Symmetry breaking patterns in QCD and the study of cold dense matter

TECHNISCHE UNIVERSITÄT DARMSTADT

Marc Leonhardt
Institut für Kernphysik, Technische Universität Darmstadt

with

Martin Pospiech and Jens Braun

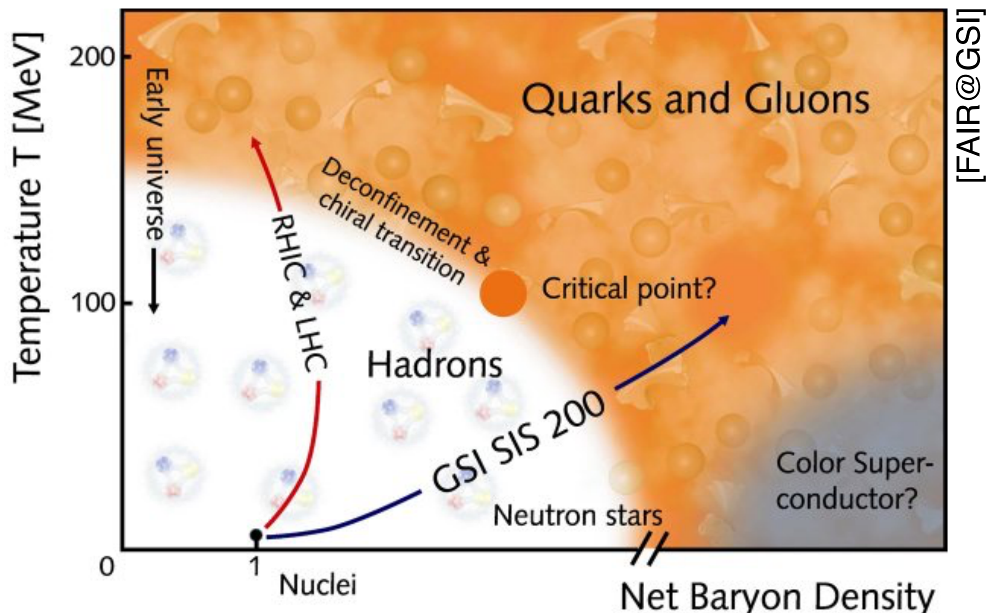
New Frontiers in QCD 2018
YITP, Kyoto University

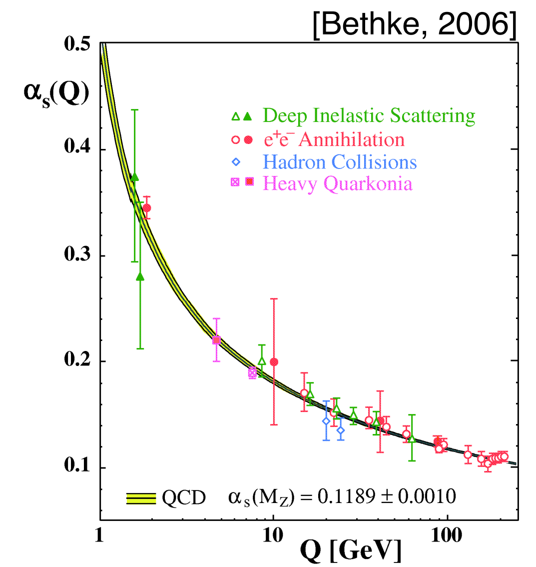
SFB 1245

[J. Braun, ML, M. Pospiech, PRD **96**, 076003 (2017)]
[J. Braun, ML, M. Pospiech, PRD **97**, 076010 (2018)]
[J. Braun, ML, J. M. Pawlowski, arXiv:1806.04432]
[J. Braun, C. Drischler, K. Hebeler, ML, M. Pospiech, A. Schwenk, in preparation]

HIC for FAIR
Helmholtz International Center

QCD phase diagram:
Neutron stars and the cold dense EoS





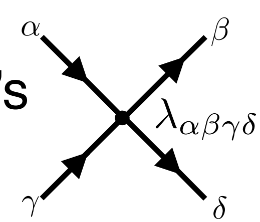
$\lesssim 35\,\rho_{\rm sat}$ [Kurkela et al., 2014]

Low-energy effective theories

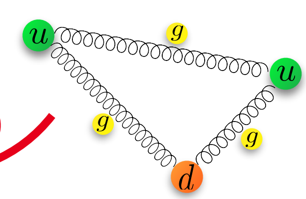
Functional methods

Perturbative methods

EoS ← (II) ← Martin's Talk



← (I) ←



Outline

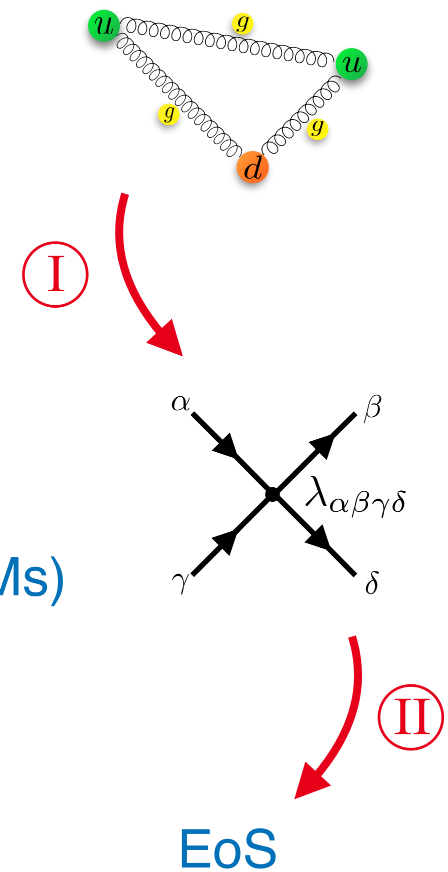
Ⓘ Including dynamic gauge fields

- Framework and structure of β functions
- Symmetry breaking patterns
- Implications on the phase diagram

