

# Generalized distribution amplitudes in meson pair production and gravitational form factors

---

Qin-Tao Song (Sokendai/KEK)  
YKIS2018b Symposium: Recent Developments in  
Quark-Hadron Sciences  
June 12, 2018, Kyoto, Japan

S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

# Outline

## **Generalized distribution amplitude (GDA) of pion**

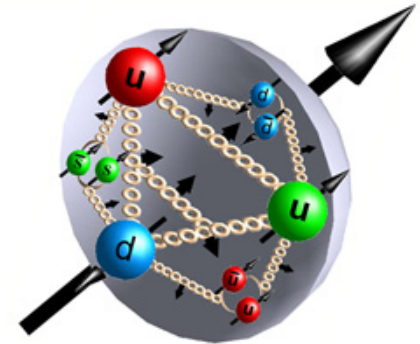
- Motivation
- GDA in two-photon process
- GDA analysis for Belle data

# Structure of hadrons: 3D structure

Spin puzzle of proton

$$\Delta u^+ + \Delta d^+ + \Delta s^+ \approx 0.3$$

$$\Delta g + \Delta L \neq 0$$



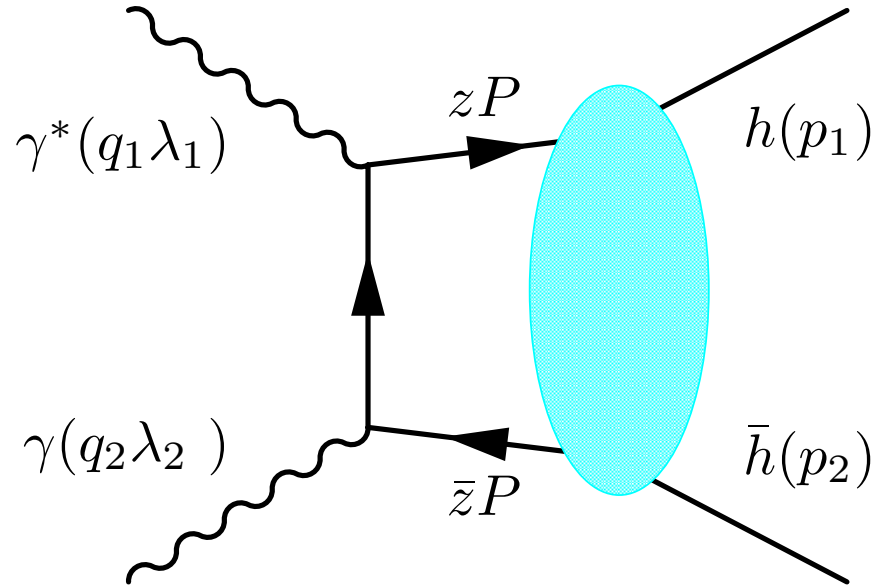
Generalized Parton Distributions (GPDs) provide information on  $\Delta L$  to solve the proton puzzle!

Generalized Distribution Amplitudes (GDAs)  $\leftrightarrow$  s-t crossing of GPDs  
Pion GDAs are investigated.

GDAs carry many important physical quantities of the hadron, such as distribution amplitudes (DAs) and timelike form factors.

# Generalized distribution amplitude for pion

In the process  $\gamma\gamma^* \rightarrow h \bar{h}$ , an hard part describing the process  $\gamma\gamma^* \rightarrow q \bar{q}$  with produced collinear and on-shell quark, and a soft part describing the production of the hadron  $h$  pair from a  $q \bar{q}$ . This soft part is called **Generalized Distribution Amplitude (GDA)**.



The process  $\gamma^* \gamma \rightarrow h \bar{h}$

GDA is an important quantity of hadron, it is defined as

$$\Phi^q(z, \xi, W^2) = \int \frac{dx^-}{2\pi} e^{-izP^+ x^-} \langle h(p) \bar{h}(p') | \bar{q}(x^-) \gamma^+ q(0) | 0 \rangle$$

$$z = \frac{k^+}{P^+}, \quad \xi = \frac{p^+}{P^+}, \quad s = W^2 = (p + p')^2 = P^2$$

GDA is closely related to generalized parton distribution (GPD) by **the s-t crossing**, so GDA could provide another way to obtain GPD information.

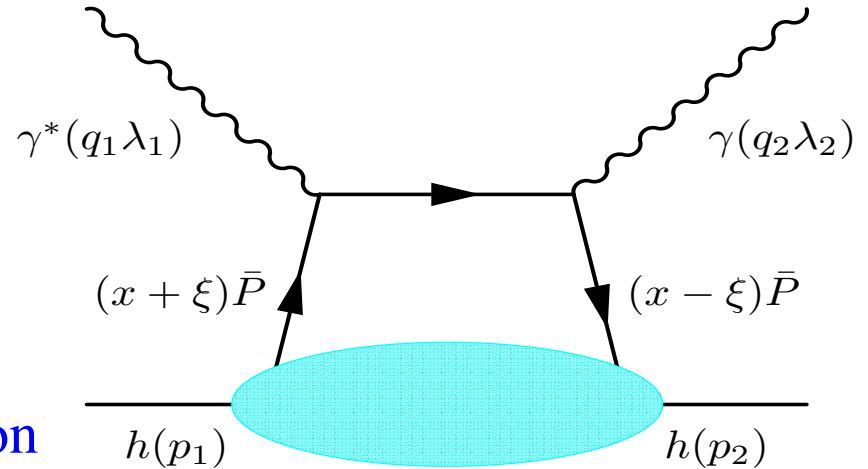
$$\Phi^q(z, \xi, W^2) \leftrightarrow H^q\left(x = \frac{1-2z}{1-2\xi}, \xi = \frac{1}{1-2\xi}, t = W^2\right)$$

