Structure of hadron resonances with a nearby zero of the amplitude

18/6/15 @YITP

Recent Developments in Quark-Hadron Sciences

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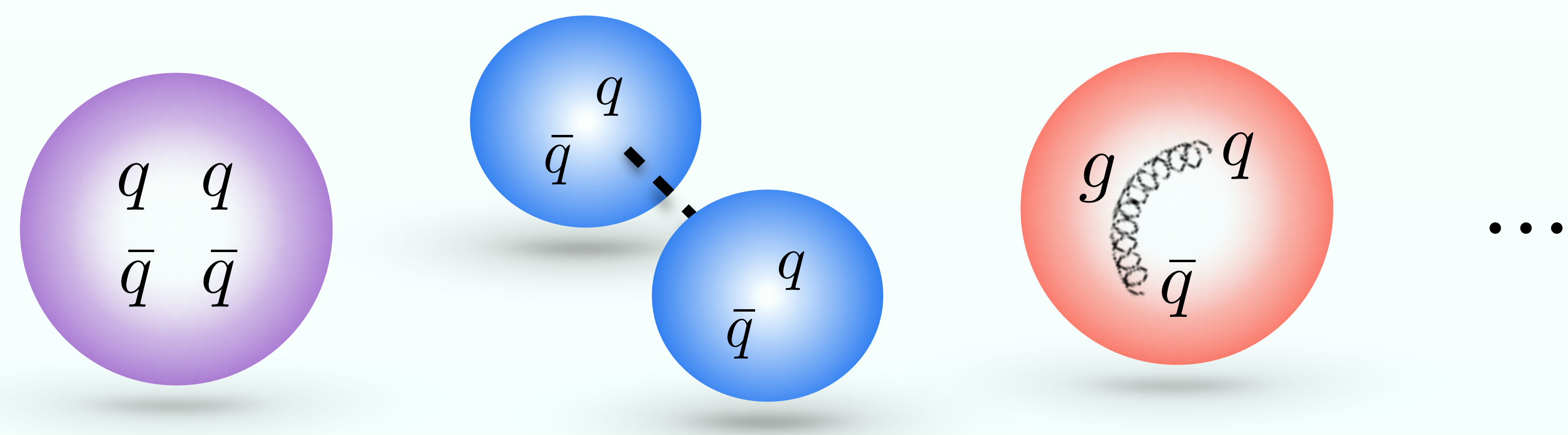
Introduction ~exotic hadrons~

- Exotic hadrons

  Hadrons which do not agree with the predictions of the quark model (qqbar, qqq).

  More complicated internal structure can be expected.

  ⇒
  - tetra quark, penta quark
  - hadron molecule ⋯

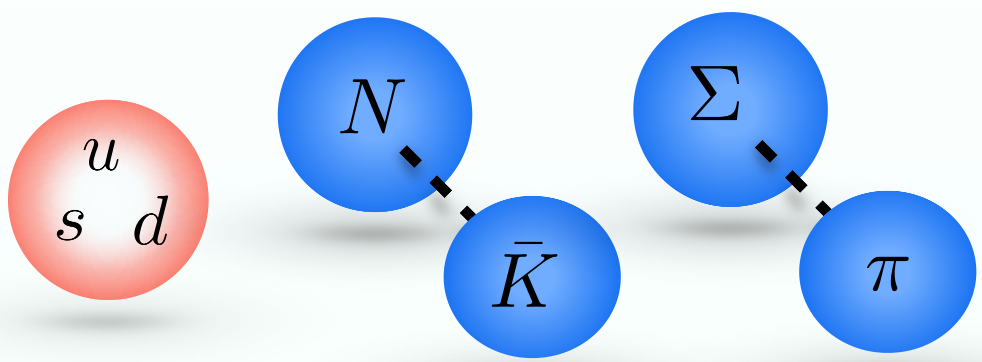


It is important to reveal the internal structure of exotics because we can acquire knowledge of strong interaction in the hadrons!

Introduction ~exotic hadrons~

$\Lambda(1405)$

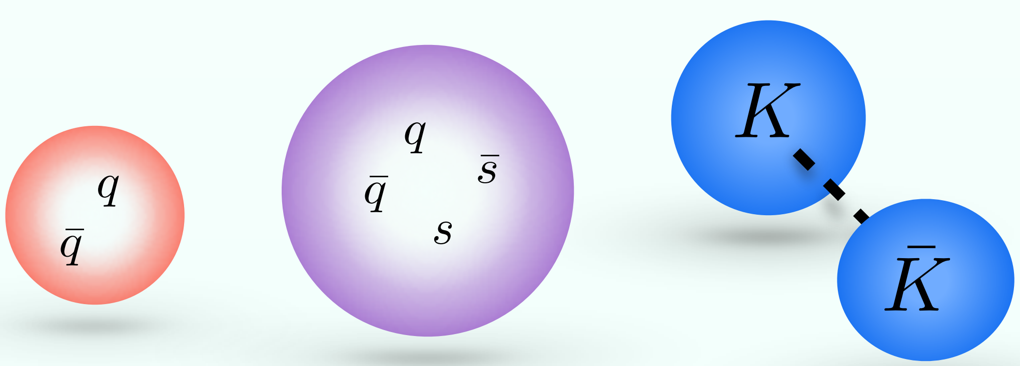
- $J^P = 1/2^-, I = 0$
- lying near $\bar{K}N$ threshold energy



or ⋯

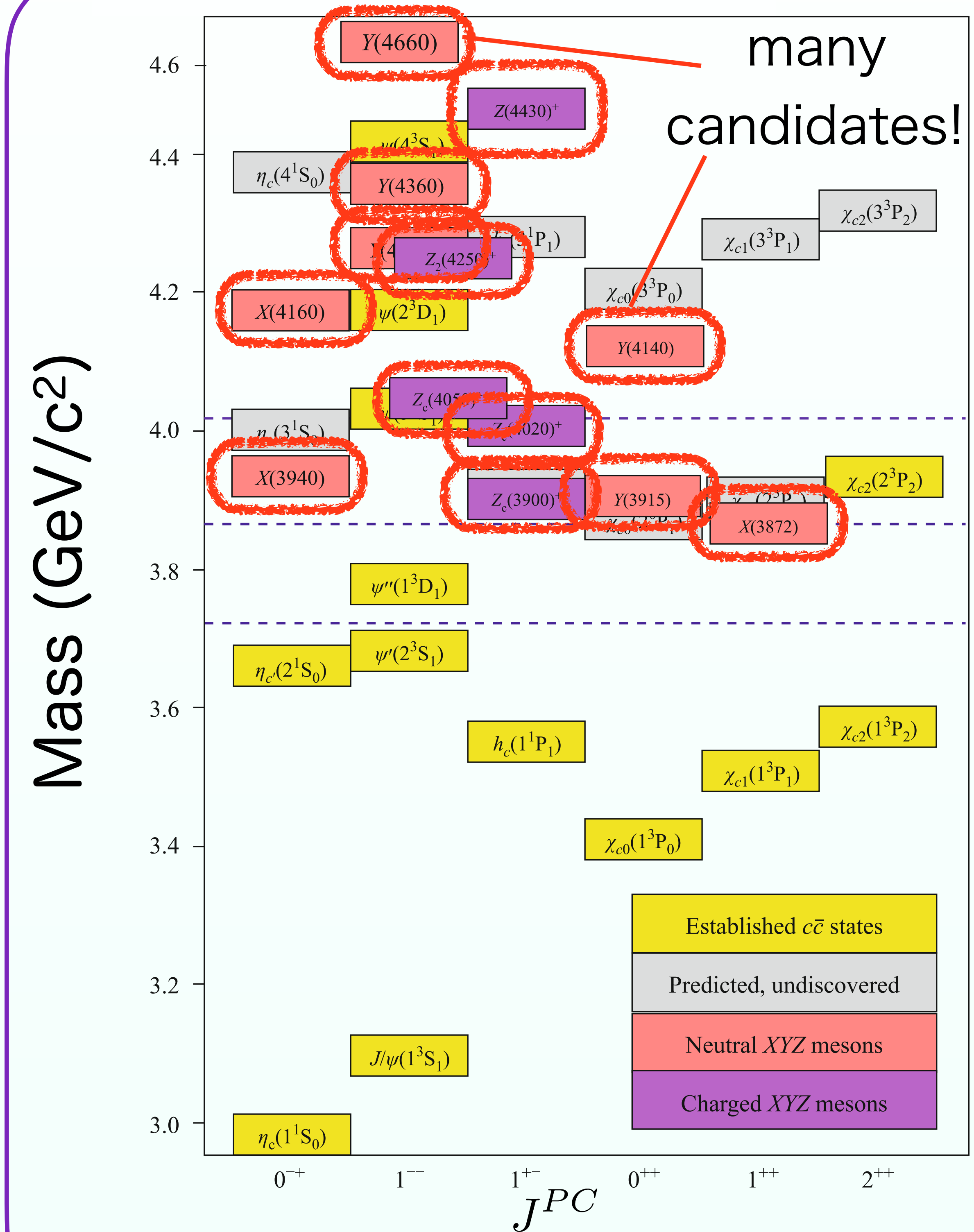
$a_0(980)$, $f_0(980)$

- $J^P = 0^+$ $\quad I = \begin{cases} 1 & \text{for } a_0(980) \\ 0 & \text{for } f_0(980) \end{cases}$
- lying near $\bar{K}K$ threshold energy