Ⓘⓘ Connecting to low-energy effective models (LEMs)

- Quark-meson-diquark model
- Findings on the equation of state (EoS)

Conclusions and outlook

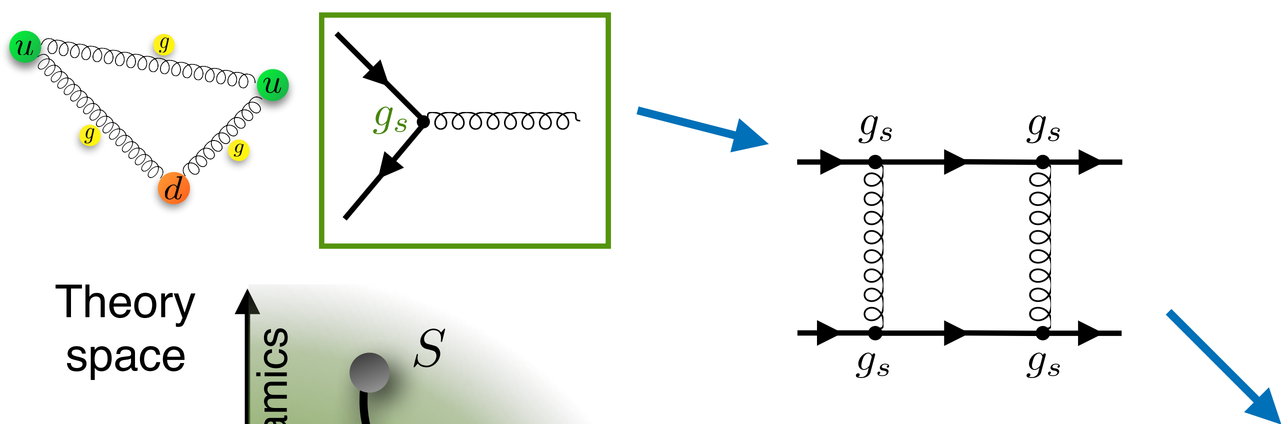


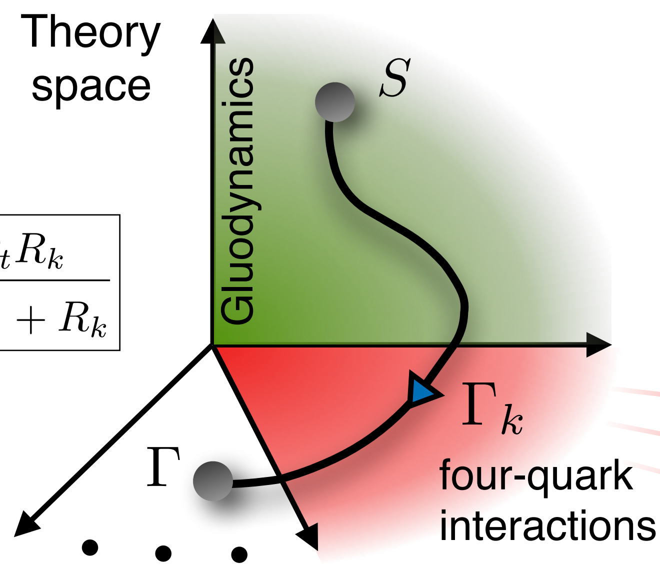
SFB 1245

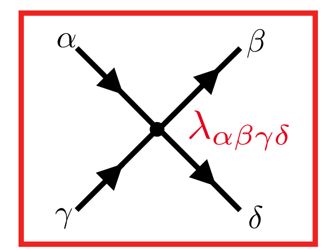
Functional renormalization group (FRG)

From high to low energies in QCD

TECHNISCHE UNIVERSITÄT DARMSTADT







$$k\frac{\partial}{\partial k}\Gamma_k = \frac{1}{2}\mathrm{STr}\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

[C. Wetterich, 1993]

$[(\bar{\psi}\gamma_\mu\psi)^2]$

$[(\bar{\psi}\gamma_\mu\gamma_5 T^a\psi)^2]$

⋮

SFB 1245

Outline

TECHNISCHE UNIVERSITÄT DARMSTADT

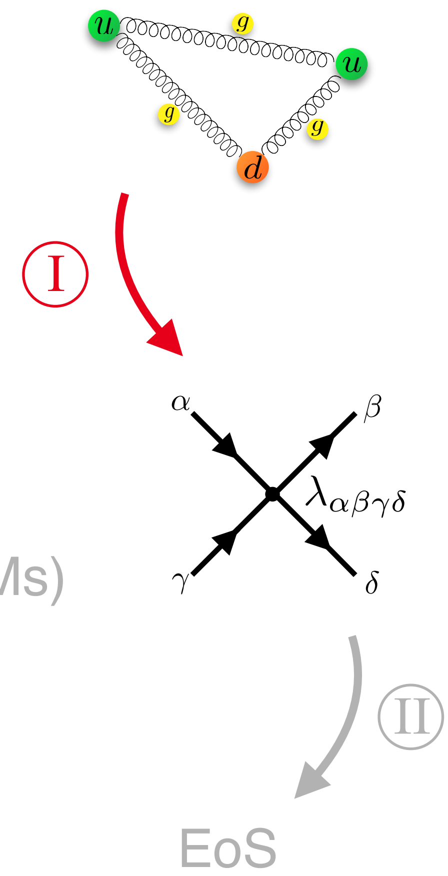
Ⓘ Including dynamic gauge fields

- Framework and structure of β functions
- Symmetry breaking patterns
- Implications on the phase diagram