GDA

GPD



GPD can be used to study the proton spin puzzle!



$$\gamma^* h \rightarrow \gamma h$$

$$\int \frac{dx^-}{2\pi} e^{-iz(\bar{P}^+ x^-)} \langle h(p_2) | \bar{q}(x^-) \gamma^+ q(0) | h(p_1) \rangle$$

$$= \frac{1}{2\bar{P}^+} \left[ H^q(x, \xi, t) \bar{u}(p_2) \gamma^+ u(p_1) + E^q(x, \xi, t) \bar{u}(p_2) \frac{i\sigma^{+\alpha} \Delta_\alpha}{2m} u(p_1) \right]$$

$$\bar{P} = (p_1 + p_2) / 2, \Delta = p_2 - p_1, x = \frac{-q_1^2}{2p_1 q_1}, \xi = \frac{\Delta^+}{p_1^+ + p_2^+}$$

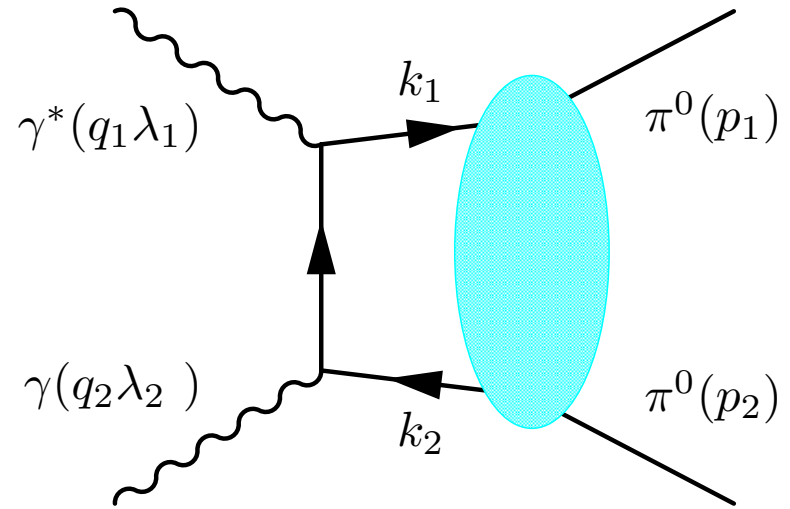
M. Diehl, Phys. Rep. 388 (2003), 41.

H. Kawamura and S. Kumano, PRD 89 (2014), 054007.

# The cross section of process $\gamma^* \gamma \rightarrow \pi^0 \pi^0$

$$d\sigma = \frac{1}{4} \frac{1}{4\sqrt{(q_1 q_2)^2 - q_1^2 q_2^2}} \sum_{\lambda_1 \lambda_2} |-iT_{\mu\nu} \varepsilon^\mu(q_1) \varepsilon^\nu(q_2)|^2 d\Phi_2$$

$$d\sigma = \frac{\pi\alpha^2 \sqrt{1 - \frac{4m^2}{s}}}{4(Q^2 + s)} |A_{++}|^2 \sin\theta d\theta$$



$A_{\lambda_1 \lambda_2}$  is the **helicity amplitude**, and there are 3 independent **helicity amplitudes**, they are  $A_{++}$ ,  $A_{0+}$  and  $A_{+-}$ . The leading-twist amplitude  $A_{++}$  has a close relation with the generalized distribution amplitude (GDA)  $\Phi^q(z, \xi, W^2)$ .

$$A_{\lambda_1 \lambda_2} = T_{\mu\nu} \varepsilon^\mu(\lambda_1) \varepsilon^\nu(\lambda_2) / e^2$$

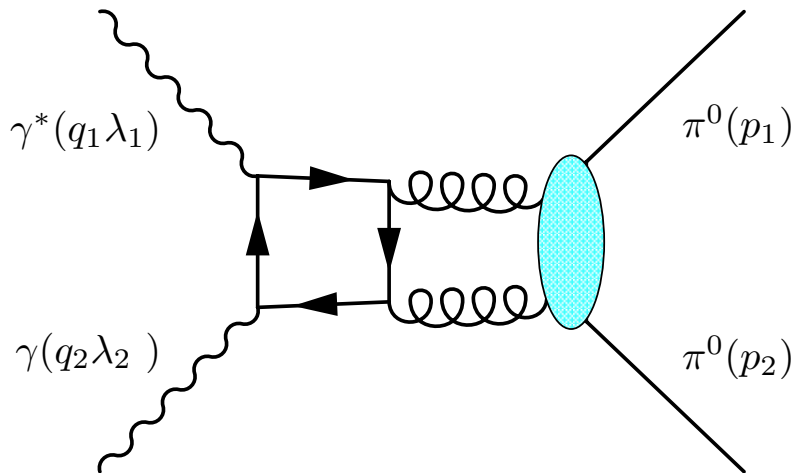
$$A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi^q(z, \xi, W^2)$$

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL **81** (1998) 1782.

M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.

# Higher twist and higher order contributions

Higher-twist contribution  $A_{0+}$  requires a helicity flip along the fermion line, and it decreases as  $1/Q$ . Higher-order contribution  $A_{+-}$  contributes with the **GDA of gluon**, since  $A_{+-}$  indicates the angular momentum  $L_z = 2$ . Therefore  $A_{+-}$  is suppressed by running coupling constant  $\alpha_s$ .



