$$

Decades of theoretical and experimental investigations provide no conclusive evidence on the fate of the H....

Experimental status

- Experiments have explored this state since :
  - 1997 - BNL E836 — No evidence of H dibaryon state.
  - 2000 - kTeV Fermilab E799 — No evidence of H dibaryon state.

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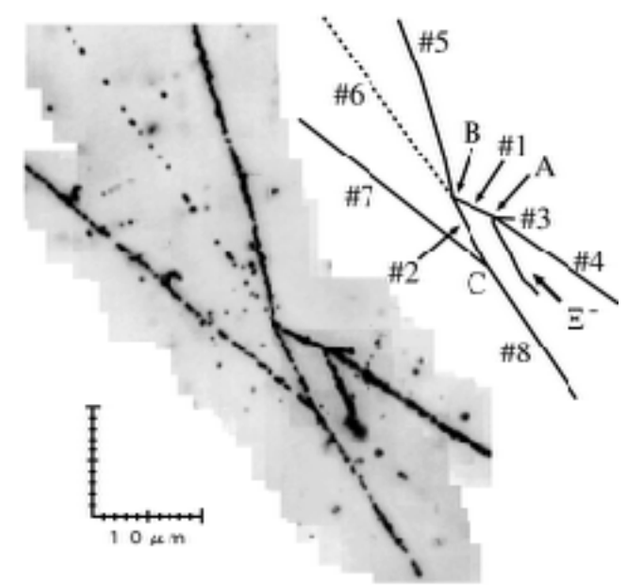
VOLUME 87, NUMBER 21 PHYSICAL REVIEW LETTERS 19 NOVEMBER 2001

Observation of a $_{\Lambda\Lambda}^{6}$He Double Hypernucleus



Strongest constraint comes from *Nagara event* E373 at KEK, which studied decays of hypernucleus $_{\Lambda\Lambda}\mathrm{He}$

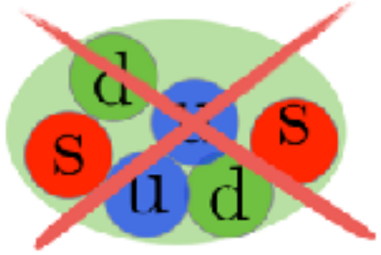
Absence of strong decay :

$$_{\Lambda\Lambda}\mathrm{He} \longrightarrow {}^{4}\mathrm{He} + \mathrm{H}$$

$$B_{\Lambda\Lambda} = 7.13 \pm 0.87 \text{ MeV}$$

(H. Takahashi et al., PRL 87, 212502 (2001))

Experimentally a deeply bound state ruled out.



Experimental status

PRL **110,** 222002 (2013) PHYSICAL REVIEW LETTERS week ending 31 MAY 2013

Search for an H-Dibaryon with a Mass near $2m_\Lambda$ in $\Upsilon(1S)$ and $\Upsilon(2S)$ Decays

(Received 16 February 2013; revised manuscript received 3 May 2013; published 31 May 2013)

We report the results of a high-statistics search for H dibaryon production in inclusive $\Upsilon(1S)$ and $\Upsilon(2S)$ decays. No indication of an H dibaryon with a mass near the $M_H = 2m_\Lambda$ threshold is seen in either the $H \to \Lambda p \pi^-$ or $\Lambda\Lambda$ decay channels and 90% confidence level branching-fraction upper limits are set that are between one and two orders of magnitude below the measured branching fractions for inclusive $\Upsilon(1S)$

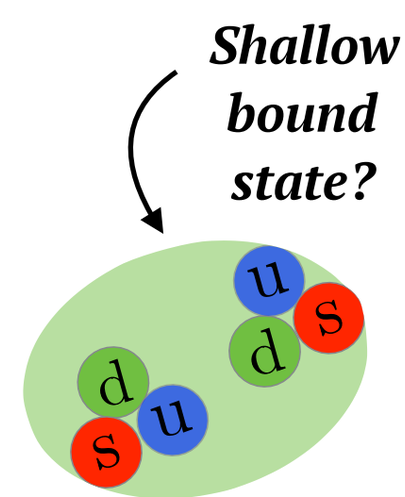
Decays of $\Upsilon(1S, 2S)$ produce flavor-SU(3)-symmetric final states.

these results put stringent constraints on H dibaryon properties.

KEK Belle Detector : No signature of bound state.

Models provide a broad range of answers !

Lattice QCD can provide some insight.



QCD on a spacetime lattice

GOAL : To compute ,

$$\langle O(t) \rangle = \frac{1}{Z_{QCD}} \int \mathcal{D}[\psi, \bar{\psi}, U] e^{-S_{QCD}(\psi, \bar{\psi}, U)} O[\psi, \bar{\psi}, U]$$

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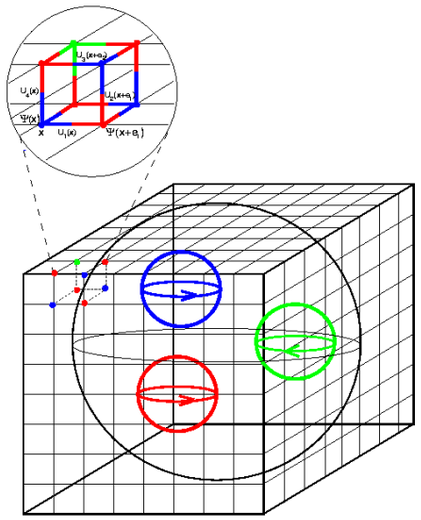

✤ Wick rotate to Euclidean space and discretise spacetime.

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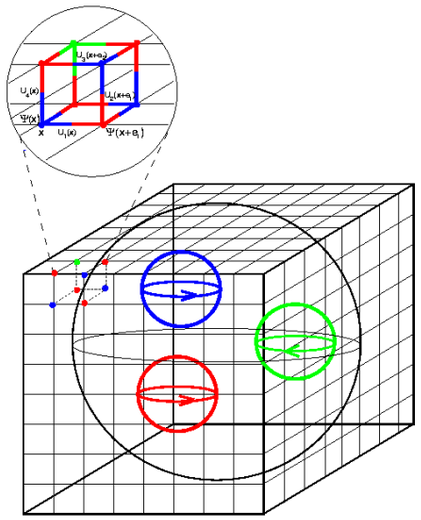

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✤ Compute path integral as a Monte-carlo integration with weight :

$$W[U^i] = \frac{e^{-S_g} \ \det(D[U^i])}{Z_{QCD}}$$

U^i is a gauge field configuration obtained from Monte-carlo

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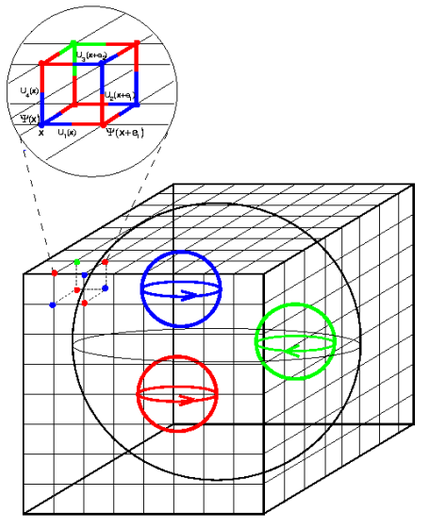

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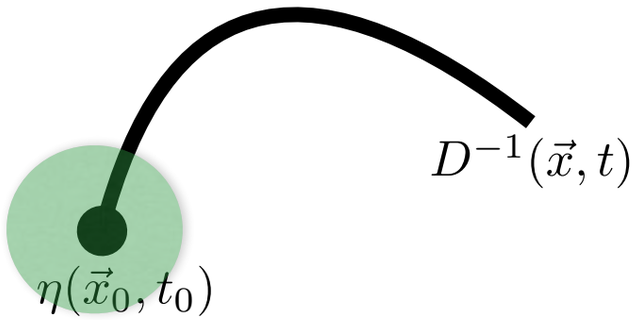
✤ Compute $\langle O \rangle$ over N gauge field configurations

$$\langle O \rangle \approx \frac{1}{N} \sum_{U^{i,N} \propto e^{-S_{QCD}}} O[U^n]$$

Methodology of lattice spectroscopy

- Compute quark propagators by inverting Dirac matrix on point sources/smeared point sources.



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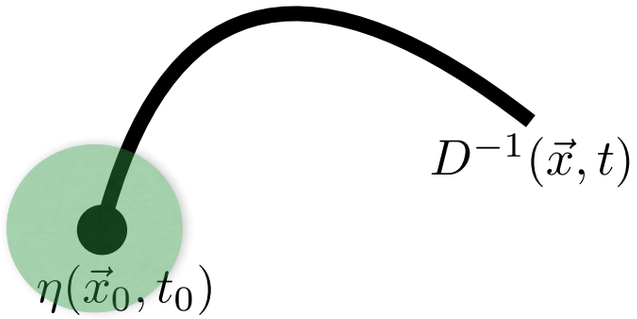
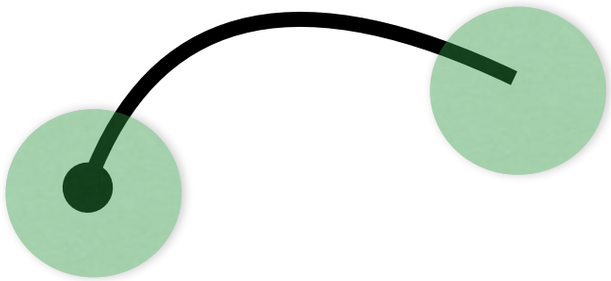


- Smear the source fields and/or the quark propagators to improve overlap onto the ground state.



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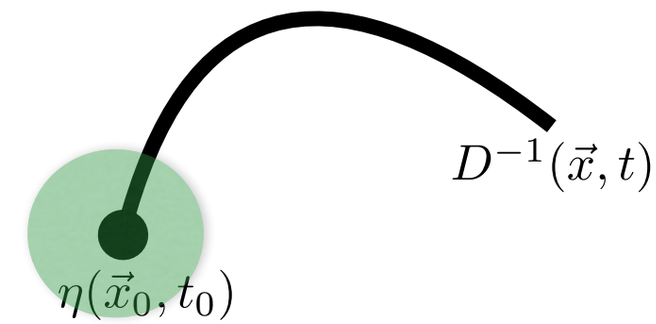

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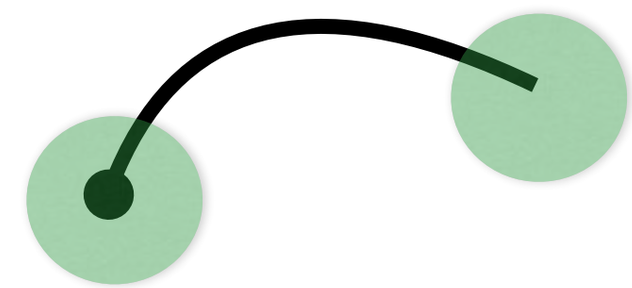

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$$\mathcal{O}(\vec{x}, t) = \bar{u}\gamma_5 d$$

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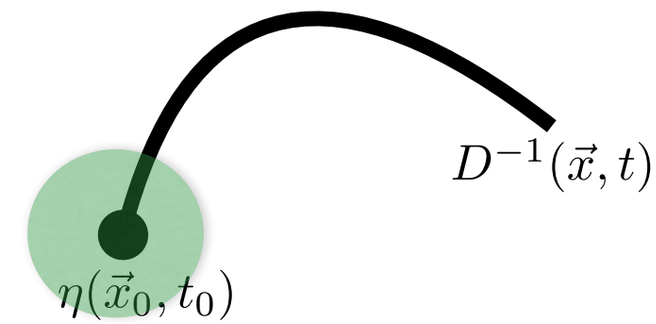

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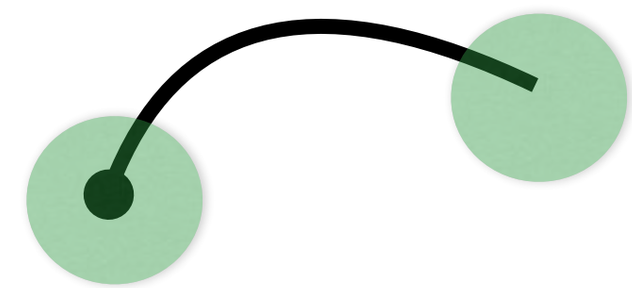

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$$\mathcal{O}(\vec{x}, t) = \bar{u}\gamma_5 d$$

- Compute expectation values of operators and determine the ground state at large t...

$$\langle 0|\mathcal{O}(\vec{x}, t)\mathcal{O}^\dagger(0)|0\rangle \longrightarrow_{t\to\infty} Ae^{-M_\pi t}$$

Challenges in multi-baryon spectroscopy

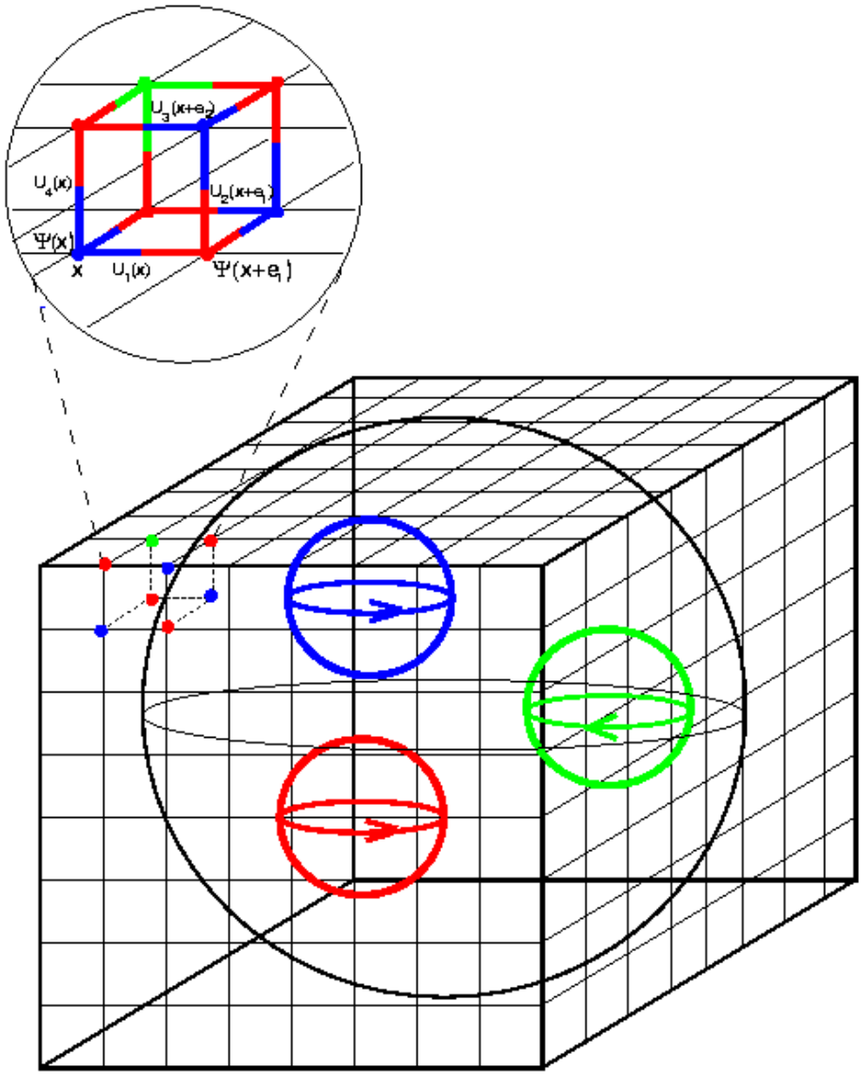
Lattice techniques offer first principle calculation of QCD observables !

