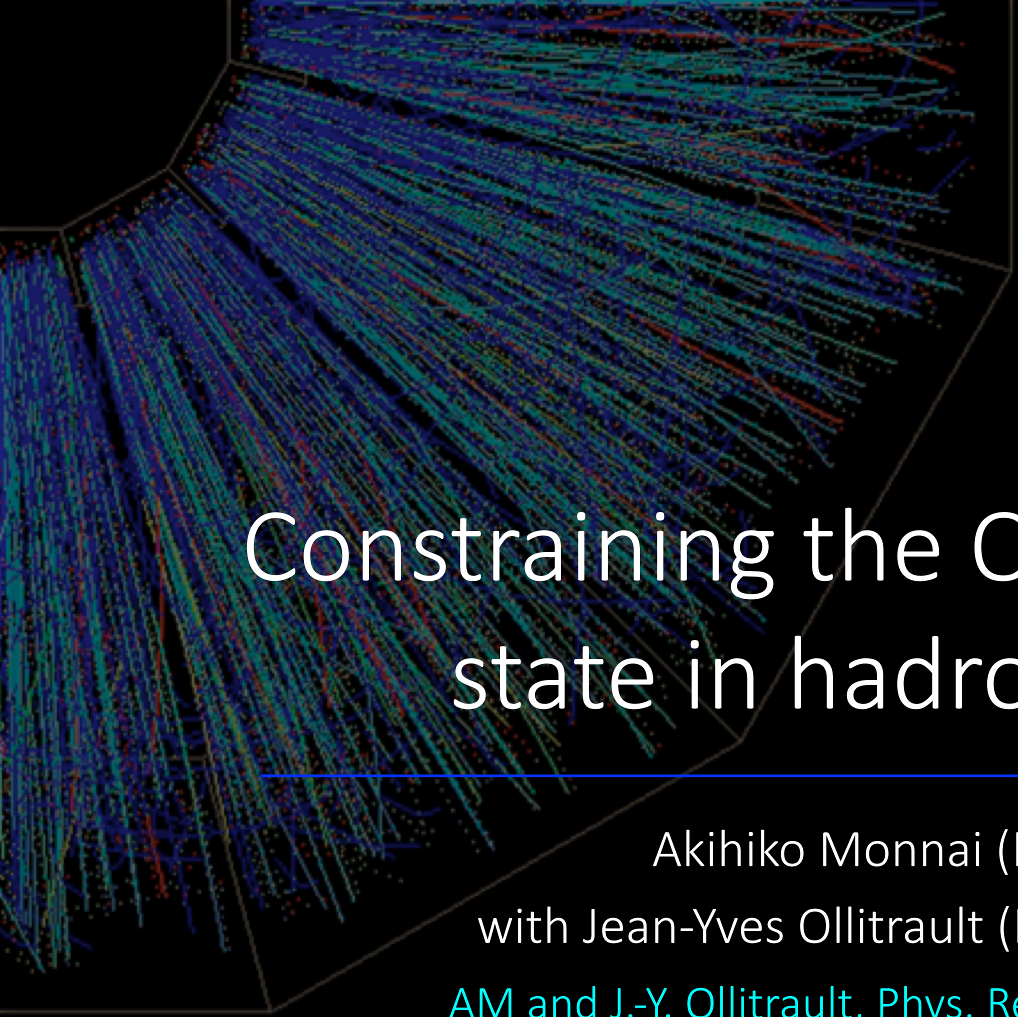


Constraining the QCD equation of state in hadron colliders

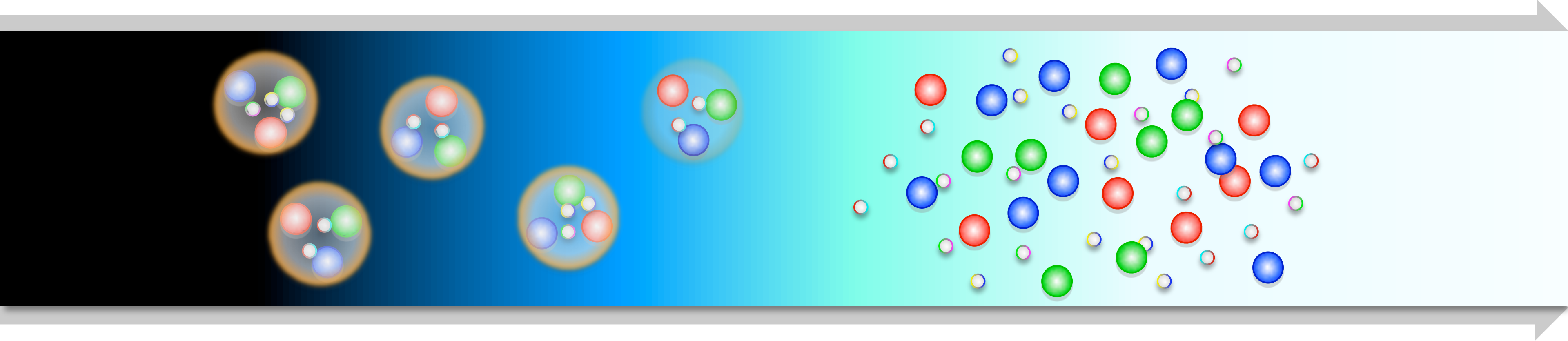
Akihiko Monnai (KEK, Japan)
with Jean-Yves Ollitrault (IPhT Saclay, France)

AM and J.-Y. Ollitrault, Phys. Rev. C 96, 044902 (2017)

New Frontiers in QCD 2018
7th June 2018, Yukawa Institute for Theoretical Physics, Kyoto, Japan

Introduction

- The quark-gluon plasma (QGP)