Gluon GDA

M. Diehl, T. Gousset and B. Pire, PRD 62 (2000) 07301.

N. Kivel, L. Mankiewicz and M.V. Polyakov PLB 467 (1999) 263.

# GDA expression

At **very high energy**  $Q^2$ , we can have the asymptotic form of the GDA

$$\begin{aligned}\sum_q \Phi_q^+(z, \xi, W^2) &= 18n_f z(1-z)(2z-1)[B_{10}(W) + B_{12}(W)P_2(2\xi-1)] \\ &= 18n_f z(1-z)(2z-1)[\tilde{B}_{10}(W) + \tilde{B}_{12}(W)P_2(\cos\theta)]\end{aligned}$$

The GDAs are related to the energy-momentum form factor in the timelike region.

$$\int dz(2z-1)\Phi_q^+(z, \xi, W^2) = \frac{2}{(P^+)^2} \langle \pi^+(p_1)\pi^-(p_2) | T_q^{++}(0) | 0 \rangle$$

where the energy-momentum form factor for quarks is defined as

$$\langle \pi^0(p_1)\pi^0(p_2) | T^{\mu\nu}(0) | 0 \rangle = \frac{1}{2} \left[ (sg^{\mu\nu} - P^\mu P^\nu) \Theta_1 + \Delta^\mu \Delta^\nu \Theta_2 \right]$$

$$P = p_1 + p_2, \Delta = p_1 - p_2$$

By using this sum rule we can obtain  $B_{12}(0) = \frac{5R_\pi}{9}$

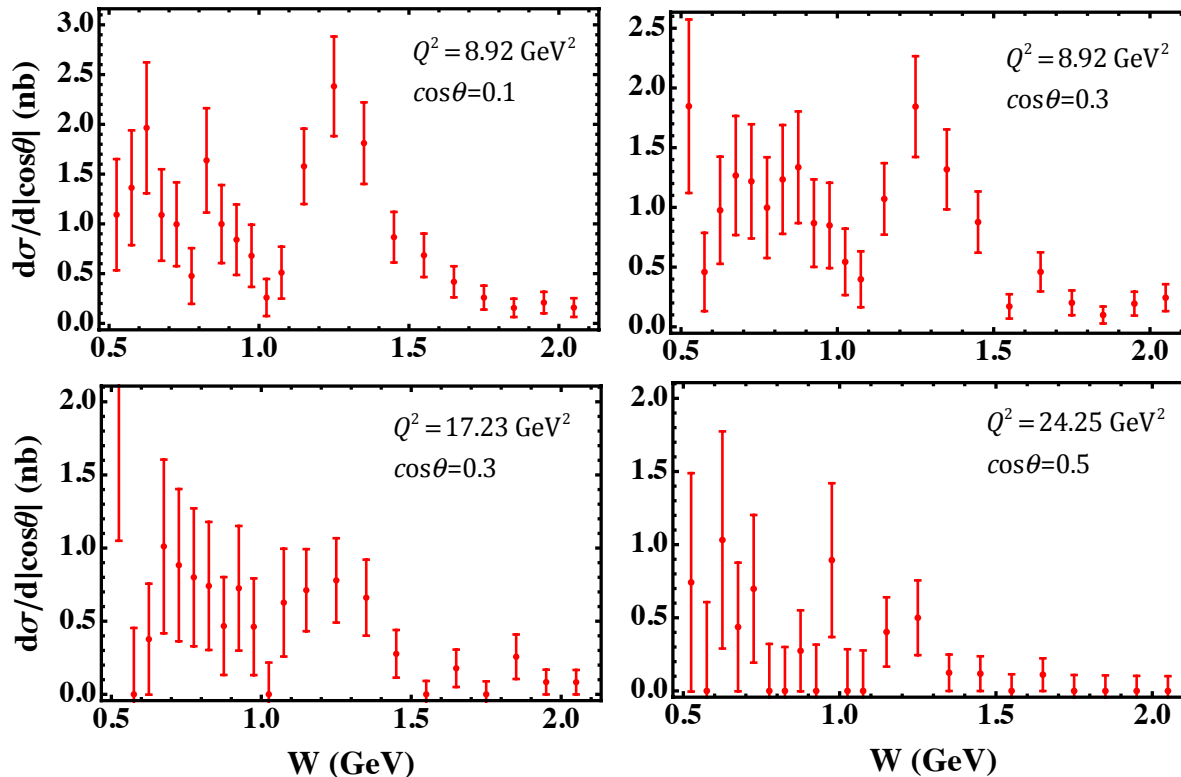
where  $R_\pi$  is the momentum fraction carried by quarks in the pion.

M. V. Polyakov, NPB **555** (1999) 231.

M. V. Polyakov and C. Weiss PRD 60 (1999) 114017.



In 2016, the Belle Collaboration released the measurements of differential cross section for  $\gamma^*\gamma \rightarrow \pi^0\pi^0$ . The GDAs can be obtained by analyzing the Belle data.