1. Unphysical quarks, finite lattice spacing and volume.

2. Non-trivial contractions.

3. Non-trivial finite volume effects.

4. ***Worse Signal/Noise problem.***



Courtesy:USQCD

Signal / Noise problem

- Baryons suffer from signal / Noise problem.
- Noise grows exponentially.

$$\frac{C_B(t)}{\sigma(t)} \sim \sqrt{N_{MC}}\; e^{-(m_B - \frac{3}{2} m_\pi)t}$$

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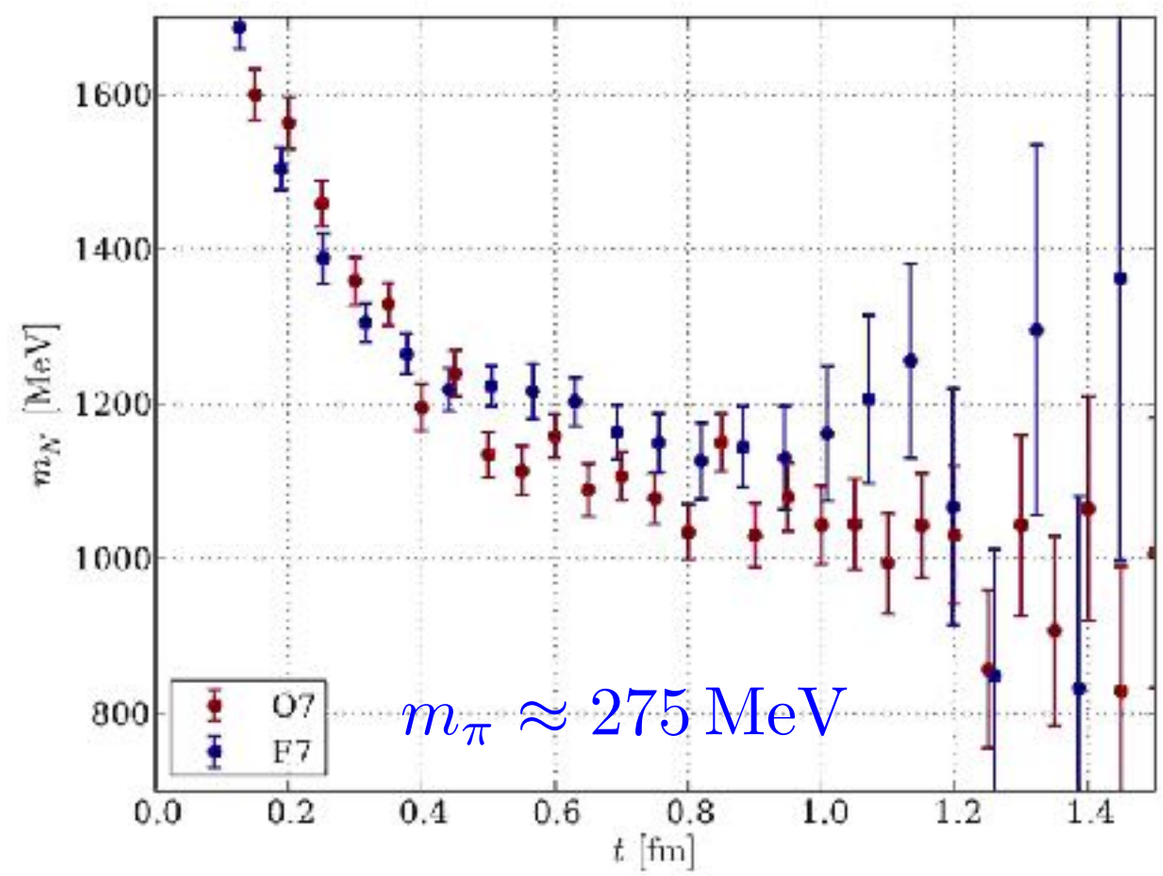
$$\frac{C_B(t)}{\sigma(t)} \sim \sqrt{N_{MC}}\; e^{-(m_B - \frac{3}{2} m_\pi)t}$$

- Impossible to reach asymptotic source-sink separations.
- Construct operators to improve overlap for fast falling excited states and access the "*GOLDEN WINDOW*".



[Capitani et al., arXiv:1504.04628]

Review of *H* dibaryon calculation

Collaboration	Method	m_π[MeV]	N_f	References
HALQCD	Baryon-baryon potential; Nambu-Bethe-Salpeter wave function	470–1170 (146*)	3	*Phys Rev Lett 106 (2011) 162002* *Nucl Phys A881 (2012) 28*
NPLQCD	Two-point correlation functions	806	3	*Phys Rev D87 (2013) 034506*
	Two-point correlation functions	230, 390 (450*)	2+1	*Phys Rev Lett 106 (2011) 162001* *Mod Phys Lett A26 (2011) 2587*
Mainz	Two-point correlation functions	(450–1000)*	2	*PoS LATTICE2013 (2014) 440* *PoS LATTICE2014 (2015) 107* *arXiv:1511.01849 [hep-lat]*

Note : (*) indicates results in proceedings, unpublished results

HALQCD results

GOAL : Compute non-local NN potential from Lattice QCD

- Solve a Schrödinger type equation :

$$-\frac{\nabla^2}{2\mu}\phi_k(\vec{r}) + \int d^3\vec{r'} U(\vec{r},\vec{r'})\phi_k(\vec{r'}) = E_k + \phi_k(\vec{r})$$

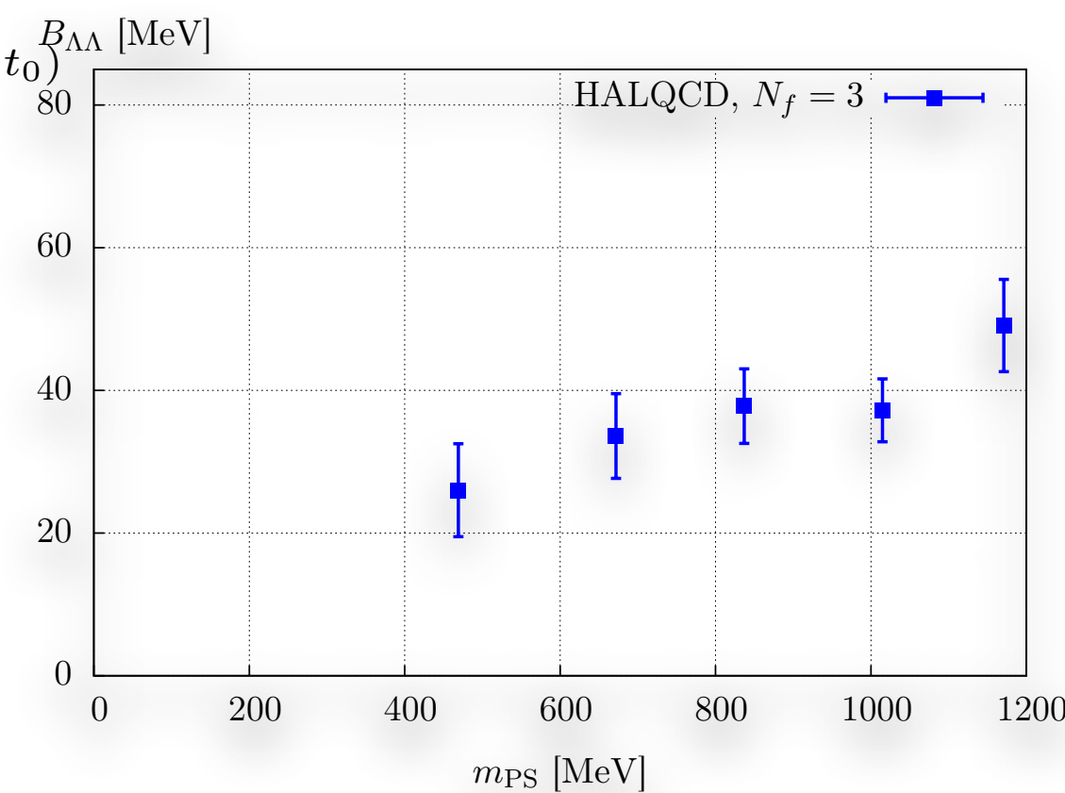
- $\phi_k(\vec{r})$ is computed in a lattice calculation :

$$G(\vec{r}, t-t_0) = \langle 0|(BB)^\alpha(\vec{r},t)(\overline{BB})^\alpha(t_0)|0\rangle = \phi(\vec{r},t)e^{-E(t-t_0)}$$

- $N_f = 3$, degenerate u,d,s

  5 pion masses

  $m_\pi = 461 - 1171$ MeV



NPLQCD results

- Compute dibaryon energy levels from multi-baryon correlation functions.

$$O_H^{BB} = \epsilon_{ijk}(r^i C\gamma_5 P_+ s^j)t^k(\vec{x},t)\ \epsilon_{lmn}(u^l C\gamma_5 P_+ v^m)w^n(\vec{y},t)$$

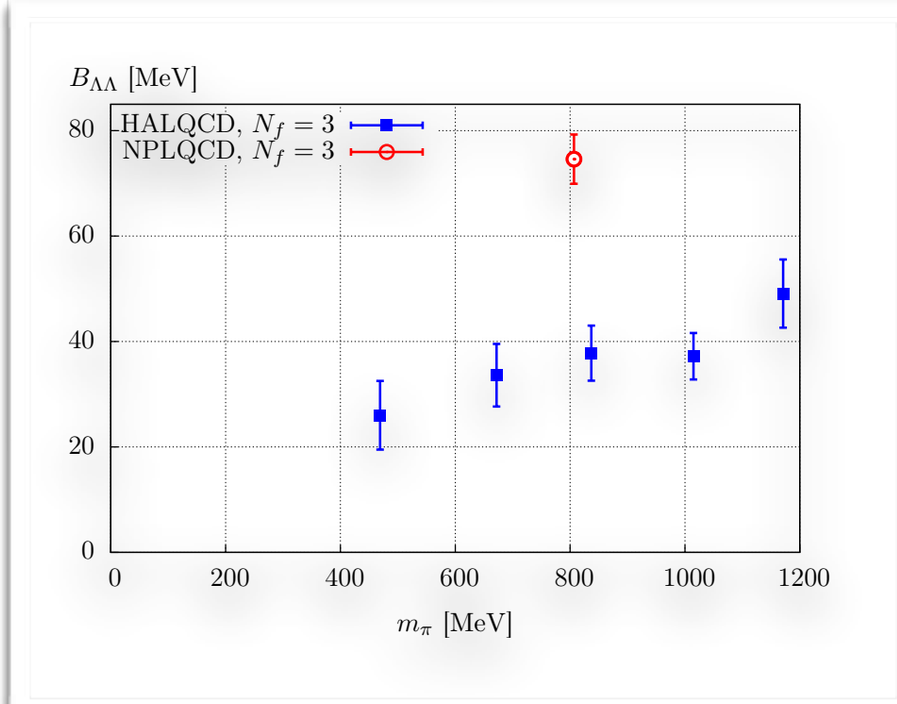
$$\langle 0|\sum_{\vec{x},\vec{y}} O_H^{BB}(\vec{x},\vec{y},t)O_H^{BB}(\vec{0},\vec{0},t)|0\rangle = A_{BB}e^{-E_{BB}t} + \ldots \quad \longrightarrow \quad B_H = 2m_\Lambda - E_{\Lambda\Lambda}$$

- Compute B_H in several volumes and establish infinite volume result.

- $N_f = 3$, degenerate u,d,s

  $m_\pi = 807$ MeV $\quad L = 3.4, 4.5, 6.7$ fm

  $\mathcal{O}(10^5)$ measurements/ensemble.



Mainz H dibaryon set-up

- Employ Lattice QCD ensembles generated by **CLS** (Coordinates Lattice Simulations) collaboration.
- $N_f = 2$, O(a) improved Wilson fermions. Quenched strange quarks.

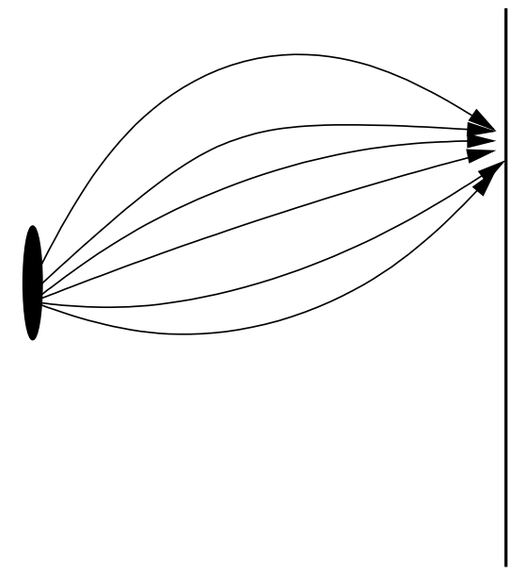
Run	a [fm]	L/a	L [fm]	m_π [MeV]	$m_\pi L$	N_{cfg}	N_{src}	N_{meas}
E1	0.066	32	2.11	1000	10.2	168	128	43 008
N1	0.049	48	2.33	858	10.1	100	128	25 600
A1	0.076	32	2.42	744	9.9	286	128	73 216
E5	0.066	32	2.11	440	4.6	1990	32	127 360

On E1, N1, A1 $m_u = m_d = m_s$, On E5 $m_u = m_d \neq m_s$

H-dibaryon interpolating operators

- Local hexaquark operators :

$$\mathcal{O}_H(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} (qqqqqq)(t,\vec{x})$$



  - Bag-model like local operator
  - Under broken SU(3), $\mathbf{1},\ \mathbf{27}-\mathrm{plet}$ are mixed.

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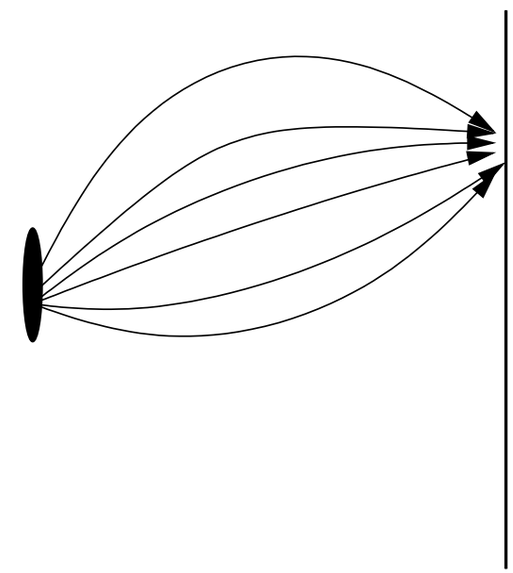
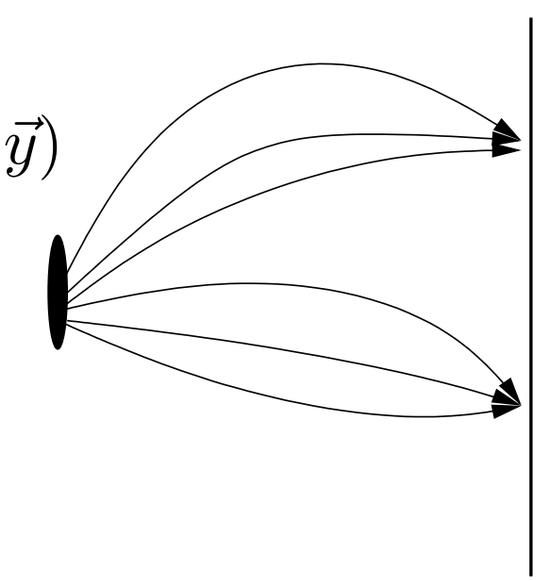


  - Bag-model like local operator
  - Under broken SU(3), $\mathbf{1},\ \mathbf{27}-\mathrm{plet}$ are mixed.