Ⓘⓘ Connecting to low-energy effective models (LEMs)

- Quark-meson-diquark model
- Findings on the equation of state (EoS)

Conclusions and outlook



Ansatz for the effective average action

Quark self-interactions and gauge degrees of freedom

TECHNISCHE UNIVERSITÄT DARMSTADT

$$\Gamma_{\mathrm{LO}}[\bar{\psi}, \psi, A] = \int_0^{\frac{1}{T}} \mathrm{d}\tau \int \mathrm{d}^3 x \left\{ \bar{\psi}\left(\mathrm{i}\gamma_0 \partial_0 + \mathrm{i}\gamma_i \partial_i - \mathrm{i}\mu\gamma_0\right)\psi + \frac{1}{2} \sum_{j \in \mathcal{B}} Z_j \bar{\lambda}_j \mathcal{L}_j \right.$$

Symmetries

Lorentz group: $\mathrm{SO}(1,3) \longrightarrow \mathrm{SO}(3)$

Flavor space: $\mathrm{SU}_L(2) \otimes \mathrm{SU}_R(2) \otimes \mathrm{U}_V(1)$

Color space: $\mathrm{SU}(N_c)$

$\downarrow$

Fierz-complete basis:
10 channels

$$\mathcal{L}^{(\sigma-\pi)}_{(\bar{\psi}\psi)^2} = \left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}\gamma_5 \vec{\tau}\psi\right)^2$$

$\leftrightarrow$ formation of chiral condensate

$$\mathcal{L}^{\mathrm{csc}}_{(\bar{\psi}\psi)^2} \sim (\mathrm{i}\bar{\psi}\gamma_5 \tau_A t_c^{A'} \mathcal{C} \bar{\psi}^T)(\mathrm{i}\psi^T \mathcal{C} \gamma_5 \tau_A t_c^{A'} \psi) \qquad J^P = 0^+$$

e.g. [Rapp, Schäfer, Shuryak, Velkovsky, 1998]

$\leftrightarrow$ formation of diquark condensate

SFB 1245

Ansatz for the effective average action

Quark self-interactions and gauge degrees of freedom

$$\Gamma_{\mathrm{LO}}[\bar{\psi},\psi,A] = \int_0^{\frac{1}{T}} \mathrm{d}\tau \int \mathrm{d}^3x \, \Big\{ \bar{\psi}\left(\mathrm{i}\gamma_0\partial_0 + \mathrm{i}\gamma_i\partial_i - \mathrm{i}\mu\gamma_0\right)\psi + \frac{1}{2}\sum_{j\in\mathcal{B}} Z_j \bar{\lambda}_j \mathcal{L}_j$$

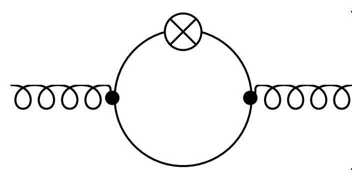
$$+\frac{Z_A}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{1}{2\xi}\left(\partial_\mu A^a_\mu\right)^2$$ kinetic term for gluons

$$+\bar{g}_s\bar{\psi}\gamma_\mu A_\mu \psi \Big\}$$ quark-gluon interaction

RG flow of the gauge coupling: $k\frac{\partial}{\partial k} g_s^2(k) = \eta_{A,k}\, g_s^2(k)$ [L. F. Abbott, 1980]

$$\eta_{A,k} = \underline{\eta^{\mathrm{YM}}_{A,k}(T)} + \Delta\eta_{A,k}(T,\mu)$$

$$\Delta\eta_{A,k}(T,\mu) = \frac{Z_A^{-1}}{3(N_c^2-1)}\left(\frac{\partial^2}{\partial p^2}\Pi^{\perp}\cdot \right)\Bigg|_{p=0}$$

