Tribaryon configuration and the inevitable three nucleon repulsion at short distance

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Attempts to understand the static energy of the 6 quark (2-baryon) and 9 quark (3-baryon) configuration in a constituent quark model and their possible relation to nuclear force at short distance

Ref: Aaron Park, Woosung Park, SHL: 1801.10350, + in preparation

I. Few words on “Multiquark states”

II. Constituent quark model, Multiquark states and Short distance 2-N static energy

III. Tribaryons and short distance 3-N static energy

Few words on “Multiquark states”

X(3872)

- 2003 -

$B^{\pm} \rightarrow K^{\pm}\pi^{+}\pi^{-}J/\psi$

$M = 3872.0 \pm 0.6 \pm 0.5\,\text{MeV}$

X(3872) $I^G(J^{PC}) = 0^+(1^{++})$

Mass $m = 3871.69 \pm 0.17$ MeV

$m_{X(3872)} - m_{J/\psi} = 775 \pm 4$ MeV

$m_{X(3872)} - m_{\psi(2S)}$

Full width $\Gamma < 1.2$ MeV, CL = 90%

Z(4430)

- 2007 -

$B \rightarrow K\pi^{\pm}\psi'$

$M = 4433 \pm 4 \pm 2\,\text{MeV}$

$\Gamma = 45^{+18}_{-13}(\text{stat})^{+30}_{-13}(\text{syst})\,\text{MeV}$

- 2014 -

Spin parity = 1+

$\eta_G = \eta_C(-1)^I$

G=+ → will look at C=-

Pentaquark - Pc

- 2015 -

$$\Lambda_b^0 \to J/\psi\ p K^-$$

$$S = 3/2 \begin{cases} M_1 = 4380 \pm 8 \pm 29\,\mathrm{MeV} \\ \Gamma_1 = 205 \pm 18 \pm 86\,\mathrm{MeV} \end{cases} \qquad S = 5/2 \begin{cases} M_2 = 4449.8 \pm 1.7 \pm 2.5\,\mathrm{MeV} \\ \Gamma_2 = 39 \pm 5 \pm 19\,\mathrm{MeV} \end{cases}$$

Baryon with ccu

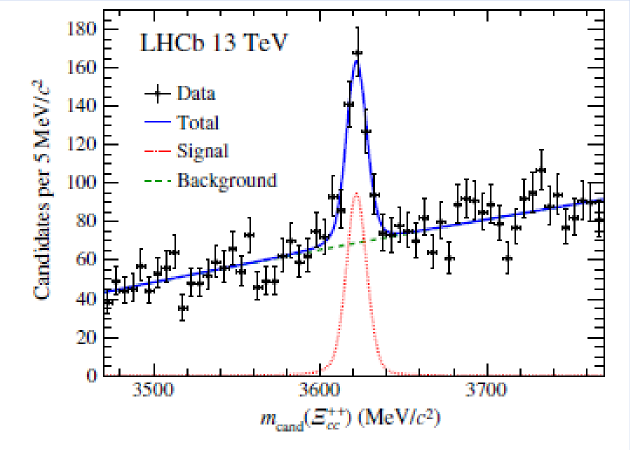
PRL119 (2017)112001

- 2017 -

$$\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$$

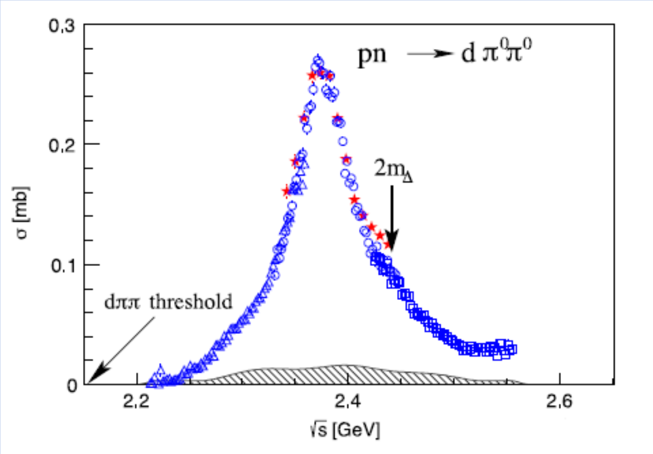
$$m_{\Xi_{cc}} - m_{\Lambda_c} = 1334.94 \pm 0.72 \pm 0.27\ \mathrm{MeV}$$

$$m_{\Xi_{cc}} = 3621.40 \pm 0.72 \pm 0.27 \pm 0.14 \left(\Lambda_c^+\right) \mathrm{MeV}$$



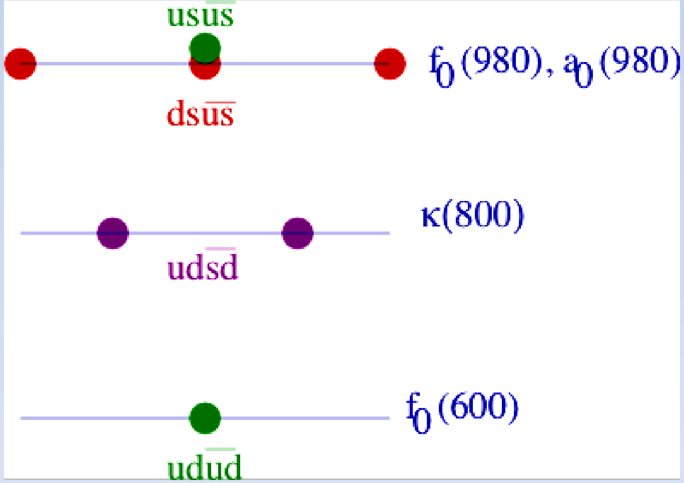
d*(2380) $I(J^P) = 0(3^+)$ Γ = 70 MeV

- WASA-at-COSY [H. Clement]-

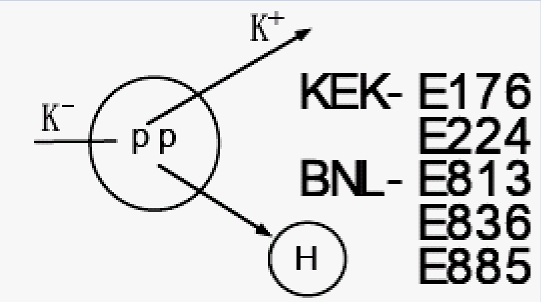


Revival of an old topic

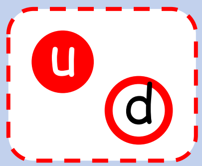
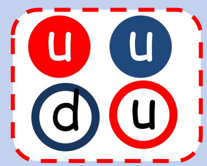
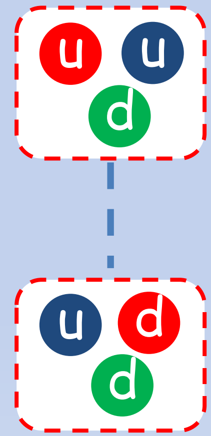
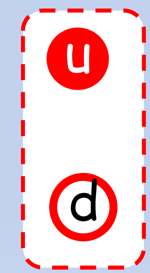
Scalar tetraquark (Jaffe 76)



Search for H dibaryon



Normal meson, compact multiquark, molecules, resonances

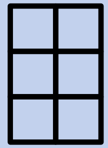
	Normal meson	Compact multiquark	Molecules	Resonance
Geometrical configuration				
Examples	Nucleon, pion, kaon	?	X(3872)	K*, rho meson