- Two-baryon operators :

$$\mathcal{O}_{BB}(t,\vec{p}) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p_1}\cdot\vec{x}} e^{-i\vec{p_2}\cdot\vec{y}}(qqq)(t,\vec{x})(qqq)(t,\vec{y})$$



  - Overlaps onto two-hadron state
  - Three SU(3) irreps contribute, $\mathbf{1},\mathbf{8},\mathbf{27}$
  - Cannot construct at source.

Operators choices at SU(3) point

- Singlet and 27-plet states decouple due to SU(3) symmetry.

- Source operator choices : *Hexaquark operators* $H_{\mathbf{1},N}, \; H_{\mathbf{1},M}$ *(N, M) smearing choices.*

- Sink operator choices :

    - Choice of smearings narrow(N) and medium (M)

    - Choice of *Hexaquark* and *Two-baryon* operators.

- Construct various 2 X 2 correlator matrices to explore the approach to ground state.

Variational method

Lüscher, Wolff **NPB B339, 1990 222-252**

B. Blossier et.al **JHEP04 (2009) 094**

Variational method provides reliable determination of ground state

Compute a correlator matrix of two point functions as,

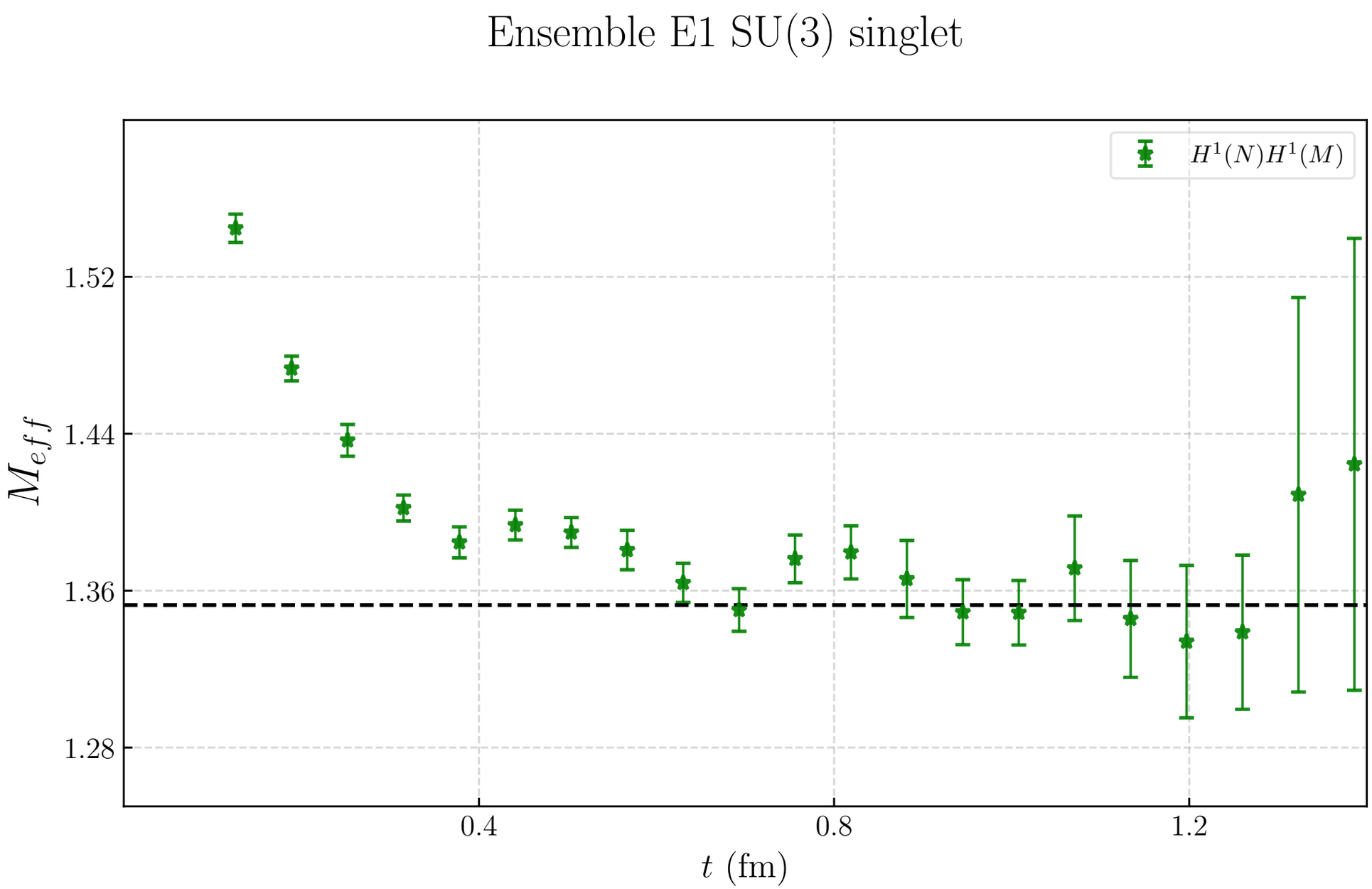
$$C_{ij} = \sum_{\vec{x}} \langle \mathcal{O}_i(t_0 + t, \vec{x}) \mathcal{O}_j^\dagger(t_0, \vec{x}_0) \rangle,$$

and solve the generalised eigenvalue problem as,

$$C_{ij}(t + \Delta t) v_j(t) = \lambda(t) C_{ij}(t) v_j(t) \qquad m_{\text{eff}} = \frac{-\log \lambda(t)}{\Delta t}$$

Cannot construct a hermitian correlator matrix with two-baryon operators.

Results - SU(3) singlet, Ensemble E1



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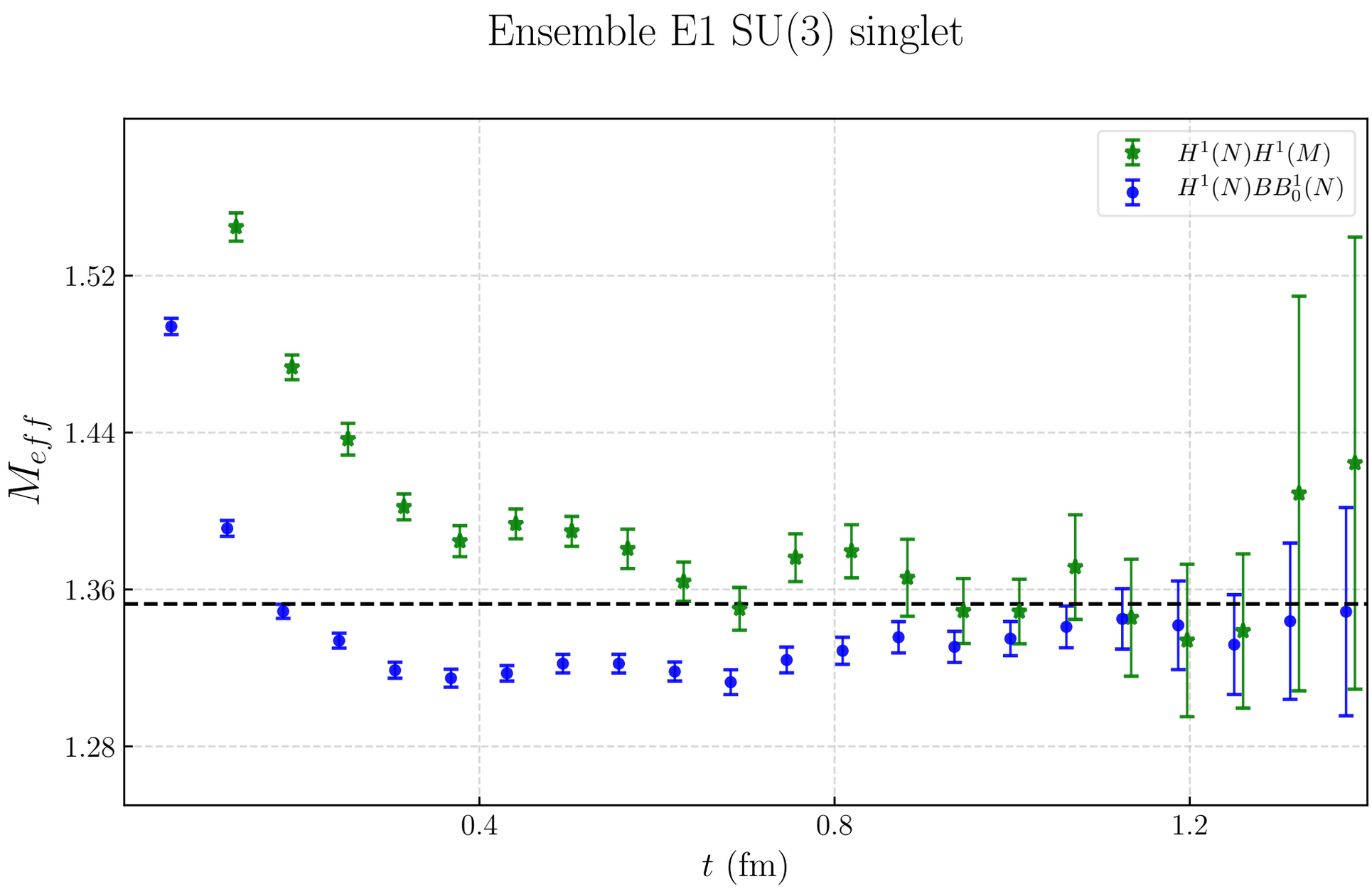

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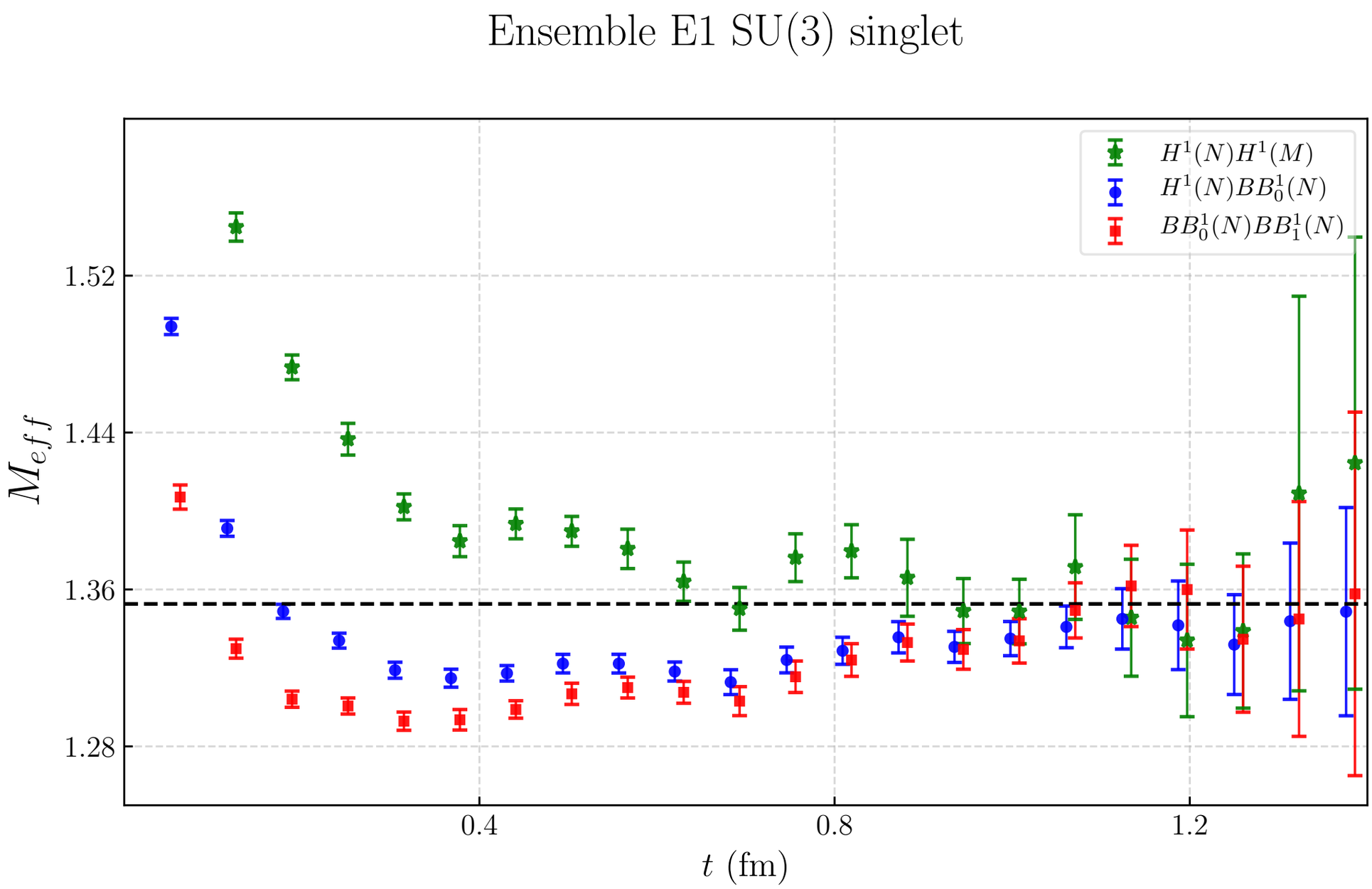

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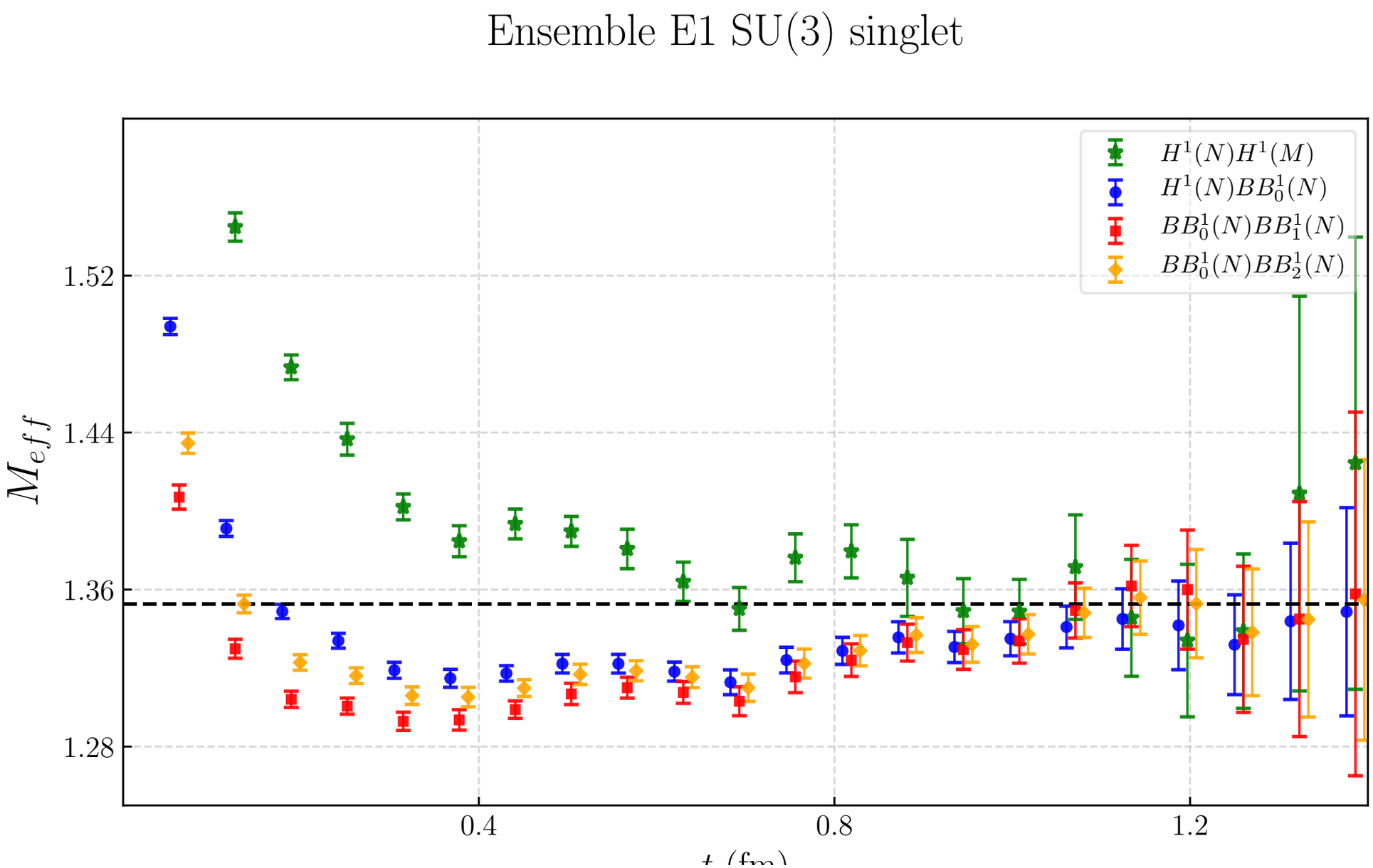

