The phase diagram of hot and dense QCD

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[J. Braun, M. Leonhardt, MP, PRD96, 076003 (2017), arXiv:1705.00074 [hep-ph]]
[J. Braun, M. Leonhardt, MP, PRD97, 076010 (2018), arXiv:1801.08338 [hep-ph]]

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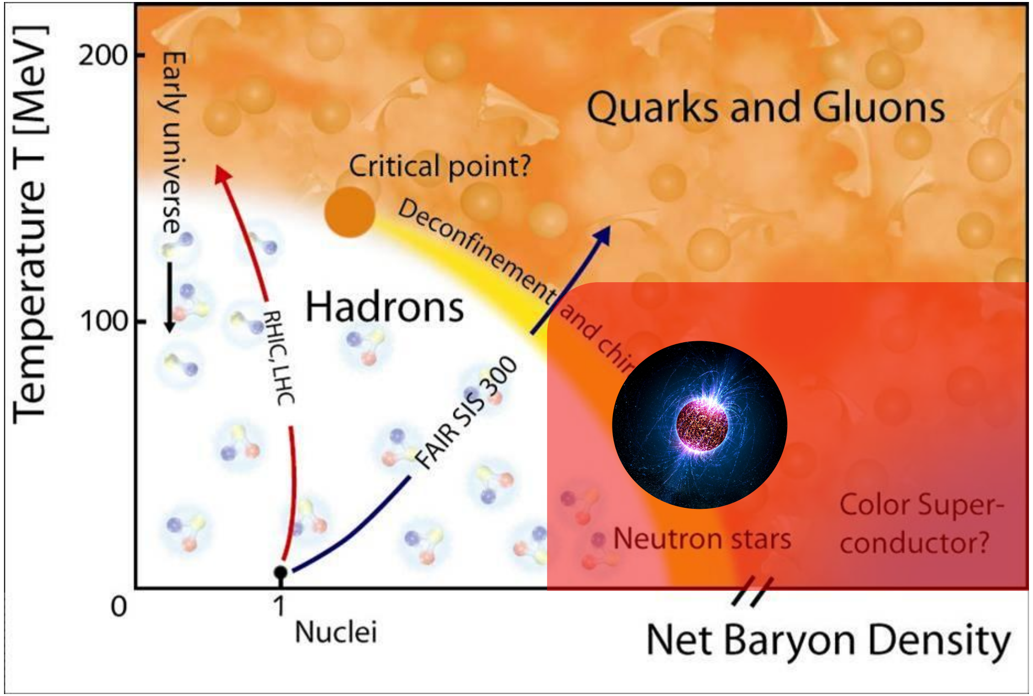
The (conjectured) phase diagram of QCD

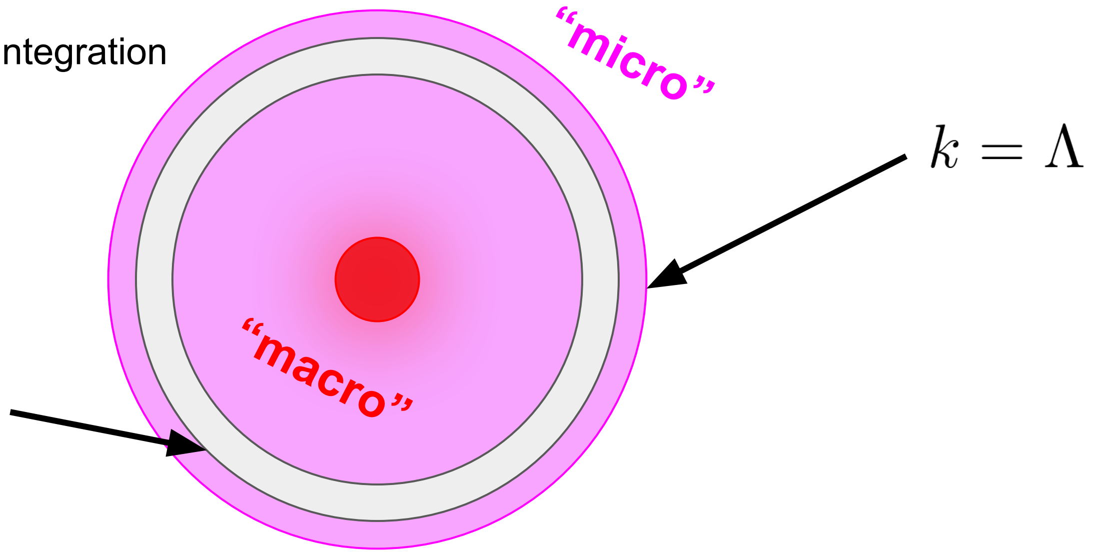
[... Kobayashi, Maskawa ‘70; Kobayashi, Kondo, Maskawa ‘71; ‘t Hooft ‘76/’78; Bailin, Love ‘84; Hatsuda, Kunihiro ‘94; Alford, Rajagopal, Wilczek ‘98; Rapp, Schäfer, Shuryak, Velkovsky ‘98; Berges, Rajagopal ‘99; Son ‘99; Pisarski, Rischke ‘00; Buballa 05’; Ratti, Thaler, Weise ‘05; Shovkovy ‘05; Fukushima ‘12; Aoki, Yamada ‘15; Andersen, Naylor, Tranberg ‘16 …]



[FAIR @ GSI; Casey Reed - PSU]

“Tool time”: The functional RG

Wilsonian momentum shell integration



momentum shell $p \sim k$

$$\partial_t \Gamma_k[\Phi] = -\frac{1}{2} \mathrm{Tr} \left\{ [\Gamma_k^{(1,1)}[\Phi] + R_k^{\psi}]^{-1} \cdot (\partial_t R_k^{\psi}) \right\}$$ [Wetterich ‘93]

(Some) Symmetries of hot and dense matter

color	chiral	baryon conservation
$SU(N_c)$	$SU_L(2) \otimes SU_R(2)$	$U_V(1)$

d u

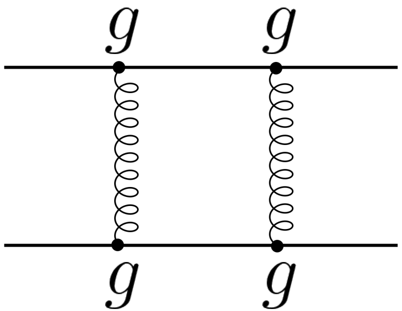
$\mu > 0$

d u $\bar{u}$ u $\bar{u}$ d u

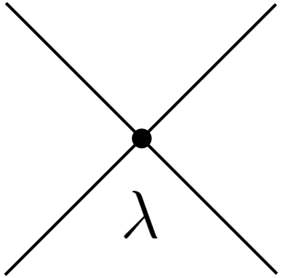
$T > 0$

		axial (anomaly)
$\mathcal{C}$	Poincaré	$U_A(1)$

Low energy QCD model [Nambu, Jona-Lasinio '61]



gluon interactions induce
four-quark interactions



$$S = \int \mathrm{d}^4x \left\{ \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \frac{1}{2} \bar{\lambda}_{(\sigma\text{-}\pi)} \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\tau_i\psi)^2 \right] \right\}$$

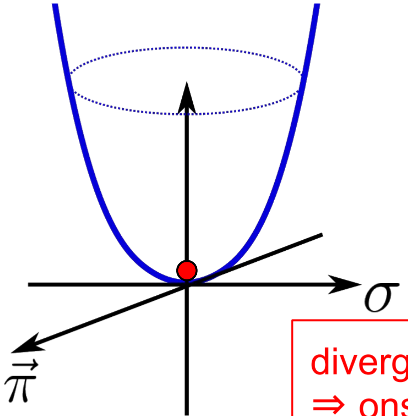
chiral condensate order parameter

$$\langle \bar{\psi}\psi \rangle \neq 0$$

Spontaneous chiral symmetry breaking

partially bosonized action: $S \sim \int \mathrm{d}^4x \, \left\{ \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \bar{\psi}(\sigma + \mathrm{i}\gamma_5 \vec{\tau} \cdot \vec{\pi})\psi + \frac{m^2}{2}(\sigma^2 + \vec{\pi}^2) \right\}$

$m^2 > 0$



ground state in the symmetric phase

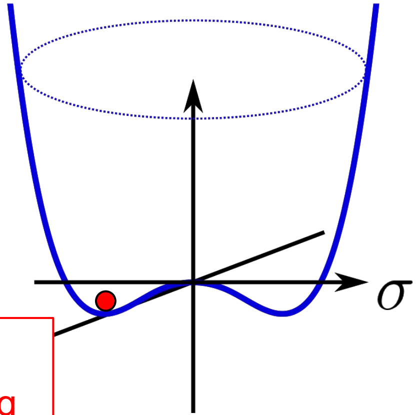
$$\sigma \sim \bar{\psi}\psi$$

$$\vec{\pi} \sim \bar{\psi}\gamma_5\vec{\tau}\psi$$

$$m^2 \sim \lambda^{-1}_{(\sigma\text{-}\pi)}$$

diverging four-fermion interaction
⇒ onset chiral symmetry breaking

$m^2 < 0$



ground state in the broken phase

Fierz-complete ansatz

ansatz for effective action:

$$\Gamma_{\mathrm{LO}} = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \left\{ \bar{\psi}\mathrm{i}(\gamma_0\partial_0 + \gamma_i\partial_i - \mu\gamma_0)\psi + \boxed{\frac{1}{2}\sum_{j\in\mathcal{B}} \bar{\lambda}_j \mathcal{L}_j} \right\}$$

all four-quark interactions compatible with symmetries

caution: Fierz ambiguity! $(\mathcal{O}^A)_{ab}(\mathcal{O}^A)_{cd} = \sum_B C^{AB}(\mathcal{O}^B)_{ad}(\mathcal{O}^B)_{cb}$

e.g. $(\bar{\psi}\psi)^2 = a_1(\bar{\psi}\psi)^2 + a_2(\bar{\psi}\gamma_5\psi)^2 + a_3(\bar{\psi}\gamma_\mu\psi)^2 + \ldots$

⇒ find minimal set of four-quark interactions!

