Rotating/magnetized fermionic matter confined in a finite-size system

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HL. Chen, K. Fukushima, XG. Huang, KM, PRD 93, 104052 (2016)
S. Ebihara, K. Fukushima, KM, PLB 764, 94 (2017)
HL. Chen, K. Fukushima, XG. Huang, KM PRD 96, 054032 (2017)

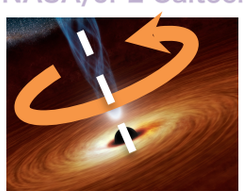
Rotating Relativistic Systems

・binary star merger

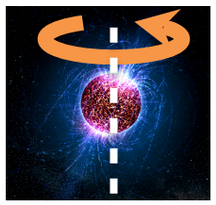
NASA/Tod Strohmayer

・black hole

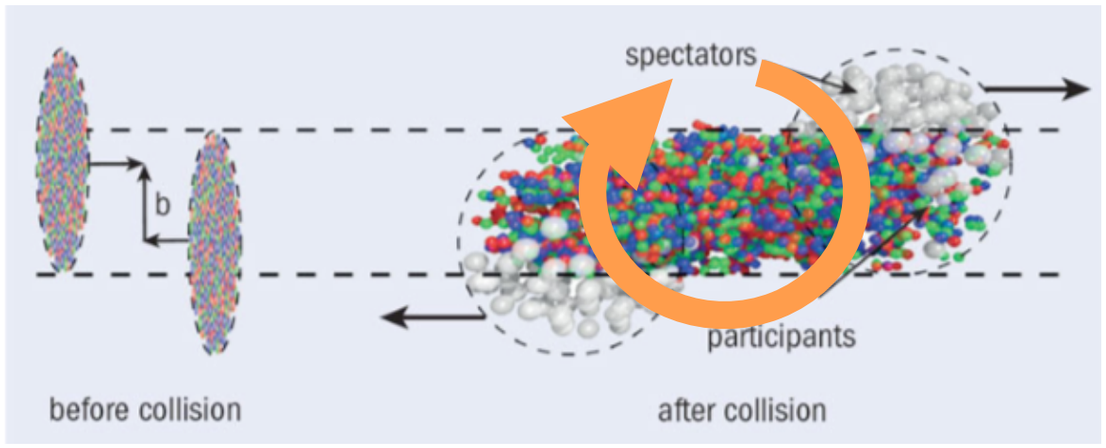
NASA/JPL-Caltech



・neutron star



・noncentral heavy-ion collision



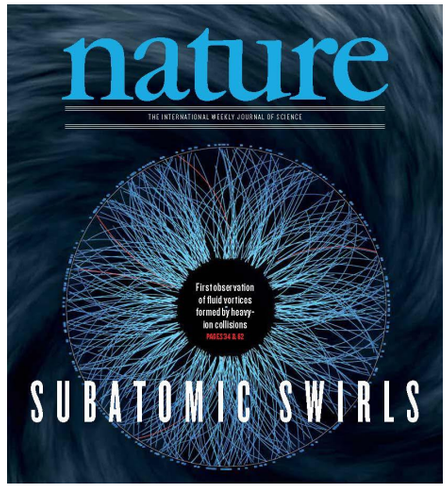
Angular momentum of heavy-ions

⟶ Vorticity (or Rotation)

First evidence:

global Λ-polarization (2017)

$$\Omega \gtrsim 10^{21}\ \mathrm{Hz}$$



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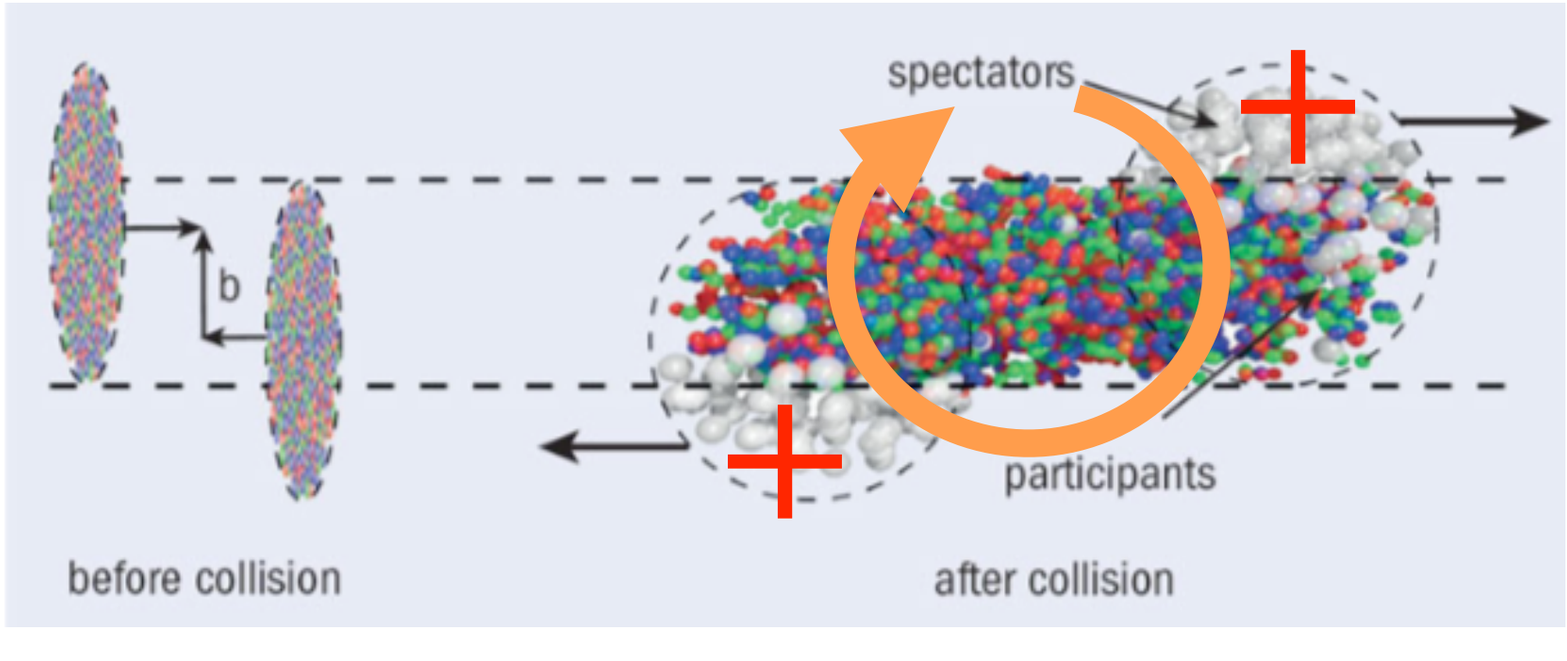
The Fastest Fluid

by Sylvia Morrow

Superhot material spins at an incredible rate.

Necessary to know Rotational effect on QFT

Rotating Magnetized System



angular momentum conservation

$$\Omega \sim 10^{22}\,\mathrm{Hz} \simeq 10\,\mathrm{MeV}$$ STAR (2017)

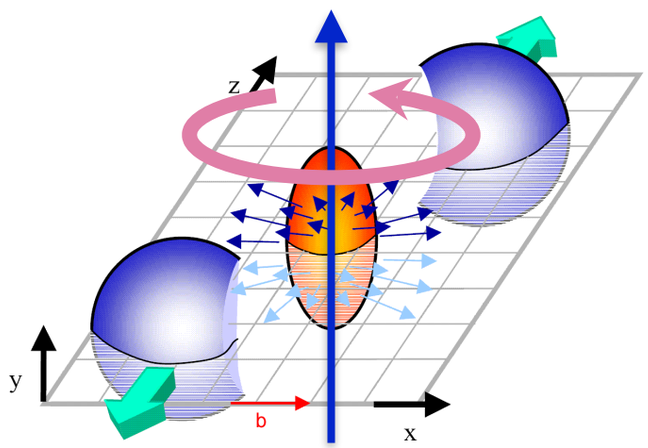
Ampere's law

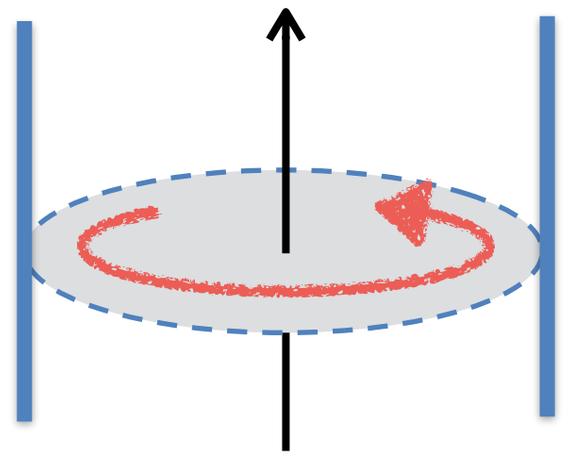
$$eB \sim 10^{18}\,\mathrm{G} \simeq m_\pi^2$$ Skokov, Illarionov, Toneev (2009)