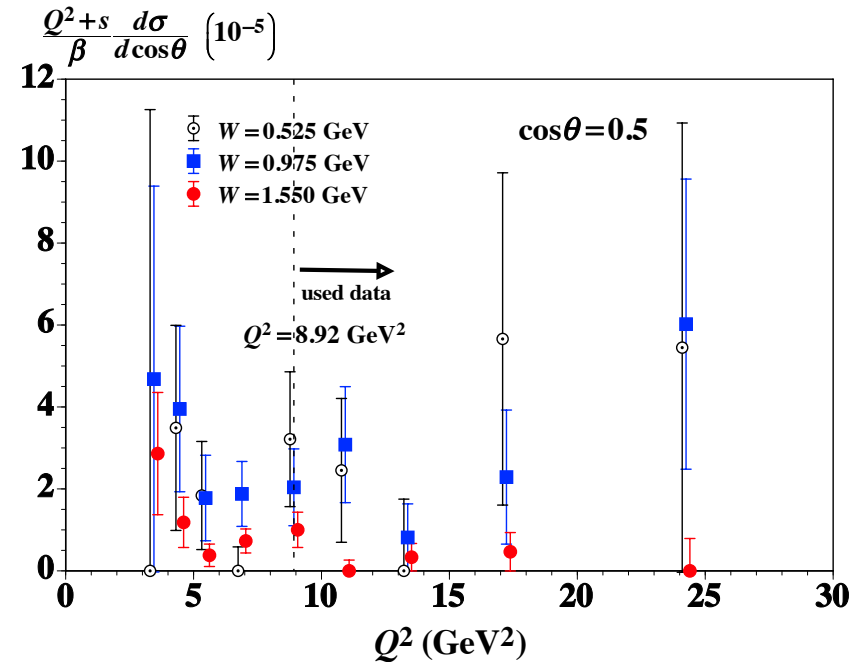
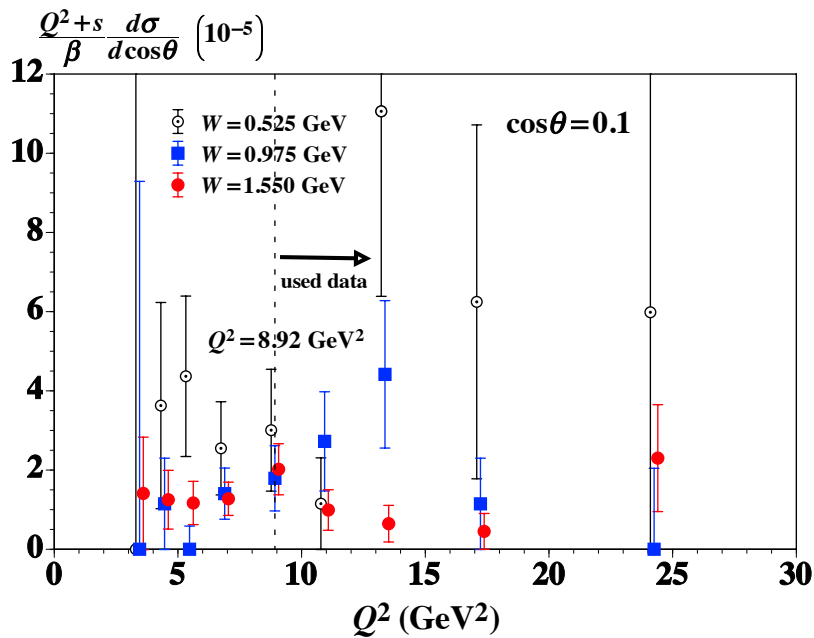


Differential cross section for  $\gamma^*\gamma \rightarrow \pi^0\pi^0$

In these figures, the resonance  $f_2(1270)$  is clearly seen around  $W = 1.25 \text{ GeV}$ , however, other resonances are not clearly seen due to the large errors.

# Scale violation of GDA based on Belle data

$$\frac{(Q^2 + s)d\sigma}{\beta d|\cos\theta|} \propto \left| \Phi^{\pi^0\pi^0}(z, \cos\theta, W, Q) \right|^2$$



The scale dependence of the Belle data. We have red color for  $W = 0.525 \text{ GeV}$ , blue color for  $W = 0.975 \text{ GeV}$ , and green color for  $W = 1.55 \text{ GeV}$ .

The scaling violation of the GDAs is not so obvious in the Belle data on account of the large errors, so that the  $Q^2$ -independent GDAs could be used in analyzing the Belle data.

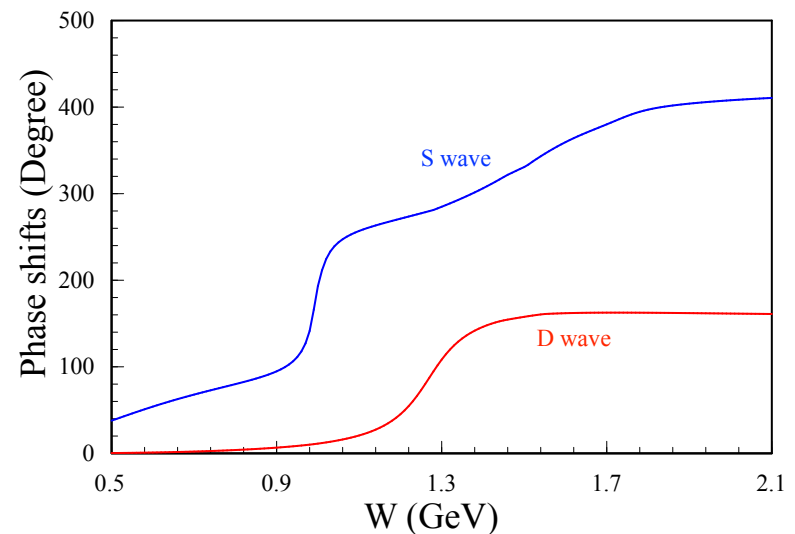
# $Q^2$ -independent (asymptotic form) GDAs

$$\begin{aligned}\sum_q \Phi_q^+(z, \xi, W^2) &= 18n_f z(1-z)(2z-1)[B_{10}(W) + B_{12}(W)P_2(2\xi-1)] \\ &= 18n_f z(1-z)(2z-1)[\tilde{B}_{10}(W) + \tilde{B}_{12}(W)P_2(\cos\theta)]\end{aligned}$$

$$\tilde{B}_{10}(W) = \bar{B}_{10}(W)e^{i\delta_0}, \tilde{B}_{12}(W) = \bar{B}_{12}(W)e^{i\delta_2}$$

In the above equation  $\delta_0$  and  $\delta_2$  are the  $\pi\pi$  elastic scattering phase shifts in the isospin=0 channel (see the figure). Above the KK threshold, the additional phase is introduced for S-wave

The S wave and D-wave  $\pi\pi$  scattering phase shifts.