or ⋯

$c\bar{c}$ quarkonium like state



S. L. Olsen, Front. Phys. China. 10, 121 (2015)

Introduction ~Methods to study structure~

- How to extract the structure from the experimental data

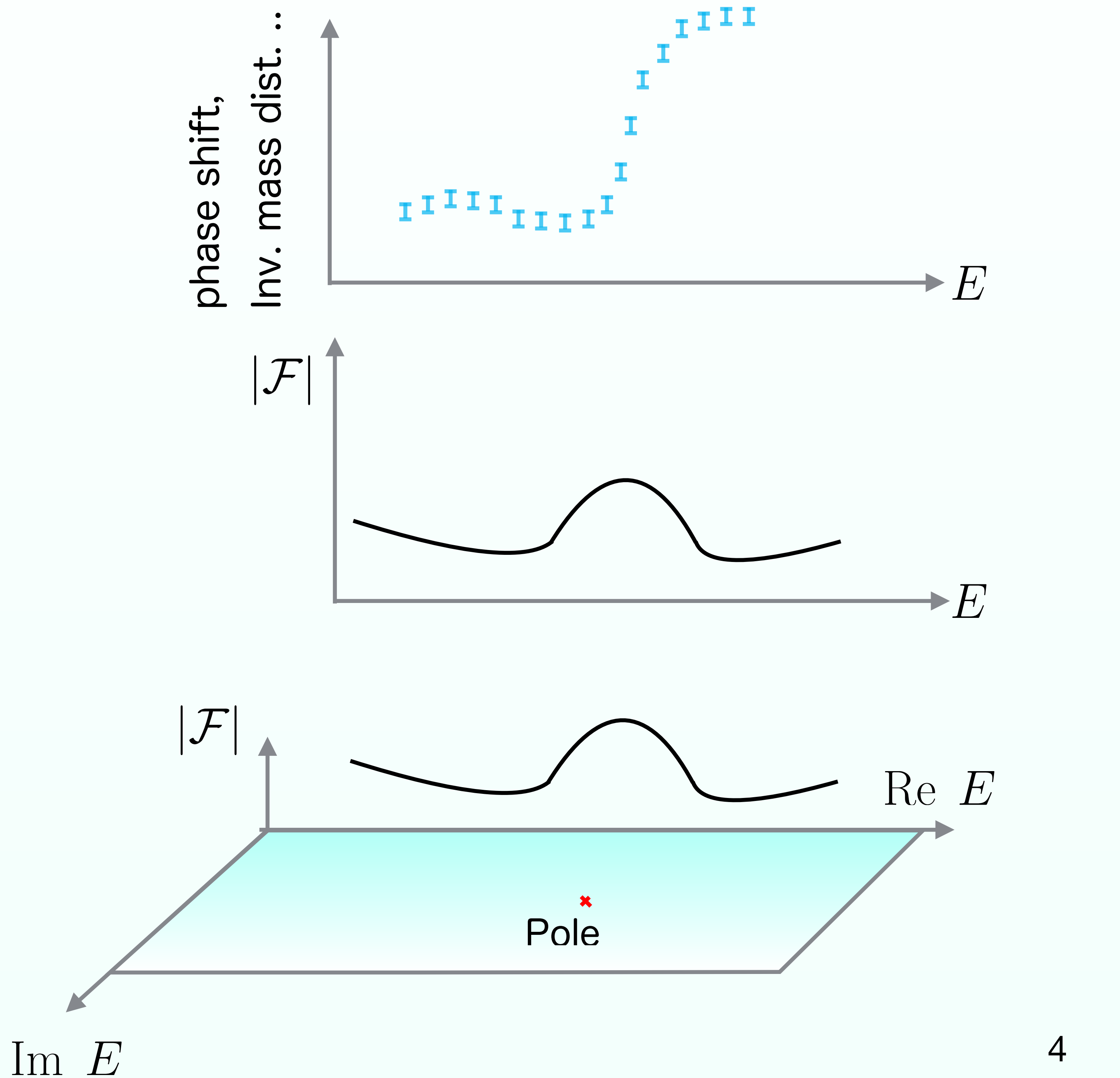
Experimental data

↓ detailed analysis

Scattering amplitude
on the physical real energy

↓ analytical continuation

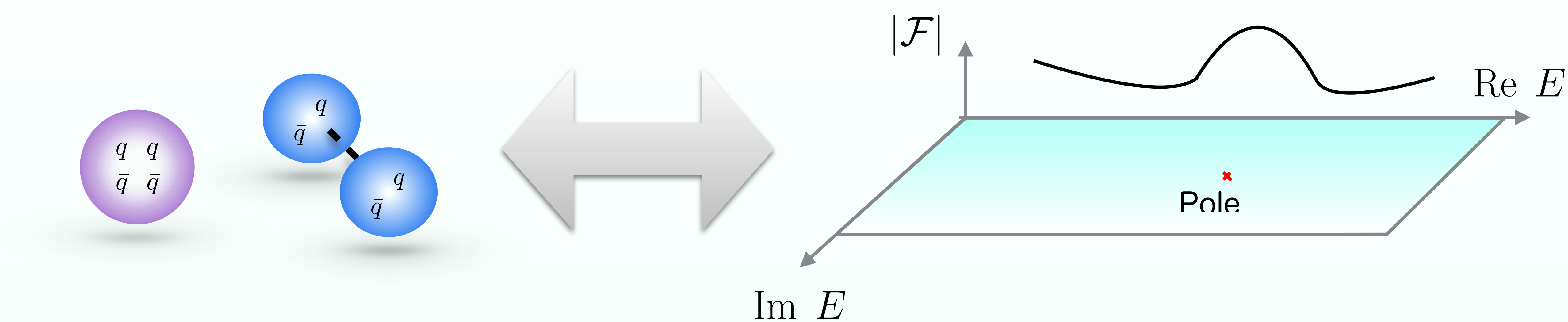
Scattering amplitude
in the complex energy plane
(or pole structure)



Introduction ~Methods to study structure~

- How to extract the structure from the experimental data

  - Relation between structure and amplitude



  - Pole counting method

    Qualitative judgement from the positions of the shadow poles.

    D. Morgan, Nucl. Phys. A543, 632 (1992)

  - Evaluation of compositeness

    Quantitative indicator of the amount of the dynamical fraction of the internal structure

    S. Weinberg, Phys. Rev. 137, B672 (1965).

Introduction ~CDD zero~

Castillejo Dalitz Dyson (CDD) Zero

- Defined as the energy point where the scattering amplitude F(E) vanishes.

CDD zero : $\mathcal{F}_{ii}(E_C) = 0$

L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. 101, 453 (1956).

- Existence indicates the contribution from outside the model space.

G. F. Chew and S. C. Frautschi, Phys. Rev. 124, 264 (1961).

- CDD zero on $\pi\Sigma$c amplitude

"CDD zero accompanied by nearby $\pi\Sigma$c thresholds performs the crucial role to reproduce the mass and width of Λc(2595)."

Z.-H. Guo and J. A. Oller, Phys. Rev. D93, 054014 (2016), 1601.00862.

