Suitable operator to test "the Abelian dominance" for sources in higher representation

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Introduction

- In the fundamental representation, the string tension between a static quark and antiquark is almost fully reproduced by the contribution of magnetic monopoles which is extracted by Abelian projection procedure. This was confirmed in lattice studies.
- In higher representations, if we adapt the same procedure as fundamental representation naively, the monopole contribution doesn't reproduce the full string tension. For example in the adjoint representation, the monopole part of the string tension seems to be zero even in the intermediate region, c.f. Del Debbio et al.(1996).

In this talk, we claim that this is because the way to extract monopole contribution is wrong, and give an appropriate operator to measure the monopole contribution. We support this claim by the lattice simulations.

Outline

1. Dual Superconductivity picture:
   The reason why we consider monopoles
2. Abelian projection and Abelian dominance:
   How to define magnetic monopoles in pure gauge theories
   How to check quarks are confined by monopoles
3. Wilson loops in higher representations:
   The problem with monopoles
4. Suitable operator to extract the monopole contribution
5. Numerical results in adj. rep. and **6** of $SU(3)$
6. Non-Abelian Stokes theorem:
   How to obtain the suitable operator

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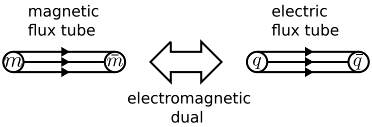
Dual superconductivity picture

- The dual superconductor picture is a promising scenario for quark confinement proposed by Nambu, 't Hooft and Mandelstam, which relates confinement and magnetic monopoles.
- In this scenario, the QCD vacuum is considered as a dual superconductor.
- The electric flux between a quark and an antiquark is squeezed into tube by the dual Meissner effect.
- The potential of the quark-antiquark pair is linear in their distance.



Ordinary superconductivity is the result of condensation of Cooper pairs. Therefore we can suppose that, in order to be a dual superconductor, condensation of magnetic monopoles have to occur.

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Abelian projection

By using the Abelian projection, which is proposed by 't Hooft(1981), we can define magnetic monopole in pure gauge theories.

- First we fix the gauge. The MA gauge is usually used, where the functional

$$\int d^4x \sum_a (A^a_\mu A^a_\mu), \quad a \text{ denotes an off diagonal component}$$

  is minimized.

- Then we extract the Cartan part A^{Abel} of the gauge field A, and by using this we define magnetic current as

$$k^\nu = \partial_\mu {}^*F_d^{\mu\nu}, \quad {}^*F_d^{\mu\nu} := \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}(\partial_\rho A_\sigma^{\mathrm{Abel}} - \partial_\sigma A_\rho^{\mathrm{Abel}})$$

For this definition, the magnetic current can be nonzero even if we don't introduce singularity to the original gauge field. This is because the gauge transf. which relates the original gauge field and the gauge fixed field can be singular and thus A_μ^{Abel} can be singular even if the original gauge field is non-singular.

Abelian dominance

On the lattices, the monopole contribution to the Wilson loop average in the fund. rep. based on the Abelian projection in MA gauge was calculated in two steps.

- First an Abelian projected Wilson loop,

$$W^{\mathrm{Abel}}[A] := \frac{1}{D_F} \operatorname{tr} \exp\left(ig \oint \sum_j 2 \operatorname{tr}(H_j A) H_j \right)$$

  (H_j is a Cartan generator) is calculated in the MA gauge as

$$\langle W^{\mathrm{Abel}}[A] \rangle_{\mathrm{MA}} \,,$$

  and it is checked that the string tension σ^{Abel} for the Abelian projected Wilson loop reproduce the full string tension. This is called the Abelian dominance.
  This was checked in $SU(2)$ (Suzuki-Yotsuyanagi(1990)) and in $SU(3)$ (Stack-Tucker-Wensley(2002)).

- Next the monopole part is extracted from the Abelian projected Wilson loop by Toussanint-DeGrand procedure, and check that the monopole part σ^{mono} of the string tension reproduce the full string tension. This is called the monopole dominance. This was checked $SU(2)$ (Shiba-Suzuki(1994)) and in $SU(3)$ (Stack-Tucker-Wensley(2002)).

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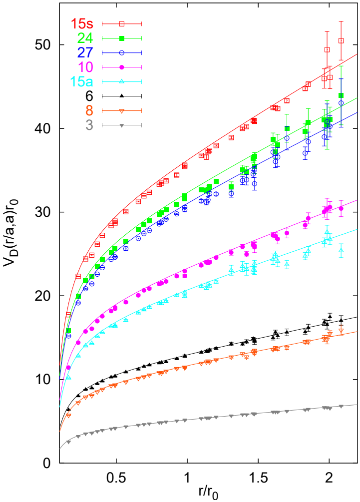
Wilson loops in higher representations

We can use Wilson loops in higher representations to test candidates of confinement mechanism by checking whether they reproduce the following behavior.