M. Diehl, T. Gousset and B. Pire, PRD 62 (2000) 07301.

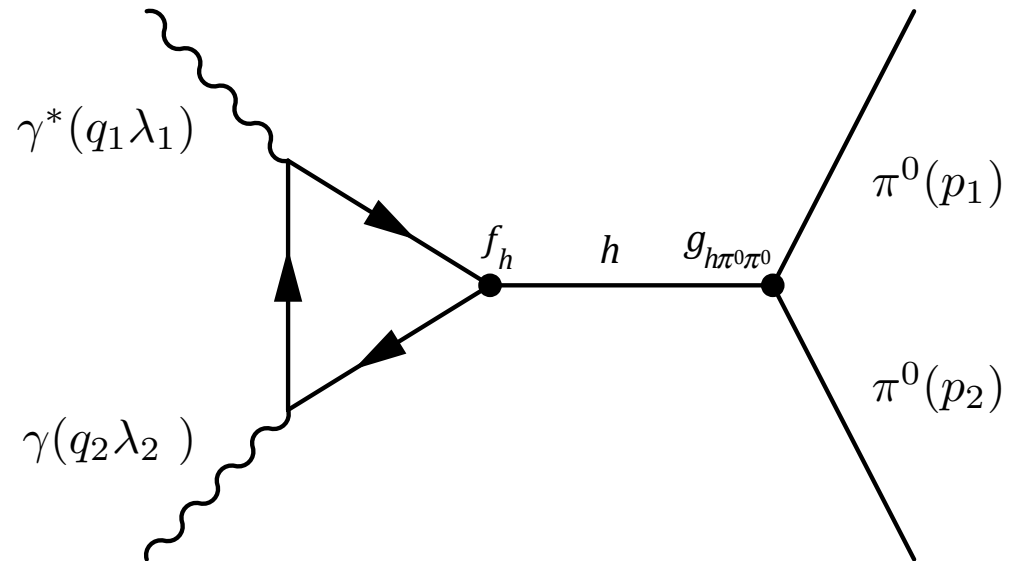
P. Bydzovsky, R. Kamiski and V. Nazari, PRD 90 (2014) , 116005; PRD 94 (2016), 116013.

# Resonance effects

In the process  $\gamma^* \gamma \rightarrow \pi^0 \pi^0$ , the  $\pi^0 \pi^0$  can be produced through intermediate meson state  $h$ . The  $q \bar{q} \rightarrow h$  amplitude should be proportional to the decay constant  $f_h$  or the distribution amplitude (DA), and the  $h \rightarrow \pi^0 \pi^0$  amplitude can be expressed by the coupling constant  $g_{h\pi\pi}$ . These resonance contributions read

$$\bar{B}_{12}(W) = \beta^2 \frac{10 g_{f_2 \pi \pi} f_{f_2} M_{f_2}^2}{9 \sqrt{2} \sqrt{(M_{f_2}^2 - W^2)^2 - \Gamma_{f_2}^2 M_{f_2}^2}}$$

$$\bar{B}_{10}(W) = \frac{5 g_{f_0 \pi \pi} f_{f_0}}{3 \sqrt{2} \sqrt{(M_{f_0}^2 - W^2)^2 - \Gamma_{f_0}^2 M_{f_0}^2}}$$



The resonance effects play an important role in the resonance regions.

We adopt a simple expression of GDA to analyze Belle data, here resonance effects of  $f_0(500)$  and  $f_2(1270)$  are introduced.

$$\Phi_q^+(z, \xi, W^2) = N_h z^\alpha (1-z)^\alpha (2z-1) [\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos\theta)]$$

$$\tilde{B}_{10}(W) = \left[ \frac{-3 + \beta^2}{2} \frac{5R_\pi}{9} F_h(W^2) + \frac{5g_{f_0\pi\pi} f_{f_0}}{3\sqrt{2} \sqrt{(M_{f_0}^2 - W^2)^2 - \Gamma_{f_0}^2 M_{f_0}^2}} \right] e^{i\delta_0}$$