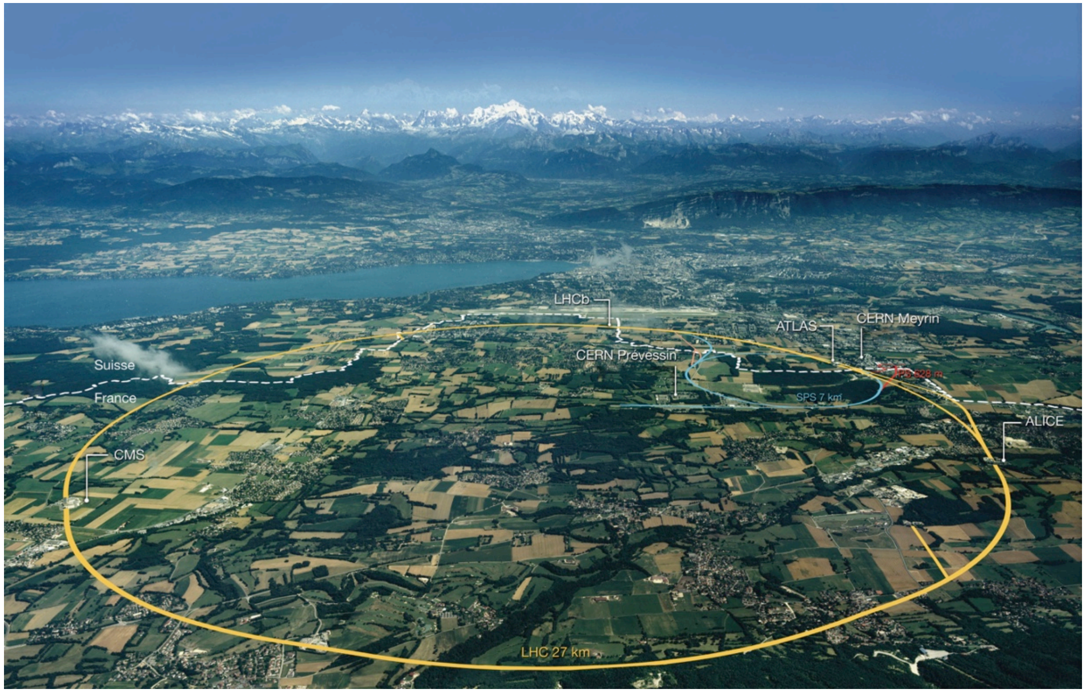


- A high-temperature phase of QCD where quarks are deconfined from hadrons (> 2×10^{12} K)
- It can be created in relativistic nuclear collider experiments

⇨ One can study the properties of the hot matter under strong interaction quantitatively

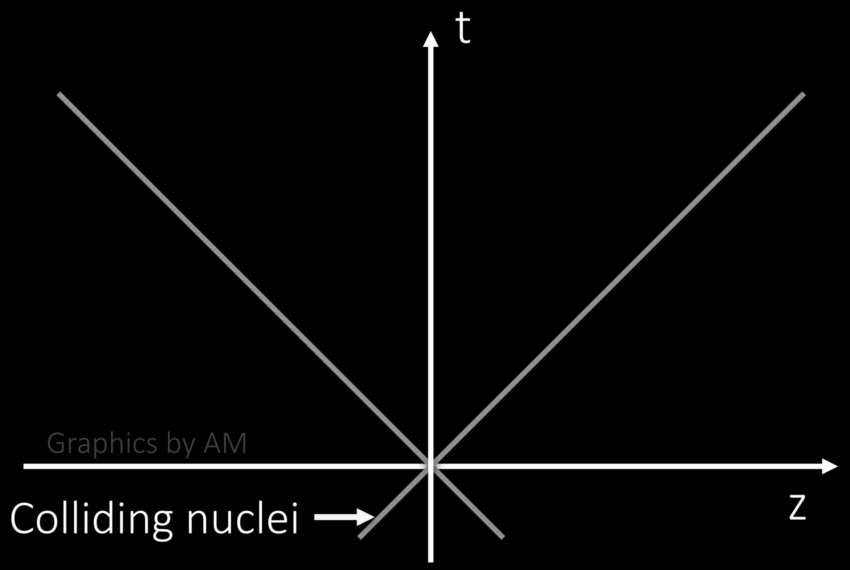
Introduction

- Relativistic nuclear colliders: a gateway to the QGP
  - Relativistic Heavy Ion Collider (RHIC)@BNL, $\sqrt{s_{NN}}$ = 5.5-200 GeV (2000-)
  - Large Hadron Collider (LHC)@CERN, $\sqrt{s_{NN}}$ = 2.76-5.44 TeV (2010-)
  - FAIR@GSI, NICA@JINR, SPS@CERN, J-PARC@JAEA/KEK … ?



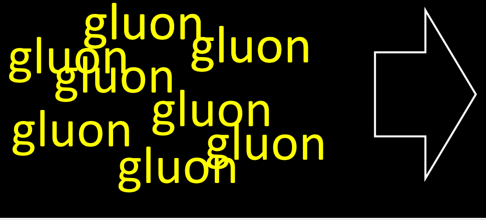
Introduction

- A “standard model” of heavy-ion collisions



Color glass condensate (< 0 fm/c)

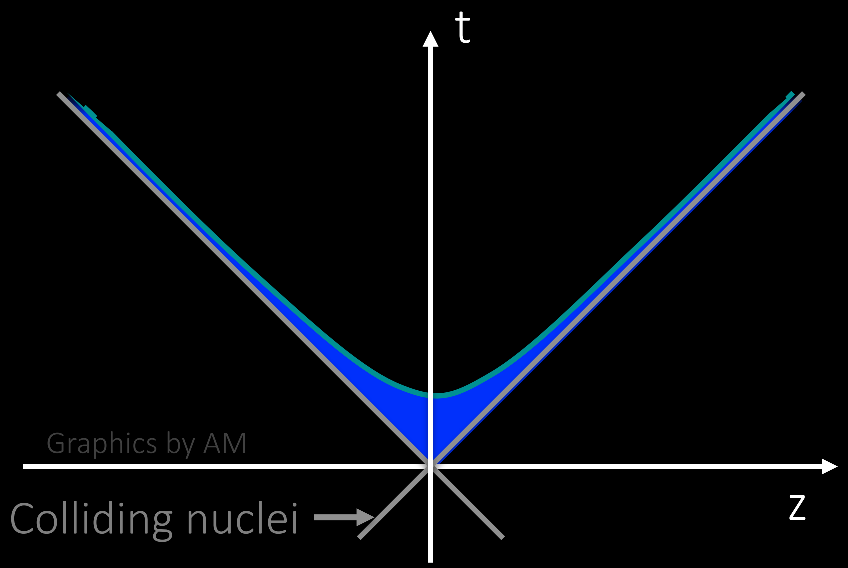
▶ Gluons are emitted in an accelerated nucleus



Gluons eventually overlap and saturate
(called *color glass condensate*)

Introduction

- A “standard model” of heavy-ion collisions



Graphics by AM

Glasma (~0-1 fm/c)

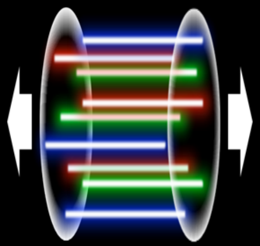
”Little bangs”

Color glass condensate (< 0 fm/c)

