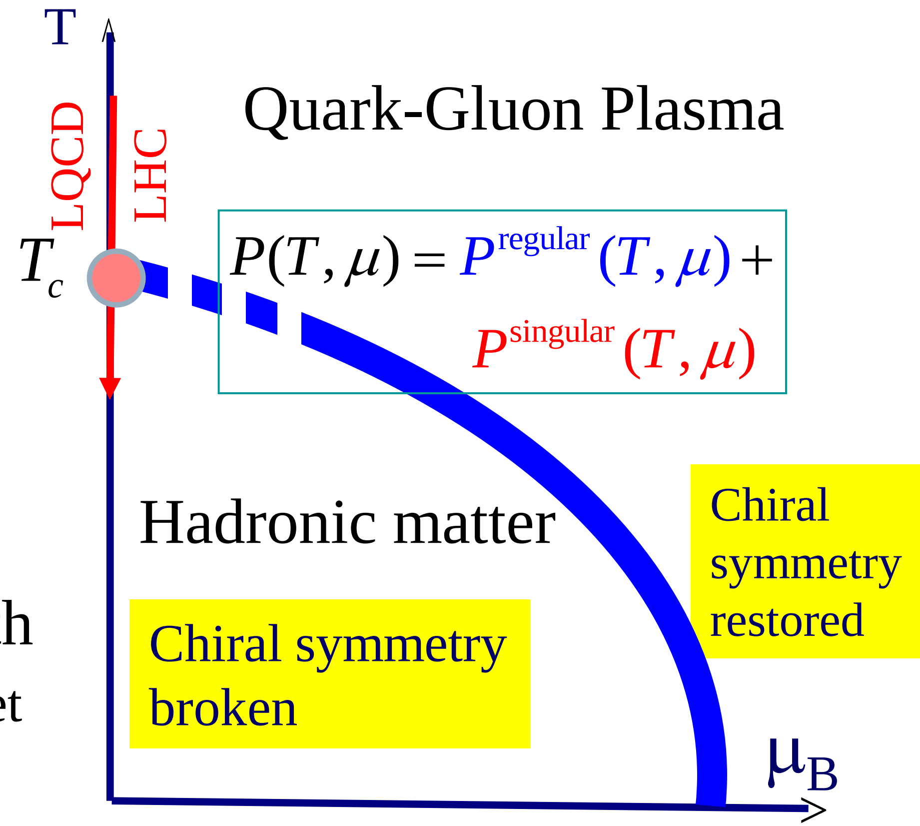
Exploring chiral symmetry restoration in heavy-ion collisions with fluctuation observables

Krzysztof Redlich (University of Wroclaw)

- Modelling regular part of pressure in hadronic phase: S-matrix approach:
  - charge-baryon correlations in LQCD
  - proton production yields at LHC
- Fluctuations of net-baryon charge:
  - probing chiral criticality systematics:
    FRG-PNJL model versus STAR data
  - decoding phase structure of QCD with a Fourier expansion coefficients of net baryon density

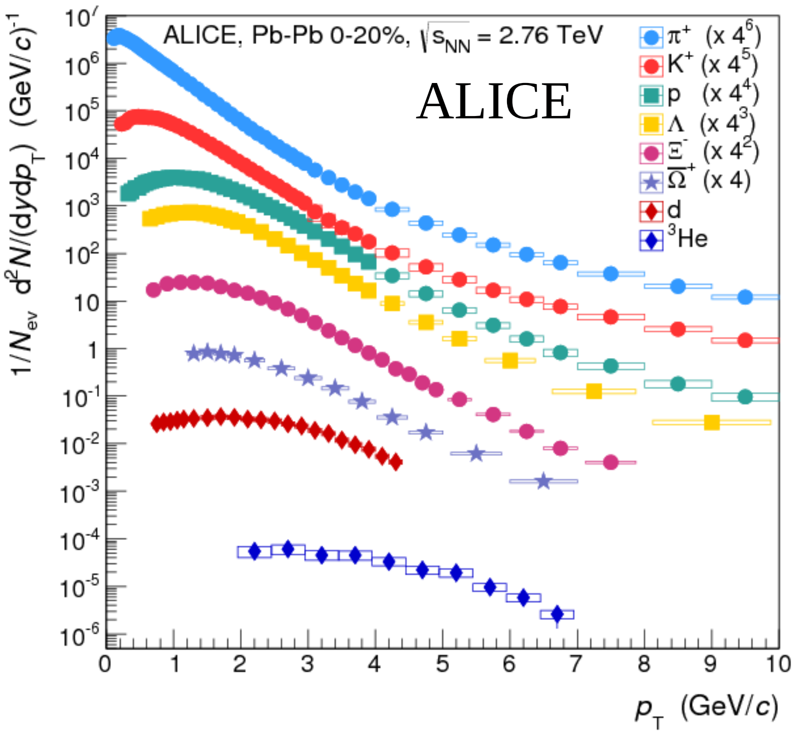


collaboration with: Gabor Almasi, Bengt Friman, Pok Man Lo, Kenji Morita, Chihiro Sasaki: Anton Andronic, Peter Braun-Munzinger, Johanna Stachel

Compare HIC data and Lattice QCD results

Can the thermal nature and composition of the collision fireball in HIC be verified ?

HIC → **Lattice QCD**



- The strategy:
  - Compare directly measured fluctuations and correlations with LGT

    F. Karsch and K. R, Phys. Lett. B 695, 136 (2011)

    F. Karsch, Central Eur. J. Phys. 10, 1234 (2012)

    A. Bazavov et al., Phys. Rev. Lett. 109, 192302 (2012):
  - Construct the 2nd order fluctuations and correlations from measured yields and compare with LGT

P. Braun-Munzinger, A. Kalweit, J. Stachel, K.R. Phys. Lett. B 47, 292 (2015), Nucl.Phys. A956, 805 (2016)

Consider fluctuations and correlations of conserved charges to be compared with LQCD

Excellent probe of:

- QCD criticality

  A. Asakawa at. al.

  S. Ejiri et al.,…

  M. Stephanov et al.,

  K. Rajagopal et al.

  B. Frimann et al.

- freezeout conditions in HIC

F. Karsch &

S. Mukherjee et al.,

C. Ratti et al.

P. Braun-Munzinger et al.

- They are quantified by susceptibilities:

  If $P(T,\mu_B,\mu_Q,\mu_S)$ denotes pressure, then

$$\frac{\chi_N}{T^2} = \frac{\partial^2(P)}{\partial(\mu_N)^2} \qquad \frac{\chi_{NM}}{T^2} = \frac{\partial^2(P)}{\partial\mu_N\partial\mu_M}$$

$$N = N_q - N_{-q}, \quad N,M = (B,S,Q), \quad \mu = \mu/T, \quad P = P/T^4$$

- Susceptibility is connected with variance

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(<N^2> - <N>^2)$$

- If $\mathrm{P}(N)$ probability distribution of N then

$$<N^n> = \sum_N N^n \mathrm{P}(N)$$

Consider special case:

$$< N_q > \equiv N_q$$ =>

Charge carrying by particles $q = \pm 1$

- Charge and anti-charge uncorrelated and Poisson distributed, then
- $P(N)$ the Skellam distribution

$$P(N) = \left(\frac{N_q}{N_{-q}} \right)^{N/2} I_N(2\sqrt{N_q N_{-q}}) \exp[-(N_q + N_{-q})]$$

- Then, the susceptibility

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(< N_q > + < N_{-q} >)$$

expressed by yields of particles and antiparticles carrying the conserved charge |q|.

Consider special case: particles carrying $q = \pm 1, \pm 2, \pm 3$

- The probability distribution

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

$< S_{-q} > \equiv S_{-q}$

$q = \pm 1, \pm 2, \pm 3$

$$P(S) = \left(\frac{S_1}{S_{\bar{1}}}\right)^{\frac{S}{2}} \exp\left[\sum_{n=1}^{3}\left(S_n + S_{\bar{n}}\right)\right]$$

$$\sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left(\frac{S_3}{S_{\bar{3}}}\right)^{\frac{k}{2}} I_k\left(2\sqrt{S_3 S_{\bar{3}}}\right)\left(\frac{S_2}{S_{\bar{2}}}\right)^{\frac{i}{2}} I_i\left(2\sqrt{S_2 S_{\bar{2}}}\right)$$

$$\left(\frac{S_1}{S_{\bar{1}}}\right)^{-i-\frac{3k}{2}} I_{2i+3k-S}\left(2\sqrt{S_1 S_{\bar{1}}}\right)$$

Fluctuations

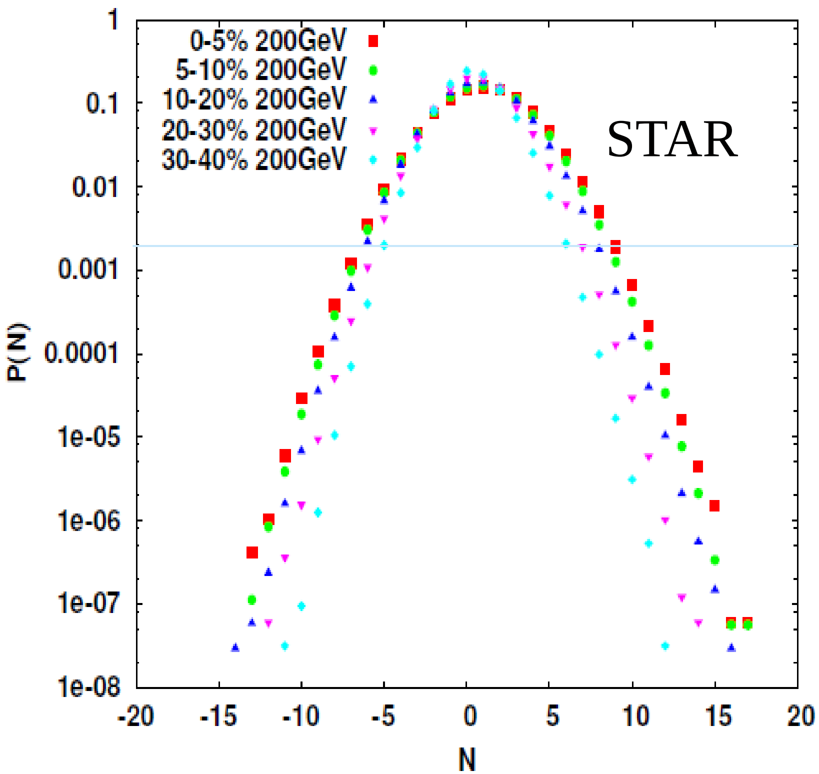
$$\frac{\chi_S}{T^2} = \frac{1}{VT^3}\sum_{n=1}^{|q|} n^2\left(\langle S_n\rangle + \langle S_{-n}\rangle\right)$$

Correlations

$$\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3}\sum_{m=-q_M}^{q_M}\sum_{n=-q_N}^{q_N} nm\langle S_{n,m}\rangle$$

$\langle S_{n,m}\rangle$ is the mean number of particles carrying charge $N = n$ and $M = m$

Variance at 200 GeV AA central coll. at RHIC

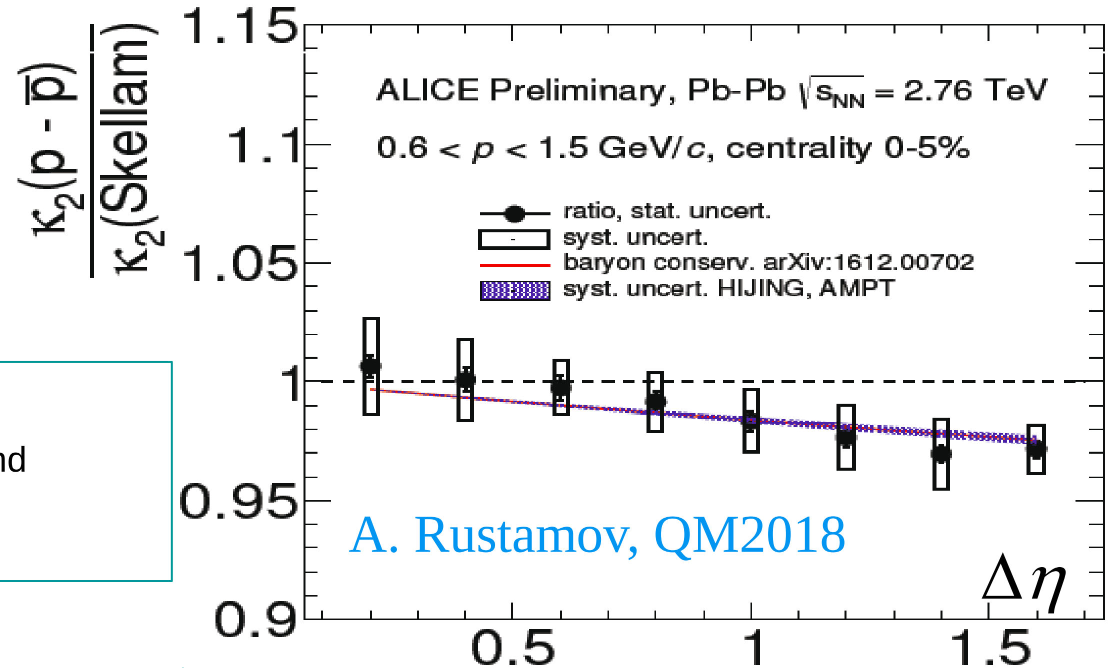


STAR Collaboration data in central coll. 200 GeV

- Consistent with Skellam distribution

$$\frac{<p>+<\bar{p}>}{\sigma^2} = 1.022 \pm 0.016 \qquad \frac{\chi_1}{\chi_3} = 1.076 \pm 0.035$$

