Model dependence of the pasta-structure effects in the quark-hadron mixed phase

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Introduction

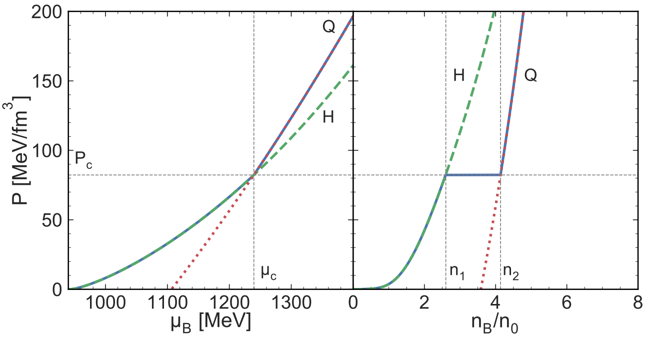
- Neutron stars are natural laboratories for studying the matter at low temperatures, large baryon densities $n_B = (5-10)n_0$ and large isospin asymmetries $\beta = \frac{n_n - n_p}{n_B} \sim 1$, where n_0 – nuclear saturation density and n_n, n_p - neutron and proton number densities
- Many possible phases of dense matter are relevant for NS physics:
  - Liquid-gas phase transition (PT) at the inner crust-core boundary
  - Hadronic phase with more baryons species - hyperons, Δ
  - Meson (K, π, ρ^-) condensates
  - Quark-hadron (QH) PT $\leftarrow$ *this work*
  - Phases of quark matter - color superconductivity, dual chiral density wave, etc.
- The most restrictive constraint comes from the maximum precisely measured NS mass of $2.01 \pm 0.04\, M_\odot$
- A lot of new data is expected from modern tools $\Rightarrow$ new constraints:
  - Simultaneous measurements of NS masses and radii
  - New gravitational wave detections

Quark-hadron phase transition

Enforced local neutrality – Maxwell construction

Local electric neutrality condition requires a specific $\mu_e = \mu_e(\mu_B)$, so only one chemical potential μ_B is left to fulfill the Gibbs conditions

$$\mu_B^{(H)} = \mu_B^{(Q)} \equiv \mu_c, \quad P^{(Q)}(\mu_c) = P^{(H)}(\mu_c) \equiv P_c$$

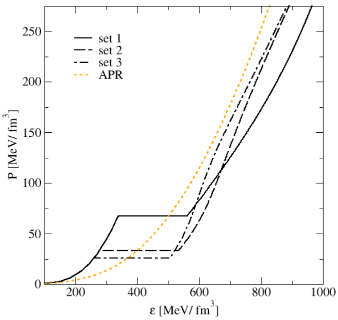


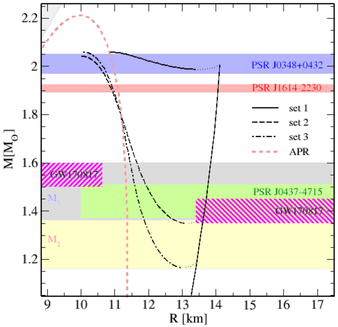
Baryon density and energy density ε is discontinuous across the phase transition with the energy jump $\Delta\varepsilon = \varepsilon(n_2) - \varepsilon(n_1)$.

Third family of compact stars (twin stars)

With a Maxwell construction a disconnected branch of compact stars may appear [U. Gerlach Phys.Rev. 172 1235 (1968)] if the quark matter is sufficiently stiff at large n_B, with the condition for the instability: [Z.F. Seidov Sov.Astron. 15 347 (1971)])

$$\frac{\Delta\varepsilon}{\varepsilon_c} \geq \frac{1}{2} + \frac{3}{2}\frac{P_c}{\varepsilon_c}, \quad \varepsilon_c \equiv \varepsilon(n_1), \quad \Delta\varepsilon = \varepsilon(n_2) - \varepsilon_c$$





[D. Alvarez-Castillo,D. Blaschke,A. Grunfeld,V. Pagura arXiv:1805.04105] - for QH PT

A measurement of NSs with same masses and different radii would prove the existence of a first-order PT, e.g. QH PT ⇒ QCD critical point, meson condensation, etc.

