Fourier coefficients of net-baryon density and chiral criticality

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Contents

QCD Phase Diagram from Imaginary μ_B

$\mu_B \to \mu_B = iT\theta_B$

No sign problem

in $P(T,V,\theta_B)$

Analytic continuation

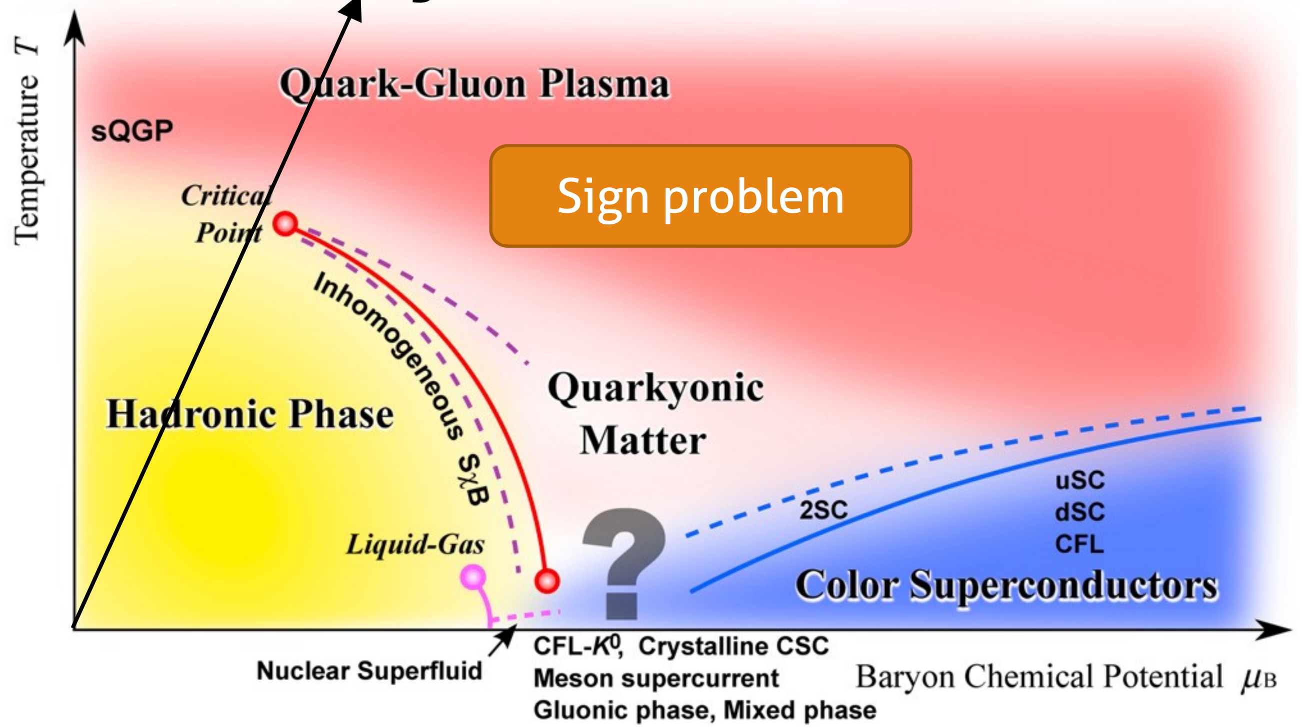
ex) $T_c(\theta) \to T_c(\mu)$

Canonical approach

$Z(T,V,\theta) \to Z_c(T,V,N)$

$T=\mu_B/\pi$

Fukushima-Hatsuda, Rep.Prog.Phys.74 '11



Fourier expansion at Imaginary μ

- Partition function – Canonical approach

$$Z(T,V,\theta) = \sum_{N=-\infty}^{\infty} Z_c(T,V,N) e^{iN\theta}$$

- Thermodynamics from $Z_c(T,V,N)$
- Probability distribution $P(N)$
  $\rightarrow$fluctuation of N: large $|N|$ sensitive to critical property of the system
  (KM et al., EPJC'14,PRC'13,PLB'15)

- Pressure (or density) – Cluster expansion

$$\mathrm{Im}\frac{\partial p/T^4}{\partial(\mu_B/T)} = \sum_{k=1}^{\infty} b_k \sin(k\theta) \quad \Rightarrow \quad \frac{n}{T^3} = \sum_{k=1}^{\infty} b_k \sinh(k\mu/T)$$

Related Works

- Lattice QCD
  - fit Im$\chi(\theta)$ (Bornyakov+ 1712.02830)
  - up to b_4 at physical point (Vovchenko+, Budapest-Wuppertal data, 1708.02852)
- Effect of deconfinement (Kashiwa and Ohnishi, 1712.06220)
  - Long tail in b_k
- Cluster Expansion Model (Vovchenko+, 1711.01261)
  - Ansatz motivated by repulsive interaction

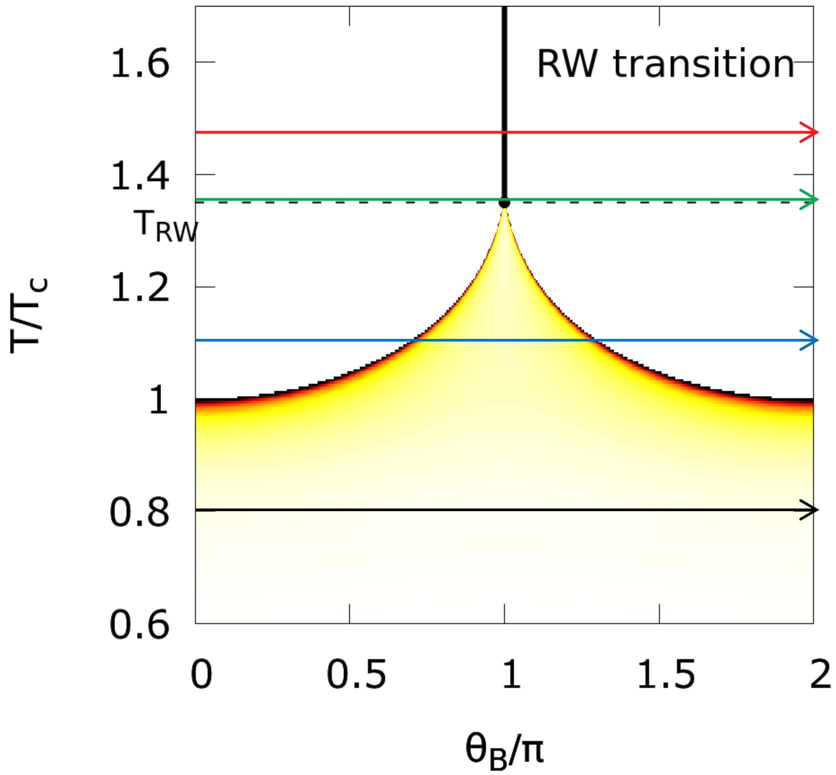
$$b_k^{\mathrm{CEM}} = \left[\frac{b_1(T)}{b_1^{\mathrm{SB}}}\right]^2 \frac{b_2^{\mathrm{SB}}}{b_2(T)} \left[\frac{b_1^{\mathrm{SB}}}{b_1(T)} \frac{b_2(T)}{b_2^{\mathrm{SB}}}\right]^k b_k^{\mathrm{SB}}$$

exponential damping

  - b_1 and b_2 from lattice QCD – consistent b_3 and b_4
  - Prediction of higher order cumulants