Fierz-complete four-quark interactions

$$\mathcal{L}_{(V+A)_\parallel},\ \mathcal{L}_{(V+A)_\perp},\ \mathcal{L}_{(V-A)_\parallel}$$
$$\mathcal{L}_{(V-A)_\perp},\ \mathcal{L}_{(V+A)^{\text{adj}}_\parallel},\ \mathcal{L}_{(V-A)^{\text{adj}}_\perp}$$

$SU(N_c)$ ✓	$U_V(1)$ ✓
$SU_L(2)\otimes SU_R(2)$ ✓	$U_A(1)$ ✓

6 channels

$$\mathcal{L}_{(\sigma\text{-}\pi)},\ \mathcal{L}_{\text{csc}},\ \mathcal{L}_{(S+P)_-},\ \mathcal{L}_{(S+P)^{\text{adj}}_-}$$

$SU(N_c)$ ✓	$U_V(1)$ ✓
$SU_L(2)\otimes SU_R(2)$ ✓	$U_A(1)$ ✗

4 channels

10 channels

"chiral" order-parameter

condensate: $\langle \bar\psi\psi \rangle$

$SU(N_c)$ ✓	$U_V(1)$ ✓
$SU_L(2)\otimes SU_R(2)$ ✗	$U_A(1)$ ✗

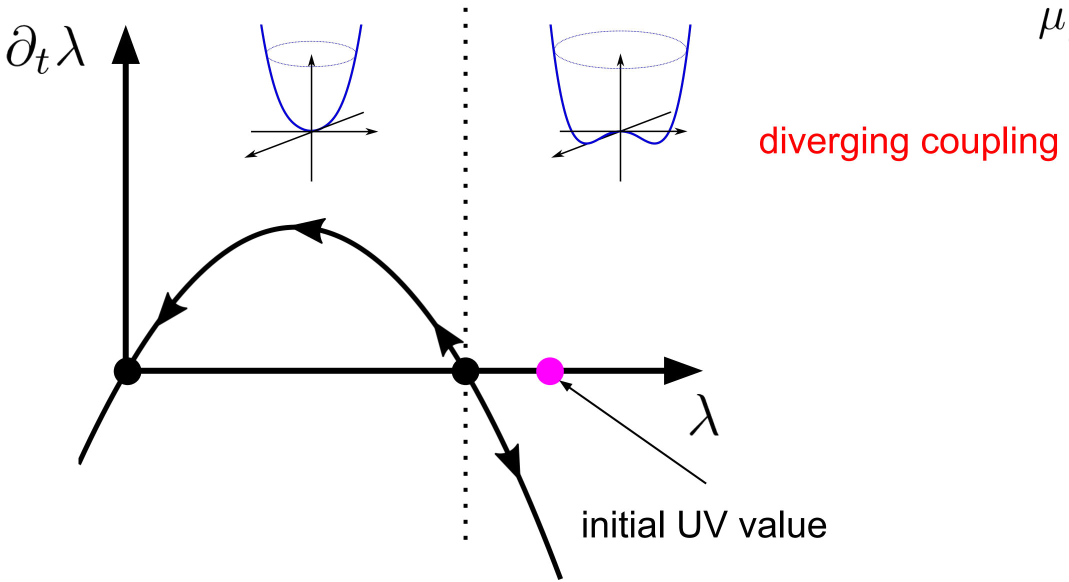
"diquark" channel

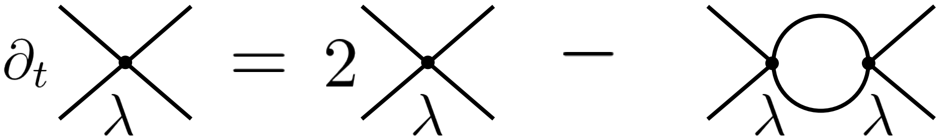
condensate: $\langle \mathrm{i}\psi^T \mathcal{C}\gamma_5 \varepsilon_f \varepsilon_c^l \psi \rangle$

$SU(N_c)$ ✗	$U_V(1)$ ✗
$SU_L(2)\otimes SU_R(2)$ ✓	$U_A(1)$ ✗

NJL-model beta functions

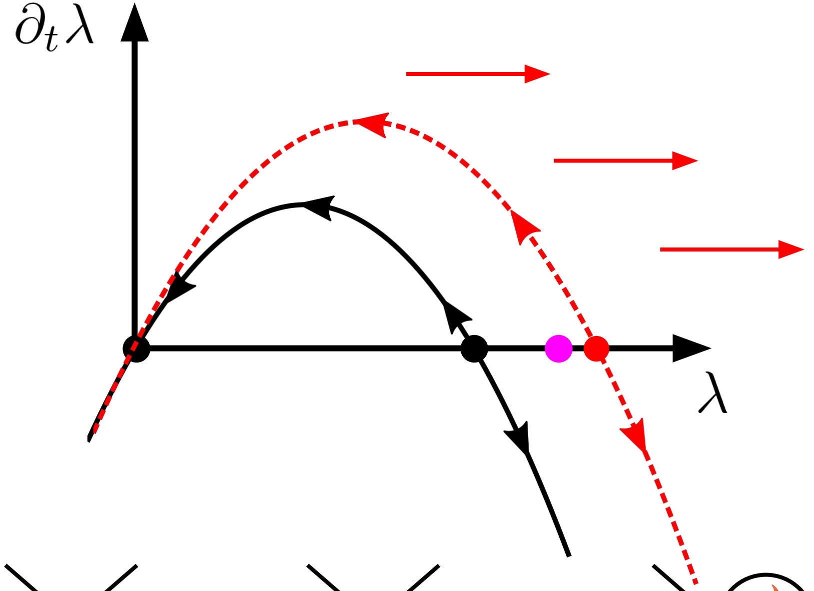
$T/k = 0$
$\mu/k = 0$





NJL-model beta functions

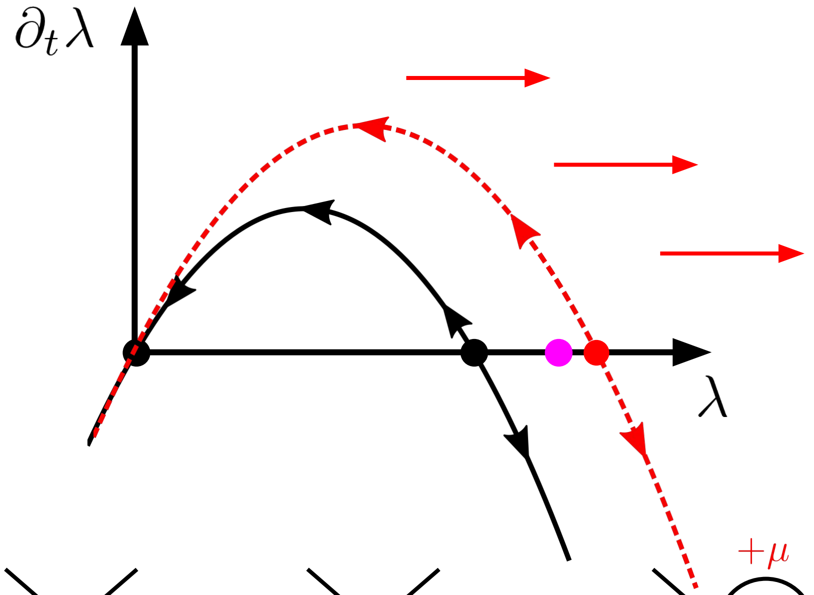
$T/k > 0$
$\mu/k = 0$



∂_t [vertex λ] $= 2$ [vertex λ] $-$ [loop λ λ]