More consequences - second neutrino burst and starquake during the NS formation; extra energy release for blowing off the supernova envelope [A.B. Migdal et al. Phys.Rept.192(1990) 179-437]

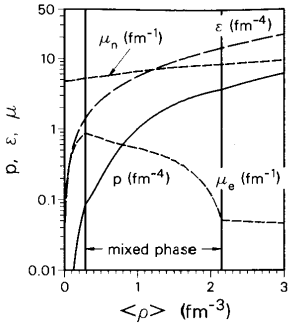
Phase transition with two conserved charges

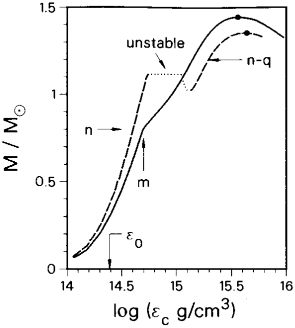
In general, the conservation laws can be obeyed globally, not locally [N.K. Glendenning Phys.Rev. D46 (1992) 1274-1287]

The Gibbs conditions

$$\mu_B^{(H)} = \mu_B^{(Q)}, \quad \mu_e^{(H)} = \mu_e^{(Q)}, \quad P^{(H)}(\mu_B^{(H)}, \mu_e^{(H)}) = P^{(Q)}(\mu_B^{(Q)}, \mu_e^{(Q)})$$

now have solutions over a range of μ_B





A puzzle - mixed phase or Maxwell construction?

Finite-size effects

Coulomb interaction

Tends to break up the like-charged regions into smaller ones

vs

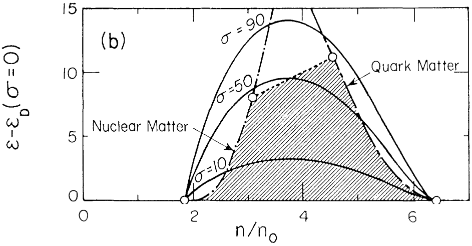
Surface tension

Requires minimization of the surface

$\Rightarrow$ formation of structures with d = 3,2,1 (droplets, rods, slabs)

Thin diffuseness layer $\sim$ 1 fm $\Rightarrow$ can describe the surface contribution using the surface tension parameter σ – not known and hard to calculate; usually treated as a parameter

For the surface tension σ larger, than some critical σ_c formation of structures becomes energetically unfavorable



solid lines - energy density of the droplets for various σ [MeV/fm^2] relative to $\sigma = 0$

dashed line - Maxwell construction

adapted from [Heiselberg Pethick Staubo Phys.Rev.Lett. 70 (1993) 1355-1359]

Critical surface tension σ_c depends on the model

Solution to the puzzle - treatment of the electric field

Wigner-Seitz approximation with the cell radius R_W (consider d=3)
Self-consistent treatment of electrostatic potential: $\mu_e \to \mu_e - V(r)$
Equations of motion for the electric field potential in a phase $p = H, Q$

$$\Delta V^{(p)}(r) = 4\pi e^2 n_{\rm ch}^{(p)}[\mu_B, \mu_e - V^{(p)}(r)]$$

$\Rightarrow$ nonuniform electron density distribution and charge screening
Linearized version defines Debye screening lengths in a phase $p = H, Q$

$$\Delta \delta V^{(p)}(r) = 4\pi e^2 n_{\rm ch}^{(p)}[\mu_B, \mu_e - V_{\rm ref}] + (\lambda_D^{(p)})^{-2} \delta V(r),$$

$$\delta V^{(p)}(r) = V(r) - V_{\rm ref}^{(p)}, \quad (\lambda_D^{(p)})^{-2} = -4\pi e^2 \left(\frac{\partial n_{\rm ch}^{(p)}}{\partial \mu_e} \right)_{\mu_B}$$