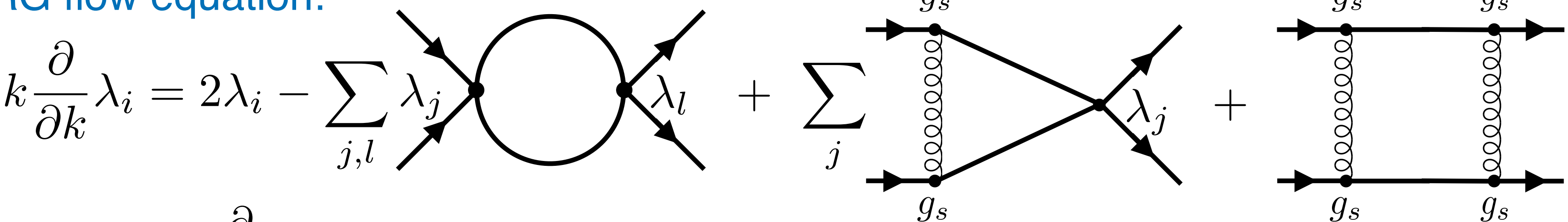


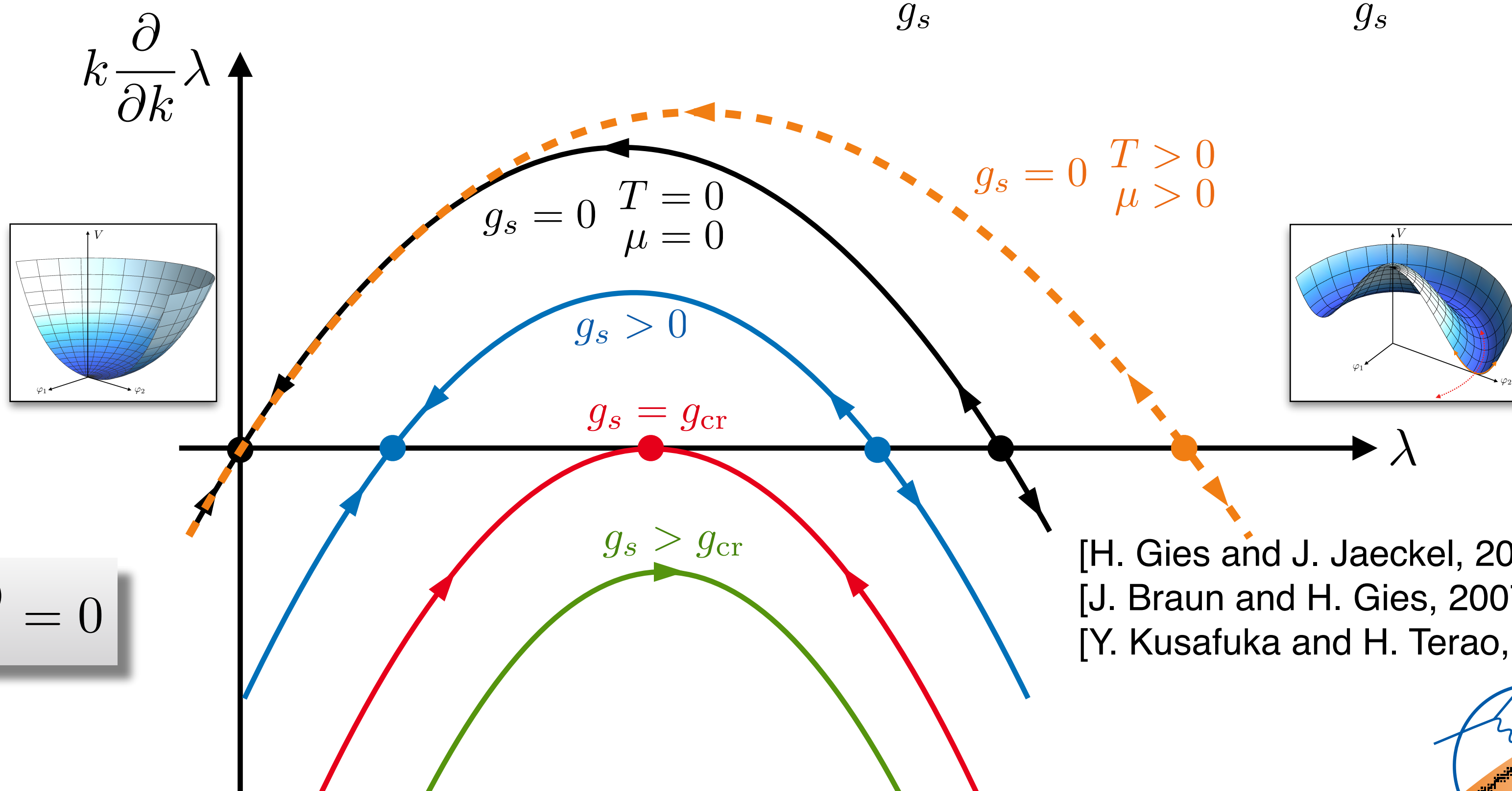
[J. Braun and H. Gies, 2007]
[J. Braun, 2009]
[J. Braun et al., 2011]
[J. M. Pawlowski, 2011]
[J. Braun et al., 2014]

RG flow of four-quark interactions

Characteristics of the β functions and the fixed-point structure

RG flow equation:





$$\lambda_i^{(\mathrm{UV})} = 0$$

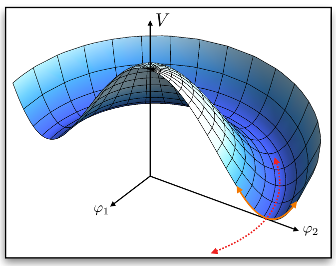
[H. Gies and J. Jaeckel, 2006]
[J. Braun and H. Gies, 2007]
[Y. Kusafuka and H. Terao, 2011]

