Constraining the QCD equation of state in hadron colliders

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Introduction

- The quark-gluon plasma (QGP)

- A high-temperature phase of QCD where quarks are deconfined from hadrons (> $2 \times 10^{12}$ K)

- It can be created in relativistic nuclear collider experiments

One can study the properties of the hot matter under strong interaction quantitatively
Introduction

- Relativistic nuclear colliders: a gateway to the QGP
  - Relativistic Heavy Ion Collider (RHIC)@BNL, $v_{s_{NN}} = 5.5-200$ GeV (2000-)
  - Large Hadron Collider (LHC)@CERN, $v_{s_{NN}} = 2.76-5.44$ TeV (2010-)
  - FAIR@GSI, NICA@JINR, SPS@CERN, J-PARC@JAEA/KEK ... ?
Introduction

- A “standard model” of heavy-ion collisions

Gluons are emitted in an accelerated nucleus → Gluons eventually overlap and saturate (called *color glass condensate*)
Introduction

- A “standard model” of heavy-ion collisions

(Color) glass + plasma = glasma

Longitudinal color electric and magnetic fields
Details of equilibration is *not known*
Introduction

- A “standard model” of heavy-ion collisions

**Hydrodynamic model** (~1-10 fm/c)

- **QGP fluid**
- **Glasma** (~0-1 fm/c)
- **“Little bangs”**
- **Color glass condensate** (< 0 fm/c)

**Hydrodynamic evolution**

The hot QCD matter behaves as a relativistic fluid
Introduction

A “standard model” of heavy-ion collisions

- Hadronic transport ($> 10 \text{ fm/c}$)
- Freeze-out
- Hydrodynamic model ($\sim 1-10 \text{ fm/c}$)
- Local equilibration
- Glasma ($\sim 0-1 \text{ fm/c}$)
- "Little bangs"
- Color glass condensate ($< 0 \text{ fm/c}$)

Graphics by AM

Hadronic decay and transport

Interaction becomes weaker when cold QCD liquid to QCD gas
Motivation and goals

**Motivation**
To investigate the **QGP dynamics using hydrodynamic models** (the first principle calculations are difficult at finite T and μ)

**Goals**
To understand (I) the **macroscopic evolution** of the QCD matter and (II) the **microscopic properties** such as the equation of state (EoS) & transport coefficients

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**Diagram**
- **Experimental data**
- **First principle calculations**
- **Hydrodynamic models**
- **EoS, transport coefficients**

THIS WORK
Relativistic hydrodynamics

Energy-momentum tensor (in the local rest frame)

\[ T^{\mu\nu} = T_{eq}^{\mu\nu} + \delta T^{\mu\nu} \]

\[
\begin{pmatrix}
\epsilon & 0 & 0 & 0 \\
0 & P & 0 & 0 \\
0 & 0 & P & 0 \\
0 & 0 & 0 & P
\end{pmatrix} +
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & \Pi + \pi^{xx} & \pi^{xy} & \pi^{xz} \\
0 & \pi^{yx} & \Pi + \pi^{yy} & \pi^{yz} \\
0 & \pi^{zx} & \pi^{zy} & \Pi + \pi^{zz}
\end{pmatrix}
\]

where

- \( \epsilon \): energy density
- \( P \): pressure
- \( \Pi \): bulk pressure
- \( \pi^{\mu\nu} \): shear stress tensor

Matching condition or AM, 1803.03318

Landau frame

In a general frame

\[ T^{\mu\nu} = (\epsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu} \]

- \( u^\mu \): flow (four-velocity)
- \( g^{\mu\nu} = \text{diag}(+, -, -, -) \)
Overview of the model

- Constraining the QCD equation of state in hadron colliders

Information of QCD
- Equation of state \( P = P(\epsilon) \)
- Transport coefficients \( \eta, \zeta \)

Initial conditions \( T^{\mu\nu} \)

Relativistic hydrodynamics
\[ \partial_\mu T^{\mu\nu} = 0 \]
\[ \partial_\mu s^{\mu} \geq 0 \]

Hadronic transport

Observables

Indirect constraining

Experimental data

Comparison
QCD equation of state (EoS)

- Static relation among thermodynamic variables; sensitive to degrees of freedom in the system

- We have lattice QCD calculations at $\mu_B = 0$

- Is it what we see in heavy-ions collisions?
QCD equation of state (EoS)

- We may see something different in HIC for various reasons:
  - Finite size effect
  - Chemical equilibration
  - Strong magnetic field

\[ p = \frac{zp}{1 - z} \]

\[ B \]
QCD equation of state (EoS)