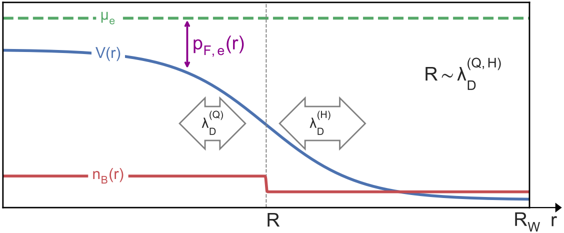
Linearized equation can be solved analytically with matching and boundary conditions (R - radius of a droplet)

$$V^{(Q)}(R) = V^{(H)}(R), \quad (\frac{d}{dr} V^{(Q)})(R) = (\frac{d}{dr} V^{(H)})(R), \quad (\frac{d}{dr} V^{(H)})(R_W) = 0$$

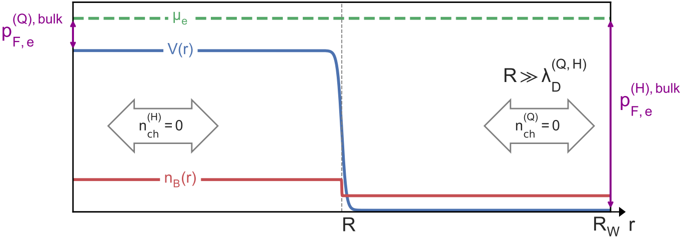
[D.N. Voskresensky, M. Yasuhira, T.Tatsumi PLB 541 (2002) 93-100, NPA 723 (2003) 291-339]

Schematic effect of the screening

1. Case $R \sim \lambda_D$: smooth non-uniform electron density distribution
$\lambda_D \sim 1/e^2 \gg$ diffuseness layer thickness $l \sim 1$ fm $\Rightarrow$ neglected in $n_B(r)$ profile



2. Case of large droplets $R \gg \lambda_D^{(Q,H)}$: electric fields contributes only in the thin border layer $\Rightarrow$ contribution to the effective surface tension



Far from the border – bulk solution with $n_{\rm ch}^{(Q,H)} = 0$ – Maxwell case

Critical surface tension

In the large droplet limit the energy per cell is (in terms of $\xi = R/\lambda_D^{(Q)}$, no muons)

$$\epsilon \simeq \frac{3}{\beta \lambda_D^{(Q)}} \frac{\sigma - \sigma_c}{\xi}$$

$$\sigma_c = \lambda_D^{(Q)} \frac{\alpha\beta(\alpha + 4/3)}{3(1+\alpha)^2},\ \alpha = \frac{\lambda_D^{(Q)}}{\lambda_D^{(H)}},\ \beta = \frac{3(\mu_e^{(H),\mathrm{bulk}})^2}{8\pi e^2 (\lambda_D^{(Q)})^2},$$

The critical value σ_c comes entirely from the electrostatic contribution

For $\sigma > \sigma_c$ energy minimization leads to

$\xi \to \infty \Rightarrow$ Maxwell construction

Model dependence resides in $\mu_e^{(H)}(\mu_B)$, $\lambda_D^{(Q,H)}(\mu_B, \mu_e)$

Recent work

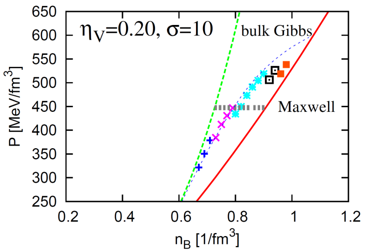
Similar structure formation has been studied for many phase transitions:

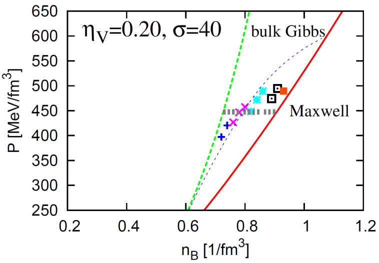
- Nuclear liquid-gas phase transition in NS crust
  Both phases within the same model $\Rightarrow$ no need to introduce the surface tension parameter [T. Maruyama et al. PRC72 (2005) 015802]
  Can be also studied using molecular dynamics
  [A.S. Scheider et al. PRC88 (2013) no.6, 065807]
  + many, many more works...
- Kaon condensation
  [T. Maruyama et al. PRC73 (2006) 035802]
- Quark-hadron phase transition
  [N. Yasutake et al. PRC89 (2014) 065803,
  X. Wu, H. Shen PRC96 (2017) no.2, 025802]

Current work: quark-hadron phase transition
How does the pasta affect the third family?
How strong is the model dependence?