RG flow of four-quark interactions

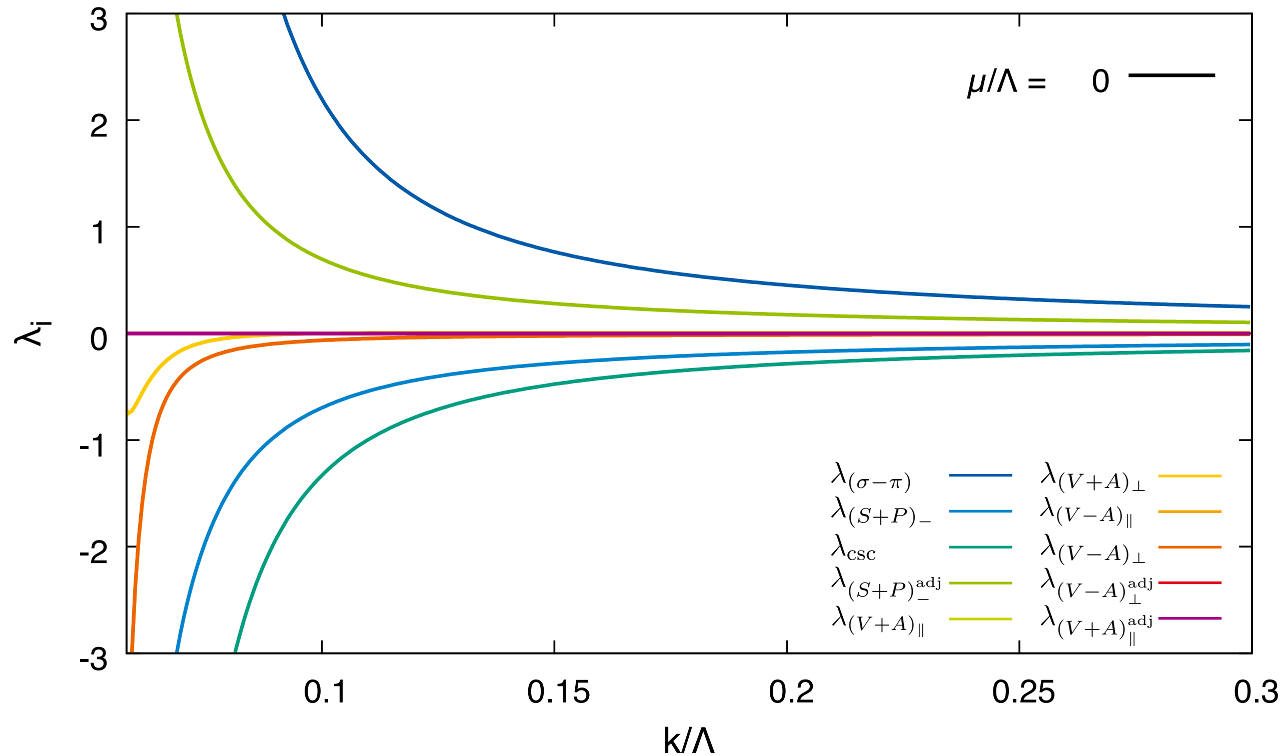
Fixed-point structure and patterns of symmetry breaking

Illustration of the RG flow of the four-quark couplings at zero temperature:

$$\left.\frac{1}{\lambda_i}\right|_{k_0} \to 0$$



$T = 0$



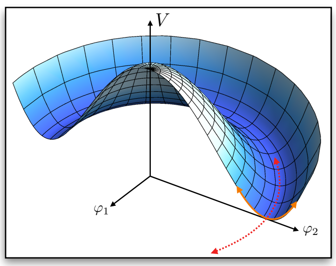
[J. Braun, 2006]
[M. Mitter, J. M. Pawlowski, N. Strodthoff, 2014]

