The $N\Omega$ Interaction:
Meson Exchanges, Inelastic Channels, and Quasi-Bound State

Takayasu SEKIHARA
(Japan Atomic Energy Agency)

in collaboration with

Yuki KAMIYA and Tetsuo HYODO
(Yukawa Inst., Kyoto Univ.)

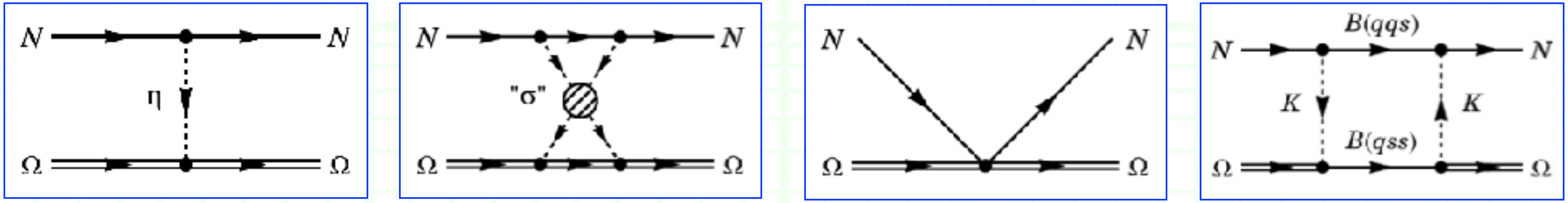
---

[1] T. S. , Y. Kamiya and T. Hyodo, arXiv:1805.04024 [hep-ph].

Contents

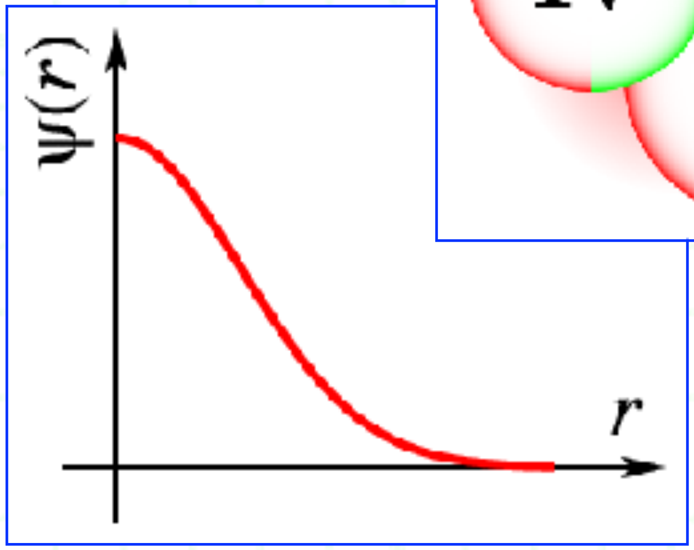
1. Introduction

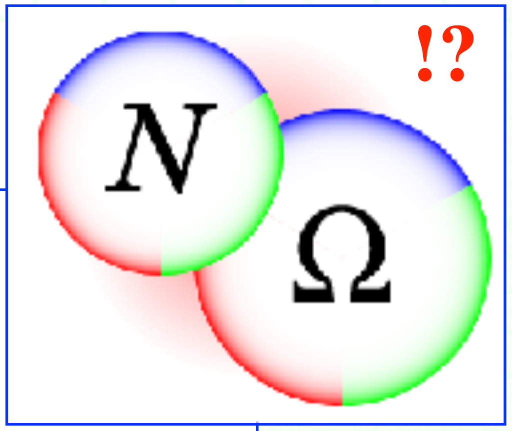
2. Model construction



3. Properties of the $N\Omega$ interaction

4. Summary and outlook



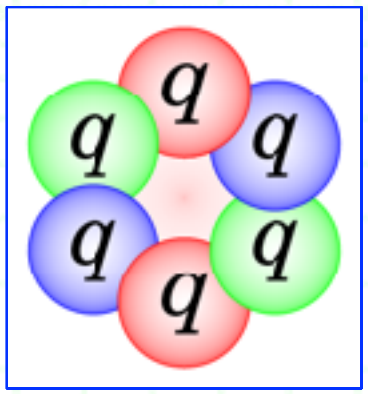


1. Introduction

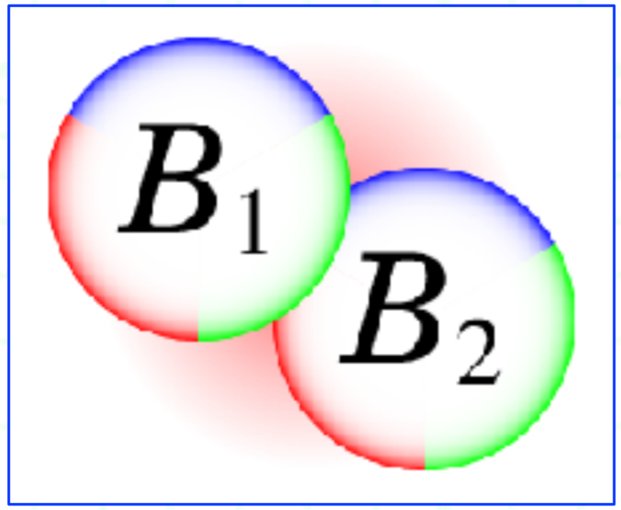
1. Introduction

++ What is dibaryon ? ++

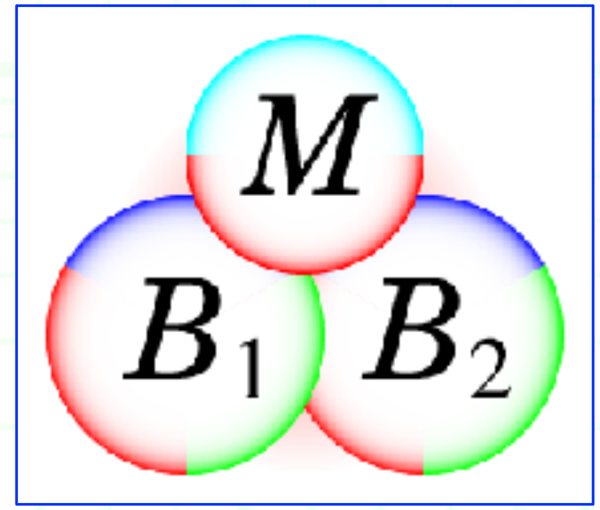
- Dibaryons: States of baryons number $B = 2$
  generated by strong interactions.



Compact hexa-quarks



Hadronic two-body molecules



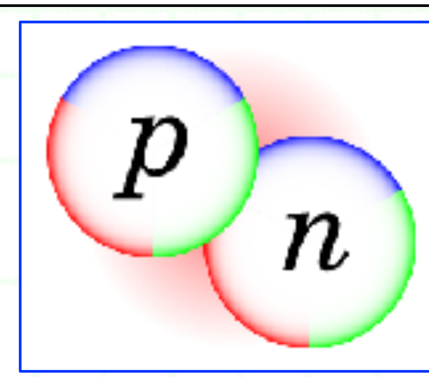
Meson-assisted dibaryon

...

--- Regardless of their structure.

Field renormalization constant ! (1965).

- There is only single well-established dibaryon, the deuteron, which is a proton-neutron molecule.

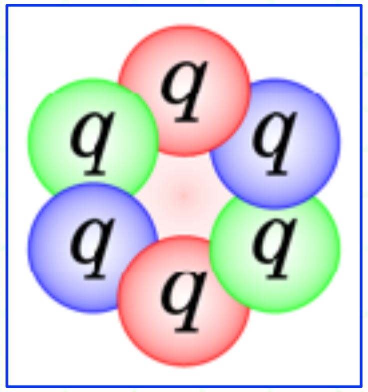


Weinberg (1965).

1. Introduction

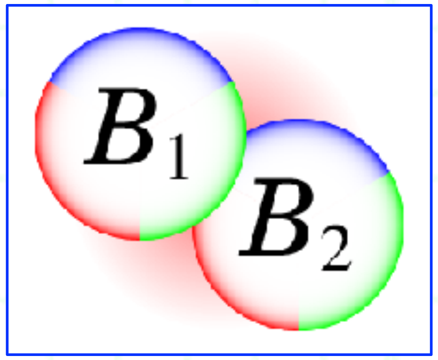
++ Why dibaryons ? ++

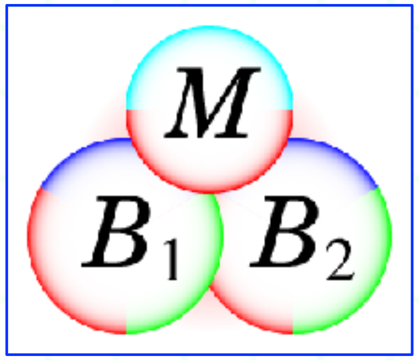
- **Motivations to study dibaryons**:
  - First of all, does such “exotic” states exist or not ?
    --- New forms of hadrons / nuclei.



  - Compact hexa-quarks:
    --- How the quark-confinement mechanism work ?
    --- Compare with typical hadrons.
    Properties of constituent quarks (such as mass $M_q \sim 300$ Me)
    are different from those in typical hadrons ?

  - Hadronic molecules (including meson-assisted dibaryons):
    --- Information on the hadron-hadron interaction.
    --- New few-body systems.





1. Introduction

++ Theor. predictions / Exp. implications ++

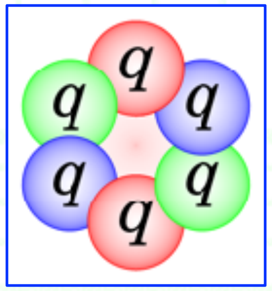
- **Many Theor. predictions / Exp. implications on the dibaryons.**

□ ***H* dibaryon** (*uuddss*). Jaffe (1977).

--- Implications by recent HAL QCD method.

K. Sasaki *et al.* [HAL QCD], *PoS* LATTICE 2016; ...

Seen near the $N\Xi$ threshold ???