- For a coupled-channel problem, both the existence and position depend on the channel.

c. f. The eigenstate pole lies the same position in the every coupled channel.

→ Can we extract information of the internal structure of the eigenstate from the position of the CDD zero?

ZCL of coupled channel amplitude

To investigate the origins of the eigenstate,
we consider the zero coupling limit of the coupled channel scattering amplitude.

Zero Coupling Limit (ZCL)

Switch off the inter-channel coupling in V_{ij}

$$V_{ij} = \begin{pmatrix} V_{11} & V_{12} & \cdots & V_{n1} \\ V_{12} & V_{22} & \cdots & V_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ V_{n1} & V_{n2} & \cdots & V_{nn} \end{pmatrix} \to \begin{pmatrix} V_{11} & 0 & \cdots & 0 \\ 0 & V_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & V_{nn} \end{pmatrix}$$

Poles and CDD zeros in the ZCL

In the ZCL, the pole exits only in the scattering amplitude of the channel whose interaction is the origin of the eigenstate.

(1) Interaction V_{ii} is the origin of the state. —> The pole remains in F_{ii}.

(2) Interaction V_{ii} is not the origin of the state. —> The pole decouples from F_{ii}.

・How about the behavior of CDD zero?

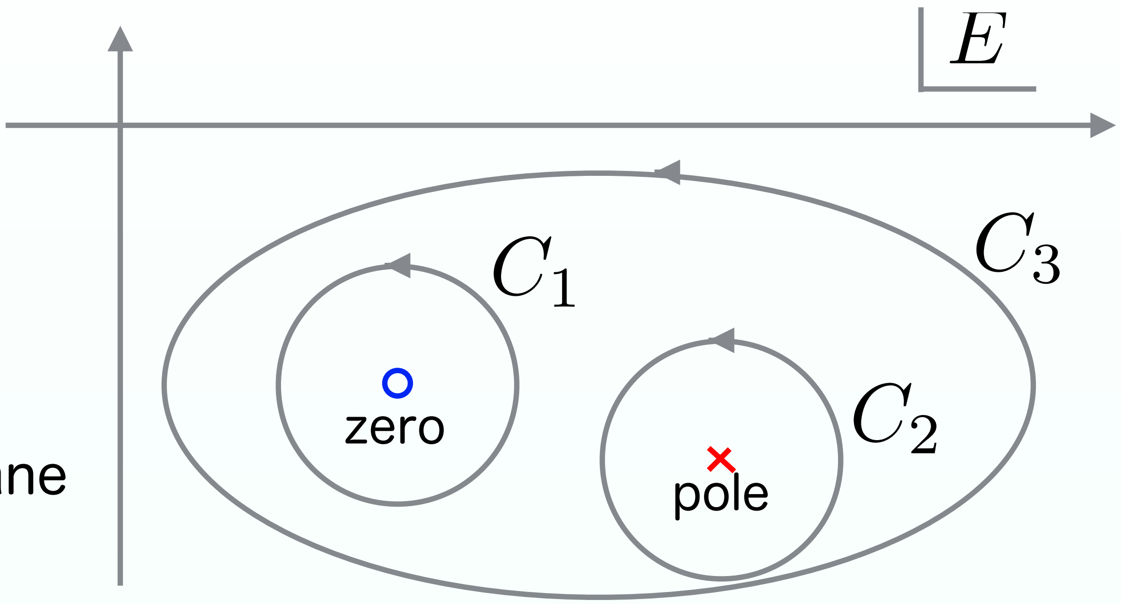
Origin of eigenstate and nearby CDD zero

- Principle of argument of scattering amplitude

$$n_C = \frac{1}{2\pi}\oint_C dz \frac{d}{dz} \arg \mathcal{F}(z)$$

F(z) : Partial-wave scattering amplitude

C : Closed integration path in the complex energy plane
(No poles and zeros lie on Path C)



→ • $n_C = (\#\text{ of CDD zeros in } \mathcal{C}) - (\#\text{ of poles in } \mathcal{C}) \in \mathbb{Z}$

$n_{C_1} = 1$

$n_{C_2} = -1$

$n_{C_3} = 0$

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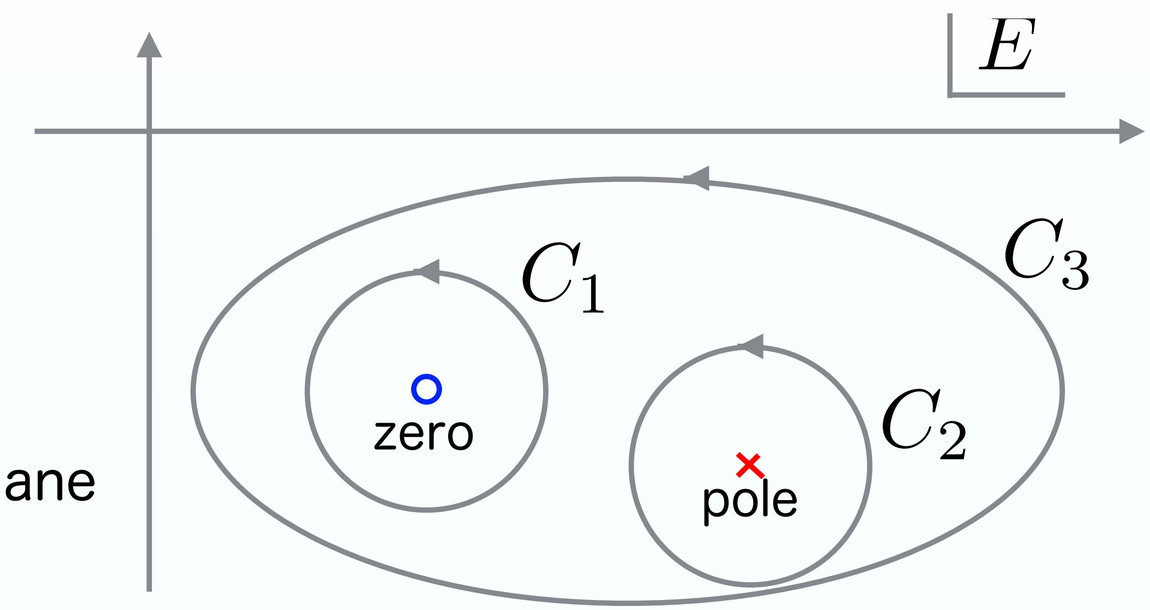

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$n_{C_1} = 1$

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$n_{C_3} = 0$

- Topological invariant ($\pi_1(U(1)) \cong \mathbb{Z}$)

—> nc is invariant under the continuous variation of amplitude (e.g. ZCL).

- Sudden vanishment of a pole or zero (nC : ±1 —> 0) is prohibited.
- The pair annihilation of a pole and a CDD zeros does not change nc.

Pole and CDD zero must encounter with each other to decouple from the scattering amplitude.

Origin of eigenstate and nearby CDD zero

- Behavior of pole of scattering amplitude $F_{ii}(E)$ in channel i in the ZCL:

  Interaction of channel i V_{ii} is origin of the state —> Pole remains in F_{ii}.

  Otherwise —> Pole decouples from F_{ii}.

- To decouple from the amplitude $F_{ii}(E)$, pole must meet CDD zero.
- Pole and CDD zero move continuously in the continuous change of amplitude.