How are the **magnetic** properties modified by **rotation**?

e.g. anomalous transport and chiral symmetry breaking

Rigidly Rotating System

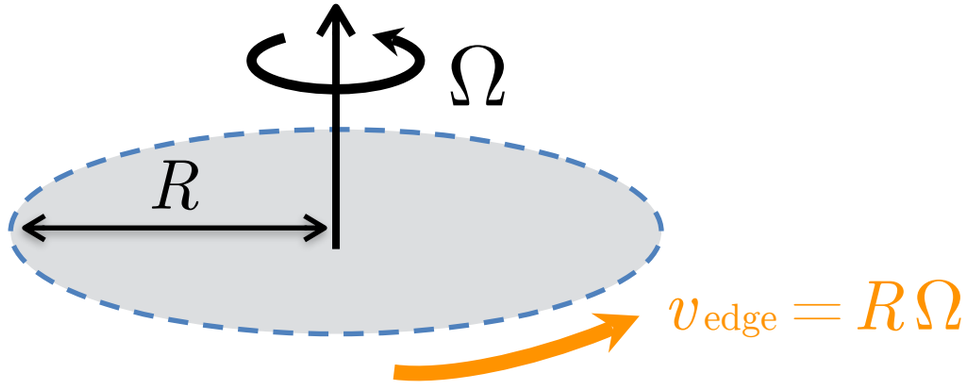




causality constraint

$$v_{\rm edge} = \Omega R \leq 1$$

$$\longrightarrow \quad R \leq 1/\Omega < \infty$$



Rotating systems must be finite-sized

coordinate transformation

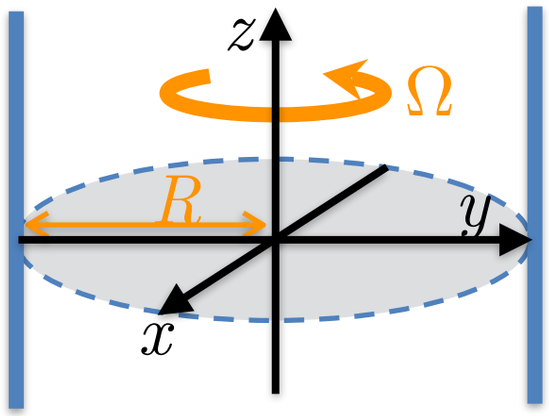
$$x^\mu_{\rm lab} \longrightarrow x^\mu_{\rm rot}$$

Contents

1. Finite-size system with Ω

Rotating Fermions

$$[i\gamma^\mu(\partial_\mu + \Gamma_\mu) - m]\psi = 0$$



$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\longrightarrow \left[i\gamma^0 \left\{ \partial_0 + \Omega \left(-x\partial_2 + y\partial_1 - \frac{i}{2}\sigma^{12} \right) \right\} + i\gamma^1\partial_1 + i\gamma^2\partial_2 + i\gamma^3\partial_3 - m \right] \psi = 0$$

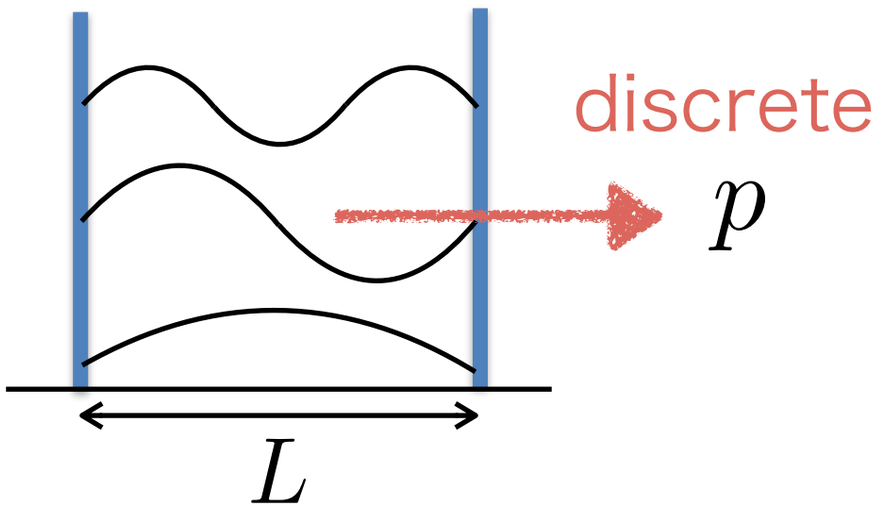
rotational energy (effective chemical potential)

Fermi distribution $$f(\varepsilon) = \frac{1}{e^{\beta\{\varepsilon - \Omega(l+1/2)\}} + 1}$$