This work : effect of criticality in b_k?

QCD Phase Diagram at Imaginary μ



$$b_k = \frac{1}{\pi}\int_0^{2\pi} [\mathrm{Im}\chi_1^B(T, i\theta_B)] \sin(k\theta_B)$$

- Temperature-dependent effect:
  - $T > T_{RW}$: 1st order transition
  - $T = T_{RW}$: Roberge-Weiss endpoint
  - $T_c < T < T_{RW}$: Phase boundary
  - $T < T_c$: No phase transition at Imμ

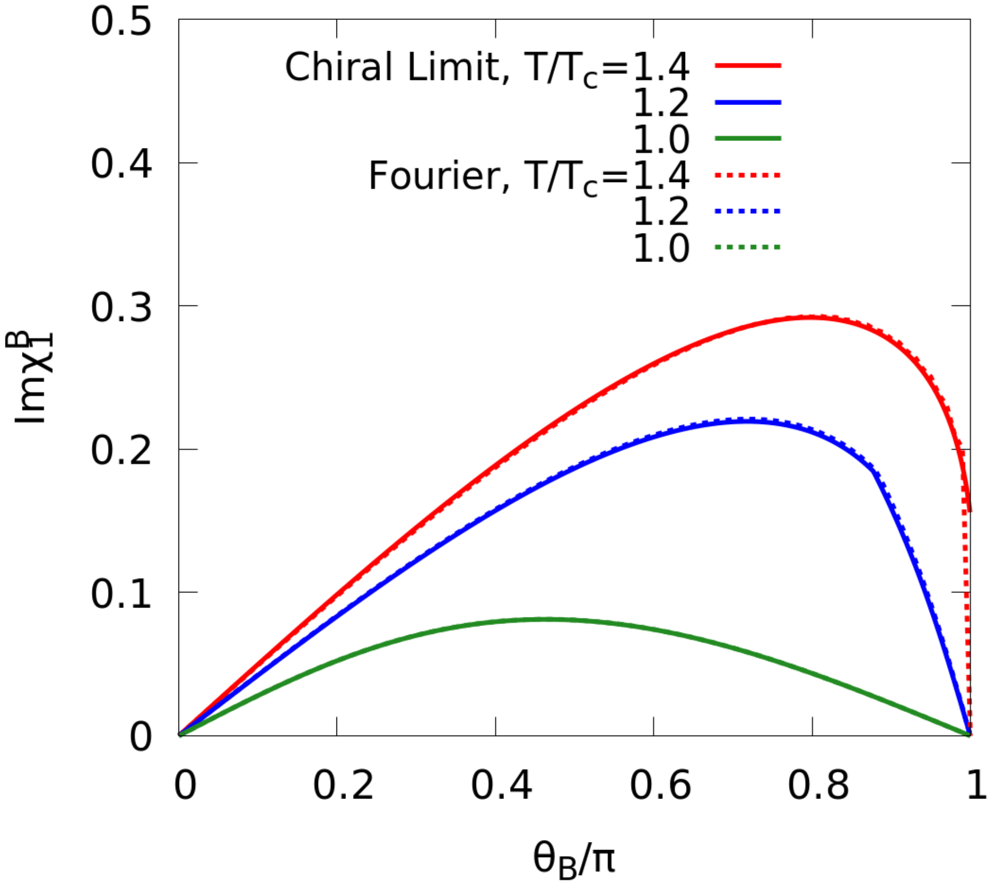
Density: PQM model (MF)