Effect on the EoS

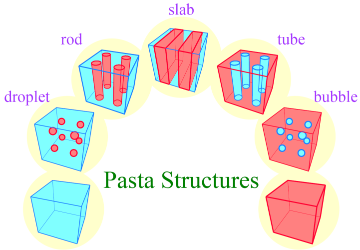
Typical result for the pressure with pasta:





[N. Yasutake et al. PRC89 (2014) 065803]

Different symbols - different structures (figure from [N. Yasutake et al. Phys.Rev. D80 (2009) 123009])



Pressure goes between bulk Gibbs ($\sigma = 0$) and Maxwell ($\sigma > \sigma_c$) constructions

Possible effect results in blurring of the phase transition

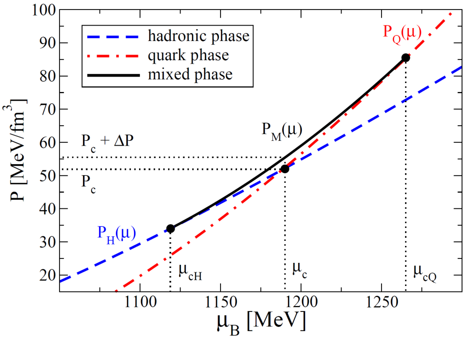
Its effect on the third family can be studied phenomenologically

Phenomenological description

[A.Ayriyan et al. Phys.Rev. C97 (2018) no.4, 045802, EPJ Web Conf. 173 (2018) 03003]

Simple parabolic interpolating construction in terms of $P(\mu)$ (here and below $\mu \equiv \mu_B$):

$$P(\mu) = \begin{cases} P^{(H)}(\mu), & \mu < \mu_{cH}, \\ a(\mu - \mu_c)^2 + b(\mu - \mu_c) + P_c + \Delta P, & \mu_{cH} < \mu < \mu_{cQ}, \\ P^{(Q)}(\mu), & \mu_{cQ} < \mu, \end{cases}$$

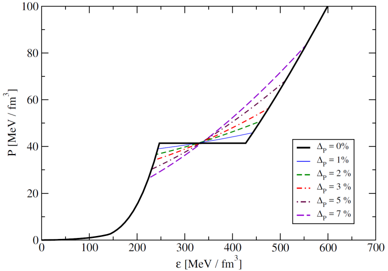


ΔP - the only parameter of the construction - excess of the pressure relative to the Maxwell caused by the structures

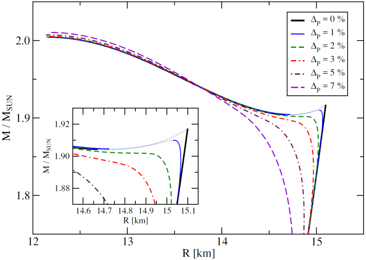
The parameters μ_{cH}, μ_{cQ}, a, b are determined by requiring the continuity of the pressure and its first derivative over μ - the baryon density

Effect on the third family

The construction in terms of $P(n)$ for a given pair of hadronic and quark models for various $\Delta_P \equiv \frac{\Delta P}{P_c}$



Disconnected third branch can disappear if the PT is blurred for a large enough Δ_P



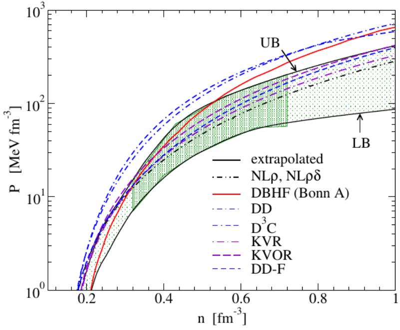
Does the construction describe the pasta adequately?
What is the relation between Δ_P and the surface tension σ?