eigenvalue of $\hat{L}_z = -i(x\partial_y - y\partial_y)$

Momentum Discretization

Bosons confined in a well



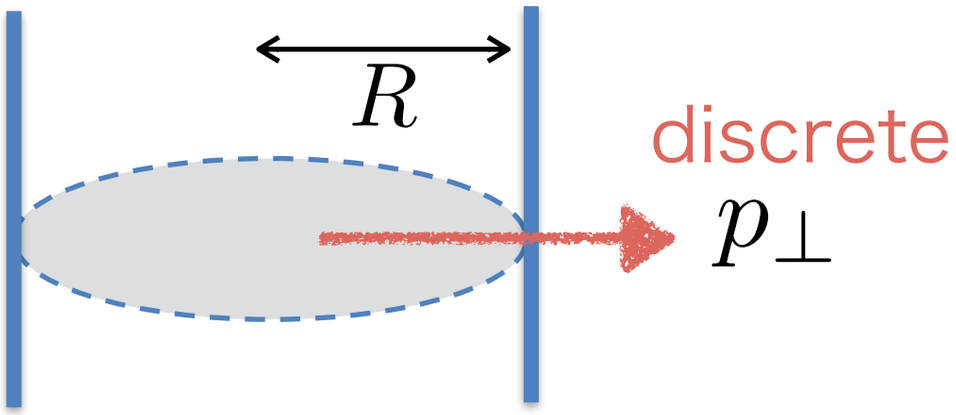
Dirichlet type

$$\sin(px)|_{x=L} = 0$$

$$\longrightarrow \quad p = \frac{n\pi}{L} \geq \frac{\pi}{L}$$

IR gap

Fermions confined in a cylinder



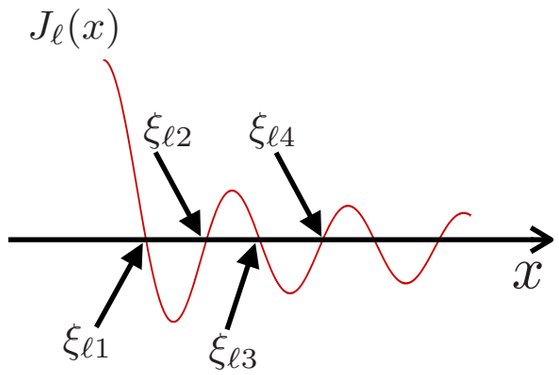
$\xi_{l,k}$: the kth root of $J_l(x)$

NO incoming current

$$\int \bar{\psi}\gamma^r\psi\Big|_{r=R} = 0$$

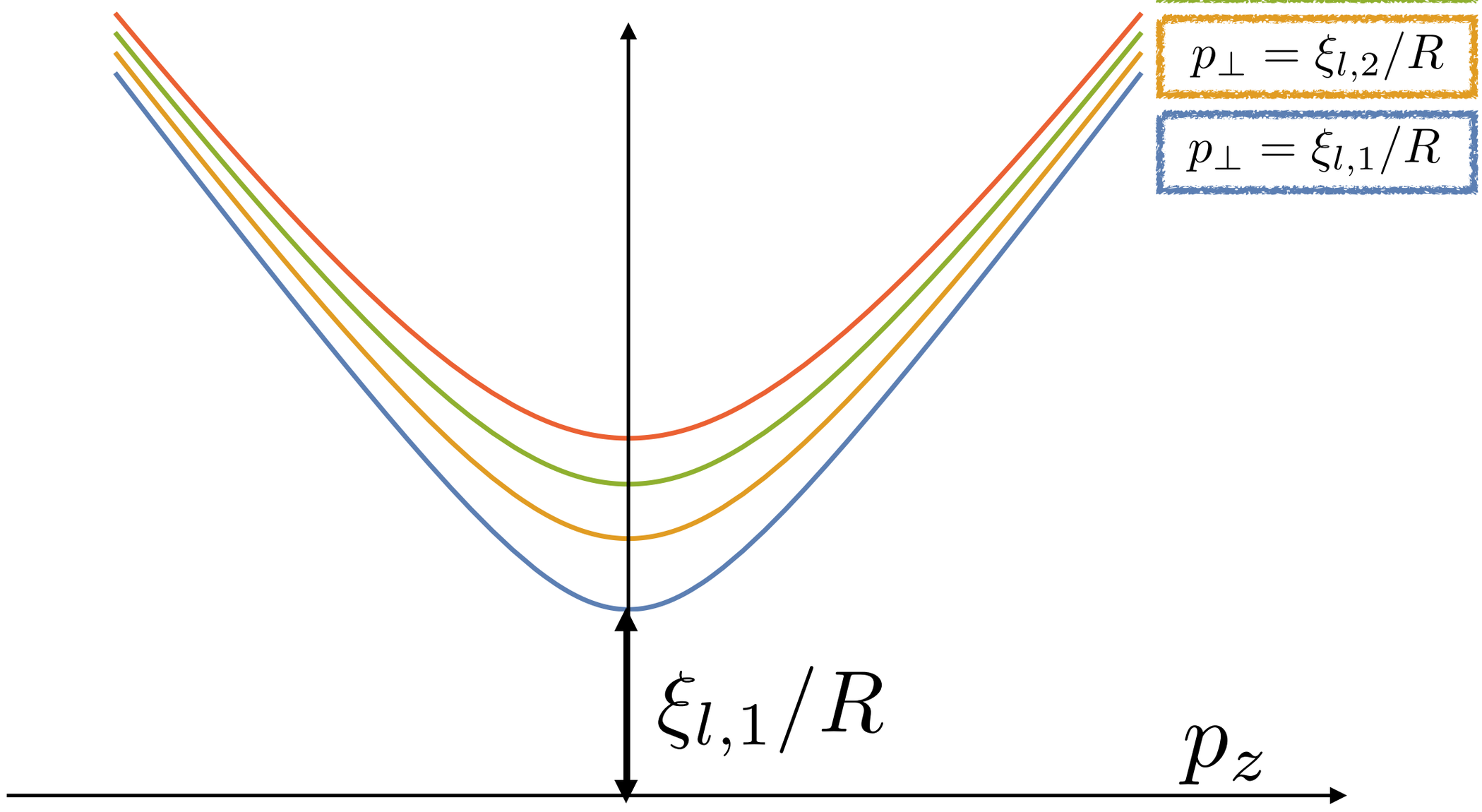
$$\longrightarrow \quad p_\perp = \frac{\xi_{l,k}}{R} \geq \frac{\xi_{l,1}}{R}$$