- ALICE data consistent with Skellam $\Delta\eta < 1$

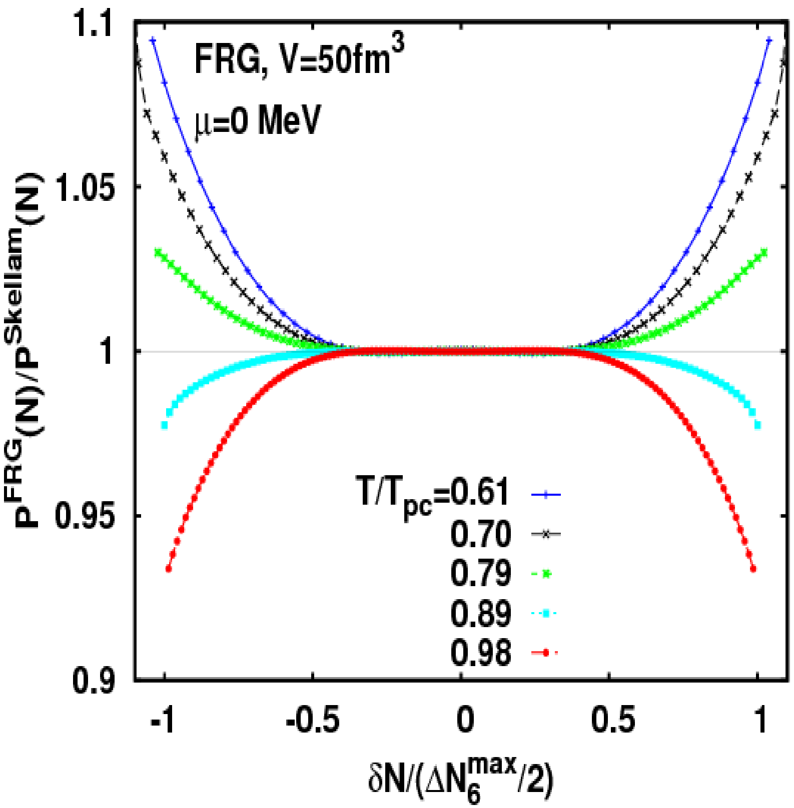


- Skellam distribution is a good approximation to calculate the 2nd order charge fluctuations in HIC

The influence of baryon number conservation: P. Braun-Munzinger, A. Rustamov, J. Stachel. Nucl Phys. A960 (2017) 114

Variance at 200 GeV AA central coll. at RHIC

K. Morita, B. Friman and K.R. Phys.Lett. B741 (2015) 178



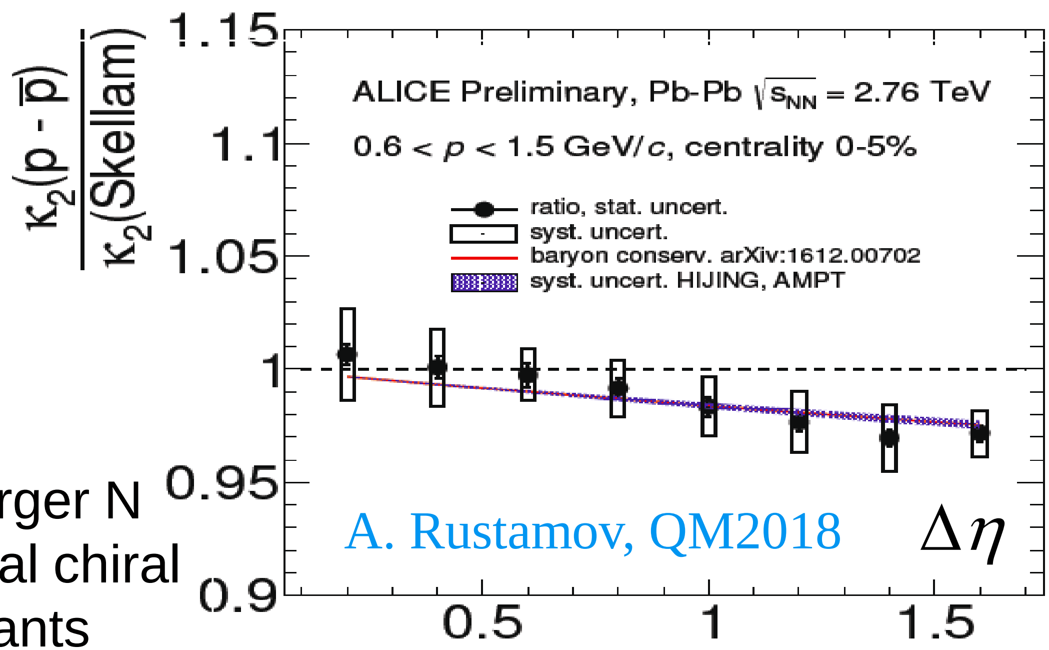
STAR Collaboration data in central coll. 200 GeV

Consistent with Skellam distribution

$$\frac{< p > + < \bar{p} >}{\sigma^2} = 1.022 \pm 0.016$$

$$\frac{\chi_1}{\chi_3} = 1.076 \pm 0.035$$

ALICE data consistent with Skellam $\Delta\eta < 1$



- Shrinking of Skellam distr. at larger N needed to capture the O(4) critical chiral properties of higher order cumulants

Variance at 200 GeV AA central coll. at RHIC

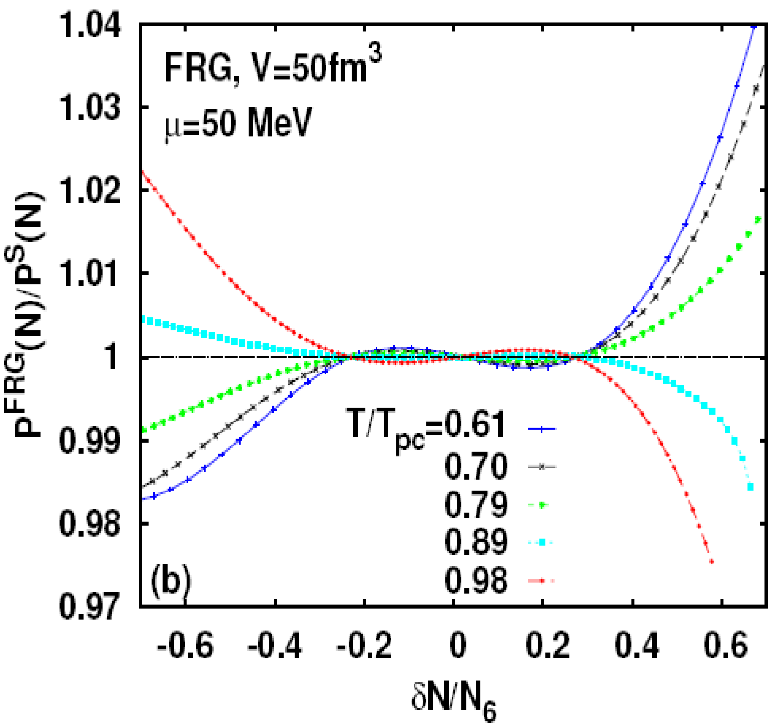
K. Morita, B. Friman and K.R. Phys.Lett. B741 (2015) 178

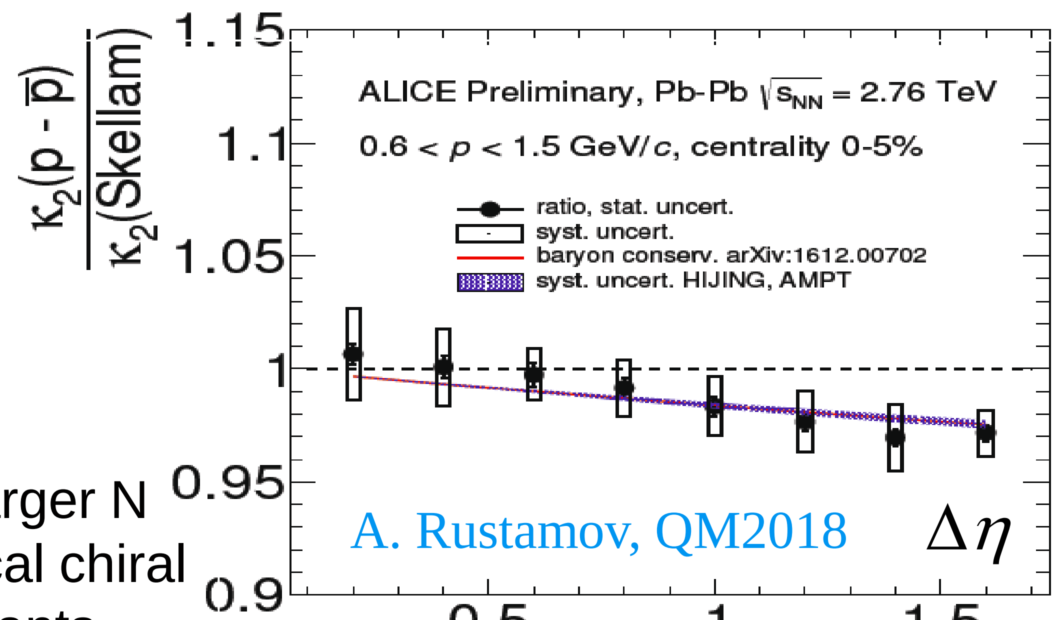
STAR Collaboration data in central coll. 200 GeV

- Consistent with Skellam distribution

$$\frac{<p>+<\overline{p}>}{\sigma^2}=1.022\pm0.016 \qquad \frac{\chi_1}{\chi_3}=1.076\pm0.035$$

- ALICE data consistent with Skellam $\Delta\eta<1$





- Shrinking of Skellam distr. at larger N needed to capture the O(4) critical chiral properties of higher order cumulants

Constructing net charge fluctuations and correlation from ALICE data

- Net baryon number susceptibility

$$\frac{\chi_B}{T^2} \approx \frac{1}{VT^3}(\langle p \rangle + \langle N \rangle + \langle \Lambda + \Sigma_0 \rangle + \langle \Sigma^+ \rangle + \langle \Sigma^- \rangle + \langle \Xi^- \rangle + \langle \Xi^0 \rangle + \langle \Omega^- \rangle + \overline{par}\ )$$

- Net strangeness

$$\frac{\chi_S}{T^2} \approx \frac{1}{VT^3}(\langle K^+ \rangle + \langle K_S^0 \rangle + \langle \Lambda + \Sigma_0 \rangle + \langle \Sigma^+ \rangle + \langle \Sigma^- \rangle + 4\langle \Xi^- \rangle + 4\langle \Xi^0 \rangle + 9\langle \Omega^- \rangle + \overline{par}$$

$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-} + \Gamma_{\varphi\to K_S^0} + \Gamma_{\varphi\to K_L^0})\langle \varphi \rangle\ )$$

- Charge-strangeness correlation

$$\frac{\chi_{QS}}{T^2} \approx \frac{1}{VT^3}(\langle K^+ \rangle + 2\langle \Xi^- \rangle + 3\langle \Omega^- \rangle + \overline{par}$$

$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-})\langle \varphi \rangle - (\Gamma_{K_0^*\to K^+} + \Gamma_{K_0^*\to K^-})\langle K_0^* \rangle\ )$$

Direct comparisons of Heavy ion data at LHC with LQCD

STAR and ALICE results => the 2nd order cumulants are consistent with Skellam distribution, thus χ_N and χ_{NM} with $N,M=\{B,Q,S\}$ are expressed by particle yields. Consider LHC data

$$\frac{\chi_B}{T^2}=\frac{1}{VT^3}(203.7\pm 11.4)$$

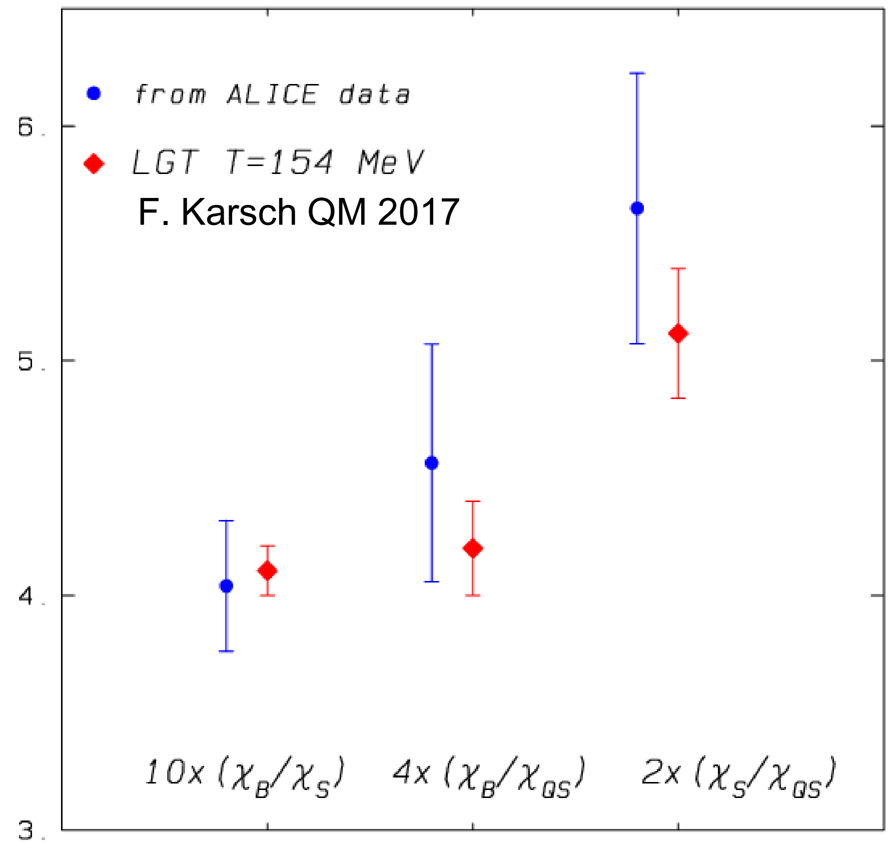
$$\frac{\chi_S}{T^2}=\frac{1}{VT^3}(504.2\pm 16.8)$$

$$\frac{\chi_{QS}}{T^2}=\frac{1}{VT^3}(191.1\pm 12)$$

- The Volume at T_c

$$V_{T_c}=3800\pm 500 \quad fm^3$$

P. Braun-Munzinger, et al.



The cumulant ratios extracted from ALICE data are consistent with LQCD at the chiral crossover: Evidence for thermalization at the phase boundary