Logic of the work

1. Perform the numerical calculation of the pasta phases for a set of hadronic and quark models, and for a range of σ
2. For each σ find the best fit parameter $\Delta_P \Rightarrow \Delta_P(\sigma)$
   Maximum possible $\Delta_P^{\max} = \Delta_P(\sigma = 0)$
3. Determine critical surface tension σ_c from the condition
   $\Delta_P(\sigma > \sigma_c) \simeq 0$
4. Investigate the model dependence of the σ_c and compare with analytical predictions
5. Evaluate properties of neutron stars and compare with known constraints

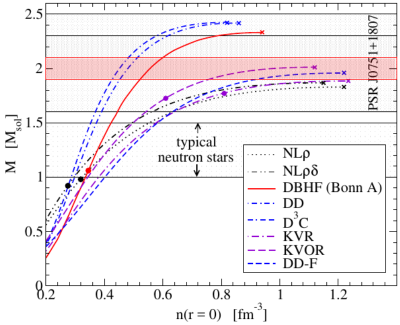
Need for a complicated model: contradicting constraints

Constraint for the pressure, obtained from analyses of transverse and elliptic flows in heavy-ion collisions
Passed by rather soft EoSs
[P. Danielewicz, R. Lacey, W.G. Lynch, Science 298 (2002)]



The maximum NS mass constraint favors stiff EoS

NS cooling data ⇒ direct URCA (DU) is not operative for most stars
⇒ constraint for the proton fraction



figures from [T. Klahn et al. PRC74 (2006)]

Hadronic models: framework

E. E. Kolomeitsev and D. N. Voskresensky NPA 759 (2005) 373

- Walecka-type model with in-medium change of masses and coupling constants of all hadrons in terms of the scalar field σ:

$$m_i^* = m_i \Phi_i(\sigma),\ g_{mB}^* = g_{mB}\chi_m(\sigma),$$
$$m = \{\text{mesons}\},\ B = \{\text{baryons}\},\ i = B \cup m$$

- Common decrease of hadron masses [Brown, Rho Phys. Rev. Lett. 66 (1991) 2720; Phys. Rept. 363 (2002) 85]:

$$\frac{m_N^*}{m_N} \simeq \frac{m_\sigma^*}{m_\sigma} \simeq \frac{m_\omega^*}{m_\omega} \simeq \frac{m_\rho^*}{m_\rho}$$

- In the infinite matter only $\eta_m(\sigma) = \frac{\Phi_m^2(\sigma)}{\chi_m^2(\sigma)}$ enter the EoS - we define them phenomenologically to pass the constraints

Below we use the dimensionless scalar field $f(n) \equiv \dfrac{g_{\sigma N}\chi_\sigma(\sigma)\sigma}{m_N}$

Working models

Initial model: KVOR [E.E.K., D.N.V. NPA 759 (2005)] described many constraints, but only without hyperons $\Rightarrow$ need for enhancement

Contributions of ω, ρ mesons to the pressure couple to the scalar field.
"Cut" mechanism: rapid decrease of $\eta_m(f)$ quenches the growth of the scalar field $f(n)$ and leads to the stiffening of an EoS
[K.A.M, E.E.K., D.N.V. PRC 92 (2015)].

KVORcut03	KVORcut02	MKVOR*
Based on KVOR Sharp decrease in $\eta_\omega(f)$ Stiff in NS matter and symmetric matter		New parameterization Sharp decrease in $\eta_\rho(f)$ Stiff in NS matter, soft in symmetric matter
Flow constraint +	Flow constraint –	Flow constraint +
Twins -	Twins +	Twins +

- In this work we use the stiffer KVORcut02 (H1) and the softer KVORcut03 (H2)
- They the maximum NS mass constraint with both **hyperons** (with help of ϕ-meson) and **Δ-isobars** included
  [E.E.K., K.A.M., D.N.V. Nucl.Phys. A961 (2017) 106-141]
- Many other constraints are also satisfied

Scalar sector version of this method was successfully employed in recent work
[H.Pais, C.Providência PRC94 (2016), M.Dutra et al. PRC93 (2016)]

Quark models

Generalized phenomenological functional for the quark matter (cf. the talk of David Blaschke)