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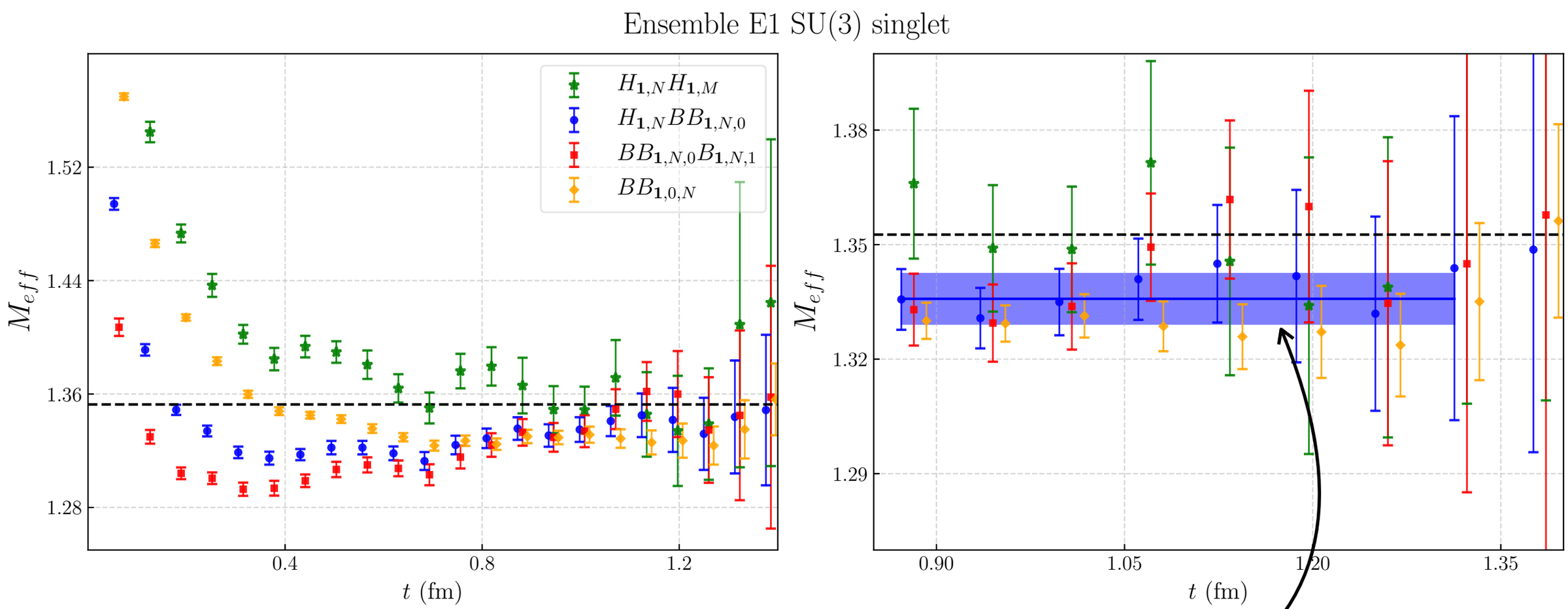
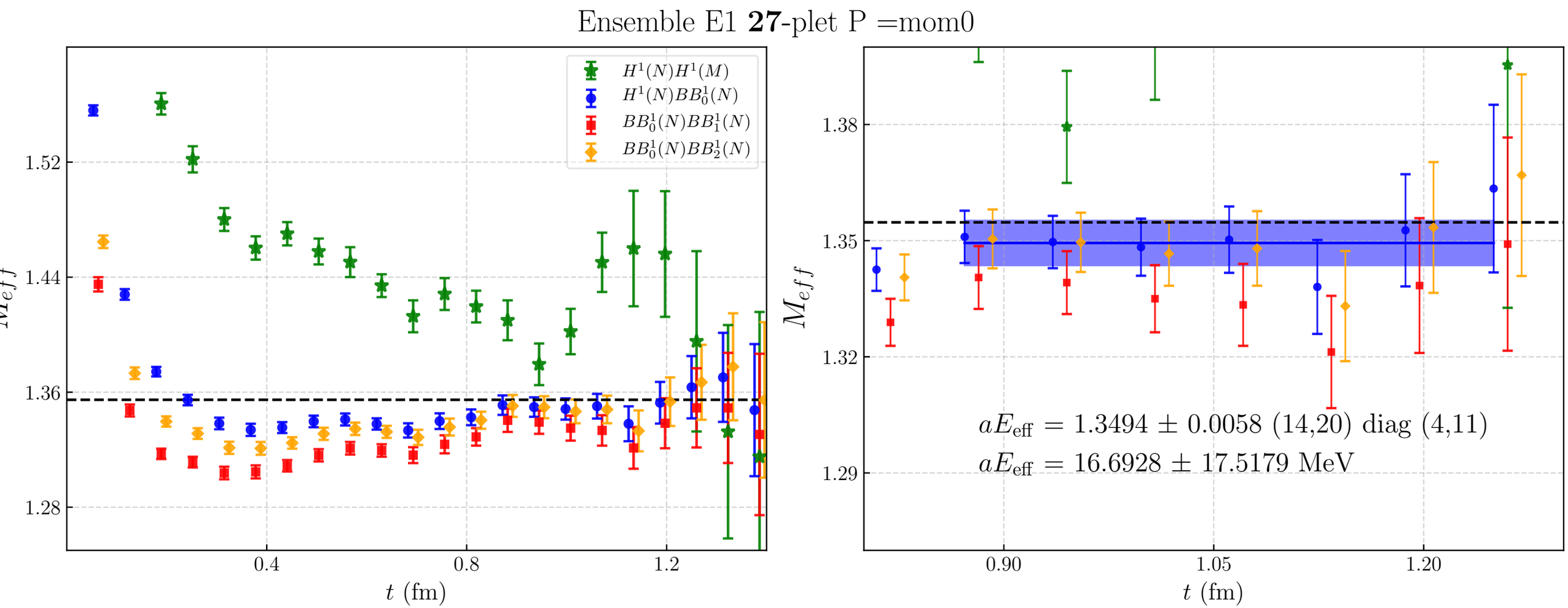


Choose $[H_{\mathbf{1},N}BB_{\mathbf{1},N,0}]$ GEVP results for fitting.

Results - SU(3) 27-plet, Ensemble E1

27-plet corresponds to NN scattering in the SU(3) limit.



Operators choices with broken SU(3)

- Singlet and 27-plet states are mixed due to SU(3) breaking.

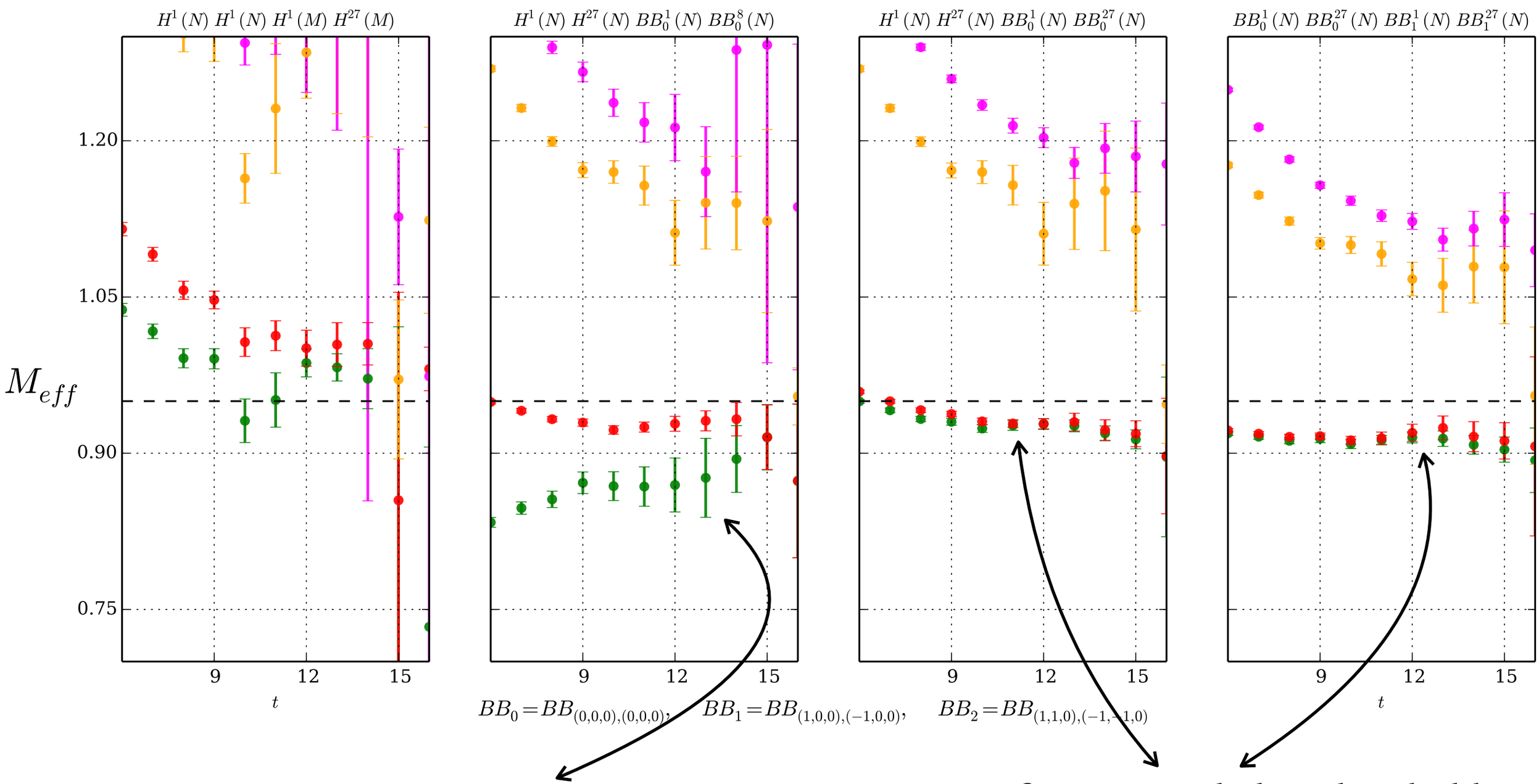
- Source operator choices : *Hexaquark operators*

$$H_{\mathbf{1},N}, \; H_{\mathbf{1},M} \; H_{\mathbf{27},N}, \; H_{\mathbf{27},M}$$

- Sink operator choices :
  - Choice of *Hexaquark* and *Two-baryon* operators.
  - *No octet operator available at the source.*

- Construct various 4 X 4 correlator matrices to explore the approach to ground state.

Results with broken SU(3)



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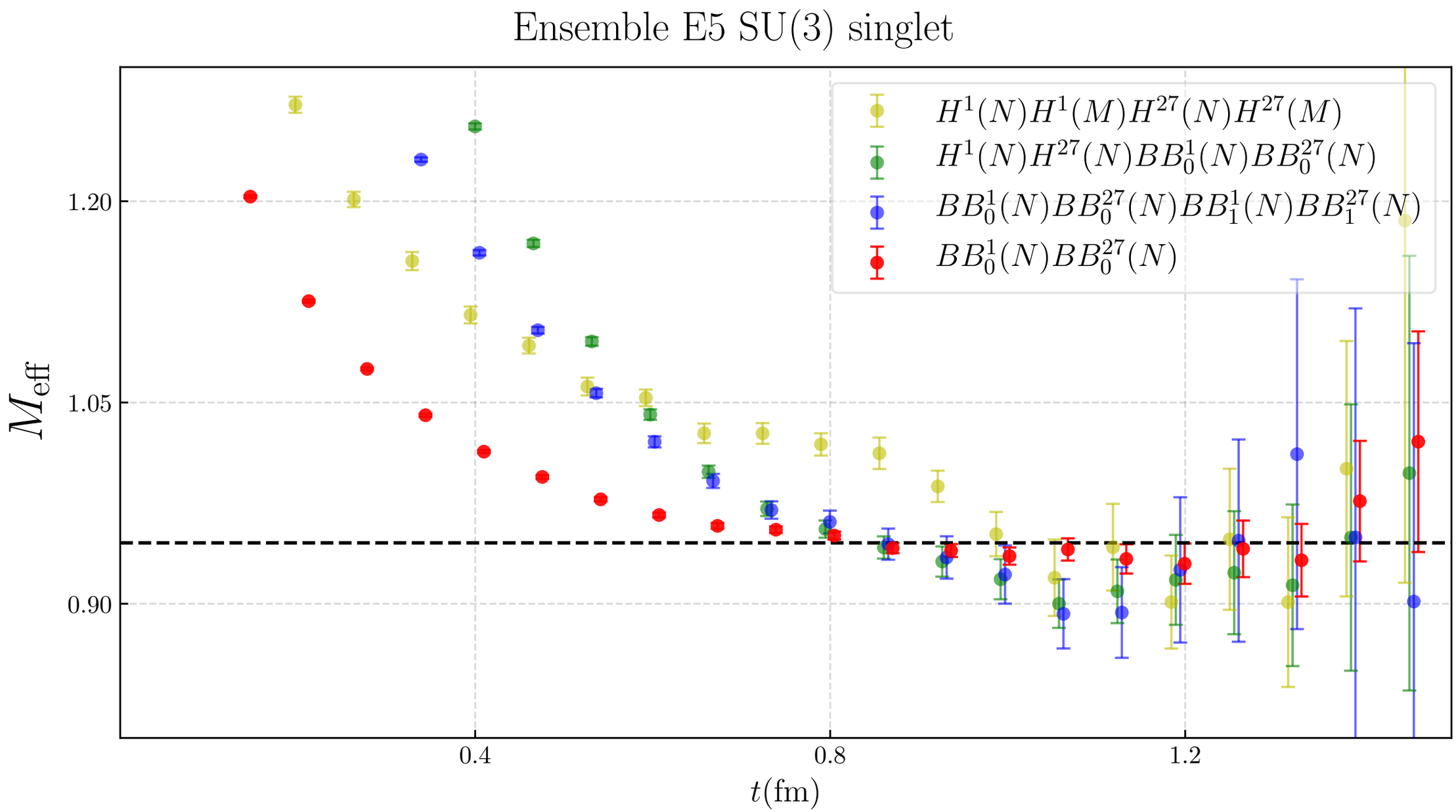