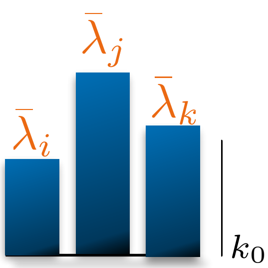
RG flow of four-quark interactions

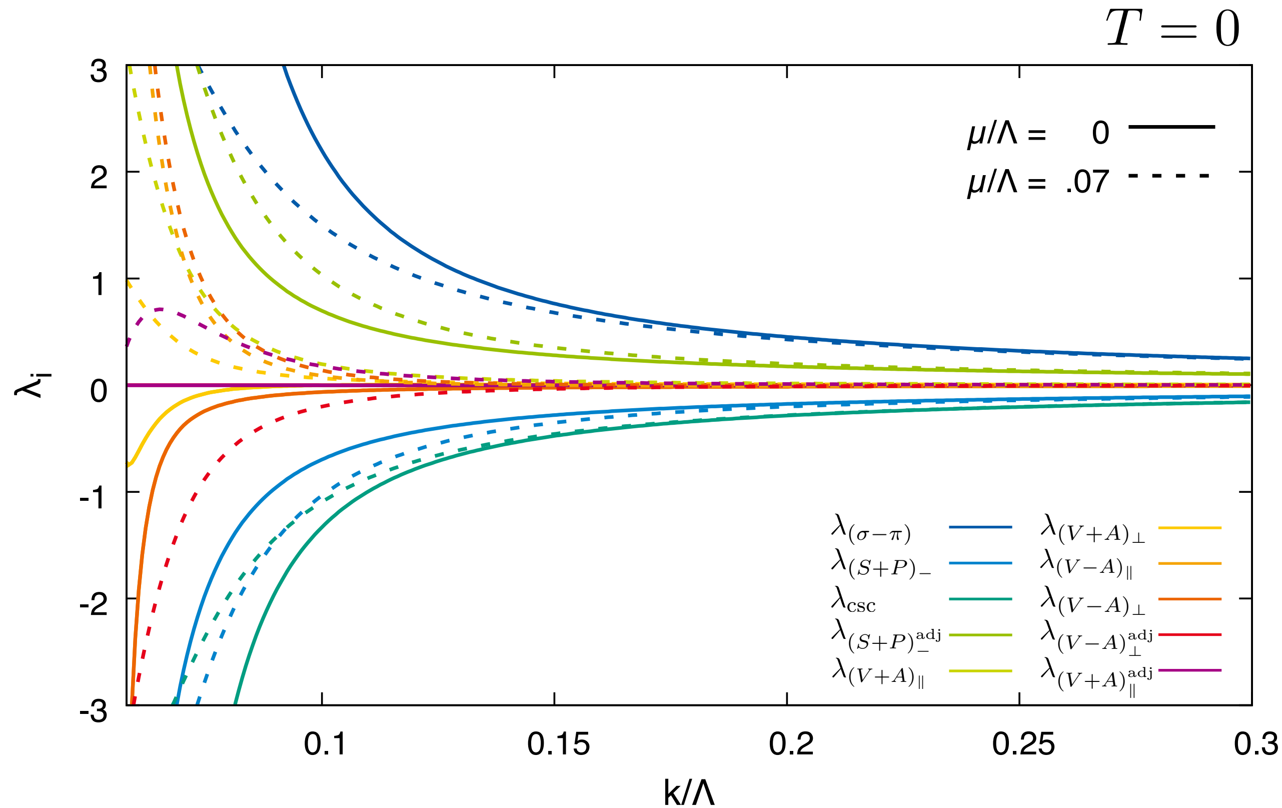
Fixed-point structure and patterns of symmetry breaking

Illustration of the RG flow of the four-quark couplings at zero temperature:

$$\left.\frac{1}{\lambda_i}\right|_{k_0} \to 0$$







Assessing relative interaction strengths → Formation of specific condensates

e.g. [J. Braun, H. Gies, L. Janssen, D. Roscher 2014]

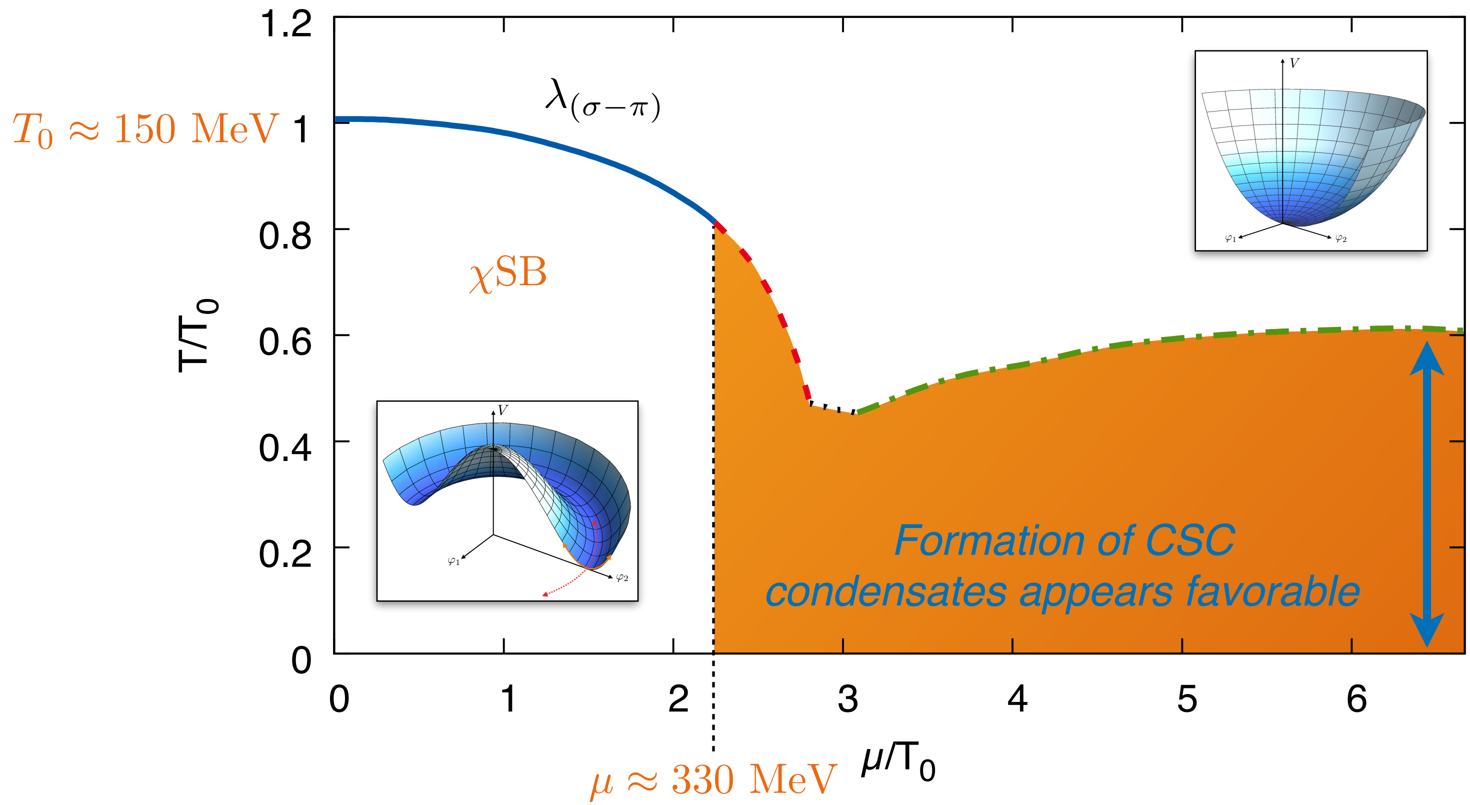
Exploring the phase diagram

Fixed-point structure and patterns of symmetry breaking

TECHNISCHE UNIVERSITÄT DARMSTADT

Scale fixing: $g_s(\Lambda)$ at Λ = 10 GeV within experimental error [Bethke, 2006]