▶ (Color) glass + plasma = glasma

Longitudinal color electric and magnetic fields

Details of equilibration is *not known*



Introduction

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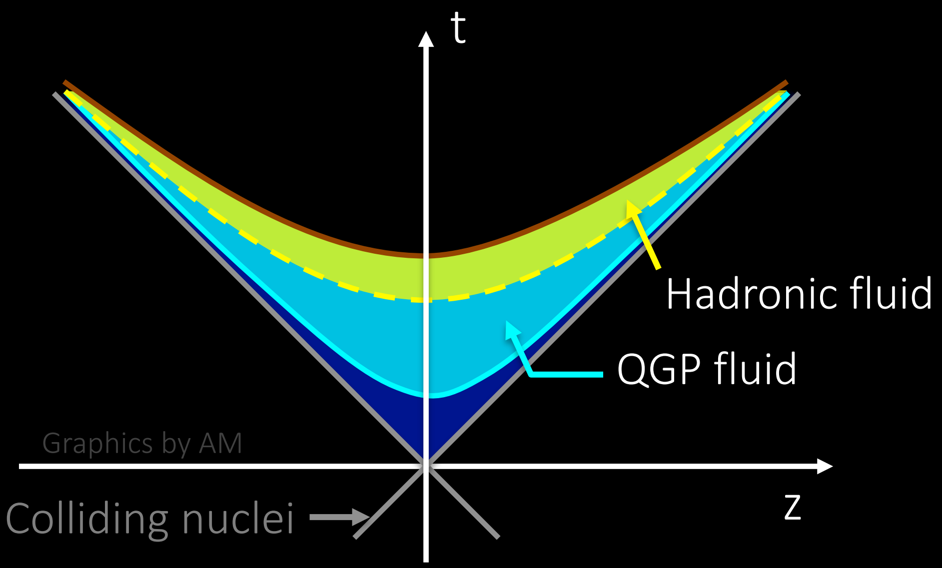


Hydrodynamic model (~1-10 fm/c)

⇧ Local equilibration

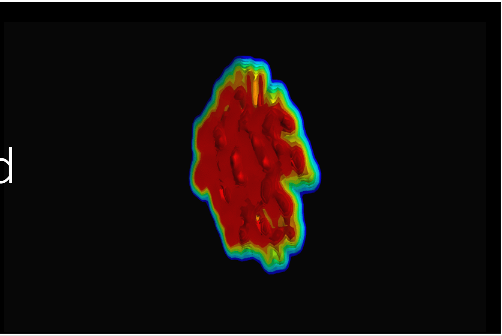
Glasma (~0-1 fm/c)

⇧ ”Little bangs”

Color glass condensate (< 0 fm/c)

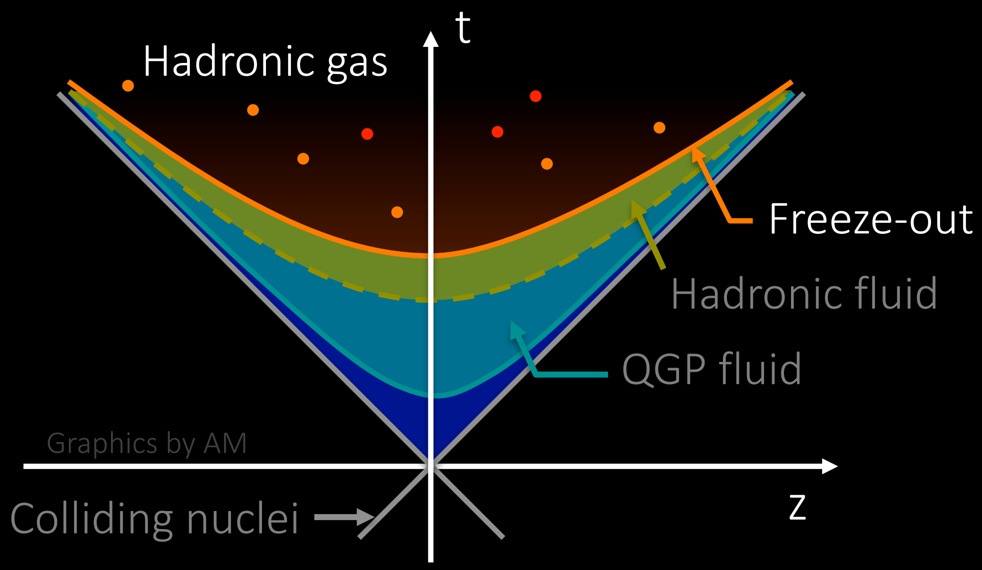
▶ Hydrodynamic evolution

The hot QCD matter behaves as a relativistic fluid



Introduction

- A “standard model” of heavy-ion collisions



Hadronic transport (> 10 fm/c)

Freeze-out

Hydrodynamic model (~1-10 fm/c)

Local equilibration

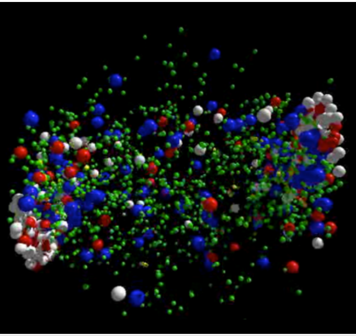
Glasma (~0-1 fm/c)

”Little bangs”

Color glass condensate (< 0 fm/c)

- Hadronic decay and transport

  Interaction becomes weaker when cold
  QCD liquid to QCD gas



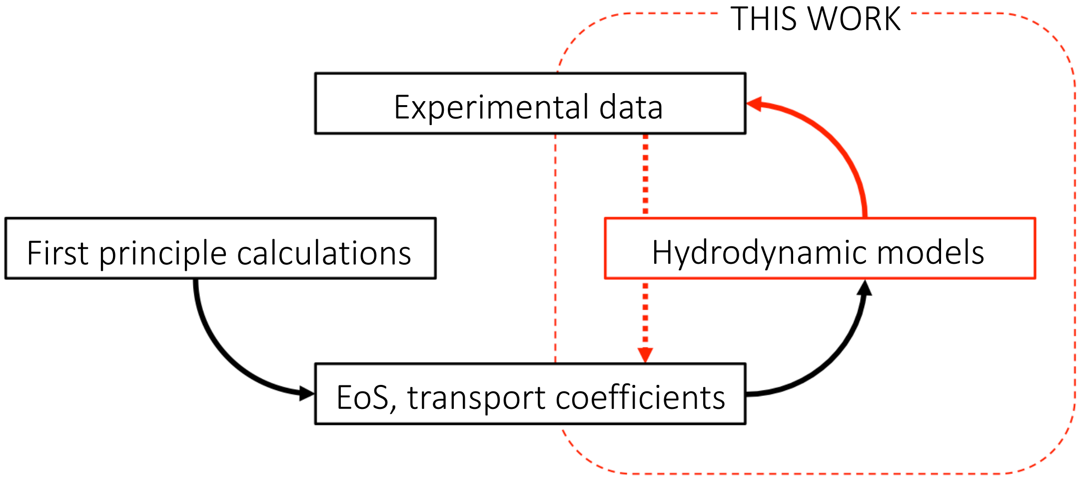
Motivation and goals

Motivation

To investigate the QGP dynamics using hydrodynamic models
(the first principle calculations are difficult at finite T and μ)

Goals

To understand (I) the macroscopic evolution of the QCD matter and (II) the microscopic properties such as the equation of state (EoS) & transport coefficients



Relativistic hydrodynamics

- Energy-momentum tensor (in the local rest frame)