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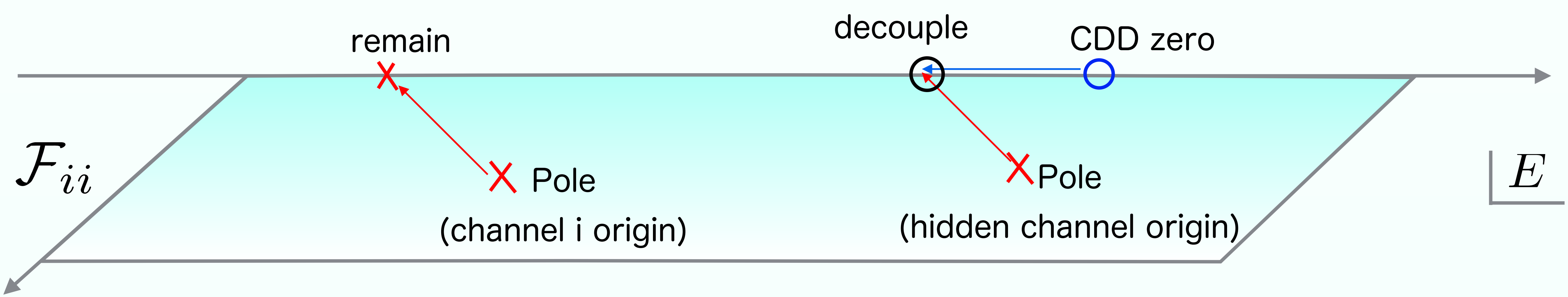
Taking the ZCL limit



Origin of eigenstate and nearby CDD zero

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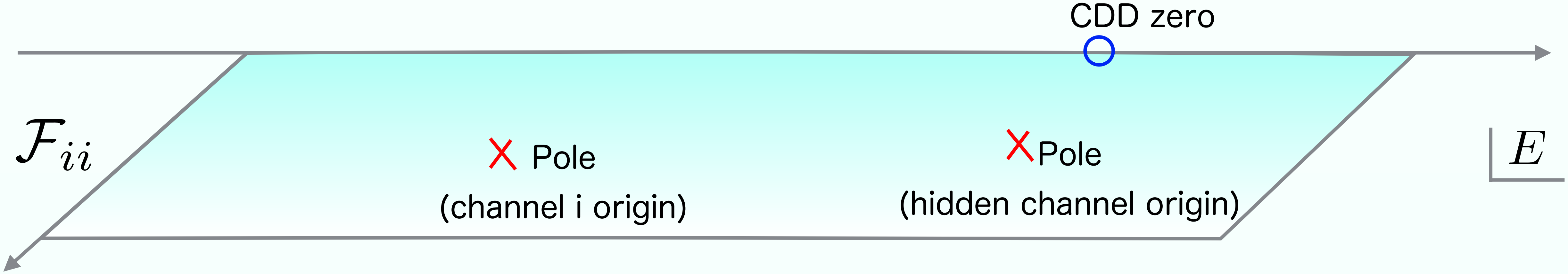
Before taking the ZCL limit



Origin of eigenstate	Near the pole in F_{ii}
Channel i	No nearby CDD zero
Not channel i (Hidden channel)	Nearby CDD zero

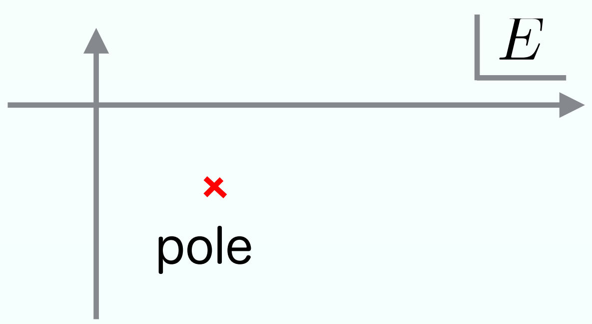
Origin of eigenstate and nearby CDD zero

Origin of eigenstate	Near the pole in F_{ii}
Channel i	No nearby CDD zero
Not channel i (Hidden channel)	Nearby CDD zero

This relation is useful to investigate the origin of the eigenstate.

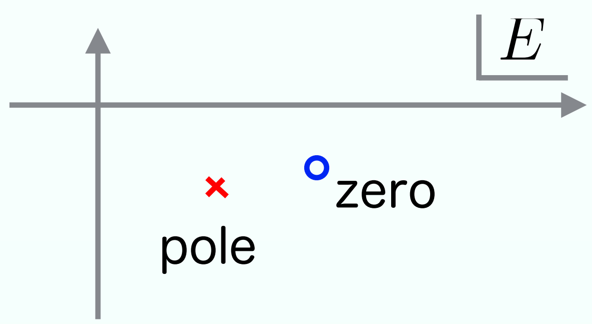
Y. K and T. Hyodo, Phys. Rev. D, 97, 054019 (2018).

(1) CDD zero does not lie around the pole.



→ The eigenstate is dynamically generated in channel i.

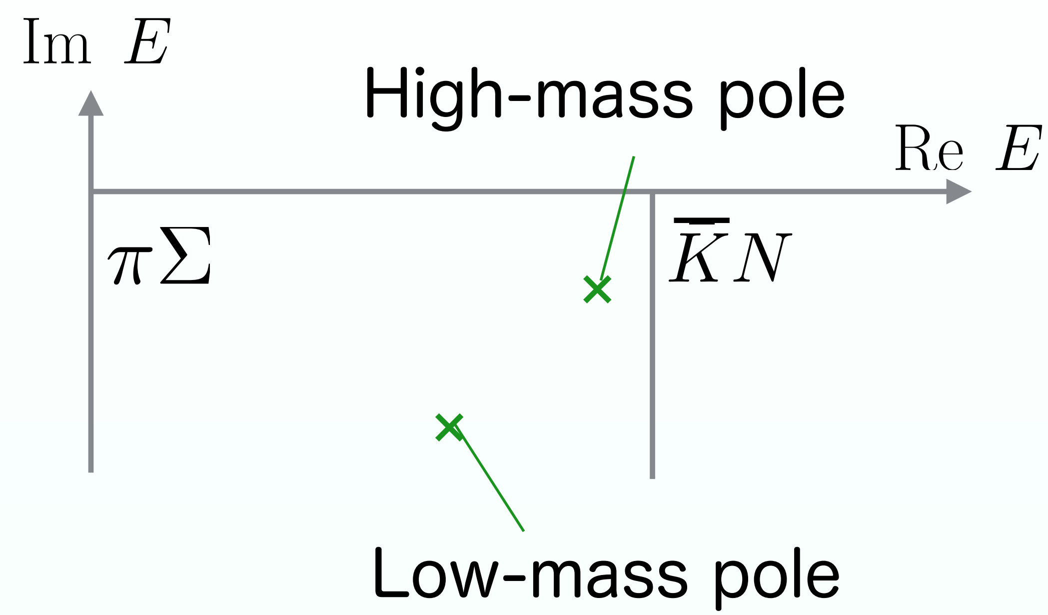
(2) Pole has a nearby CDD zero.



→ The origin of the eigenstate is not in channel i.

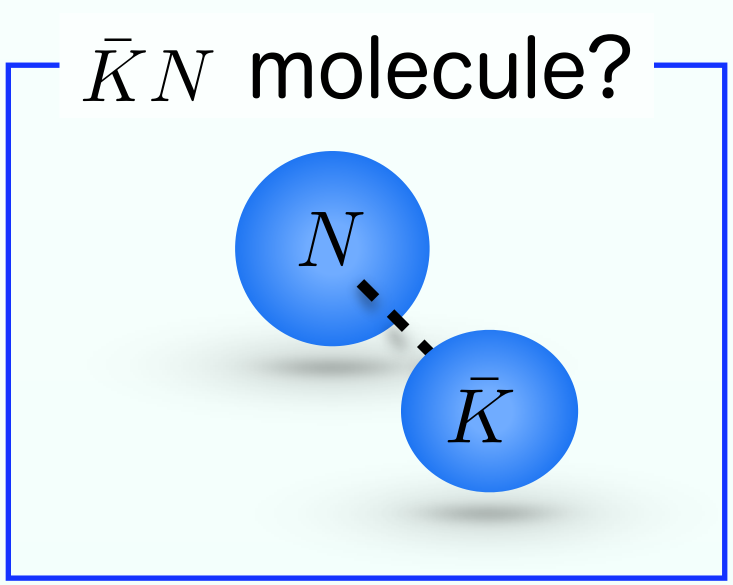
Application to Λ(1405)

- $\Lambda(1405)$ ($I=0$ $\bar{K}N$ scattering)
  - $J^P = \frac{1}{2}^-$
  - Analysis with chiral dynamics
    —> Two pole structure
    D. Jido et al Nucl. Phys. A 725, 181 (2003)