NJL-model beta functions

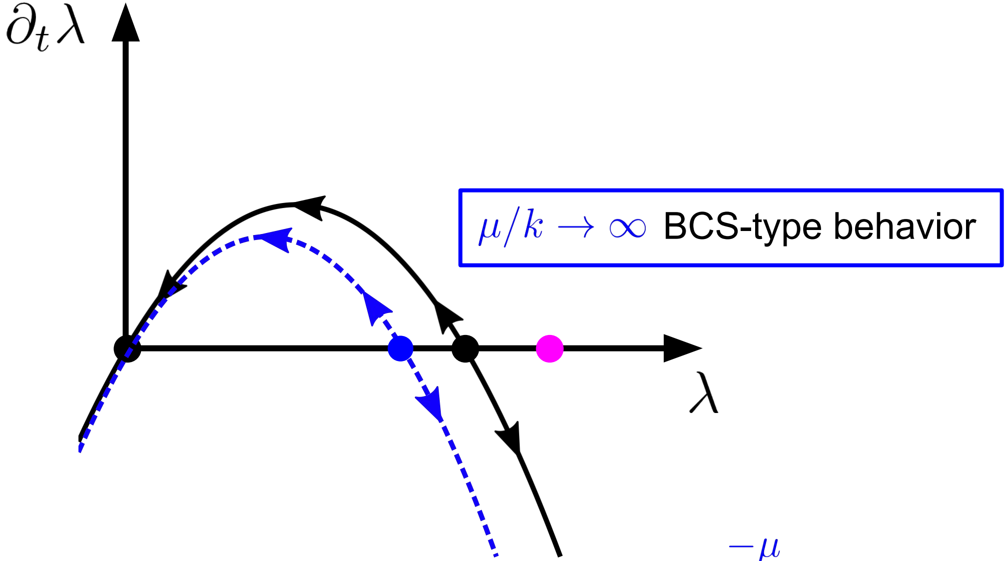
$T/k = 0$
$\mu/k > 0$

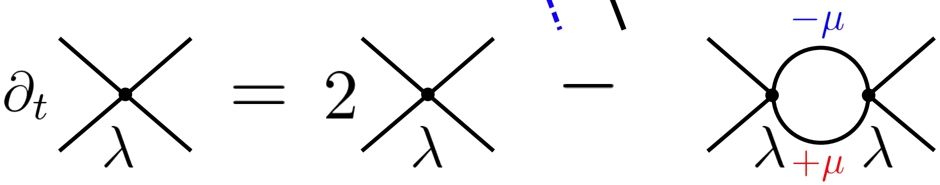


$$\partial_t \lambda = 2\lambda - \lambda\,(+\mu)\,(+\mu)\,\lambda$$

NJL-model beta functions

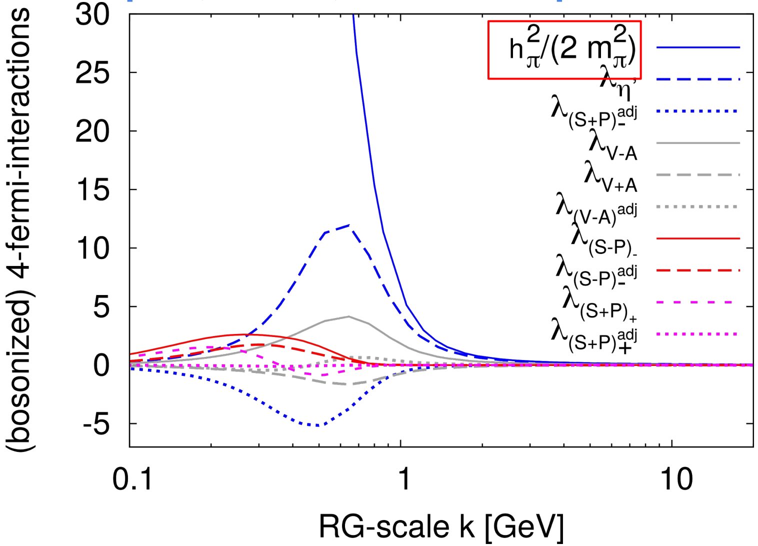
$T/k = 0$
$\mu/k > 0$





Scale fixing procedure

[Mitter, Pawlowsk, Strodthoff '14/'17]



$(\sigma\text{-}\pi)$ - channel dominates low-energy physics

choice: $\lambda^{(\mathrm{UV})}_{(\sigma\text{-}\pi)} \neq 0$, else $\lambda^{(\mathrm{UV})}_i = 0 \quad (\Lambda = 1\,\mathrm{GeV})$

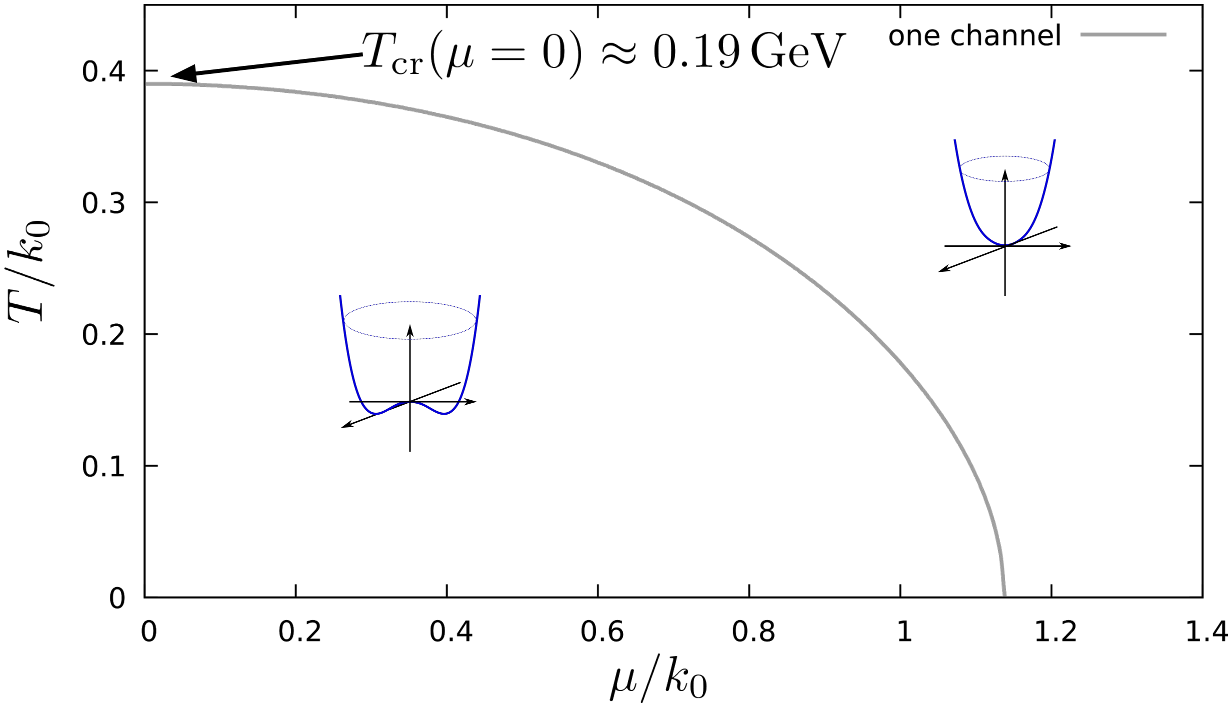
Idea: map one-channel RG flow on corresponding mean-field gap equation (vacuum, one-channel) [Braun '11]

$m_\psi \approx 0.3\,\mathrm{GeV}$ ⟷ $k_0 \approx 0.48\,\mathrm{GeV}$ (scale chiral SB)

tune $\lambda^{(\mathrm{UV})}_{(\sigma\text{-}\pi)}$ such that $k_{\mathrm{cr}} \approx k_0$

One-channel phase diagram

[Braun, Leonhardt, MP ‘18]



Fierz-complete calculations: analytical methods

How to assess for 10 channels which condensate arises?

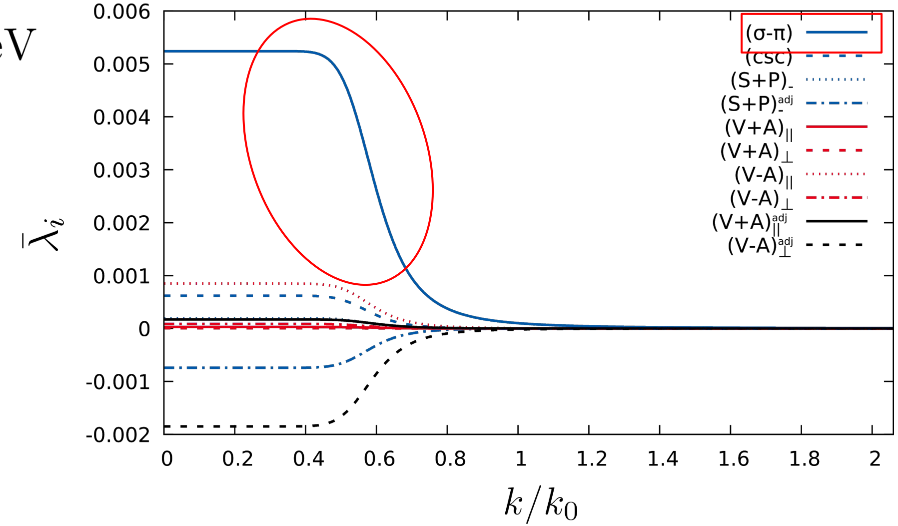
$$\langle ? \rangle$$

General: Check relative “dominance” of a channel [e.g. Braun, Gies, Janssen, Roscher ‘14]

[Braun, Leonhardt, MP ‘18]