Matching condition or AM, 1803.03318

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}} + \delta T^{\mu\nu}$$

$$= \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix} + \begin{pmatrix} \boxed{0} & 0 & 0 & 0 \\ 0 & \Pi + \pi^{xx} & \pi^{xy} & \pi^{xz} \\ 0 & \pi^{yx} & \Pi + \pi^{yy} & \pi^{yz} \\ 0 & \pi^{zx} & \pi^{zy} & \Pi + \pi^{zz} \end{pmatrix}$$

Landau frame

where

ϵ : energy density

P : pressure

Π : bulk pressure

$\pi^{\mu\nu}$: shear stress tensor

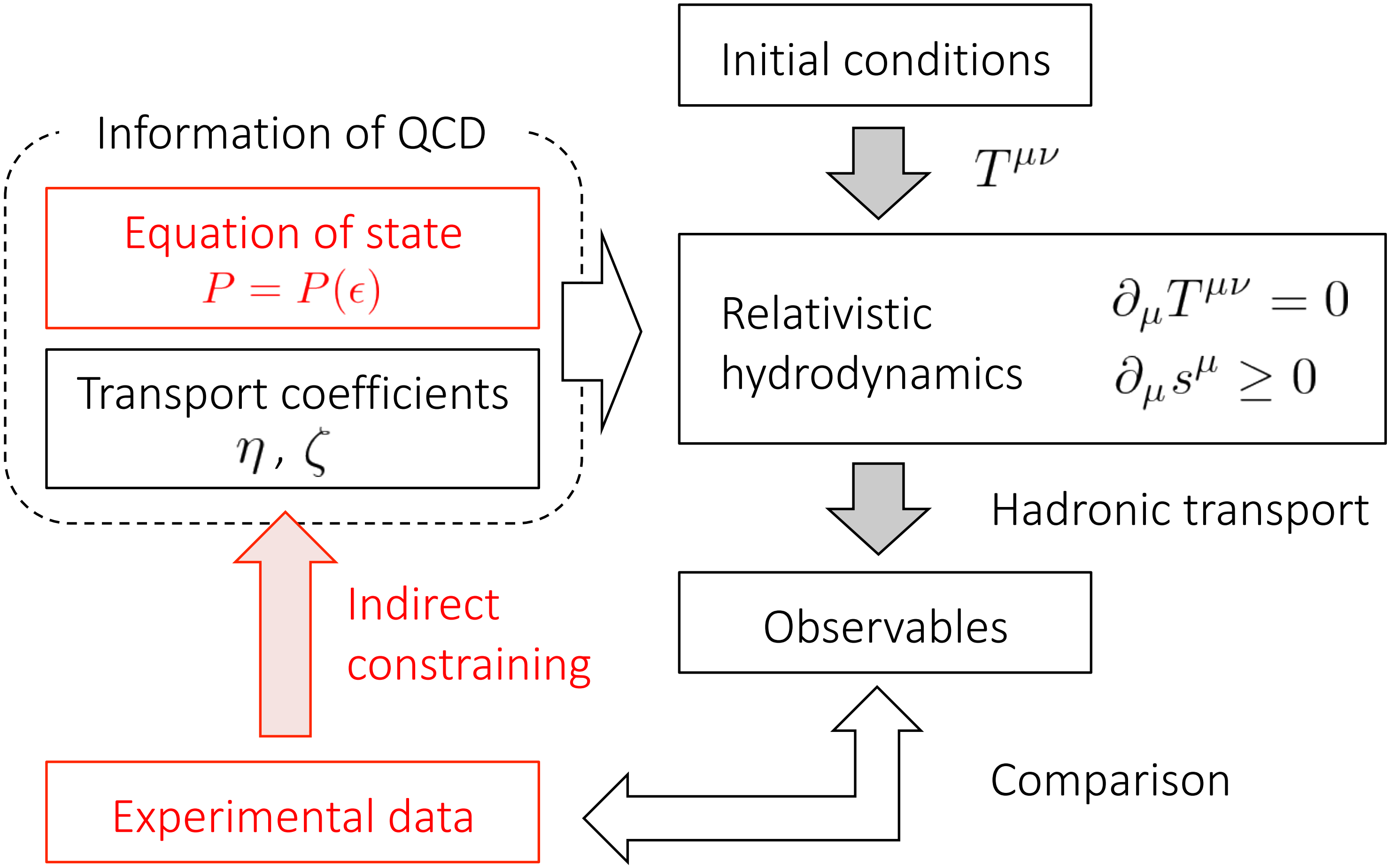
In a general frame

$$\boxed{T^{\mu\nu} = (\epsilon + P + \Pi) u^\mu u^\nu - (P + \Pi) g^{\mu\nu} + \pi^{\mu\nu}}$$

u^μ : flow (four-velocity) $\quad g^{\mu\nu} = \mathrm{diag}(+,-,-,-)$

Overview of the model

- Constraining the QCD equation of state in hadron colliders



Information of QCD

Equation of state

$P = P(\epsilon)$

Transport coefficients

$\eta\,,\ \zeta$

Initial conditions

$T^{\mu\nu}$

Relativistic hydrodynamics

$\partial_\mu T^{\mu\nu} = 0$

$\partial_\mu s^\mu \geq 0$

Hadronic transport

Observables

Comparison

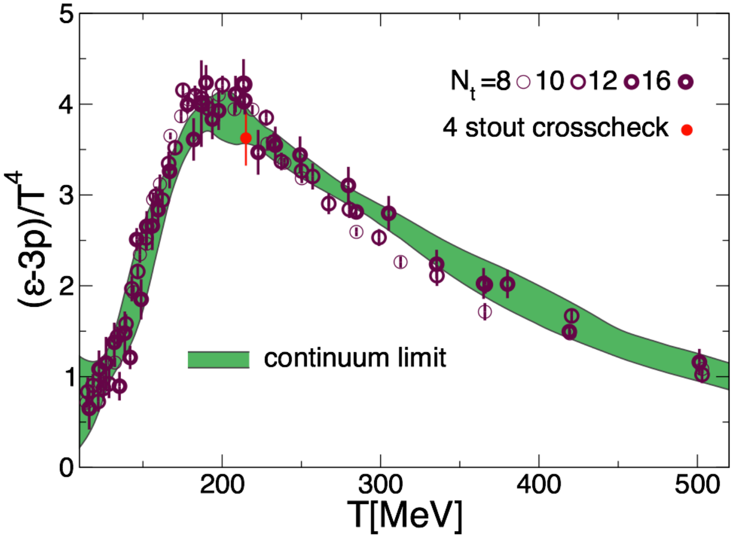
Indirect constraining

Experimental data

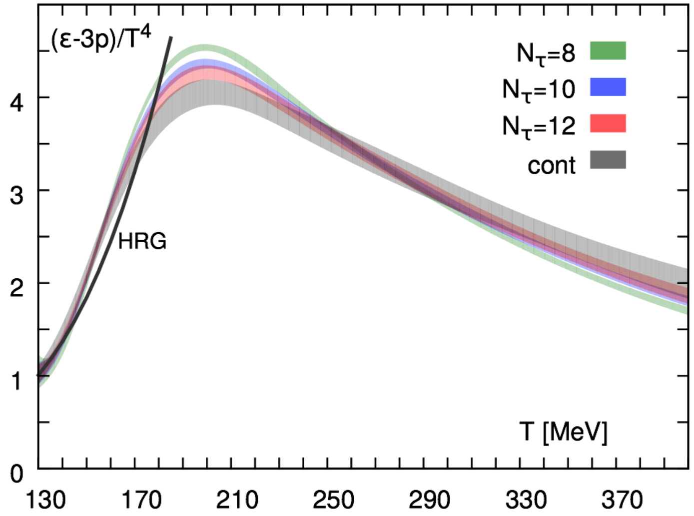
QCD equation of state (EoS)