Charge - Strangeness correlations

- The ratio

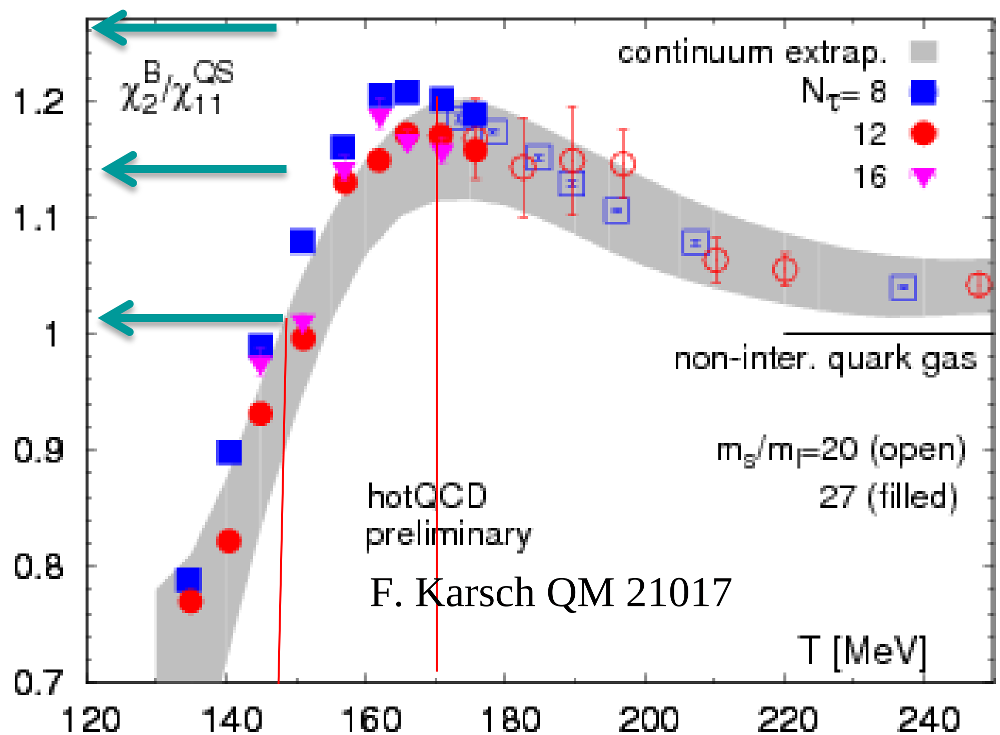
$$1.014 \le \frac{\chi_2^B}{\chi_2^{QS}} \le 1.267$$

extracted from ALICE data is consistent with LQCD for

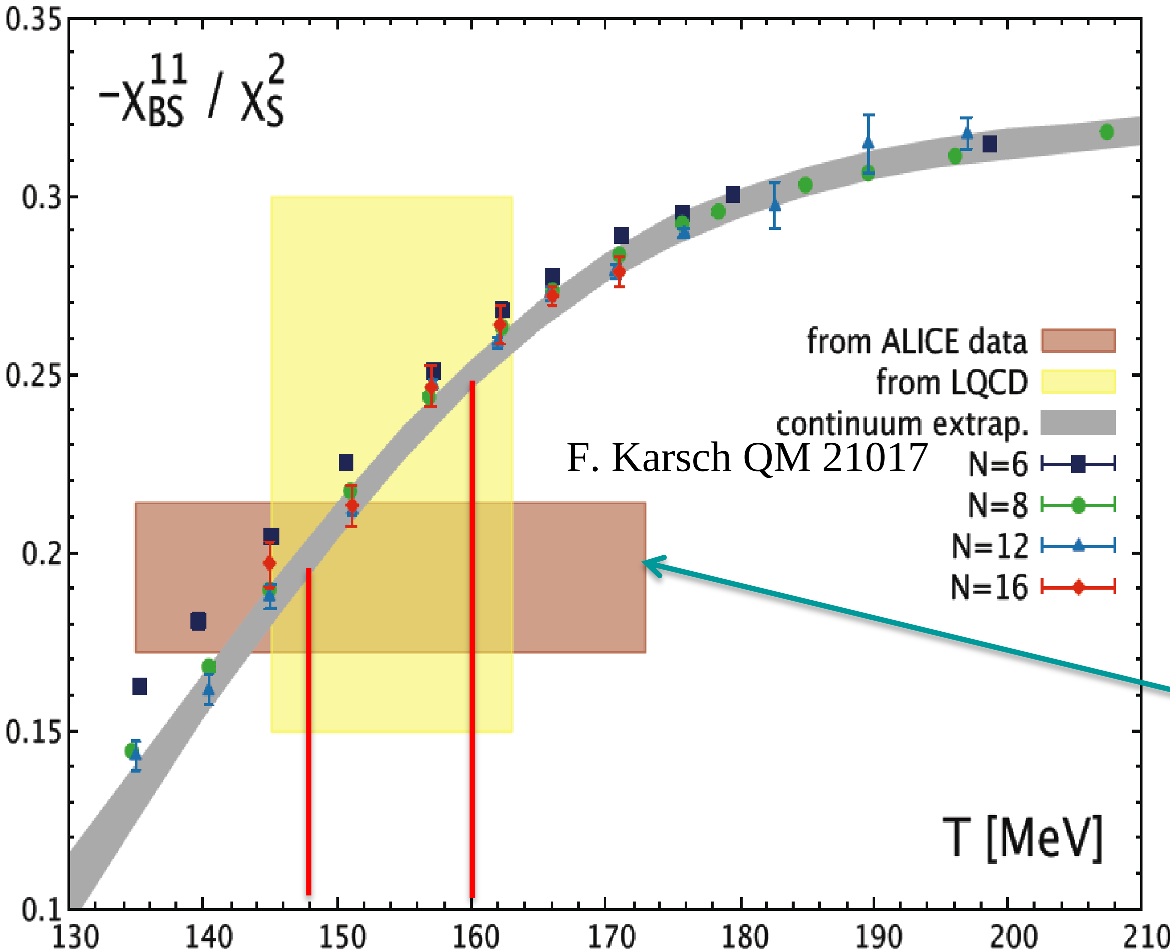
$$148 < T_f \le 170 \quad \text{MeV}$$

when combined with T_f obtained from χ_2^B / χ_2^S one concludes that, data consistent with LGT for

$$148 < T_f \le 160$$



Constraining chemical freezeout temperature at the LHC



At the LHC energy the fireball created in HIC is a QCD medium at the chiral cross over temperature.

$$C_{BS} = -\frac{<(\delta B)(\delta S)>}{<(\delta S)^2>} = -\frac{\chi_{BS}}{\chi_S}$$

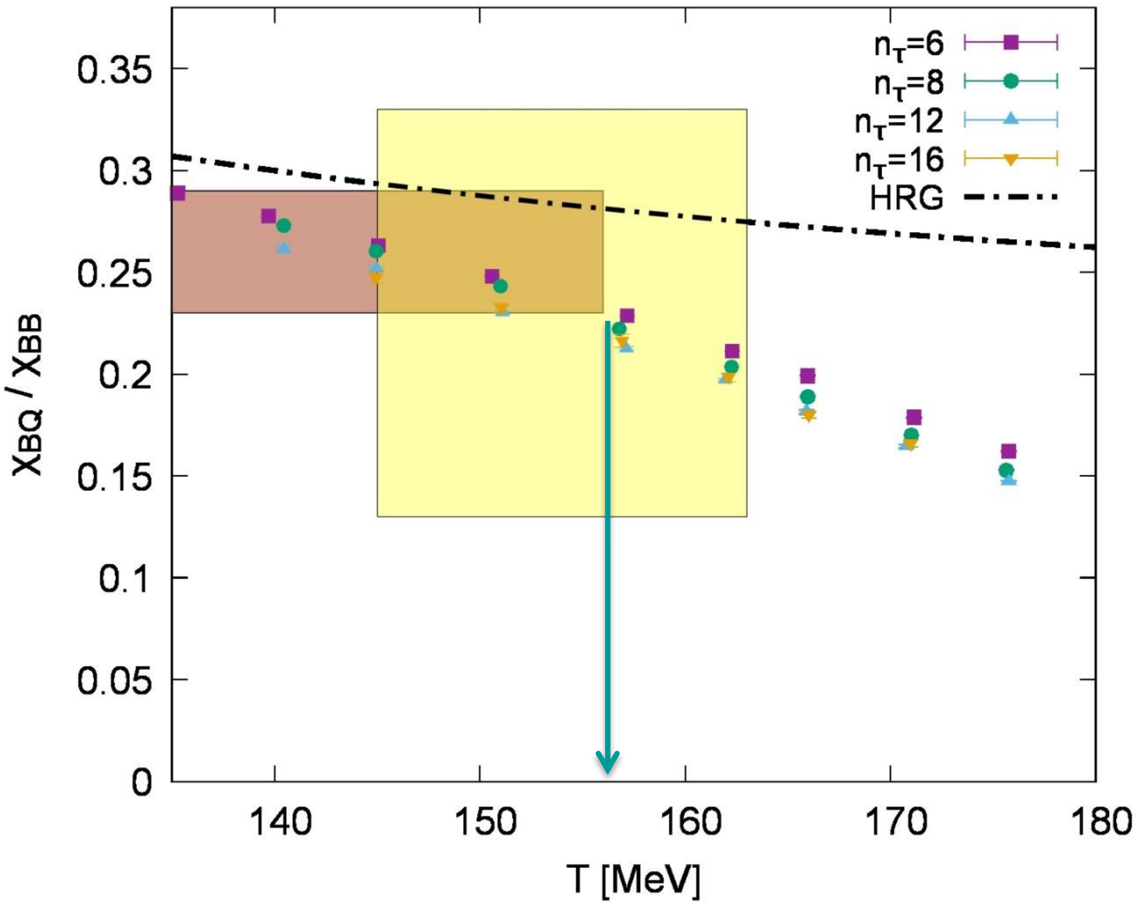
- Excellent observable to fix the temperature

$$-\frac{\chi_{BS}}{T^2} \approx \frac{1}{VT^3}[2<\Lambda+\Sigma^0>+4<\Sigma^+>$$
$$+8<\Xi>+6<\Omega^->] = (97.4 \pm 5.8)/VT^3$$

However, this is the lower limit since e.g. $\Sigma^*(\geq 1660) \to N\overline{K}$ $\Lambda^*(\geq 1520) \to N\overline{K}$ are not included

- Data on χ_B / χ_S and χ_B / χ_{QS} consistent with LQCD results for $0.148 \leq T_f < 160\ MeV$

Constraining the upper value of the chemical freeze-out temperature at the LHC



- Considering the ratio

$$\frac{<(\delta B)(\delta Q)>}{<(\delta B)^2>} = \frac{\chi_{BQ}}{\chi_B} = 0.26 \pm 0.03$$

one gets $T < 156\ MeV$

- From the comparison of 2nd order fluctuations and correlations observables constructed from ALICE data and LQCD, one gets agreement at

$$148 \leq T_f < 158\ MeV$$

Particle yields data at the LHC consistent with LQCD at the phase boundary

Thermal origin of particle yields with respect to HRG

Rolf Hagedorn => the Hadron Resonace Gas (HRG):
"uncorrelated" gas of hadrons and resonances

$$P^{regular}(T,\vec{\mu}) = \sum_H P_H^{id} + \sum_R P_R^{id}$$

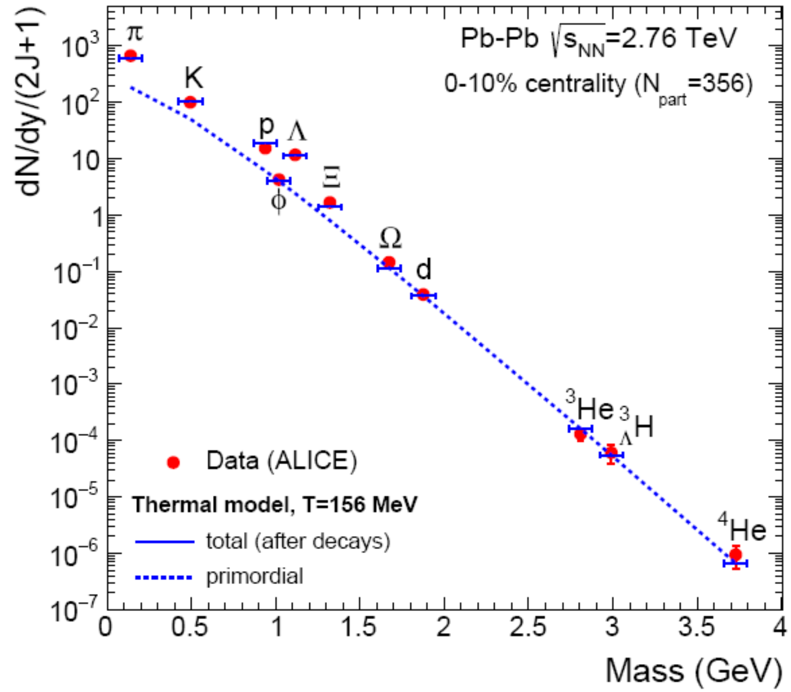
$$< N_i > = V\,[n_i^{th}(T,\vec{\mu}) + \sum_K \Gamma_{K\to i}\, n_i^{th-\mathrm{Res.}}(T,\vec{\mu})]$$

A. Andronic, Peter Braun-Munzinger, Johanna Stachel & K.R.

Particle yields with no resonance decay contributions:

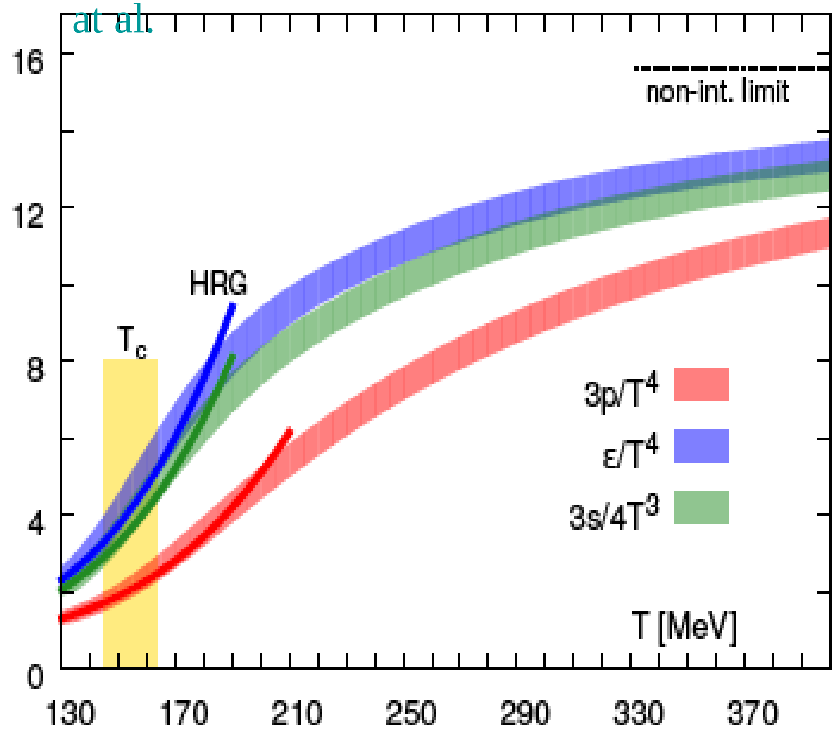
$$\frac{1}{2j+1}\frac{dN}{dy} = V(m/T)^2 K_2(m/T)$$

