NFQCD2018, Kyoto, June 12, 2018

The entangled 3D structure of QCD

Piet Mulders

European Research Council

Netherlands Organisation for Scientific Research

Nikhef

VU UNIVERSITY AMSTERDAM

mulders@few.vu.nl

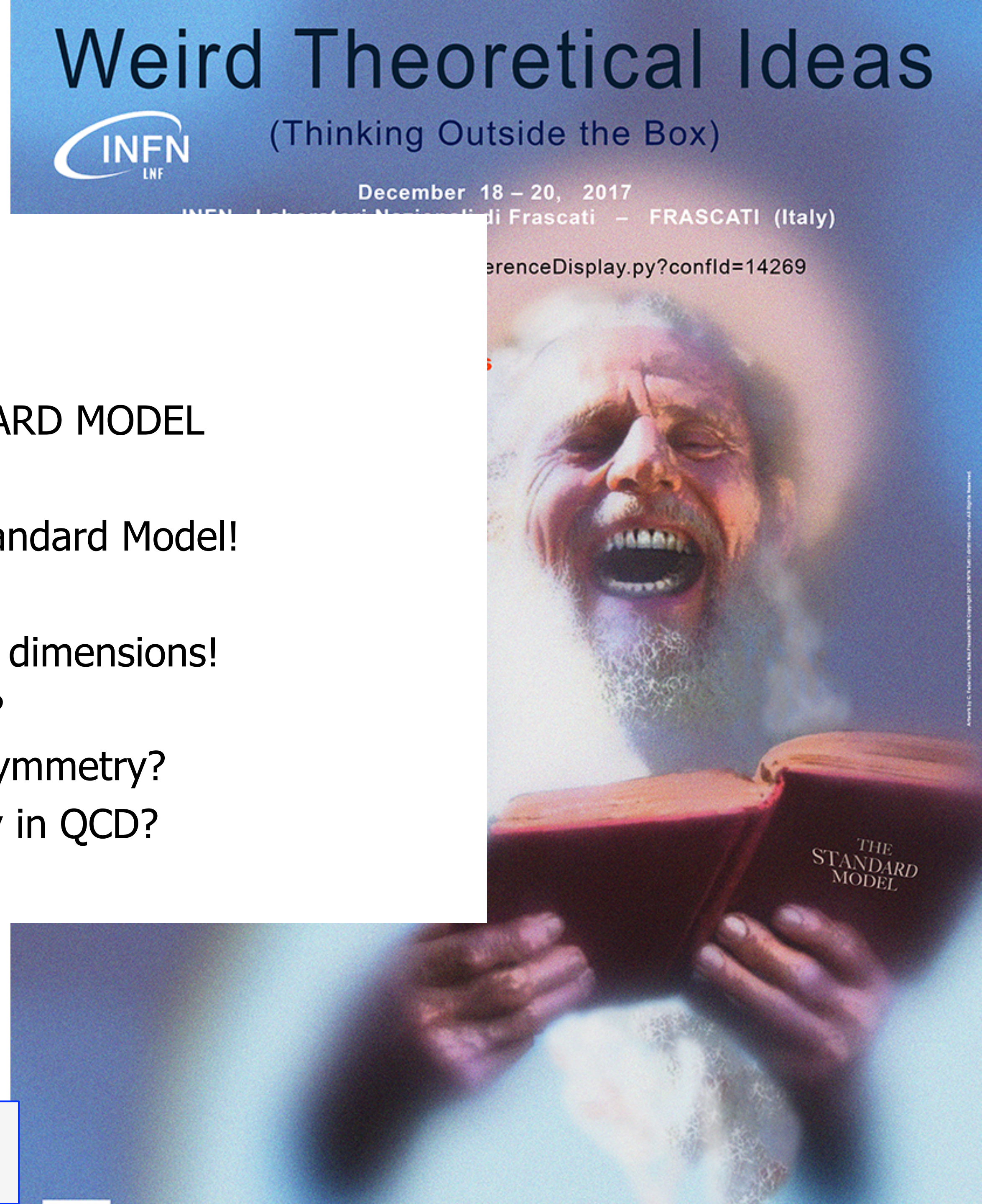
abstract

YKIS 2018b (Kyoto, 11-15 June 2018):

The entangled 3D structure of QCD

P.J. Mulders

We discuss a new idea to understand the emergence of symmetries in the standard model starting with less than three space dimensions. Even if at this stage it does not upset the present successful phenomenology or make striking predictions , the hope is that it could shed light on the many peculiarities and simplify our understanding of the 3D quark and gluon structure of hadrons at low and high energies.



- Motivation
  (NOT) HAPPY WITH STANDARD MODEL

- In spite of the success of Standard Model!

- Three families, colors, space dimensions!
- Left-right (a)symmetry? B-L?
- Naturalness? Missing supersymmetry?
- Confinement and Collinearity in QCD?

PJM 1601.00300
PJM 1801.03664

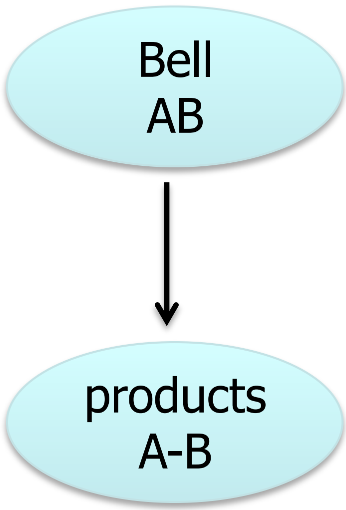
QCD – entangled states and QIT

- Parton-hadron duality in hard QCD scattering: PDFs x FFs
  - nucleon is pure state → ensemble of partons (good light-front states) [see for instance Kharzeev & Levin (1702.03489)]
  - hard (short distance) process: partons → partons
  - emerging partons are pure state(s) → ensemble of hadron states

- Entangled (pure) states $|\Phi\rangle$ in $\mathcal{H}^A \otimes \mathcal{H}^B$ with a density matrix $\rho = |\Phi\rangle\langle\Phi|$ lead to ensembles (non-pure state) in the reduced spaces.
  - EPR bipartite pure state leads to a 50% - 50% ensemble in both subspaces.
- Maybe both hadrons and partons live in a multipartite Hilbert space !
- Possibly combined with a principle of maximal entanglement (MaxEnt), such as hinted at in Cervera-Lierta, Latorre, Rojo & Rottoli (1703.02989): maximally entangled chiral left/right two-particle states are consistent with QED (g_A=0) & electroweak (g_V=0), at least if $\sin \Theta_W = ½$

Bipartite entangled states

- Bell states are maximally entangled (MaxEnt) states: $|RR\rangle \pm |LL\rangle$ or $|RL\rangle \pm |LR\rangle$
- They belong to the same class (SLOCC, for us local unitary, local = subspace)