- Static relation among thermodynamic variables; sensitive to degrees of freedom in the system
  - We have lattice QCD calculations at $\mu_B = 0$

Wuppertal-Budapest, PLB 730, 99 (2013)



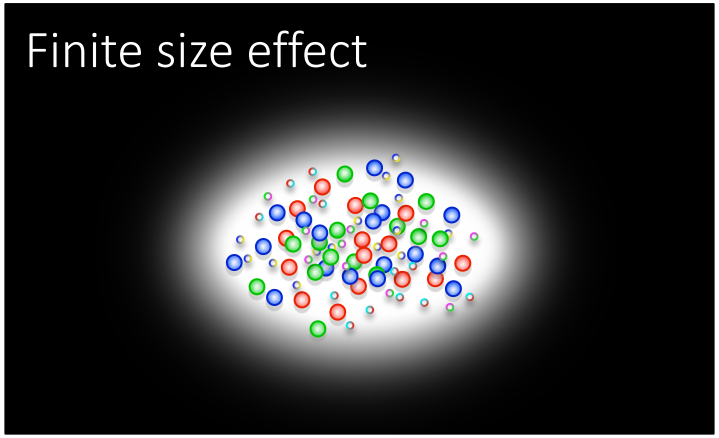
HotQCD, PRD 90, 094503 (2014)

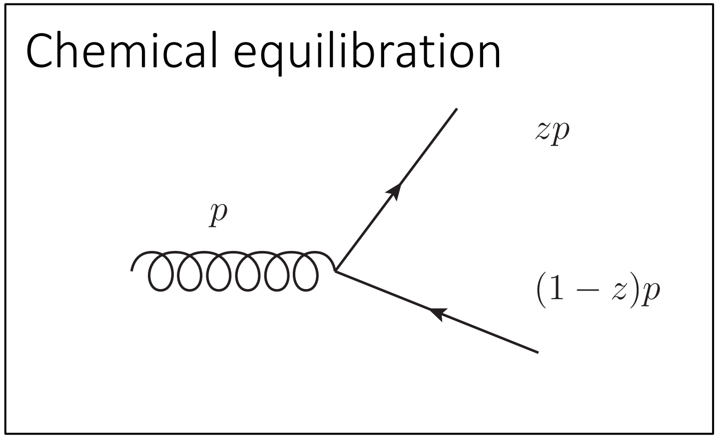


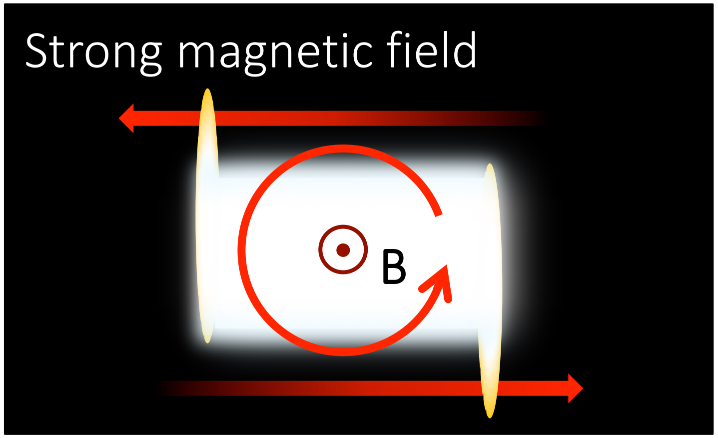
  - *Is it what we see in heavy-ions collisions?*

QCD equation of state (EoS)

- We may see something different in HIC for various reasons:


Finite size effect

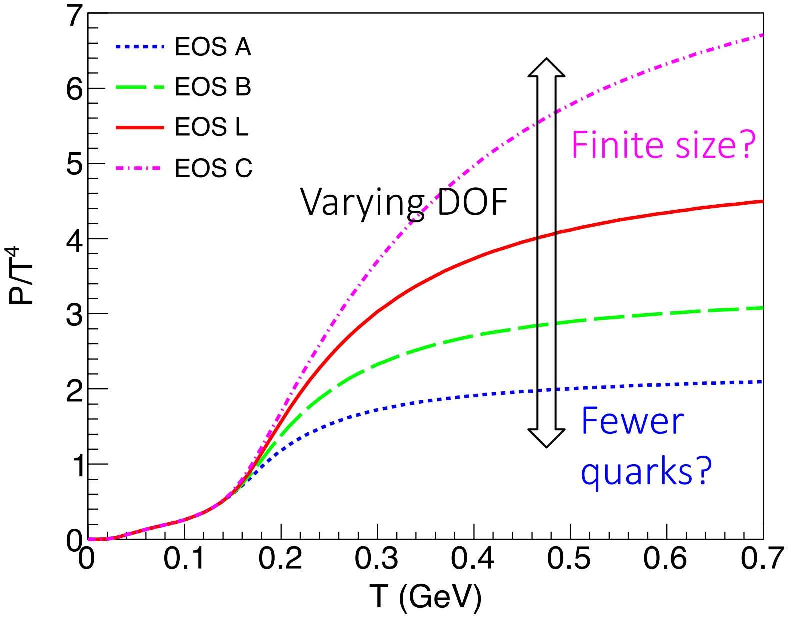

Chemical equilibration


Strong magnetic field

…

QCD equation of state (EoS)