Recent determinations are tabulated in PDG

C. Patrignani et al. (Particle Data Group), Chin. Phys. C40, 100001 (2016)



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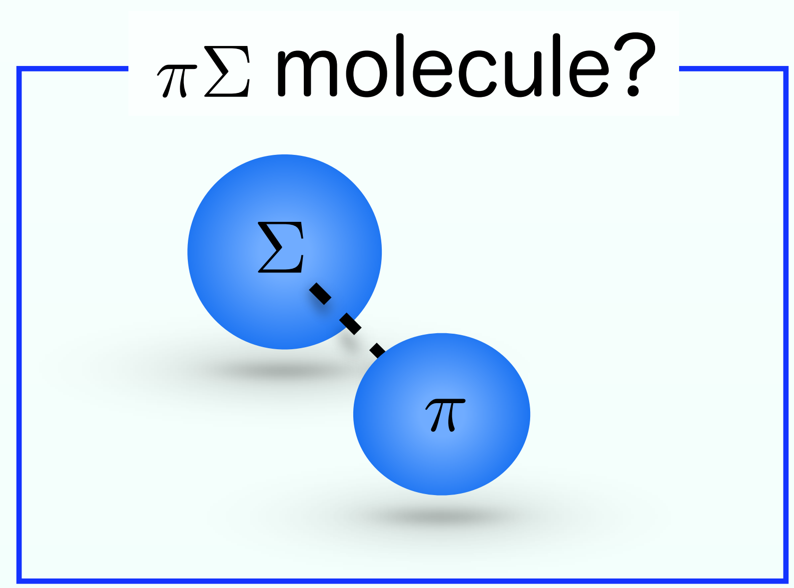


Application to Λ(1405)

Effective Tomozawa-Weinberg model

Y. Ikeda, T. Hyodo, and W. Weise, Nucl. Phys. A881, 98 (2012)

- Isospin basis

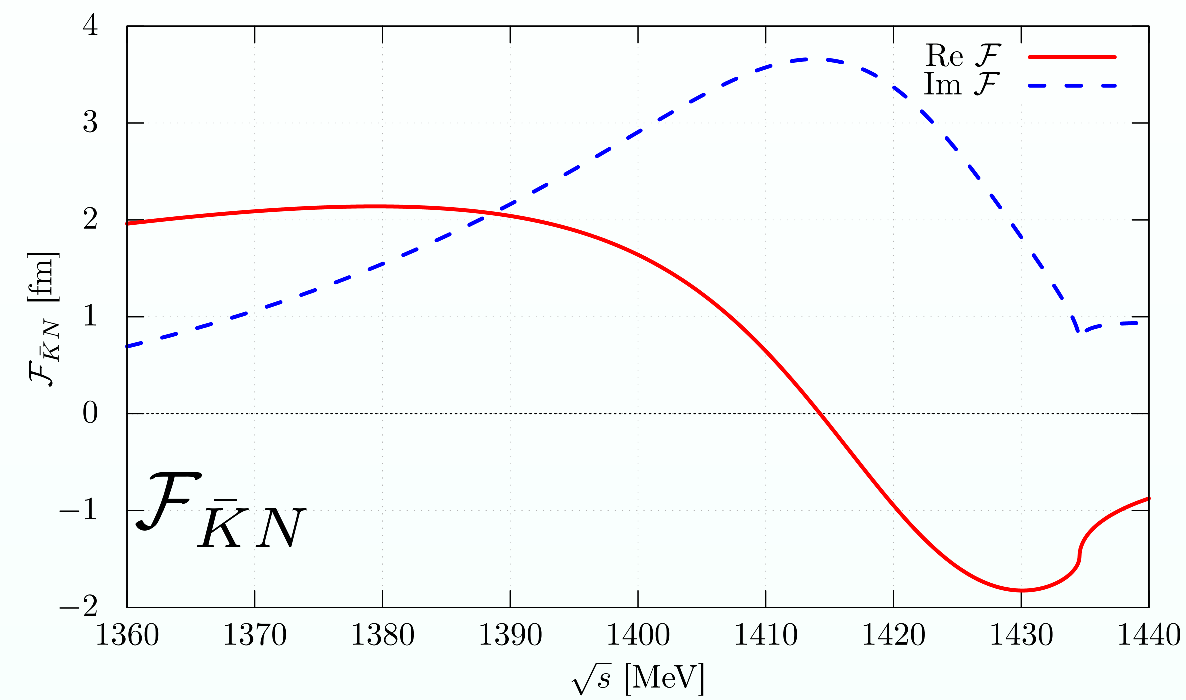
  Coupled channel : $\bar{K}N$-$\pi\Sigma$

- interaction : Tomozawa-Weinberg interaction

- Pole position;