$T \simeq T_{\rm cr} \simeq 0.19\,\mathrm{GeV}$

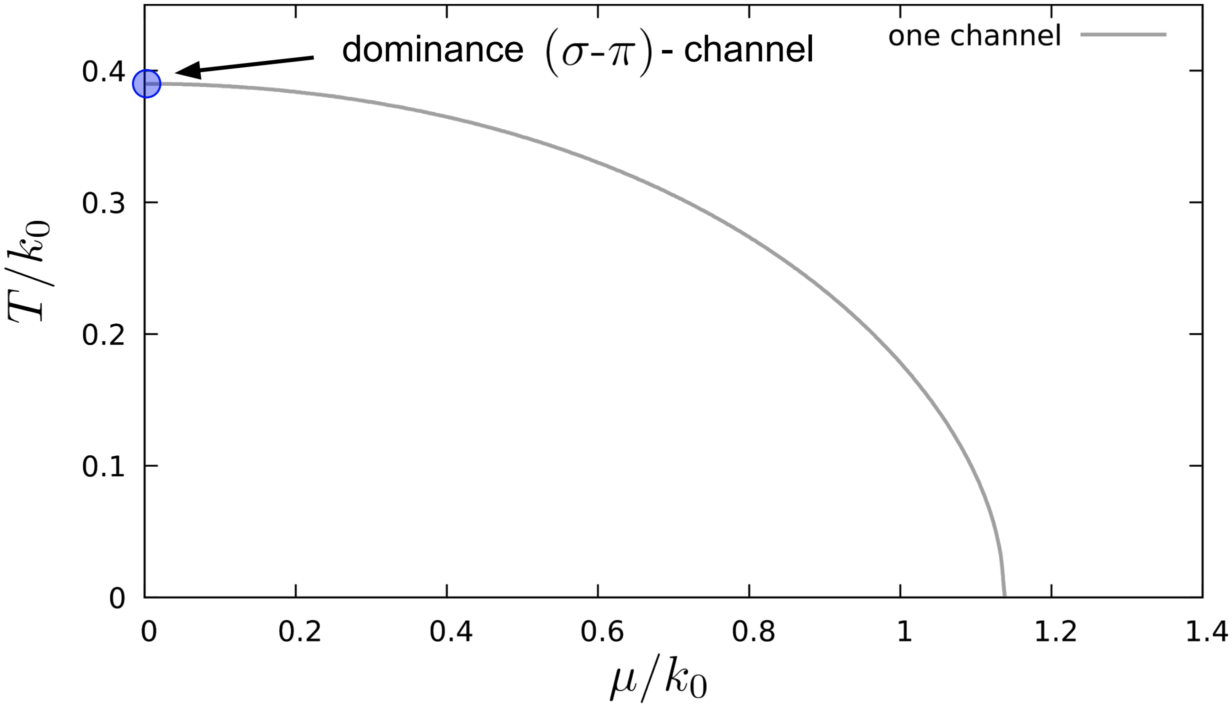
$\mu = 0$



$$\lambda^{(\mathrm{UV})}_{(\sigma\text{-}\pi)} \neq 0$$
$$\lambda^{(\mathrm{UV})}_{i} = 0$$

Fierz-complete phase diagram

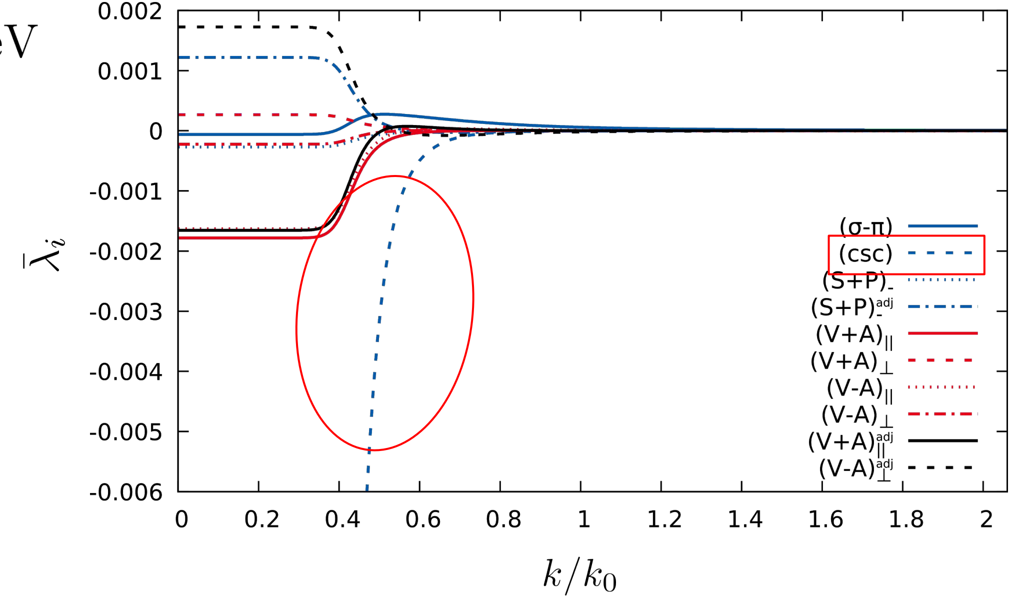
[Braun, Leonhardt, MP ‘18]



Fierz-complete phase diagram

$T \simeq T_{\rm cr} \simeq 0.09\,\text{GeV}$
$\mu \simeq 0.54\,\text{GeV}$

[Braun, Leonhardt, MP '18]



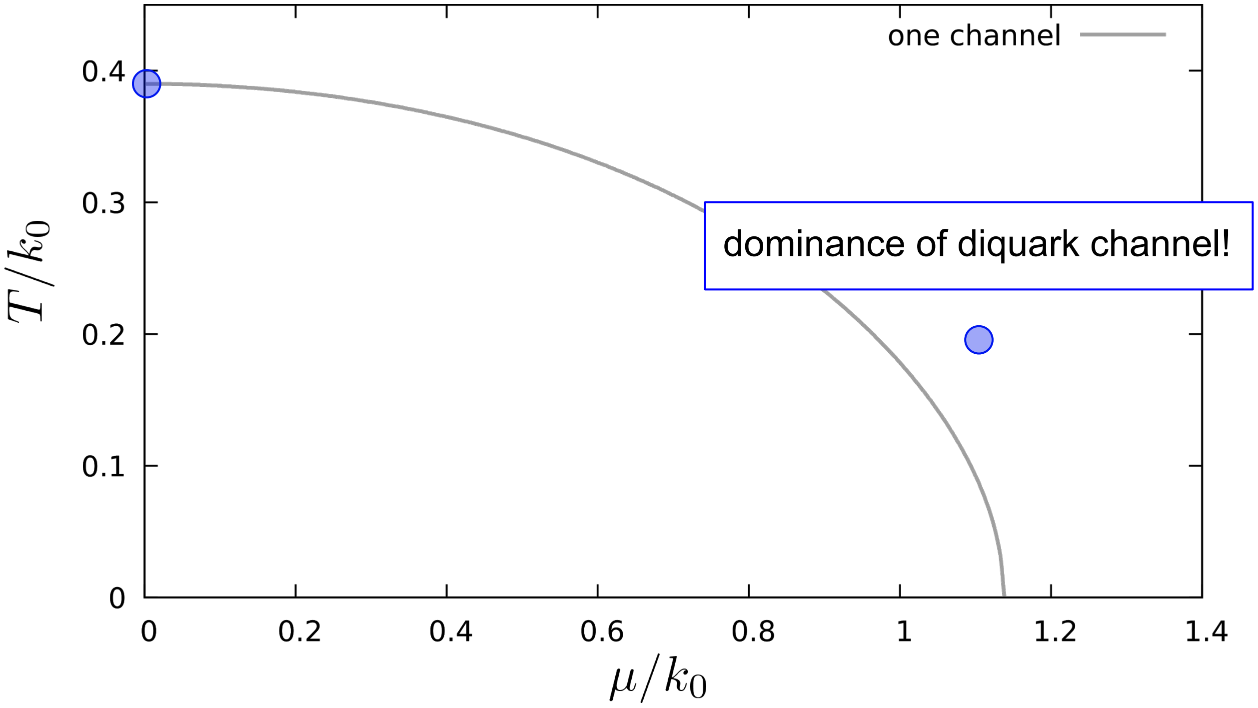
$\langle\ ?\ \rangle$

$\lambda^{(\rm UV)}_{(\sigma\text{-}\pi)} \neq 0$

$\lambda^{(\rm UV)}_{i} = 0$

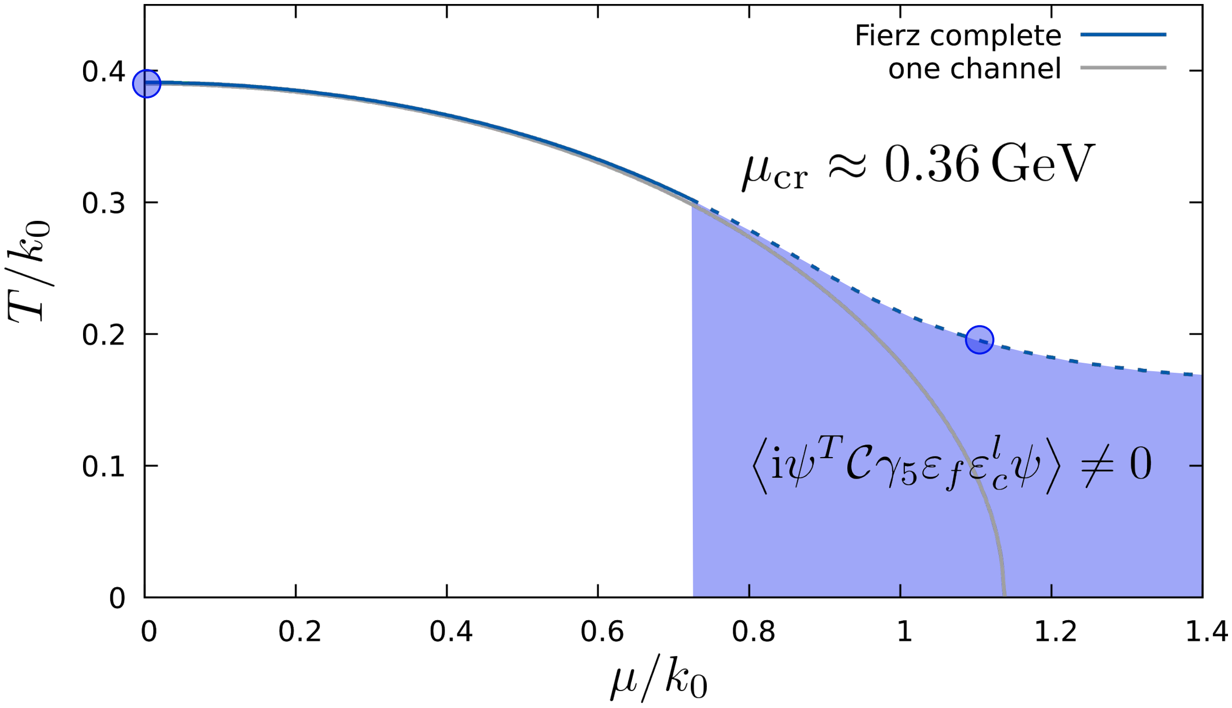
Fierz-complete phase diagram

[Braun, Leonhardt, MP ‘18]