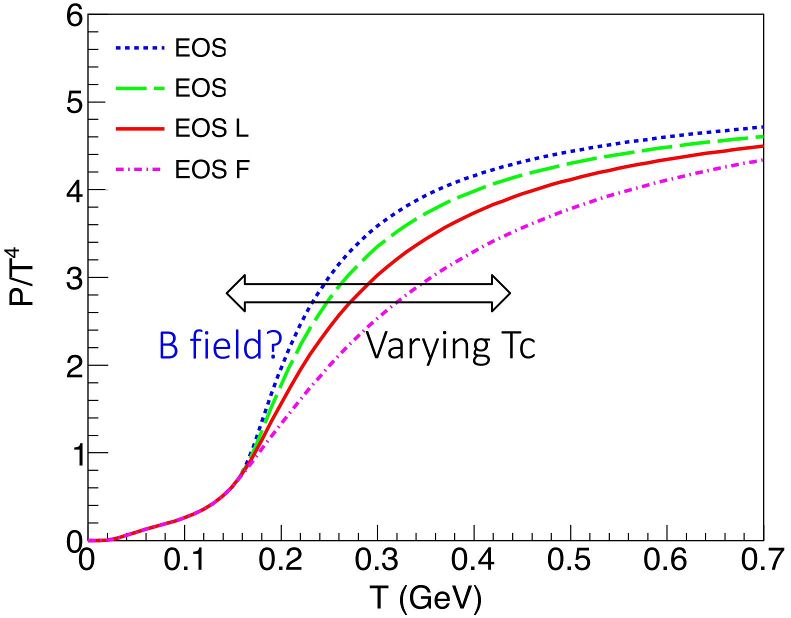
- We systematically generate variations of EoS at $\mu_B = 0$:



Varying DOF

Finite size?

Fewer quarks?



B field?

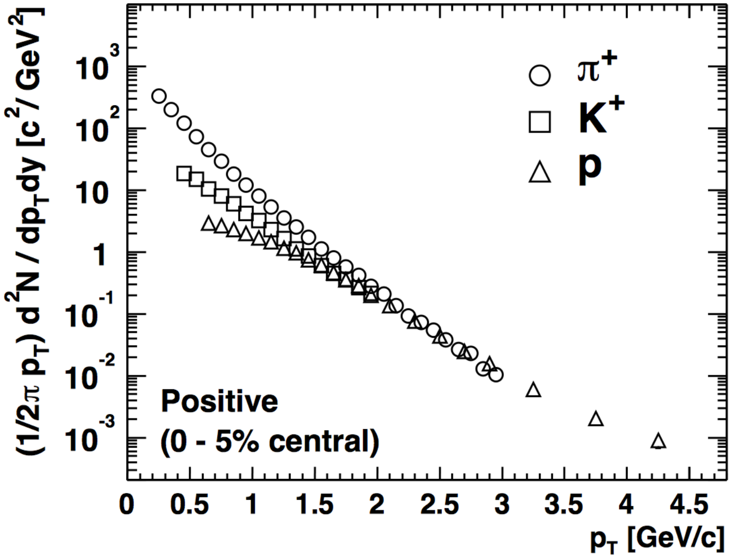
Varying Tc

- We estimate observables using hydrodynamic models for each EoS and find the correspondence between them and the EoS

Observable sensitive to EoS

- Particle spectra

PHENIX, PRC 69, 034909 (2004)

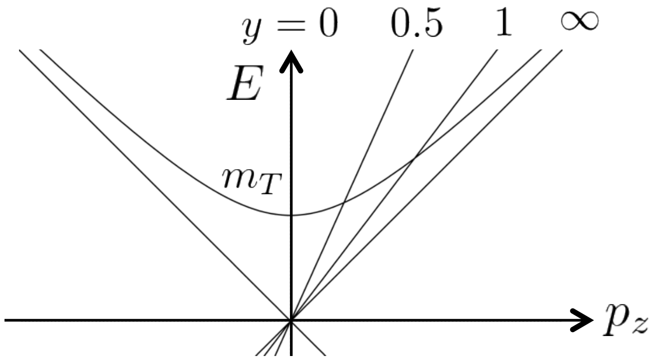


(I) Integrate p_T out
→ particle number at y = 0

~~(II) Integrate p_T out *with*~~
~~p_T as weight~~
~~→ momentum per particle~~

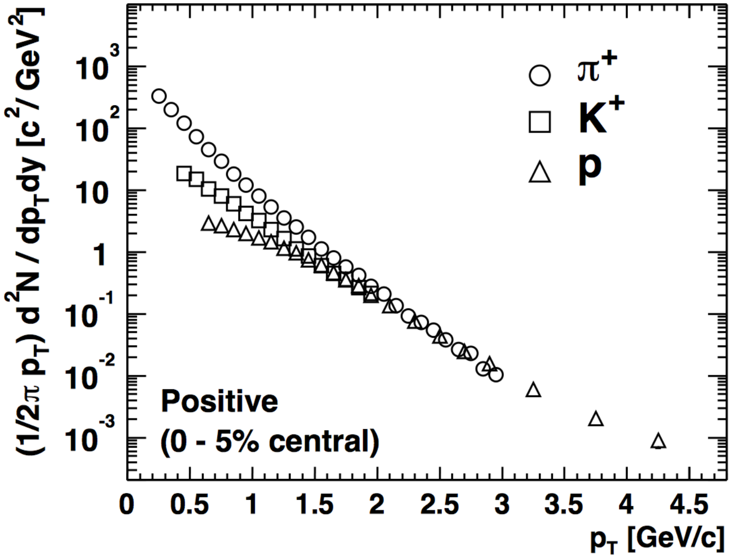
Rapidity y: related to momentum
in the beam axis direction

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$



Observable sensitive to EoS

- Particle spectra


PHENIX, PRC 69, 034909 (2004)

(I) Integrate p_T out
→ particle number at y = 0

(II) Integrate m_T out *with*
m_T as weight
→ energy per particle

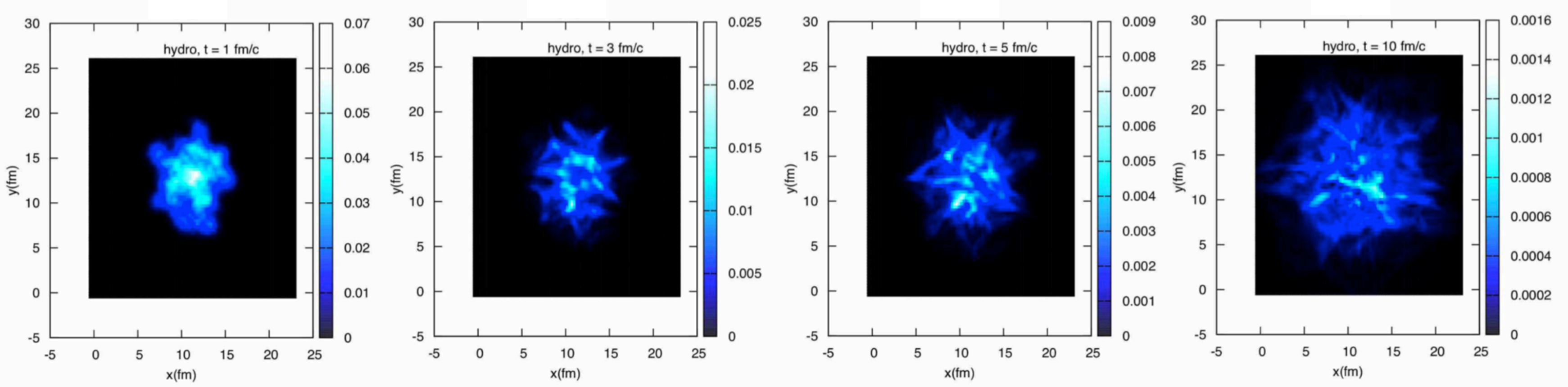
Transverse mass m_T : effective mass of a particle where p_T is "incorporated"

$$m_T = \sqrt{p_T^2 + m^2} = \sqrt{E^2 - p_z^2}$$

*At $z = 0$, $m_T \sim E$ except for thermal blurring

Rough picture of hydro expansion

- Transverse expansion and τ_{eff}



$$\tau \sim \tau_{\mathrm{eff}}$$

Longitudinal expansion is dominant ($R \sim R_0$)