□ ***$\bar{K}NN$*.**

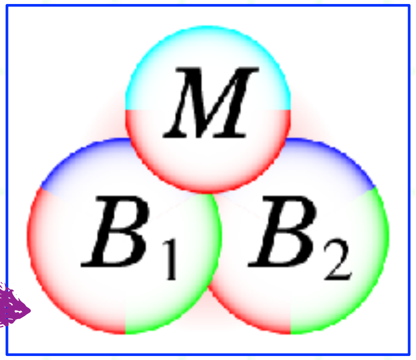
--- Bound by the strongly attractive $\bar{K}N$ interaction.

Akaishi-Yamazaki (2003); Dote-Hyodo-Weise (2008); ...

--- "Peak" seen in FINUDA, J-PARC E27 & E15 *etc*.

Agnello *et al.* (2005); Ichikawa *et al.* (2015); Sada *et al.* (2016); ...

!?

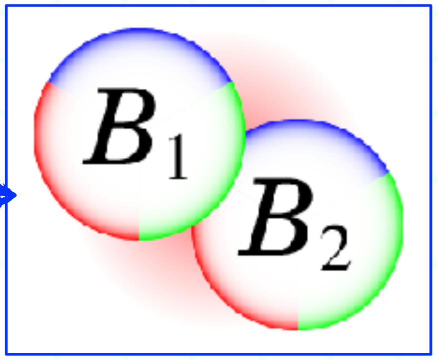


□ *d**(2380) [$I\,(J^P) = 0\,(3^+)$].

--- Found in the $p\,n \to d\,\pi\,\pi$ reaction. WASA-at-COSY (2011).

ΔΔ molecules ??? Dyson-Xuong (1964).

???



□ ...

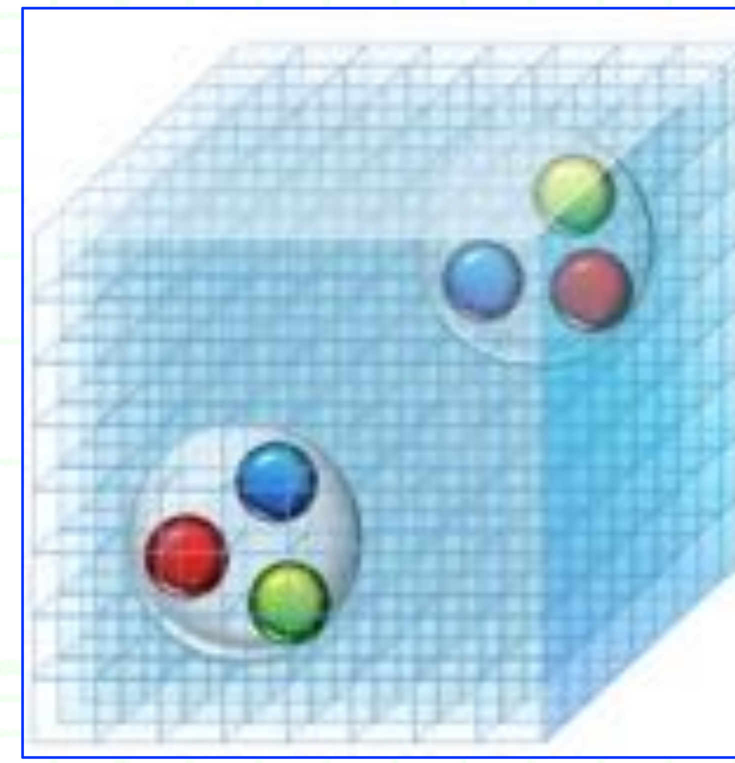
1. Introduction

++ Theor. predictions / Exp. implications ++

- We are **now in a very good time to discuss dibaryons**.
  - Recent remarkable progress in hadron Exp. enables us to examine "traditional" ideas of dibaryons.

J-PARC.

HAL QCD.



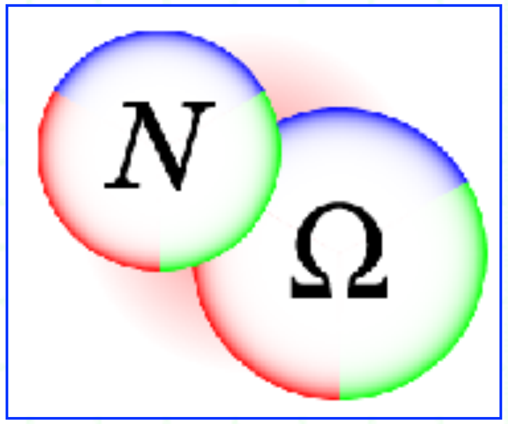
  - Further information is available from numerical simulations of lattice QCD, especially with the physical quark masses.

- **More hadron-hadron pairs !**
- **More binding energy to be "stable" !**

1. Introduction

++ Predictions of the $N\Omega$ bound state ++

- $N\Omega$ **dibaryon system.**
  - □ Combination of $N(uud\,/\,udd)$ [octet] + $\Omega(sss)$ [decuplet]. No same flavor. --> No repulsive core !?



  - □ Calculations in quark models.

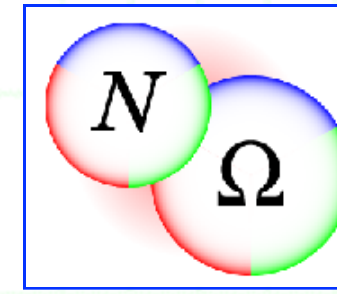
Goldman, Maltman, Stephenson, Schmidt and F. Wang, *Phys. Rev. Lett.* 59 (1987) 627;
Oka, *Phys. Rev.* D38 (1988) 298;
Li and Shen, *Eur. Phys. J.* A8 (2000) 417;
Pang, Ping, Wang, Goldman and Zhao, *Phys. Rev.* C69 (2004) 065207;
Zhu, Huang, Ping and F. Wang, *Phys. Rev.* C92 (2015) 035210;
Huang, Ping and Wang, *Phys. Rev.* C92 (2015) 065202; ...

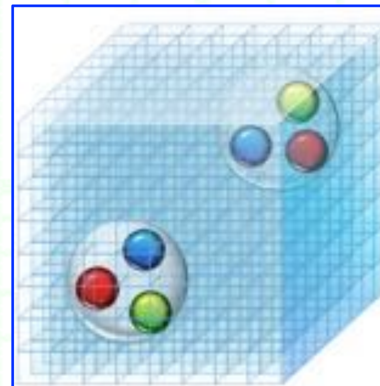
--- Although the details are different, these calculations indicate the existence $N\Omega$ bound state.

1. Introduction

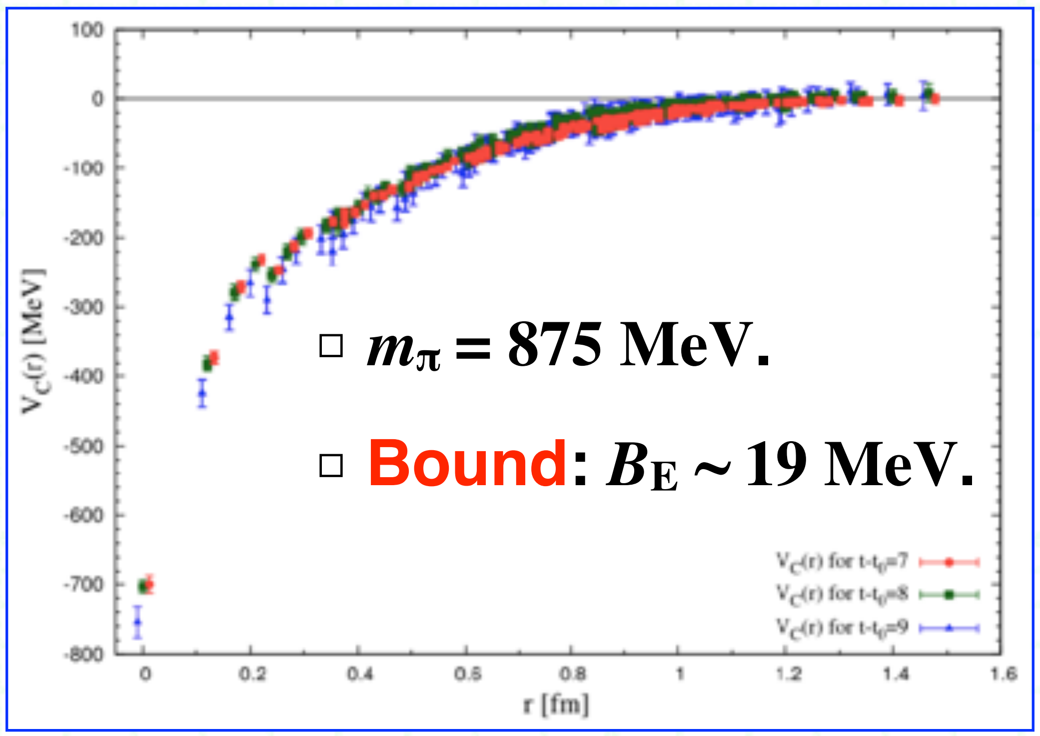
++ The $N\Omega$ system from lattice QCD ++

- **The S-wave $N\Omega$ interaction ($J^P = 2^+$) in HAL QCD analysis of lattice QCD data.**