Fierz-complete phase diagram

[Braun, Leonhardt, MP '18]



Two-channel approximation

Motivation: Understanding "diquark" dominances for large chemical potential

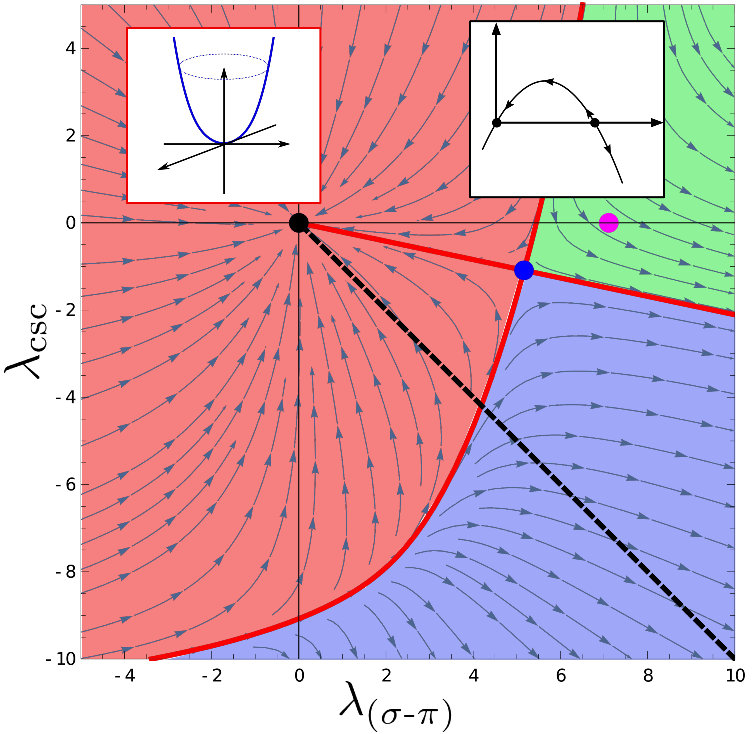
$$\Gamma_{\text{2-chan.}} = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \left\{ \bar{\psi}\mathrm{i}(\gamma_0\partial_0 + \gamma_i\partial_i - \mu\gamma_0)\psi + \boxed{\frac{\lambda_{(\sigma\text{-}\pi)}}{2}\mathcal{L}_{(\sigma\text{-}\pi)} + \frac{\lambda_{\text{csc}}}{2}\mathcal{L}_{\text{csc}}} \right\}$$

Keep only two most dominant channels

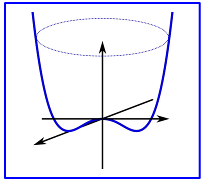
⇒ Scale-fixing procedure analogue to Fierz-complete calculation

$\lambda^{(\mathrm{UV})}_{(\sigma\text{-}\pi)} \neq 0$, else $\lambda^{(\mathrm{UV})}_{\text{csc}} = 0 \quad (\Lambda = 1\,\mathrm{GeV})$