Density in imaginary μ

Dotted: Reconstructed

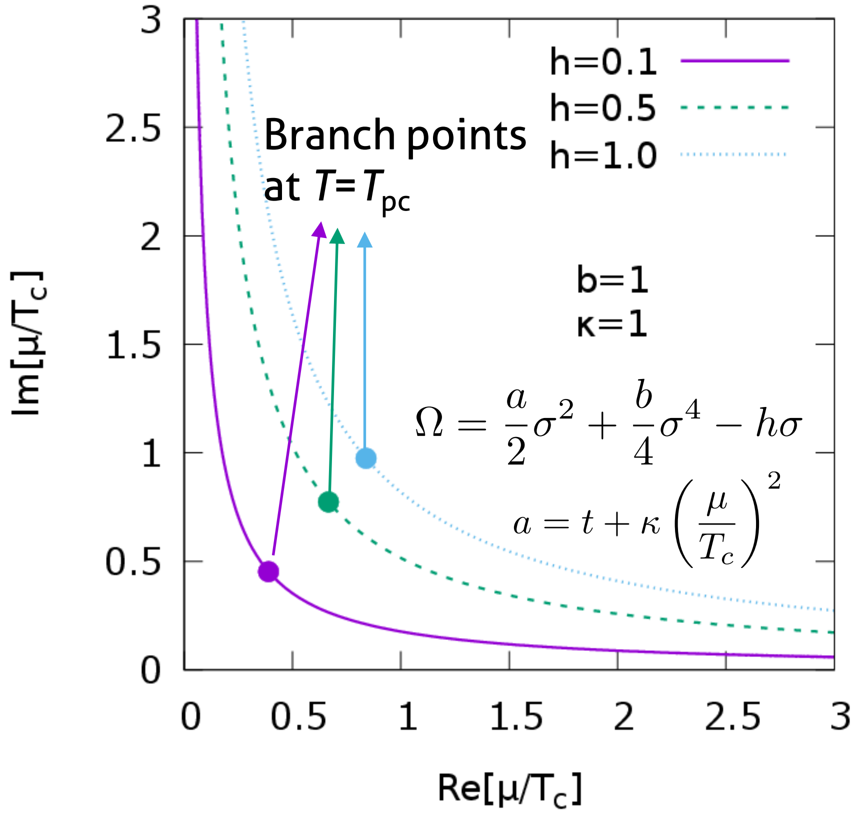
Kink: phase boundary

Discontinuity: RW transition

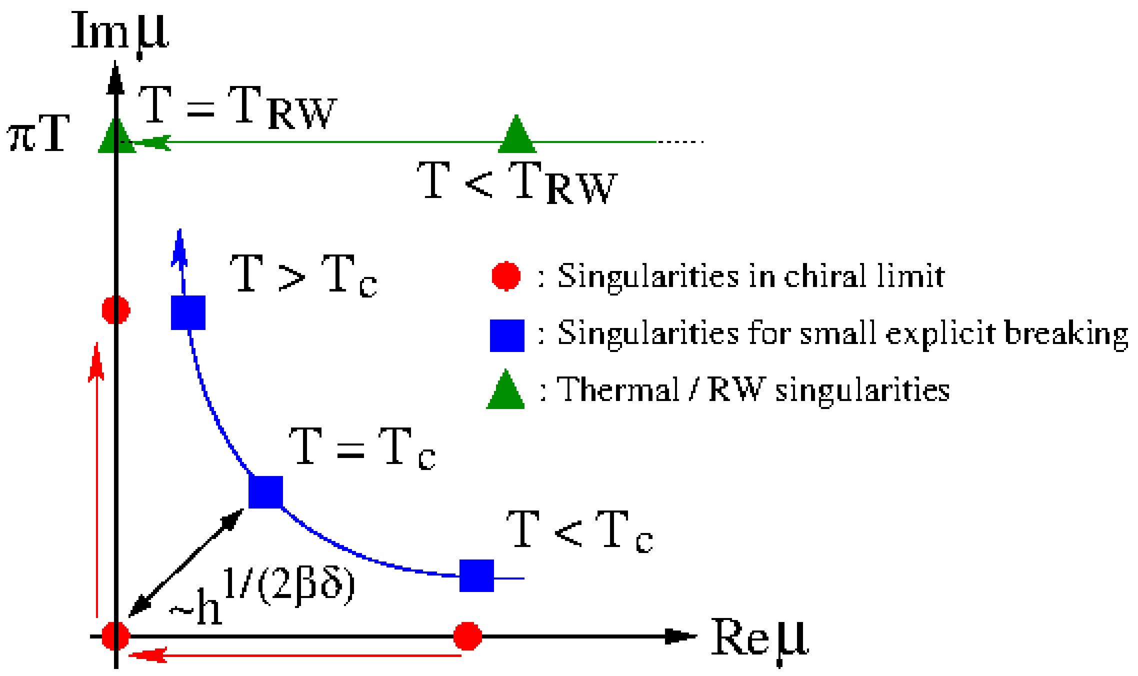


Singularities in complex μ

- Landau Theory



- Schematic picture



First order PT: Cuts across real or imaginary axis

Asymptotic b_k: Regular part

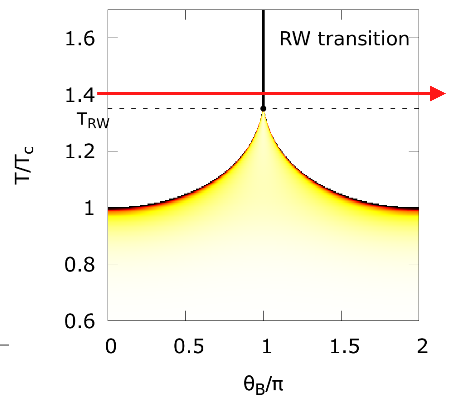
Regular part (massive Fermi gas)

$$b_k \sim (-1)^{k+1} \frac{K_2(km/T)}{k} \sim (-1)^{k+1} \frac{e^{-km/T}}{k^{3/2}}$$

Exponential damping

Sign change : thermal singularity

Asymptotic b_k: 1st order PT ($T > T_c$)



First order transition → Discontinuty at θ_c in Im$\chi_1{}^B$

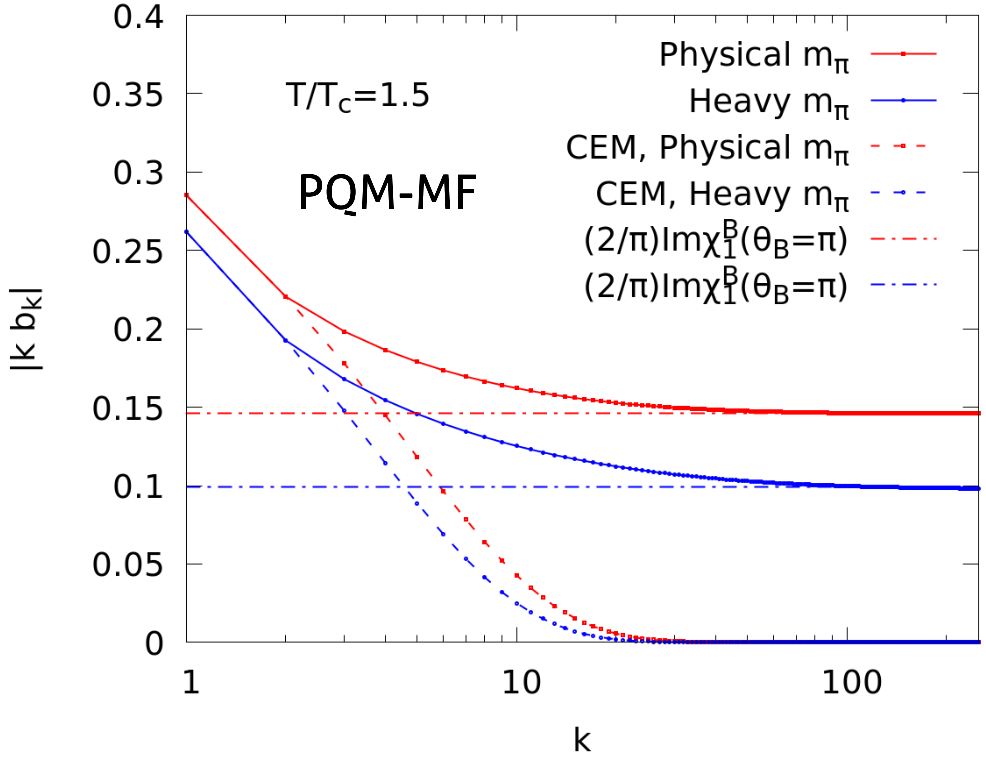
$$\mathrm{Im}\chi_1^B(T, i\theta_B) = f(\theta_B)\Theta(\theta_c - \theta_B) + g(\theta_B)\Theta(\theta_B - \theta_c)$$

$$b_k = \frac{2\cos(k\theta_c)}{\pi k}(g(\theta_c) - f(\theta_c)) + \text{subleading terms}$$