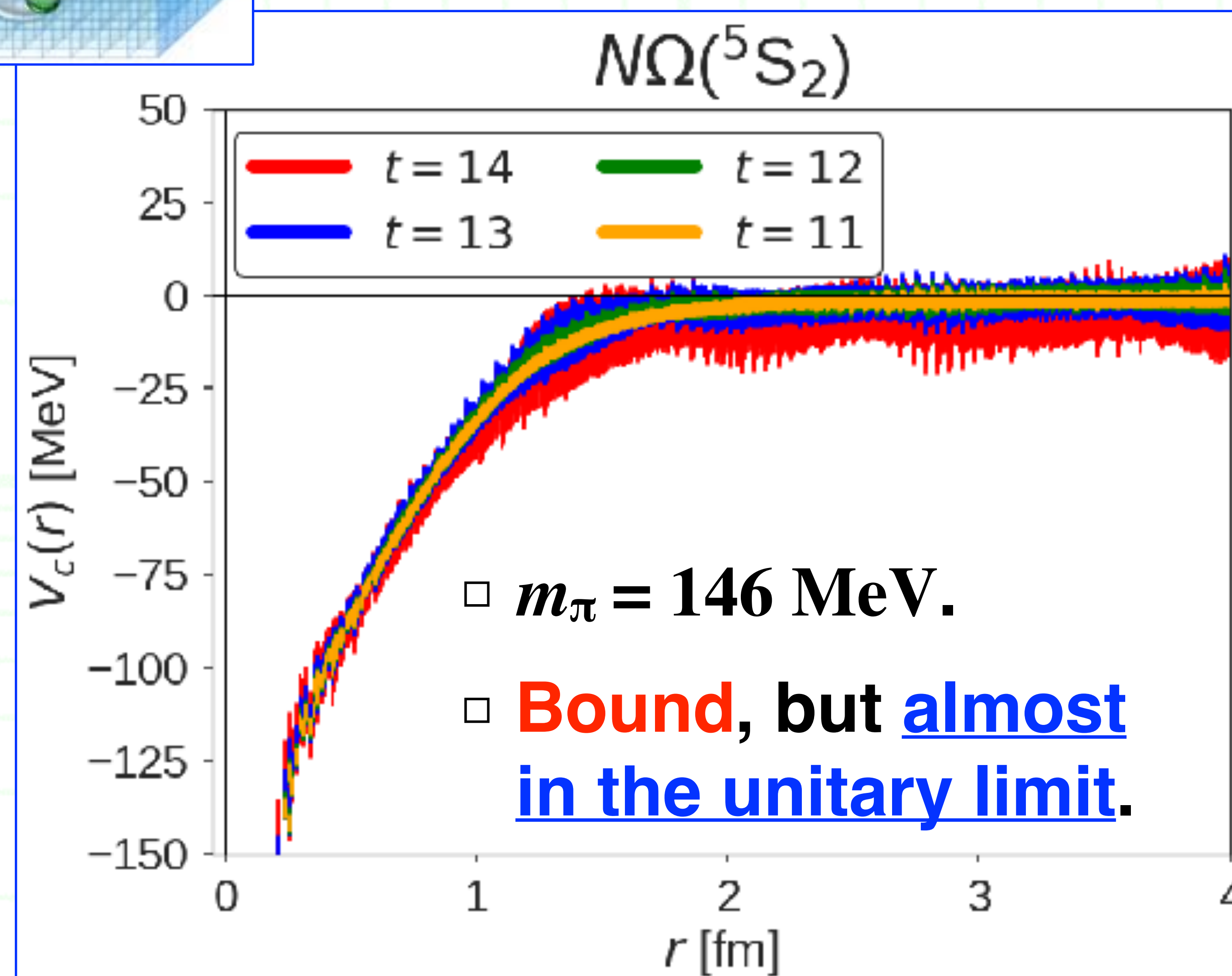
Etminan *et al.* [HAL QCD], *Nucl. Phys.* A928 (2014) 89.



□ m_π = 875 MeV.

□ **Bound**: $B_E \sim 19$ MeV.

Doi *et al.* [HAL QCD], *EPJ Web of Conf.* 175 (2018) 05009.



□ m_π = 146 MeV.

□ **Bound**, but almost in the unitary limit.

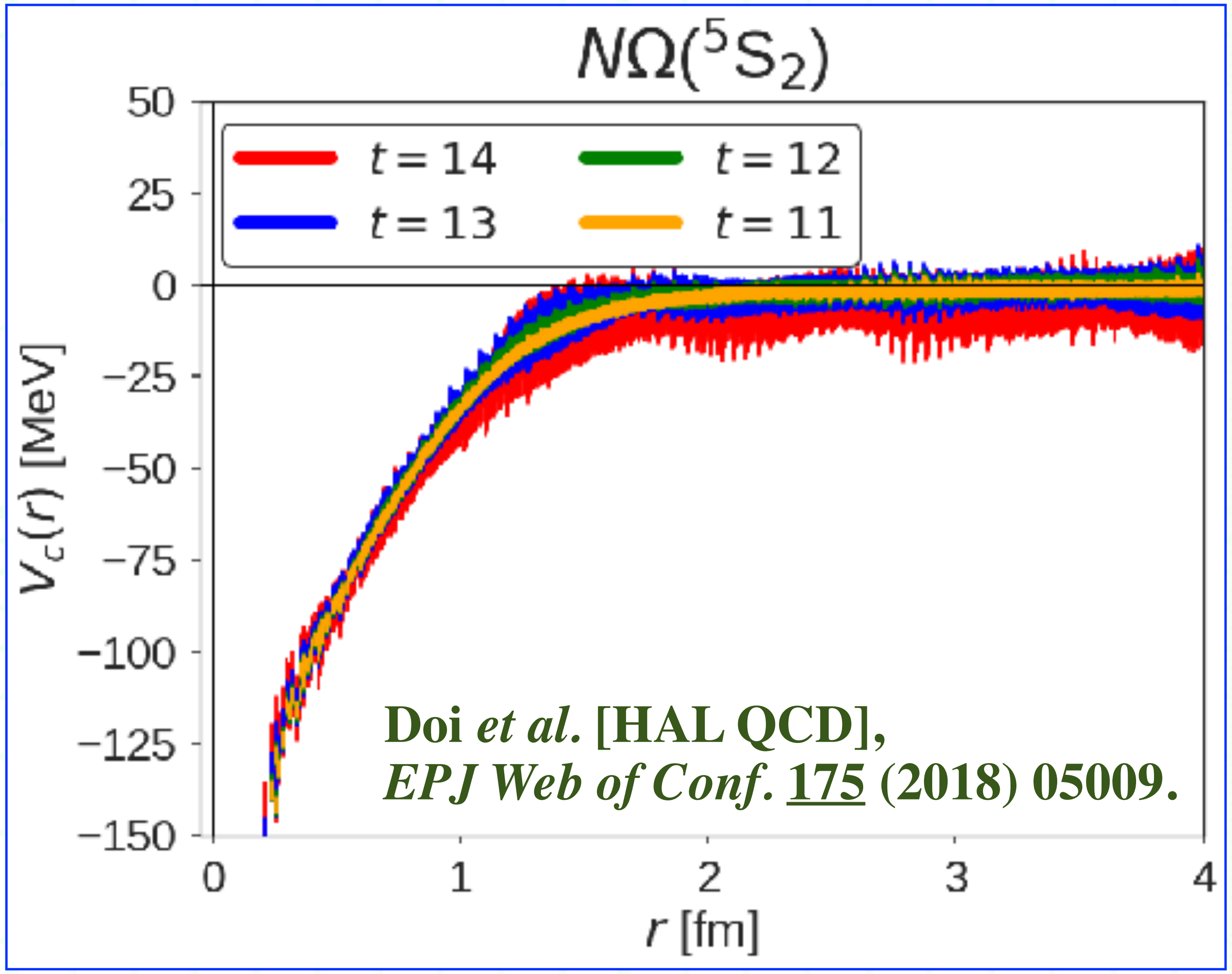
□ No repulsive core !

□ Implying $N\Omega$ bound state.

1. Introduction

++ Motivation ++

- We want to understand the $N\Omega$ ($^5S_2$) interaction.
  - What is the origin of the attraction ? <-- Physics behind it.
  - Connect lattice-QCD quark masses and physical quark masses.
  - Discuss decay modes.



- We construct a baryon-baryon interaction model with meson exchanges.
  - We expect that the meson exchange will play an important rule to generate the attraction.

2. Model construction

2. Model construction

++ $N\Omega$ and coupled channels ++

- Consider the S-wave $N\Omega$ channel of $J^P = 2^+$ and coupled channels.
  - Baryon-baryon systems in $S = -3$ & $I = 1/2$:

Channel	Threshold [MeV]
$\Lambda\Xi$	2434
$\Sigma\Xi$	2511
$N\Omega$	2611
$\Lambda\,\Xi(1530)$	2649
$\Sigma(1385)\,\Xi$	2703
$\Sigma\,\Xi(1530)$	2727
$\Sigma(1385)\,\Xi(1530)$	2918

Channel	$J^P = 2^+$
1	$N\Omega\ (^5S_2)$
2	$\Lambda\Xi\ (^3D_2)$
3	$\Lambda\Xi\ (^1D_2)$
4	$\Sigma\Xi\ (^3D_2)$
5	$\Sigma\Xi\ (^1D_2)$
6	$\Lambda\Xi^*\ (^5S_2)$

($^{2S+1}L_J$)

  - We take into account the decay channels ($\Lambda\Xi$, $\Sigma\Xi$) and one nearest closed channel [$\Lambda\Xi(1530)$] in addition to the elastic channel.

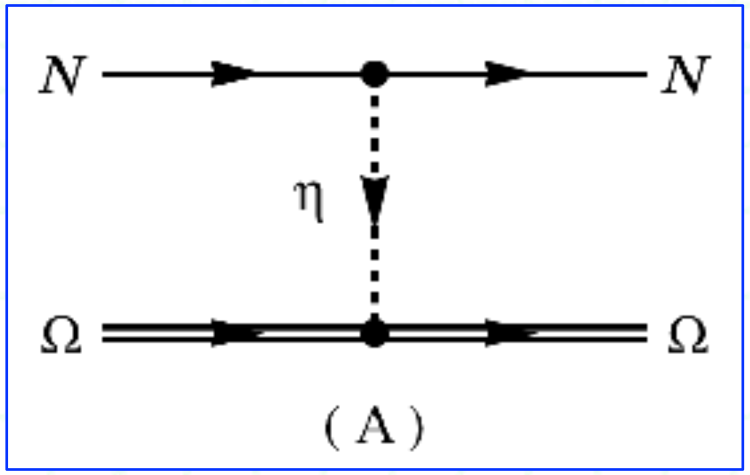
--- In particular, the $N\Omega$ ($J^P = 2^+$) couples to the decay modes $\Lambda\Xi$ and $\Sigma\Xi$ only in the D *wave*. --> Expect small decay width.