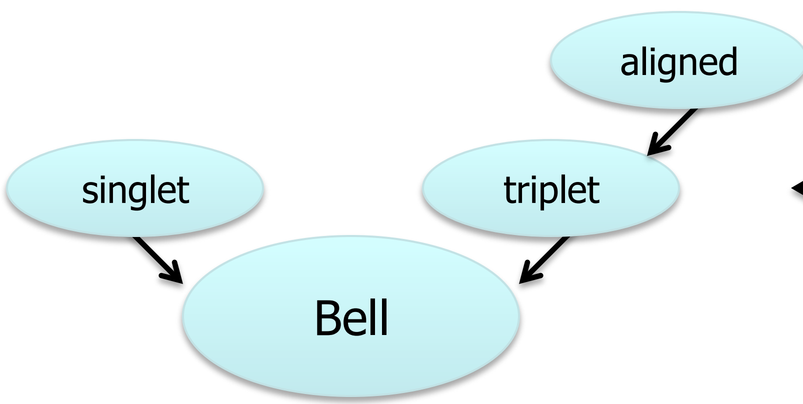
$$\rho = |\mathrm{Bell}\rangle\langle\mathrm{Bell}| \implies \rho_A = \frac{1}{2}\left(|R\rangle\langle R| + |L\rangle\langle L|\right)$$

$$|\Phi\rangle = a|RR\rangle + b|RL\rangle + c|LR\rangle + d|LL\rangle$$

entanglement measure:

$$0 \leq \Delta = 2\sqrt{1 - \mathrm{Tr}(\rho^2)} = 2|ad - bc| \to 2\sqrt{p_1 p_2} \leq 1$$

- Symmetry eigenstates are in general entangled



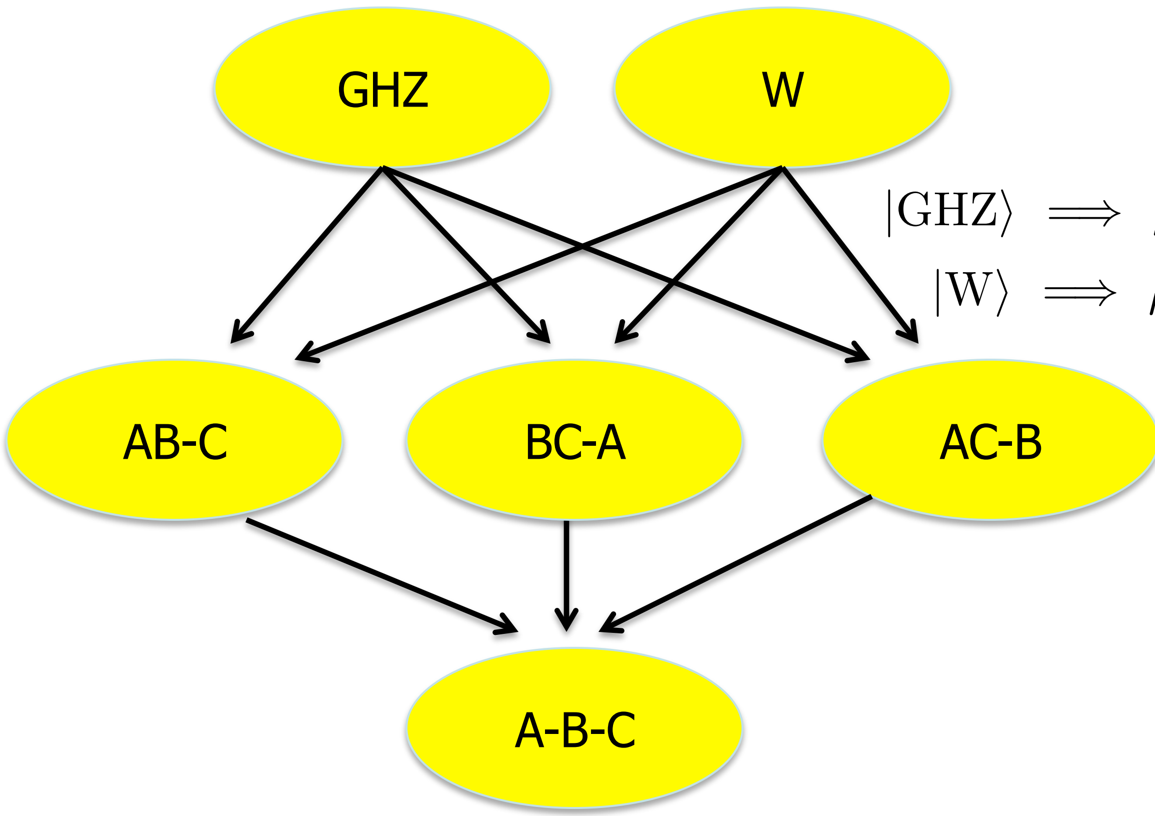
← generators $\tau \times \tau$

← generators $\tau \times 1 + 1 \times \tau$

Tripartite entangled states

- Two classes of maximally entangled states: (Dur, Vidal, Cirac 2000)

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$$
$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|LRR\rangle + |RLR\rangle + |RRL\rangle)$$



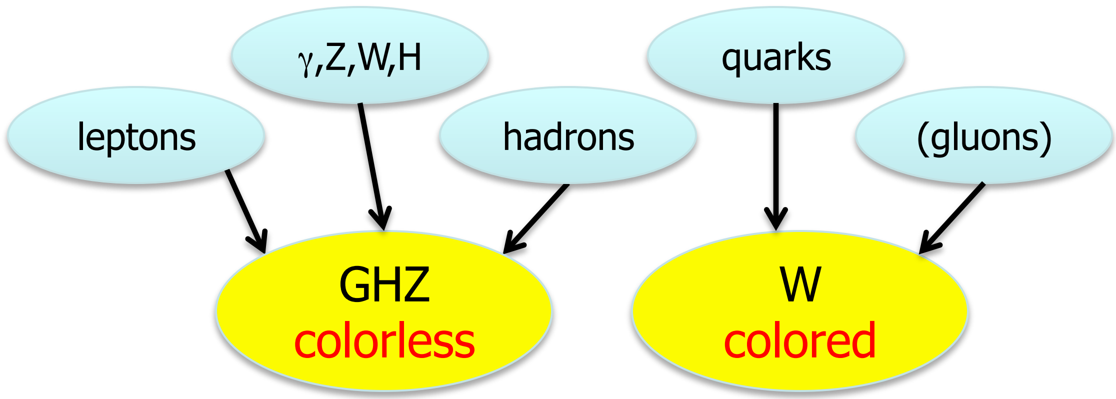
$$|\mathrm{GHZ}\rangle \implies \rho_{AB} = \frac{1}{2}\left(|RR\rangle\langle RR| + |LL\rangle\langle LL|\right)$$
$$|\mathrm{W}\rangle \implies \rho_{AB} = \frac{2}{3}|\mathrm{Bell}\rangle\langle \mathrm{Bell}| + \frac{1}{3}|RR\rangle\langle RR|$$