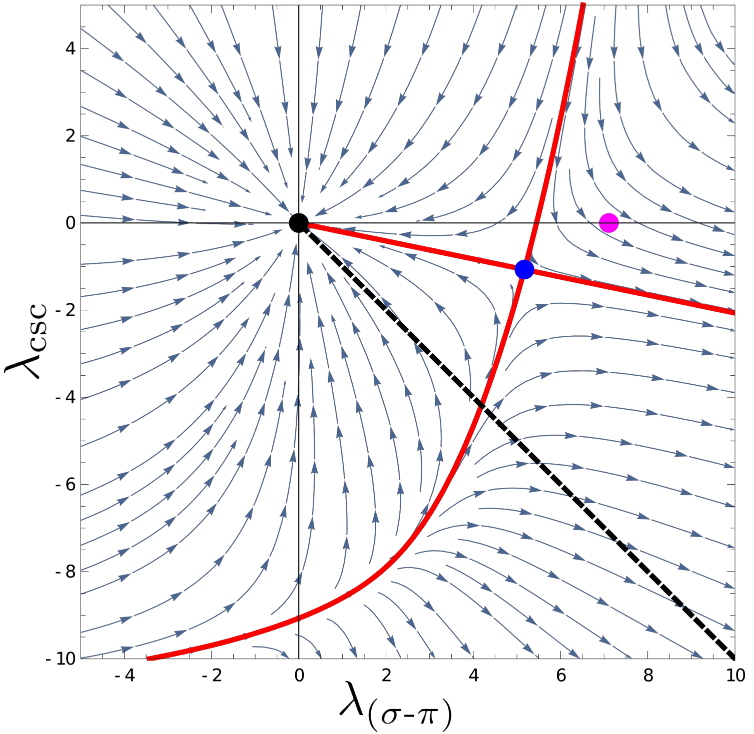
Two-channels: Finite temperature



$T/k = 0$
$\mu/k = 0$

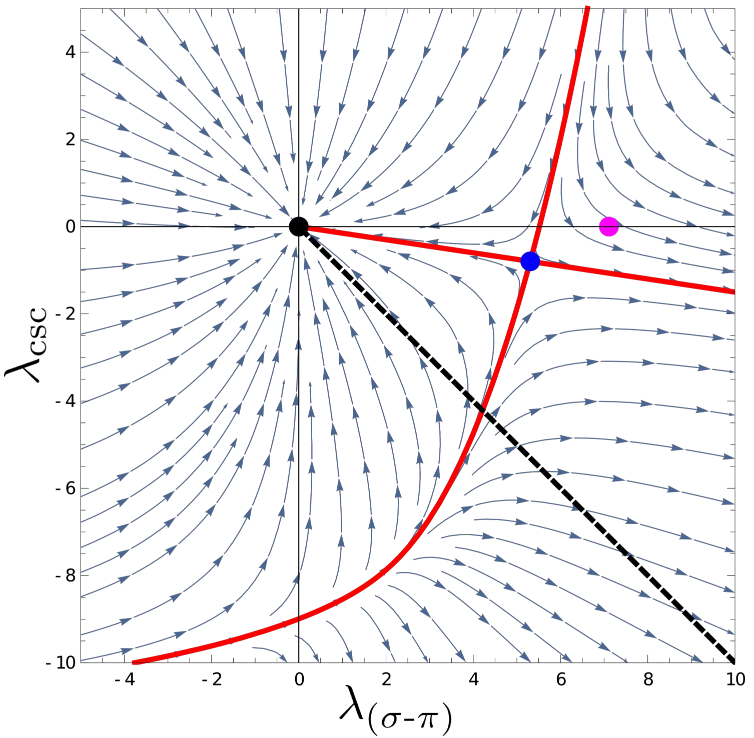


Two-channels: Finite temperature



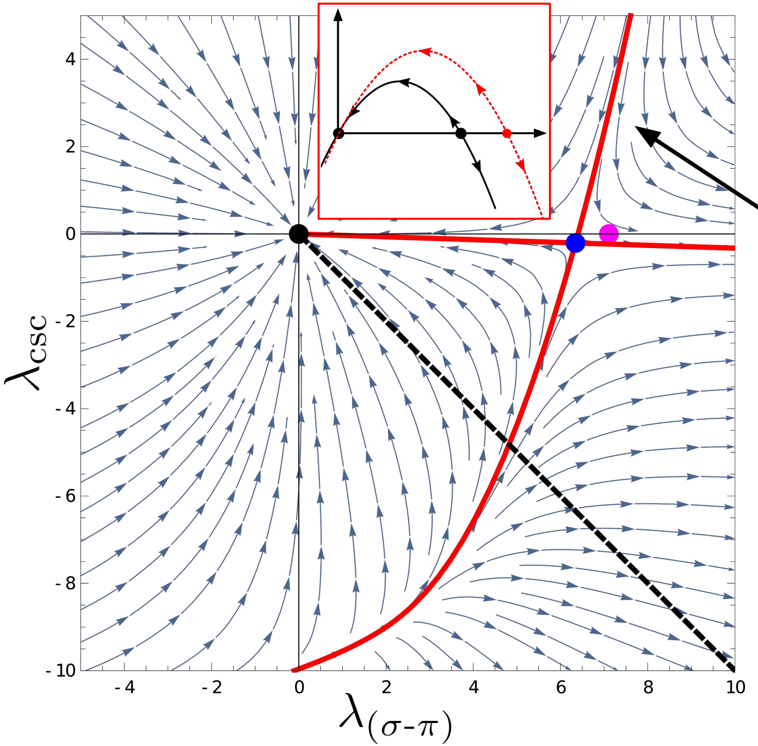
$$T/k = 0.1$$
$$\mu/k = 0$$

Two-channels: Finite temperature



$T/k = 0.2$
$\mu/k = 0$

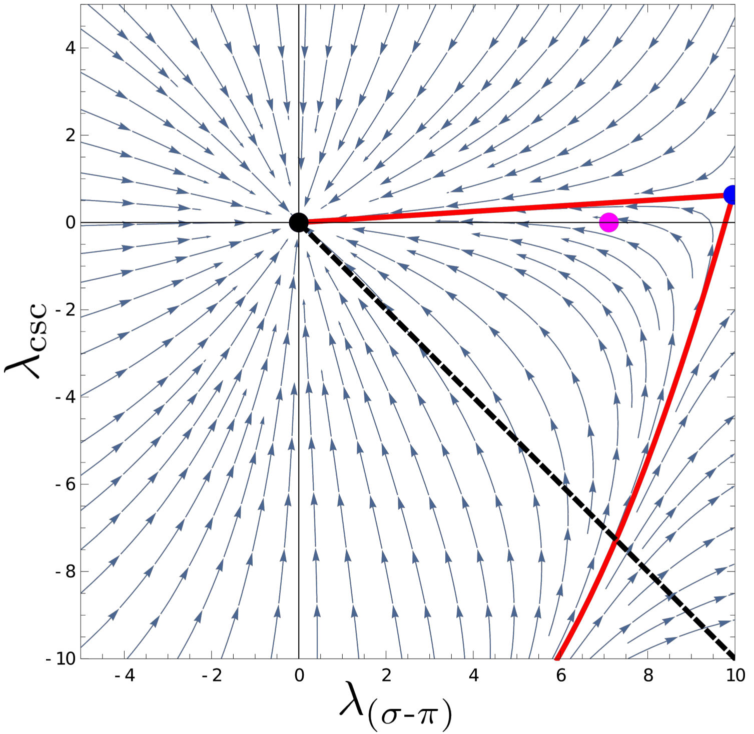
Two-channels: Finite temperature



$$T/k = 0.3$$
$$\mu/k = 0$$

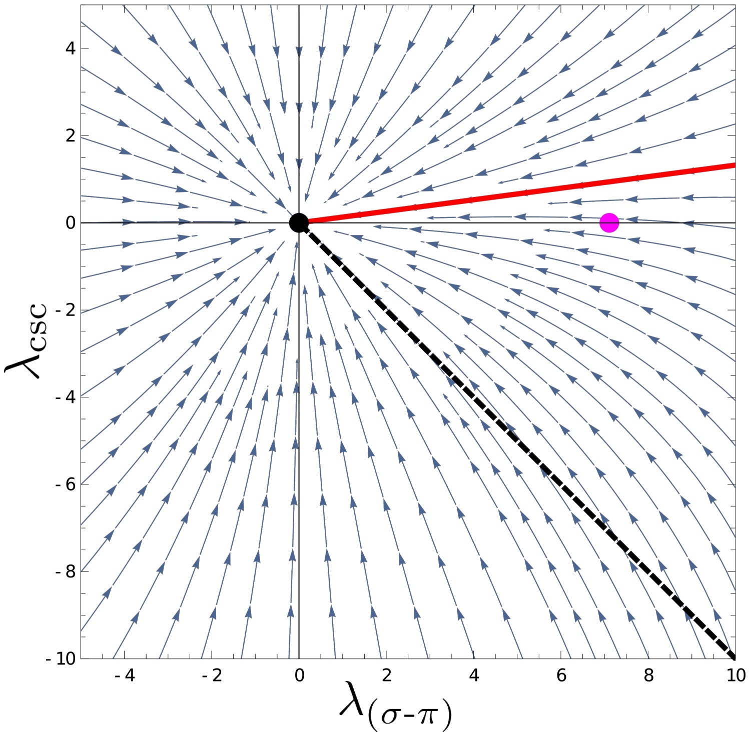
Separatrix is shifted away from Gaussian fixed point!