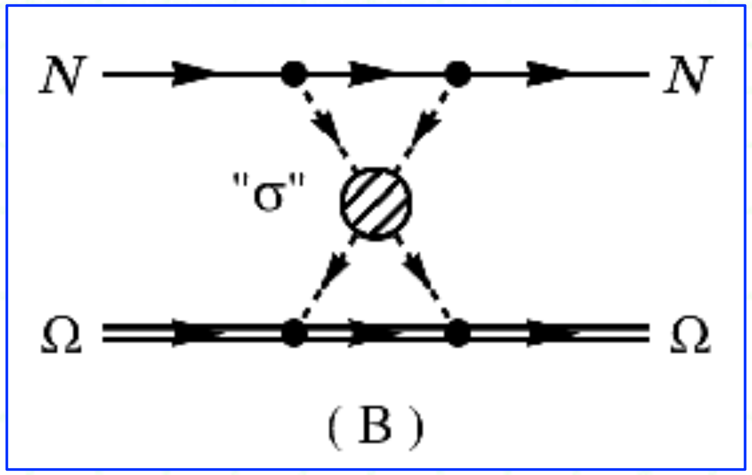
2. Model construction

++ Elastic $N\Omega$ interaction ++

- For the elastic $N\Omega$ interaction, possible mediating mesons are only those with quantum numbers $I = 0$ and Charge $= 0$:
  - Pseudoscalar: the η meson.
  - Scalar: the " σ " meson, which should be treated as the correlated two pseudoscalar mesons (*cf.* NN force).
  - Vector: NO light vector mesons. Both ω and φ cannot mediate owing to OZI rule.

- As a consequence, we have the following diagrams in the conventional meson exchange:



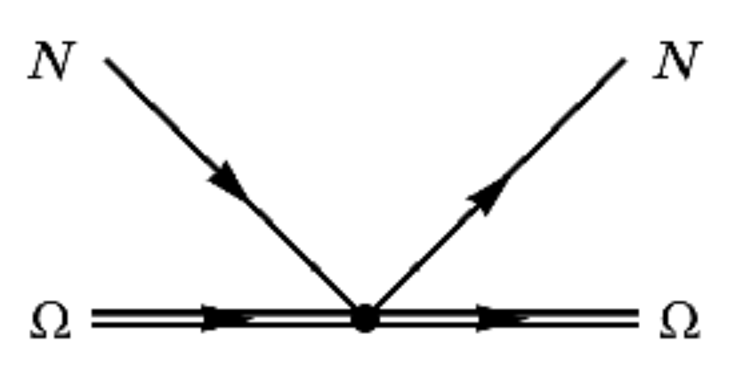


2. Model construction

++ Elastic $N\Omega$ interaction ++

- Besides, we may consider further contributions at short ranges:
  - Exchanges of heavier mesons.
  - Color magnetic interactions at quark-gluon level.
  - ...

--> They are treated as a contact term:



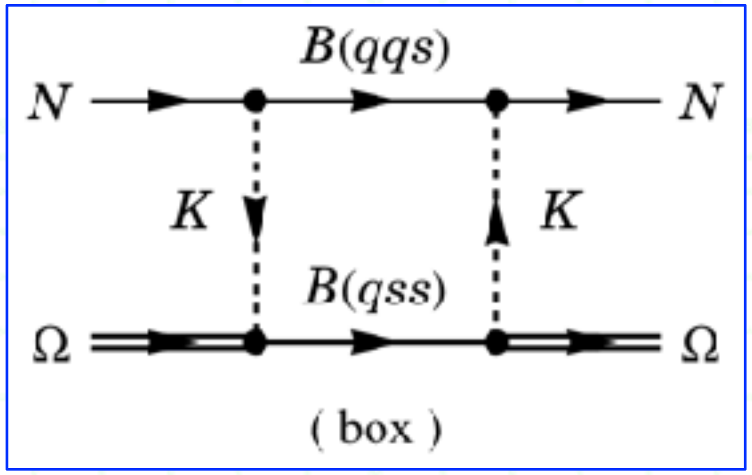
(C)

2. Model construction

++ Inelastic $N\Omega$ interaction ++

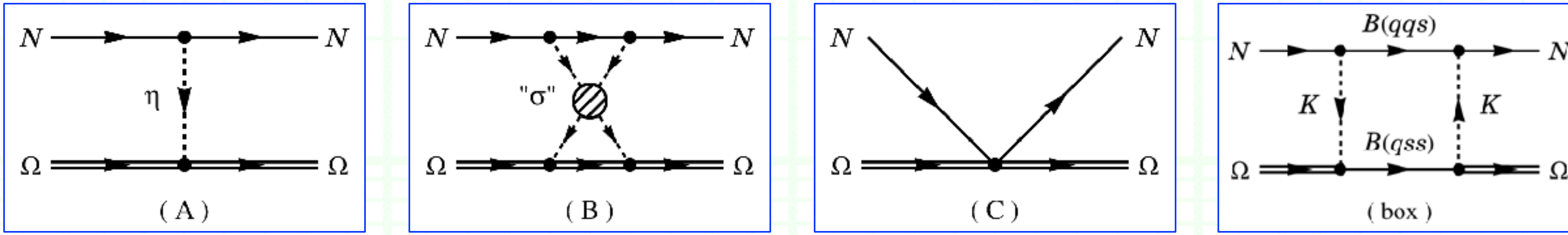
- In addition, we take into account **the inelastic channels**: **$\Lambda\Xi$, $\Sigma\Xi$, and $\Lambda\Xi^*$**.
  - We consider **the simplest** coupling: the K meson exchange.
  - To concentrate on the $N\Omega$ interaction around its threshold, we neglect the transitions between inelastic channels such as $\Lambda\Xi$ –> $\Lambda\Xi$, which will be subdominant contributions.

--> Our $N\Omega$ interaction contains **the inelastic-channel contributions as a box diagram**:



2. Model construction

++ Summary of the $N\Omega$ interaction ++



- We evaluate **the $N\Omega$ ($^5S_2$) interaction with the above four diagrams.**

$$V(E;p',p) = V_{\rm A}(p',p) + V_{\rm B}(p',p) + V_{\rm C}(p',p) + \sum_{j=2}^{6} V_{{\rm box}(j)}(E;p',p)$$

--- Note: **Non-local** interaction.

The interaction has **energy dependence** coming from the **box** terms, where **the inelastic channels are integrated out**.

Channel	$J^P = 2^+$
1	$N\Omega$ ($^5S_2$)
2	$\Lambda\Xi$ ($^3D_2$)
3	$\Lambda\Xi$ ($^1D_2$)
4	$\Sigma\Xi$ ($^3D_2$)
5	$\Sigma\Xi$ ($^1D_2$)
6	$\Lambda\Xi^*$ ($^5S_2$)

2. Model construction

++ Effective Lagrangians ++

- Each vertex is governed by **effective Lagrangians**.

□ ***MBB* coupling**: $\mathcal{L}_{MBB} = -\frac{F}{\sqrt{2}f}\langle \bar{\mathcal{B}}\gamma^{\mu}\gamma_5 [\partial_\mu \Phi, \mathcal{B}]\rangle - \frac{D}{\sqrt{2}f}\langle \bar{\mathcal{B}}\gamma^{\mu}\gamma_5 \{\partial_\mu \Phi, \mathcal{B}\}\rangle$

--- Parameter fixed by the octet-baryon decay.

□ ***MBD* coupling**: $\mathcal{L}_{MBD} = -\frac{f_{MBD}}{m_\pi}\langle (\bar{\Delta}_\mu \cdot \partial^\mu \Phi) \mathcal{B} + \text{h.c.}\rangle$

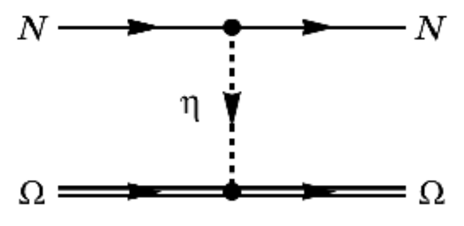
--- Parameter fixed by the decuplet-baryon decay.

□ ***MDD* coupling**: $\mathcal{L}_{MDD} = -\frac{f_{MDD}}{m_\pi}\langle (\bar{\Delta}^\mu \cdot \gamma^\nu \gamma_5 \Delta_\mu)\partial_\nu \Phi\rangle$

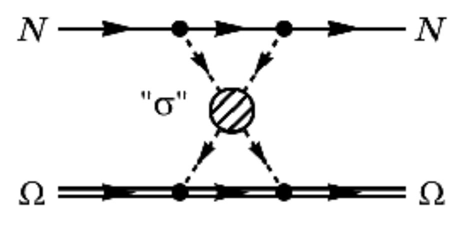
--- Parameter fixed by the *SU*(6) quark model.

□ **Contact coupling**: $\mathcal{L}_{\text{contact}} = c\,(\bar{\Omega}\Omega)\,(\bar{p}p + \bar{n}n)$

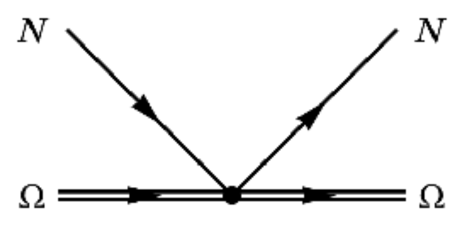
--- **Model parameter *c*.**



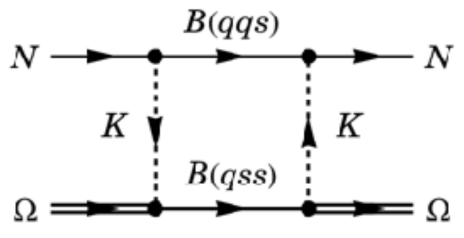
(A)



(B)



(C)



(box)

2. Model construction

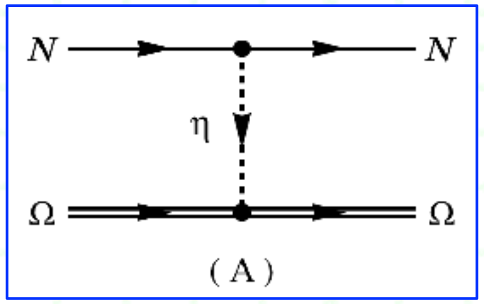
++ Details of the interaction ++

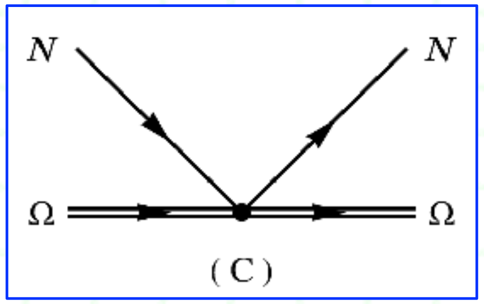
- We introduce **a form factor for every diagram**:

$$F(q)^2 = \left(\frac{\Lambda^2}{\Lambda^2 + q^2}\right)^2$$

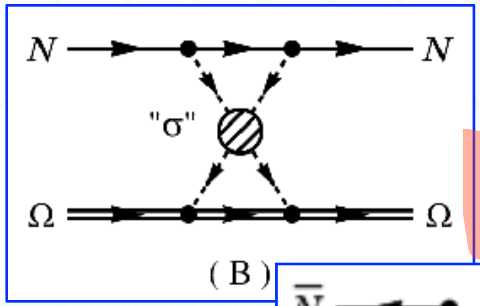
--- Cut-off is fixed to be a hadronic scale: $\Lambda = 1$ GeV.