IR gap



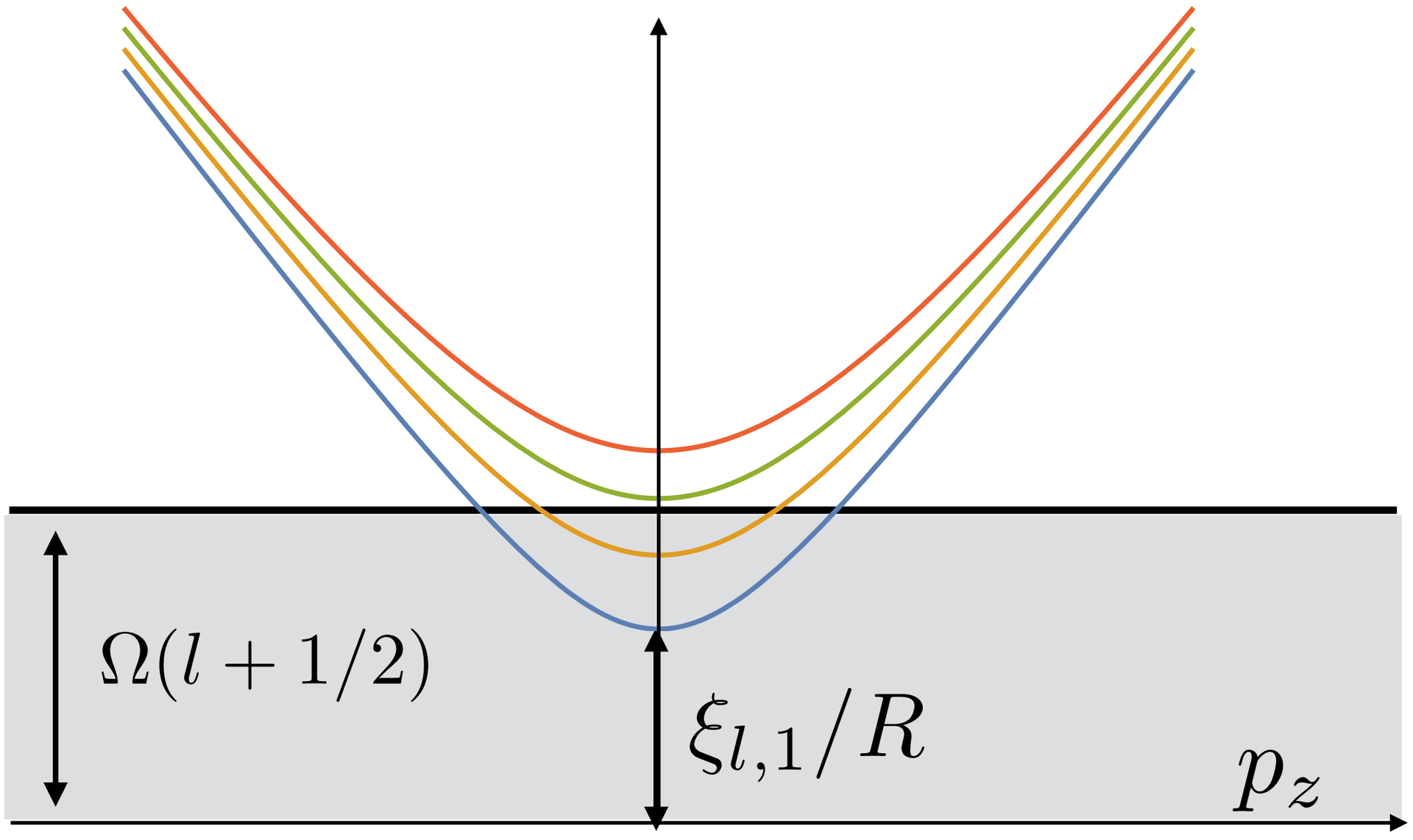
Rotational Effect at $T = 0$

$$\varepsilon = \sqrt{p_\perp^2 + p_z^2}$$



Rotational Effect at $T = 0$

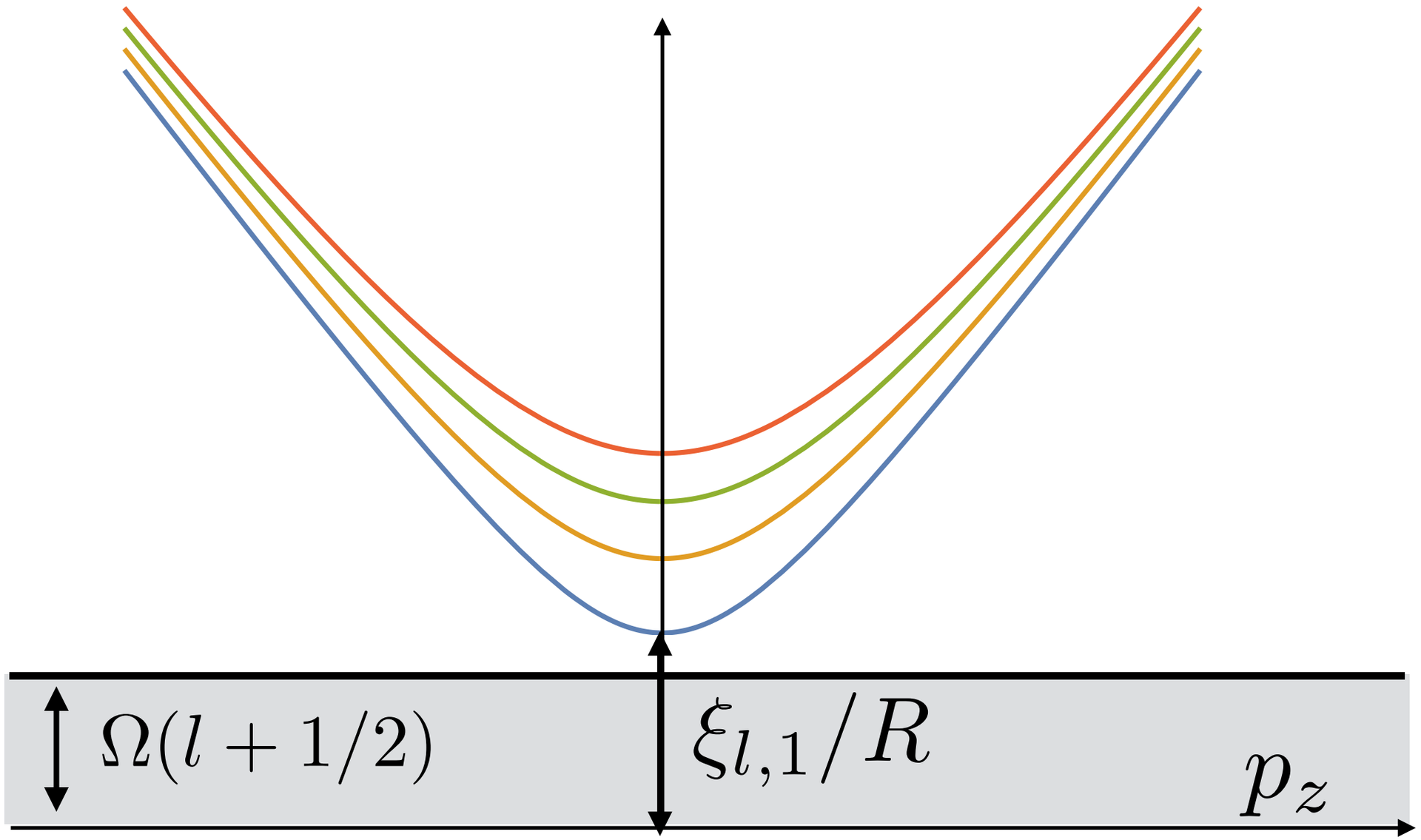
$$f(\varepsilon) = \frac{1}{e^{\beta\{\varepsilon - \Omega(l+1/2)\}} + 1} \longrightarrow f(\varepsilon) = \theta\big(\Omega(l+1/2) - \varepsilon\big)$$



visible

Rotational Effect at $T = 0$

$$f(\varepsilon) = \frac{1}{e^{\beta\{\varepsilon - \Omega(l+1/2)\}} + 1} \longrightarrow f(\varepsilon) = \theta\big(\Omega(l+1/2) - \varepsilon\big)$$



invisible

cf. Silver Blaze

Rotational Effect at $T = 0$

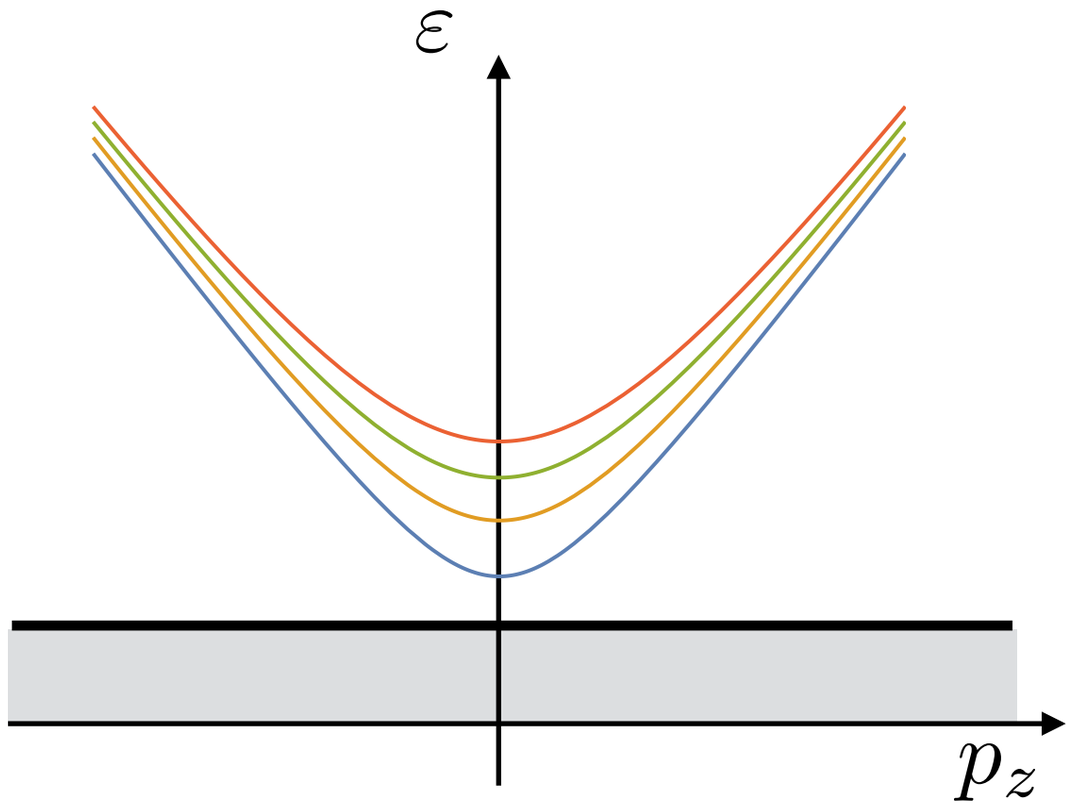
Ebihara, Fukushima, KM (2017)

causality $\Omega \leq 1/R$

$$\xi_{l,1}/R > \Omega(l + 1/2)$$

for arbitrary l

NO rotational effect at $T = 0$



* same discussion with the MIT b.c. Chernodub, Gongyo (2017)

Note : visible at finite temperature

CVE $j_5 = \dfrac{T^2}{12}\Omega$

Vilenkin (1979)