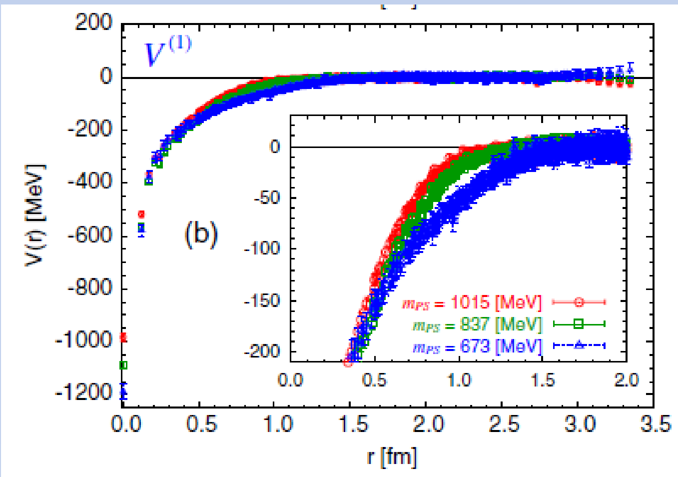
Pc, d*

II: Constituent quark model, Multiquark states and Short distance 2-N static energy

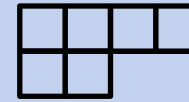
- Lattice Results : HAL QCD collaboration for H dibaryon in SU(3) symmetric limit

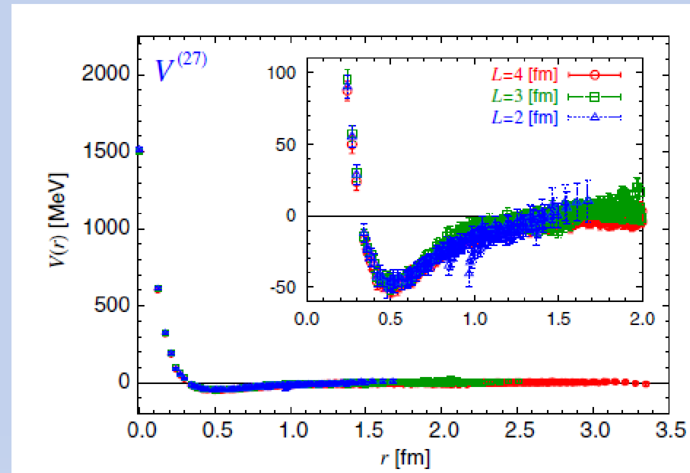
SU(3) flavor 1 state





SU(3) flavor 27 state





→ Flavor 1 channel could give compact configuration

Compact multiquark states could exists if the static energy at short range is attraction

The r→0 can be understood from quark model: Oka et al. quark cluster model

Quark wave function for multiquark states *(W.Park, A.Park, S.Cho, SHL)*

- Some Previous works have limited Fock space: diquark picture …

- Hard to picture interplay between various contribution

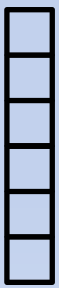
- Hard to understand SU(3) breaking effects.

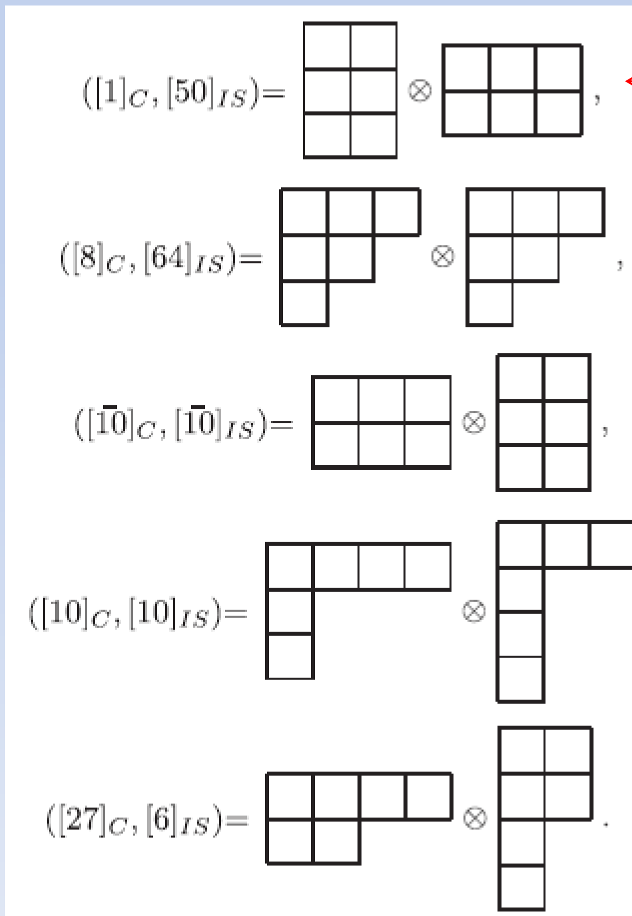
→ Work out the full (color) x (spin) x (flavor) wave function for all multiquark configurations at least for the ground state and with s-wave quark states only

- Choose the spatial part to be symmetric

- Choose the Color-Flavor-Spin part to be antisymmetric : SU(12)

$$[1^6]_{CIS} = ([1]_C,[50]_{IS}) \oplus ([8]_C,[64]_{IS}) \oplus ([10]_C,[10]_{IS}) \oplus ([10]_C,[10]_{IS}) \oplus ([27]_C,[6]_{IS})$$





Physical State

- Dibaryon: 5 Independent color singlet bases

$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline 5 & 6 \\ \hline \end{array}$ $$\left|C_1\right\rangle = \left\{\left[(12)_6 3\right]_8 \left[4(56)_6\right]_8\right\}_1$$

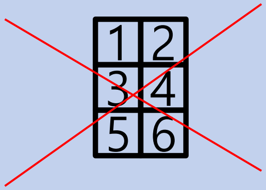
$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline 5 & 6 \\ \hline \end{array}$ $$\left|C_2\right\rangle = \left\{\left[(12)_{\bar{3}} 3\right]_8 \left[4(56)_6\right]_8\right\}_1$$

$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 5 \\ \hline 4 & 6 \\ \hline \end{array}$ $$\left|C_3\right\rangle = \left\{\left[(12)_6 3\right]_8 \left[4(56)_{\bar{3}}\right]_8\right\}_1$$

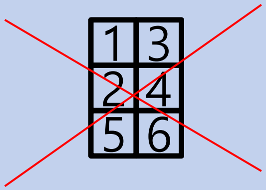
$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 5 \\ \hline 4 & 6 \\ \hline \end{array}$ $$\left|C_4\right\rangle = \left\{\left[(12)_{\bar{3}} 3\right]_8 \left[4(56)_{\bar{3}}\right]_8\right\}_1$$

$\begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 & 6 \\ \hline \end{array}$ $$\left|C_5\right\rangle = \left\{\left[(12)_{\bar{3}} 3\right]_1 \left[4(56)_{\bar{3}}\right]_1\right\}_1$$

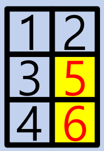
- Pentaquark: 3 Independent color singlet bases (W.Park, A. Park, S.Cho, SHL PRD95,054027)