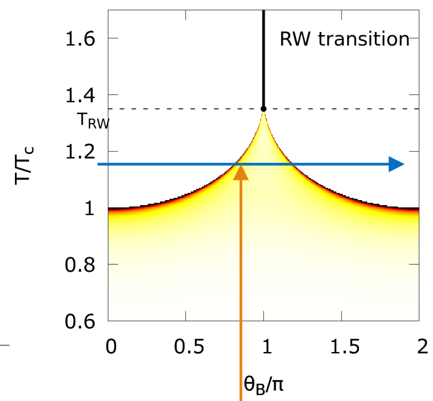
Oscillation×1/k×density gap

For $T > T_{RW}$, $\theta_c = \pi$

$$b_k \sim (-1)^{k+1}\frac{2}{\pi k}\mathrm{Im}\chi_1^B(\theta_B = \pi)$$



Asymptotic b_k: 2nd Order ($T > T_c$)



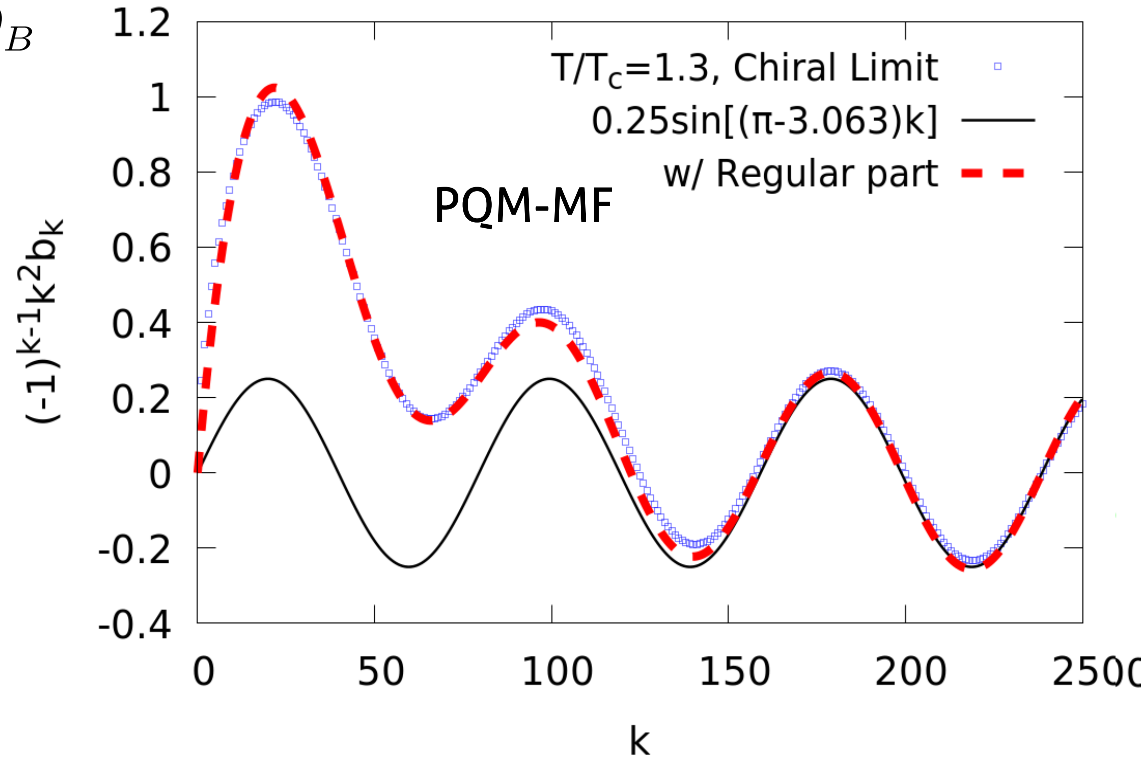
Second order transition in imaginary μ : Branch point

$$p^{\text{singular}}(T,\theta_B) \sim |t-\kappa\theta_B^2|^{\phi+1}, \quad \text{Im}\chi_1^B \sim |t-\kappa\theta_B^2|^{\phi}\theta_B$$

$$\theta_c = \sqrt{\frac{t}{\kappa}} \qquad \phi = \begin{cases} 1-\alpha & (T \neq T_c) \\ 3-2\alpha & (T = T_c) \end{cases}$$

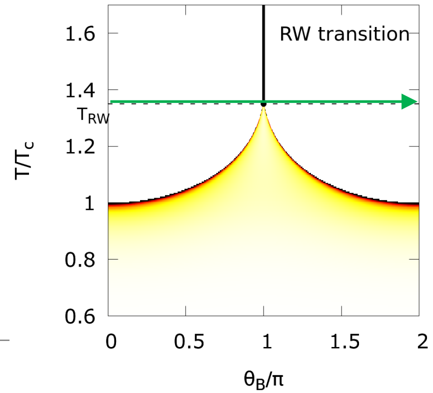
Steepest descent method for large k

$$b_k \sim \text{Im}\int dy\, e^{ik(y+\theta_c)+\phi\ln y} \sim \frac{\sin(k\theta_c + (\phi+1)\frac{\pi}{2})}{k^{\phi+1}}$$



Oscillation × power law

Asymptotic b_k: $T=T_{RW}$ (2nd order)