- We systematically generate variations of EoS at $\mu_B = 0$:

We estimate observables using hydrodynamic models for each EoS and find the correspondence between them and the EoS.
Observable sensitive to EoS

- Particle spectra

PHENIX, PRC 69, 034909 (2004)

(1) Integrate $p_T$ out → particle number at $y = 0$

(II) Integrate $p_T$ out with $p_T$ as weight → momentum per particle

Rapidity $y$: related to momentum in the beam axis direction

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$
Observable sensitive to EoS

- Particle spectra

(I) Integrate $p_T$ out → particle number at $y = 0$

(II) Integrate $m_T$ out with $m_T$ as weight → energy per particle

Transverse mass $m_T$: effective mass of a particle where $p_T$ is “incorporated”

$$m_T = \sqrt{p_T^2 + m^2} = \sqrt{E^2 - p_z^2}$$

*At $z = 0$, $m_T \sim E$ except for thermal blurring
Rough picture of hydro expansion

- Transverse expansion and $\tau_{\text{eff}}$

\[ \tau \sim \tau_{\text{eff}} \]

Longitudinal expansion is dominant ($R \sim R_0$)  
Transverse expansion is relevant ($R > R_0$)

Effective radius of the medium:  
\[ R^2 = 2(\langle |x(\tau)|^2 \rangle - |\langle x(\tau) \rangle|^2) \]

Cf: J.-Y. Ollitrault, PLB 273, 32 (1991)
Imprint of the EoS

- Entropy density vs. Particle number

\[ s(T_{\text{eff}}) = a \frac{1}{R_0^3} \frac{dN}{dy} \]

\[ \frac{S(T_f)}{N(T_f)} \times \frac{1}{V(T_{\text{eff}})} \]

\[ V = f \pi R_0^3 \]

Total entropy conservation \( S(T_f) = S(T_{\text{eff}}) \)

- \( T_{\text{eff}} \): effective temperature when transverse expansion starts
- \( R_0 \): effective radius of the medium where \( R_0^2 = 2(\langle |\mathbf{x}(\tau_0)|^2 \rangle - |\langle \mathbf{x}(\tau_0) \rangle|^2) \)
- \( T_f \): freeze-out temperature (constant)
- \( a \): dimensionless constant factor
Imprint of the EoS

- Energy density over entropy density vs. Mean $m_T$

$$\frac{\epsilon(T_{\text{eff}})}{s(T_{\text{eff}})} = b\langle m_T \rangle$$

$N(T_f)/S(T_f)$

- Once transverse expansion sets in ($T \sim T_{\text{eff}}$), longitudinal work becomes smaller and total energy is conserved $E(T_f) \sim E(T_{\text{eff}})$

$[b : \text{dimensionless constant factor}]$
Input for hydrodynamic model

- Transport coefficients
  - Shear viscosity $\eta/s = 1/4\pi$  
    P. Kovtun et al., PRL 94, 111601 (2005)
  - Bulk viscosity $\zeta/s = 2(1/3 - c_s^2)\eta/s$  
    A. Buchel, PLB 663, 286 (2008)

*Minimalistic choices from the gauge-string correspondence

- Initial conditions
  - Monte-Carlo Glauber model  
    M. L. Miller et al., ARNPS 57, 205 (2007); AM, 1408.1410
  - Monte-Carlo KLN model  
    H.-J. Drescher and Y. Nara, PRC 75, 034905 (2007)

We show the results of the MC Glauber model here
The scaling relations not affected; may affect comparison to data
Determination of $a$ and $b$

- Ideal hydro calculations, Au-Au collisions, 0-5% central events

- A single set of $(a, b)$ fits hydro results on to all the EoS
  There is correspondence between the EoS and observables
Before comparing to data

- Must considered are the effects of hadronic decay and viscosity

- \((a, b)\) can be determined so that all the EoS are satisfied
- Hadronic decay reduces \(<m_T>\) and increases \(dN/dy\)
- Viscosity increases \(dN/dy\) by entropy production
Comparisons to experimental data

- Viscous hydro results with hadronic decays, Au-Au, 0-5% events

Conclusions

- Compatible with the lattice QCD equation of state within errors
- Larger effective # of degrees of freedom allowed by the data
Summary and outlook

- We have probed collective properties of the hot QCD matter
  - QCD equation of state is constrained using dN/dy and \(<m_T>\)
  - The EoS with the effective number of DOF equal to or larger than that of lattice QCD is favored

- We may extract the finite-density EoS out of Beam Energy Scan experimental data

AM and J.-Y. Ollitrault, in preparation
The end

Thank you for listening!