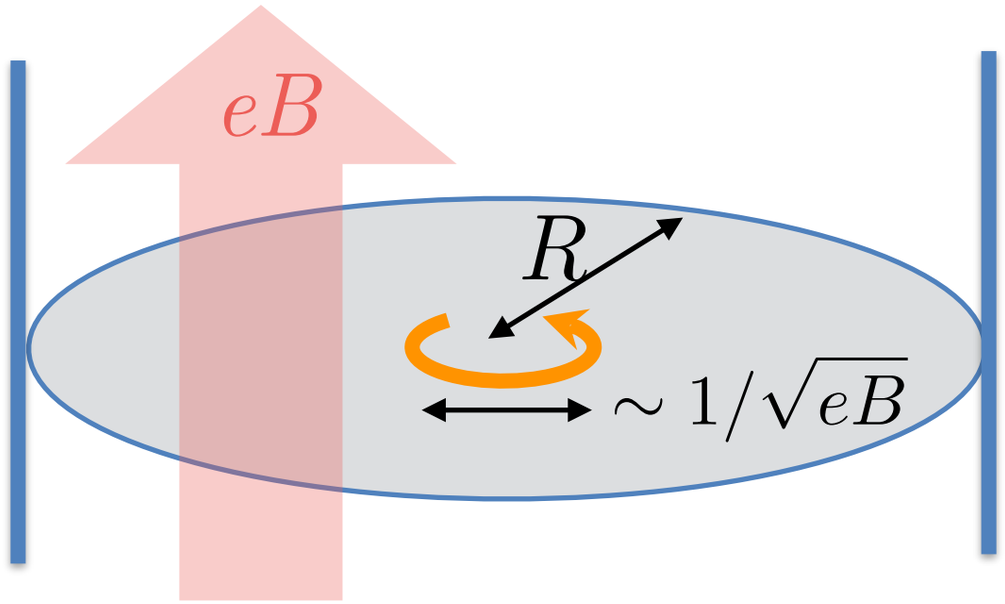
2. Finite-size system with eB

Cyclotron Motion

(1) $1/\sqrt{eB} \ll R$

→ $p_\perp = \sqrt{2neB}$

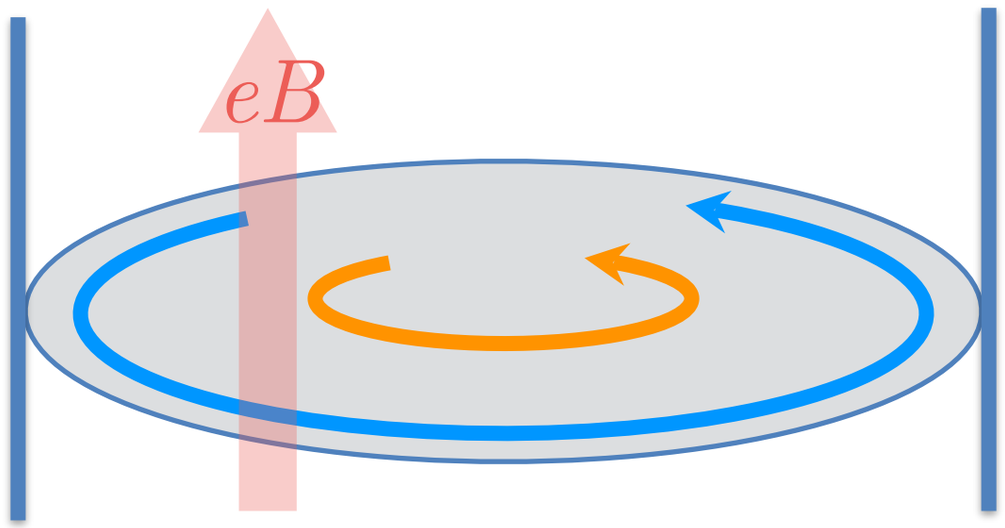
independent of l



(2) $1/\sqrt{eB} \lesssim R$

small l → still $p_\perp \simeq \sqrt{2neB}$

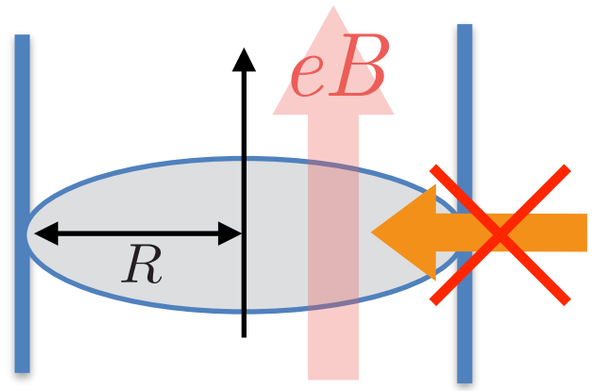
large l → modified



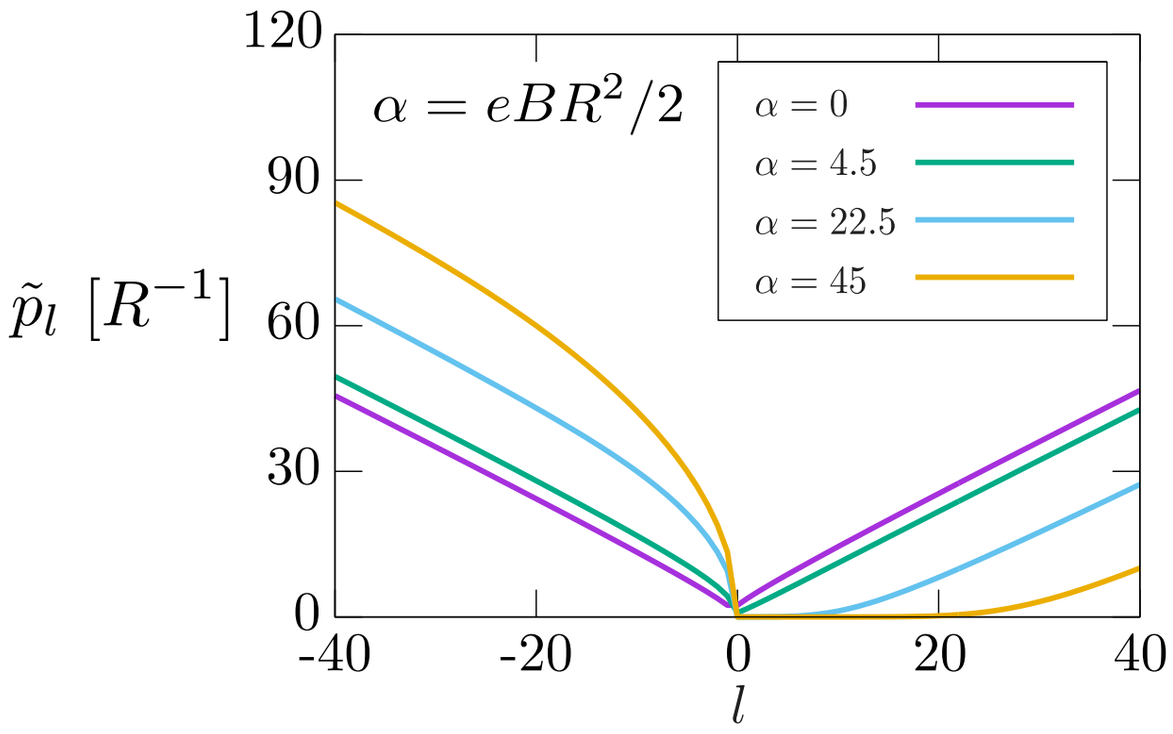
“Incomplete Landau quantization”