In $SU(3)$ Source: Bali(2000)



The potential between color sources in a higher representation has two characteristic features depending on the distance.

- At intermediate distance, the string tension is proportional to the quadratic Casimir.
- At asymptotic region, due to the screening by gluons, the string tension depends only on the N-ality of the representation.

Naively extended Abelian projection in higher reps.

Naively extended Abelian projection does not reproduce the correct behavior of Wilson loops in higher representation.

For example, in the adjoint rep. in $SU(2)$ gauge theory, the Abelian projected Wilson loop,

$$W^{\mathrm{Abel}} = \frac{1}{3}\left(\exp\left(ig\oint A^3\right) + \exp\left(-ig\oint A^3\right) + 1\right),$$

approaches $1/3$ other than 0.

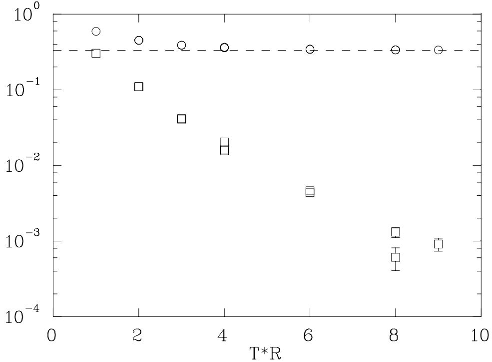


FIG. 7. The adjoint Wilson loop $W_{j=1}$ (2) versus the adjoint diagonal Wilson loop $W^d_{j=1}$ (o) in MA projection. The dashed line corresponds to the asymptotic value for the latter, $W^d_{j=1} = 1/3$.

Source: Poulis(1996)

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The suitable operator to check "the Abelian dominance"

In an arbitrary representation of an arbitrary group, we claim that the suitable operator is

$$\tilde{W}_R = \exp\left(ig\oint \langle\Lambda| A_\mu |\Lambda\rangle\right), \quad |\Lambda\rangle : \text{ the highest weight state}$$

which is different from the naively defined Abelian projected Wilson loop in the representation R

$$\begin{aligned} W_R^{\mathrm{Abel}} &= \mathrm{tr}_R \exp\left(ig\oint 2\,\mathrm{tr}\left(H_j A_\mu\right) H_j\right) \\ &= \sum_\mu \exp\left(ig\oint \langle\mu| A_\mu |\mu\rangle\right), \end{aligned}$$

where the sum is over the whole weights of R and

H_j: the Cartan generators

Later, I will explain why we can consider this is the suitable operator.

Example: adj. rep. in $SU(2)$ and adj. and $\mathbf{6}^*$ in $SU(3)$

In adj. rep. of $SU(2)$,

$$\tilde{W}_A = e^{ig\oint A^3},$$
$$W_A^{\mathrm{Abel}} = e^{ig\oint A^3} + e^{-ig\oint A^3} + 1$$

(c.f. Poulis(1996))
In adj. rep. of $SU(3)$,

$$\tilde{W}_A = e^{ig\oint A^3}, \quad (\Lambda = (1,0))$$
$$W_A^{\mathrm{Abel}} = e^{ig\oint A^3} + e^{-ig\oint A^3} + e^{ig\oint\left(\frac{1}{2}A^3+\frac{\sqrt{3}}{2}A^8\right)} + e^{-ig\oint\left(\frac{1}{2}A^3+\frac{\sqrt{3}}{2}A^8\right)}$$
$$+ e^{ig\oint\left(\frac{1}{2}A^3-\frac{\sqrt{3}}{2}A^8\right)} + e^{-ig\oint\left(\frac{1}{2}A^3-\frac{\sqrt{3}}{2}A^8\right)} + 2$$

In $\mathbf{6}^*$ of $SU(3)$

$$\tilde{W}_{\mathbf{6}} = e^{i\oint\frac{2}{\sqrt{3}}A^8}, \quad (\Lambda = (0, 1/2\sqrt{3}))$$
$$W_{\mathbf{6}}^{\mathrm{Abel}} = e^{ig\oint\frac{2}{\sqrt{3}}\oint A^8} + e^{ig\oint\left(A^3-\frac{1}{\sqrt{3}}A^8\right)} + e^{ig\oint\left(-A^3-\frac{1}{\sqrt{3}}A^8\right)}$$
$$+ e^{-ig\oint\frac{1}{\sqrt{3}}A^8} + e^{ig\oint\left(\frac{1}{2}\oint A^3+\frac{1}{2\sqrt{3}}A^8\right)} + e^{ig\oint\left(-\frac{1}{2}\oint A^3+\frac{1}{2\sqrt{3}}A^8\right)}$$