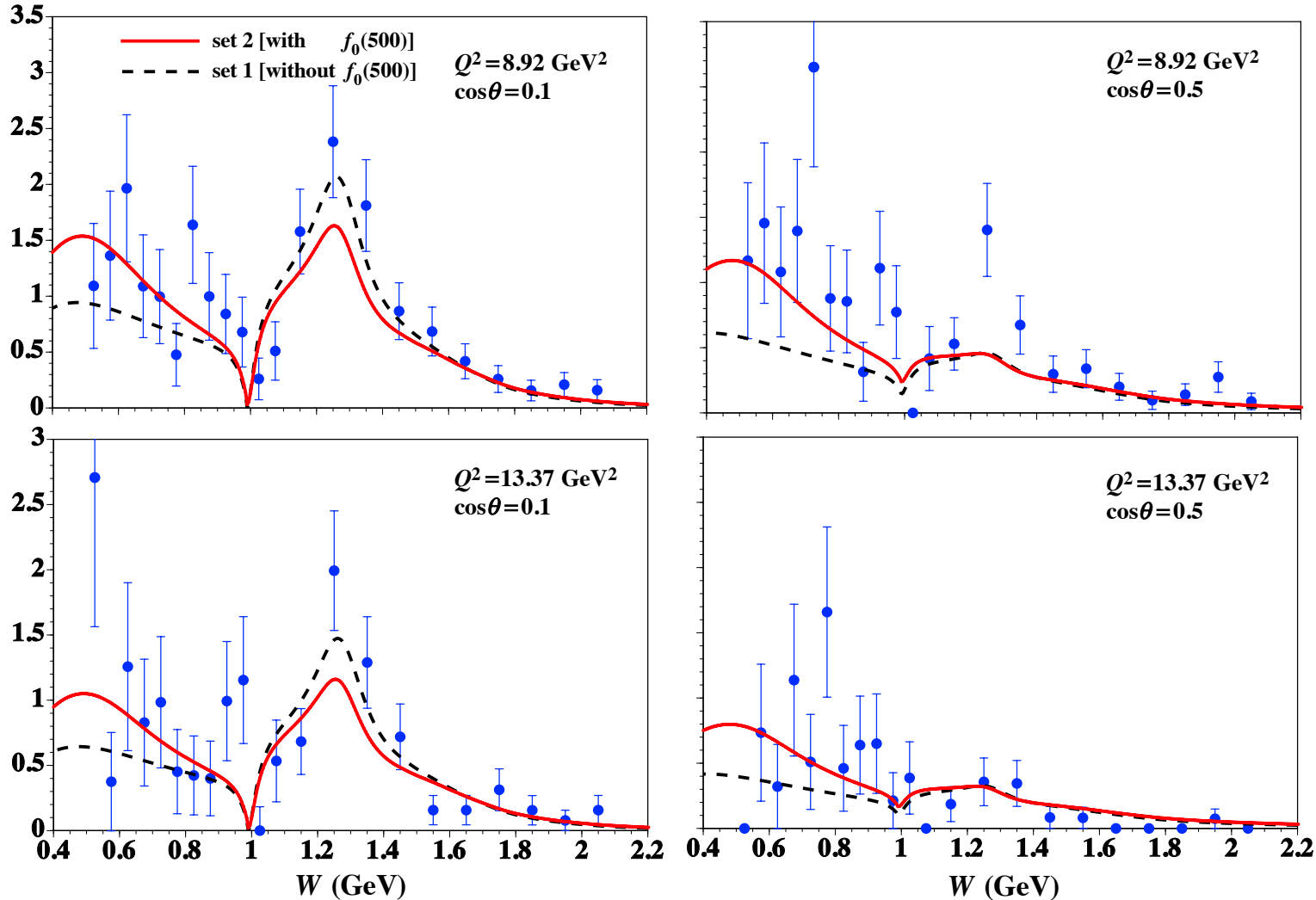
$$\tilde{B}_{12}(W) = \left[ \beta^2 \frac{5R_\pi}{9} F_h(W^2) + \beta^2 \frac{10g_{f_2\pi\pi} f_{f_2} M_{f_2}^2}{9\sqrt{2} \sqrt{(M_{f_2}^2 - W^2)^2 - \Gamma_{f_2}^2 M_{f_2}^2}} \right] e^{i\delta_2}$$

$$F_h(W^2) = \frac{1}{\left[ 1 + \frac{W^2 - 4m_\pi^2}{\Lambda^2} \right]^{n-1}}$$

The function  $F_h(W^2)$  is the form factor of the quark part of the energy-momentum tensor, and the parameter  $\Lambda$  is the momentum cutoff in the form factor. The parameter  $n$  is predicted as  $n = 2$  at very high energy, because we have  $d\sigma/d|\cos\theta| \sim 1/W^6$  by the counting rule. In the asymptotic limit,  $\alpha = 1$ .