Second order RW endpoint : Branch point at $\theta_B = \pi$

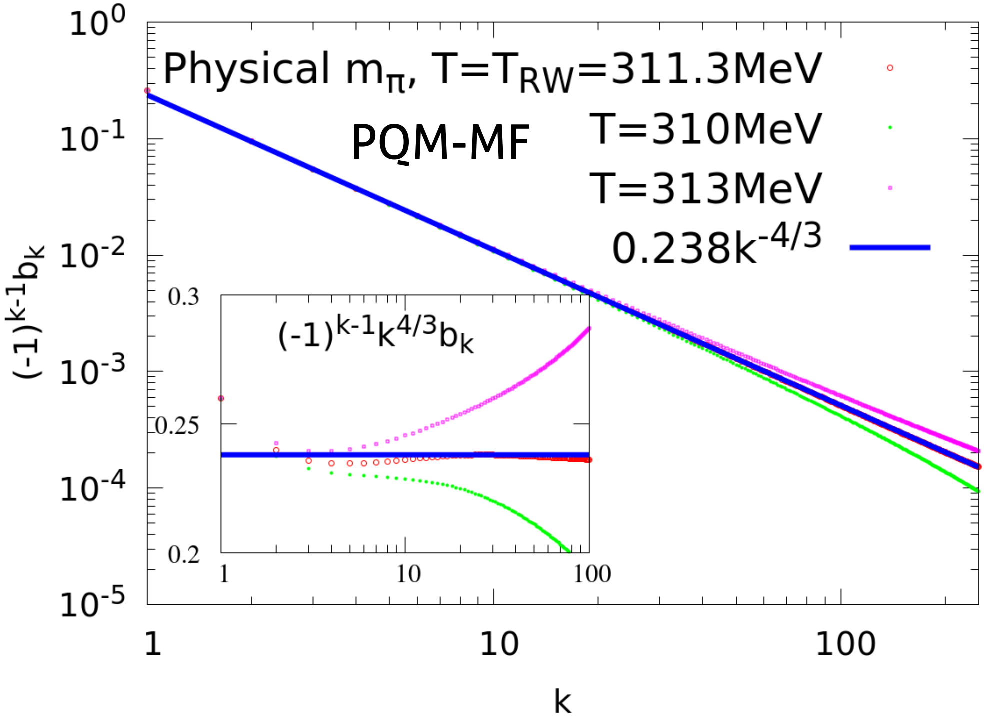
Z(2) universality class: $\mathrm{Im}\chi_1^B \sim (\pi - \theta_B)^{1/\delta}$

$$b_k \sim \frac{(-1)^{k+1}}{k^{1+1/\delta}}$$

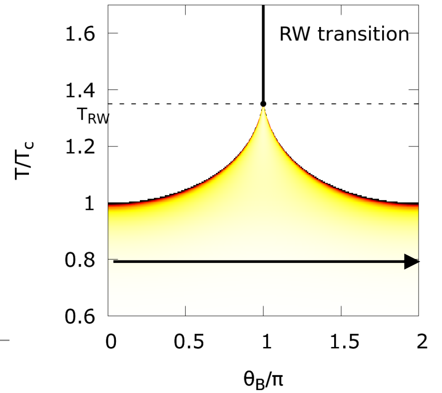
Steepest descent method for large k

Sign change × power law

Special case of the oscillation: singularity at $\theta_B = \pi$



Asymptotic b_k: 2nd order ($T < T_c$)

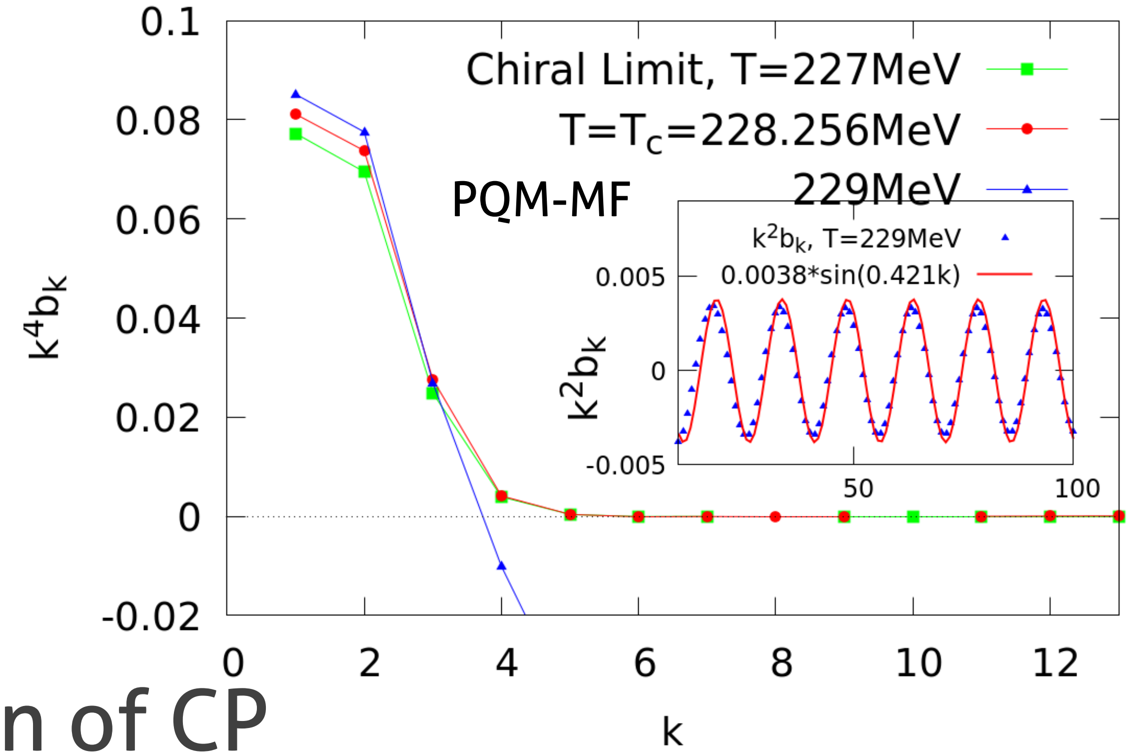


Below T_c : second order transition at real $\mu_B = \pm\mu_c$

- Branch point at $\theta_B = \pm i\mu_c / T$

- $$b_k \sim \frac{e^{-k\mu_c/T}}{k^{\phi+1}}$$

Steepest descent method for large k:
Choose integration contour to satisfy R-L lemma