- Measured yields are well reproduced within HRG with $T = 156 \pm 1.5\ MeV$

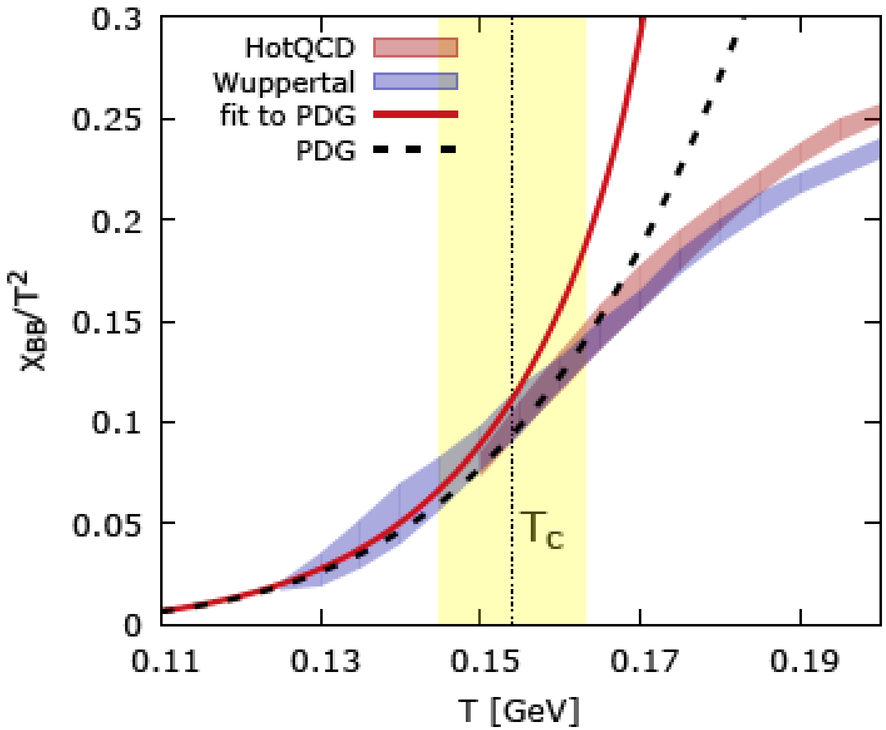


Good description of the QCD Equation of States by Hadron Resonance Gas

A. Bazavov et al. HotQCD Coll. July 2014 at al.

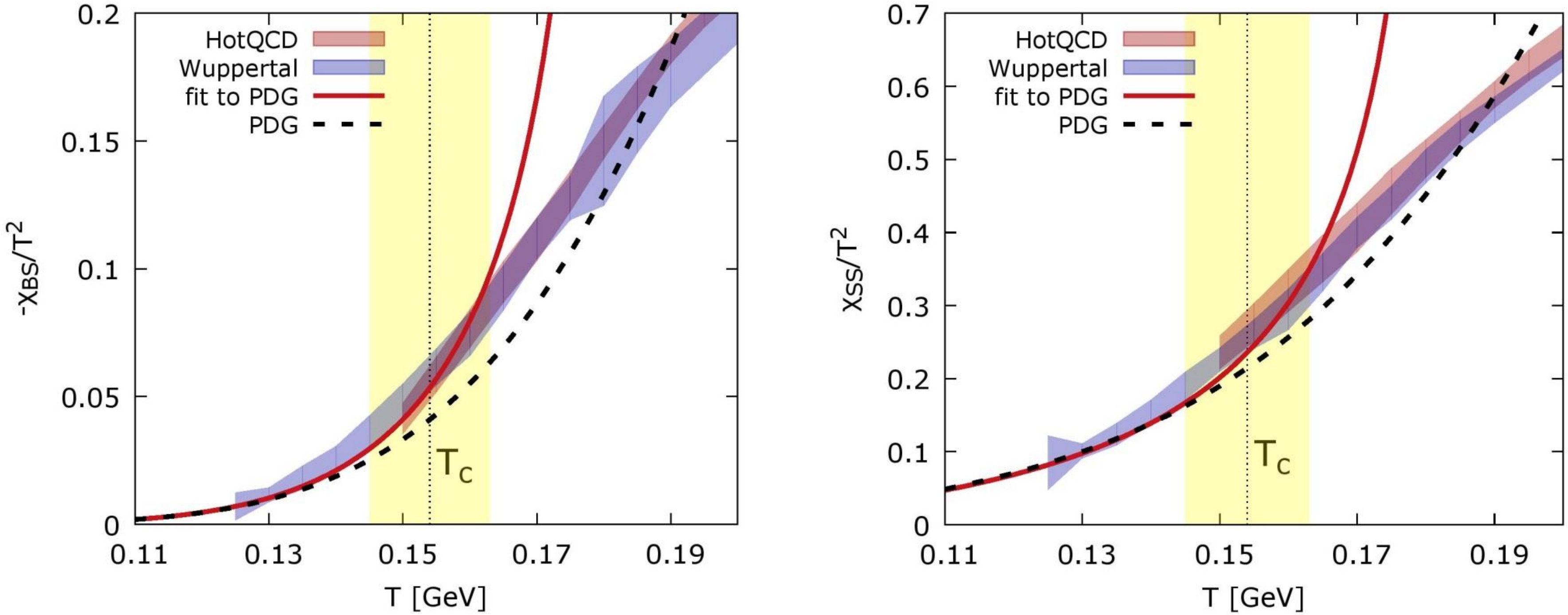


P.M. Lo, M Marczenko et al. Eur. Phys.J. A52 (2016)



- Hadron Gas thermodynamic potential provides an excellent approximation of the QCD equation of states in confined phase
- As well as, good description of the net-baryon number fluctuations which can be improved by adding baryonic resonances expected in the Hagedorn exponential mass spectrum

Hagedorn's continuum mass spectrum contribution to strangeness fluctuations



- Missing strange baryon and meson resonances in the PDG

F. Karsch, et al., Phys. Rev. Lett. 113, no. 7, 072001 (2014)

P.M. Lo, M. Marczenko, et al. Eur. Phys.J. A52 (2016)

- Satisfactory description of LGT with asymptotic states from Hagedorn's exponential mass spectrum $\rho^H(m) = (m^2 + m_0^2)^{-5/2} e^{m/T_H}$ fitted to PDG
- However, HRG provides 1st approximation of QCD free energy in hadronic phase

HRG in the S-MATRIX APPROACH

Pressure of an interacting, a+b ⇔ a+b, hadron gas in an equilibrium

$$P(T) \approx P_a^{id} + P_b^{id} + P_{ab}^{\text{int}}$$

The leading order interactions, determined by the two-body scattering phase shift, which is equivalent to the second virial coefficient

$$P^{\text{int}} = \sum_{I,j} \int_{m_{th}}^{\infty} dM \; B_j^I(M) P^{id}(T,M)$$

$$B_j^I(M) = \frac{1}{\pi} \frac{d}{dM} \delta_j^I(M)$$

Effective weight function ↓   *Scattering phase shift* ↓

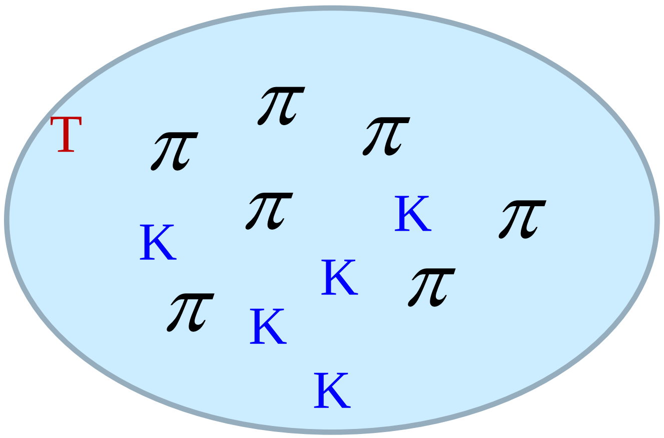
R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. 187, 345 (1969)
R.Venugopalan, and M. Prakash, Nucl. Phys. A 546 (1992) 718.
W. Weinhold,, and B. Friman, Phys. Lett. B 433, 236 (1998).
Pok Man Lo, Eur. Phys.J. C77 (2017) no.8, 533

- Interactions driven by narrow resonance of mass M_R

$$B(M) = \delta(M^2 - M_R^2) \quad \Rightarrow \quad P^{\text{int}} = P^{id}(T, M_R) \;\Rightarrow HRG$$

- For non-resonance interactions or for broad resonances the HRG is too crude approximation and $P^{\text{int}}(T)$ should be linked to the phase shifts

S-MATRIX APPROACH



- Consider interacting pions and kaons gas in thermal equilibrium at temperature T
- Due to *K*π scattering resonances are formed

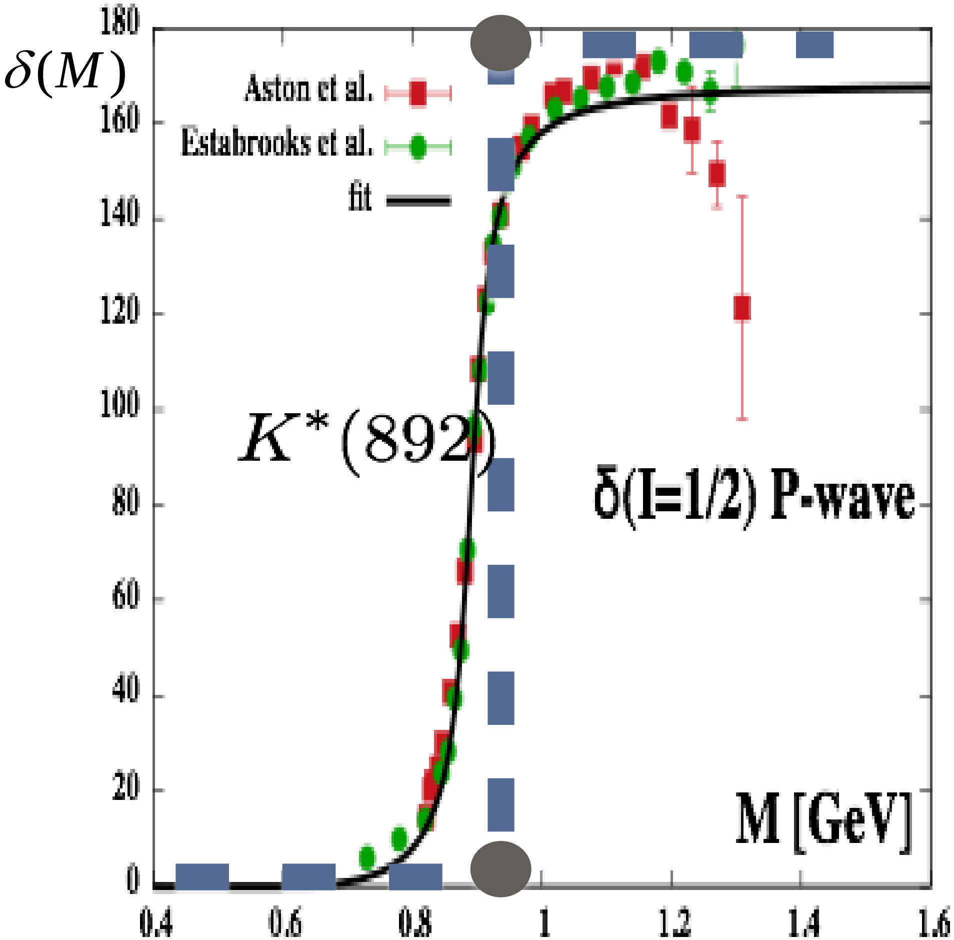
  I =1/2, s -wave : κ(800), K0*(1430) [*JP* = 0+]

  I =1/2, p -wave : *K**(892), *K**(1410), *K**(1680) [*JP* =1−]

  I =3/2 purely repulsive interactions
- In the S-matrix approach the thermodynamic pressure in the low density approximation

$$P(T) \approx P_{\pi}^{id} + P_{K}^{id} + P_{\pi K}^{\text{int}}$$

Experimental phase shift in the P-wave channel



B. Friman, P. M. Lo, M. Marczenko, K. Redlich and C. Sasaki, Phys. Rev. D 92, no. 7, 074003 (2015)

For narrow resonance

$$B(M) = 2\frac{d}{dM}\delta(M)$$

very well described by the Breit-Wigner form

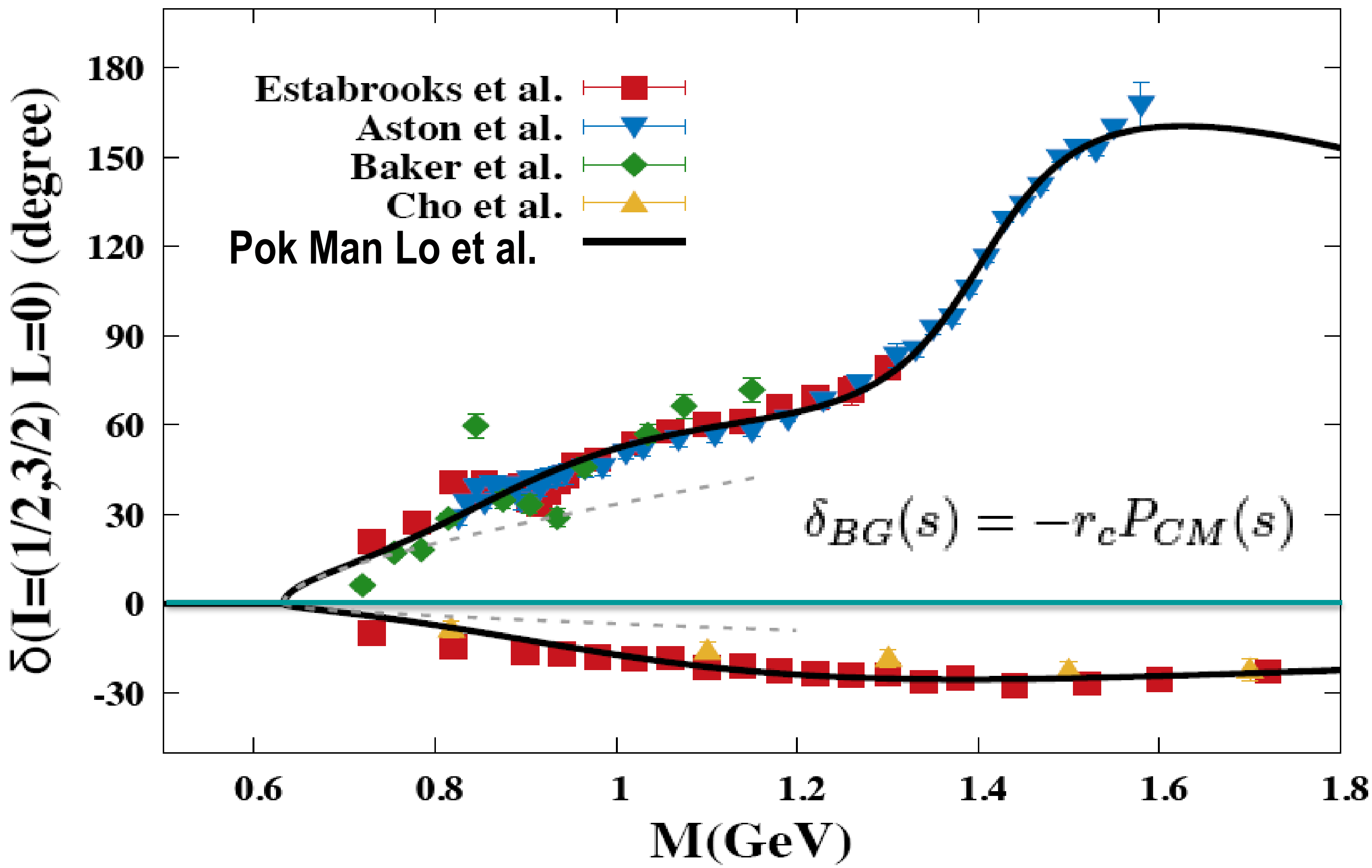
$$B(M) \approx M \frac{2M\gamma_{\rm BW}}{(M^2 - M_0^2)^2 + M^2\gamma_{\rm BW}^2}$$