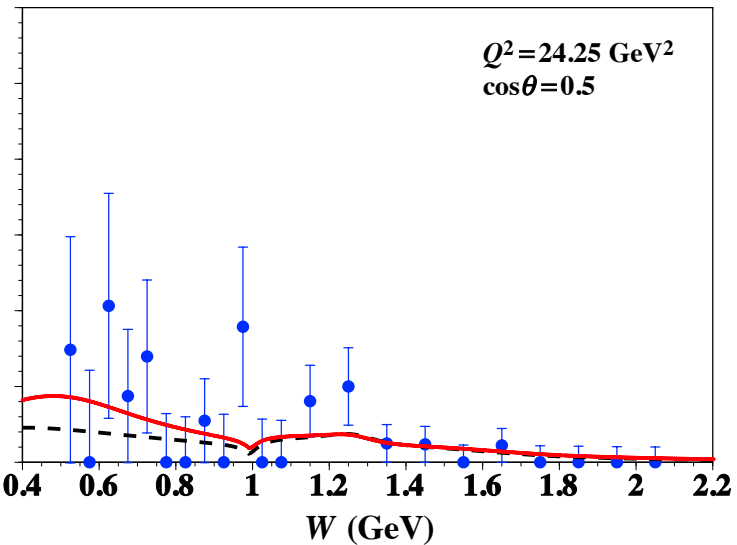
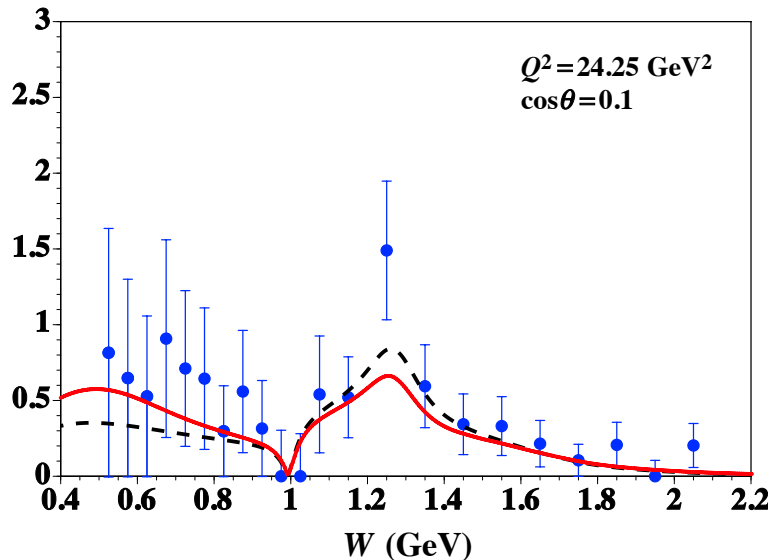
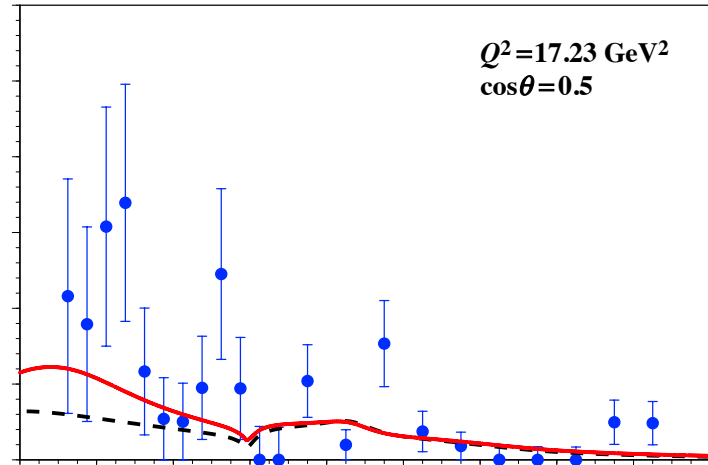
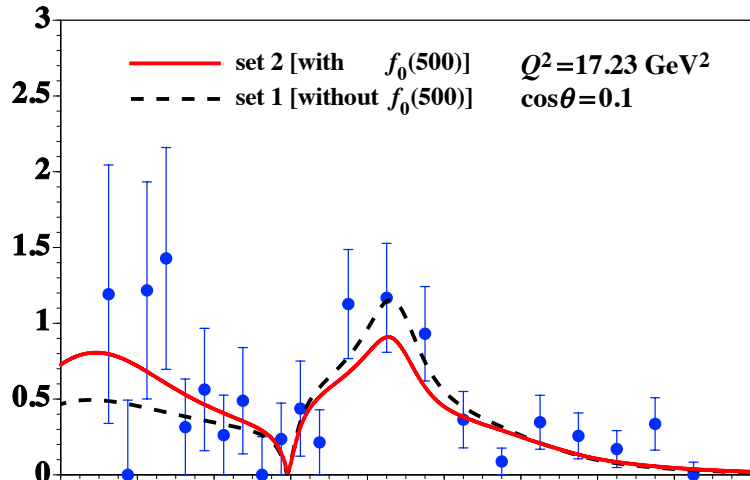
By analyzing the Belle data, the values of parameters are obtained.

$d\sigma/d\cos\theta$  (nb)



The  $W$  dependence of the differential cross section (in units of nb), and in comparison with Belle data.

$d\sigma/d\cos\theta$  (nb)



The  $W$  dependence of the differential cross section (in units of nb), and in comparison with Belle data.

By considering the following sum rule, we can also obtain the energy-momentum form factors for pion.

$$\int dz(2z-1)\Phi_q^+(z,\xi,W^2) = \frac{2}{(P^+)^2} \langle \pi^0(p_1)\pi^0(p_2) | T_q^{++}(0) | 0 \rangle$$

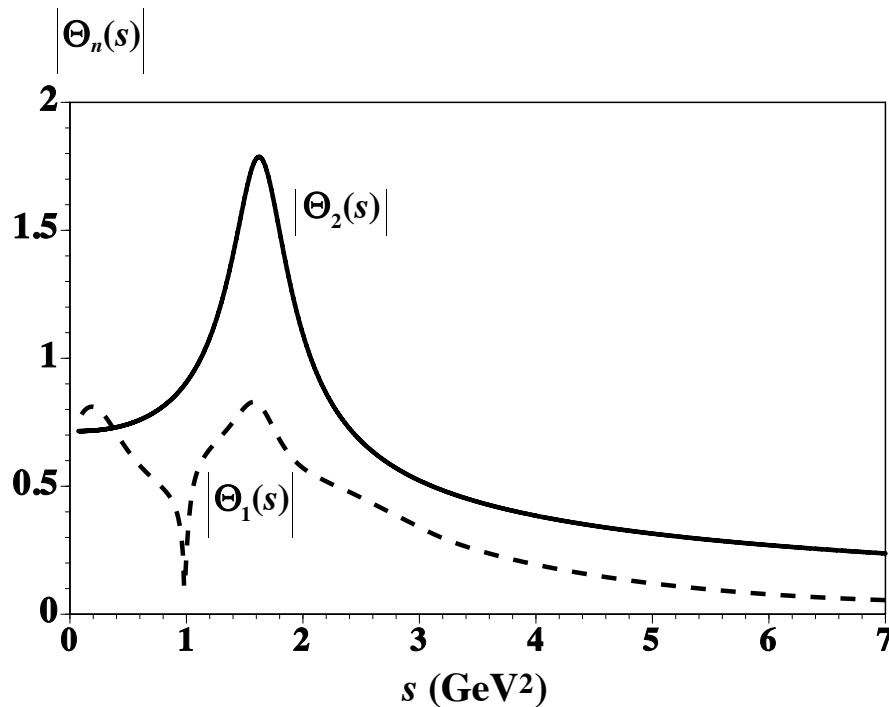
$$\langle \pi^0(p_1)\pi^0(p_2) | T^{\mu\nu}(0) | 0 \rangle = \frac{1}{2} \left[ (sg^{\mu\nu} - P^\mu P^\nu) \Theta_1 + \Delta^\mu \Delta^\nu \Theta_2 \right]$$

$$\Theta_1 = \frac{3}{5}(\tilde{B}_{12} - 2\tilde{B}_{10}), \quad \Theta_2 = \frac{9}{5\beta^2} \tilde{B}_{12}$$