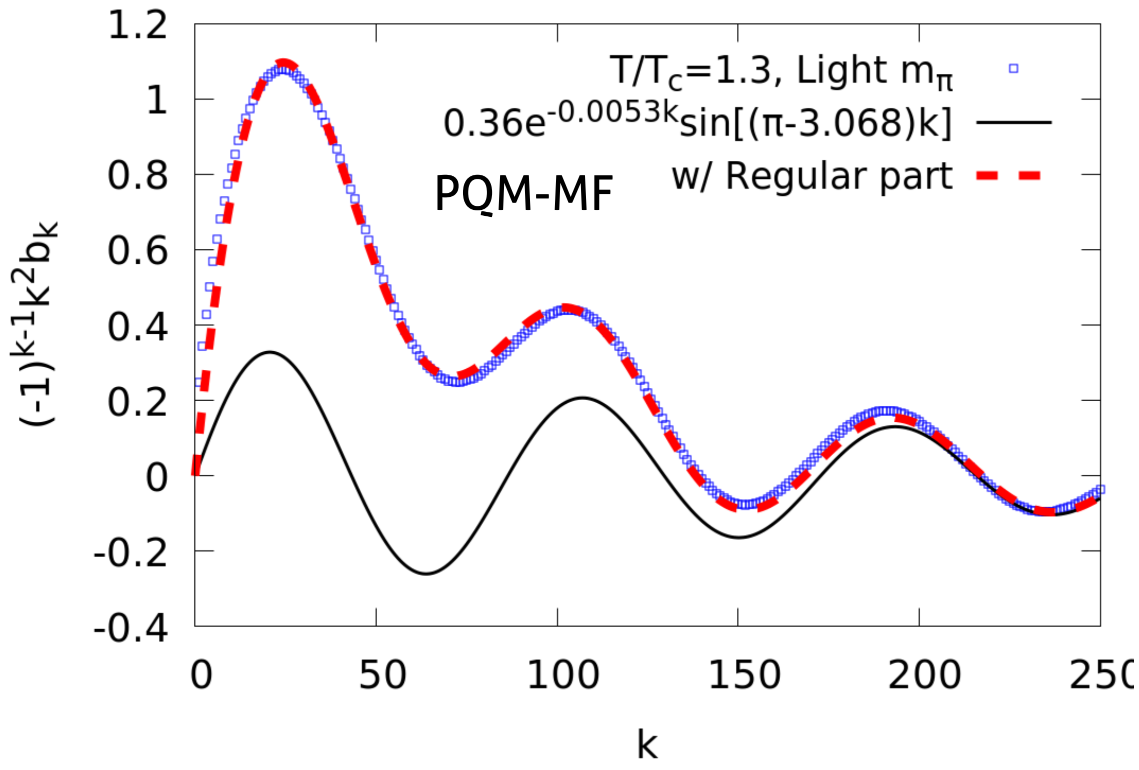
Exponential damping: location of CP

Asymptotic b_k: Crossover

Crossover: Branch points at $\theta_B = \pm\theta_c \pm i\mu_c/T$

$$b_k \sim \frac{e^{-k\mu_c/T}}{k^{\phi+1}} \sin\left(k\theta_c + (\phi+1)\frac{\pi}{2}\right)$$

Steepest descent method for large k: Choose integration contour to satisfy R-L lemma



Exponential damping: location of real part of CP

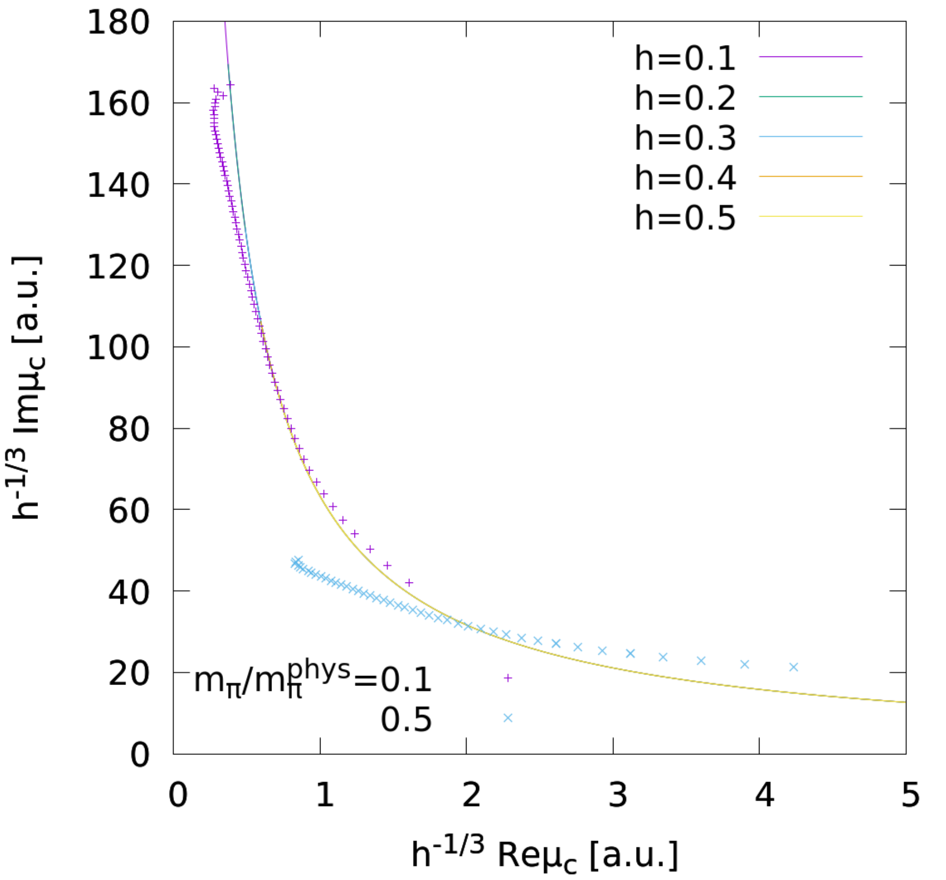
Oscillation: location of imaginary part of CP