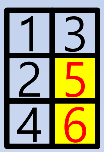
$$\left|C_1\right\rangle = \left\{\left[(12)_6 3\right]_8 \left[4(56)_6\right]_8\right\}_1$$



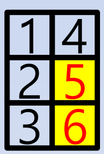
$$\left|C_2\right\rangle = \left\{\left[(12)_{\bar{3}} 3\right]_8 \left[4(56)_6\right]_8\right\}_1$$



$$\left|C_3\right\rangle = \left\{\left[(12)_6 3\right]_8 \left[4(5)_{\bar{3}}\right]_8\right\}_1$$



$$\left|C_4\right\rangle = \left\{\left[(12)_{\bar{3}} 3\right]_8 \left[4(5)_{\bar{3}}\right]_8\right\}_1$$



$$\left|C_5\right\rangle = \left\{\left[(12)_{\bar{3}} 3\right]_1 \left[4(5)_{\bar{3}}\right]_1\right\}_1$$

- Heptaquark: 11 Independent color singlet bases (W.Park, A. Park, SHL PRD96,034029)

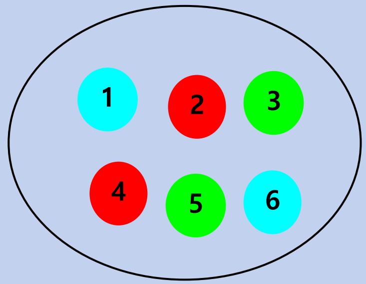
$$|C_1\rangle = \left(\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline 5 & \\ \cline{1-1}\end{array}, \begin{array}{|c|}\hline \bar{6} \\ \hline \bar{7} \\ \hline\end{array}\right), \quad |C_2\rangle = \left(\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline 5 & \\ \cline{1-1}\end{array}, \begin{array}{|c|}\hline \bar{6} \\ \hline \bar{7} \\ \hline\end{array}\right), \quad |C_3\rangle = \left(\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 5 \\ \hline 4 & \\ \cline{1-1}\end{array}, \begin{array}{|c|}\hline \bar{6} \\ \hline \bar{7} \\ \hline\end{array}\right), \quad |C_4\rangle = \left(\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 5 \\ \hline 4 & \\ \cline{1-1}\end{array}, \begin{array}{|c|}\hline \bar{6} \\ \hline \bar{7} \\ \hline\end{array}\right), \quad |C_5\rangle = \left(\begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 & \\ \cline{1-1}\end{array}, \begin{array}{|c|}\hline \bar{6} \\ \hline \bar{7} \\ \hline\end{array}\right),$$

$$|C_6\rangle = \left(\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & & \\ \cline{1-1} 5 & & \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline \bar{6} & \bar{7} \\ \hline\end{array}\right), \quad |C_7\rangle = \left(\begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 & & \\ \cline{1-1} 5 & & \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline \bar{6} & \bar{7} \\ \hline\end{array}\right), \quad |C_8\rangle = \left(\begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 & & \\ \cline{1-1} 5 & & \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline \bar{6} & \bar{7} \\ \hline\end{array}\right), \quad |C_9\rangle = \left(\begin{array}{|c|c|c|}\hline 1 & 2 & 5 \\ \hline 3 & & \\ \cline{1-1} 4 & & \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline \bar{6} & \bar{7} \\ \hline\end{array}\right),$$

$$|C_{10}\rangle = \left(\begin{array}{|c|c|c|}\hline 1 & 3 & 5 \\ \hline 2 & & \\ \cline{1-1} 4 & & \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline \bar{6} & \bar{7} \\ \hline\end{array}\right), \quad |C_{11}\rangle = \left(\begin{array}{|c|c|c|}\hline 1 & 4 & 5 \\ \hline 2 & & \\ \cline{1-1} 3 & & \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline \bar{6} & \bar{7} \\ \hline\end{array}\right).$$

In quark model: wave function should follow Pauli Principle

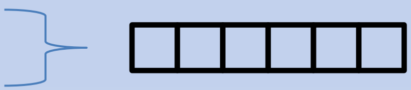
- Totally antisymmetric (color x spin x flavor) wave function (s-wave quarks)



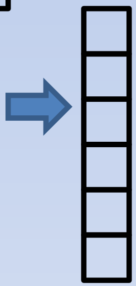
Example: ΩΩ in the Spin=3 channel is highly repulsive because

→ Flavor is totally symmetric

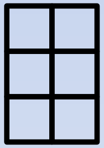
→ Spin is totally symmetric



→ Remaining part should be totally antisymmetric



→ But color singlet implies



→ Hence, assuming all quarks are in the S wave, Pauli principle forbids compact configuration.

Such forbidden configuration are highly repulsive at r→0 **(Oka et al quark cluster model)**

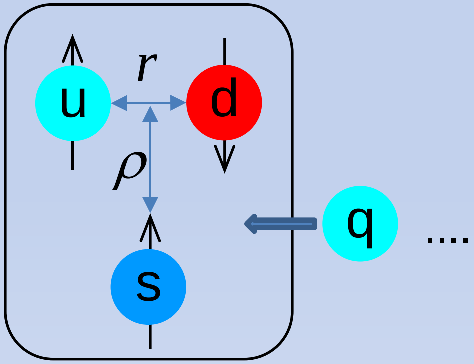
what about states that are allowed?

Constituent quark model

- In Constituent quark model (Can fit experimental hadron spectrum well)

$$H = \sum_{i=1}^{n}\left(m_i + \frac{p_i^2}{2m_i}\right) - \sum_{i<j}^{n}\left(\lambda_i^c \lambda_j^c\right) V_{ij}^C (r_{ij}) - \sum_{i<j}^{n}\frac{\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i \sigma_j\right)}{m_i m_j} V_{ij}^{SS}(r_{ij})$$

$$|\psi_M\rangle =$$



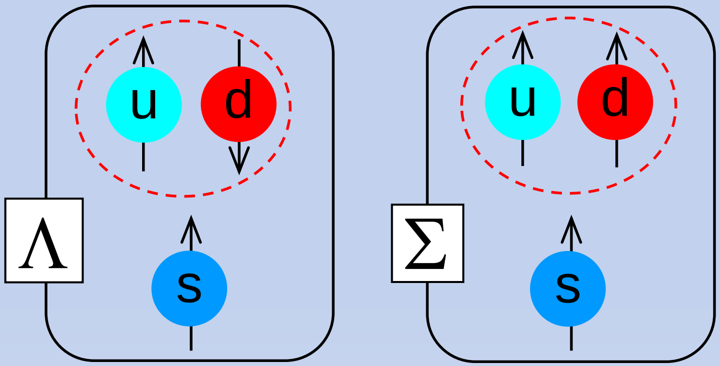
$$m_M = \langle \psi_M | H | \psi_M \rangle$$
$$= \langle \psi_M (space) | H | \psi_M (space) \rangle \times \langle \psi_M (C-S-F) | H | \psi_M (C-S-F) \rangle$$

→ In this talk, Will concentrate on the Color-Spin-Flavor part

- Baryon Mass splitting in a simplified version

$$\text{Mass} = \text{Kinetic} + \text{confining..} + \sum_{i,j} \frac{C_B}{m_i m_j} \left[\lambda_i \lambda_j s_i \cdot s_j \right]$$

Example



$$\Lambda_c \ \text{Mass} = \text{Kinetic} + \text{conf.} - \frac{3}{4} \frac{C_B}{m_u m_d}$$

$$\Sigma_c \ \text{Mass} = \text{Kinetic} + \text{conf.} + \frac{1}{4} \frac{C_B}{m_u m_d} - \frac{C_B}{m_u m_s}$$

$$m_u = m_d = 300\,\text{MeV}, \quad m_s = 500\,\text{MeV}, \quad m_c = 1500\,\text{MeV}, \quad m_b = 4700\,\text{MeV}$$

Mass diff	$M_\Delta - M_N$	$M_\Sigma - M_\Lambda$	$M_{\Sigma c} - M_{\Lambda c}$	$M_{\Sigma b} - M_{\Lambda b}$
Formula	290 MeV	77 MeV	154 MeV	180 MeV
Experiment	290 MeV	75 MeV	170 MeV	192 MeV

When allowed, Where are the Compact multiquark configuration?