Transverse expansion is relevant ($R > R_0$)

Effective radius of the medium: $R^2 = 2(\langle|\mathbf{x}(\tau)|^2\rangle - |\langle\mathbf{x}(\tau)\rangle|^2)$

Cf: J.-Y. Ollitrault, PLB 273, 32 (1991)

Imprint of the EoS

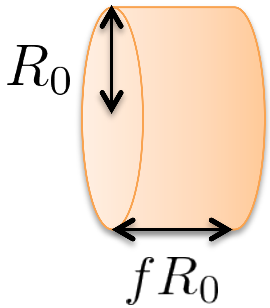
- Entropy density vs. Particle number

Effective entropy per volume

$$s(T_{\text{eff}}) = a\frac{1}{R_0^3}\frac{dN}{dy}$$

Particle number at T_f

$$\frac{S(T_f)}{N(T_f)} \times \frac{1}{V(T_{\text{eff}})}$$



$$V = f\pi R_0^3$$

- Total entropy conservation $S(T_f) = S(T_{\text{eff}})$

T_{eff} : effective temperature when transverse expansion starts
R_0 : effective radius of the medium where $R_0^2 = 2(\langle|\mathbf{x}(\tau_0)|^2\rangle - |\langle\mathbf{x}(\tau_0)\rangle|^2)$
T_f : freeze-out temperature (constant)
a : dimensionless constant factor

Imprint of the EoS

- Energy density over entropy density vs. Mean m_T

Effective energy per entropy

$$\frac{\epsilon(T_{\mathrm{eff}})}{s(T_{\mathrm{eff}})} = b\langle m_T \rangle$$

Energy per number at T_f

$$\frac{N(T_f)}{S(T_f)}$$

- Once transverse expansion sets in ($T \sim T_{\mathrm{eff}}$), longitudinal work becomes smaller and total energy is conserved $E(T_f) \sim E(T_{\mathrm{eff}})$

[b : dimensionless constant factor

Input for hydrodynamic model

- Transport coefficients

  - Shear viscosity $\eta/s = 1/4\pi$ P. Kovtun et al., PRL 94, 111601 (2005)
  - Bulk viscosity $\zeta/s = 2(1/3 - c_s^2)\eta/s$ A. Buchel, PLB 663, 286 (2008)

*Minimalistic choices from the gauge-string correspondence

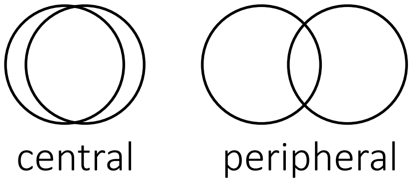
- Initial conditions

  - Monte-Carlo Glauber model M. L. Miller et al., ARNPS 57, 205 (2007); AM, 1408.1410
  - Monte-Carlo KLN model H.-J. Drescher and Y. Nara, PRC 75, 034905 (2007)

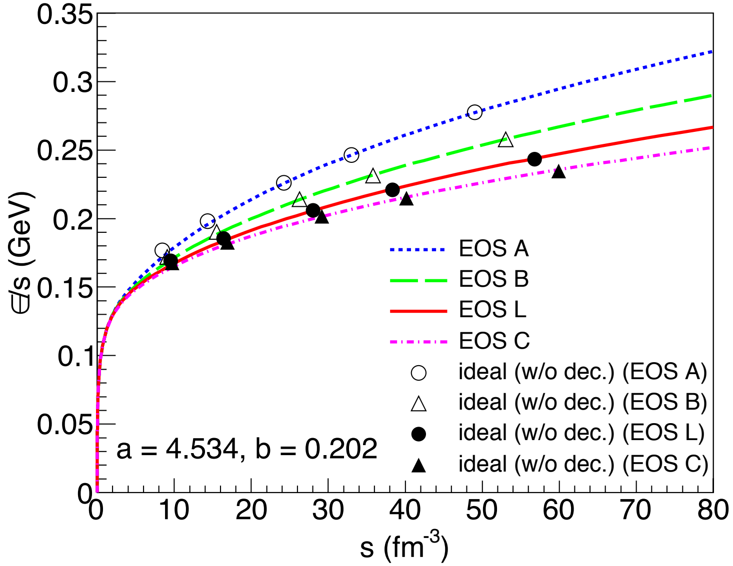
⇨ We show the results of the MC Glauber model here

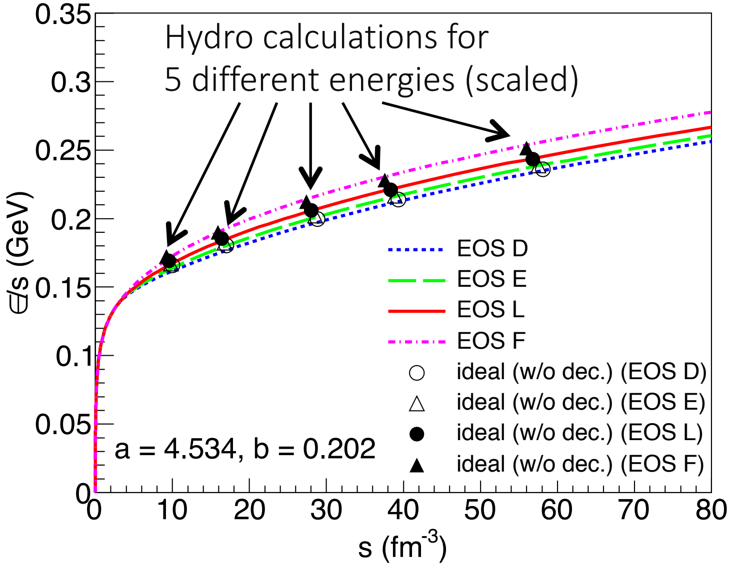
The scaling relations not affected; may affect comparison to data

