# Probing the Cold & Dense Pressure With pQCD

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# Outline

# Introduction

# What: Pressure of Cold QCD

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- Difficult to compute, need to simplify
  - Here, we mostly care about the $\mu$-dependence
- Will eventually need a limit: Zero-temperature and massless
  - Good approximation for dense and cold systems (NS)

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  - Severe sign problem $\implies$ $T = 0$, large $\mu$ out of reach of lattice, no complete solution
  - $\chi$PT stops being an applicable EFT
  - Dimensional reduction fails when temperature no longer dominates the Matsubara frequencies ($T/\mu$ small)
  - ...

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## How: Applying Perturbation Theory at Large $\mu$

- Instead use: QCD perturbative for asymptotically large $\mu$
  - First-principles method! We use the QCD Lagrangian...
  - ...Almost: we use some HTL at times, but only to get rid of uninteresting higher-order bits
  - Requires high-order corrections to reach physical densities, convergence suboptimal
  - Suffers from IR-ambiguities

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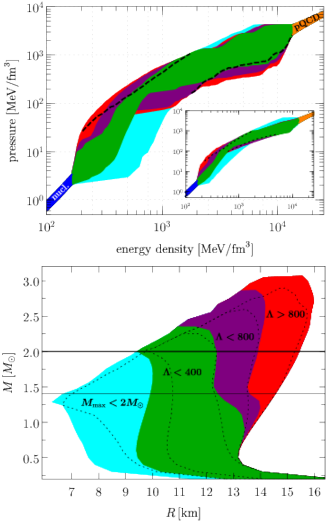
- Despite bad convergence, the pQCD EoS can be used *together* with a Nuclear EoS ($\chi$PT...) to interpolate to the intermediate regime [a]
- $M - R$-plane is constrained:
  - Thermodynamic consistency, subluminality, ...
  - Mass measurements, tidal deformabilities from GWs [b]...
- Eg. The interpolation methods can still be improved




[a] Kurkela et al. astro-hep/1402.6618

[b] Annala et al. astro-hep/1711.02644, figures borrowed from there

# Three-loop Pressure

# Thermal pQCD History Lesson pt. I

$$p = c_0 + c_2 g^2 + \tilde{c}_4 g^4 \log g + c_4 g^4 + \tilde{c}_6' g^6 \log^2 g + \tilde{c}_6 g^6 \log g + c_6 g^6 + \ldots$$

- First three-loop ($\mathcal{O}(g^4)$, NNLO) results at finite density from the 70s (!) [1]
- Included everything for massless quarks at $T = 0$: $c_0, c_2, \tilde{c}_4, c_4$

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- Included everything for massless quarks at $T = 0$: $c_0, c_2, \tilde{c}_4, c_4$
- Later generalised to $m \neq 0$ [2] and small but finite $T$ [3]

[1] Freedman & McLerran, Phys. Rev. D 16 1977

[2] Kurkela et al., hep-ph/0912.1856

[3] Kurkela & Vuorinen, hep-ph/1603.00750

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- For comparison, consider "hot QCD", the large-T limit:
- $\tilde{c}_6' = 0$ and $\tilde{c}_6$ are known [4]

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- For comparison, consider "hot QCD", the large-T limit:
- $\tilde{c}_6' = 0$ and $\tilde{c}_6$ are known [4]
- The theory is fundamentally different:
  - A 3d EFT description is available
  - However, there is is a *magnetic scale* $g^2 T$
  - As a consequence, $c_6$ *cannot* be computed in perturbation theory!

[4] Kajantie et al. hep-ph/0211321

# Scale Hierarchy

- Naïve diagrammatic expansion valid in vacuum

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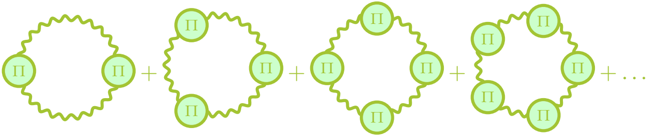
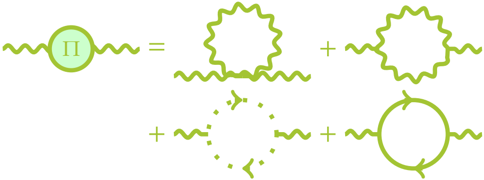
- In the intermediate region, when ratios of scales are involved, this also causes nonanalytic terms $\mathcal{O}(g^m \log^n g)$ to be generated