  High-mass pole ; $1423 - 22i$ MeV

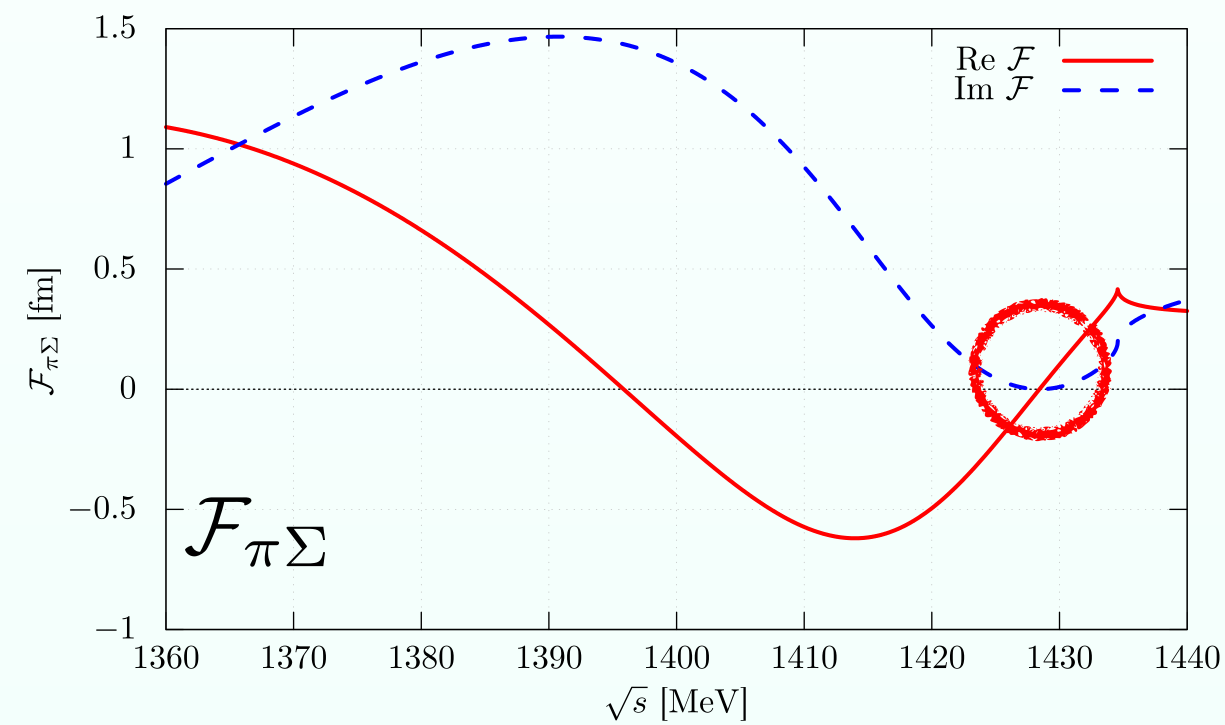
  Low-mass pole ; $1375 - 65i$ MeV



$\mathcal{F}_{\pi\Sigma}$

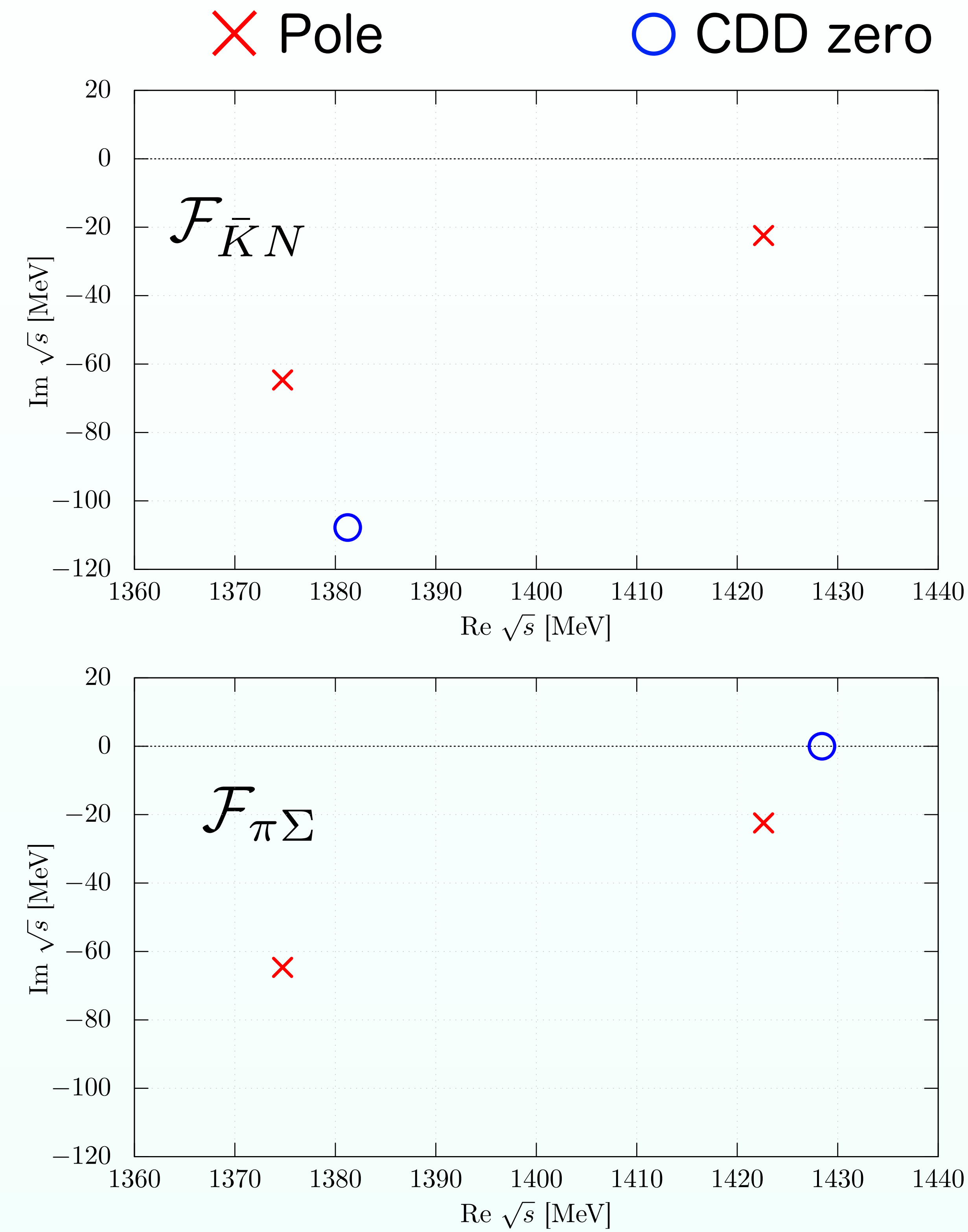
CDD zero lies near high-mass pole.

Y. K. and T. Hyodo, PTEP 2017, 023D02 (2017)



Application to Λ(1405)

Position of poles and CDD zeros



Application to Λ(1405)

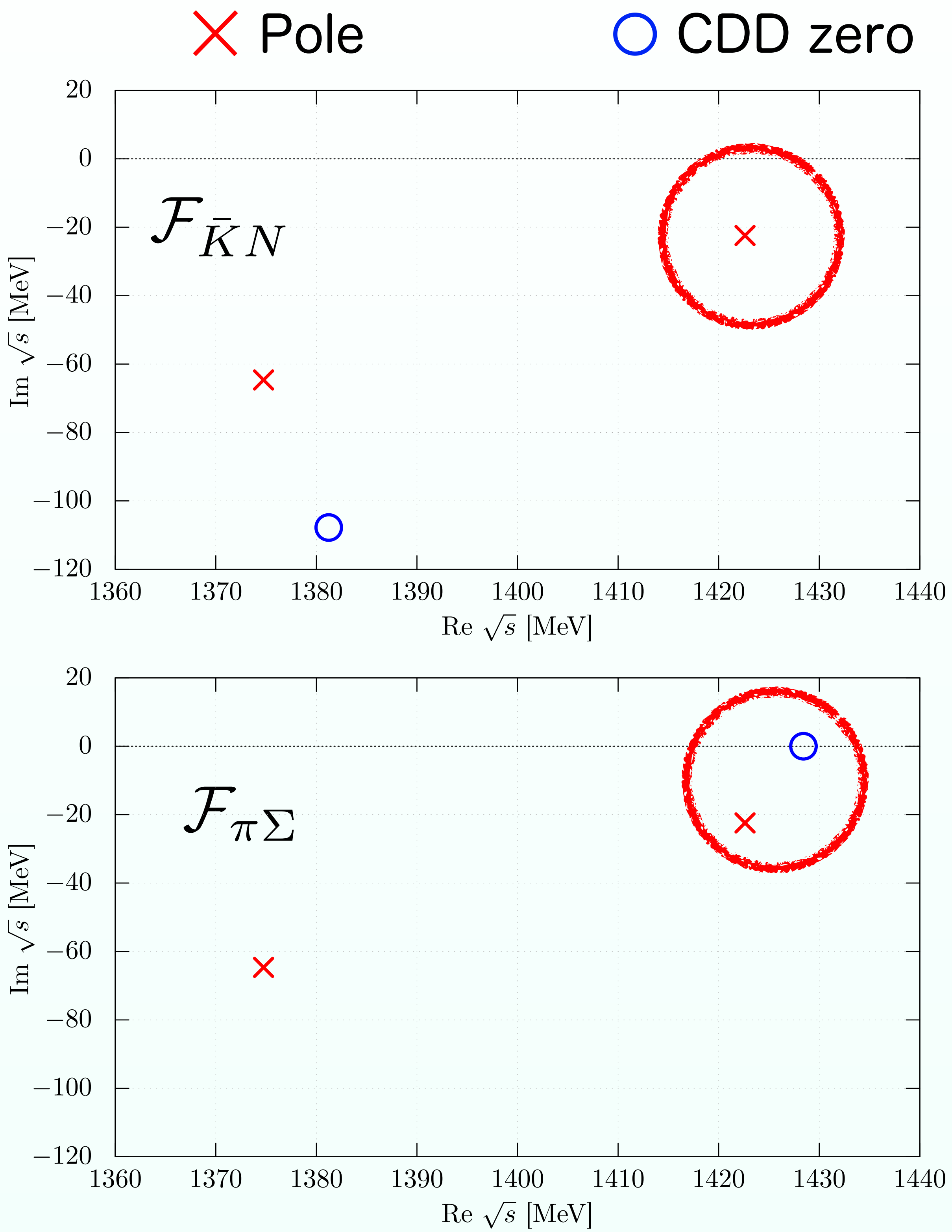
Position of poles and CDD zeros

High-mass pole

No nearby CDD zero in $\bar{K}N$ amplitude

Nearby CDD zero in $\pi\Sigma$ amplitude

⇨ Origin is in $\bar{K}N$ channel.



Application to Λ(1405)