Power law: Critical exponent

Location of branch point from b_k

Fit to b_k yields real and imaginary part of the location of the singularity

Scaling theory : Branch point location scales with $h^{-1/3}$

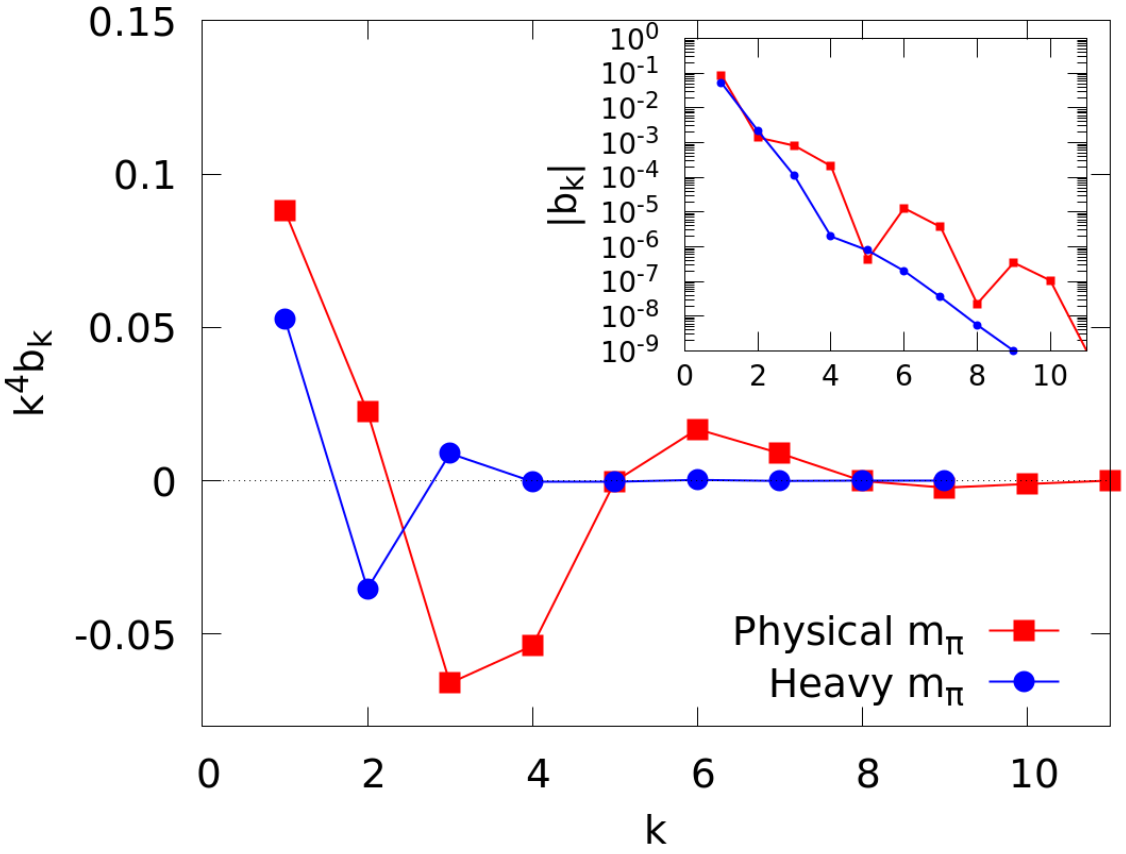


Physical and Heavy m_π

b_k at physical m_π near T_c

- Stronger damping
  - hard to get large k
- Oscillation still visible

Heavier m_π close to CEM



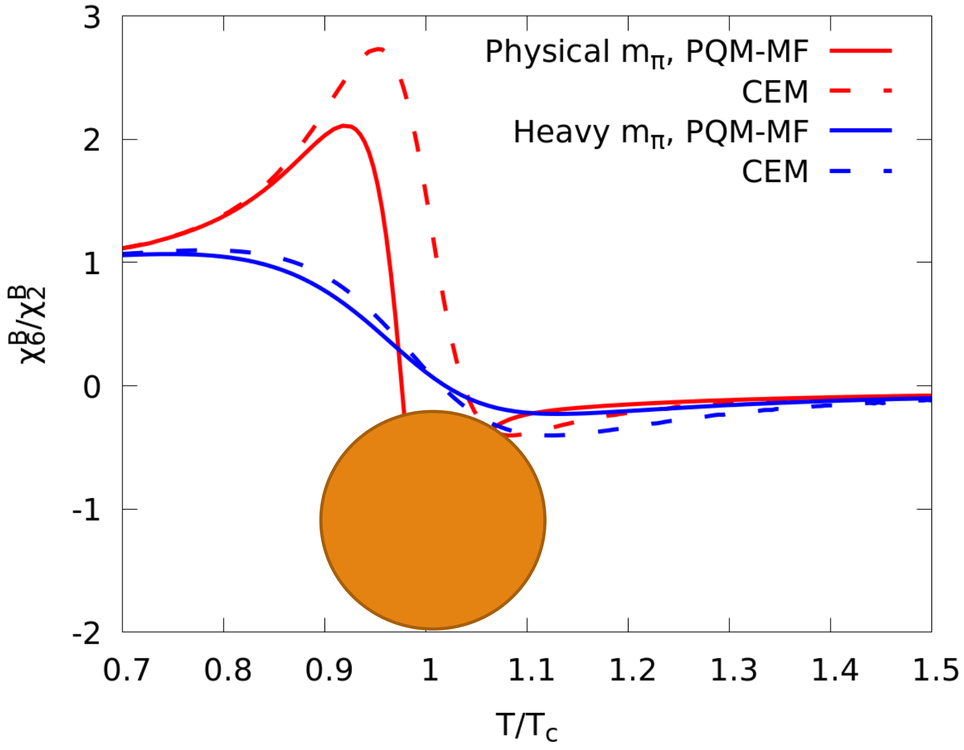
Net-baryon fluctuations from b_k

Observable consequence of characteristic b_k

$$\chi_{2n}^B = \sum_{k=1}^{k_{\max}} k^{2n-1} b_k$$

Negative χ_6 – Remnant of O(4) 2nd order transition

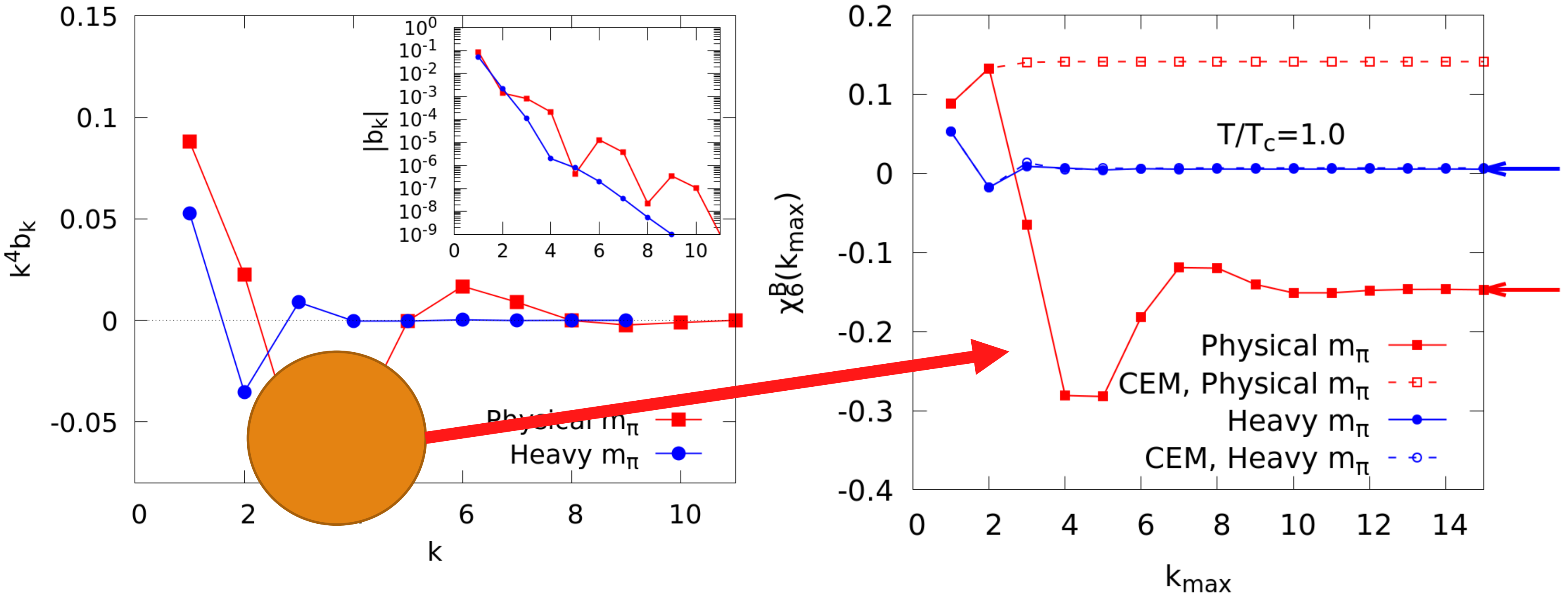
Relation to b_k?