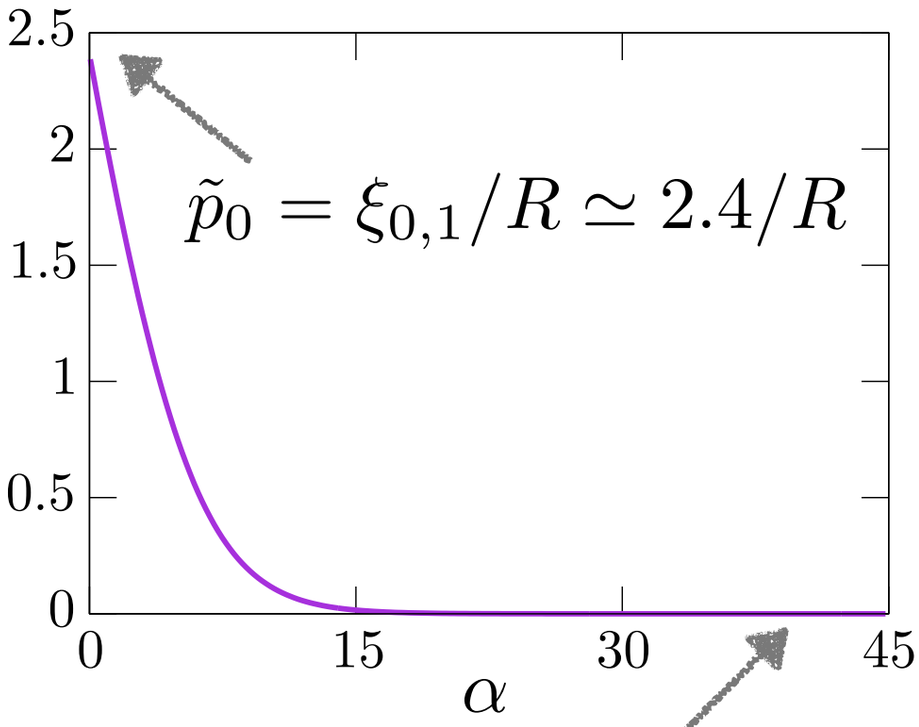
Incomplete Landau Level

$$[i\gamma^\mu(\partial_\mu + ieA_\mu) - m]\psi = 0 \text{ with}$$



$\tilde{p}_l$ = lowest transverse momentum for l





disfavor
$l < 0$

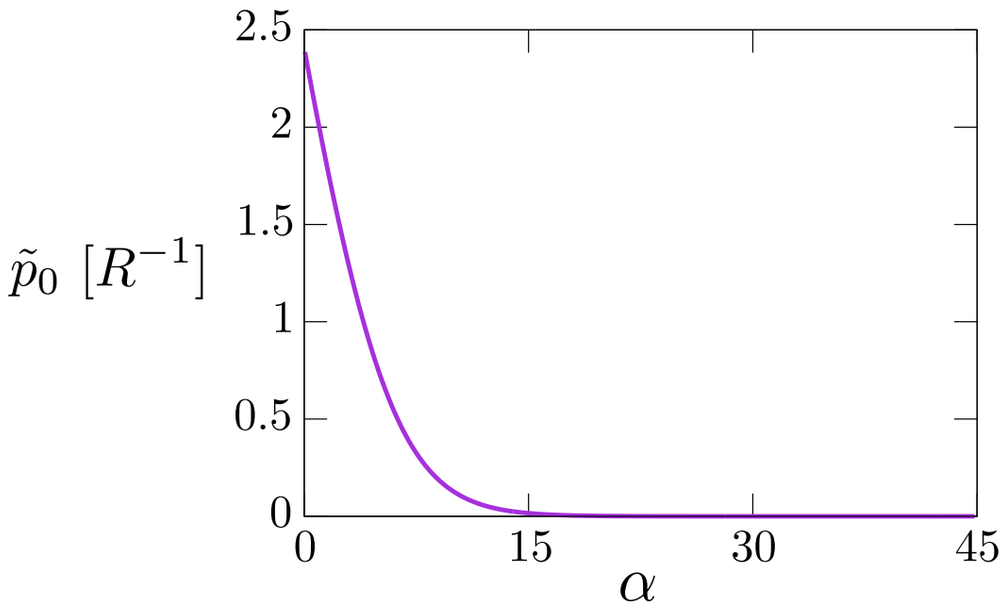
favor
$l \geq 0$

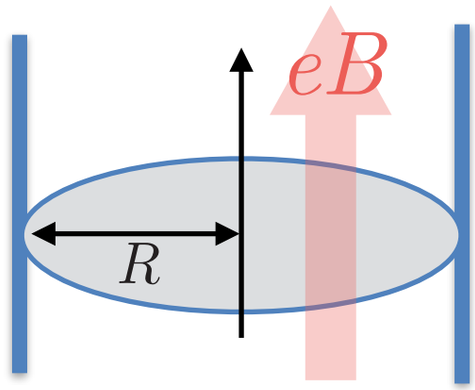
eB

$\tilde{p}_0 = 0$
Landau zero mode

3. Finite-size system with Ω and eB

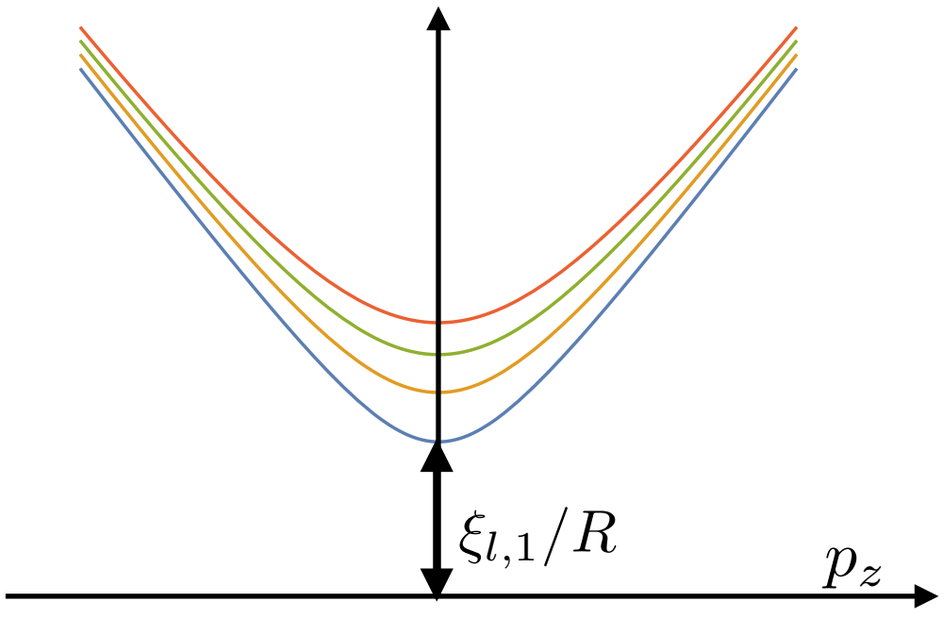
Gapped to Gapless

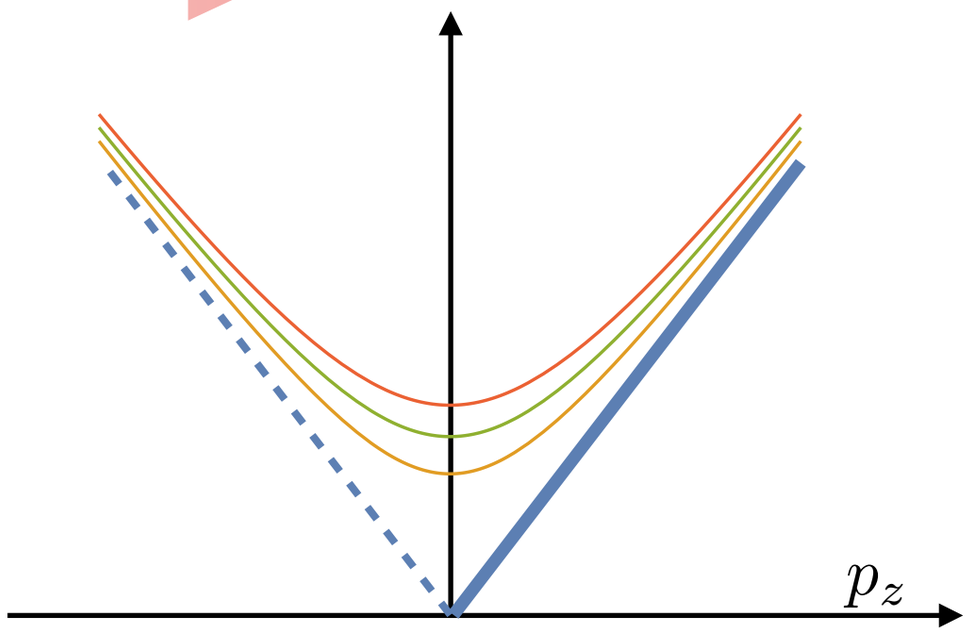




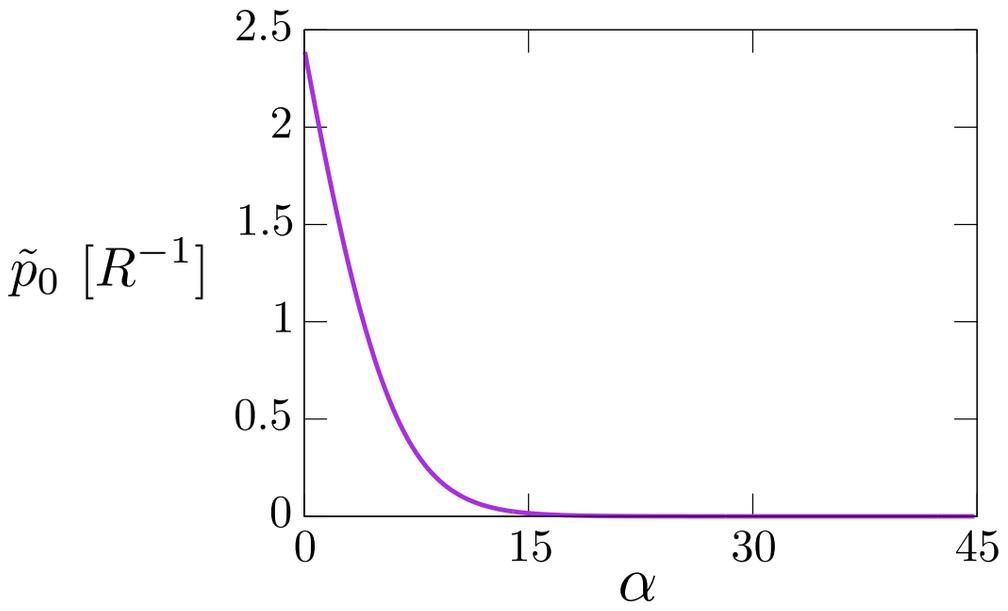
$$\alpha = eBR^2/2$$

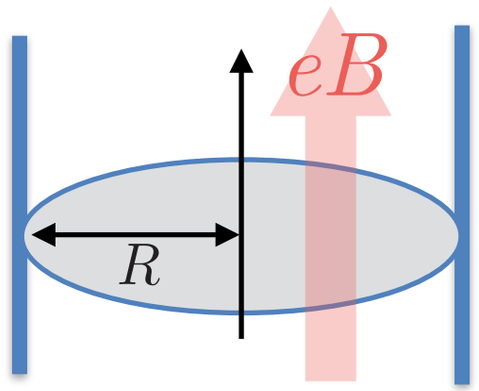
weak magnetic field strong





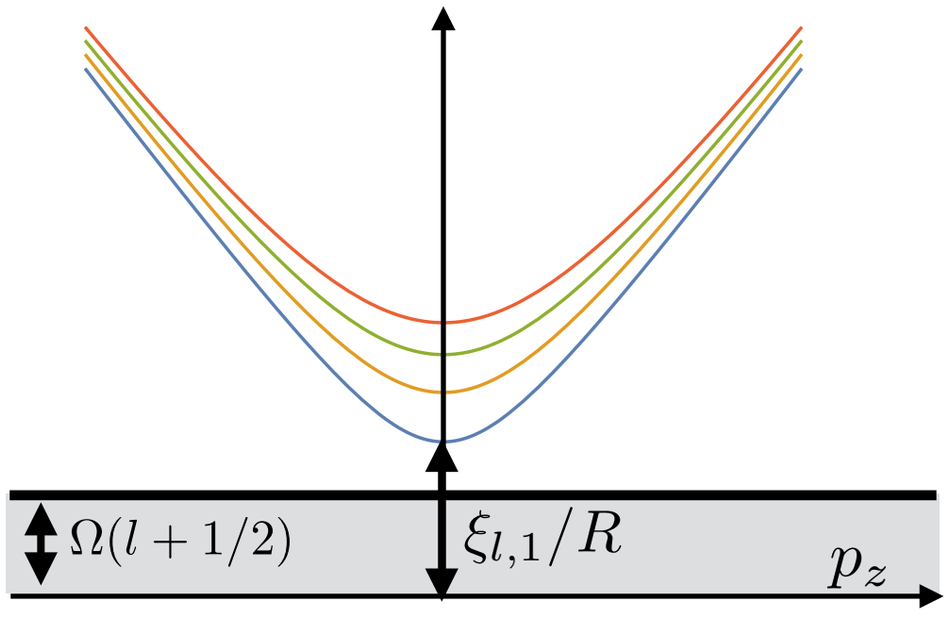
Gapped to Gapless

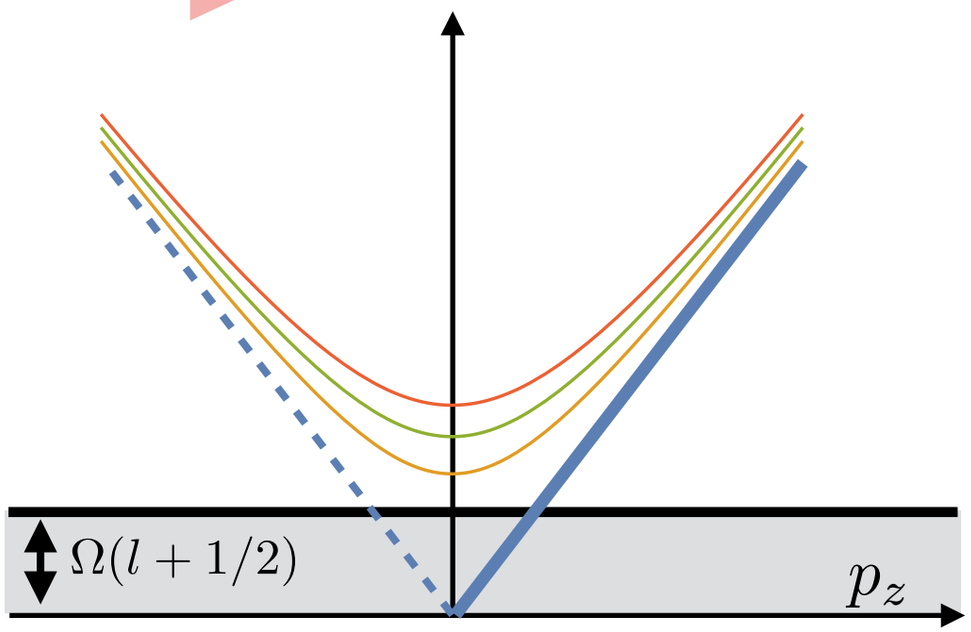