Position of poles and CDD zeros

High-mass pole

No nearby CDD zero in $\bar{K}N$ amplitude

Nearby CDD zero in $\pi\Sigma$ amplitude

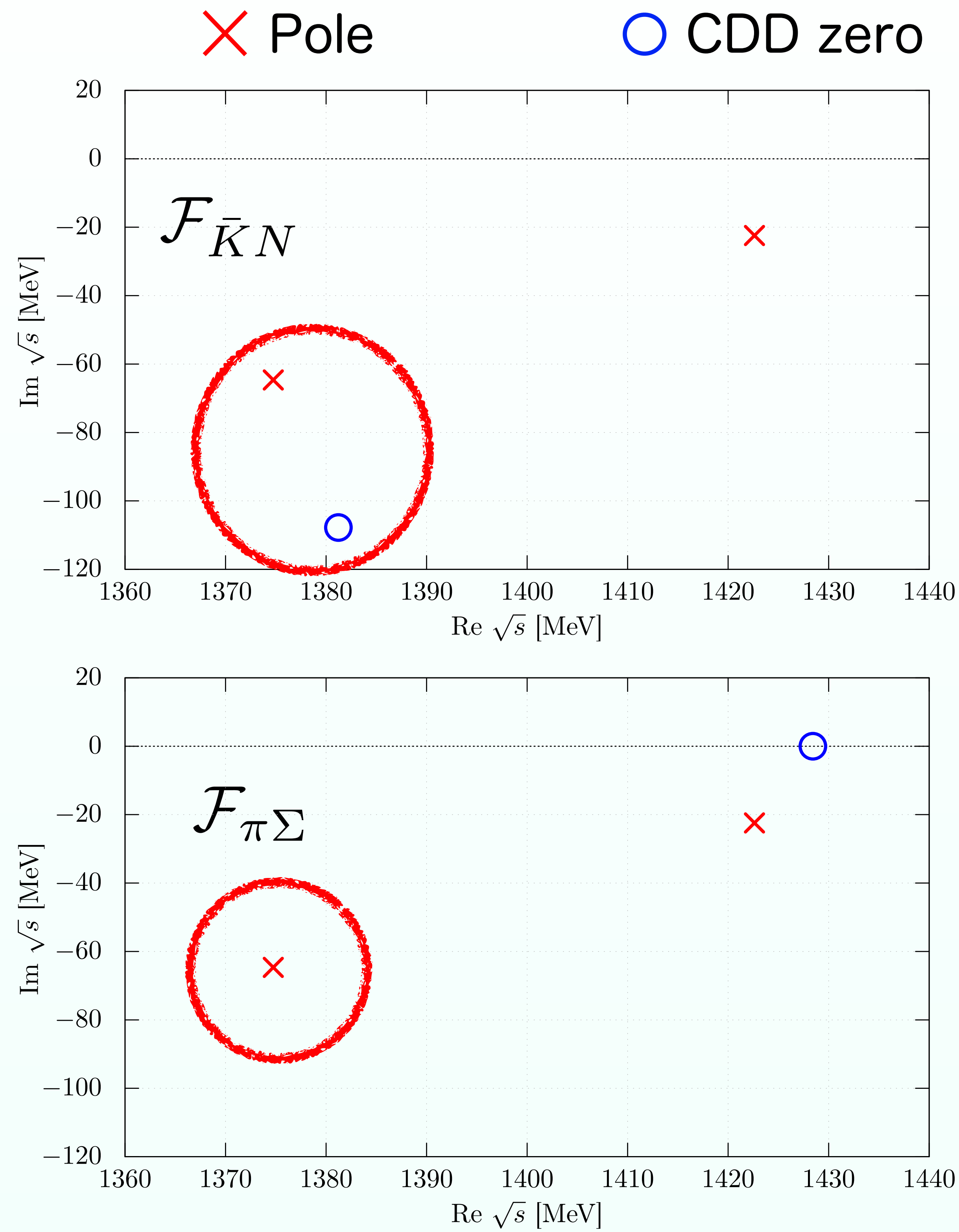
Origin is in $\bar{K}N$ channel.

Low-mass pole

Nearby CDD zero in $\bar{K}N$ amplitude

No Nearby CDD zero in $\pi\Sigma$ amplitude

Origin is in $\pi\Sigma$ channel.

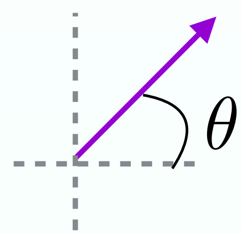


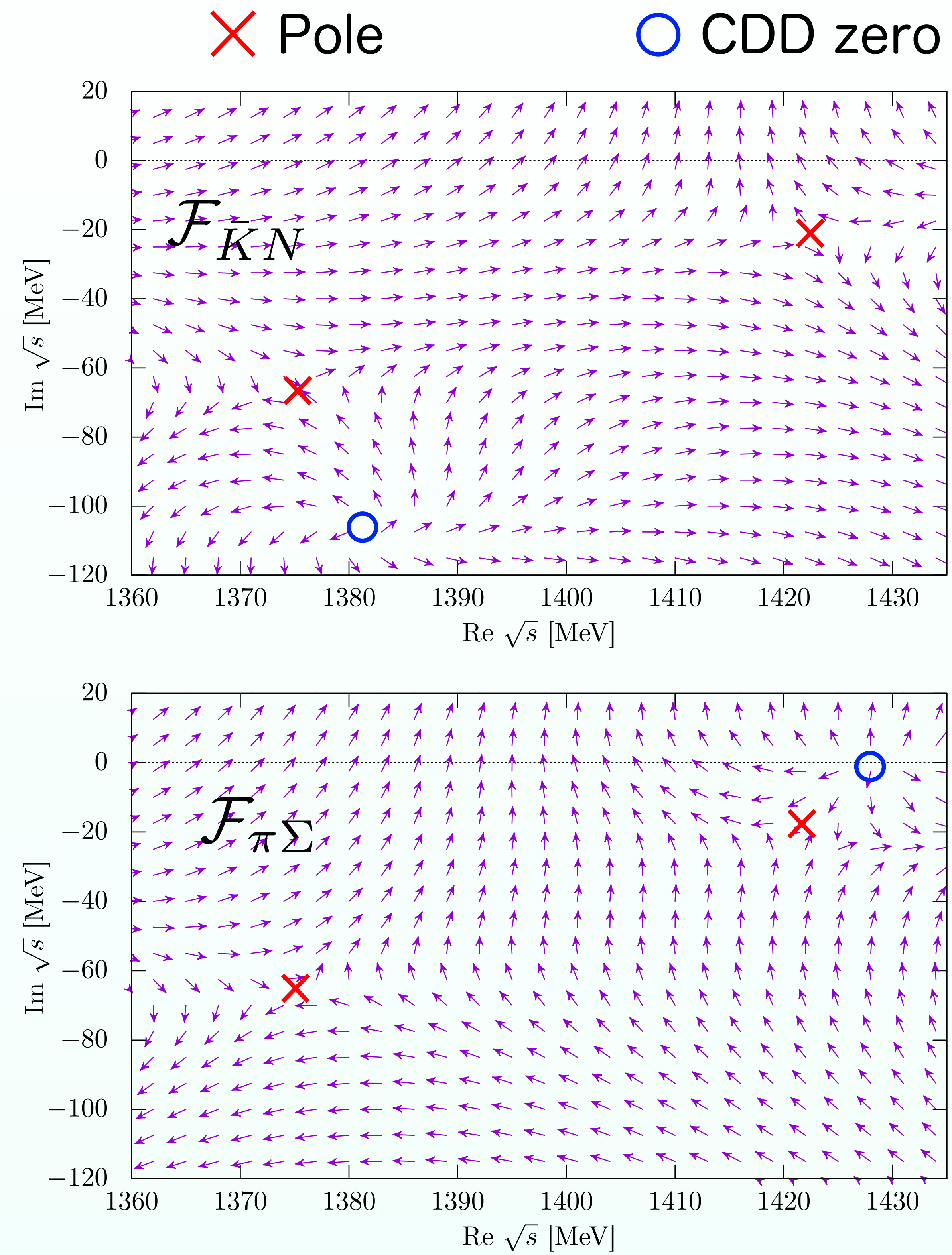
Application to Λ(1405)

Vector map of phase structure of amplitude

The existence of the pole and CDD zero can be confirmed by the phase structure of amplitude.