for $\gamma_{BW} \to 0$

$$B(M) = \delta(M^2 - M_0^2)$$ **and**

$$P_{\pi K}^{\rm int}(T) \approx P_{K^*}^{id}(T)$$

Non-resonance contribution- negative phase shift in S-wave channel

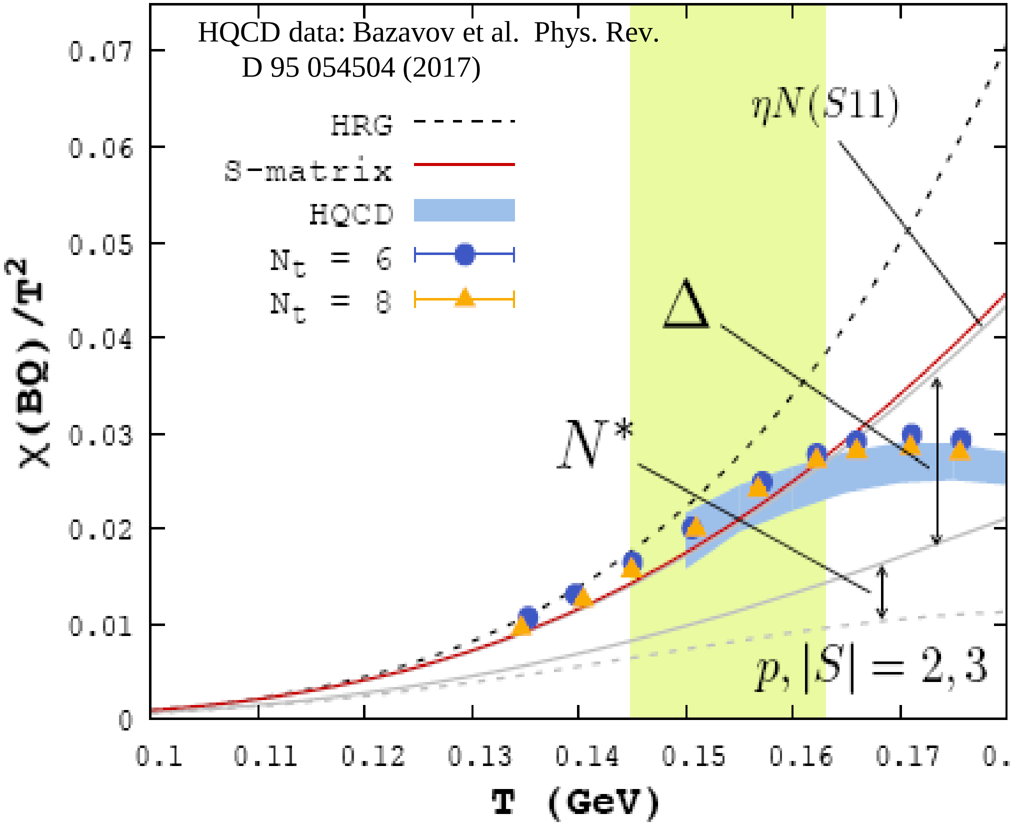


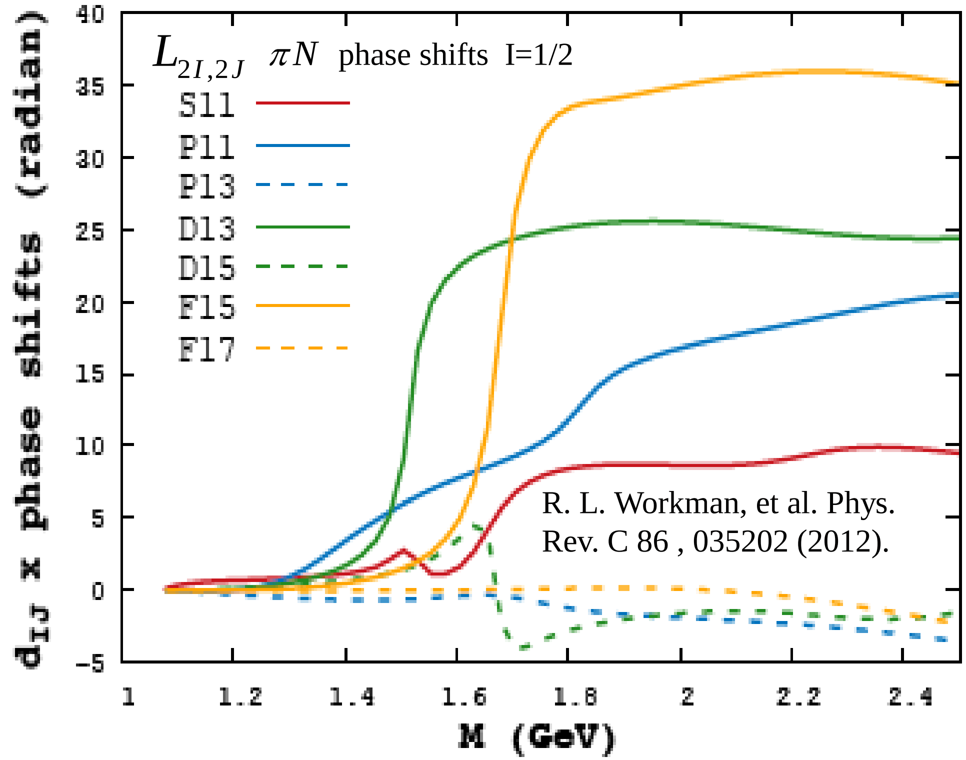
$$\delta_0^{1/2} = \delta_\kappa + \delta_{K_0^*} + \delta_{BG}.$$ ⇨ $$B(M) = 2\frac{d}{dM}\delta(M)$$ ⇨ $$\chi_{SS}(T)$$

Probing non-strange baryon sector in πN- system

Pok Man Lo, B. Friman, C. Sasaki & K.R. , Phys.Lett. B778 (2018)

$$\chi_{BQ} = (\chi_{BB} - |\chi_{BS}|)/2$$



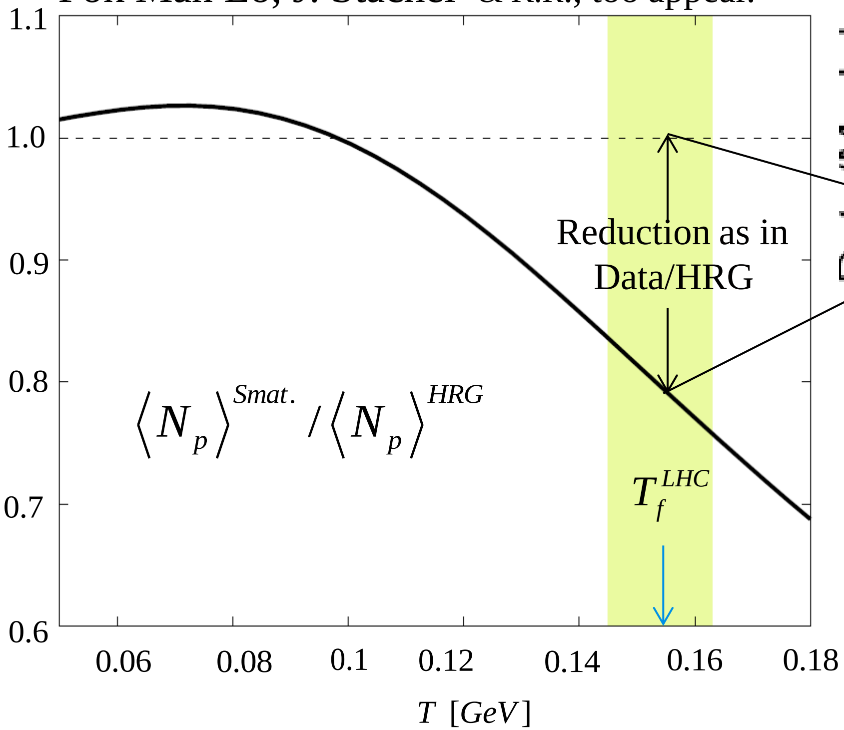


$$\Delta\chi_{BQ} \approx \sum_{I_z, j, B} d_j BQ \int dM \int d^3p \frac{1}{T} \frac{d\delta_j^I}{dM} \times e^{-\beta\sqrt{p^2+M^2}} (1+e^{-\beta\sqrt{p^2+M^2}})^{-2}$$

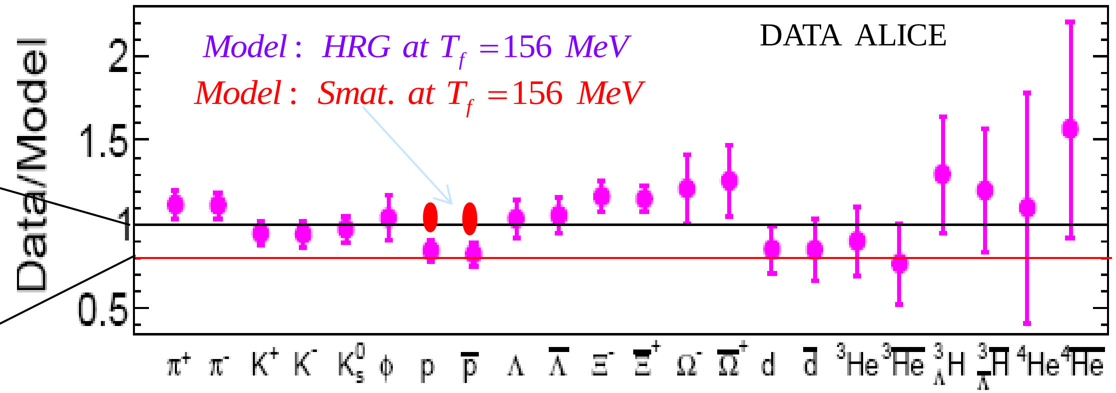
- Considering contributions of all πN $\delta_j^{I=(1/2),\ (3/2)}$ (N^*, Δ^* resonances) to χ_{BQ} within S-matrix approach, reduces the HRG predictions towards the LQCD in the chiral crossover $0.15 < T < 0.16\ GeV$

Phenomenological consequences: proton production yields

A. Andronic, P. Braun-Munzinger, B. Friman, Pok Man Lo, J. Stachel & K.R., too appear.

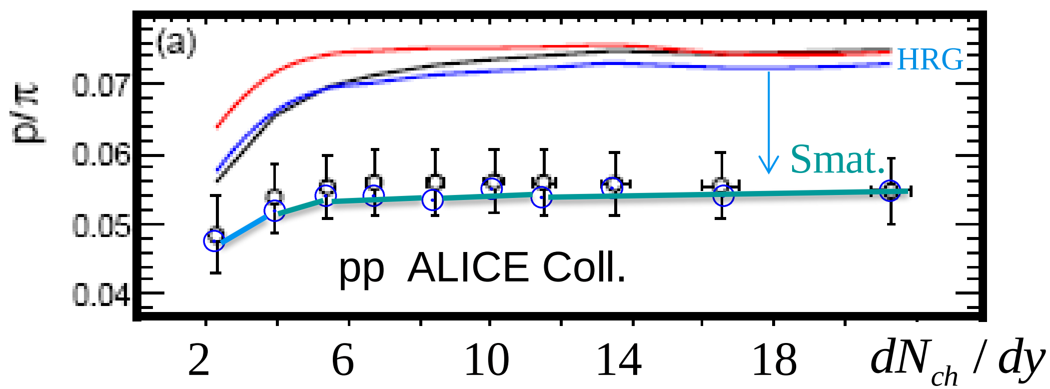


HRG: A. Andronic, P. Braun-Munzinger, J. Stachel & K.R.