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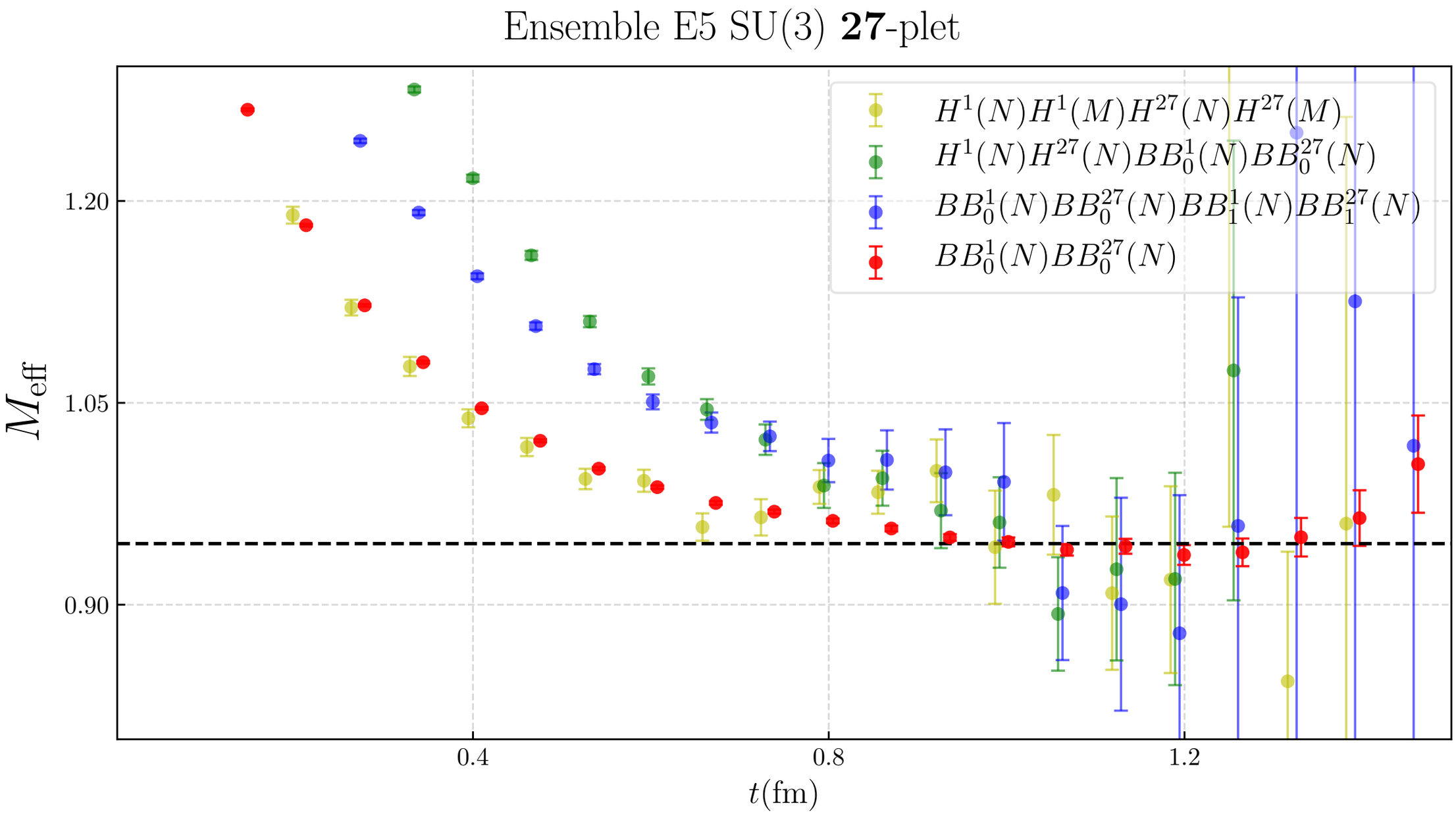


Overview with point sources

- ✤ Hexaquark operators provide a poor overlap onto the ground state
- ✤ Two-baryon operators provide an improved overlap at the cost of analysing non-hermitian correlator matrices.
- ✤ Non-hermitian correlator matrices may provide different approaches to the ground state. (agreeing asymptotically)
- ✤ Octet contributions cannot be accounted for.
- ✤ A possible reliable method : ***Distillation.***

Distillation

M. Peardon et.al, **Phys Rev D80 (2009) 054506**

J. Bulava, et.al, **Phys Rev D83 (2011) 114505**

Distillation is a smearing scheme enabling timeslice-to-all propagator computation.

Compute N low eigenvectors of spatial lattice laplacian,

$$\Delta(t)\ V(t) = \lambda(t)\ V(t)$$

Employ a smeared diluted source and invert the Dirac matrix on it,

$$\tilde{\rho}(t_0) = V(t_0)P(t_0)\rho(t_0)$$

Dilution projector

$$\tau(t) = V^\dagger(t)D^{-1}(t', t_0)\tilde{\rho}(t_0)$$

Perambulator

A stochastic estimate of the Quark propagator is then given by,

$$\phi(t) = V(t)\tau(t)$$

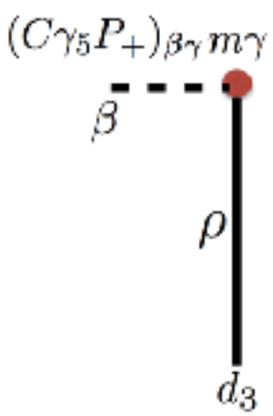
Quark sink field

$$D^{-1}(t', t_0) \sim E\Big(\phi(t)\ \tilde{\rho}^\dagger(t_0)\Big)$$

Noise expectation value

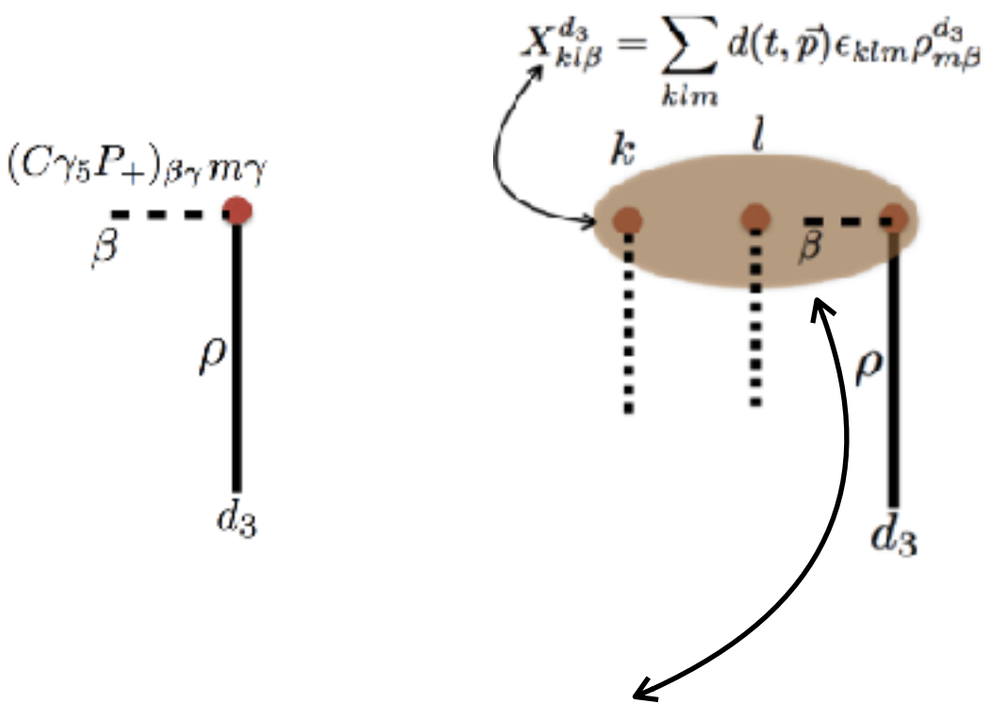
Baryons in Distillation

Compute color singlets in distillation space. Use 1 overall source vector.



Baryons in Distillation

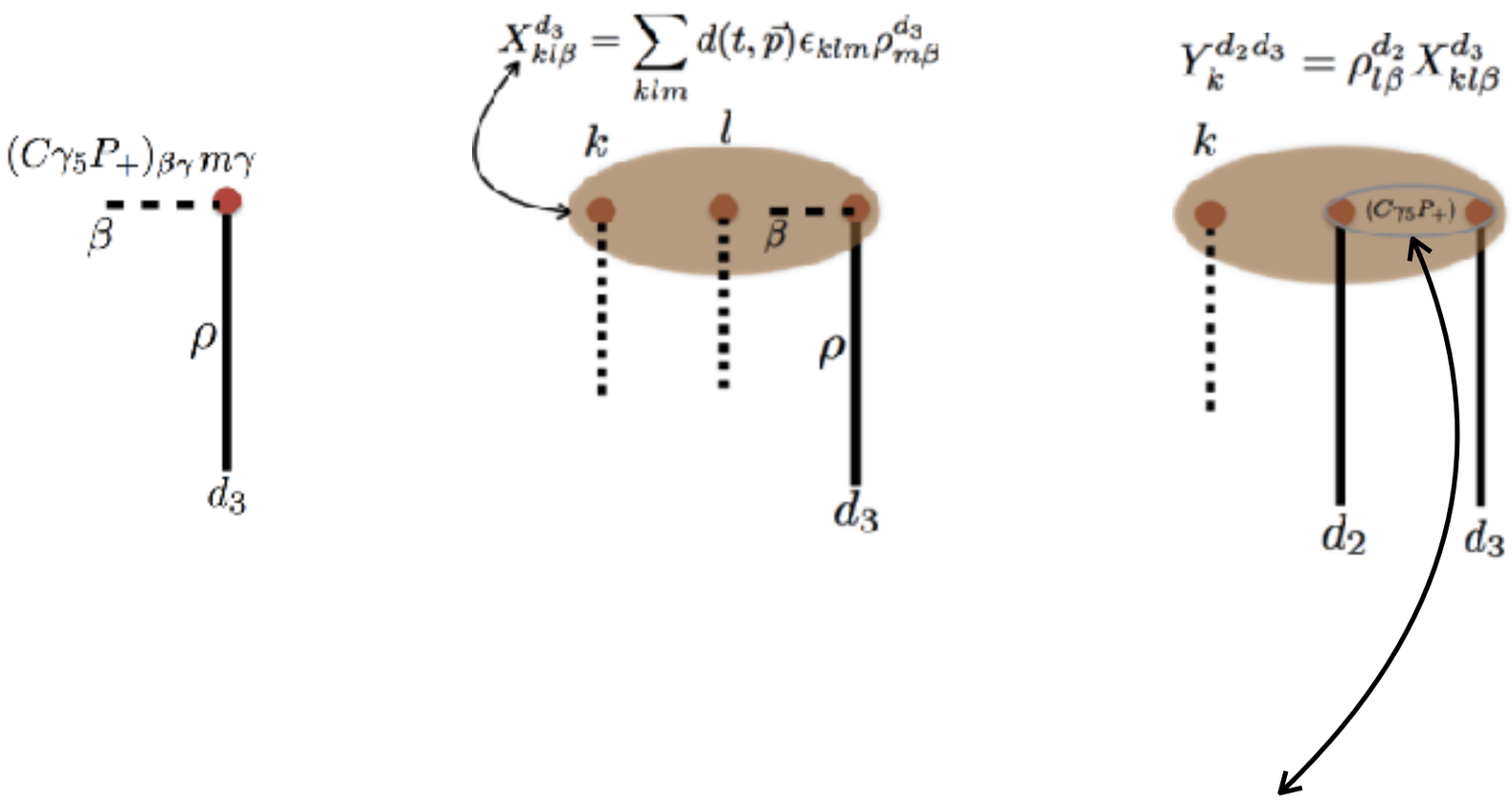
Compute color singlets in distillation space. Use 1 overall source vector.



Use colour triplets in distillation space

Baryons in Distillation

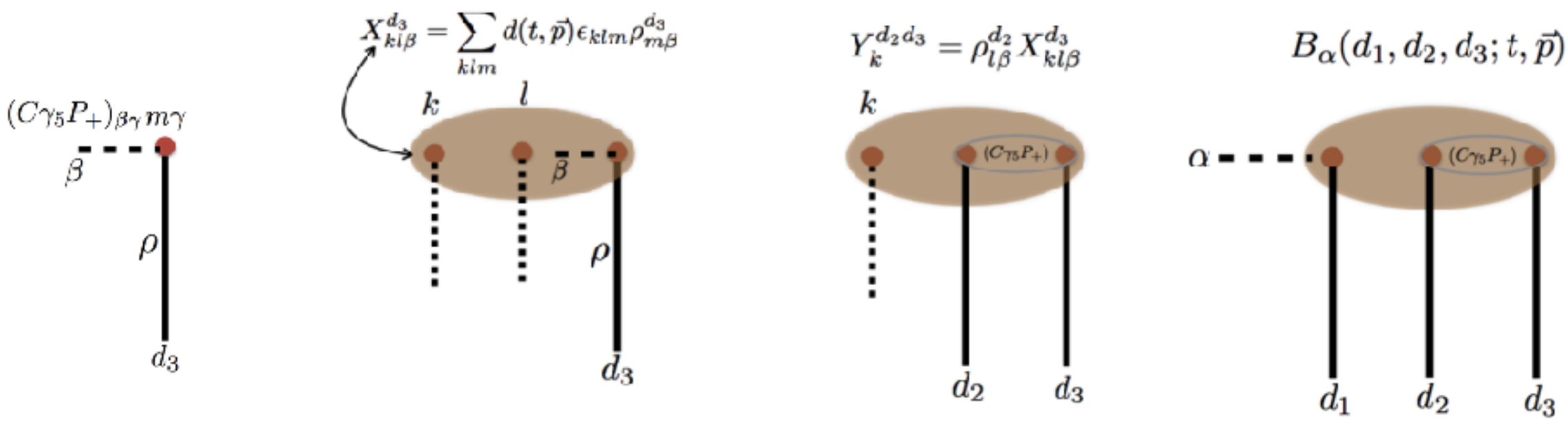
Compute color singlets in distillation space. Use 1 overall source vector.



Construct diquark with second line.