$$\mathcal{F}(\sqrt{s}) = |\mathcal{F}|e^{i\theta}$$





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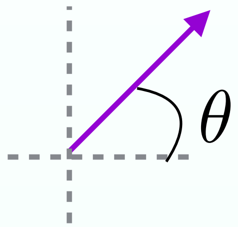
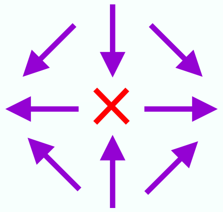
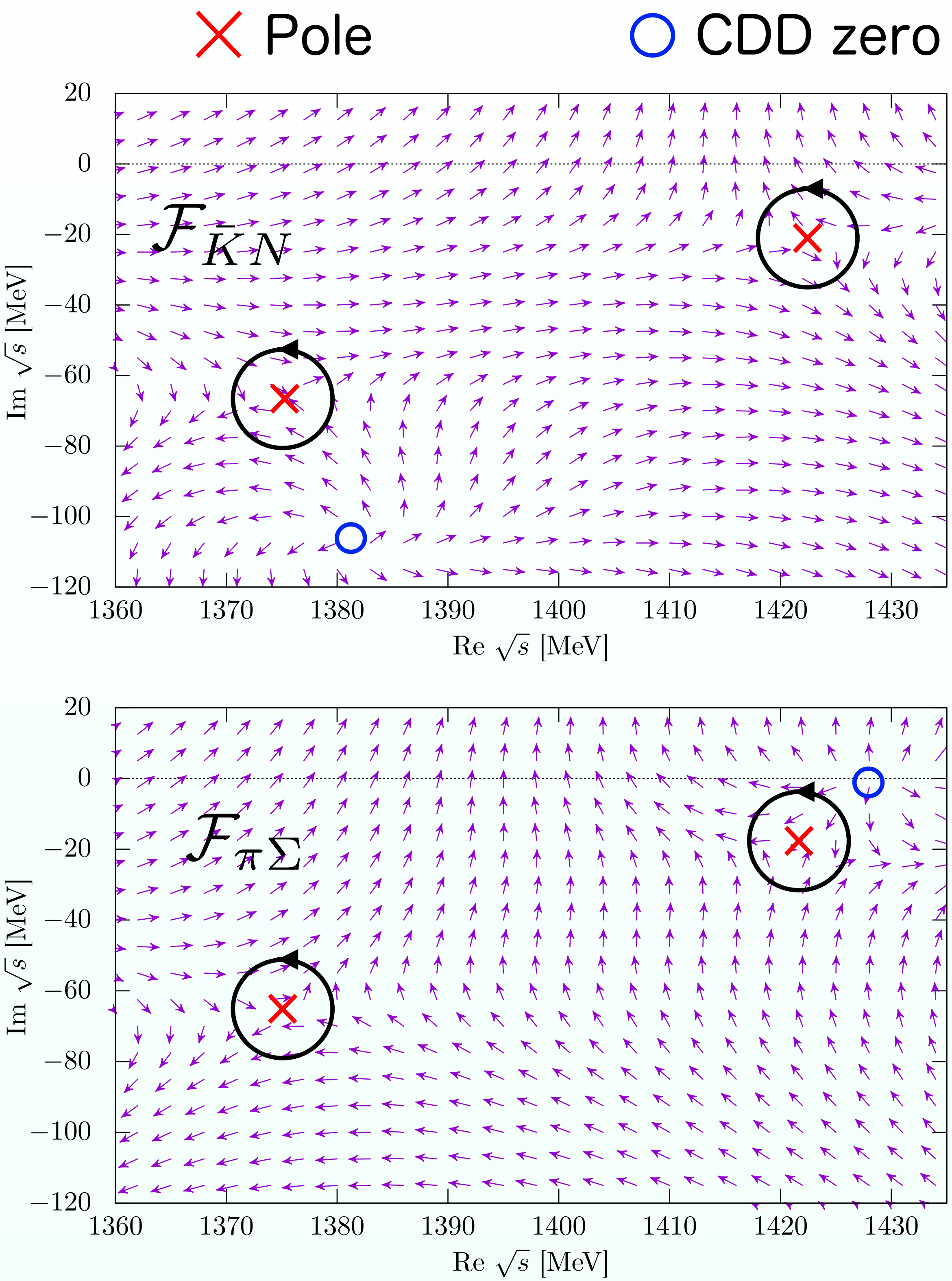


Along contour around pole



The vector of phase turns clockwise.



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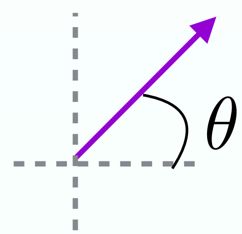
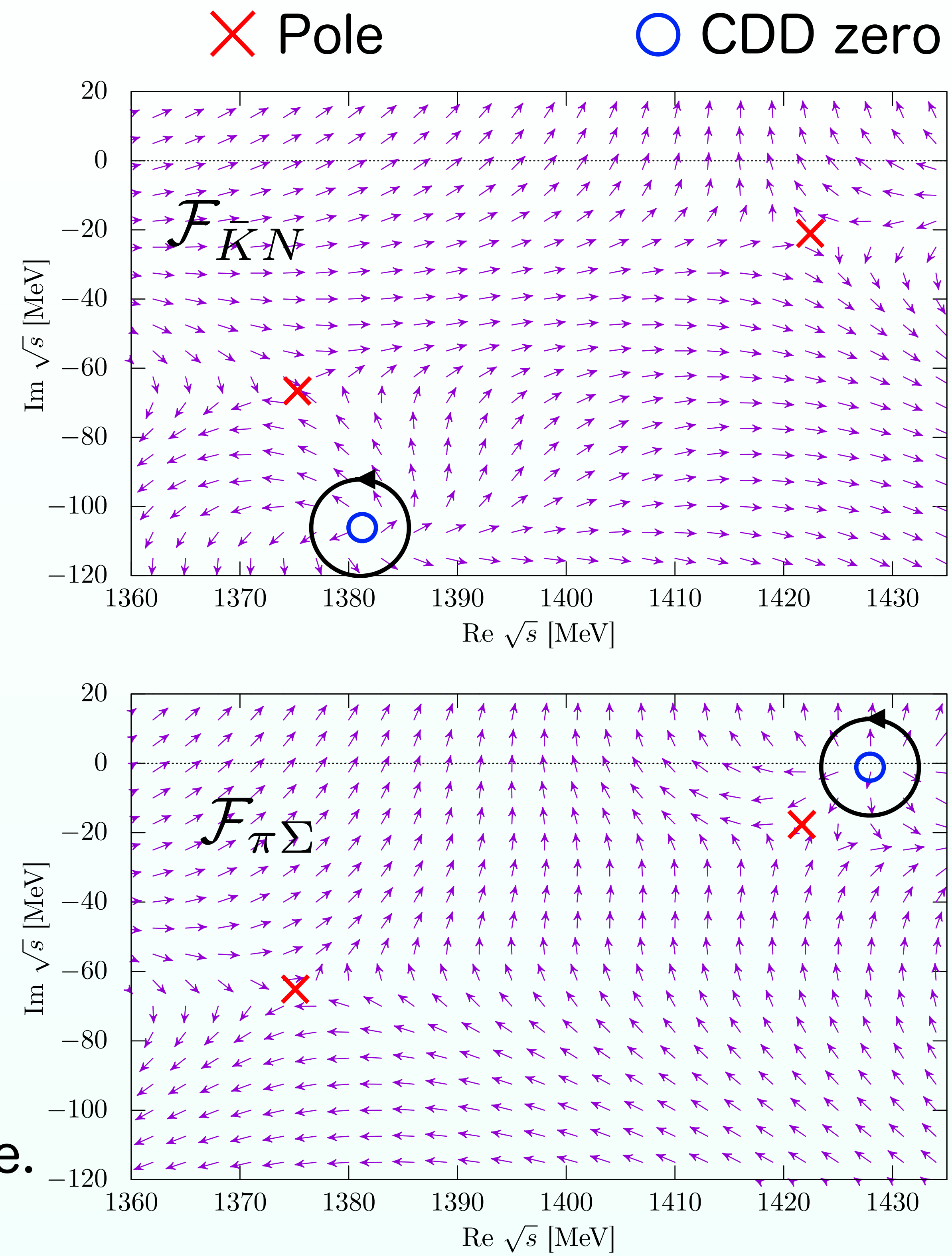
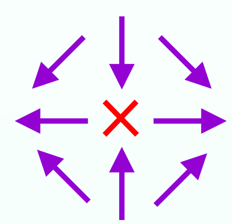


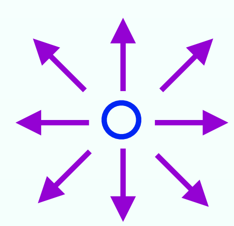


Along contour around pole



The vector of phase turns clockwise.

Along contour around CDD zero



The vector of phase turns counterclockwise.

Conclusion

- The eigenstate pole should decouple from the amplitude in the ZCL, if the eigenstate originates in the other channel.
- We show that the pole must annihilate with CDD zero to decouple.
- New method to study the origin of the eigenstate ;

(1) Pole without a nearby CDD zero —> Dynamical origin.

(2) Pole with a nearby CDD zero —> Origin is the other channel.

- Application to Λ(1405)

Y. K. and T. Hyodo, PRD 97, 054019 (2018).