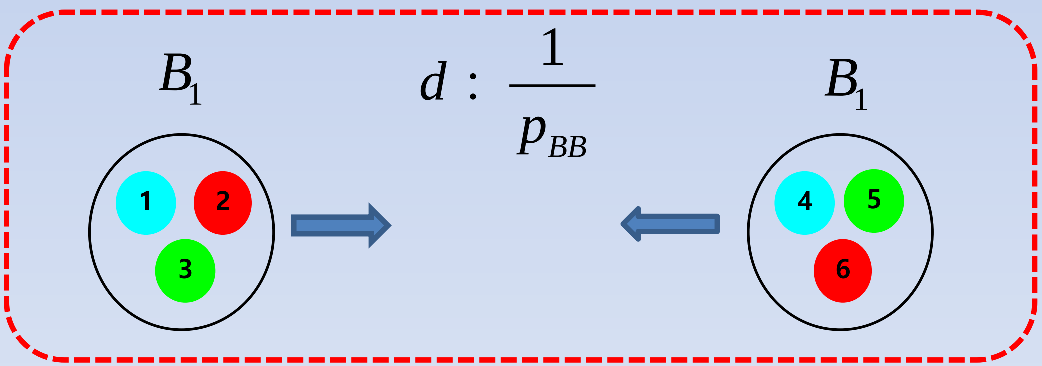
- Kinetic energy part

$$H=\sum_{i=1}^{n}\left(m_i+\frac{p_i^2}{2m_i}\right)-\sum_{i<j}^{n}\left(\lambda_i^c\lambda_j^c\right)V_{ij}^C\left(r_{ij}\right)-\sum_{i<j}^{n}\frac{\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i\sigma_j\right)}{m_im_j}V_{ij}^{SS}\left(r_{ij}\right)+V^{3-body}$$

Compact state 6 quark state vs 2 separated baryons → Additional kinetic energy

$$\left|\psi_{6-quark}\right\rangle=$$

B_1 $\quad d \sim \frac{1}{p_{BB}}$ $\quad B_1$

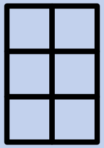


$$\frac{p_{BB}^2}{2\mu_{BB}}\approx 100\ \text{MeV}\quad \text{for}\ \ \mu_{BB}=0.5\ \text{GeV},\ \ d=0.6\ \text{fm}$$

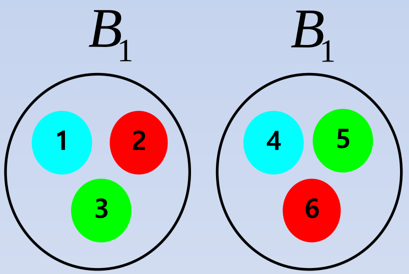
- Color-Color interaction

$$H = \sum_{i=1}^{n}\left(m_i + \frac{p_i^2}{2m_i}\right) - \sum_{i<j}^{n}\left(\lambda_i^c\lambda_j^c\right)V_{ij}^C\left(r_{ij}\right) - \sum_{i<j}^{n}\frac{\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i\sigma_j\right)}{m_i m_j}V_{ij}^{SS}\left(r_{ij}\right) + V^{3-body}$$

Related to Casimir



$$\underbrace{\left(\lambda_1^c + \lambda_2^c + \;\ldots\; \lambda_n^c\right)^2}_{\text{If color singlet}} = 2\sum_{i<j}\lambda_i^c\lambda_j^c + \sum_i\left(\lambda_i^c\right)^2$$



→ Sum of color-color interaction is additive

when the total number of quarks $N_{Total} = N_{B1} + N_{B2}$

$$\sum_{i<j}\lambda_i^c\lambda_j^c = -\frac{1}{2}\times\sum_{i=1}^{N_{Total}}\left(\lambda_i^c\right)^2 = -\frac{2}{3}N_{Total} = -\frac{2}{3}\left(N_{B1} + N_{B2}\right)$$

- Quark 3-body force

$$H=\sum_{i=1}^{n}\left(m_i+\frac{p_i^2}{2m_i}\right)-\sum_{i<j}^{n}\left(\lambda_i^c\lambda_j^c\right)V_{ij}^C\left(r_{ij}\right)-\sum_{i<j}^{n}\frac{\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i\sigma_j\right)}{m_im_j}V_{ij}^{SS}\left(r_{ij}\right)+V^{3-body}$$

Two types

$$V_1^{3-body}=c\sum_{i\neq j\neq k}f^{abc}F_i^aF_j^bF_k^c,\quad V_2^{3-body}=d\sum_{i\neq j\neq k}d^{abc}F_i^aF_j^bF_k^c$$

- Note, related to 2 Casimir

$$C_1=\left(F^a\right)^2=-\frac{2i}{3}f^{abc}F^aF^bF^c,\quad C_2=d^{abc}F^aF^bF^c=C_1\left(2C_1-\frac{11}{6}\right)$$

→ 3-quark interaction are additive: For color singlet state composed of $N_{Total}=N_{B1}+N_{B2}$ *quarks*

$$V_1^{3-body}=0$$

$$V_2^{3-body}=-\left(N_{B1}+N_{B2}\right)C_1^q\left(2C_1^q-\frac{13}{3}\right)$$

- Color spin interaction

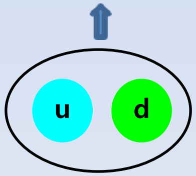
$$H=\sum_{i=1}^{n}\left(m_i+\frac{p_i^2}{2m_i}\right)-\sum_{i<j}^{n}\left(\lambda_i^c\lambda_j^c\right)V_{ij}^C\left(r_{ij}\right)-\sum_{i<j}^{n}\frac{\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i\sigma_j\right)}{m_i m_j}V_{ij}^{SS}\left(r_{ij}\right)+V^{3-body}$$

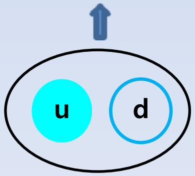
Color-spin interaction for quark-quark and quark antiquark

$$K=\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s)$$

quark-quark and quark-antiquark

	qq				$\bar{q}q$			
Color	A	S	A	S	1	8	1	8
Flavor	A	A	S	S				
Spin	A(1)	S(3)	S(3)	A(1)	1	1	3	3
K	-8	-4/3	8/3	4	-16	2	16/3	-2/3





- Color spin interaction - General remarks $K = -\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s)$