GHZ: fragile
W: robust

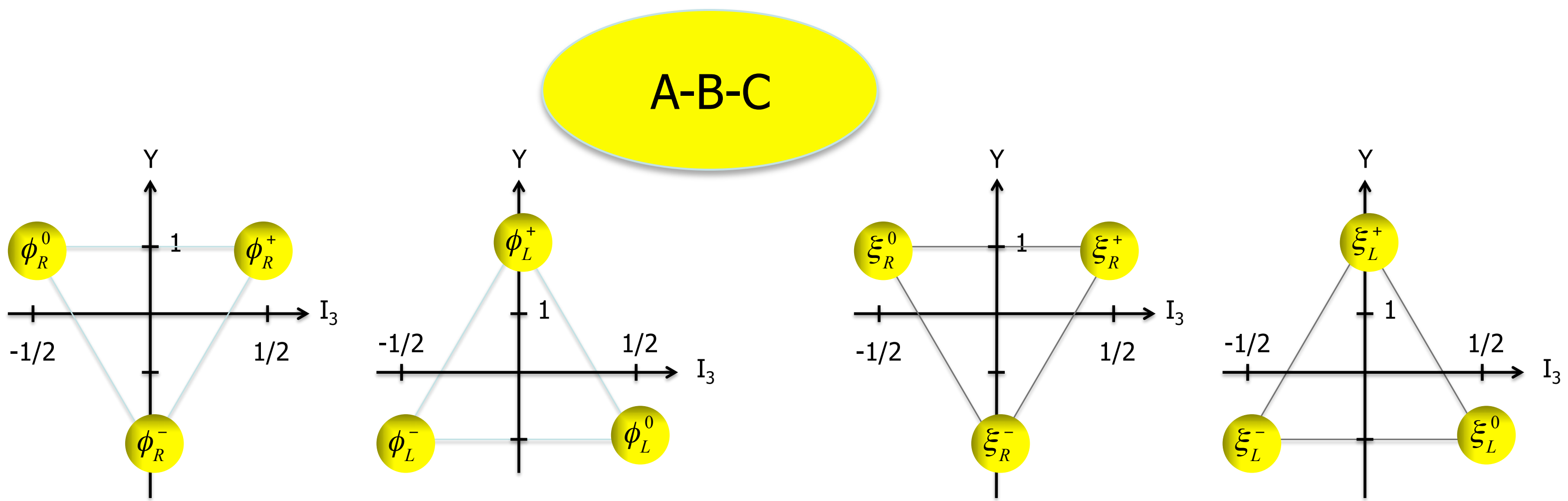
- Beyond tripartites there is an infinite number of classes!

Quarks and leptons as entangled states

- Symmetry eigenstates are in general entangled



- Conjecture is that physically relevant states correspond to MaxEnt tripartite states with basic product states being simple (CP) and even supersymmetric



PJM 1601.00300, PJM 1801.03664

EMERGENCE OF SPACE-TIME DEPENDENCE

All Possible Symmetries of the S Matrix*

SIDNEY COLEMAN† AND JEFFREY MANDULA‡
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 16 March 1967)

We prove a new theorem on the impossibility of combining space-time and internal symmetries in any but a trivial way. The theorem is an improvement on known results in that it is applicable to infinite-parameter groups, instead of just to Lie groups. This improvement is gained by using information about the S matrix; previous investigations used only information about the single-particle spectrum. We define a symmetry group of the S matrix as a group of unitary operators which turn one-particle states into one-particle states, transform many-particle states as if they were tensor products, and commute with the S matrix. Let G be a connected symmetry group of the S matrix, and let the following five conditions hold: (1) G contains a subgroup locally isomorphic to the Poincaré group. (2) For any $M>0$, there are only a finite number of one-particle states with mass less than M. (3) Elastic scattering amplitudes are analytic functions of s and t, in some neighborhood of the physical region. (4) The S matrix is nontrivial in the sense that any two one-particle momentum eigenstates scatter (into something), except perhaps at isolated values of s. (5) The generators of G, written as integral operators in momentum space, have distributions for their kernels. Then, we show that G is necessarily locally isomorphic to the direct product of an internal symmetry group and the Poincaré group.

I. INTRODUCTION

UNTIL a few years ago, most physicists believed that the exact or approximate symmetry groups of the world were (locally) isomorphic to direct products of the Poincaré group and compact Lie groups. This world-view changed drastically with the publication of the first papers on $SU(6)$[1]; these raised the dazzling possibility of a relativistic symmetry group which was not simply such a direct product. Unfortunately, all attempts to find such a group came to disastrous ends, and the situation was finally settled by the discovery of symmetry group of the S matrix, which contains the Poincaré group and which puts a finite number of particles in a supermultiplet. Let the S matrix be nontrivial and let elastic scattering amplitudes be analytic functions of s and t in some neighborhood of the physical region. Finally, let the generators of G be representable as integral operators in momentum space, with kernels that are distributions. Then G is locally isomorphic to the direct product of the Poincaré group and an internal symmetry group. (This is a loose statement of the theorem; a more precise one follows below.)

Basic symmetries including SUSY

- Hilbert space

$$\{(a^\dagger)^n|0\rangle, b^\dagger|0\rangle\} \qquad [a, a^\dagger] = 1,\ \{b, b^\dagger\} = 1$$

- Supercharges

$$Q^\dagger_{ik} = b_i\, a^\dagger_k \text{ and } Q_{ik} = b^\dagger_i a_k \qquad \{Q^\dagger_{ik}, Q_{jl}\} = \tfrac{1}{2}\,\delta_{ij}\{a^\dagger_l, a_k\} + \tfrac{1}{2}\,\delta_{kl}[b^\dagger_i, b_j]$$

$$a^\dagger_k \xrightarrow{Q_{ik}} b^\dagger_i \qquad a^\dagger_k \xleftarrow{Q^\dagger_{ik}} b^\dagger_i$$

hamiltonian/number operators (i=j, k=l) & unitary rotations

- For boson and fermion fields

$$\varphi = \frac{1}{\sqrt{2\omega}}\,(a + a^\dagger) \quad \text{and} \quad \xi = \frac{1}{\sqrt{2}}\,(b + b^\dagger)$$

$$Q = \sqrt{\omega}(a^\dagger b - b^\dagger a)$$

$$[Q, \varphi] = \xi \qquad \{Q, \xi\} = \{Q, [Q, \varphi]\} = F = iD\varphi$$

$$[Q, F] = [Q, \{Q, \xi\}] = iD\xi$$

Single (free) field

$$F = [\varphi, H] = iD\varphi = i\dot{\varphi}$$

$$iD = i\partial + gA$$

↑ unitary rotations

- Implement symmetries via constraints F

... and a nontrivial vacuum (not everything is for free!)

$$\phi(t) = \mathcal{T}\exp\left(-i\int_0^t ds\cdot D\right)\phi$$

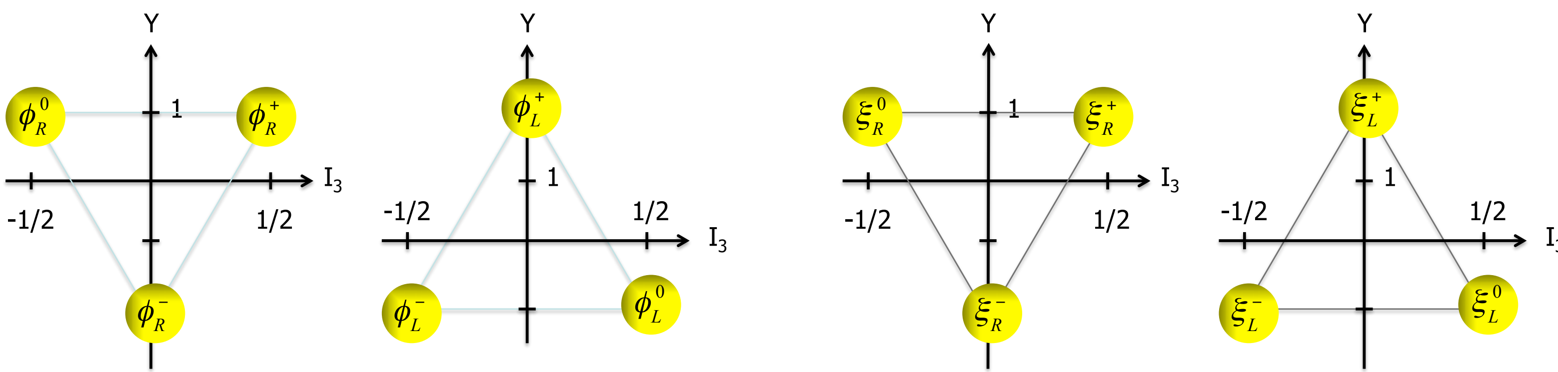
Basic starting point

- Right-Left symmetry
- Supersymmetry (Wess-Zumino structure)