Two-channels: Finite temperature



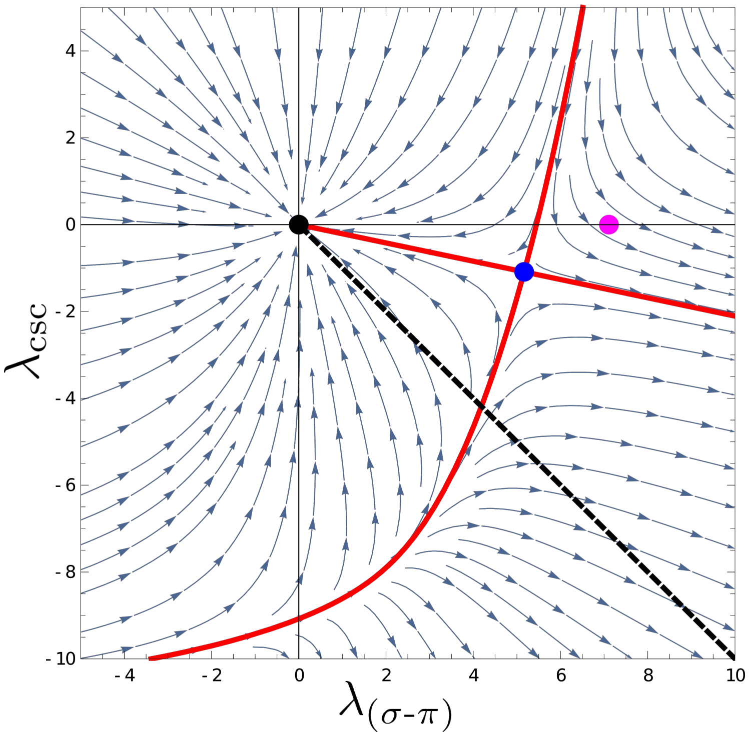
$T/k = 0.4$
$\mu/k = 0$

Two-channels: Finite temperature



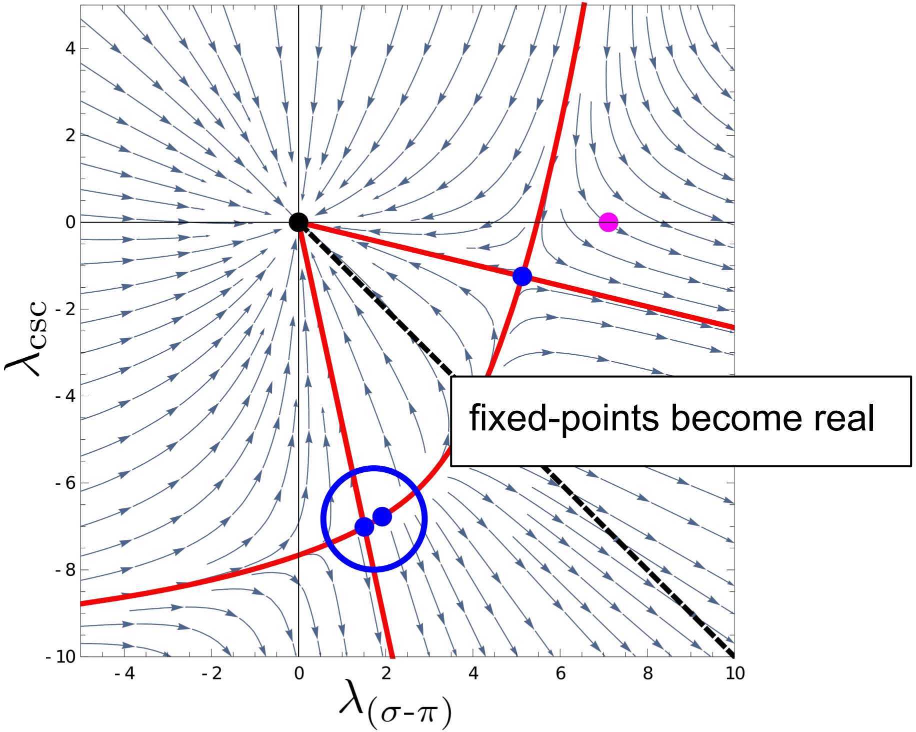
$T/k = 0.5$
$\mu/k = 0$

Two-channels: Finite chemical potential



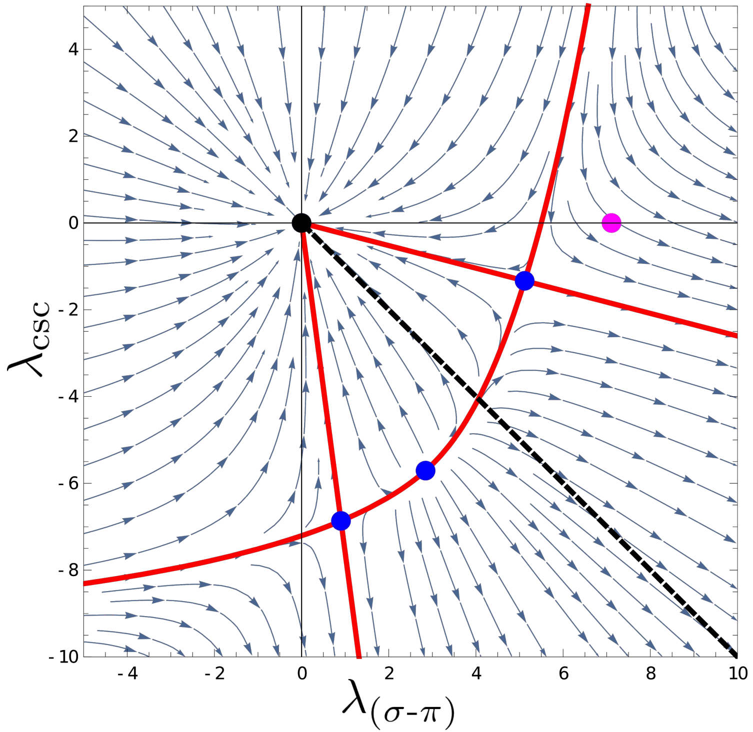
$$T/k = 0$$
$$\mu/k = 0$$

Two-channels: Finite chemical potential



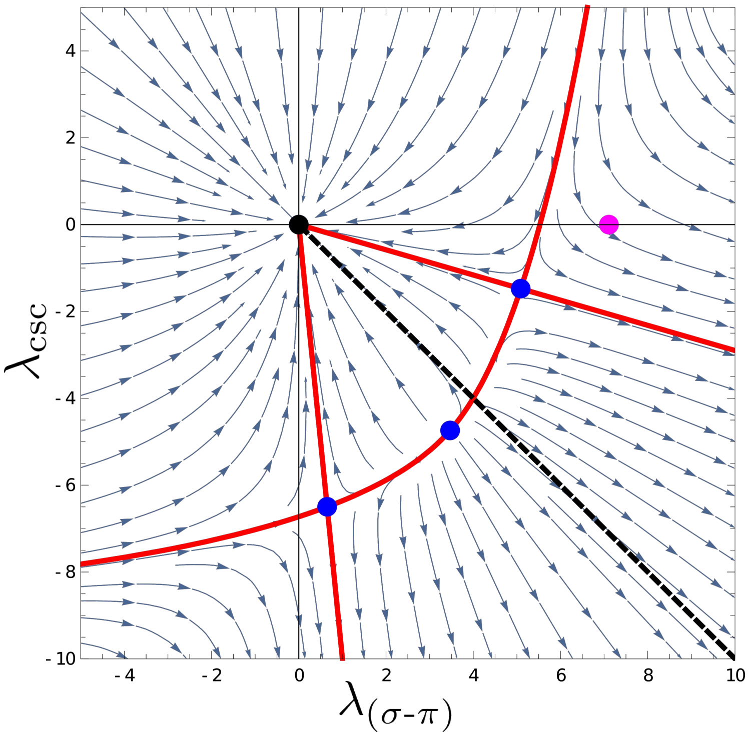
$T/k = 0$
$\mu/k = 0.3$

Two-channels: Finite chemical potential



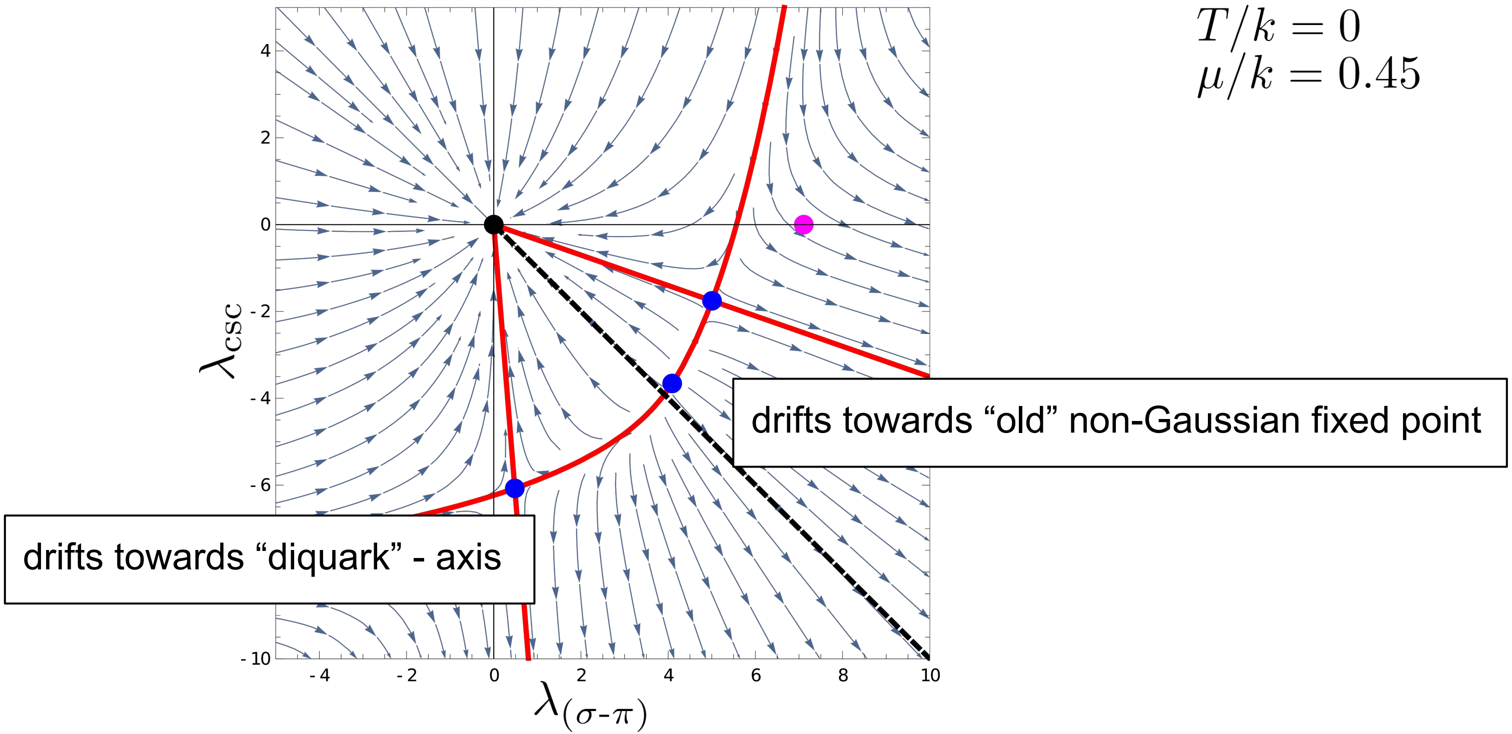
$T/k = 0$
$\mu/k = 0.35$

Two-channels: Finite chemical potential



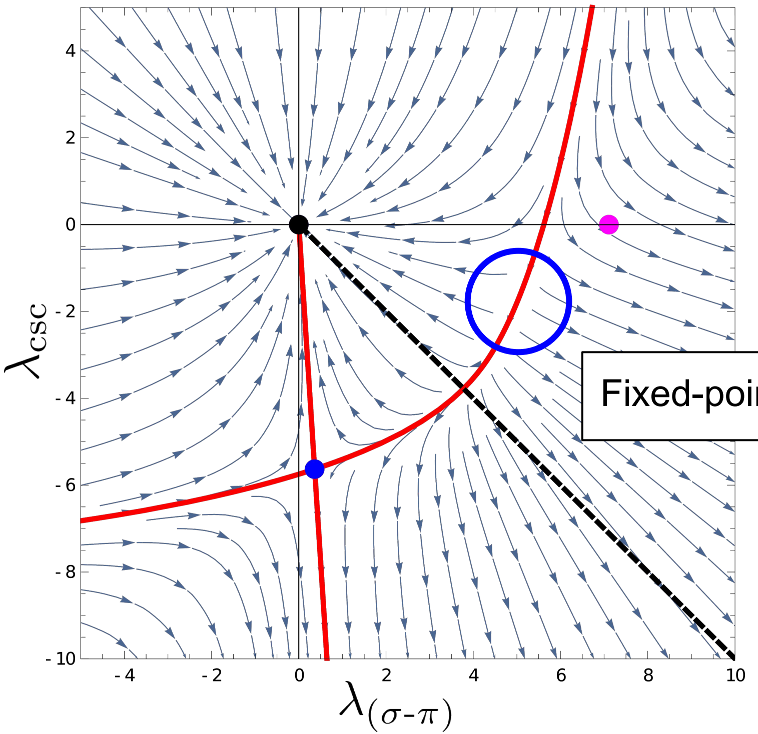
$T/k = 0$
$\mu/k = 0.4$

Two-channels: Finite chemical potential



$T/k = 0$
$\mu/k = 0.45$

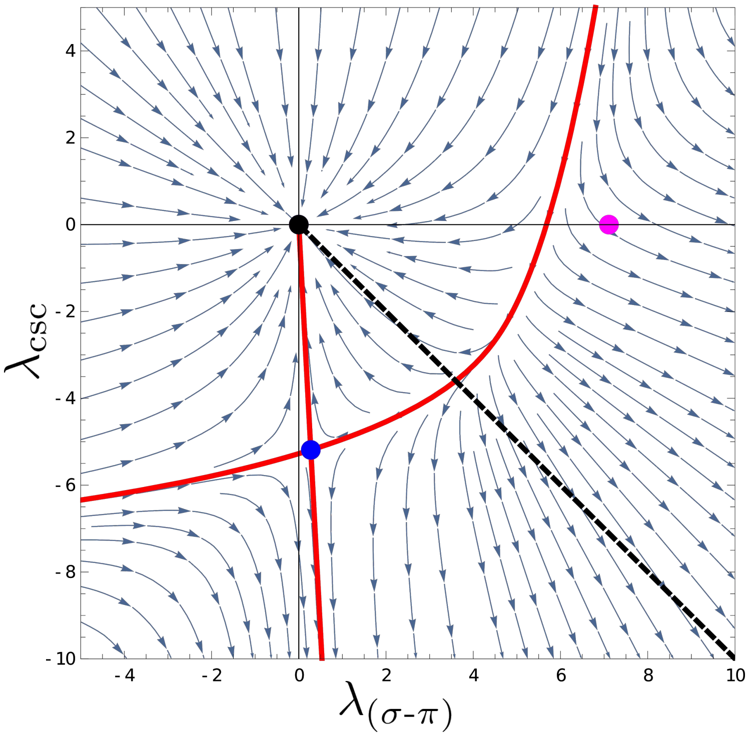
Two-channels: Finite chemical potential



$T/k = 0$
$\mu/k = 0.5$

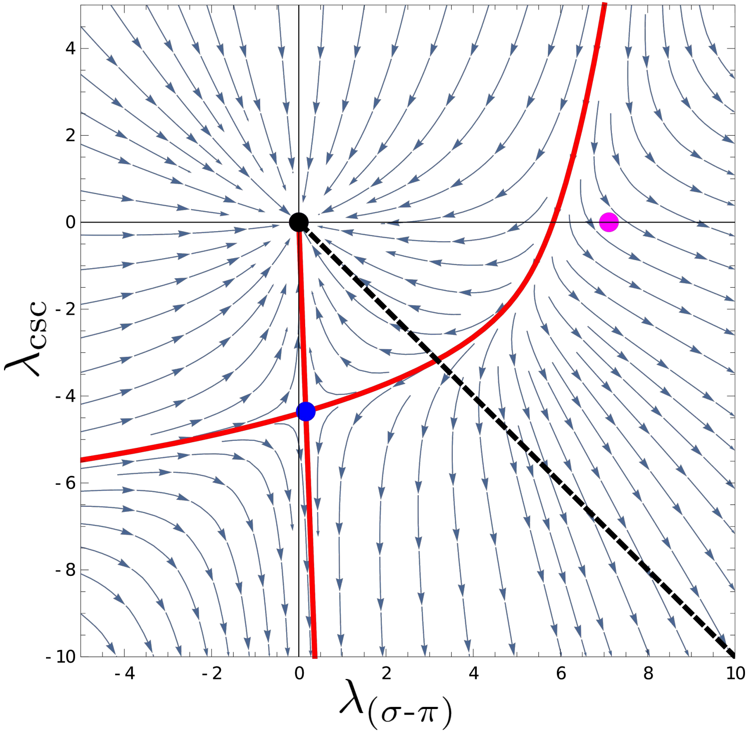
Fixed-point annihilation