1) Part of a larger group

- Color: SU(3) and 8 generators λ^c

- Spin: SU(2) and 3 generators σ^s

- SU(6) generator: $\lambda^c \times \sigma^s$ (24) + $\lambda^c \times 1$ (8) + $1 \times \sigma^s$ (3) = A (35 generators)

Therefore SU(6) Casmir of N quarks $C_6 = \sum (A_1 + \cdots A_N)^2 = 2\sum_{i<j} A_i A_j + N\,(A_1^2)$

where $(A_1^2) = \frac{35}{6}$ and $2\sum_{i<j} A_i A_j = \sum_{i<j}(\frac{1}{3}\sigma_i\sigma_j + \frac{1}{2}\lambda_i\lambda_j + \frac{1}{3}(\lambda\sigma)_i(\lambda\sigma)_j)$

→ $\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s) \propto C_6 - N\frac{35}{6} - \frac{1}{2}(\sigma_{Total}^2 - \sum\sigma_i^2) - \frac{3}{4}(\lambda_{Total}^2 - \sum\lambda_i^2)$

total spin−N x(quark spin) , total color-N x(quark color)

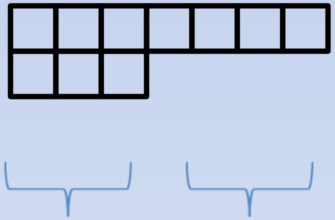
• Color spin interaction - General remarks II

$$\rightarrow \sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s) \propto C_6 - N\frac{35}{6} - \frac{1}{2}\left(\sigma_{Total}^2 - \sum\sigma_i^2\right) - \frac{3}{4}\left(\lambda_{Total}^2 - \sum\lambda_i^2\right)$$

2) Color –flavor-spin wave function should be totally antisymmetric. (Aerts, Mulders, de Swart 78)

For SU(2) flavor: $-\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s) = \frac{4}{3}N(N-6) + 4I(I+1) + \frac{4}{3}S(S+1) + 2C_c$

For SU(3) flavor: $-\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s) = N(N-10) + 4C_F + \frac{4}{3}S(S+1) + 2C_c$



$$4C_F = \frac{4}{3}(p^2 + q^2 + 3p + 3q + qp)$$

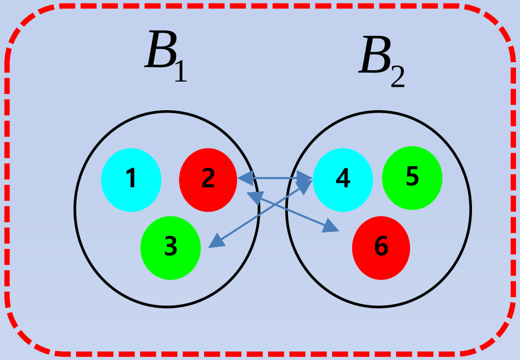
q p using $p + 2q = N$ $\rightarrow 4C_F = (4I(I+1) + N(N+6)/3)$

For SU(4) flavor: $-\sum_{i<j}(\lambda_i^c\lambda_j^c)(\sigma_i^s\sigma_j^s) = \frac{5}{6}N\left(N - \frac{72}{5}\right) + 4C_F^{SU(4)} + \frac{4}{3}S(S+1) + 2C_c$

Short distance energy is dominated by color-spin interaction

$$H = \sum_{i=1}^{n}\left(m_i + \frac{p_i^2}{2m_i}\right) - \sum_{i<j}^{n}\left(\lambda_i^c \lambda_j^c\right) V_{ij}^C\left(r_{ij}\right) - \sum_{i<j}^{n}\frac{\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i \sigma_j\right)}{m_i m_j} V_{ij}^{SS}\left(r_{ij}\right) + V^{3-body}$$

$$\left|\psi_{6-quark}\right\rangle =$$



$$m_M = \left\langle \psi_M \right| H \left| \psi_M \right\rangle$$
$$= \left\langle \psi_M(space) \right| H \left| \psi_M(space) \right\rangle \times \left\langle \psi_M(C-S-F) \right| H \left| \psi_M(C-S-F) \right\rangle$$

Most important part $K = -\sum_{i<j}(\lambda_i^c \lambda_j^c)(\sigma_i^s \sigma_j^s)$

- Comparison with lattice – NN interaction in SU(2)

For SU(2) flavor: $K = -\sum_{i<j}(\lambda_i^c \lambda_j^c)(\sigma_i^s \sigma_j^s) = \frac{4}{3}N(N-6) + 4I(I+1) + \frac{4}{3}S(S+1) + 2C_c$

$$K_{2-N} = K_{6-quark} - \left(K_{1N} + K_{1N}\right)$$

$K_{1N} = -8$ For P, N and Λ

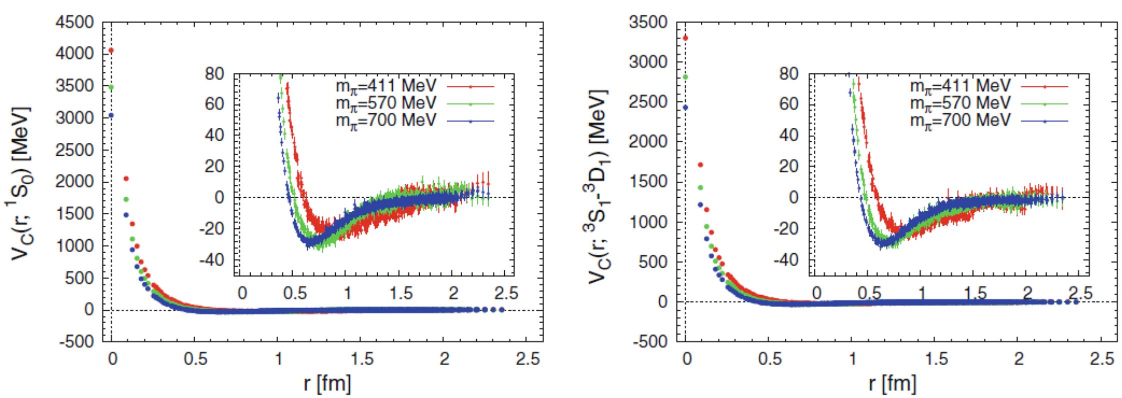
Positive for all 6 quark configuration

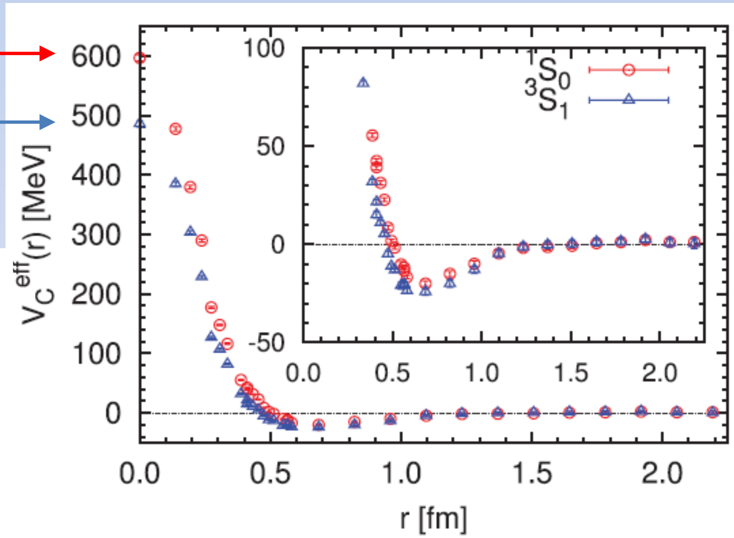
K of 6 quark state and their decays (W Park, A Park, Lee 2015)