$$\alpha = eBR^2/2$$

weak magnetic field strong





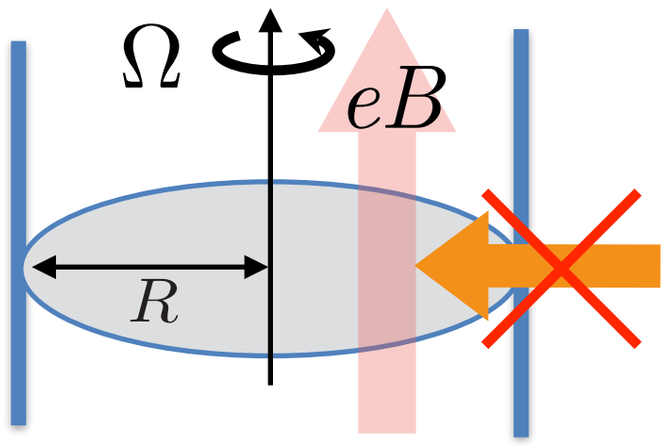
visible rotational effect due to **magnetic field**

Ex.1) Anomalous Density

Ebihara, Fukushima, KM (2017)
Hattori, Huang, KM (in preparation)

magnetic field　　rotation

$$[i\gamma^\mu(\partial_\mu + ieA_\mu + \Gamma_\mu)]\psi = 0 \quad \text{with}$$



$$n(r) = \langle \psi^\dagger(x)\psi(x)\rangle \qquad f_\pm(\varepsilon) = \frac{1}{e^{\beta(\varepsilon \mp \Omega j)} + 1}$$

$$= \sum_{p_z, p_\perp} \Big[f_+(\varepsilon) - f_-(\varepsilon) \Big] \times \Big(r\text{-dependence} \Big) \longrightarrow n(r=0) \underset{\sqrt{eB} \gg \Omega}{\longrightarrow} \frac{eB\Omega}{4\pi^2}$$

temperature independent

Cf. Hattori, Yin (2016)