- Yields of protons in AA collisions at LHC is consistent with S-matrix result within 1σ

HRG: N. Sharma, J. Cleymans, B. Hippolite, arXiv: 1803.05409



- S-matrix results well consistent with pp data

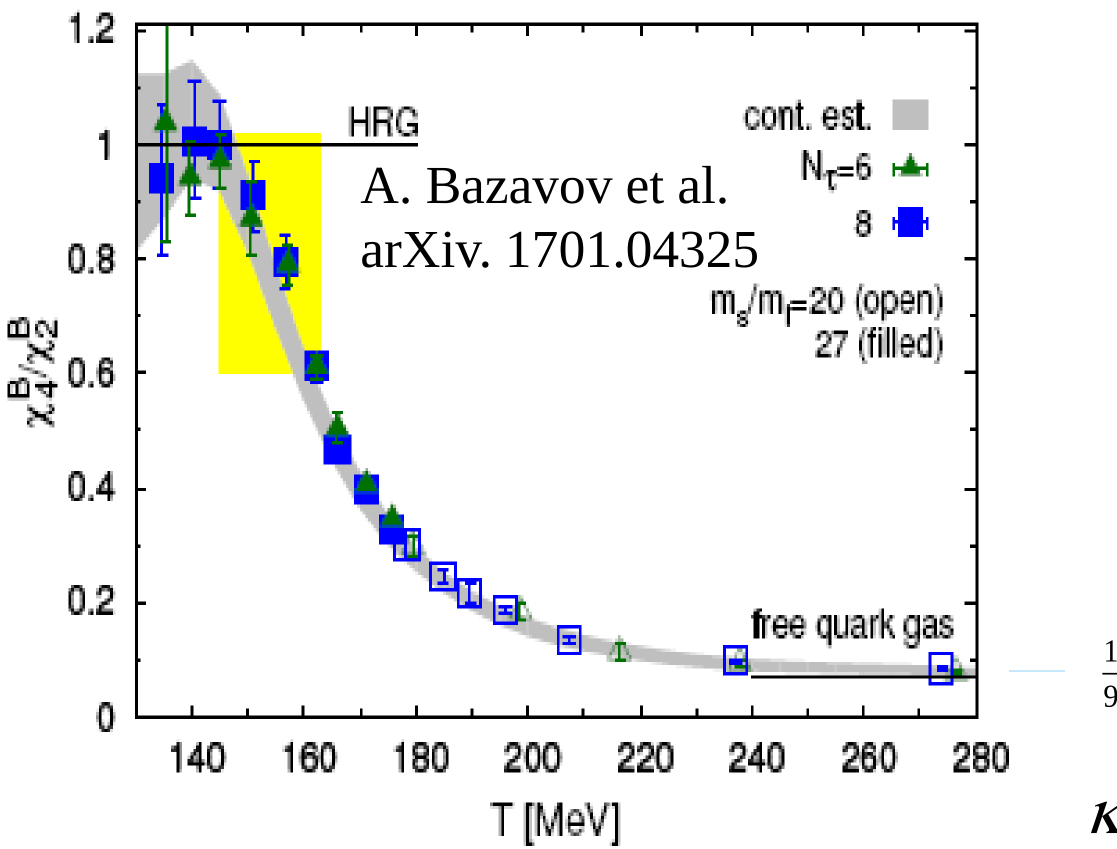
- Yields of protons in the S-matrix is suppressed relative to HRG

For further consequences of smat. See also: P. Huovinen, P. Petreczky Phys. Lett. B77 (2018) P. Huovinen, poster QM2018

Deviations of Fluctuations of net charges

due to deconfinement and partial chiral symmetry restoration in QCD

$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$$



- HRG factorization of pressure:

$$P^B(T,\mu_q) = F(T)\cosh(B\mu_B/T)$$

- Kurtosis measures the squared of the baryon number carried by leading particles in a medium

S. Ejiri, F. Karsch & K.R. Phys.Lett. B633 (2006) 275
S. Ejiri et al., Nucl.Phys.Proc.Suppl. 140 (2005) 505 , Phys.Rev. D71 (2005) 054508

$$\kappa\,\sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{pmatrix}$$

Modelling $P^{s-gular}(T,\mu_B)$ in the O(4)/Z(2) universality class

Gabor Almasi, Bengt Friman & K.R., Phys. Rev. D96 (2017) no.1, 014027

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - g(\sigma + i\gamma_5\vec{\tau}\vec{\pi}) - g_\omega\gamma^\mu\omega_\mu\right)q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - U_m(\sigma,\vec{\pi}) - \mathcal{U}(\Phi,\bar{\Phi};T) - \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Effective potential is obtained by solving *the exact flow equation* (Wetterich eq.) with the approximations resulting in the O(4)/Z(2) critical exponents

$$\partial_k\Omega_k(\sigma) = \frac{Vk^4}{12\pi^2}\left[\sum_{i=\pi,\sigma}\frac{d_i}{E_{i,k}}[1+2n_B(E_{i,k})] - \frac{2\nu_q}{E_{q,k}}[1-n_F(E_{q,k}^+)-n_F(E_{q,k}^-)]\right]$$

$$E_{\pi,k} = \sqrt{k^2+\Omega_k'}$$

$$E_{\sigma,k} = \sqrt{k^2+\Omega_k'+2\rho\Omega_k''}$$

$$E_{q,k} = \sqrt{k^2+2g^2\rho}$$

$$\Omega_k' \equiv \frac{\partial\Omega_k}{\partial(\sigma^2/2)}$$

Full propagators with k < q < Λ

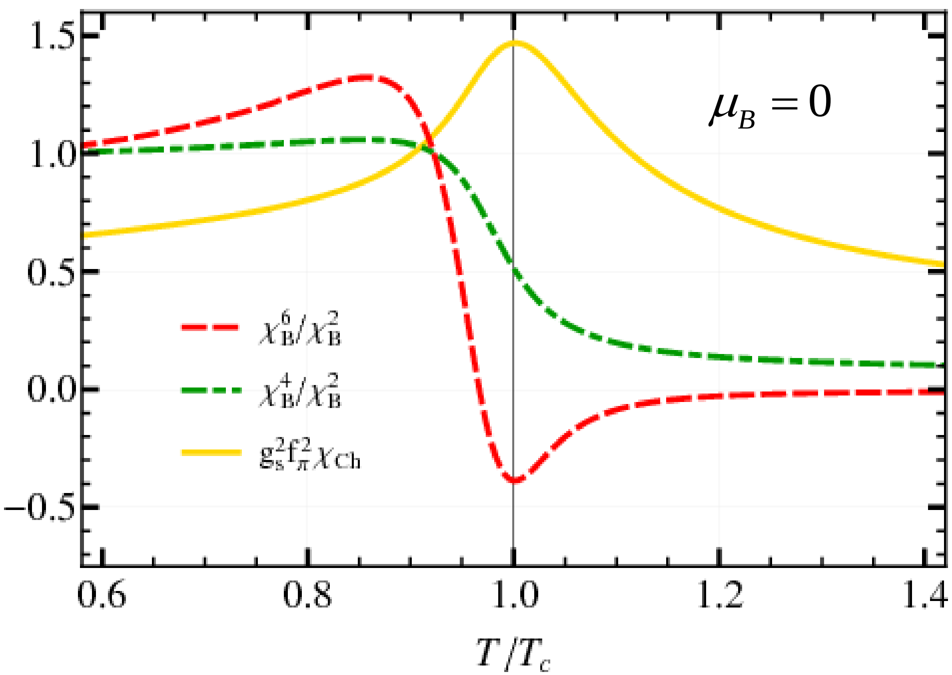
Γ_Λ=S classical

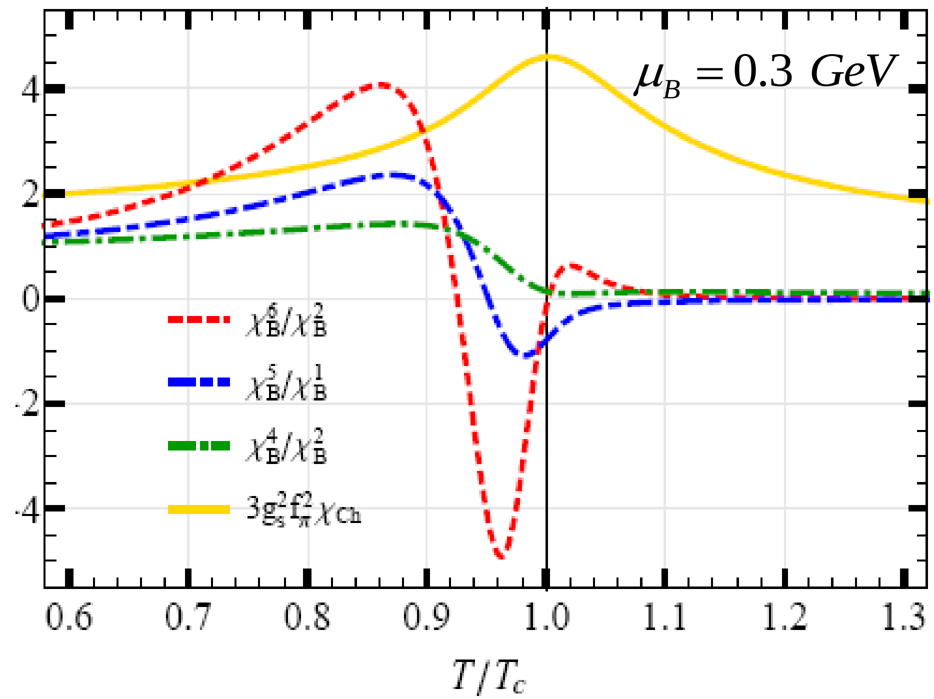
Integrating from k=Λ to k=0 gives full quantum effective potential

Higher order cumulants in effective chiral model within FRG approach, belongs to the O(4)/Z(2) universality class

B. Friman, V. Skokov &K.R. Phys. Rev. C83 (2011) 054904

G. Almasi, B. Friman &K.R. Phys. Rev. D96 (2017) no.1, 014027

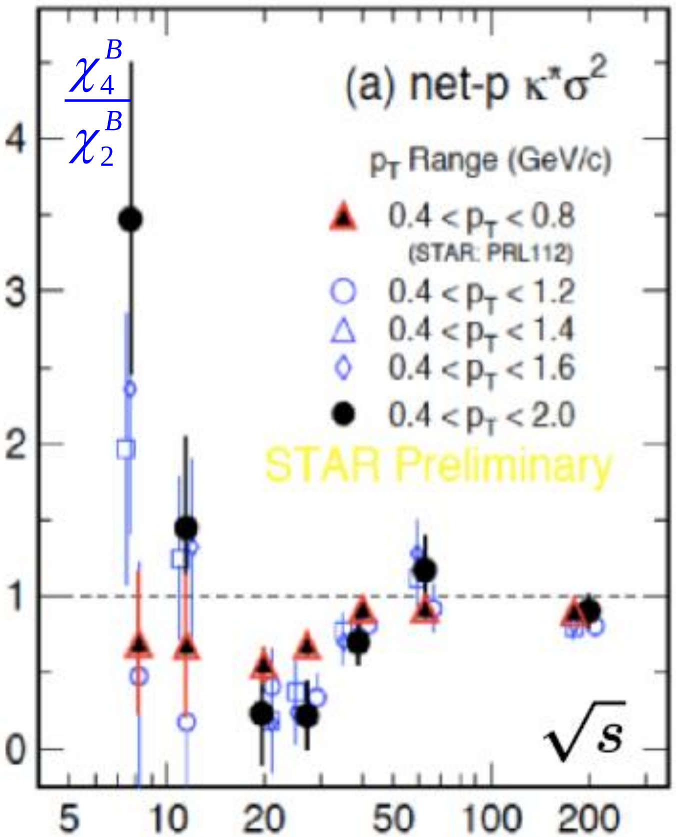




Deviations of cumulant ratios from Skellam distribution are increasing with the order of the cumulants and can be used to identify the chiral QCD phase boundary in HIC

Net-baryon fluctuations as a probe of chiral criticality

X. Luo et al. (2015), STAR Coll.



G. Almasi, B. Friman & K.R, Phys. Rev. D96 (2017) 014027

- An excellent observable of the chiral criticality

$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} \quad \text{and} \quad R^{n,m} = \frac{\chi_n^B}{\chi_m^B}$$

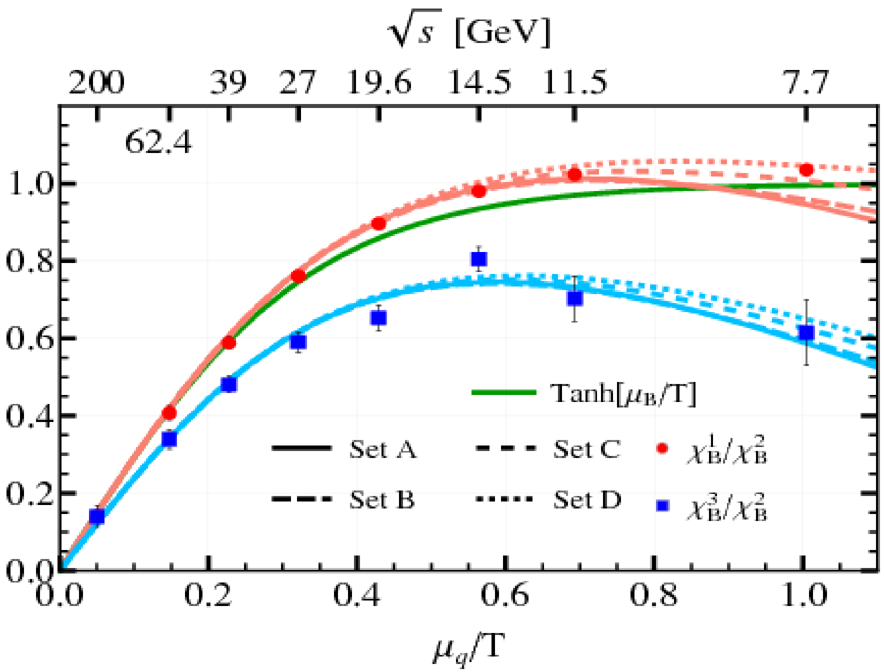
- Modelling chiral properties of QCD in PNJL model within FRG approach.

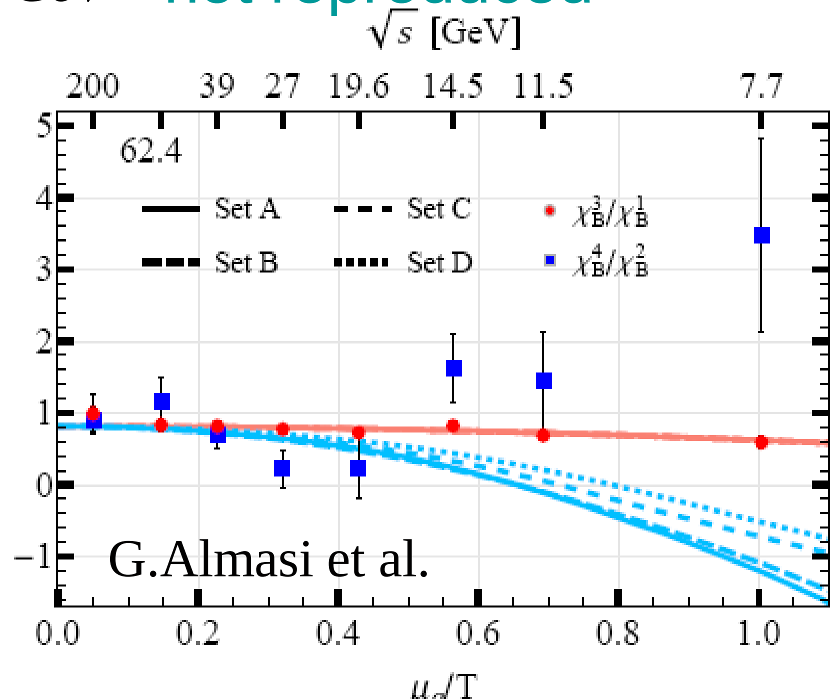
Consider systematics of $R^{n,m}$ in relation to STAR data

Are the above deviations an indication of the chiral criticality and the existence of the CEP?