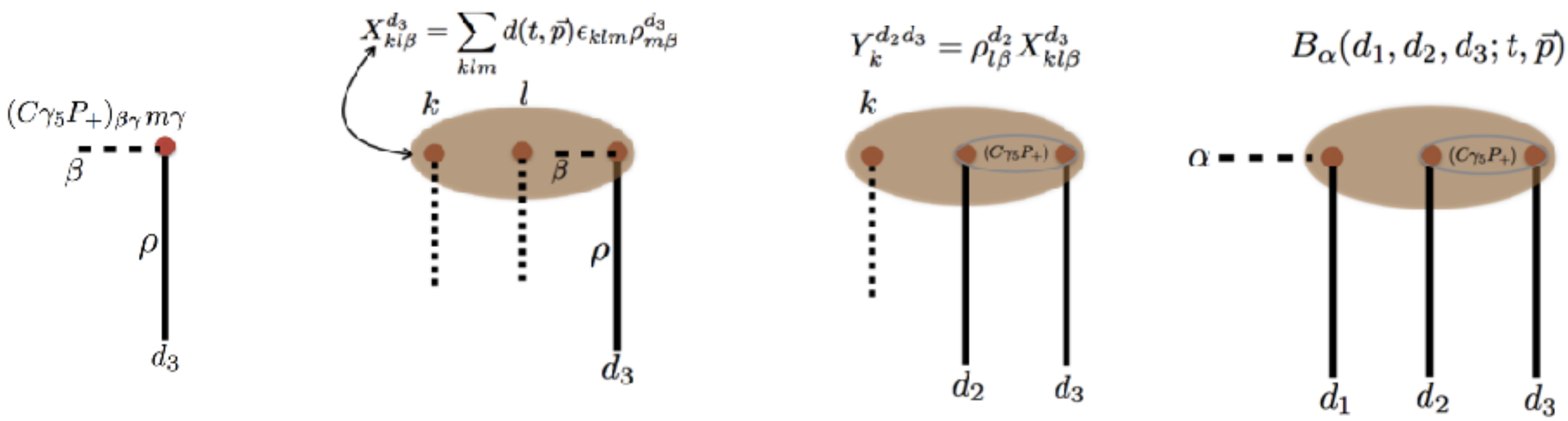
Baryons in Distillation

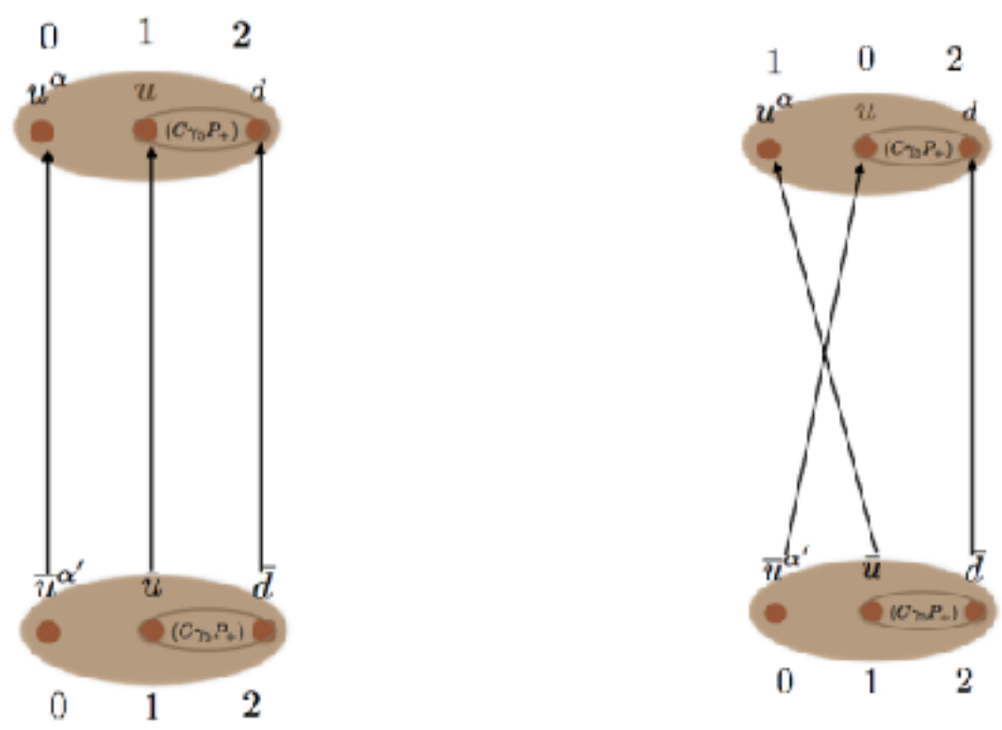
Compute color singlets in distillation space. Use 1 overall source vector.



Baryons in Distillation

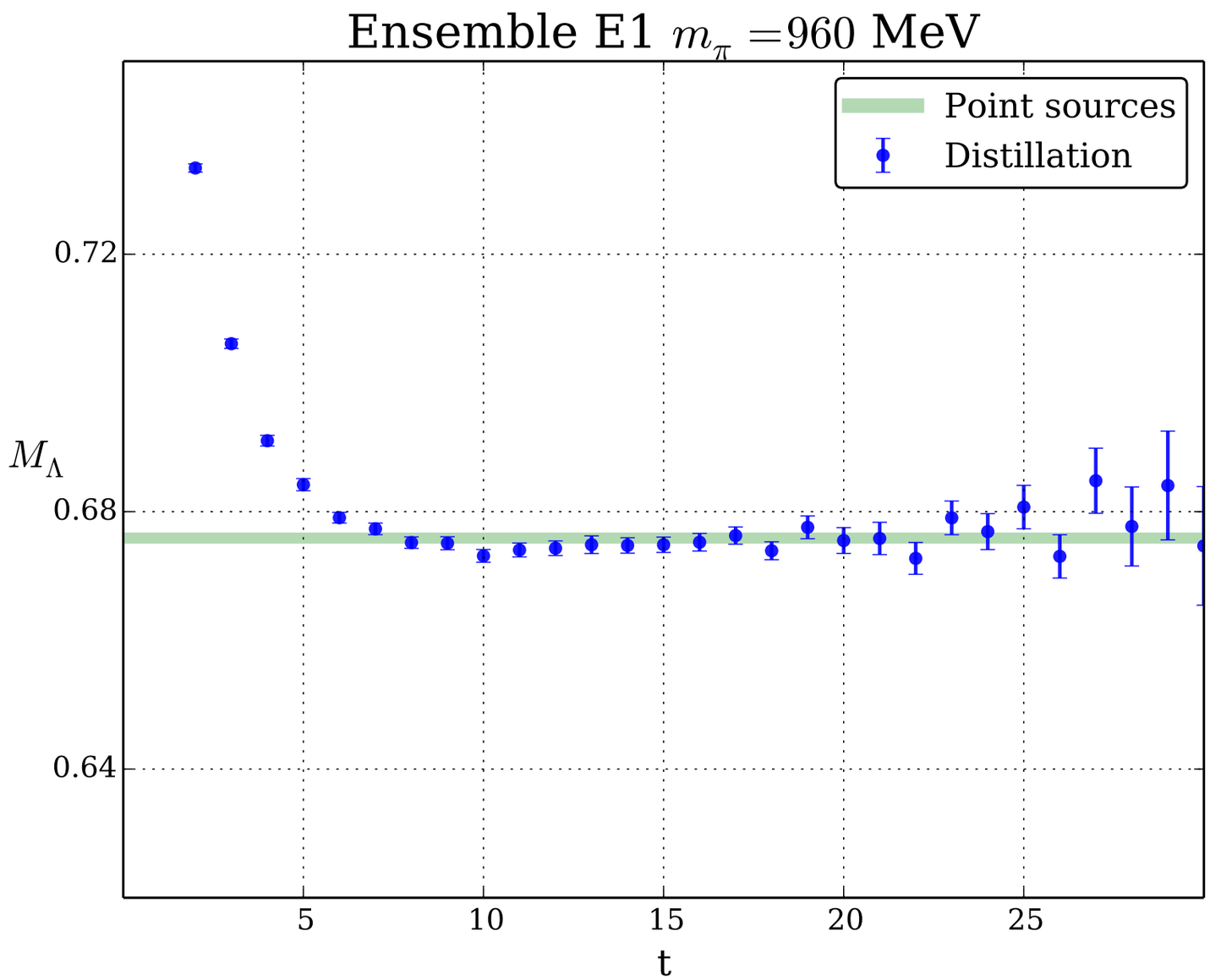
Compute color singlets in distillation space. Use 1 overall source vector.





Results on single Lambda

Distillation results compatible with point sources for single baryons

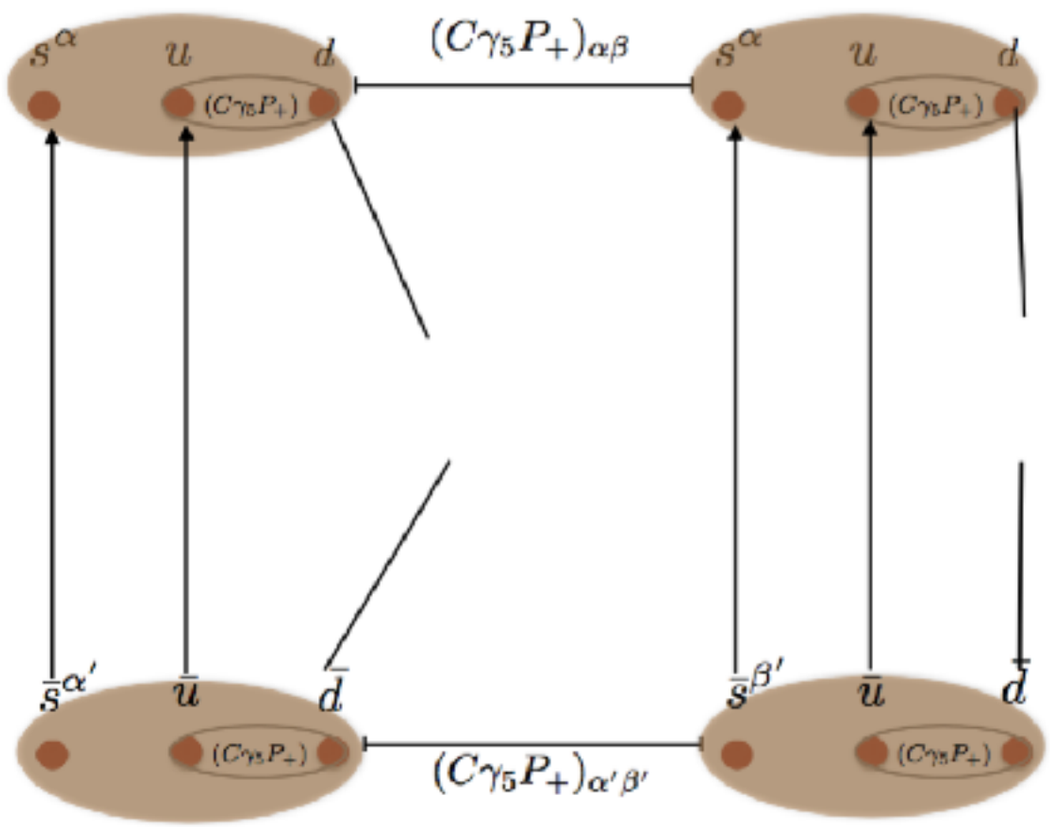


Dibaryon contractions

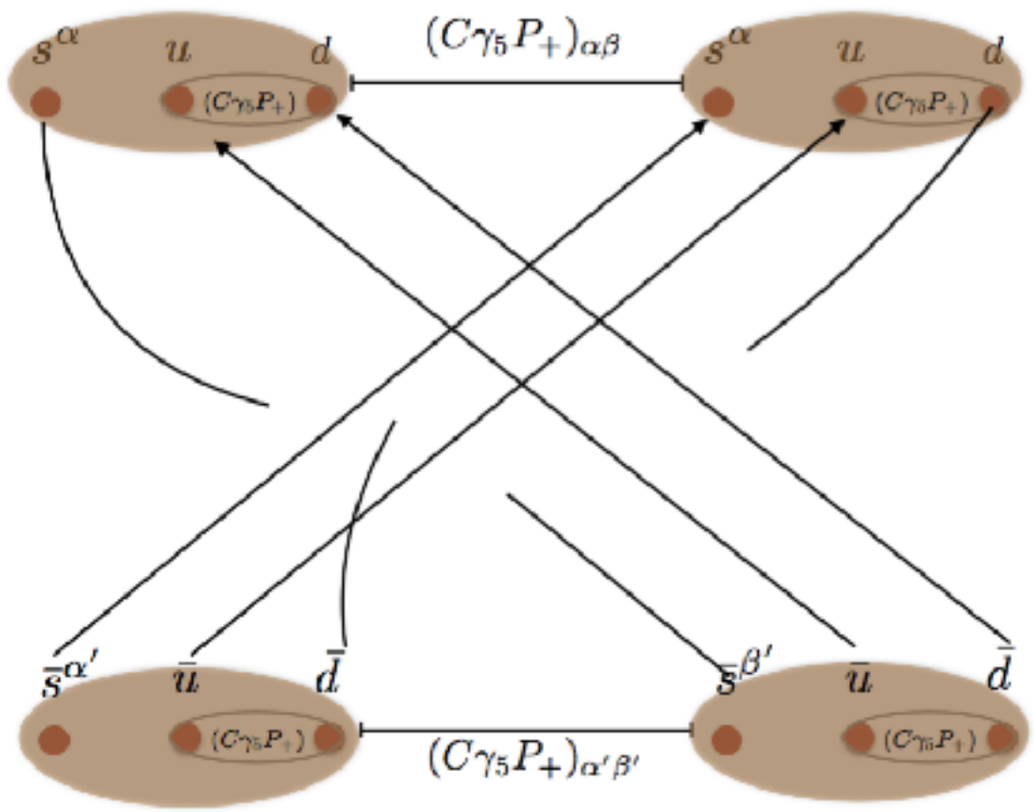
Use baryons objects to construct dibaryons.

Construct N^2 baryon blocks and contract them.

Two classes of contractions :