  - Bosons: $$\phi\sqrt{2} = e^{i\pi/4}\phi_R + e^{-i\pi/4}\phi_L$$ $$= \phi_S + i\phi_P$$

  - Fermions: $$\xi\sqrt{2} = \begin{bmatrix} \xi_R \\ -i\,\xi_L \end{bmatrix}$$

- Space-time structure via covariant derivatives and supercharges
- Different 'kinds' of particles correspond to SLOCC equivalence classes where the fun starts in tripartite space with the following basis:



Emerging symmetries & space-time

Fields

- Real/Majorana: $\phi \quad \xi \quad \text{and} \ \langle\phi\rangle = 1$
- $\phi_{R/L} \quad \xi_{R/L} \quad \text{and} \ \langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$ (Wess-Zumino)

Generators

Space-time & Internal

- H
- P^+, P^- K, SU(3)

$U(1)_R \times U(1)_L \times SU(3)$

Emerging symmetries & space-time

Fields

- Real/Majorana: $\phi \quad \xi \quad \text{and} \ \langle\phi\rangle = 1$
- $\phi_{R/L} \quad \xi_{R/L} \quad \text{and} \ \langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$
  (Wess-Zumino → gauge theory)
- 1D: $\phi_S \quad \phi_P \longrightarrow A_3^a \quad \psi$

$$iD_\sigma\phi^i = i\partial_\sigma\phi^i + g_0 \sum_{a=1,\dots 8} A_\sigma^a (T_a)^i_j \phi^j$$

Generators

Space-time & Internal

- H
- P^+, P^- K, SU(3)

Z(2)

- H, P, K SU(3)

P(1,1) x SU(3)

Emerging symmetries & space-time

Fields

- Real/Majorana: $\phi \quad \xi \quad \text{and } \langle\phi\rangle = 1$
- $\phi_{R/L} \quad \xi_{R/L} \quad \text{and } \langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$
  (Wess-Zumino → gauge theory)
- 1D: $\phi_S \quad \phi_P \longrightarrow A_3^a \quad \psi$

$$iD_\sigma\phi^i = i\partial_\sigma\phi^i + g_0 \sum_{a=1,\ldots 8} A_\sigma^a (T_a)^i_j \phi^j$$

- 3D: $\phi_S \quad A_k^a \quad \psi$

$$iD_\mu\phi^i = i\partial_\mu\phi^i + g \sum_{a=1,2,3,8} A_\mu^a (T_a)^i_j \phi^j$$

Generators

Space-time & Internal

- H
- P+, P– — K, SU(3)

Z(2)

- H, P, K — SU(3) = [SO(3),SU(2)xU(1)]

Z(3)

- H, **P**, **K**, **J** — SU(2)xU(1)

P(3,1) x SU(2) x U(1)

Emerging symmetries & space-time

Fields

- Real/Majorana: $\phi \quad \xi \quad \text{and } \langle\phi\rangle = 1$
- $\phi_{R/L} \quad \xi_{R/L} \quad \text{and } \langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$
  (Wess-Zumino → gauge theory)
- 1D: $\phi_S \quad \phi_P \longrightarrow A_3^a \quad \psi$

$$iD_\sigma\phi^i = i\partial_\sigma\phi^i + g_0 \sum_{a=1,\ldots 8} A_\sigma^a (T_a)_j^i \phi^j$$

- 3D: $\phi_S \quad A_k^a \quad \psi$

$$iD_\mu\phi^i = i\partial_\mu\phi^i + g \sum_{a=1,2,3,8} A_\mu^a (T_a)_j^i \phi^j$$

- and ....

$$n_\pm^\sigma \longrightarrow n_\alpha^\mu \quad \gamma^\sigma = \begin{bmatrix} 0 & n_-^\sigma \\ n_+^\sigma & 0 \end{bmatrix} \longrightarrow \gamma^\mu = \begin{bmatrix} 0 & \bar\sigma^\mu \\ \sigma^\mu & 0 \end{bmatrix}$$

in order to match space-time and field symmetries and respect Coleman-Mandula

Generators

Space-time & Internal

- H
- P^+, P^- — K, SU(3)

Z(2)

- H, P, K — SU(3) = [SO(3),SU(2)xU(1)]

Z(3)

- H, **P**, **K**, **J** — SU(2)xU(1)

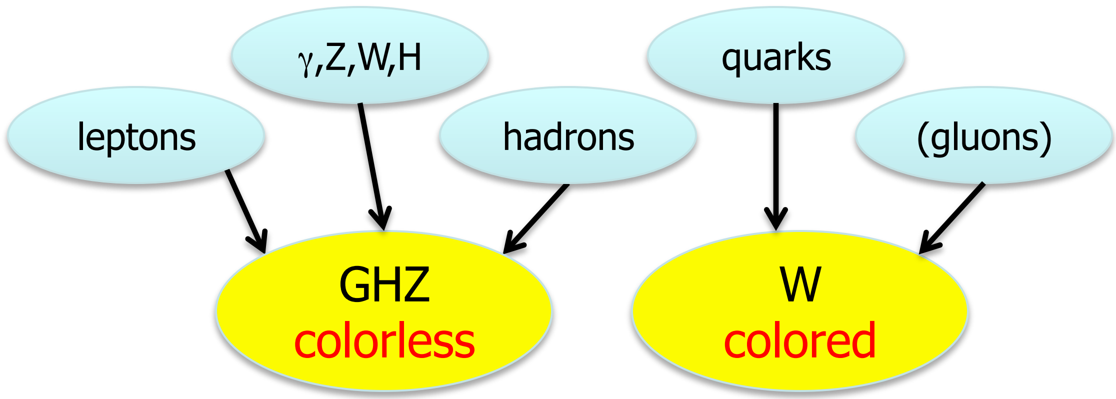
P(3,1) x SU(2) x U(1)

A(4)

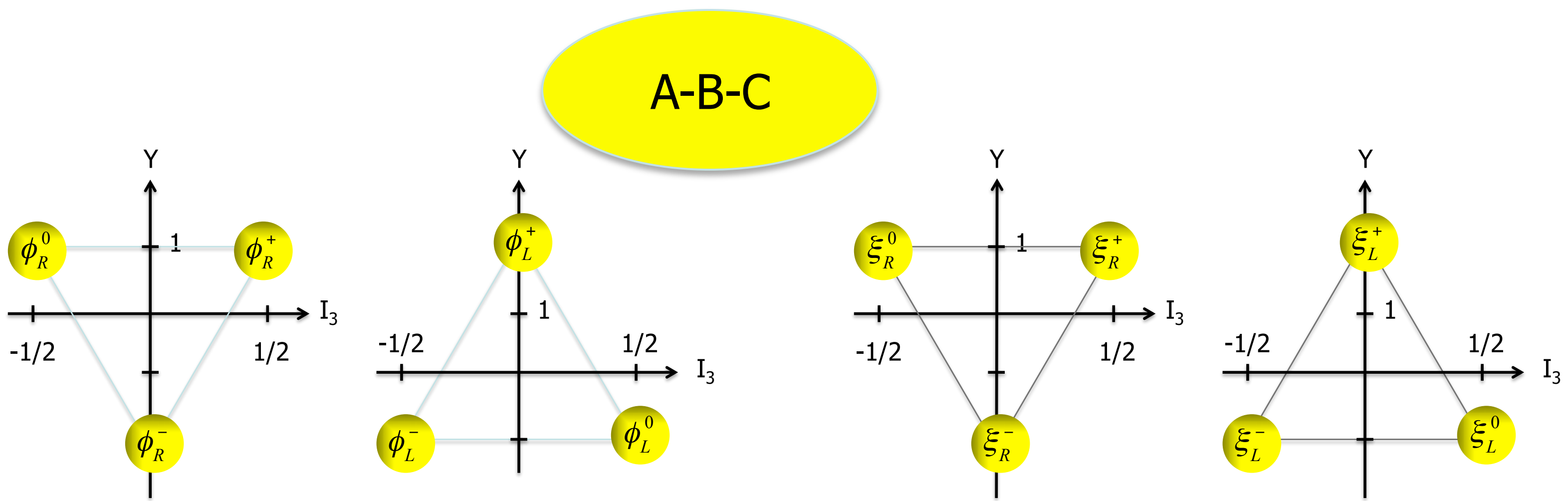
FERMIONS AND BOSONS AS TRIPARTITE STATES

Quarks and leptons as entangled states

- Symmetry eigenstates are in general entangled



- Conjecture is that physically relevant states correspond to MaxEnt tripartite states with basic product states being simple and even supersymmetric.