imply a **tensorial** anomalous term

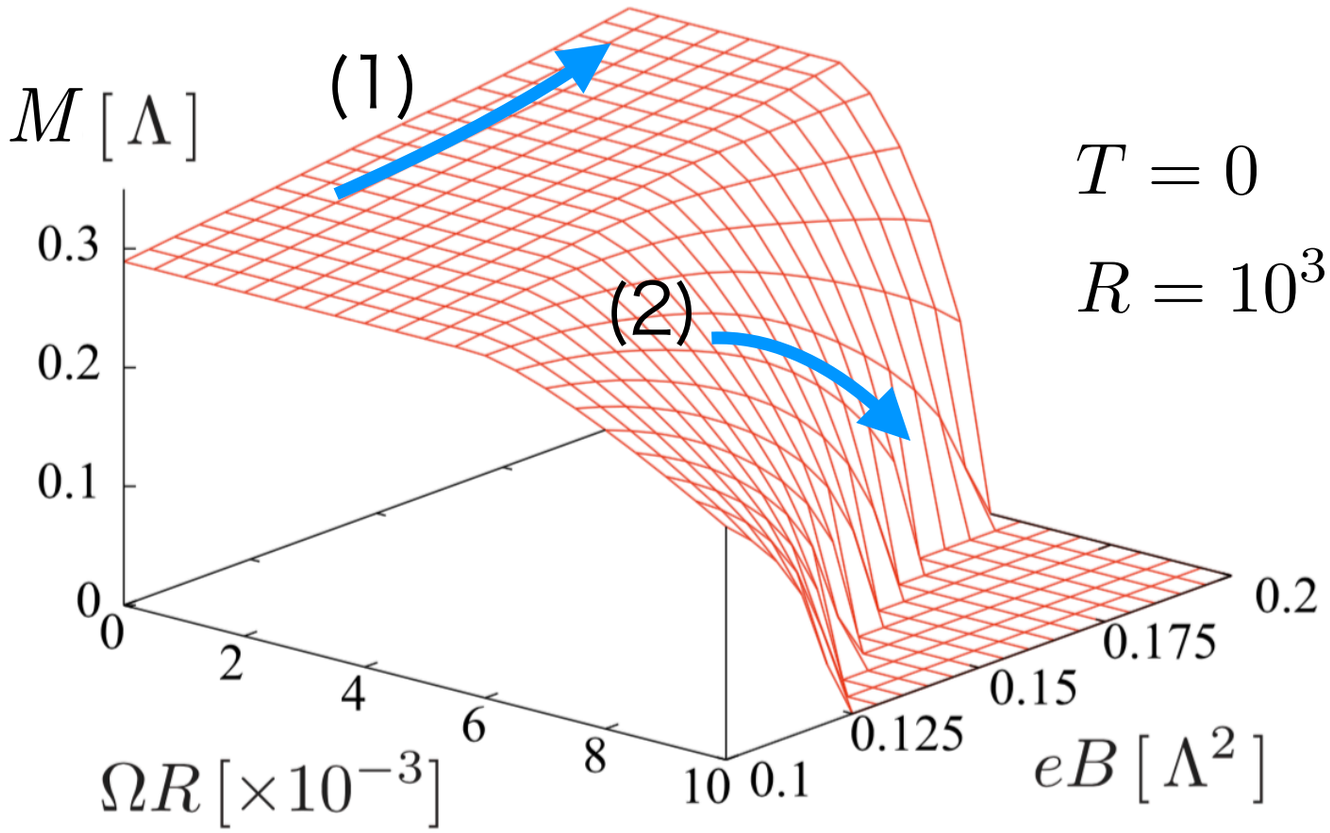
$$\Delta S \sim \mu \boldsymbol{B} \cdot \boldsymbol{\Omega} = \mu g_{ij} B^i \Omega^j \xrightarrow{\dfrac{\delta \Delta S}{\delta g_{ij}}} T^{ij} = \# \, eB^i \Omega^j$$

Cf. Hermandez, Kovtun (2017)

Ex.2) Chiral Symmetry

Chen, Huang, Fukushima, KM (2016)

NJL model (mean field approx.) + neglecting inhomogeneity



$T = 0$

$R = 10^3\ [\Lambda^{-1}]$

(1) eB increases ⟶ M increases Magnetic Catalysis

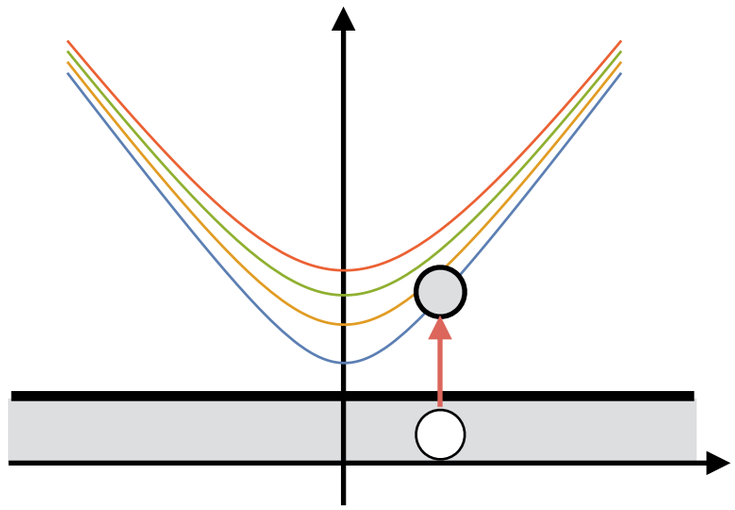
(2) eB increases ⟶ M decreases **Inverse of MC**

‘Rotational magnetic inhibition’

Summary: Rotation effect requires …

- **temperature**

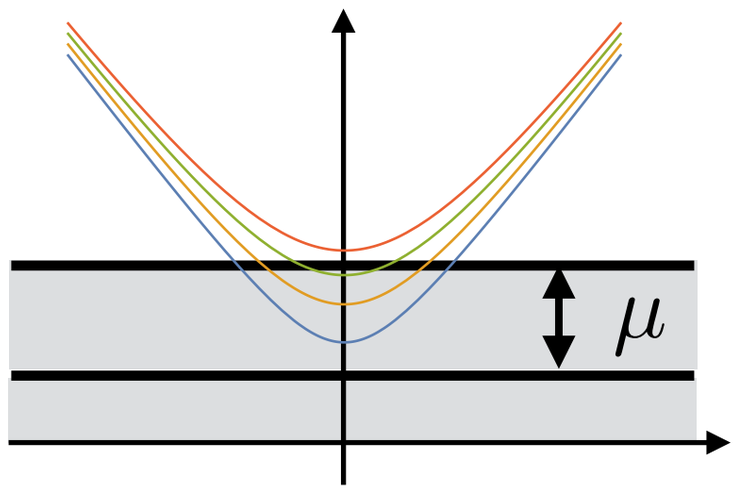
  thermal excitation



Jiang, Liao (2016)
Chernodub, Gongyo (2016)

- **density**

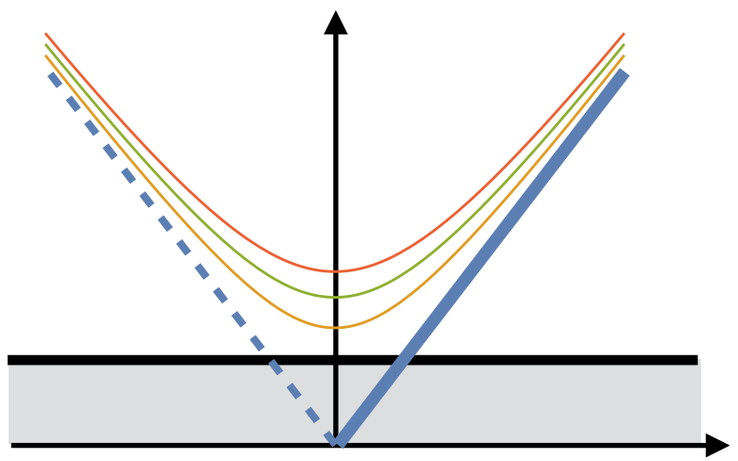
  Fermi surface shift



Huang, Nishimura,
Yamamoto (2017)

- **magnetic field**

  Landau zero mode



Chen, Huang,
Fukushima, KM (2016)
Liu, Zahed (2017)

Outlook

- Rotation yields abundant phase structures
  Chen, Huang, Fukushima, KM (2016)
  Jiang, Liao (2016) Chernodub, Gongyo (2016)
  Liu, Zahed (2017) Huang, Nishimura, Yamamoto (2017)

- Rotational magnetic inhibition : spacial dependent chiral condensate

- Novel anomalous (magneto-vorticical) correction $T^{ij} = \# eB^i\Omega^j$
  Hattori, Huang, KM (in preparation)

- Finite-size system under B-field EdH effect for chiral fermions
  Fukushima, Hirono, Huang, Kharzeev, KM (in preparation)