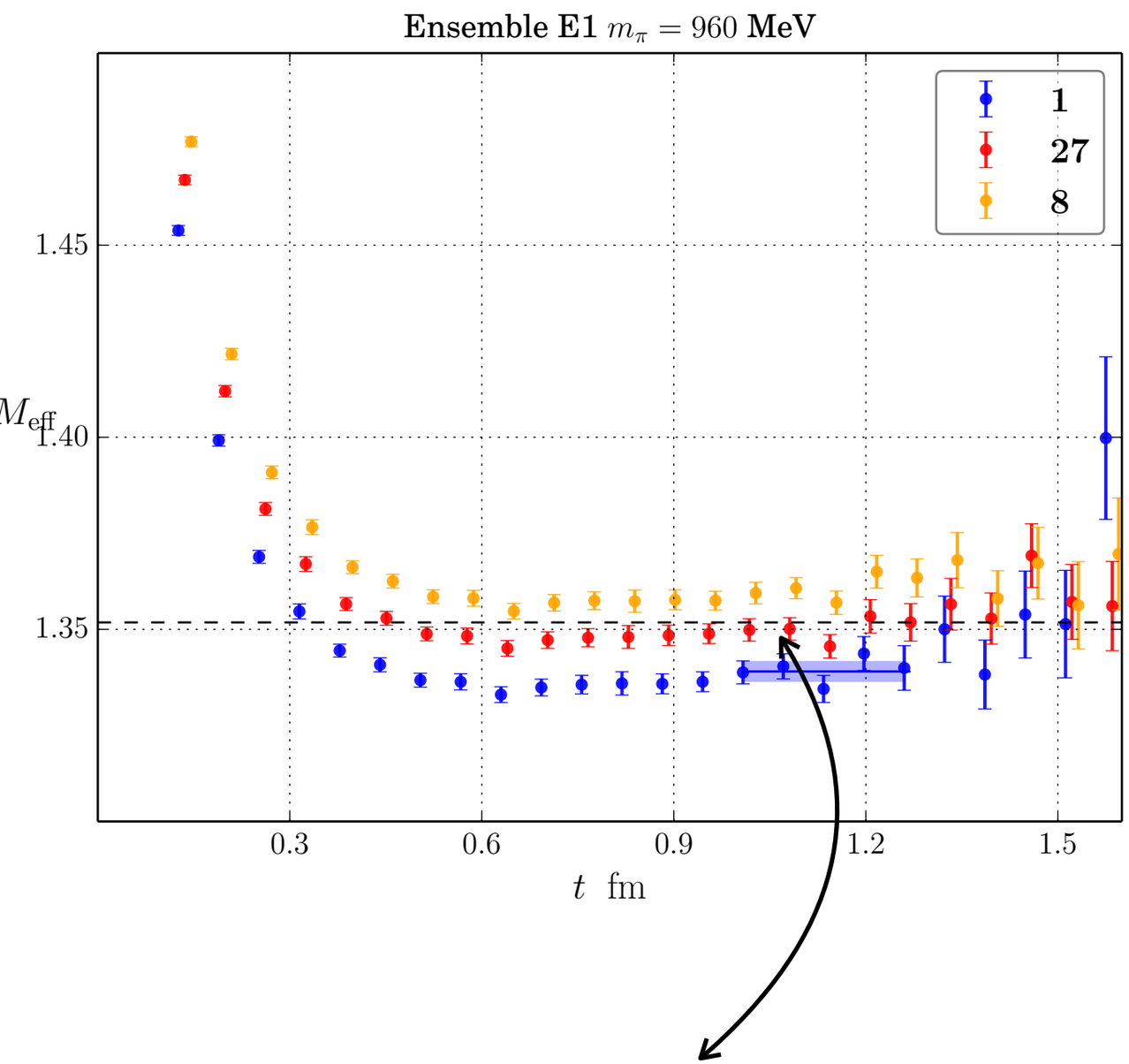


Straight type



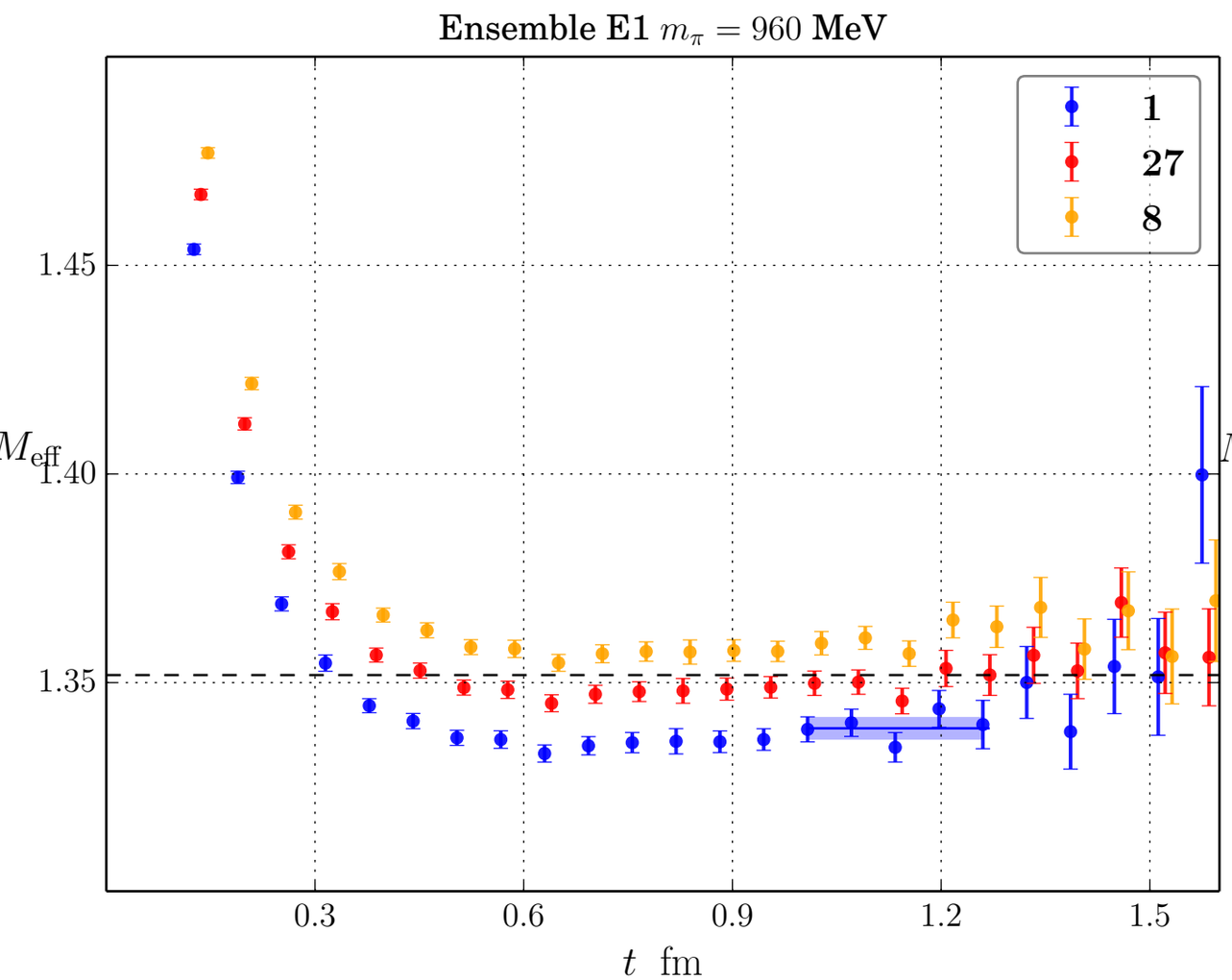
Cross type

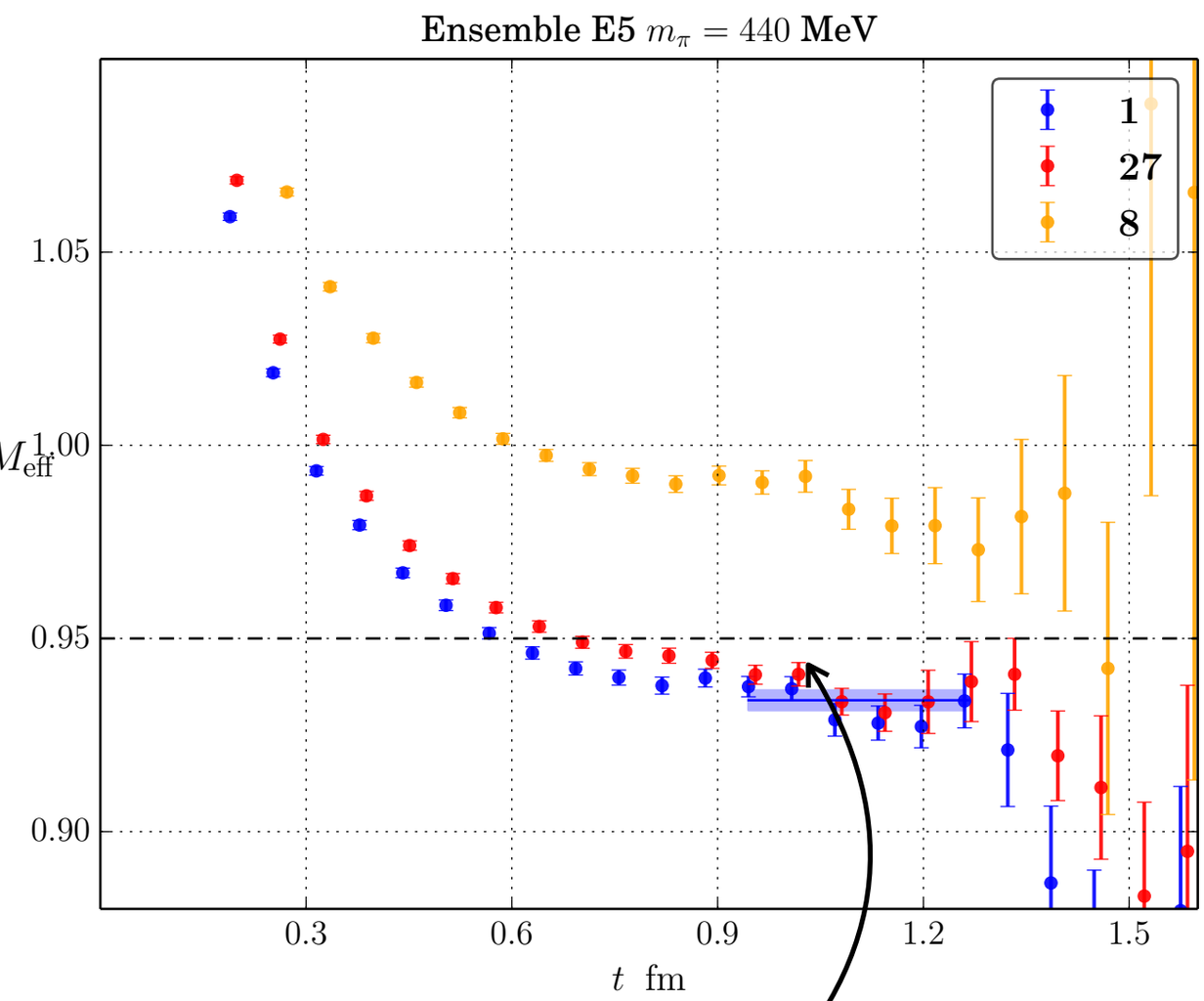
Dibaryon results



Three clearly separated levels.

Dibaryon results

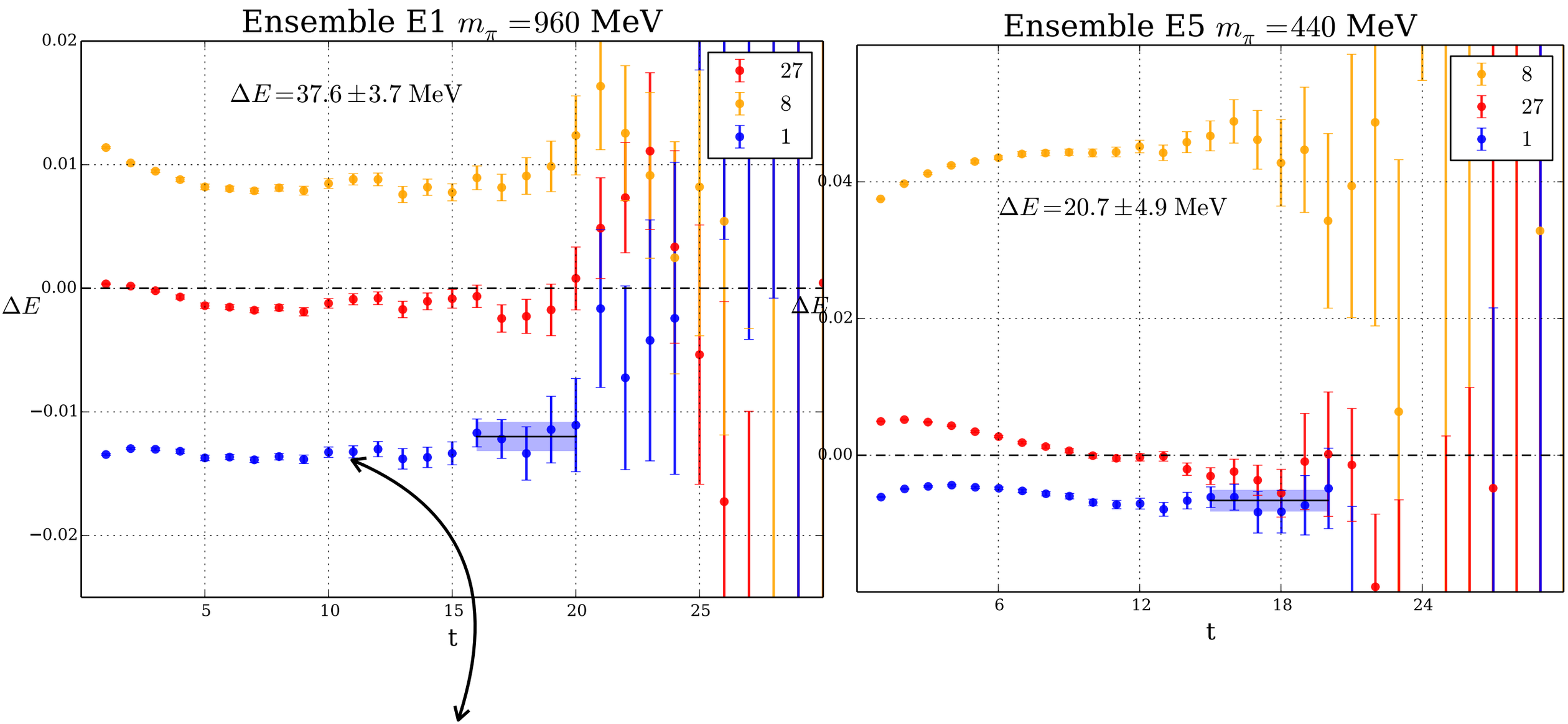




Dominant SU(3) breaking effects.

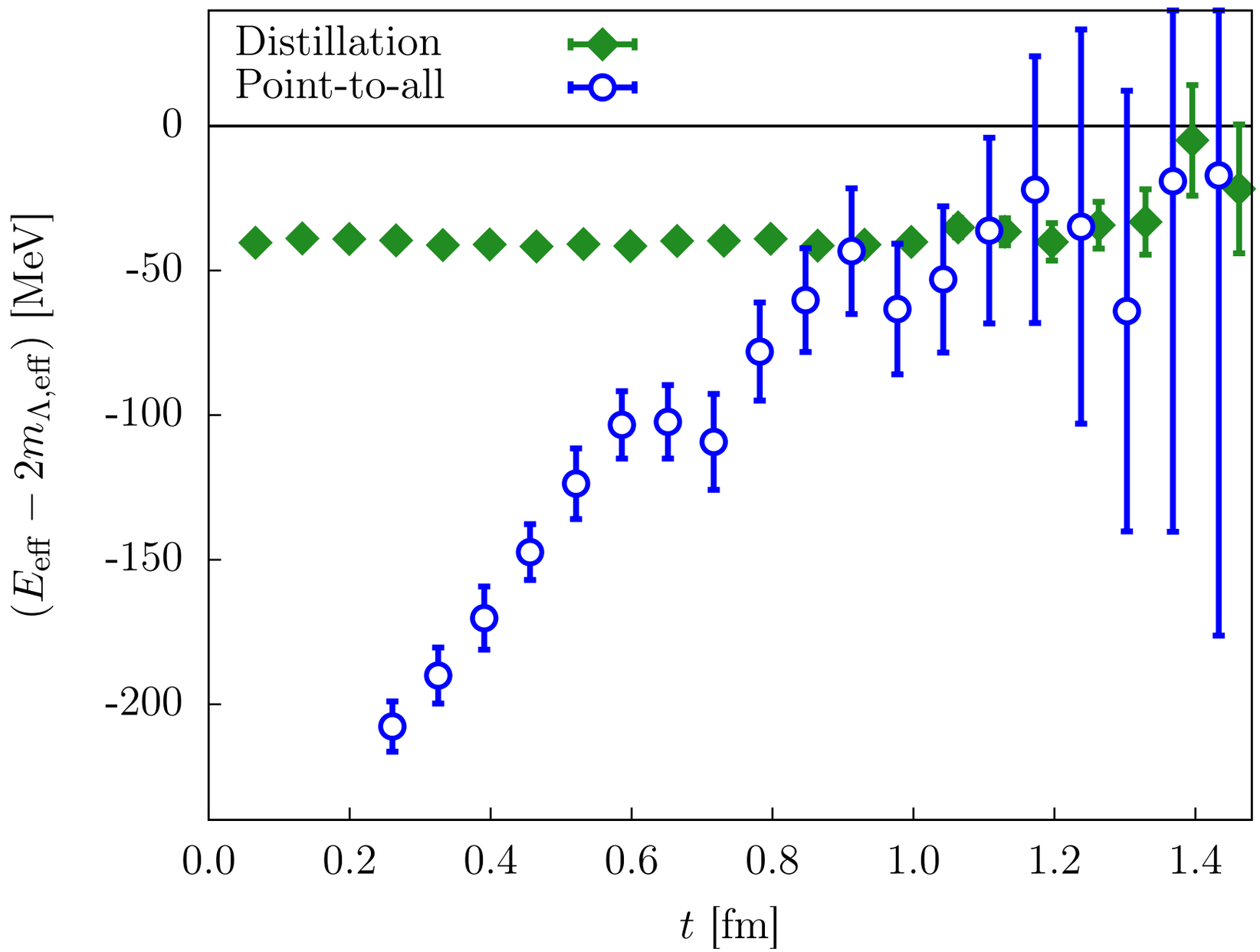
Dibaryon results

Effective binding energies



Cancellation of dibaryon excited states from single baryons.