Two-channels: Finite chemical potential



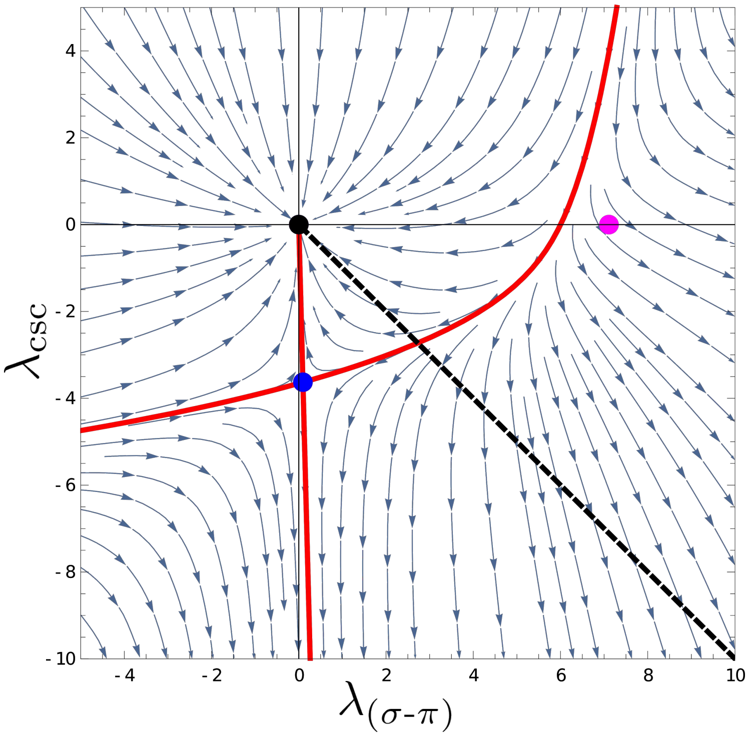
$$T/k = 0$$
$$\mu/k = 0.55$$

Two-channels: Finite chemical potential



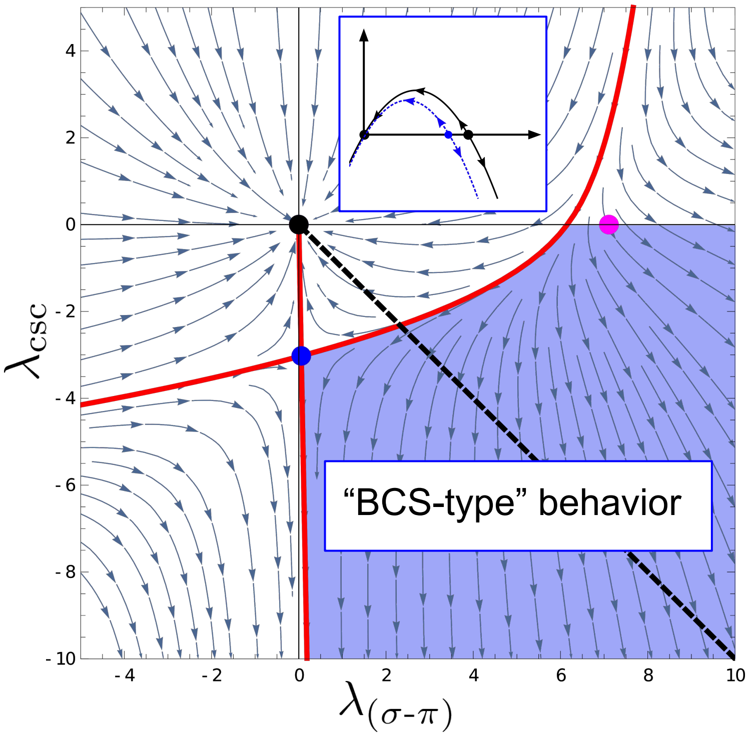
$$T/k = 0$$
$$\mu/k = 0.65$$

Two-channels: Finite chemical potential



$T/k = 0$
$\mu/k = 0.75$

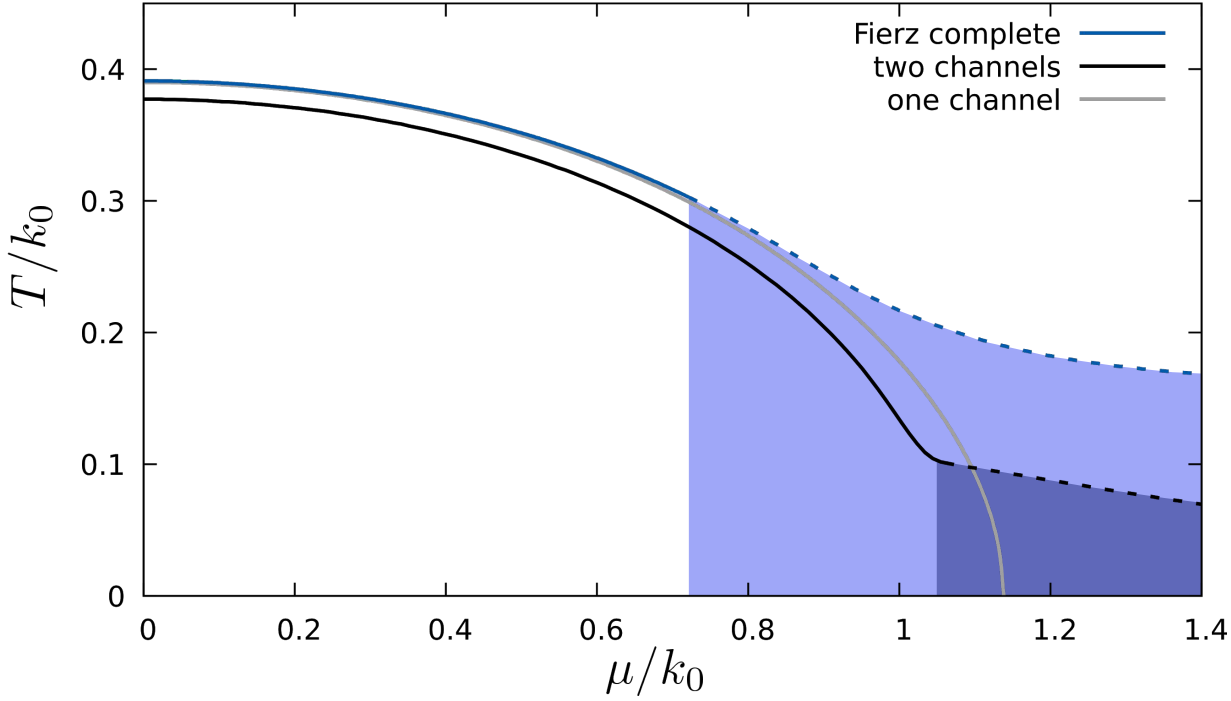
Two-channels: Finite chemical potential



$$T/k = 0$$
$$\mu/k = 0.85$$

Two-channel phase diagram

[Braun, Leonhardt, MP ‘18]



Summary

- Fierz-complete NJL-model study (2 flavors)
- Indications for diquark-dominated low energy physics at large quark-chemical potential
- Analysis of fixed-point structure at finite temperature and quark-chemical potential
- Possible mechanism for diquark condensation
- At large chemical potential critical temperature of Fierz-complete study is significantly higher compared to two-channel approximation