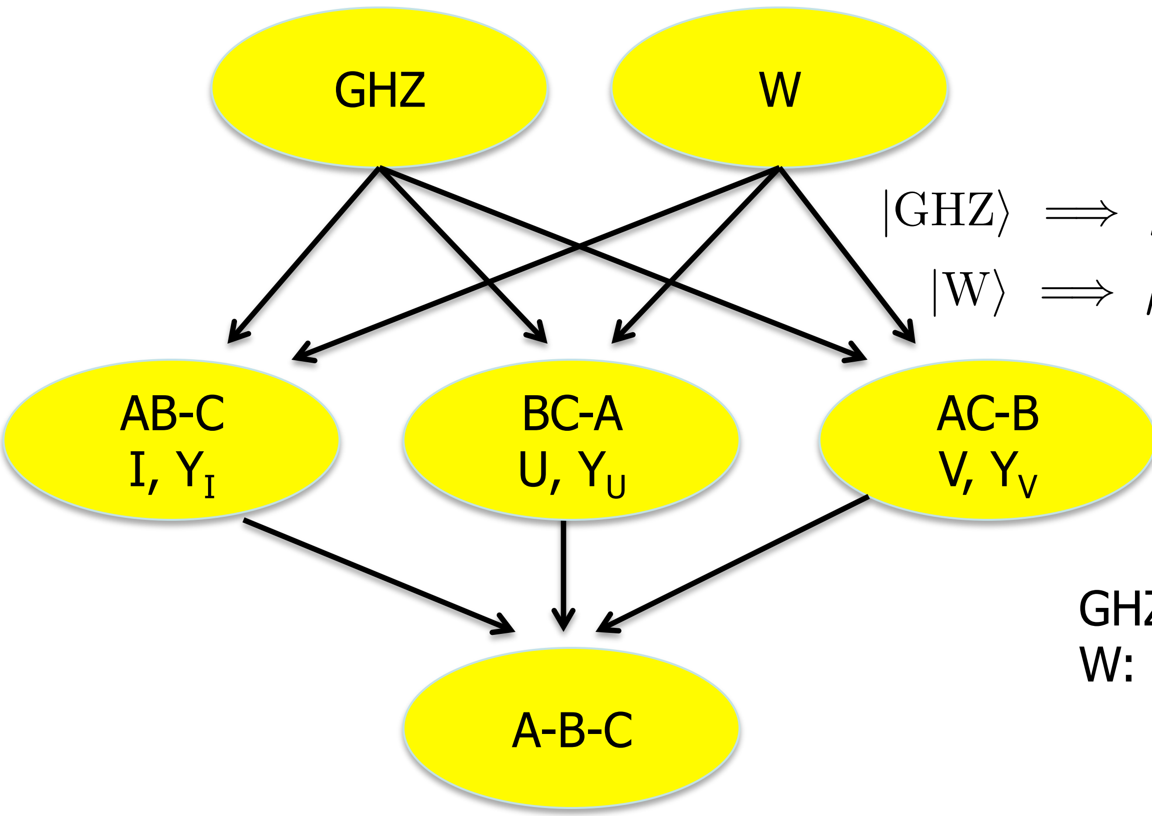
PJM 1601.00300, PJM 1801.03664

Tripartite entangled states

- Two classes of maximally entangled states:
  (Dur, Vidal, Cirac 2000)

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$$

$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|LRR\rangle + |RLR\rangle + |RRL\rangle)$$



$$|\mathrm{GHZ}\rangle \Longrightarrow \rho_{AB} = \frac{1}{2}(|RR\rangle\langle RR| + |LL\rangle\langle LL|)$$

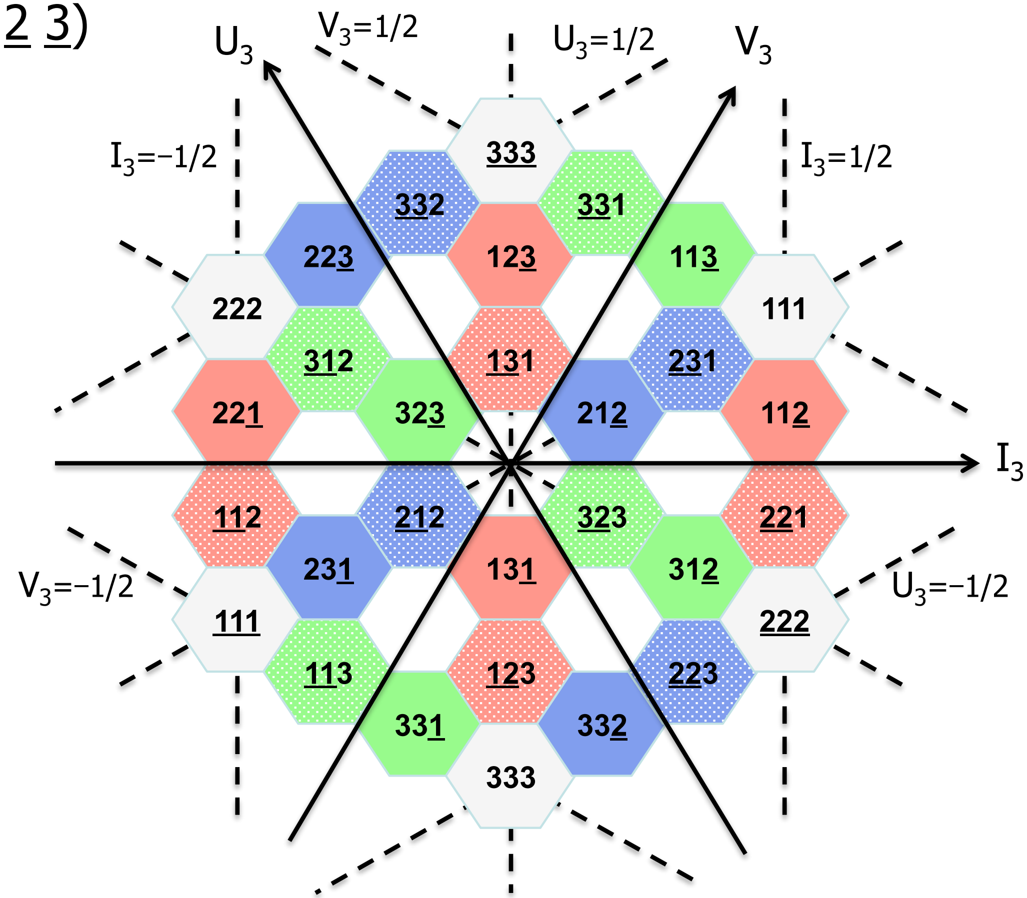
$$|\mathrm{W}\rangle \Longrightarrow \rho_{AB} = \frac{2}{3}|\mathrm{Bell}\rangle\langle\mathrm{Bell}| + \frac{1}{3}|RR\rangle\langle RR|$$