Comparison with point sources



Scattering phase shift from energy levels

The two-particle scattering/binding momenta is given by:

$$p^2 = \frac{1}{4}\left(E^2 - \vec{P}\cdot\vec{P}\right) - M_\Lambda^2$$

is related to scattering phases in the continuum via,

$$p\ \cot\delta_0(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}^d_{0,0}(1,q^2) \quad q = \frac{pL}{2\pi}$$ [Lüscher (1991)]

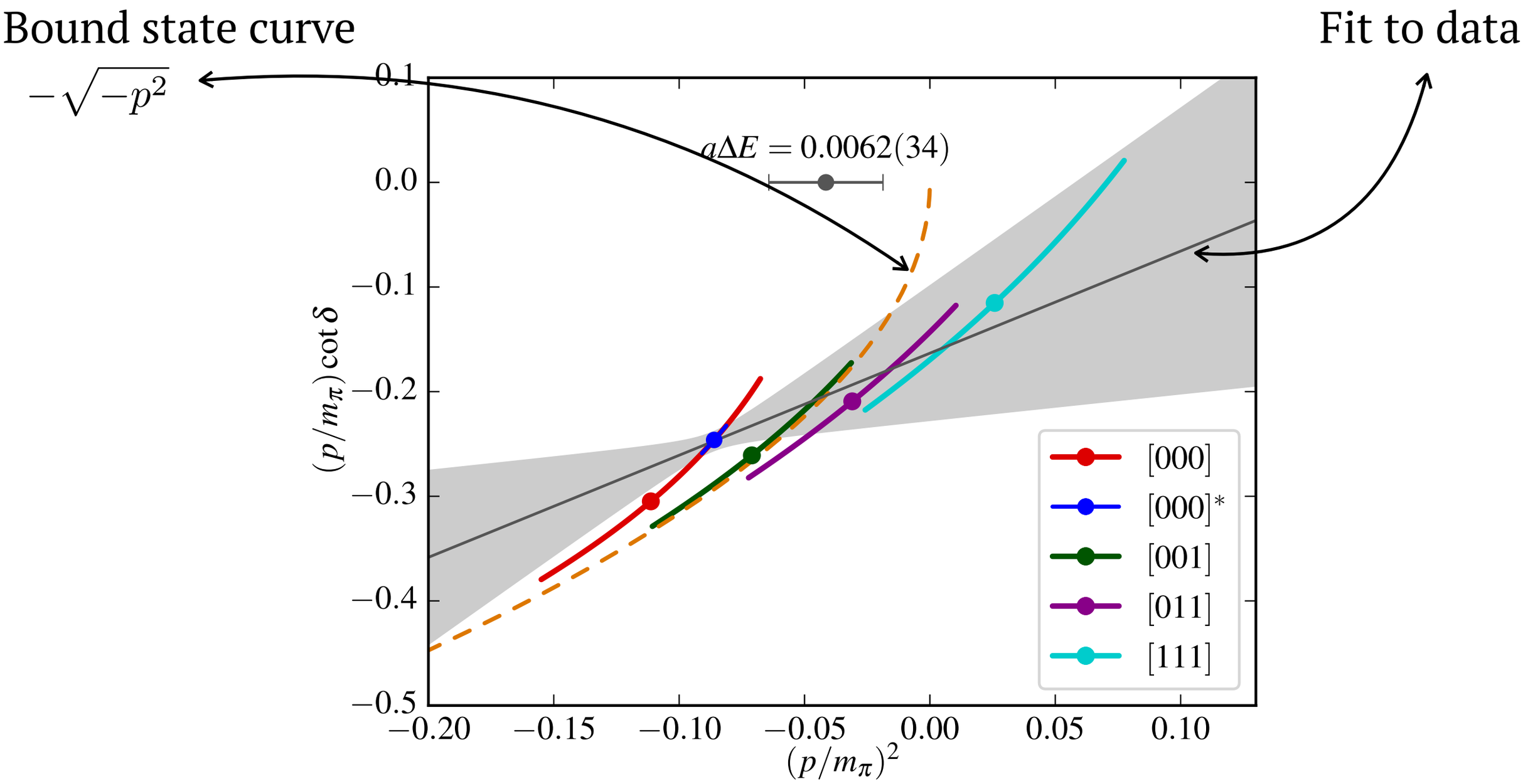
Use scattering information to locate the pole in the scattering amplitude,

$$\mathcal{A} \propto \frac{1}{p\ \cot\delta_0(p) - ip} \qquad p\ \cot\delta_0(p) = -\frac{1}{a} + \frac{1}{2}r_0 p^2 + \ldots$$

Constraints: slope of $p\ \cot\delta_0(p)$ $<$ slope of $-\sqrt{-p^2}|_{p=p*}$

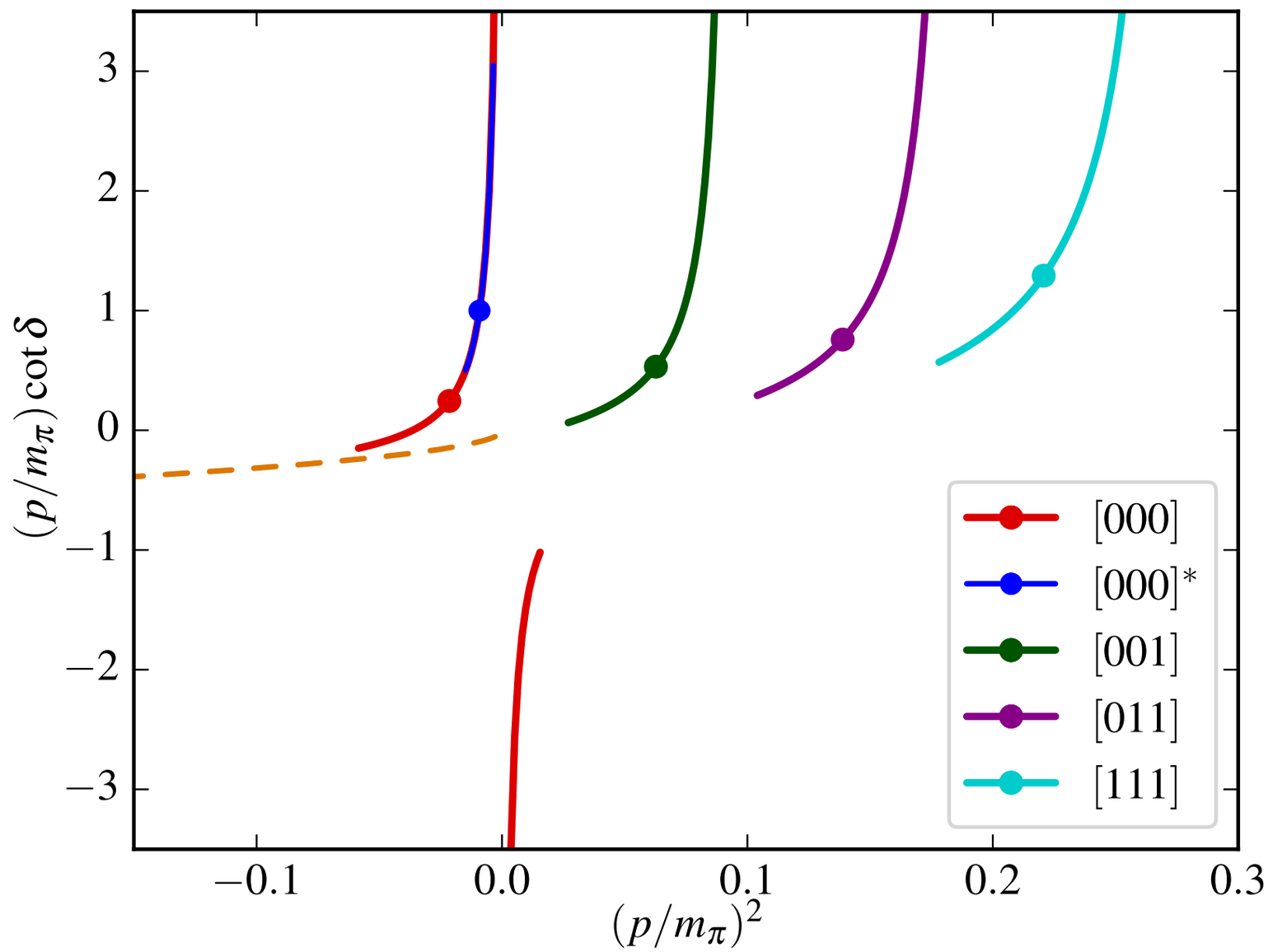
[HALQCD (1703.07210)]

Phase shift on E1 - SU(3) singlet

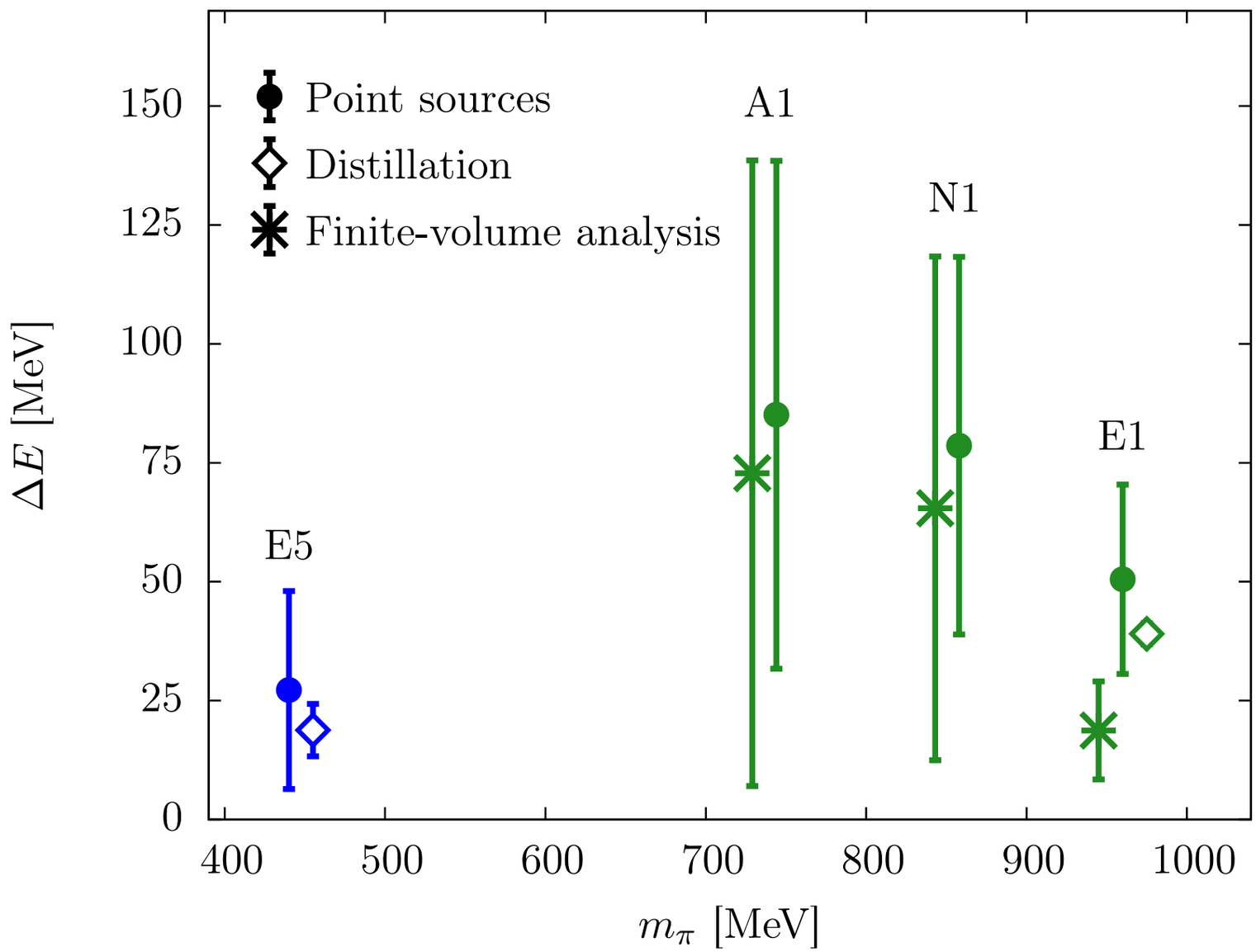


$$a = 1.3(5)\ \text{fm},\ \ r_0 = 0.4(3)\ \text{fm}$$

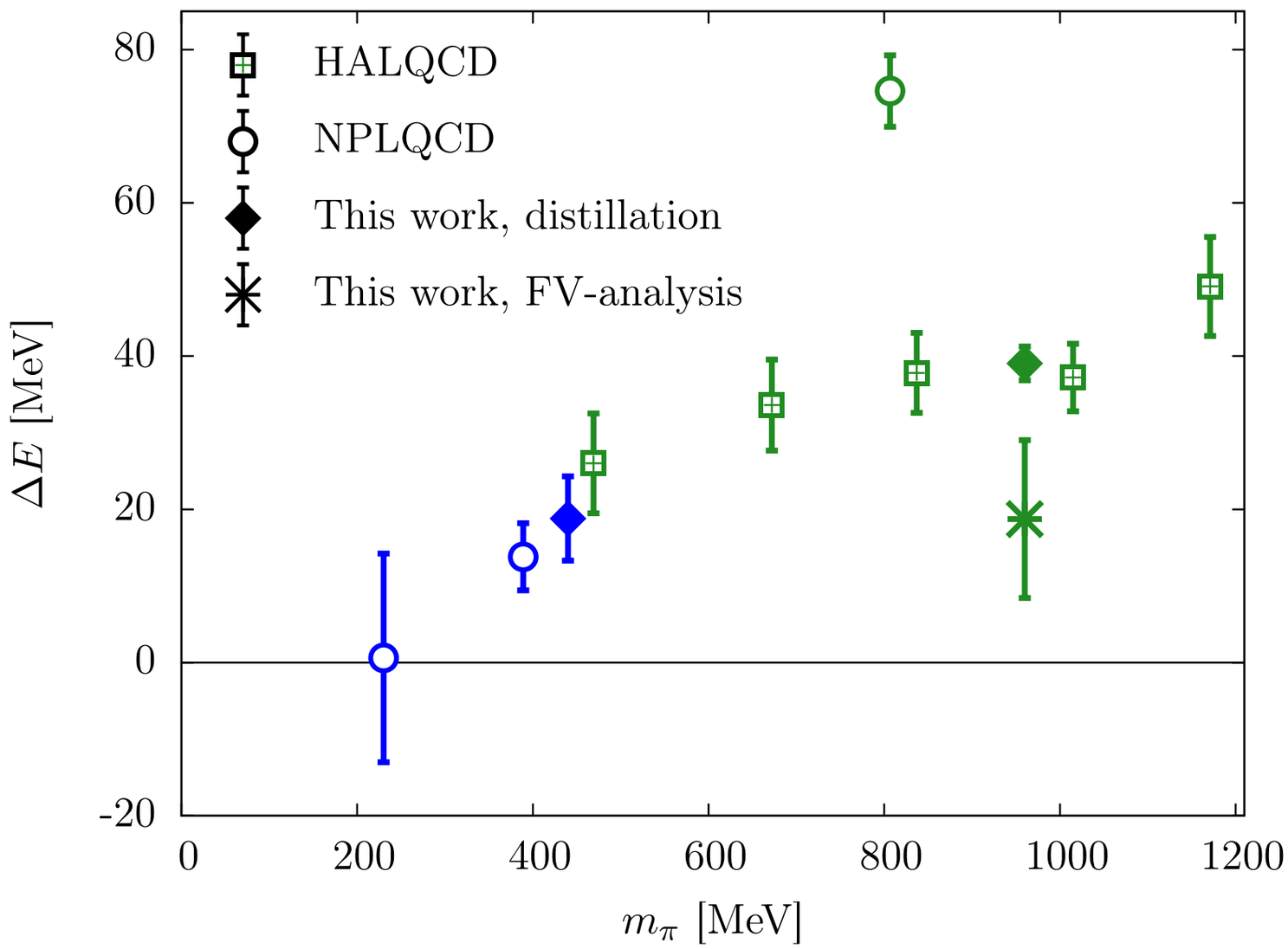
Phase shift on E1 - 27-plet



Summary of results



Comparison of different calculations



Conclusions

- Hexaquark operators are not relevant for *H* dibaryon spectroscopy.
- Two-baryon operators while providing an improved overlap introduce non-hermitian correlator matrices.
- Distillation provides a precise determination of energy levels at comparable cost with hermitian correlator matrices.
- Finite volume analysis on E1 indicates non-trivial corrections to naive binding energies.
- Currently studying extensions to NN.