M. V. Polyakov, NPB **555** (1999) 231.

M. V. Polyakov and C. Weiss PRD **60** (1999) 114017.

$\Theta_1 \rightarrow$  Mechanical (pressure and shear force)  
 $\Theta_2 \rightarrow$  Mass

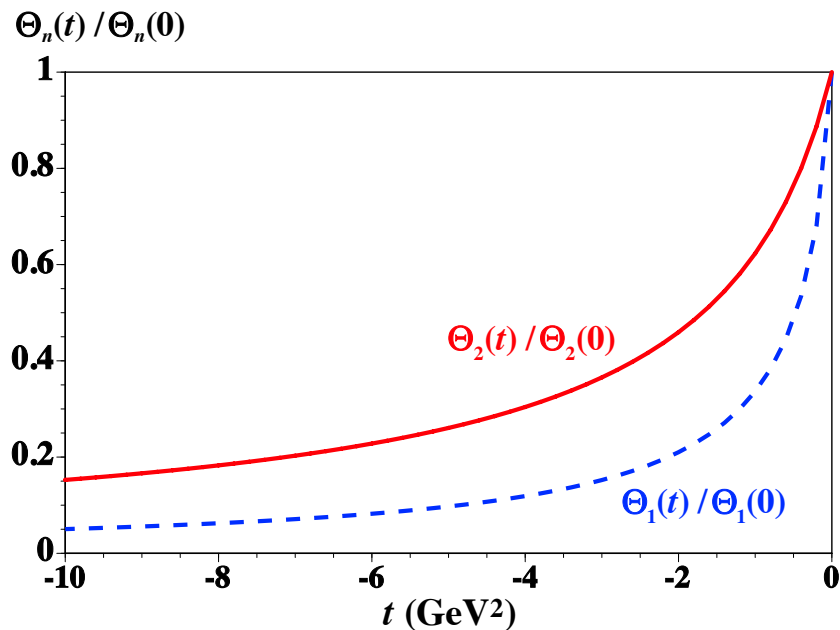


The timelike form factors  $\Theta_1$   
and  $\Theta_2$

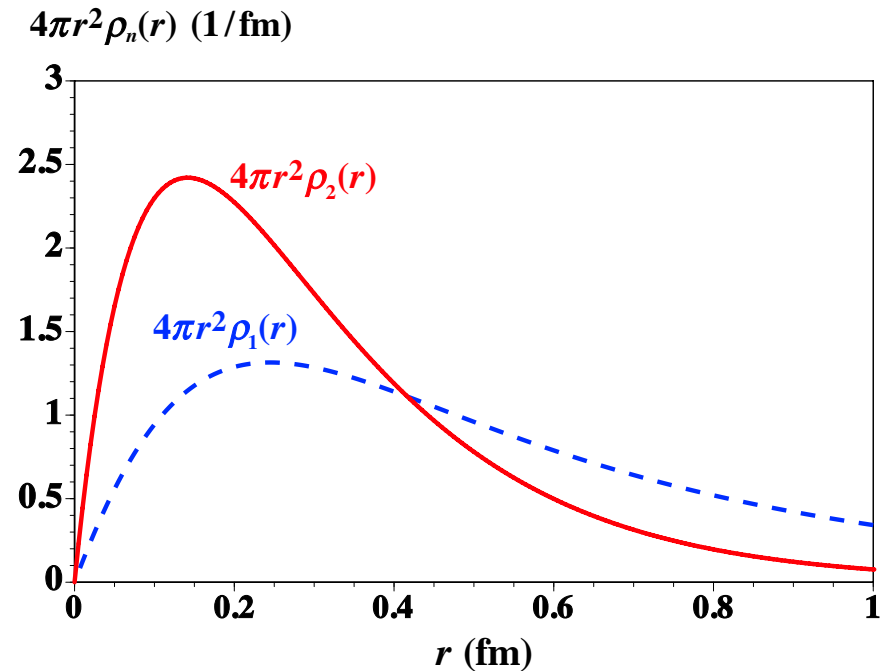


Timelike form factor  $\rightarrow$  Spacelike form factor (pion radius) : dispersion relation

$$F(t) = \int_{4m^2}^{\infty} \frac{ds}{\pi} \frac{\text{Im}(F(s))}{s-t-i\epsilon}$$



The spacelike form factors  $\Theta_1$  and  $\Theta_2$



Fourier Transform of  $\Theta_1$  and  $\Theta_2$

Radius can be obtained by the following equation

$$\langle r^2 \rangle = 6 \int_{4m^2}^{\infty} \frac{\text{Im}(F(s))}{s^2}$$

$$\sqrt{\langle r^2 \rangle} = 0.69 \text{ fm for } \Theta_2 \text{ Mass radius}$$

$$\sqrt{\langle r^2 \rangle} = 1.45 \text{ fm for } \Theta_1 \text{ Mechanical radius (pressure and shear force)}$$

In our analysis we introduce the additional phase for S-wave above the KK threshold. However, the additional phase could be add to D-wave phase above the threshold, in this case we have

**Mass radius: 0.56-0.69 fm, Mechanical radius: 1.45-1.56 fm**

## Summary

- ◆ By analyzing the Belle data the pion GDAs are obtained, and the obtained GDAs can also give a good description of experimental data.
- ◆ The energy-momentum form factors for pion are calculated from the GDA of pion.
- ◆ This is the first finding on gravitational radii of hadrons from actual experimental measurements: The mass radius (0.56-0.69fm) is obtained.

Thank you very much