(I,S)	(3,0)	(2,1)	(1,2)	(1,0)	(0,3)	(0,1)
$K_{6-quark}$	48	$\frac{80}{3}$	16	8	16	$\frac{8}{3}$
K_{2-N}	32	$\frac{80}{3}$	16	24	0	$\frac{56}{3}$

$$\frac{K_{2-N}^{S=0}}{K_{2-N}^{S=1}} \approx 1.29$$ → comparison

(HAL QCD Collaboration)



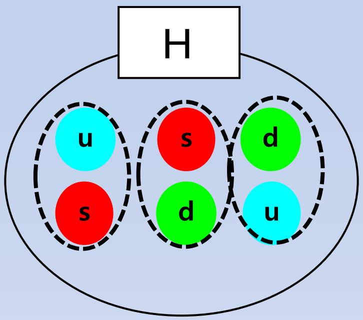


- H dibaryon and Color spin interaction for 3 flavors

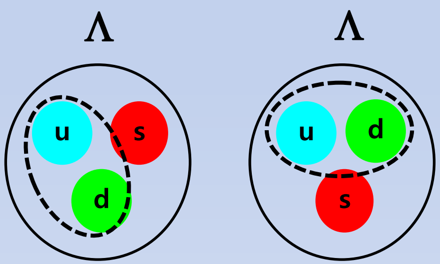
For SU(3) flavor: $K = -\sum_{i<j}(\lambda_i^c \lambda_j^c)(\sigma_i^s \sigma_j^s) = N(N-10) + 4C_F + \frac{4}{3}S(S+1) + 2C_c$

Nucleon and Λ → $K = -8$ even in the SU(3) broken limit

- Jaffe (77) : K for H-dibaryon vs two Λ



vs



$K = -24$

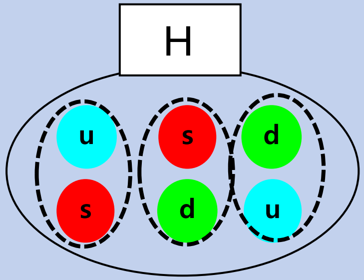
$K = (-8) + (-8) = -16$

→ using Nucleon(K=-8) to Delta (K=+8) mass difference of 290 MeV

ΔK=-8 corresponds to about 145 MeV attraction ≫ additional Kinetic energy of 100 MeV

- H dibaryon in SU(3) symmetric limit

W Park, A. Park, Lee PRD2016



$$|\psi_N\rangle = \begin{pmatrix} F^1 \\ F^{27} \end{pmatrix}$$

TABLE III. The matrix element of $-\langle\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\rangle$ for hyperfine potential of the dibaryon with respect to isospin and flavor.

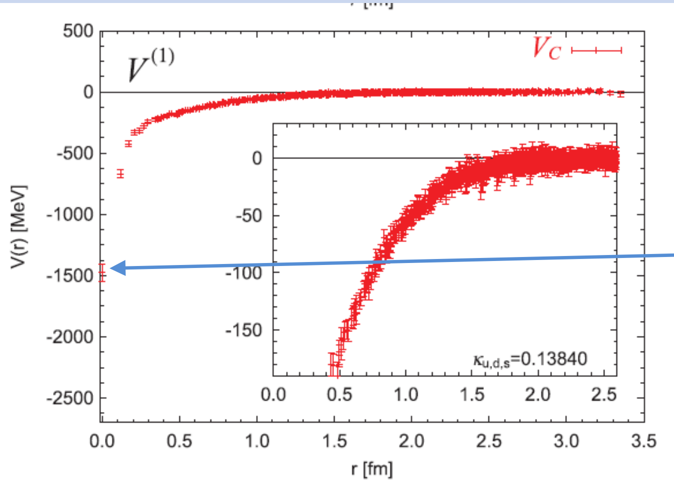
Isospin Flavor	$-\langle\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\rangle$ $i<j=1$–4 (u,d)	$-\langle\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\rangle$ $i=1$–4, $j=5,6$ (u,s)	$-\langle\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\rangle$ $i=5$, $j=6$ (s,s)
$I=0$, F^1	$-5/6$	$-11/4$	3
$I=0$, F^{27}	$-13/18$	$13/12$	$11/3$
Cross terms	$1/(6\sqrt{3})$	$-1/(4\sqrt{3})$	$1/\sqrt{3}$
$I=1$, F^{27}	$4/9$	$1/3$	$8/3$
$I=2$, F^{28}	$16/5$	$16/5$	$16/5$
$I=2$, F^{27}	$146/45$	$-28/15$	$52/15$
Cross terms	$-2\sqrt{2}/(15\sqrt{3})$	$\sqrt{2}/(5\sqrt{3})$	$-4\sqrt{2}/(5\sqrt{3})$

Diquark picture does not work

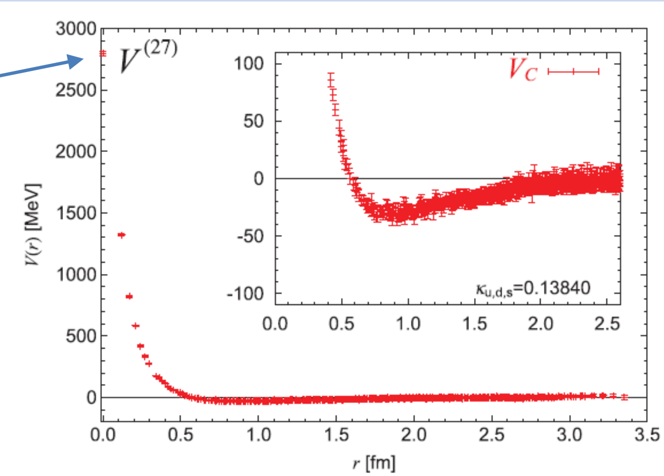
$$\frac{K}{m_i m_j} = \begin{pmatrix} -\frac{5}{m_u^2} - \frac{22}{m_u m_s} + \frac{3}{m_s^2} & \frac{1}{\sqrt{3}m_u^2} - \frac{2}{\sqrt{3}m_u m_s} + \frac{1}{\sqrt{3}m_s^2} \\ \frac{1}{\sqrt{3}m_u^2} - \frac{2}{\sqrt{3}m_u m_s} + \frac{1}{\sqrt{3}m_s^2} & -\frac{13}{3m_u^2} + \frac{26}{3m_u m_s} + \frac{11}{3m_s^2} \end{pmatrix} \xrightarrow{SU(3)} \begin{pmatrix} -\frac{24}{m_u^2} & 0 \\ 0 & \frac{8}{m_u^2} \end{pmatrix}$$

→ Ratio between F^1 and F^{27}

$$K_{2-N} = K_{6-quark} - \left(K_{1N} + K_{1N}\right)$$



$$\frac{K_{2-N}^{F=27}}{K_{2-N}^{F=1}} = -3$$



(HAL QCD Collaboration)

- H dibaryon with realistic quark masses: W.Park, A. Park, SHL, PRD93(2016)074007

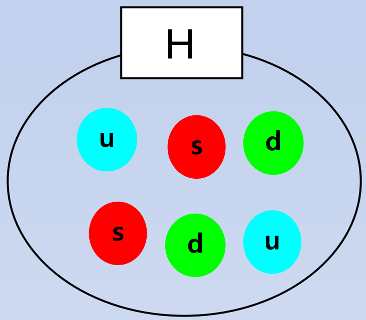
$$m_{u,d} = 300 \text{ MeV},$$
$$m_s = 500 \text{ MeV}$$

$$K = -\sum_{i<j} \lambda_i^c \lambda_j^c \sigma_i^s \sigma_j^s$$

TABLE III. The matrix element of $-\langle \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j \rangle$ for hyperfine potential of the dibaryon with respect to isospin and flavor.