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Generalized MA gauge

Before showing the numerical results, we introduce generalized MA gauges (c.f. Stuck-Tucker-Wensley(2002)).
The gauge fixing functional of the MA gauge is the form of a mass term for the gauge fields.

$$\int \mathrm{tr}\left(A^a_\mu A^a_\mu\right) \quad (a \text{ denotes off-diagonal components})$$

In $SU(3)$ case, we can generalize it as

$$\int \left({m_1}^2\left((A^1_\mu)^2 + (A^2_\mu)^2\right) + {m_2}^2\left((A^4_\mu)^2 + (A^5_\mu)^2\right) + {m_3}^2\left((A^6_\mu)^2 + (A^7_\mu)^2\right)\right).$$

In the following we use

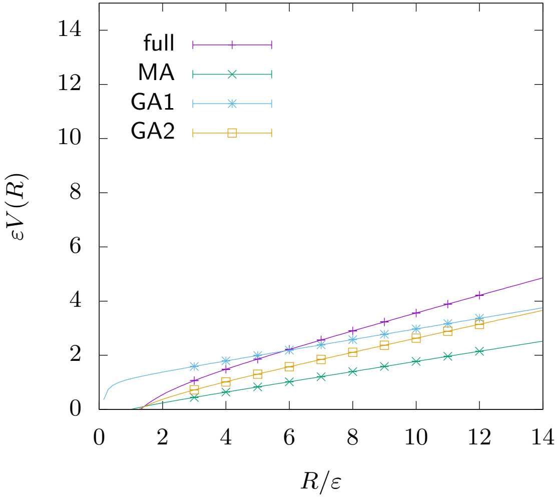
$$m_1 = 0,\ m_2 = m_3 = m,\ \text{(GA1)}$$
$$m_1 = 2m,\ m_2 = m_3 = m.\ \text{(GA2)}$$

GA1 is special because the symmetry breaking pattern is different from the MA gauge as $SU(3) \to U(2)$. Therefore we cannot use GA1 in every case, for example we can use it in fund. rep. and $\mathbf{6}$ and cannot use it in adj. rep.

$SU(3)$ fund. rep. $(\mathbf{3},\ [1,0])$

24^4lattice
$\beta = 6.2$
APE smearing

Preliminary



$\sigma_{\text{full}} \simeq 0.32$
$\sigma_{\text{MA}} \simeq 0.19 \simeq 0.59\sigma_{\text{full}}$
$\sigma_{\text{GA1}} \simeq 0.19 \simeq 0.59\sigma_{\text{full}}$
$\sigma_{\text{GA2}} \simeq 0.25 \simeq 0.78\sigma_{\text{full}}$
c.f.
Suganuma-Sakumichi(2016)
Perfect Abelian dominance in MA gauge
32^4lattice $\beta = 6.4$

$SU(3)$ adj. rep. $(\mathbf{8}, [1,1])$

24^4lattice
$\beta = 6.2$
APE smearing

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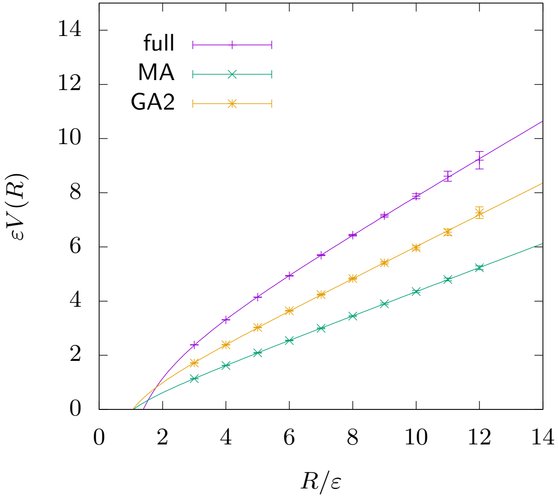


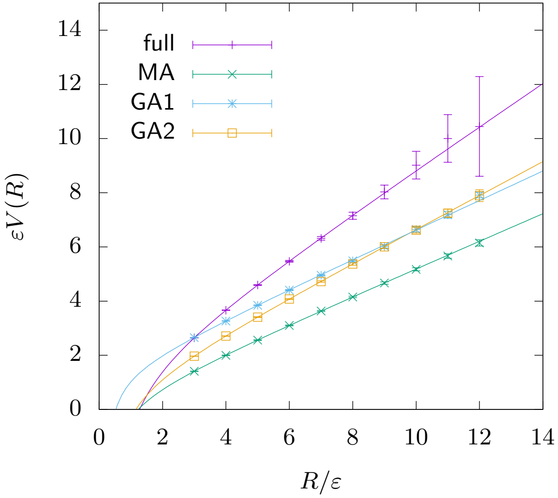
$C_2(A)/C_2(F) = 2.25$

$\sigma_{\text{full}} \simeq 0.67$
$\sigma_{\text{MA}} \simeq 0.44 \simeq 0.66\sigma_{\text{full}}$
$\sigma_{\text{GA2}} \simeq 0.58 \simeq 0.87\sigma_{\text{full}}$