GHZ: I, U, and V symmetry states
W: I, U, or V symmetry states

- For symmetry considerations we employ SU(3) and SU(2) x U(1) subgroups (I, U, V) in intermediate classes
- For space-time embedding we employ A(4) symmetry: three singlets and three triplets

Fermionic excitations: tripartite entanglement

- Tripartite states (R: 1 2 3 & L: 1 2 3)
- Aligned (RRR, LLL): GHZ states
  - I, U, *and* V allowed
  - SO(3) → asymptotic/space
  - Three A(4) singlets
- Mingled (RRL, RLL): W-states
  - I, U, *or* V allowed
  - non-asymptotic
  - Three A(4) triplets (color)

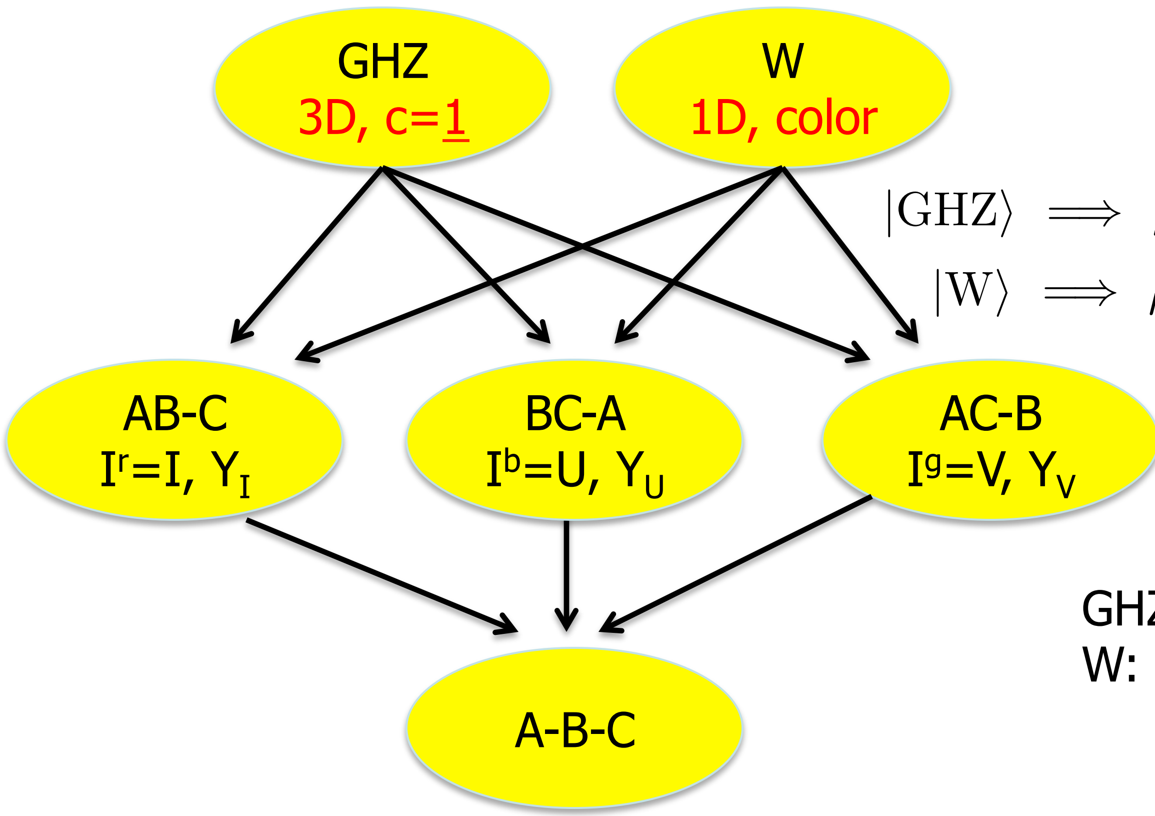


Tripartite entangled states

- Two classes of maximally entangled states: (Dur, Vidal, Cirac 2000)

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$$

$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|LRR\rangle + |RLR\rangle + |RRL\rangle)$$



$$|\mathrm{GHZ}\rangle \implies \rho_{AB} = \frac{1}{2}\left(|RR\rangle\langle RR| + |LL\rangle\langle LL|\right)$$

$$|\mathrm{W}\rangle \implies \rho_{AB} = \frac{2}{3}|\mathrm{Bell}\rangle\langle\mathrm{Bell}| + \frac{1}{3}|RR\rangle\langle RR|$$

GHZ: I, U, and V symmetry states
W: I, U, or V symmetry states

- For symmetry considerations we employ SU(3) and SU(2) x U(1) subgroups (I, U, V) in intermediate classes
- For space-time embedding we employ A(4) symmetry: three singlets and three triplets

S(3), SO(3) and SU(3) in tripartite space

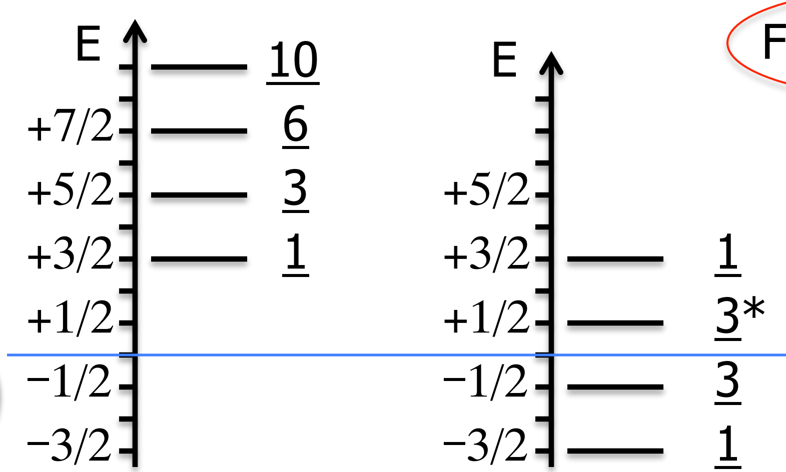
Relevant symmetry in tripartite space $\mathcal{H}^A \otimes \mathcal{H}^B \otimes \mathcal{H}^C$ is S(3), SO(3), and SU(3)

- Example: a 3D harmonic oscillator: states $|n_x n_y n_z>$ or $|n_r l m>$ or $|\underline{n}>$

level	degeneracy	(n_x, n_y, n_z)	SO(3) $(n_r \ell)$	SU(3) $(\underline{n})$
0	1	(0,0,0)	0s	$\underline{1}$
1	3	(1,0,0), ...	0p	$\underline{3}$
2	6	(2,0,0), (1,1,0), ...	1s ⊕ 0d	$\underline{6}$
3	10	(3,0,0), (2,1,0), (1,1,1), ...	1p ⊕ 0f	$\underline{10}$
4	15	...	2s ⊕ 1d ⊕ 0g	$\underline{15}_s$

- 3D HO is separable, has rotational [SO(3)] and more [SU(3)] symmetry. Symmetry eigenstates (involving ABC) are in general entangled states.
- Bosons (R or L)