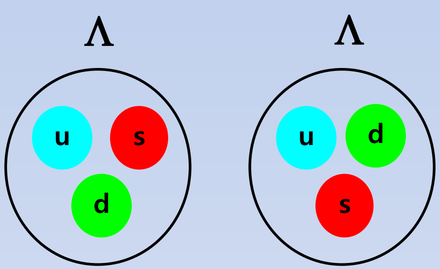
Isospin Flavor	$-\langle \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j \rangle$ $i < j = 1$–4 (u, d)	$-\langle \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j \rangle$ $i = 1$–4, $j = 5, 6$ (u, s)	$-\langle \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j \rangle$ $i = 5$, $j = 6$ (s, s)
$I = 0$, F^1	$-5/6$	$-11/4$	3
$I = 0$, F^{27}	$-13/18$	$13/12$	$11/3$
Cross terms	$1/(6\sqrt{3})$	$-1/(4\sqrt{3})$	$1/\sqrt{3}$
$I = 1$, F^{27}	$4/9$	$1/3$	$8/3$
$I = 2$, F^{28}	$16/5$	$16/5$	$16/5$
$I = 2$, F^{27}	$146/45$	$-28/15$	$52/15$
Cross terms	$-2\sqrt{2}/(15\sqrt{3})$	$\sqrt{2}/(5\sqrt{3})$	$-4\sqrt{2}/(5\sqrt{3})$

→ If the SU(3) breaking is taken into account. Color spin with constituent quark mass



$$\leftarrow \frac{p_{BB}^2}{2\mu_{BB}} \approx 100 \text{ MeV}$$

VS



$-\sum_{i<j}^{n} \frac{K}{m_i m_j}$	H dibaryon	Λ + Λ	Δ $E_{\text{color spin}}$	Δ E_{kinetic}
$m_{u,d} = m_s$	$-\frac{24}{m_u^2}$	$-\frac{8}{m_u^2} - \frac{8}{m_u^2}$	-145 MeV	+100MeV
$m_{u,d} \approx \frac{3}{5} m_s$	$\left(-\frac{5}{m_u^2} - \frac{22}{m_u m_s} + \frac{3}{m_s^2}\right) \approx -\frac{17.12}{m_u^2}$	$-\frac{8}{m_u^2} - \frac{8}{m_u^2}$	-20 MeV	+ 84 MeV

III. Tribaryons

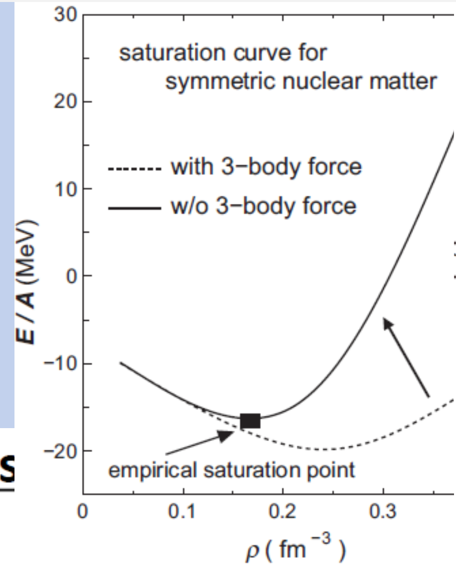
and

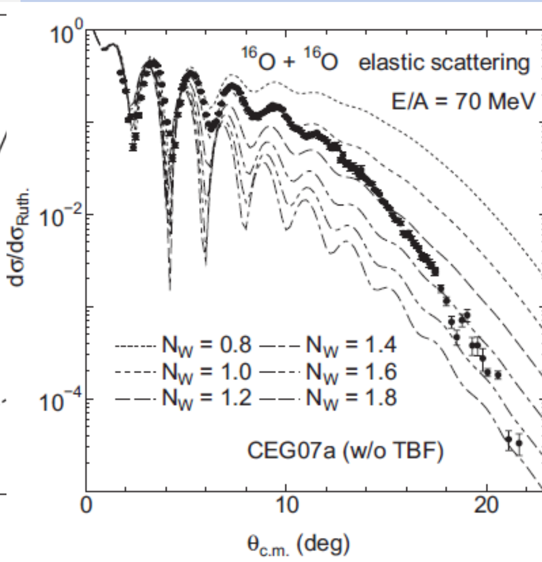
Short distance repulsive three body nuclear force

- Three body nuclear force is repulsive at short distance

1) Review by Sakuragi

(PTEP 2016,06A106)



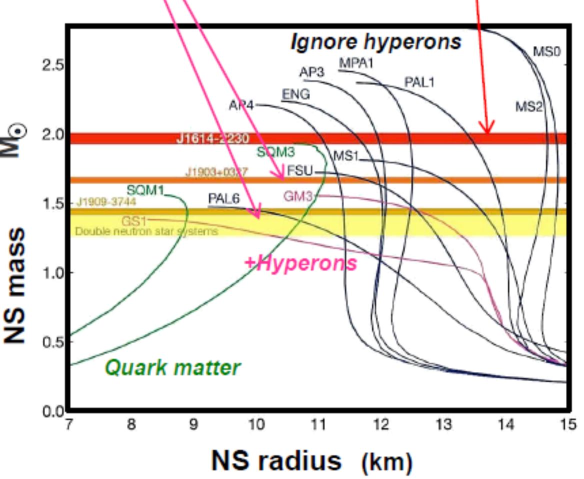


2) Hyperon Puzzle : slide by Tamura

"Hyperon puzzle" in neutron stars

- Hyperons (Λ at least) should appear at $\rho \sim 2\text{-}3\ \rho_0$
  EOS's with hyperons or kaons too soft => cannot support $M > 1.5\ M_{sun}$
- Heavy NS's (~2.0 M_{sun}) were observed.