$SU(3)$ $\mathbf{6}^*$ ($[0,2]$)

24^4lattice
$\beta = 6.2$
APE smearing

Preliminary



$$C_2(\mathbf{6})/C_2(F) = 2.5$$

$\sigma_{\text{full}} \simeq 0.79$
$\sigma_{\text{MA}} \simeq 0.50 \simeq 0.63\sigma_{\text{full}}$
$\sigma_{\text{GA1}} \simeq 0.54 \simeq 0.68\sigma_{\text{full}}$
$\sigma_{\text{GA2}} \simeq 0.62 \simeq 0.78\sigma_{\text{full}}$

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Gauge inv. projection

Through the Non-Abelian Stokes theorem, we obtain another procedure to extract monopole contribution. This procedure has two merits.

- We can confirm that the Abelian projection procedure gives the gauge invariant definition of magnetic monopoles.
- We can obtain the suitable operator to test "the Abelian dominance" in a higher representation.

Non-Abelian Stokes theorem

Diakonov and Petrov version of the Non-Abelian Stokes theorem gives us the gauge invariant projection which is essentially same as the Abelian projection in the fundamental representation but is different from that in higher representations. According to the theorem, the Wilson loop for the representation R can be written as

Non-Abelian Stokes theorem (Diakonov, Petrov(1989))

$$W_R[A] = \int DU \exp\left(\oint ig \langle\Lambda| A^U |\Lambda\rangle\right)$$
$$= \int DU \exp\left(\int_S ig\, d\left(\langle\Lambda| A^U |\Lambda\rangle\right)\right),$$

where

DU	is the product of the Haar measure over the loop or a surface
$A^{U^\dagger}$	$:= UAU^\dagger + ig^{-1}UdU^\dagger$, and
$\|\Lambda\rangle$	is the highest weight state of the representation R.

Gauge inv. projection

Corresponding to Abelian projection, we assume we can approximate the right hand side of NAST as

$$W_R[A] \simeq \exp\left(\oint ig \langle\Lambda| A^{\Theta_*[A]} |\Lambda\rangle\right) =: \tilde{W}_R[A],$$

where $\Theta_*[A]$ is a group-valued functional of A which determined by imposing the condition that $\Theta_*[A]$ minimizes the functional

$$F[\Theta; A] = \begin{cases} \int \sum_j \mathrm{tr}\left(D_\mu \boldsymbol{n}_j D_\mu \boldsymbol{n}_j\right) & \text{(corresponding to MA)} \\ \int \mathrm{tr}\left(D_\mu \boldsymbol{n}_8 D_\mu \boldsymbol{n}_8\right) & \text{(corresponding to GA1)} \\ \int \mathrm{tr}\left(D_\mu \boldsymbol{n}_3 D_\mu \boldsymbol{n}_3\right) & \text{(corresponding to GA2)} \end{cases}$$

for given A, where

$$\boldsymbol{n}_j := \Theta^\dagger H_j \Theta,$$

H_j is a Cartan generators, and

$$D_\mu \boldsymbol{n}_j := \partial_\mu \boldsymbol{n}_j - ig[A_\mu, \boldsymbol{n}_j].$$

$F[\Theta; A]$ is not gauge fixing functional. By using this we determine $\Theta_*[A]$ as the functional of A.
If $\Theta_*[A]$ transforms as

$$\Theta_*[UAU^\dagger + ig^{-1}UdU^\dagger] = U\Theta_*[A],$$

the operator $\tilde{W}_R[A]$ is gauge invariant:

$$\tilde{W}_R[UAU^\dagger + ig^{-1}UdU^\dagger] = \tilde{W}_R[A].$$

We take the average of this operator over configurations which is not gauge fixed.

$$\langle \tilde{W}_R[A] \rangle_{full}$$

Summary

- The Abelian projected Wilson loop for a higher representation does not reproduce the correct behavior of the original Wilson loop.
- Through the NAST, we obtain another projected Wilson loop, which is essentially same as the Abelian projected Wilson loop in the fundamental representation, and is different from that in in higher representations.
- According to the lattice simulation, the gauge invariant projected Wilson loop reproduce the correct behavior in the adjoint representation in the $SU(2)$ gauge theory and in the adjoint and $\mathbf{6}$ in $SU(3)$ gauge theory.