- The η exchange and contact interactions are straightforwardly calculated.

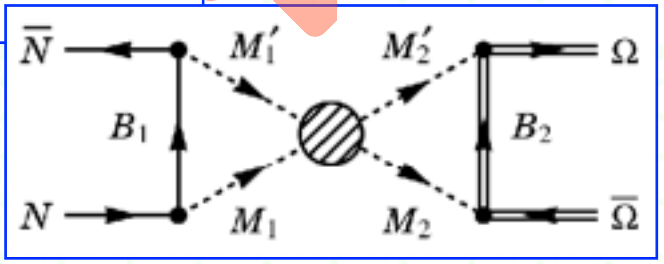




- The "σ" exchange interaction is evaluated with the dispersion relation of $N\bar{N} \to \Omega\bar{\Omega}$.



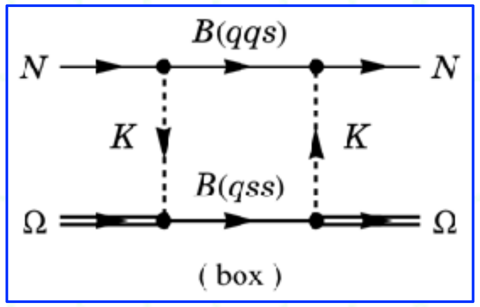
$$\mathrm{Im}T_{N\bar{N}\to\Omega\bar{\Omega}}(t) = \sum_{j=\pi\pi, K\bar{K}, \eta\eta} \rho_j(t) T_{N\bar{N}\to j}(t) T^*_{\Omega\bar{\Omega}\to j}(t)$$



- The box interaction is:

$$V_{\mathrm{box}(j)}(E; p', p) = \int_0^\infty \frac{dp''}{2\pi^2} p''^2 \frac{V_{1j}(p', p'') V_{j1}(p'', p)}{E - \mathcal{E}_j(p'') + i0}$$

$$\mathcal{E}_j(p) = \sqrt{p^2 + m_j^2} + \sqrt{p^2 + m_j'^2}$$

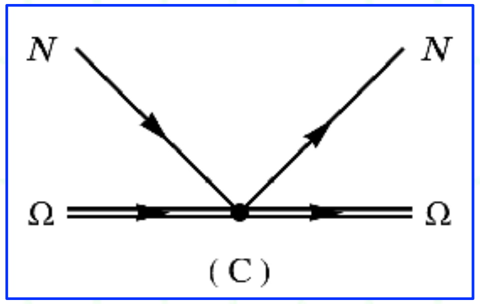


2. Model construction

++ Model parameter ++

- In our model, **only the contact coupling constant c is a free parameter.**

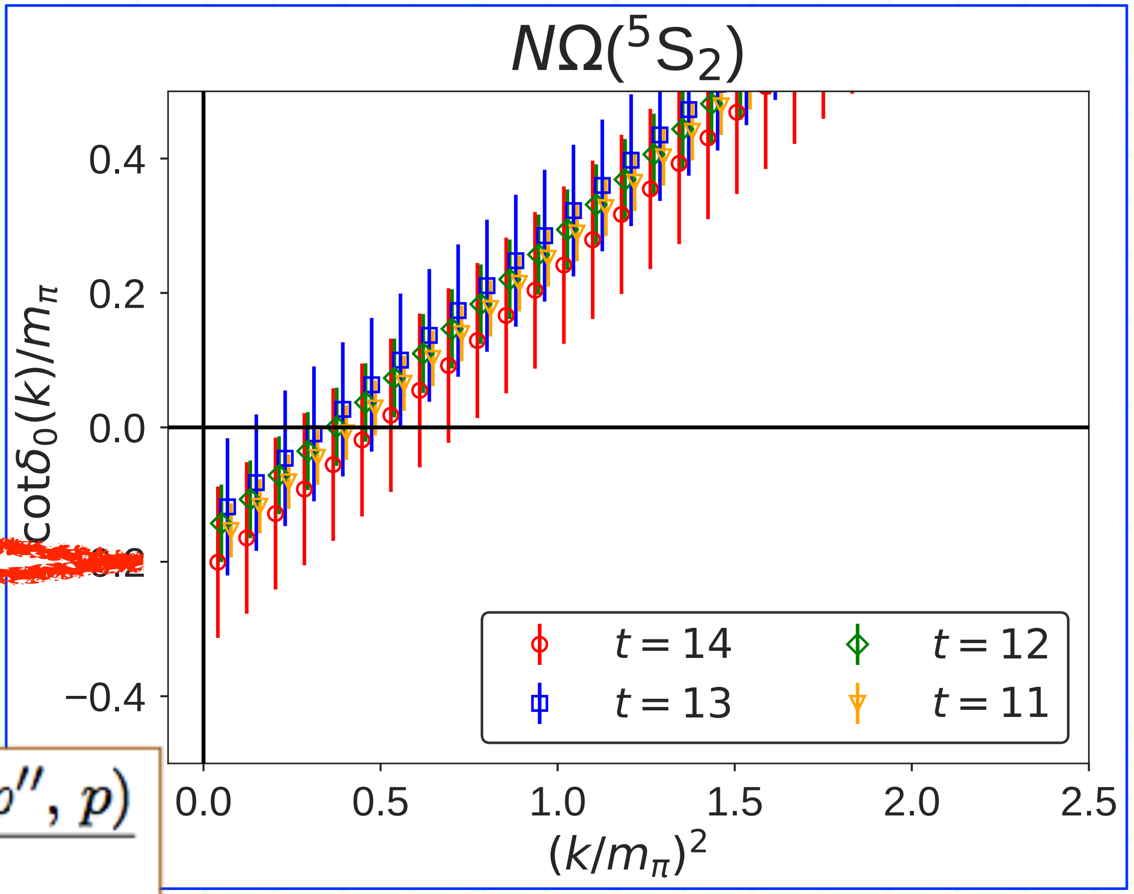
$$\mathcal{L}_{\rm contact} = c\,(\bar{\Omega}\Omega)\,(\bar{p}p + \bar{n}n)$$



--- Fixed by information from recent HAL QCD analysis !

- We reproduce **the scattering length of the HAL QCD analysis on $N\Omega$ ($^5S_2$).**

Scattering length
$a = 7.4 \pm 1.6$ fm at $t = 11$.



Doi *et al.* [HAL QCD], *EPJ Web of Conf.* 175 (2018) 05009.
* HAL QCD's scattering length has opposite sign compared to ours.

--- Scattering Amp. at the threshold:

$$T(E;\,p',\,p) = V(E;\,p',\,p) + \int_0^\infty \frac{dp''}{2\pi^2}p''^2\frac{V(E;\,p',\,p'')T(E;\,p'',\,p)}{E-\mathcal{E}_{N\Omega}(p'')}$$

--- But with the nearly physical quark masses on the lattice.

--> We fix $c = -22.1\ \mathrm{GeV}^{-2}$ to reproduce $a = 7.4$ fm with lattice masses.

3. Properties of the $N\Omega$ interaction

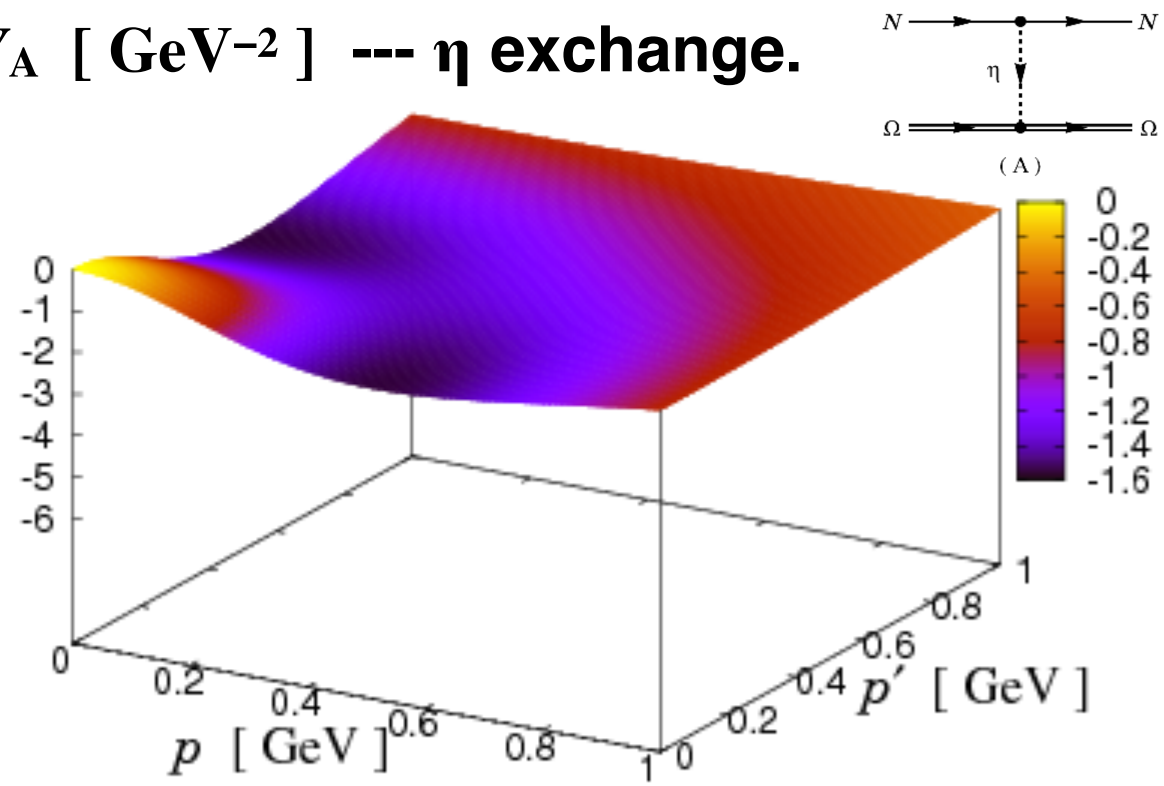
3. Properties of the $N\Omega$ interaction