PSR J1614-2230 (2010) $1.97 \pm 0.04\ M_{sun}$
PSR J0348-0432 (2013) $2.01 \pm 0.04\ M_{sun}$



=> Unknown repulsion at high ρ

- Strong repulsion in three-body force including hyperons,
  NNN, YNN, YYN, YYY ?
  Chiral EFT is successful in NNN force.
  Extension to include hyperons requires **high quality YN scattering data.**
- Phase transition to quark matter ?
  (quark star or hybrid star)

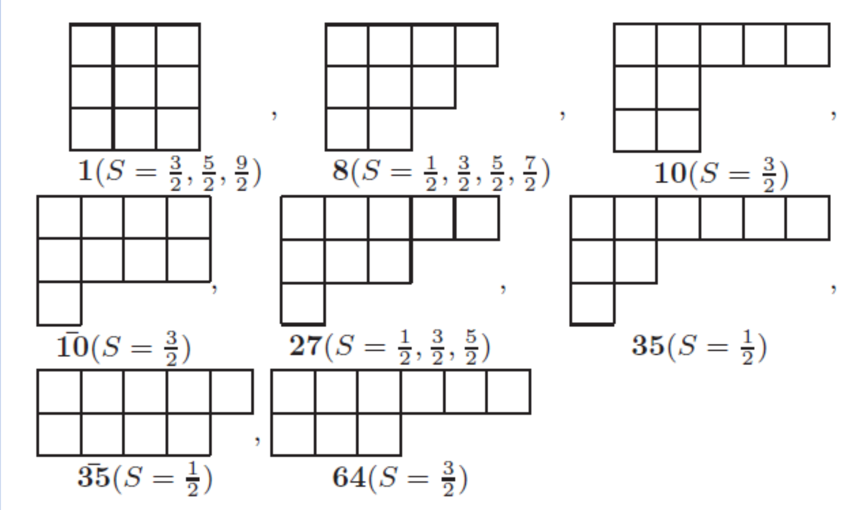
We need to know YN, YY, $K^{bar}N$ interactions both <u>in free space</u> and <u>in nuclear medium</u>

- Tribaryon configurations (Aaron Park, W. Park, SH Lee 1801.10350)

1) Color singlet x Flavor spin state

$$[333]_{FS} = [63]_F \otimes [63]_S + [54]_F \otimes [54]_S + [621]_F \otimes [54]_S + [531]_F \otimes [72]_S + [531]_F \otimes [63]_S + [531]_F \otimes [54]_S + [522]_F \otimes [63]_S + [441]_F \otimes [63]_S + [432]_F \otimes [81]_S + [432]_F \otimes [72]_S + [432]_F \otimes [63]_S + [432]_F \otimes [54]_S + [333]_F \otimes [9]_S + [333]_F \otimes [72]_S + [333]_F \otimes [63]_S.$$

2) Possible Flavor state



- Color spin interaction - General remarks
  Tribaryon configuration (Aaron Park)

$$K = -\sum_{i<j} (\lambda_i^c \lambda_j^c)(\sigma_i^s \sigma_j^s)$$

1) SU(2): Three nucleons

For SU(2) flavor: $-\sum_{i<j}(\lambda_i^c \lambda_j^c)(\sigma_i^s \sigma_j^s) = \frac{4}{3}N(N-6) + 4I(I+1) + \frac{4}{3}S(S+1) + 2C_c$

For nucleon *K*= *-8*, *But* But Tribaryon (N=9) K>>0

2) SU(3): Including hyperons

$$K_{9-quarks}$$

$$\left(K_{1N} + K_{1N} + K_{1N}\right) \rightarrow$$

Flavor	$-\sum_{i<j} \lambda_i \lambda_j \sigma_i \cdot \sigma_j$				
	$S=\frac{1}{2}$	$S=\frac{3}{2}$	$S=\frac{5}{2}$	$S=\frac{7}{2}$	$S=\frac{9}{2}$
1		-4	$\frac{8}{3}$		24
8	4	8	$\frac{44}{3}$	24	
10		20			
$\overline{\mathbf{10}}$		20			
27	24	28	$\frac{104}{3}$		
35	40				
$\overline{\mathbf{35}}$	40				
64		56			
V	-24	-24	-8	8	24

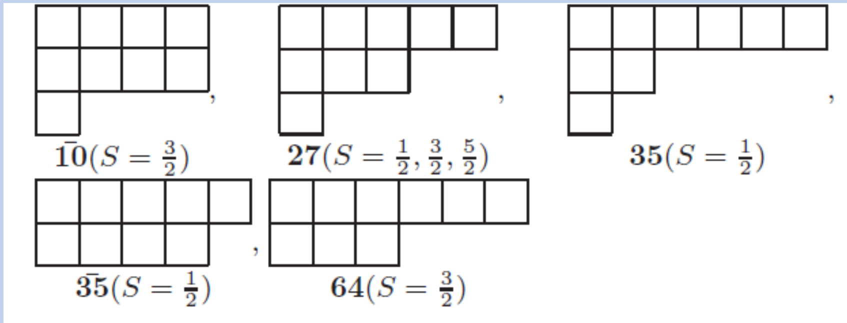
For SU(3) flavor: $-\sum_{i<j}(\lambda_i^c \lambda_j^c)(\sigma_i^s \sigma_j^s) = N(N-10) + 4C_F + \frac{4}{3}S(S+1) + 2C_c$

All tribaryon channel is very repulsive

→ Three Baryon force should be repulsive with or without strangeness

- Color spin interaction - Tribaryon configuration with broken SU(3)

1) With one strangeness and isospin 0



Flavor	$-\sum_{i<j}\lambda_i\lambda_j\sigma_i\cdot\sigma_j$				
	$S=\frac{1}{2}$	$S=\frac{3}{2}$	$S=\frac{5}{2}$	$S=\frac{7}{2}$	$S=\frac{9}{2}$
1		-4	$\frac{8}{3}$		24
8	4	8	$\frac{44}{3}$	24	
10		20			
$\overline{\mathbf{10}}$		20			
27	24	28	$\frac{104}{3}$		
35	40				
$\overline{\mathbf{35}}$	40				
64		56			
V	-24	-24	-8	8	24

With one strangeness: least repulsive state is S=3/2, Flavor antidecuplet

$$\overline{10}\left(S=\frac{3}{2}\right) \quad \rightarrow \quad K_9-\left(\sum K_{1N}\right)=20+\frac{20}{3}\left(1-\frac{m_u}{m_s}\right)+24 \quad ? \quad 0$$

$$\overline{35}\left(S=\frac{1}{2}\right) \quad \rightarrow \quad K_9-\left(\sum K_{1N}\right)=40-\frac{40}{3}\left(1-\frac{m_u}{m_s}\right)+24 \quad ? \quad 0$$

- Purely three body repulsion

$$K = -\sum_{i<j} \lambda_i^c \lambda_j^c \sigma_i^s \sigma_j^s$$

→ 2-Nucleon $$K_{2-N} = K_6 - \left(K_{1N} + K_{1N} \right)$$

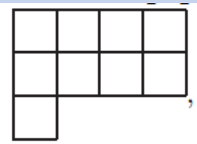
→ 3-Nucleon $$K_{3-N} = K_9 - \left(\sum K_{2-N} \right) - \left(\sum K_{1N} \right)$$

1) SU(2): Triton configuration (I,S)=(1/2,1/2)

$\overline{35}(S = \frac{1}{2})$

$$K_{3-N} = K_9 - \left(\sum K_{2-N} \right) - \left(\sum K_{1N} \right) = 8$$

2) Configuration with one strange quark



$\overline{10}(S = \frac{3}{2})$

$$K_{3-N} = K_9 - \left(\sum K_{2-N} \right) - \left(\sum K_{1N} \right) = 12$$

Intrinsic three nucleon forces are repulsive

Summary

- Constituent quark model can be used to study multiquark configurations: d*, Pc, X(3872), Z are most likely molecular states

- Qualitative features of the short distance Nuclear two-body repulsion from HAL QCD can be understood from a very simple constituent quark picture

- Short distance Nuclear three-body interactions seems to be repulsive in the NNN and NNΛ configurations even in the SU(3) symmetry breaking case as required from phenomenology

$\rightarrow$ more work in progress and looking forward to more lattice data on this