$T_{cr}(\mu{=}0) \equiv T_0 \approx 150$ MeV



$T_c \sim \Delta_{csc}(T=0)$

[BCS theory]

Implications for EoS!

Outline

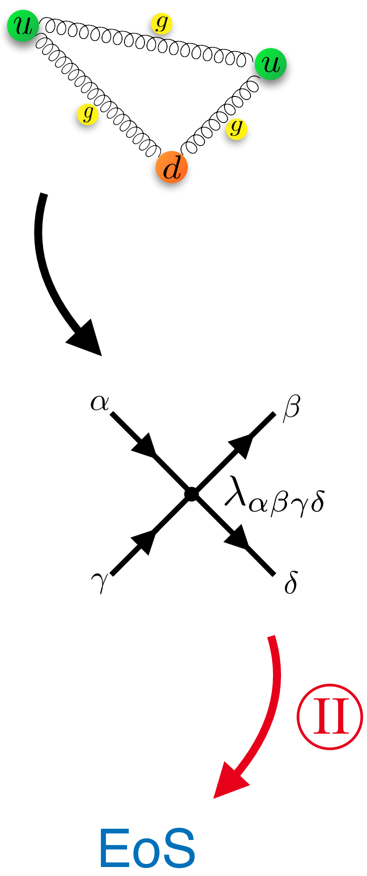
Ⅰ Including dynamic gauge fields

- Framework and structure of β functions
- Symmetry breaking patterns
- Implications on the phase diagram

Ⅱ Connecting to low-energy effective models (LEMs)

- Quark-meson-diquark model
- Findings on the equation of state (EoS)

Conclusions and outlook



Connecting to low energy effective models

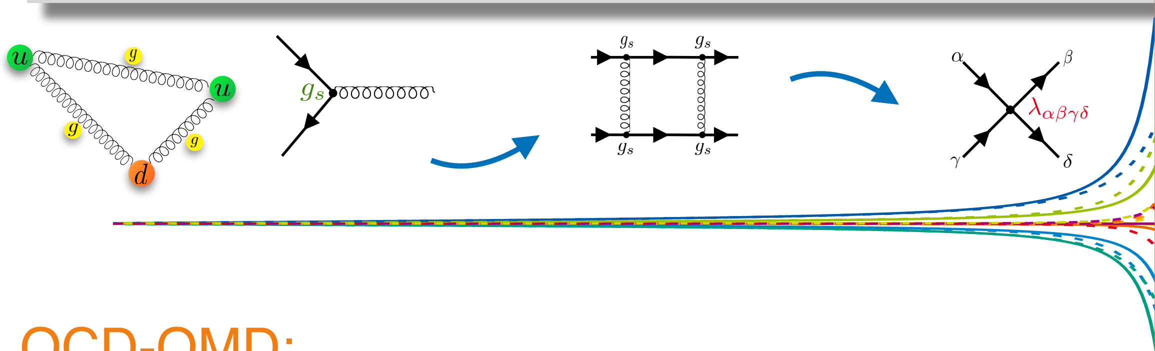
Determining parameters at high densities

TECHNISCHE UNIVERSITÄT DARMSTADT

Quark-meson-diquark model (QMD)

$$S = \int_x \left\{ \bar{q}\Big(\mathrm{i}\not{\partial} - \mathrm{i}\mu\gamma_0 + \mathrm{i}(\sigma + \mathrm{i}\vec{\tau}\cdot\vec{\pi}\gamma_5) \Big) q + \frac{1}{2}\frac{1}{\bar{\lambda}_{(\sigma-\pi)}}\varphi^2 \right. \quad \text{QM part}$$

$$\left. + \bar{q}\gamma_5\tau_2\Delta_A^* T^A \mathcal{C}\bar{q}^T - q^T\mathcal{C}\gamma_5\tau_2\Delta_A T^A \mathcal{C} q + \frac{1}{2}\frac{1}{\bar{\lambda}_{\mathrm{csc}}}|\Delta|^2 \right\} \quad \text{diquarks}$$



$$\Gamma_{\Lambda_0} = \frac{1}{2}\frac{1}{\bar{\lambda}^{(\Lambda_0)}_{(\sigma-\pi)}}\bar{\varphi}^2 + \frac{1}{2}\frac{1}{\bar{\lambda}^{(\Lambda_0)}_{\mathrm{csc}}}|\bar{\Delta}|^2$$

e.g.
[K. Rajagopal and F. Wilczek, 2000]
[M. Buballa, 2005]
[I. A. Shovkovy, 2005]
[M. G. Alford et al., 2008]

QCD-QMD:

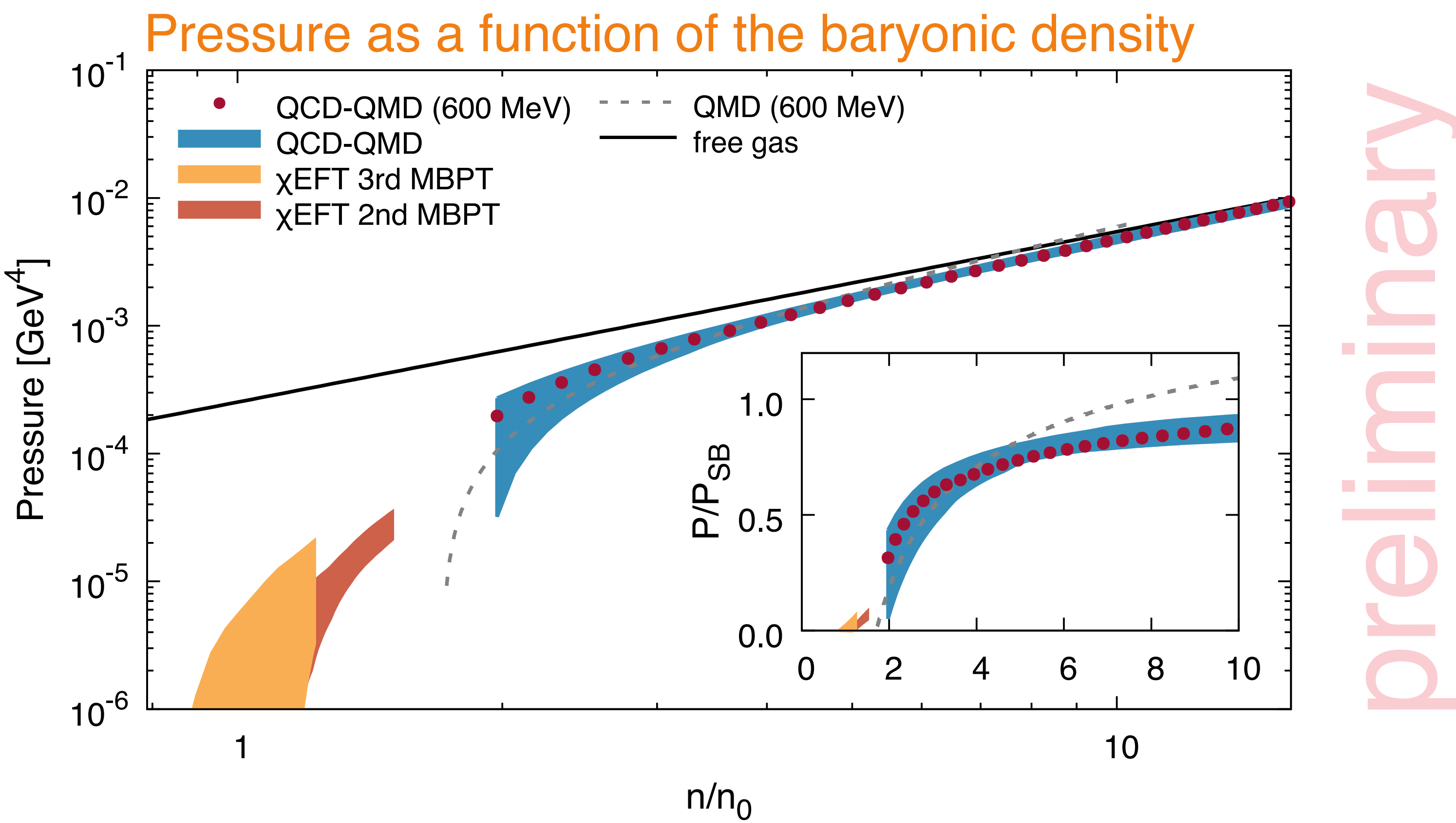
$$\Gamma_{\Lambda_0} = \frac{1}{2}\frac{1}{\bar{\lambda}_{(\sigma-\pi)}(k=\Lambda_0)}\bar{\varphi}^2 + \frac{1}{2}\frac{1}{\bar{\lambda}_{\mathrm{csc}}(k=\Lambda_0)}|\bar{\Delta}|^2$$

SFB 1245

Implications on the equation of state

Connecting to the quark-meson-diquark model

TECHNISCHE UNIVERSITÄT DARMSTADT



[χEFT data by C. Drischler, K. Hebeler and A. Schwenk]

- Compatible to pQCD [E. S. Fraga, A. Kurkela, and A. Vuorinen, 2015]
- Degrees of freedom at smaller densities [M. Drews and W. Weise, 2016]

SFB 1245

Conclusions and outlook

TECHNISCHE UNIVERSITÄT DARMSTADT

Dynamic gauge fields

- Dynamical generation of four-quark interactions
- Importance of Fierz completeness at high quark chemical potential and low temperature
- Formation of CSC condensates appears to be favoured at high densities

Connecting to LEMs: quark-meson-diquark model

- RG flow of four-quark couplings to fix parameters at high densities
- Equation of state:
  - Connects to free gas pressure at asymptotically large densities
  - Pressure towards smaller densities lifted to higher values; compatibility with χEFT results appears to be improved

Outlook

- Improve connection between high- and low-energy regime by employing dynamical hadronization techniques

SFB 1245