Self - consistent freeze-out and STAR data

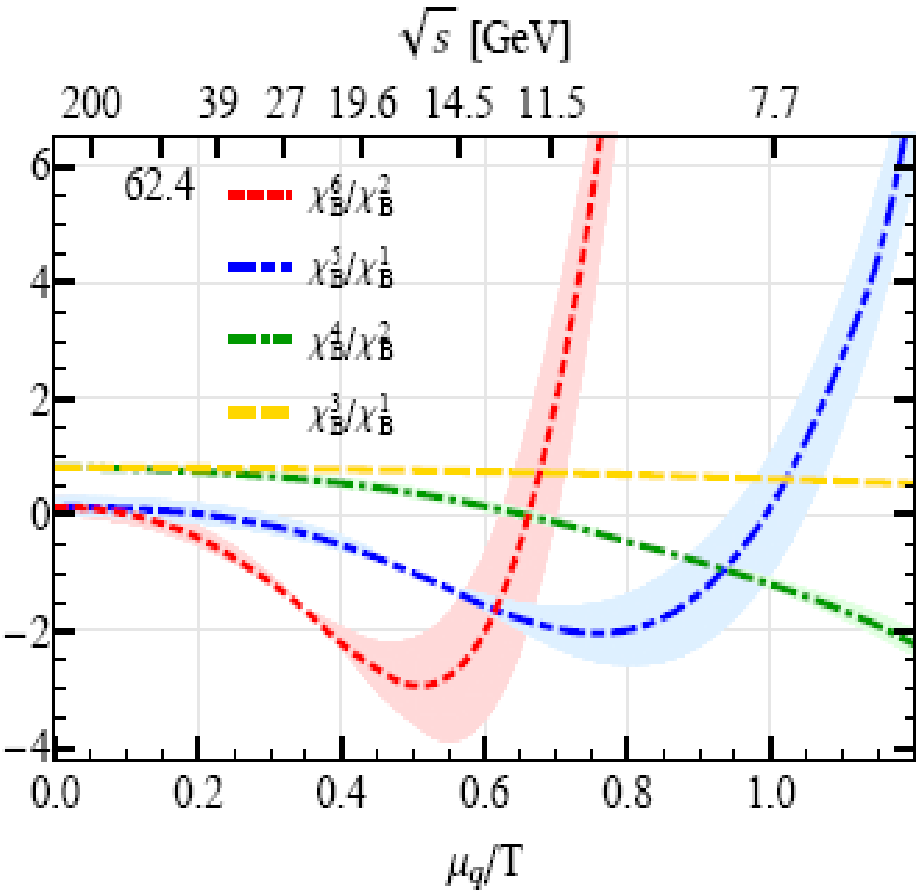
- Freeze-out line in $(T,\mu)-$ plain is fixed by χ_B^3/χ_B^1 to data
- Ratio $\chi_B^1/\chi_B^2 \approx \tanh(\mu/T)$ => further evidence of equilibrium and thermalisation at $7\ GeV \le \sqrt{s} < 5\ TeV$
- Ratio $\chi_B^1/\chi_B^2 \neq \chi_B^3/\chi_B^2$ expected due to critical chiral dynamics
- Enhancement of χ_B^4/χ_B^2 at $\sqrt{s} < 20\ GeV$ not reproduced



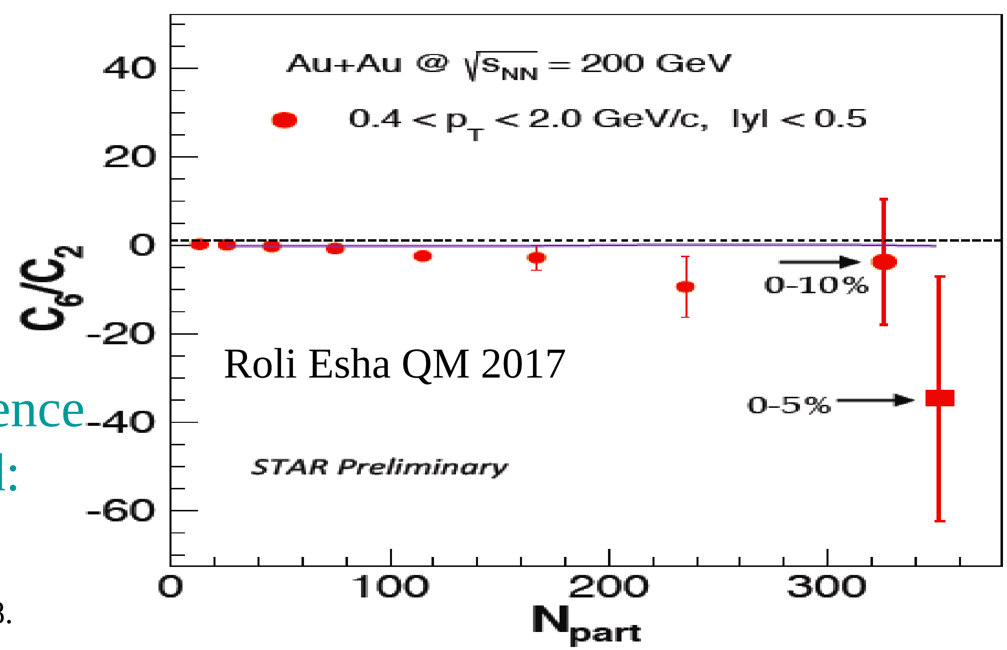


Similar conclusions as in the previous comparison of LQCD results with STAR data:
Frithjof Karsch J. Phys. Conf. Ser. 779, 012015 (2017)

Higher order cumulants - energy dependence



- Strong non-monotonic variation of higher order cumulants at lower $\sqrt{s}$
- Equality of different ratios excellent probes of equilibrium evolution in HIC
- At freeze-out , the ratio $\chi_B^6 / \chi_B^2 \approx 0$ in agreement with preliminary STAR data, albeit within still very large error



However, to make final conclusions the influence of non-critical fluctuations must be analyzed:

See e.g. P. Braun-Munzinger, A. Rustamov and J. Stachel Nucl. Phys. A 960, 114 (2017),
A. Bzdak, V. Koch, V. Skokov, Eur.Phys.J. C77 (2017) 288.

M. Kitazawa et al. (2015,16,17)

Fourier coefficients of $\chi_B^1(T,\mu)$ and chiral criticality

G. Almasi, B. Friman, P.M. Lo, K. Morita & K.R. arXiv: **1805.04441**

- Considering the Fourier series expansion* of baryon density

$$\chi_B^1(T,\mu)=\sum_{k=1}^{\infty} b_k(T)\sinh(k\mu) \quad \text{with} \quad b_k(T)=\frac{2}{\pi}\int_0^{\pi} d\theta[\mathrm{Im}\,\chi_B^1(T,i\theta)]\sin(k\theta)$$

and $\mu=(\mu/T)$, $\theta=\mathrm{Im}\,\mu$

- At $\mu=0$,the susceptibility $\chi_B^n(T)$ expressed by Fourier coefficients

$$\chi_B^n(T,\mu)=\sum_{k=1}^{\infty} b_k(T)\frac{\partial^{n-1}}{\partial\mu^{n-1}}\sinh(k\mu), \quad \text{thus} \quad \chi_B^n(T,\mu=0)=\sum_{k=1}^{\infty} k^{2n-1} b_k(T)$$

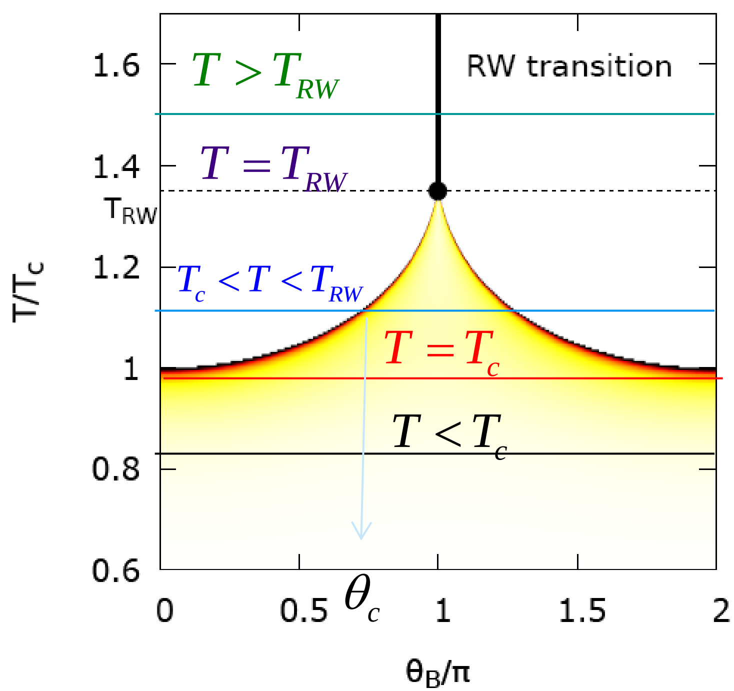
- Since $b_k(T)$ are carrying information on chiral criticality, thus their $T-$ and $k-$ dependence must inform about phase transition

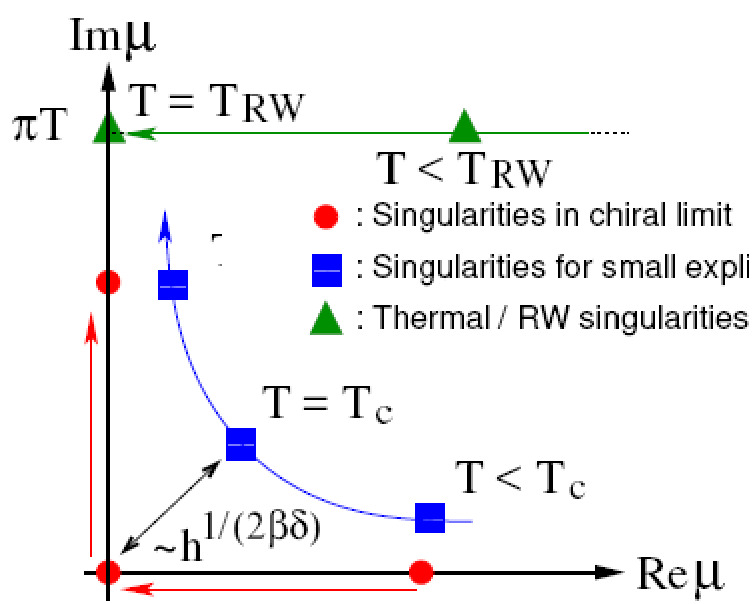
* The first four $b_k(T)$ obtained recently in LQCD: V. Vovchenko, A. Pasztor, Z. Fodor, S. D. Katz, and H. Stoecker, Phys. Lett. B **775**, 71 (2017).

* see also K. Kashiwa and A. Ohnishi (2017) hep-1712.06220, for $b_k(T)$ properties related with deconfinement transition

* Modelling $b_k(T)$: V. Vovchenko, Jan Steiheimer et al. (2017), 1711.01261

Chiral limit: scaling of $b_k(T)$





- $T < T_c$, $P(T,\mu)$ dominated by

  $P(T,\mu) \approx f(m)\cosh(\mu)$, thus

  exponential damping ⇨ $b_k(T) \approx K_2(km)$

- $T = T_c$, $P(T,\mu)$ dominated by $P^{\sin gular}$

  $\chi_1^B \approx \theta \left| \frac{T - T_c}{T_c} - \kappa\theta^2 \right|^{1-\alpha}$ ⇨ $b_k(T_c) \approx k^{2\alpha-4}$

- $T_c < T < T_{RW}$

  ⇨ $b_k(T > T_c) \approx k^{\alpha-2}\sin(k\theta_c - \alpha\pi/2)$

- $T = T_{RW}$, $P^{\sin gular}$ in Z(2) universality class

  $\chi_1^B \approx (\pi - \theta_B)^{1/\delta}$ ⇨ $b_k(T_{RW}) \approx (-1)^{k-1} k^{-1-1/\delta}$

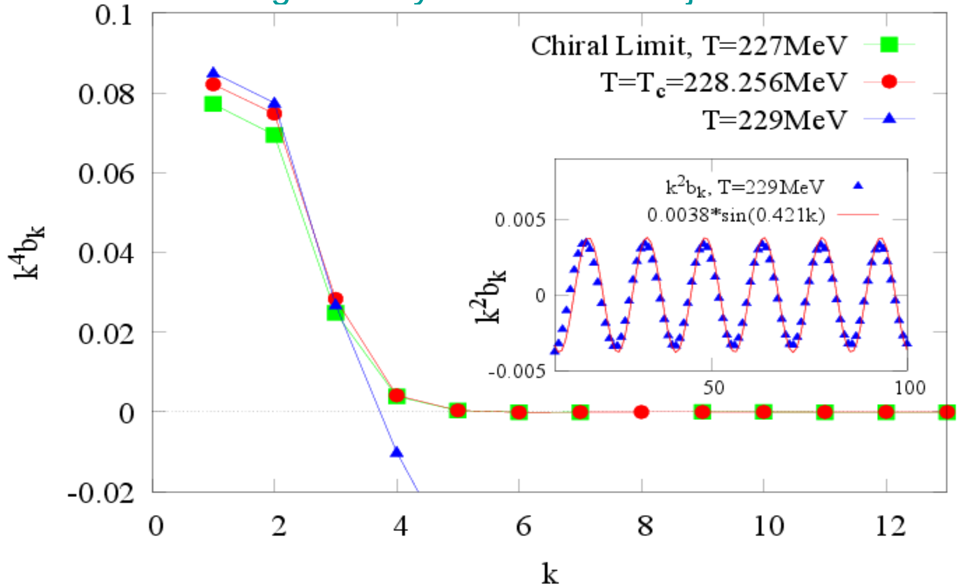
- $T > T_{RW}$, 1st order transition at $\theta = \pi$.