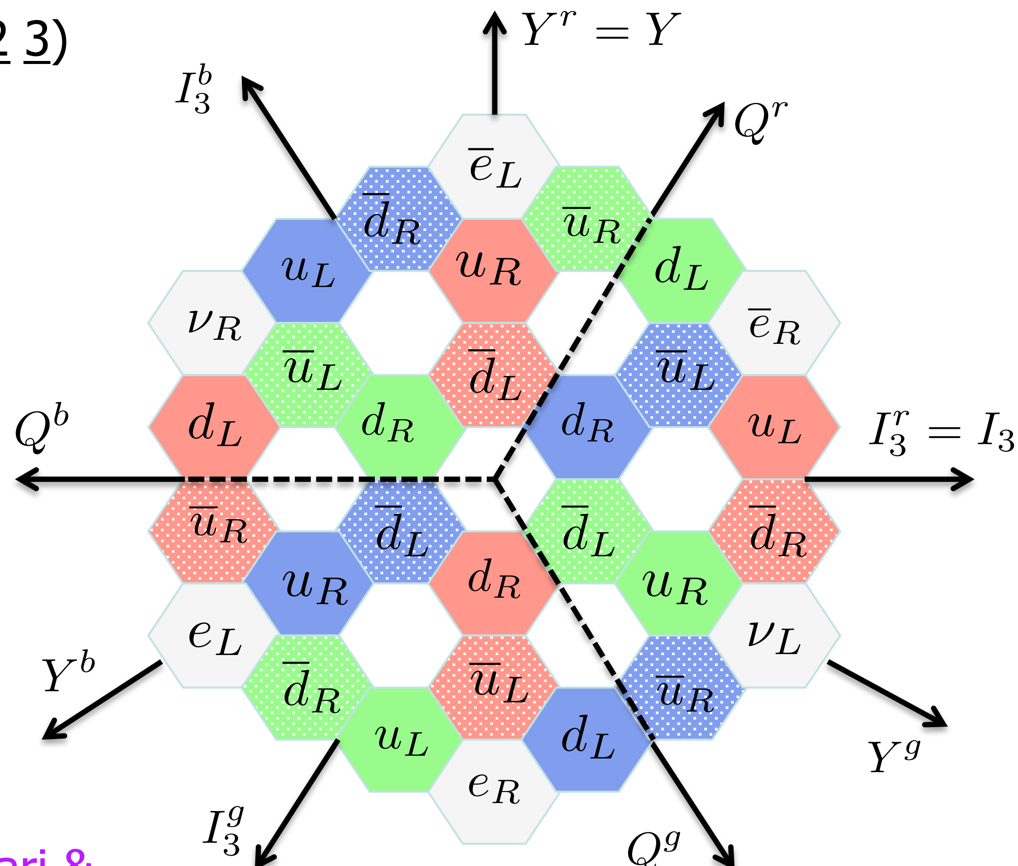
Fermions (R/L)



intermezzo

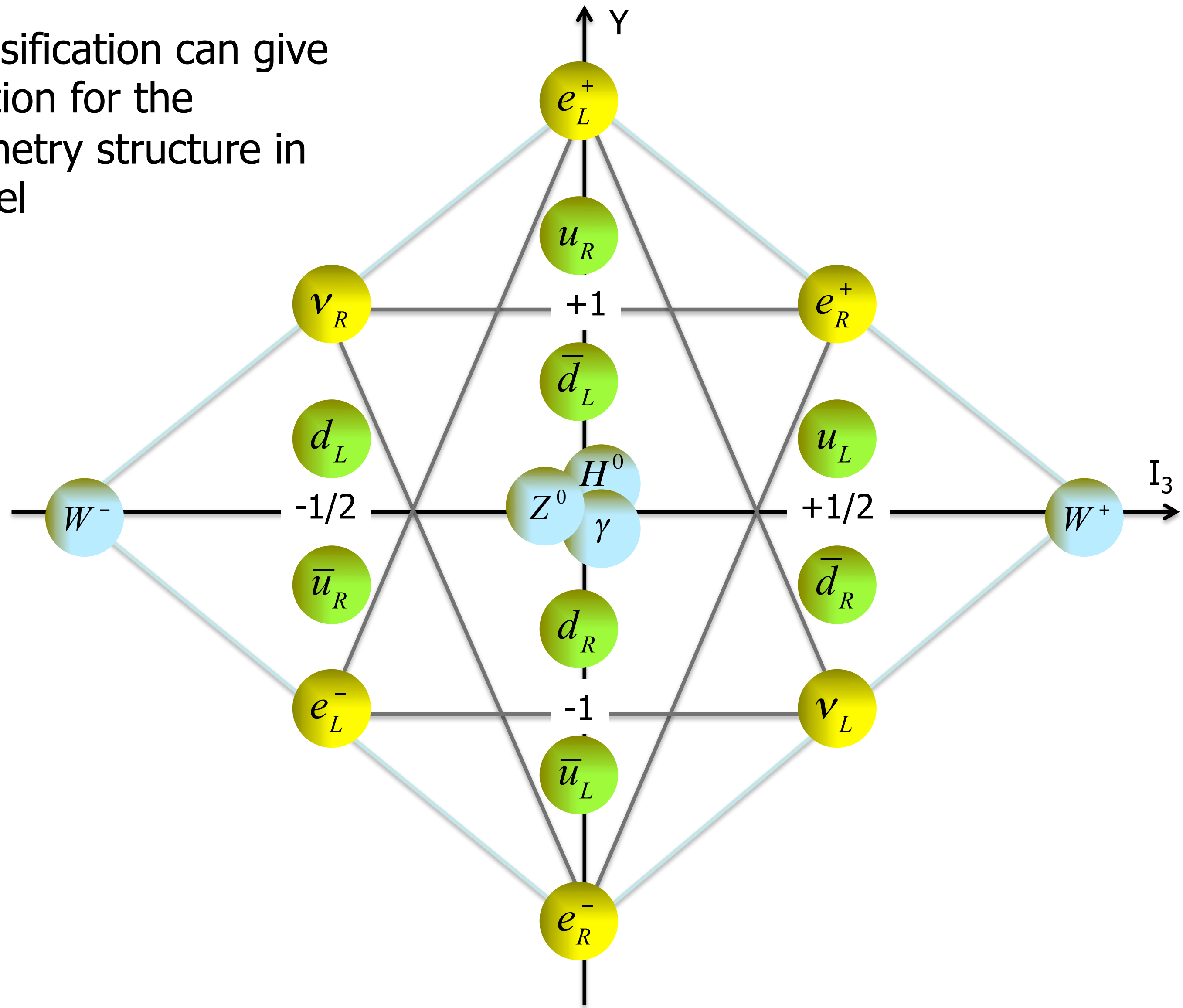
Fermionic excitations: electroweak identification

- Tripartite states (R: 1 2 3 & L: $\underline{1}$ $\underline{2}$ $\underline{3}$)
- Aligned (RRR, LLL): LEPTONS
  - I, U, *and* V allowed
  - SO(3) → asymptotic/space
  - Three A(4) singlets
- Mingled (RRL, RLL): QUARKS
  - I, U, *or* V allowed
  - non-asymptotic
  - Three A(4) triplets (color)
- Resembles the rishon model (Harari & Seiberg 1982)



Standard model particle content

A multipartite classification can give a natural explanation for the electroweak symmetry structure in the standard model



Bosonic excitations: electroweak and strong

- Looking for physical bosonic degrees of freedom depends on S(3) symmetric vacuum structure in tripartite space and the introduction of locality (3D or 1D) and remaining gauge freedom.
- Leptonic sector

$$iD_\mu\phi = i\partial_\mu\phi + \frac{g}{2}\left(\sum_{i=1}^{3} W_\mu^i\lambda_i + B_\mu\lambda_8\right)\phi$$
$$= i\partial_\mu\phi + \frac{g}{\sqrt{2}}(W_\mu^+ I_- + W_\mu^- I_+)\phi + (g\,W_\mu^0 I_3 + \frac{g}{2\sqrt{3}}B_\mu Y)\phi$$