From a generic Lagrangian [M.Kaltenborn et al. Phys.Rev. D96 (2017) no.5, 056024]

$$\mathcal{L} = \mathcal{L}_{\text{free}} - U(\bar{q}q, \bar{q}\gamma_0 q)$$

by expanding around the mean-field values $\langle \bar{q}q \rangle = n_S, \langle \bar{q}\gamma_0 q \rangle = n_V$ and neglecting the fluctuations one gets by the standard way

$$P = \sum_{f=u,d} P_{\text{quasi}}(\{\mu_f^*\}, \{m_f^*\}) - (U - n_S\Sigma_S - n_V\Sigma_V),$$

$$\Sigma_S(n_S, n_V) = \frac{\partial U}{\partial n_S}, \quad \Sigma_V(n_S, n_V) = \frac{\partial U}{\partial n_V}, \quad m_f^* = m_f + \Sigma_S, \quad \mu_f^* = \mu_f - \Sigma_V$$

Values of the n_S, n_V for a given set of μ_f follows from the self-consistency condition

$$n_S = \frac{3}{\pi^2} \sum_{f=u,d} \int_0^{p_{\text{F},f}} dp p^2 \frac{m_f^*}{E_f^*}, \quad n_V = \sum_{f=u,d} \frac{p_{\text{F},f}^3}{\pi^2}, \quad p_{\text{F},f} = \sqrt{(\mu_f^*)^2 - (m_f^*)^2},$$

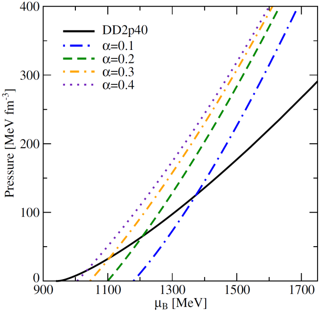
$$E_f^* = \sqrt{p^2 + m_f^{*2}}$$

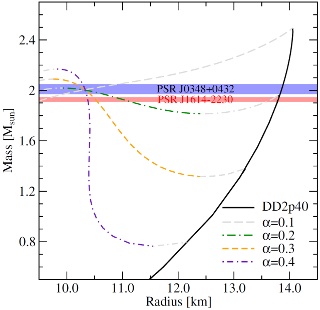
Parameterization of the effective potential

$$U(n_S, n_V) = D(n_V) n_S^{2/3} + a n_V^2 + \frac{b n_V^4}{1 + c n_V^2}.$$

- $\Sigma_S = D(n_V) n_S^{-1/3}$ – quark effective mass diverges at $n_B \to 0$: simulation of the confinement; $D(n_V) = D_0 \exp(-\alpha(n_V \cdot \mathrm{fm}^{-3})$ – reduction of the effective string tension in the medium.
- $a n_V^2$ – ordinary vector repulsion
- Nonlinear repulsion – needed to control the existence of the third family.

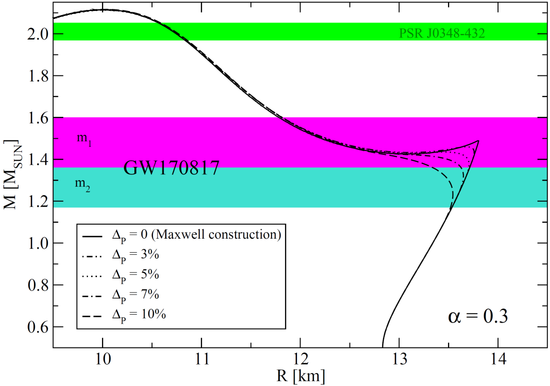
Two models we use differ by the value of α: Q1: $\alpha = 0.2$ and Q2: $\alpha = 0.3$





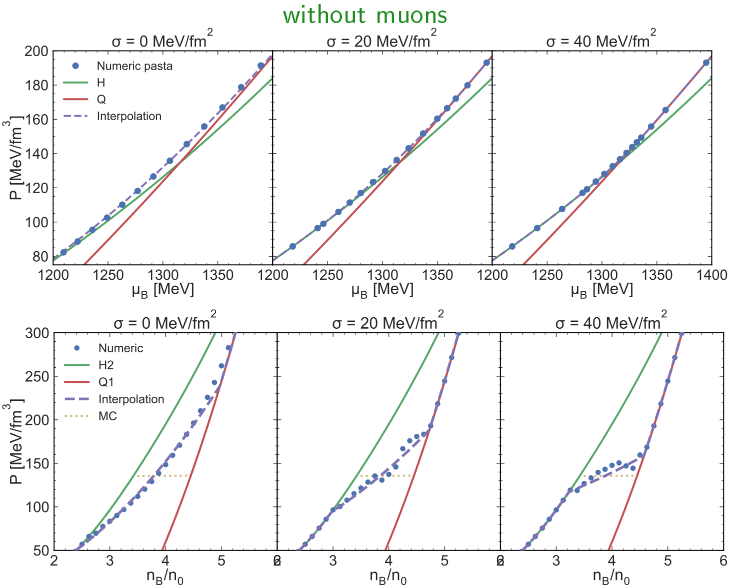
[M.Kaltenborn et al. Phys.Rev. D96 (2017) no.5, 056024]