++ Elastic $N\Omega$ interaction ++

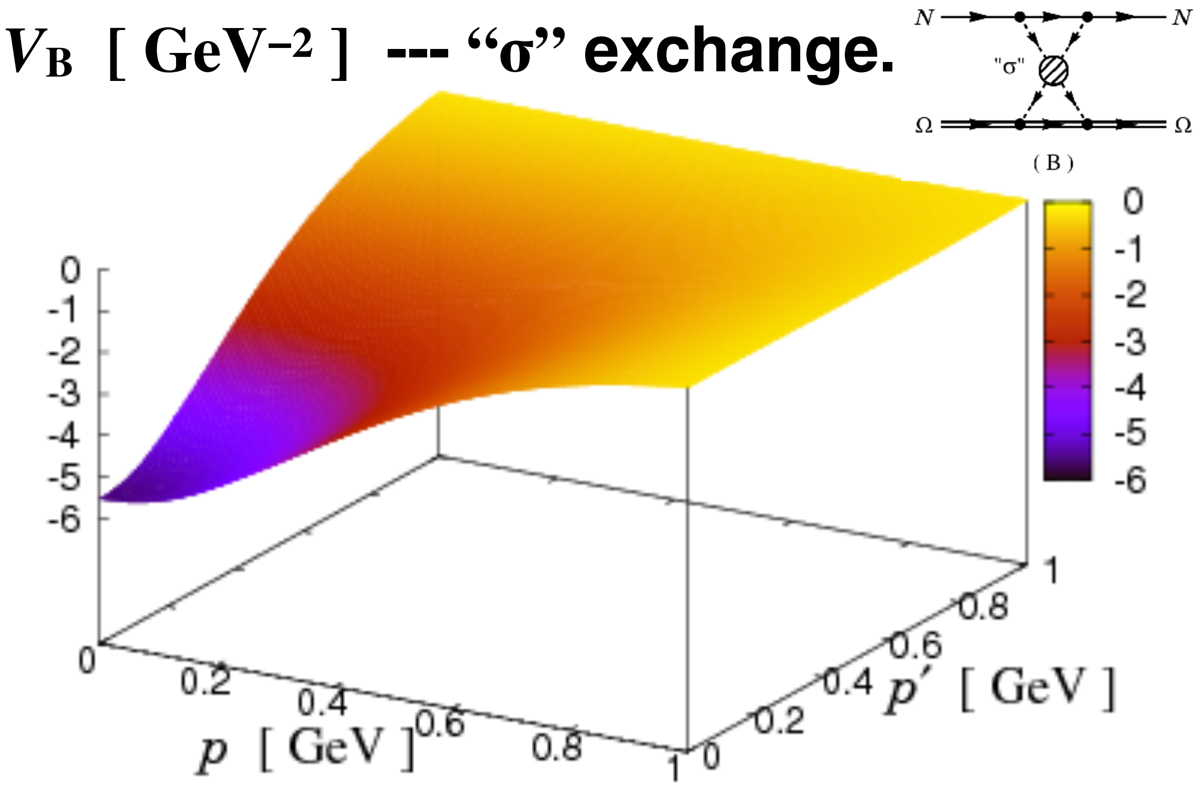
- First, we show **the $N\Omega$ ($^5S_2$) interaction in elastic channels**:

$$V(E;p',p) = V_{\rm A}(p',p) + V_{\rm B}(p',p) + V_{\rm C}(p',p) + \sum_{j=2}^{6} V_{{\rm box}(j)}(E;p',p)$$

$V_{\rm A}$ [GeV^{-2}] --- η exchange.



$V_{\rm B}$ [GeV^{-2}] --- "σ" exchange.

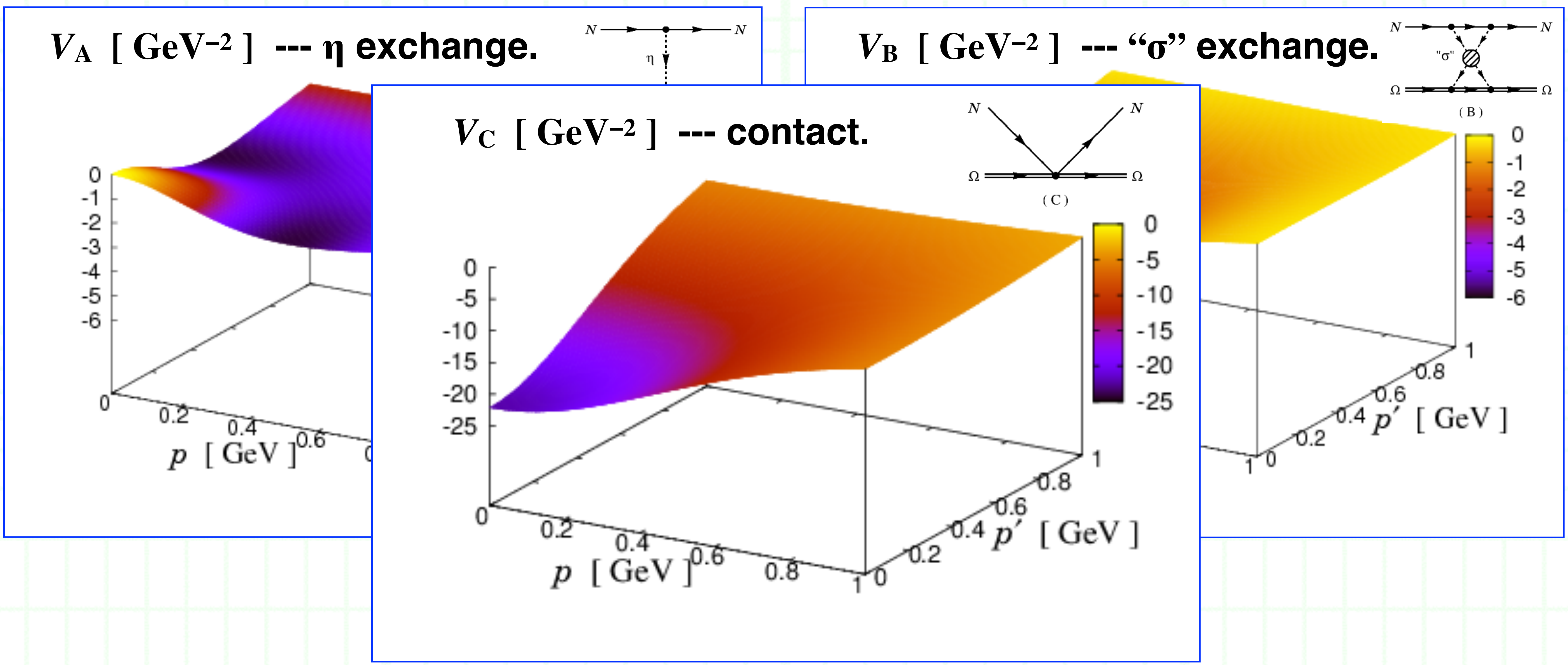


3. Properties of the $N\Omega$ interaction

++ Elastic $N\Omega$ interaction ++

- First, we show **the $N\Omega$ ($^5S_2$) interaction in elastic channels**:

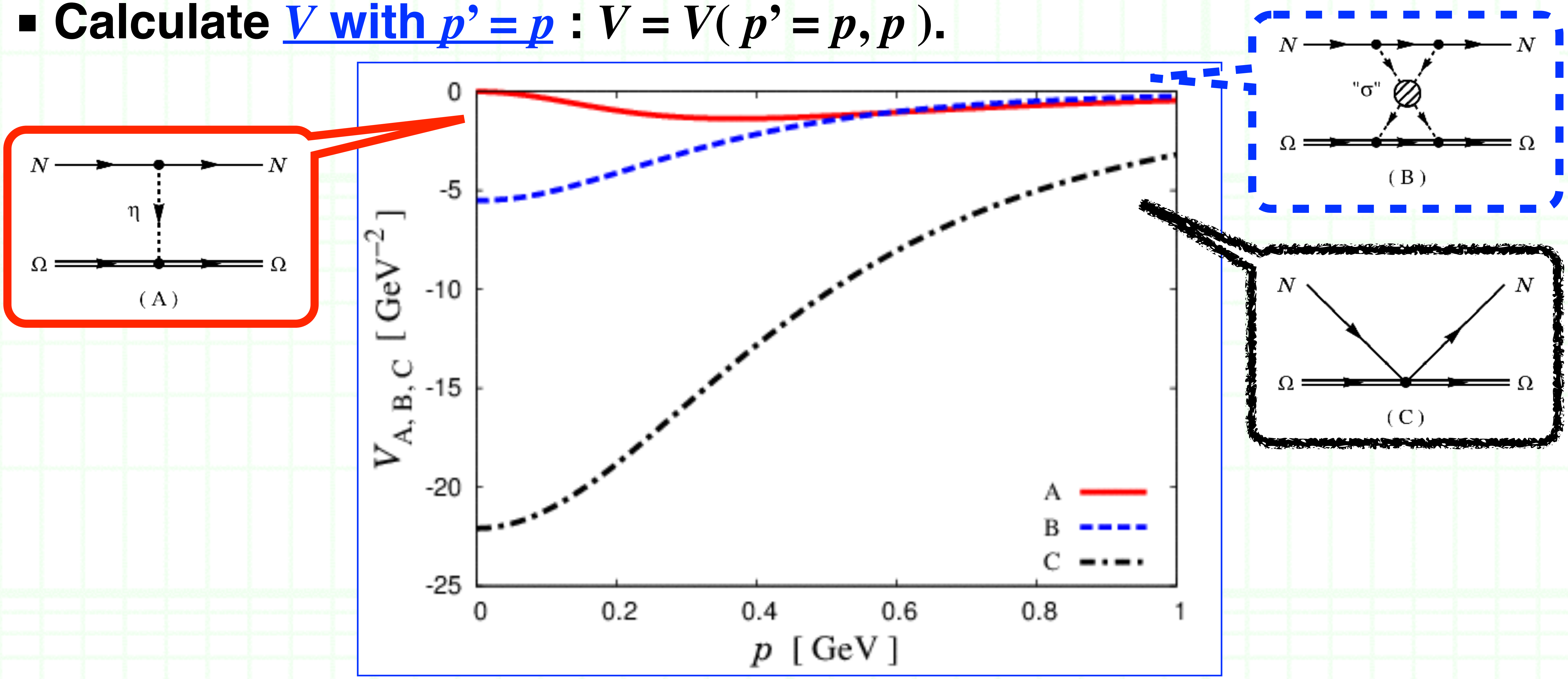
$$V(E;p',p) = V_{\rm A}(p',p) + V_{\rm B}(p',p) + V_{\rm C}(p',p) + \sum_{j=2}^{6} V_{{\rm box}(j)}(E;p',p)$$



3. Properties of the $N\Omega$ interaction

++ Elastic $N\Omega$ interaction ++

- **Calculate V with $p' = p$: $V = V(p' = p, p)$.**



- **The contact term is dominant.** --- This includes the parameter.
- **The η meson gives moderate attraction due to small ηNN coupling.**
- **The "σ" exchange is also moderate due to small "σ" ΩΩ coupling.**

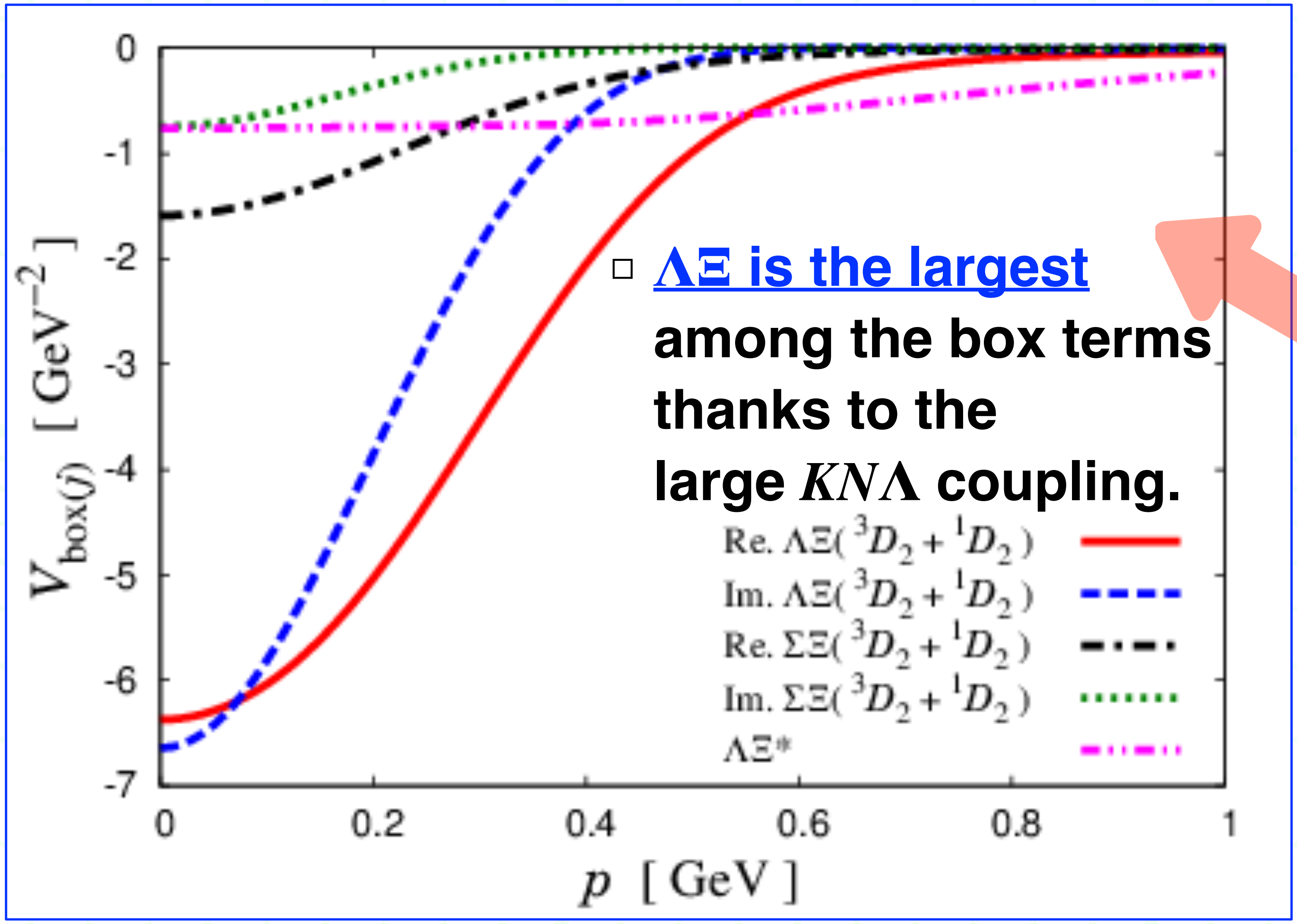
3. Properties of the $N\Omega$ interaction

++ Inelastic $N\Omega$ interaction ++

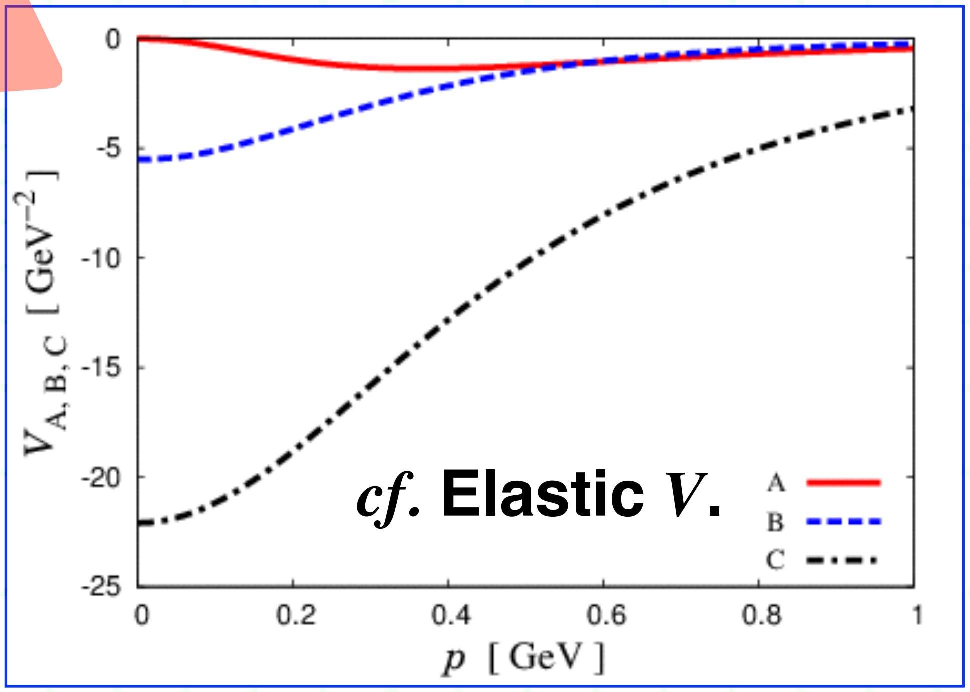
- Next, we show **the $N\Omega$ ($^5S_2$) interaction from inelastic channels**:

$$V(E;p',p) = V_{\rm A}(p',p) + V_{\rm B}(p',p) + V_{\rm C}(p',p) + \sum_{j=2}^{6} V_{{\rm box}(j)}(E;p',p)$$

- Calculate $V_{\rm box}$ with $p' = p$ and $E = m_N + m_\Omega$: $V = V_{\rm box}(m_N + m_\Omega; p' = p, p)$.



□ The attraction by inelastic channels is similar strength to the η / "σ" exchange.