- 3D electroweak symmetry breaking is SU(2) x U(1) → $U(1)_{QED}$ with usual relations in SM and some new ones such as $M_H^2 = 2M_Z^2$ and sin Θ_W = ½ (Weinberg 1972) and Yukawa coupling to top quark being 1
- QCD sector

$$iD_\sigma\phi^i = i\partial_\sigma\phi^i + g_0 \sum_{a=1,\ldots 8} A_\sigma^a (T_a)^i_j \phi^j$$

- Resembles $XQCD_{1+1}$ (analogous to Kaplan 1306.5818), while dynamics governed by via Wilson loop

$$W[C] = \exp\left(-ig\oint_C ds^\mu A_\mu(s)\right) \qquad gF_{\tau\sigma} = \delta W[C]/\delta\sigma^{\tau\sigma}$$

GENERAL IMPLICATIONS

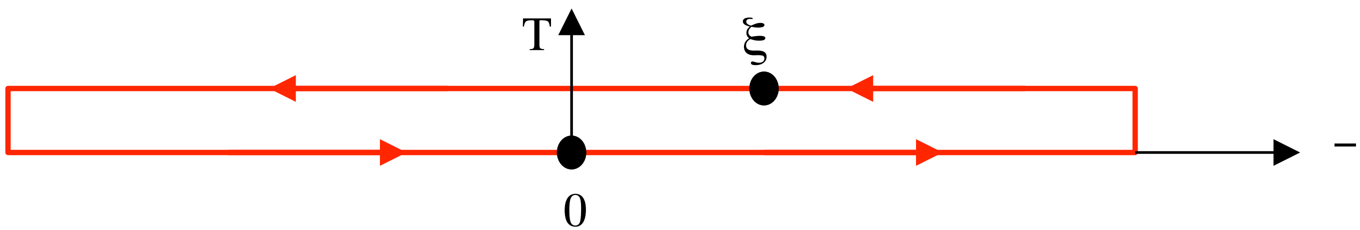
Emergence in the Standard Model

- Basic degrees of freedom can also be described in less dimensions while the extension of 1+1 $\rightarrow$ 1+3 proceeds via tripartite entanglement.
  - Advantageous for convergence: $d[\phi] = (d-2)/2 \rightarrow 0$, $d[\xi] = (d-1)/2 \rightarrow 1/2$, naturalness, … [see Stojkovic – 1406.2696]
  - Consequences in looking at higher order corrections for e.g. g-2 ?
  - Supersymmetry included, but invisible in 3D !
  - Gravity also emerges in 1D $\rightarrow$ 3D.
- A natural framework to look at family structure and symmetries of standard model as emergent phenomena
  - e.g. tribimaximal mixing for leptons
- Tripartite space for quarks naturally has color dual to space/electroweak.
  - explains why color naturally is decoupled from electroweak interactions
  - color invisible in 3D: local gauge invariance! No asymptotic quarks or gluons!
  - global color remains visible in 3D via color factors such as N vs 1/N, f x D (distribution x fragmentation)
- Gravity also emerges in 1D to 3D transition

IMPLICATIONS FOR QCD

Emergence in QCD

- Natural arena for light-front approach with a 'preferred' space direction: quantization of good fields, dominating the OPE at high energies, these are asymptotic (free) fields (Kogut & Soper): $\frac{1}{2}\gamma^{-}\gamma^{+}\psi \;\; \text{and} \;\; g_T^{\alpha\mu} A_\mu^a$
- Natural role for Wilson loops generating (gluon) TMDs (1805.05219), process dependence linked to global color flow from initial to final states



$$F^{\alpha\beta} = \frac{\delta W[C]}{\delta \sigma_{\alpha\beta}}$$

- Possible new ways to look at color-kinematic duality, soft collinear effective theory (SCET) via embedding in the entanglement schemes

Emergence of hadrons in QCD

- Include hadrons: construct color singlets in tripartite space (naively make three-particle baryonic states) and use local invariance for the 3D embedding. This is analogous to choosing the appropriate basis of quantum states in multipartite space [cf ontological basis of 't Hooft – 1405.1548]
- Example: a 3D harmonic oscillator: states $|n_x n_y n_z\rangle$ or $|n_r l m\rangle$ or $|\underline{n}\rangle$

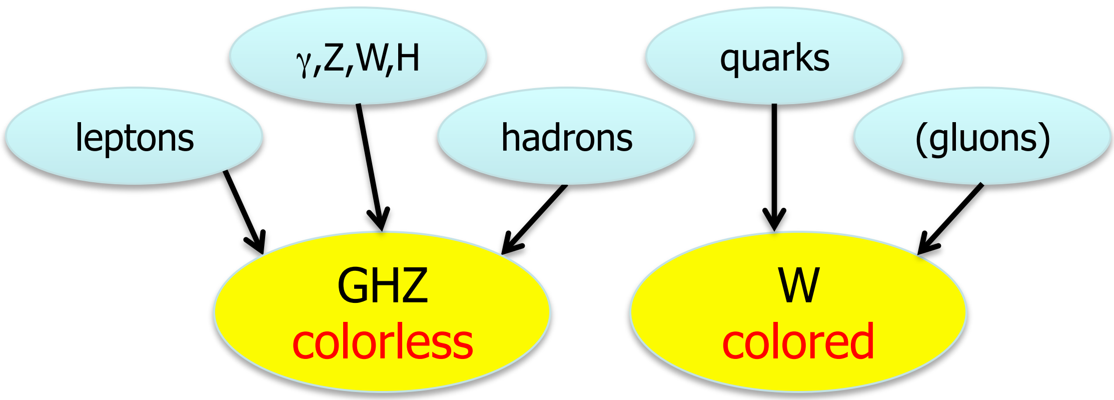
level	degeneracy	(n_x, n_y, n_z)	SO(3) $(n_r \ell)$	SU(3) $(\underline{n})$
0	1	(0,0,0)	0s	$\underline{1}$
1	3	(1,0,0), ...	0p	$\underline{3}$
2	6	(2,0,0), (1,1,0), ...	1s $\oplus$ 0d	$\underline{6}$
3	10	(3,0,0), (2,1,0), (1,1,1), ...	1p $\oplus$ 0f	$\underline{10}$
4	15	...	2s $\oplus$ 1d $\oplus$ 0g	$\underline{15}_s$

- Compare the well-known NRQM classification of baryons

N	configuration	SU(6) $\times$ O(3) multiplets
0	$(0s)^3$	$[56, 0^+]$
1	$(0s)^2(1p)$	$(56, 1^-)$ $[70, 1^-]$
2	$(0s)^2(2s)$	$(56, 0^+)$ $[70, 0^+]$
	$(0s)^2(2d)$	$(56, 2^+)$ $[70, 2^+]$
	$(0s)(1p)^2$	$[56, 0^+]$ $[56, 2^+]$ $(70, 0^+)$ $(70, 1^+)$ $(70, 2^+)$ $[20, 1^+]$

Concluding remarks

- Conjecture: quarks, leptons and hadrons in multipartite spaces
- Starts with a simple (even supersymmetric basis) which at tripartite level enables embedding of known standard model symmetries



- May provide an embedding of all ways to look at QCD, e.g. valence vs current, chiral dynamics, SCET, ...