Phenomenological results for models of this class



We found that the critical value of Δ_P for the twin disappearance is $6-7\%$ within these types of models [A.Ayriyan et al. Phys.Rev. C97 (2018) no.4, 045802]

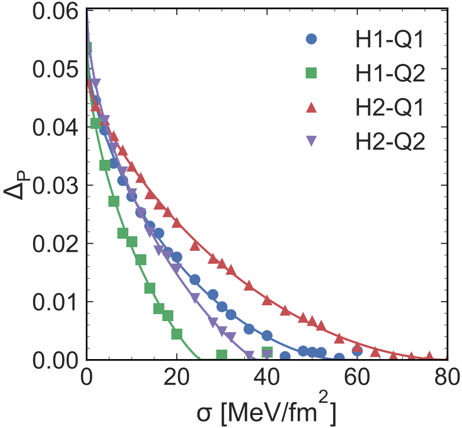
Fitting the pasta results: example (Q1–H2)



Fit works reasonably well, so the polynomial construction indeed can be used to describe the effect of the structures

Determination of the critical surface tension

Thus obtained $\Delta_P(\sigma)$ decreases with increase of σ – shown by symbols



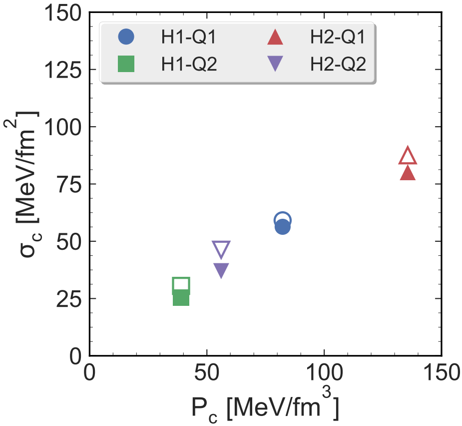
- There exists a systematic error due to numeric limitations: spatial grid step, maximum WS cell size and limited precision. Δ_P cannot be exactly zero for any σ
- Possible definition of the critical surface tension σ_c: filter out the error using the smoothing fit function

$$\Delta_P = \Delta_P^{(0)} \left(1 - (\frac{\sigma}{\sigma_c})^{\alpha}\right)^{\beta}$$

shown by lines

Critical surface tension: model dependence

Parameter of the phase transition, characterizing a pair of models: pressure on the Maxwell line P_c - different for all the models.
Filled symbols – numeric result,
Empty symbols – analytical result using the expression above

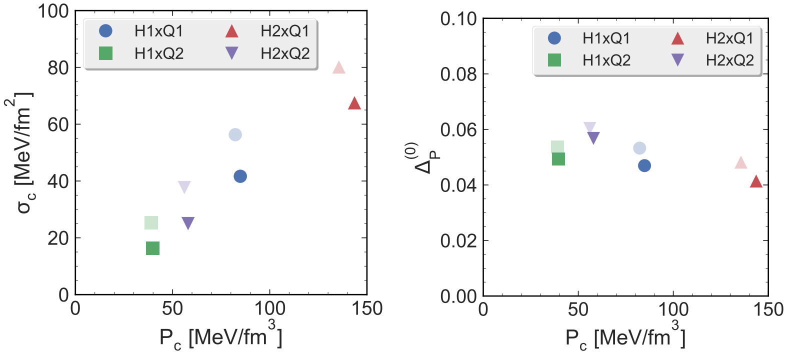


Observations:

- Critical surface tension grows almost linearly with P_c
- Numeric result agrees well with the analytical estimate
- If the energy jump is sufficiently large, low $P_c \Rightarrow$ low-mass twins. So these models should be less affected by the structures