$$\chi_{2n}^B = \sum_{k=1}^{k_{\max}} k^{2n-1} b_k$$

Net-baryon fluctuations from b_k

Observable consequence of characteristic b_k



Summary and Outlook

- Fourier coefficients provide interesting insights into phase structure and critical property
- Complex singularities dictate large order behavior
  - One may be able to locate the singularity from b_k

<table>
<tr><th>Temperature</th><th>$T < T_c$</th><th>$T_c < T < T_{\mathrm{RW}}$</th><th>$T = T_{\mathrm{RW}}$</th><th>$T > T_{\mathrm{RW}}$</th></tr>
<tr><td>Chiral limit</td><td>$\frac{e^{-k\mu_c/T}}{k^{2-\alpha}}$</td><td>$\frac{\sin(k\theta_c - \alpha\pi/2)}{k^{2-\alpha}}$</td><td rowspan="2">$\frac{(-1)^{k+1}}{k^{1+1/\delta}}$</td><td rowspan="2">$\frac{(-1)^{k+1}}{k}\Delta n|_{\theta_B=\pi}$</td></tr>
<tr><td>Nonzero m_π</td><td colspan="2">$\frac{e^{-k\mu_c/T}\sin(k\theta_c - \alpha\pi/2)}{k^{2-\alpha}}$</td></tr>
</table>

Summary and Outlook

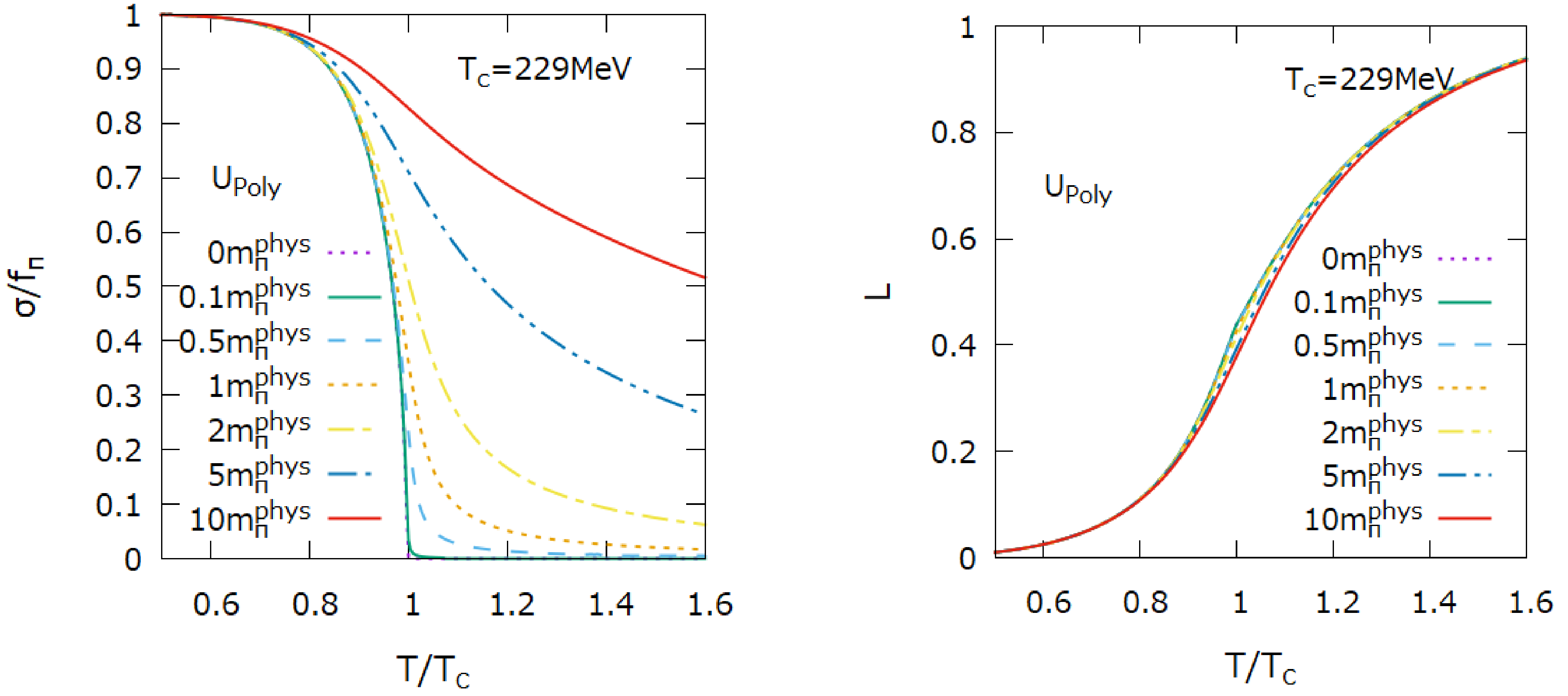
- Related to criticality in net-baryon fluctuations
  - Negative χ_6 from complex singularity

To apply to Lattice QCD…

- Effect of Lee-Yang zeros in finite size systems
- How large k is practically possible?

Backup Slides

PQM model: setup



Temperature dependence