- Hard Scale $p \sim \mu$:
  - Energetic excitations $\rightarrow$ medium effects small
  - Naïve loop expansion OK
- Semisoft Scale $g\mu \ll p \ll \mu$:
  - "Somewhat" energetic excitations $\rightarrow$ medium effects "average"
  - Need to resum; correction to naïve diagrams suppressed, but not enough
- Soft Scale $p \sim g\mu$:
  - Low-energy excitations $\rightarrow$ medium effects large
  - Resum IR-sensitive objects (gluons) to all orders

# 1-loop Ring Sum

- First logs at $\mathcal{O}(g^4)$: The gluonic ring sum



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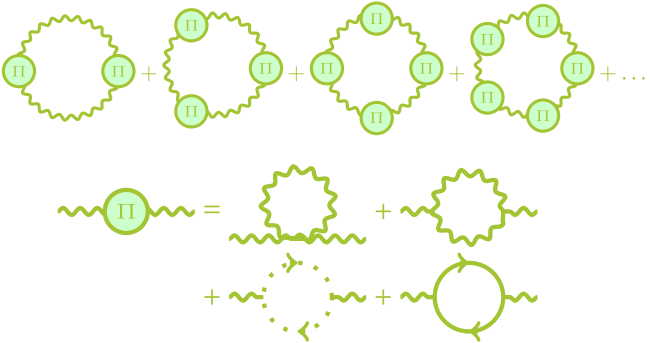


- Contributes non-analytic terms $\mathcal{O}(g^4 \log g)$: $\tilde{c}_4 \neq 0$
- Some diagrams / terms easiest to handle with naïve loop expansion, watch out for double counting

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is complicated
- IR DoFs admit an effective description
- Hard Thermal Loops [5] suitable since hard modes dominate the self-energies


- Physical motivation: Accounts for the medium
- Computing the resummed correction from the ring sum yields $\mathcal{O}(g^4), \mathcal{O}(g^4 \log g)$ vs. naïve first term $\mathcal{O}(g^4)$: Logarithmic enhancement at 3-loop order from the semisoft modes

[5] Hard Dense Loops, really, but abbreviated HTL

# HTL Propagators & $m_E$

- After scalarisation, transversal and longitudinal gluons:

$$G_T(P) = \frac{-1}{P^2 + \frac{m_E^2}{d-1} - \frac{P^2}{p^2}\Pi_{\mathrm{HTL}}(P)},\ G_L(P) = \frac{1}{p^2 + \Pi_{\mathrm{HTL}}(P)}$$

- HTL structure is given by the function

$$\Pi_{\mathrm{HTL}}(P) = m_E^2 \left( 1 - \int_{S^{d-1}} \Omega_{\mathbf{v}} \frac{iP_0}{iP_0 - \mathbf{p}\cdot\mathbf{v}} \right)$$

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- Without quark masses, the only scale is the effective mass

$$m_E^2 \equiv \mathrm{Tr}\Pi \overset{T\to 0}{\propto} (g\mu)^2$$

# Double-log at Four-loop Order

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- $\implies$ Try to compute the non-analytic terms $\tilde{c}_6, \tilde{c}_6'$ first
  - Non-hard modes need to be resummed again
  - Analogous with hot QCD development

## Two-loop Ring Sums: Gluonic Diagrams

- There are now double logs $\mathcal{O}(g^6 \log^2 g), \tilde{c}_6'$ , and single logs $\mathcal{O}(g^6 \log g)$, $\tilde{c}_6$.
- Five classes of IR-sensitive 2-loop diagrams. Two of them are gluonic and contribute to both logs:

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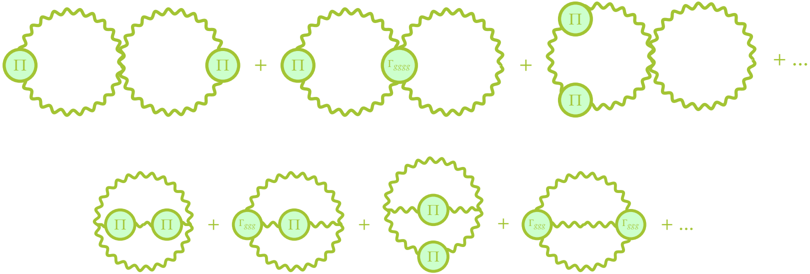

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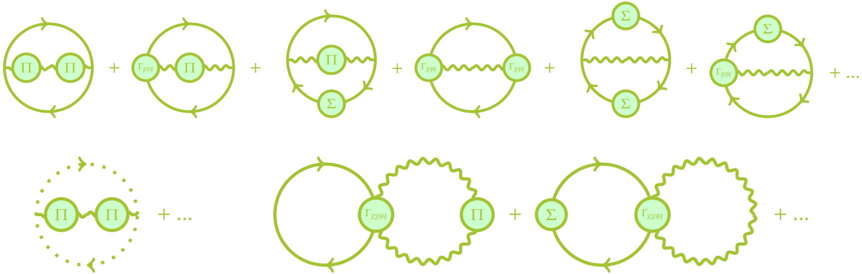

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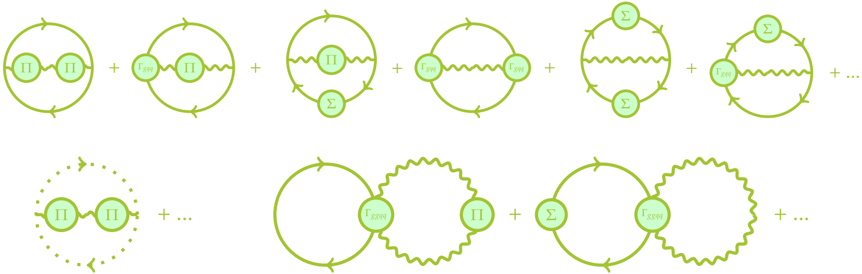


- Gluons IR-sensitive $\Longrightarrow$ resummed, but HTL-approximated
- Fermions IR-insensitive $\Longrightarrow$ not resummed, but loop-corrected

# Two-loop Ring Sums: Double-log

- Double log $\tilde{c}_6'$ easier to extract [6]
- Need two resummed gluon lines, so only the first two diagram classes contribute

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# Two-loop Ring Sums: Double-log

- Double log $\tilde{c}_6'$ easier to extract [6]
- Need two resummed gluon lines, so only the first two diagram classes contribute
- The HTL corrections to the *ggg* and *gggg* vertices are required
- However, for the leading log only one-loop (HTL) self-energies and vertices are required...
  - ...But for the subleading log $\tilde{c}_6$ this is not enough
  - ...And the other diagrams are needed
  - Ensuring correct coefficients, no double-counting etc. takes quite a lot of diagrammatics (aka "SCIENCE CLOUDS")

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## Our Calculation

- Relevant HTL diagrams appear in the literature [7]
- The double-log could be extracted from the semisoft regime with some tricks and we obtained the $\mathcal{O}(g^6 \log^2 g)$-term

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$$\tilde{c}_6' g^6 \log^2 g = -\frac{11}{96} \frac{N_c d_A}{(2\pi)^4} g^2 m_E^4 \log^2 g$$

- Surprisingly simple result
- No renormalisation scale dependence yet...

[7] Andersen et al., hep-ph/0205085

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- Physical intuition: In the semisoft region transverse gluons massive with mass $m_\infty^2 = \frac{1}{d-1} m_E^2$, longitudinal gluons massless
- This is a sufficient approximation for the $\log^2 g$! Can remove complicated $\Pi_{\text{HTL}}$ leaving just a constant mass
- Much easier to compute, verified that it gives the same $\log^2 g$

# Conclusions

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- 3-loop pQCD pressure understood in many regimes
- IR-sector requires resummation starting at 3-loops - But this lets us compute part of the 4-loop result
- $\mathcal{O}(g^6 \log^2 g)$ done, paper out soon
- $\mathcal{O}(g^6 \log g)$ will require eg. higher order HTL-corrections...
- Resummation schemes, small-T corrections etc. also under consideration...