Determination of *a* and *b*



- Ideal hydro calculations, Au-Au collisions, 0-5% central events





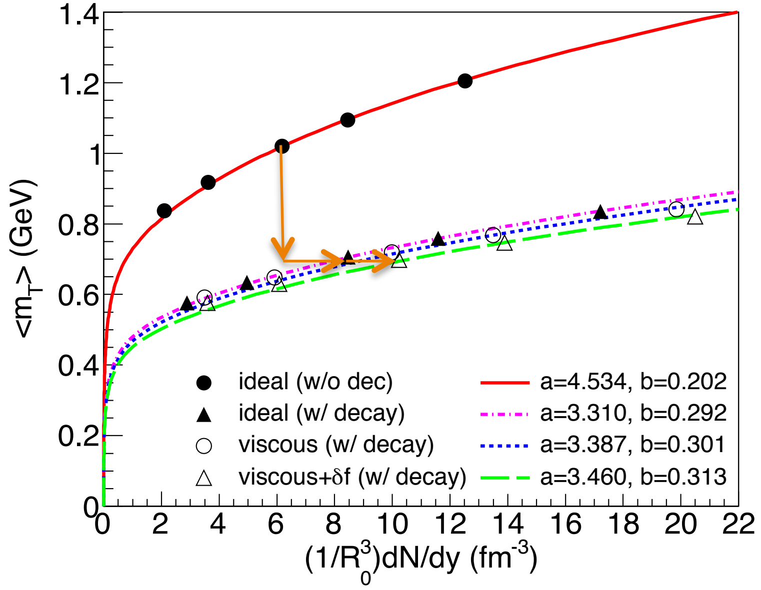
Hydro calculations for
5 different energies (scaled)

- A single set of (a, b) fits hydro results on to all the EoS

  There is correspondence between the EoS and observables

Before comparing to data

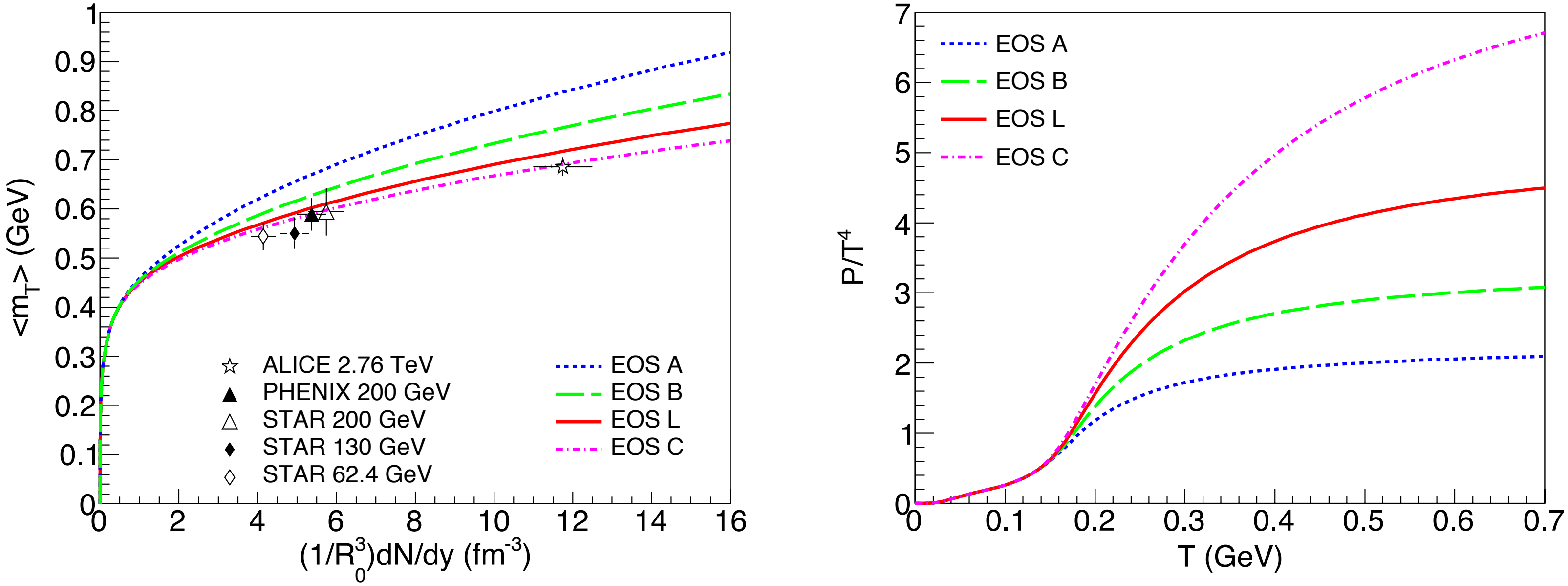
- Must considered are the effects of hadronic decay and viscosity



- (a, b) can be determined so that all the EoS are satisfied
- Hadronic decay reduces <m_T> and increases dN/dy
- Viscosity increases dN/dy by entropy production

Comparisons to experimental data

- Viscous hydro results with hadronic decays, Au-Au, 0-5% events



Conclusions

- Compatible with the lattice QCD equation of state within errors
- Larger effective # of degrees of freedom allowed by the data

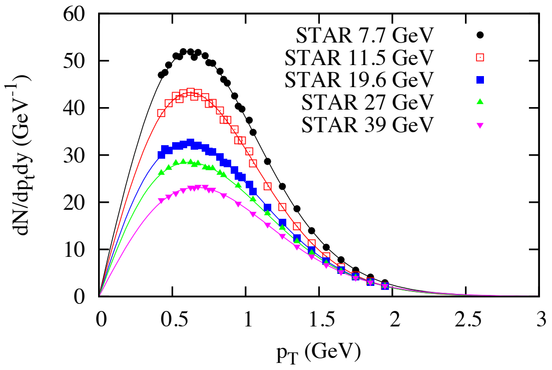
Summary and outlook

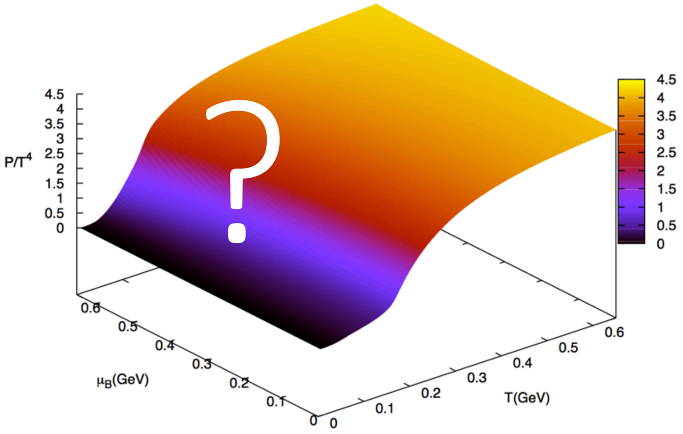
- We have probed collective properties of the hot QCD matter

  - QCD equation of state is constrained using dN/dy and $<m_T>$
  - The EoS with the effective number of DOF equal to or larger than that of lattice QCD is favored

  - We may extract the finite-density EoS out of Beam Energy Scan experimental data

AM and J.-Y. Ollitrault, in preparation





The end

Thank you for listening!