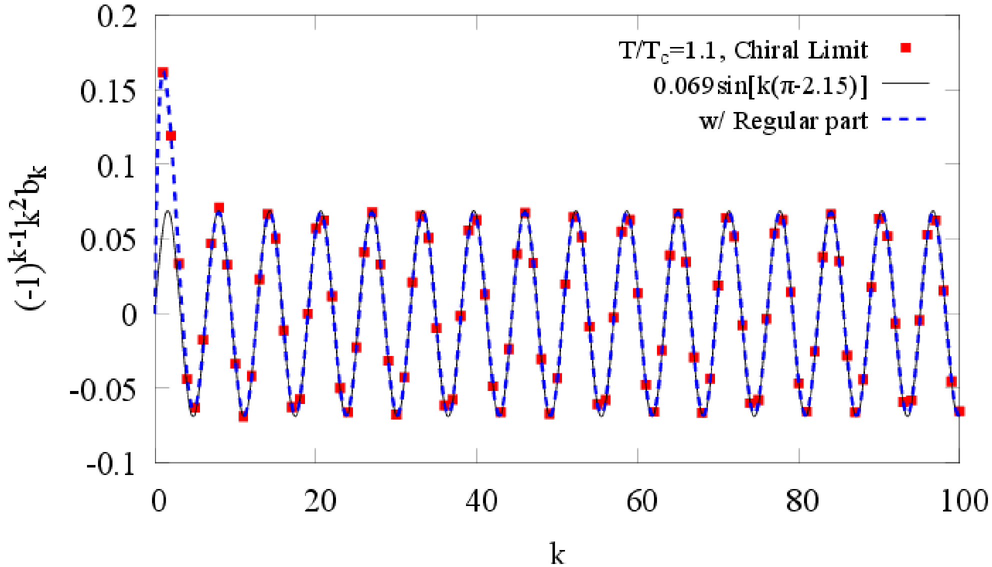
  ⇨ $b_k(T) \approx (-1)^{k-1} k^{-1}$

Scaling of Fourier coefficients: PNJL MF-results

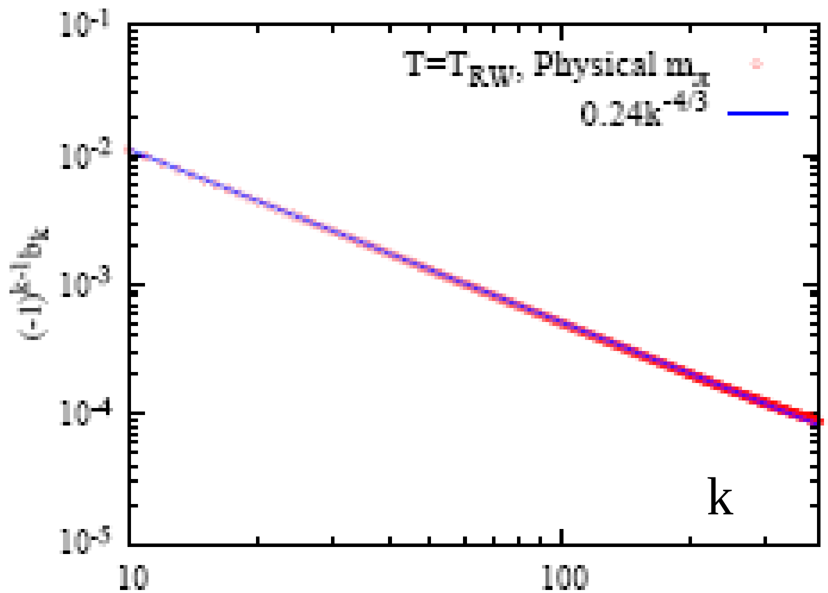
K. Morita et al.

- In the chiral limit, the phase transition is signaled by oscillations of just above



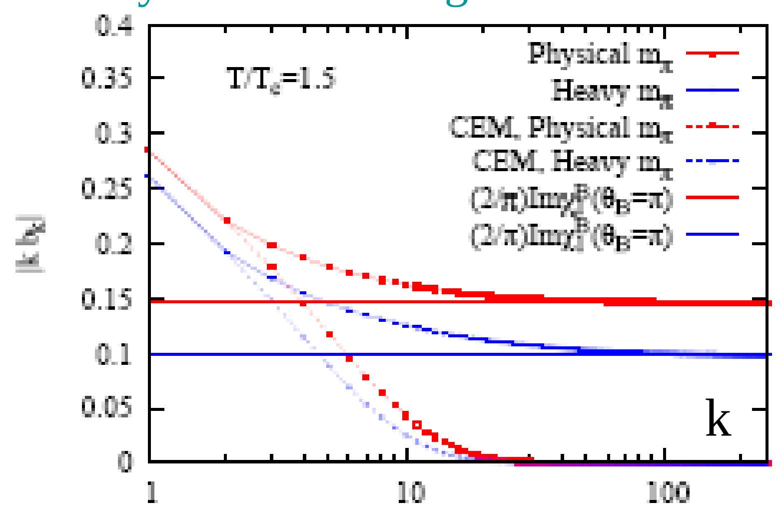


Scaling at the Roberge-Weiss transition

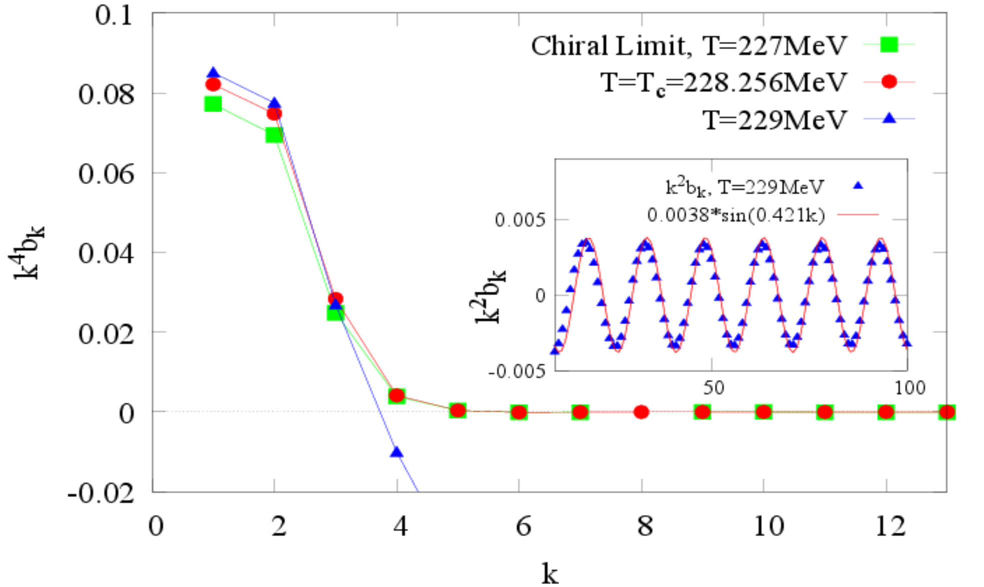


T_c

beyond the Roberge-Weiss transition

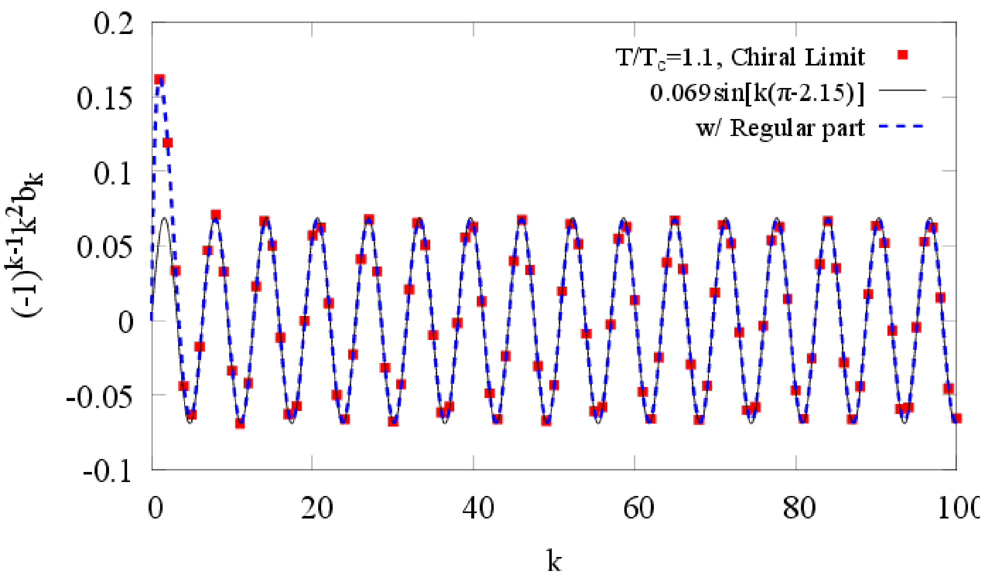


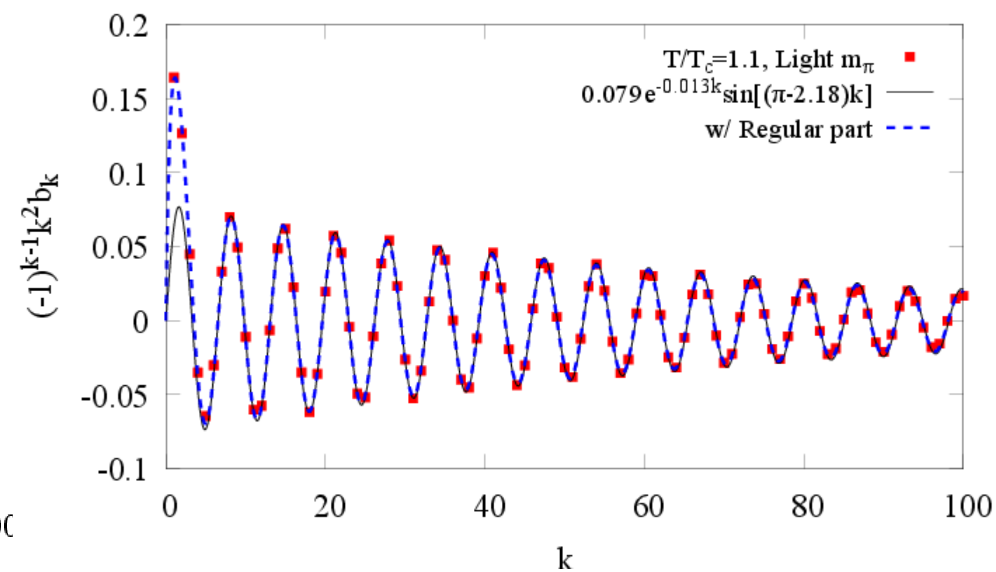
Scaling of Fourier coefficients: PNJL MF-results



- In the chiral limit, i.e. $m_\pi = 0$, the phase transition is signaled by oscillations of $b_k(T)$ just above T_c
- For $m_\pi > 0$ the singularity moves to the complex $\mu -$ plain resulting in an additional dumping of oscillations

$$b_k \simeq k^{-2} e^{-k \operatorname{Re} \mu_c(m_\pi, T)} \sin(k\theta_c)$$



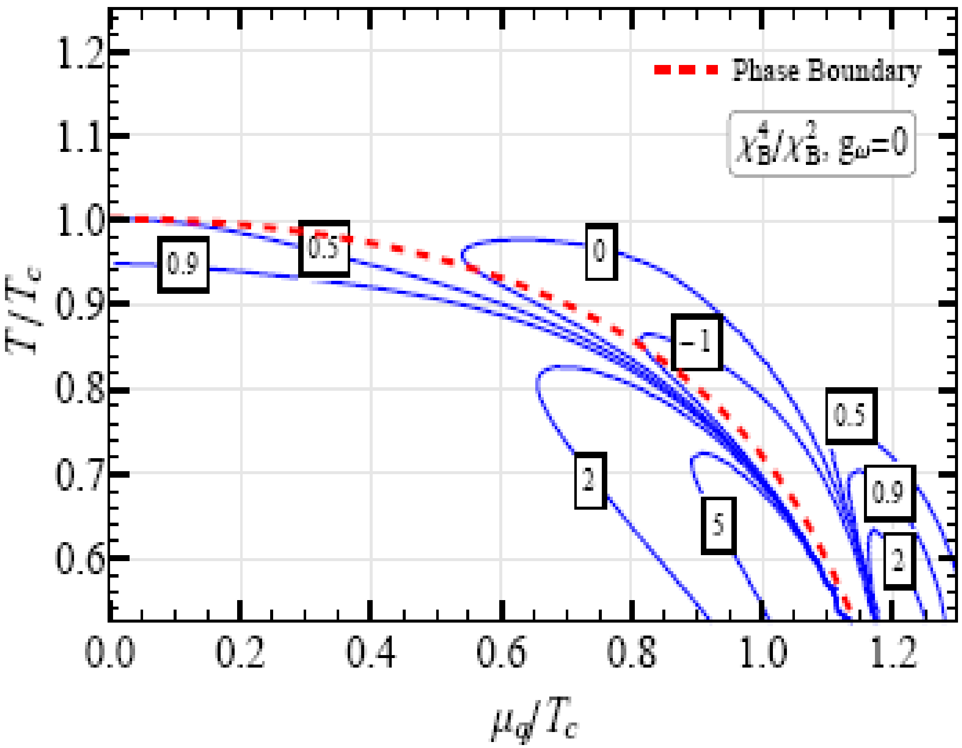


Conclusions:

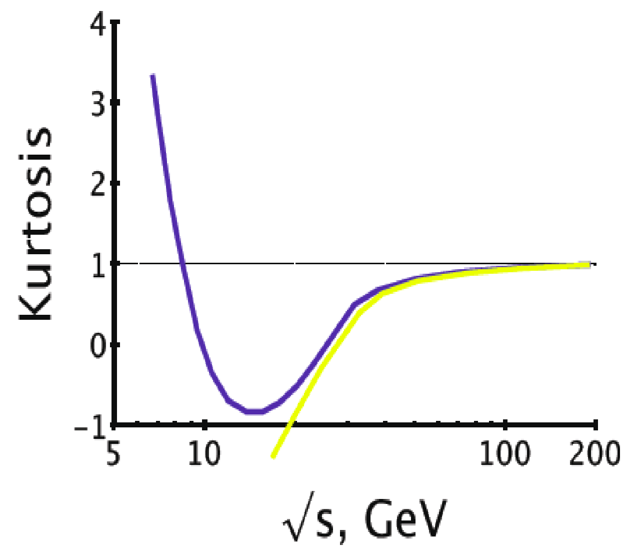
- The S-matrix approach to hadron gas thermodynamics with empirical scattering phase shifts provides consistent description of LQCD results on the 2^{nd} fluctuations and correlations in the chiral crossover, and particle production yields in AA and pp collisions at the LHC
- Systematics of net-proton number fluctuations at $\sqrt{s} > 20\ GeV$ measured by STAR in HIC at RHIC is qualitatively consistent with the expectation, that they are influenced by the critical chiral dynamics and deconfinement,

  however, possible contributions to fluctuation observables from effects not related to critical phenomena have to be understood
- The Fourier expansion coefficients of baryon density exhibit rich structure to probe the QCD phase diagram and chiral criticality in the complex chemical potential

Modelling critical fluctuations



- It is possible to find the freeze-out line such that kurtosis exhibits the energy dependence as seen in data.



- However, are other cumulants consistent?