Compact star structure: effect of muons

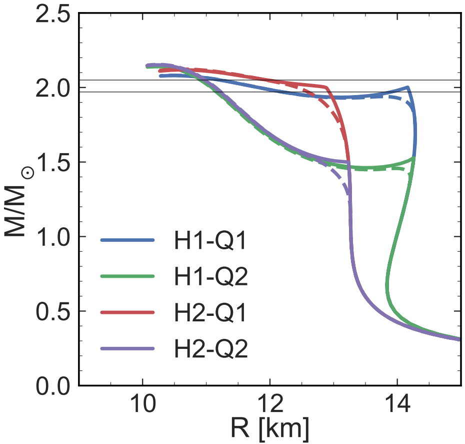
Usually in literature the contribution of muons to the pasta is neglected. Results with muons – normal symbols; without muons – transparent symbols



- Muon contribution is indeed important
- Linear relation between σ_c and P_c holds
- Maximum possible $\Delta_P^{(0)}$ decreases and is less than 6%.

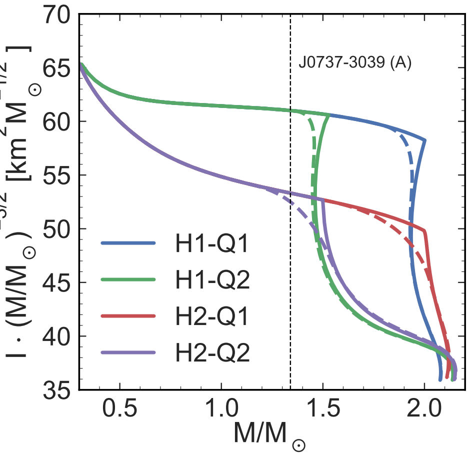
Results for MR relation

Compare the Maxwell case (solid) with the maximum Δ_P case (dashed)



- H1 model: The radius difference shrinks, but both high-mass and low-mass twins survive the inclusion of the pasta phases
  Consistent with results of [A.Ayriyan et al. Phys.Rev. C97 (2018) no.4, 045802]

Moment of inertia

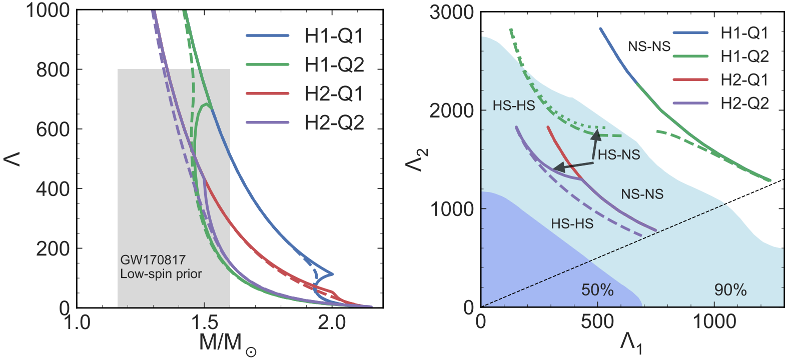


Vertical line - upcoming measurement of the moment of inertia

Tidal deformabilities

Can be constrained using gravitational-wave signals

Chirp mass $\mathcal{M} \equiv \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}} = 1.188\, M_\odot$ for GW170817



solid – Maxwell, dashed – bulk Gibbs   dotted line: NS-HS with Maxwell H1-Q2

Models not passing the constraint still can be reconciled with it by the phase transition

Inclusion of mixed phase ⇒ the GW170817 could be a HS-HS merger

Summary

Results and conclusions

- The effect of the structures can be approximately described by a simple phenomenological construction
- We found the critical surface tension for a class of models in a realistic calculation with inclusion of screening

  Critical surface tension is proportional to the Maxwell construction pressure

  It is consistent with analytical result
- Maximum possible effect of the structures does not destroy the third family
- GW170817 could be produced by two hybrid stars with mixed phase inside
- Muons cannot be neglected

Outlook

- Inclusion of strangeness
- Dependence on the symmetry energy? today's talk of Xuhao Wu
- Another class of models?
- Calculation of the surface tension is still needed to make a decisive conclusion