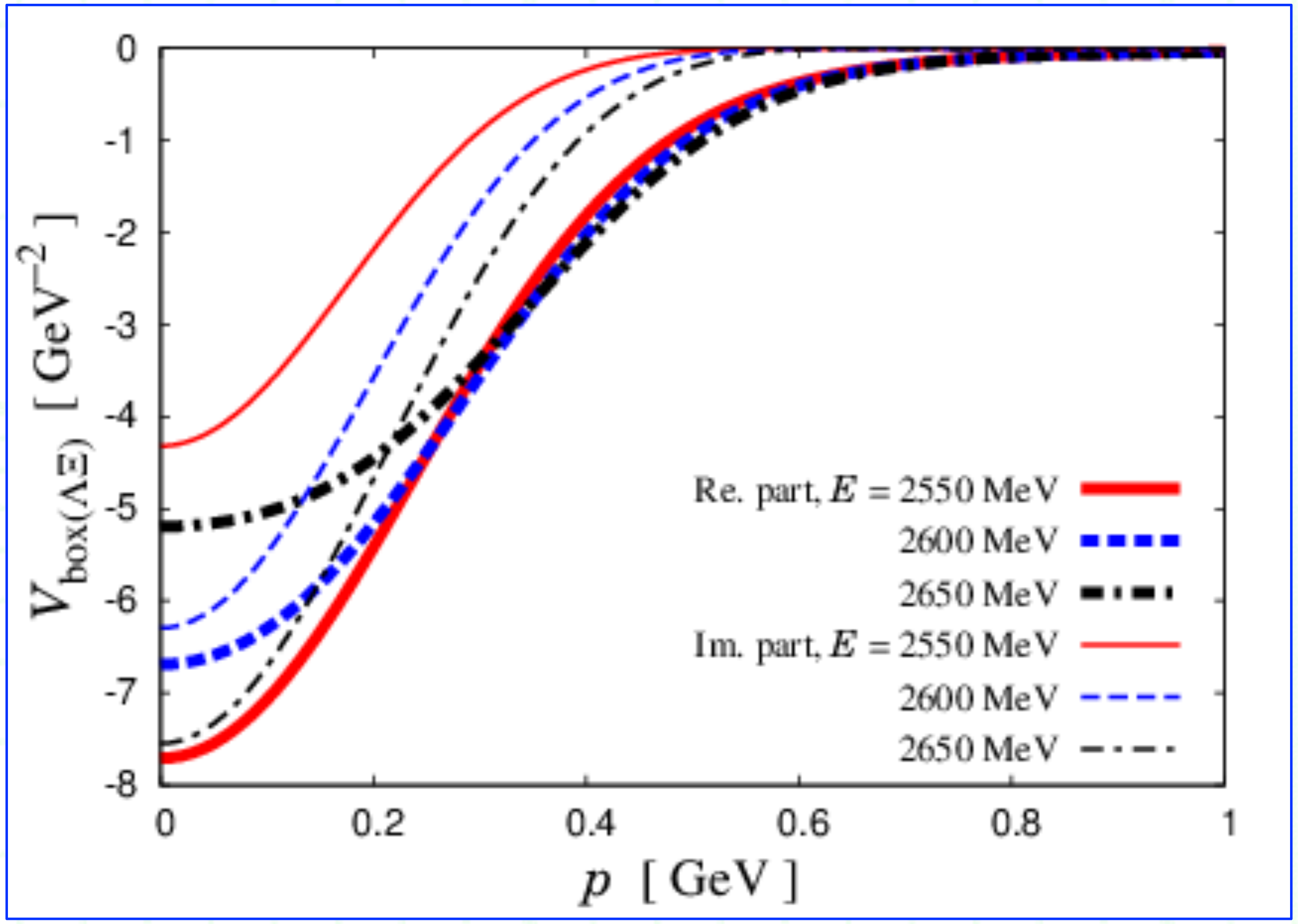


3. Properties of the $N\Omega$ interaction

++ Inelastic $N\Omega$ interaction ++

- Next, we show **the $N\Omega$ ($^5S_2$) interaction from inelastic channels**:

$$V(E; p', p) = V_A(p', p) + V_B(p', p) + V_C(p', p) + \sum_{j=2}^{6} V_{box(j)}(E; p', p)$$



- We change the energy E.

--- **The energy dependence** of the box interaction is **not significant.**

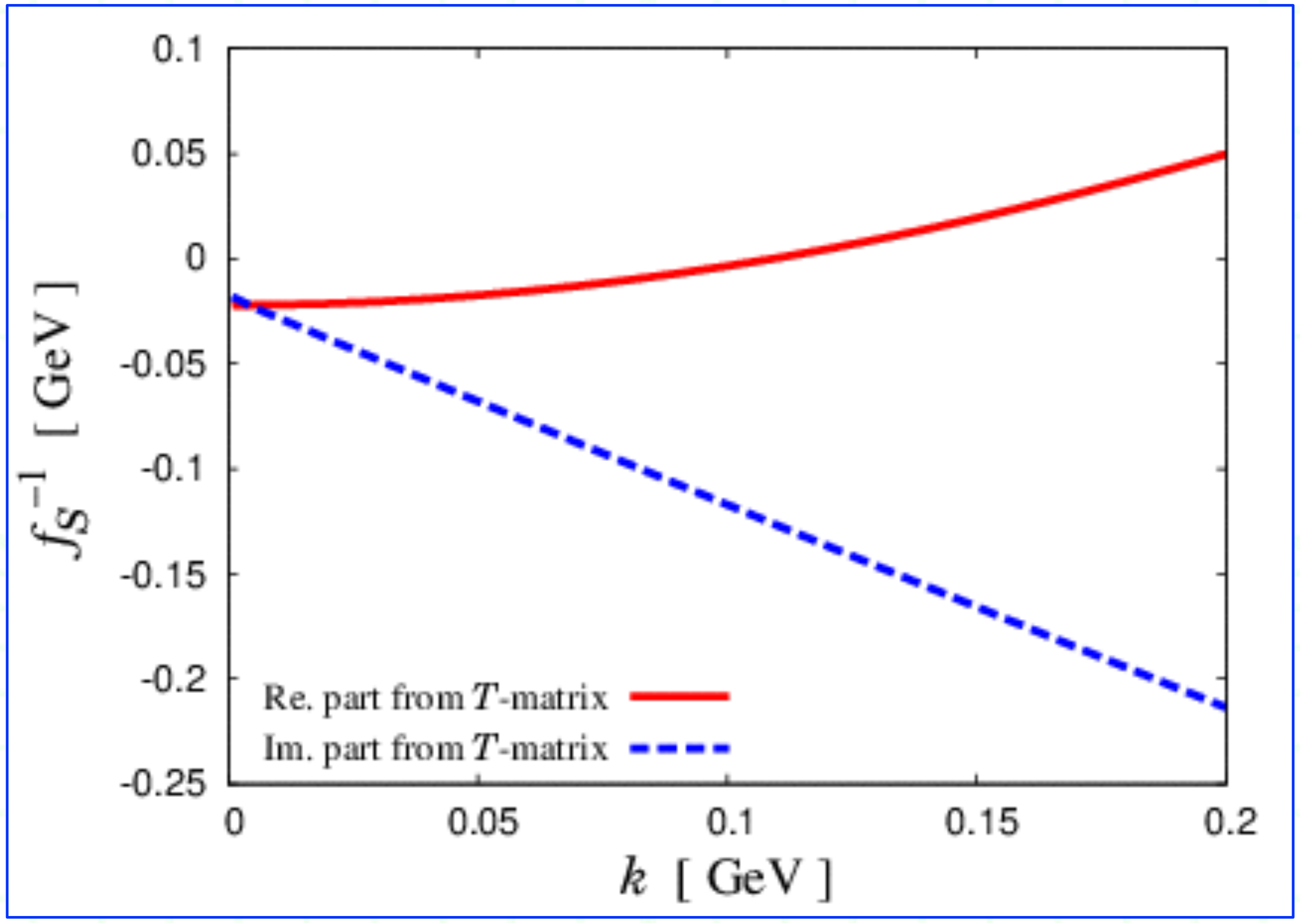
3. Properties of the $N\Omega$ interaction

++ $N\Omega(^5S_2)$ scattering amplitude ++

- **Information of the $N\Omega$ ($^5S_2$) system is reflected in its scattering amplitude f_S as a function of relative momentum k:**

$$T(E;p',p) = V(E;p',p) + \int_0^\infty \frac{dp''}{2\pi^2} p''^2 \frac{V(E;p',p'')T(E;p'',p)}{E - \mathcal{E}_{N\Omega}(p'')}$$

--> $f_S(k) = (\text{kinematical factor}) \times T_{\text{on-shell}}$



- **Threshold parameters of the $N\Omega$ ($^5S_2$) scattering:**
  - □ **Scattering length a:**

$$a = -f_S(k=0) = 5.3 - 4.3i \text{ fm}$$

--- Complex due to decay Chan.
Positive real part implies existence of a bound state.

  - □ **Effective range r_{eff}:**

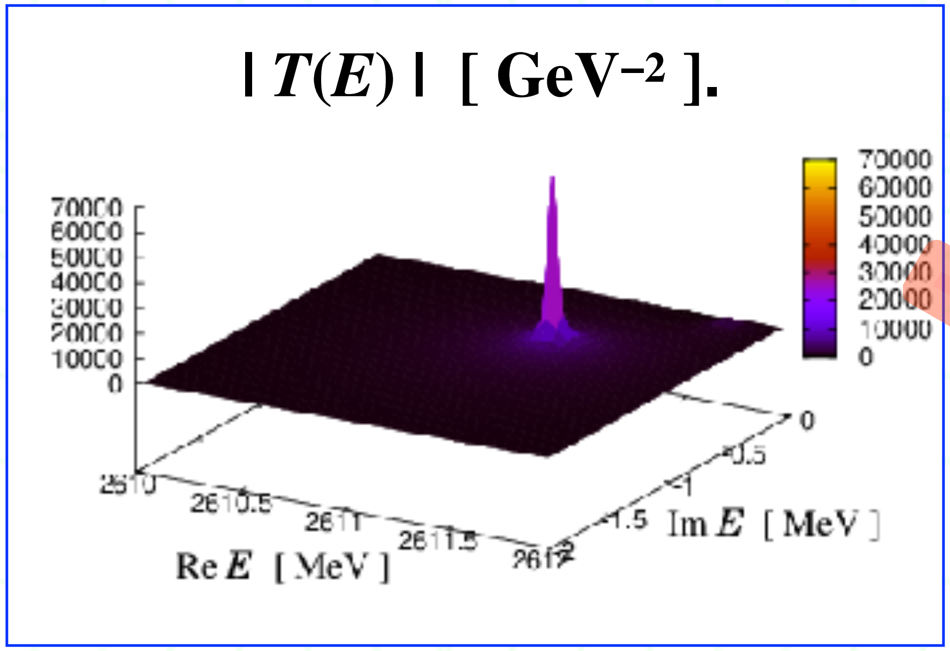
$$r_{\text{eff}} = \left[\frac{d^2 f_S^{-1}}{dk^2}\right]_{k=0} = 0.74 + 0.04i \text{ fm}$$

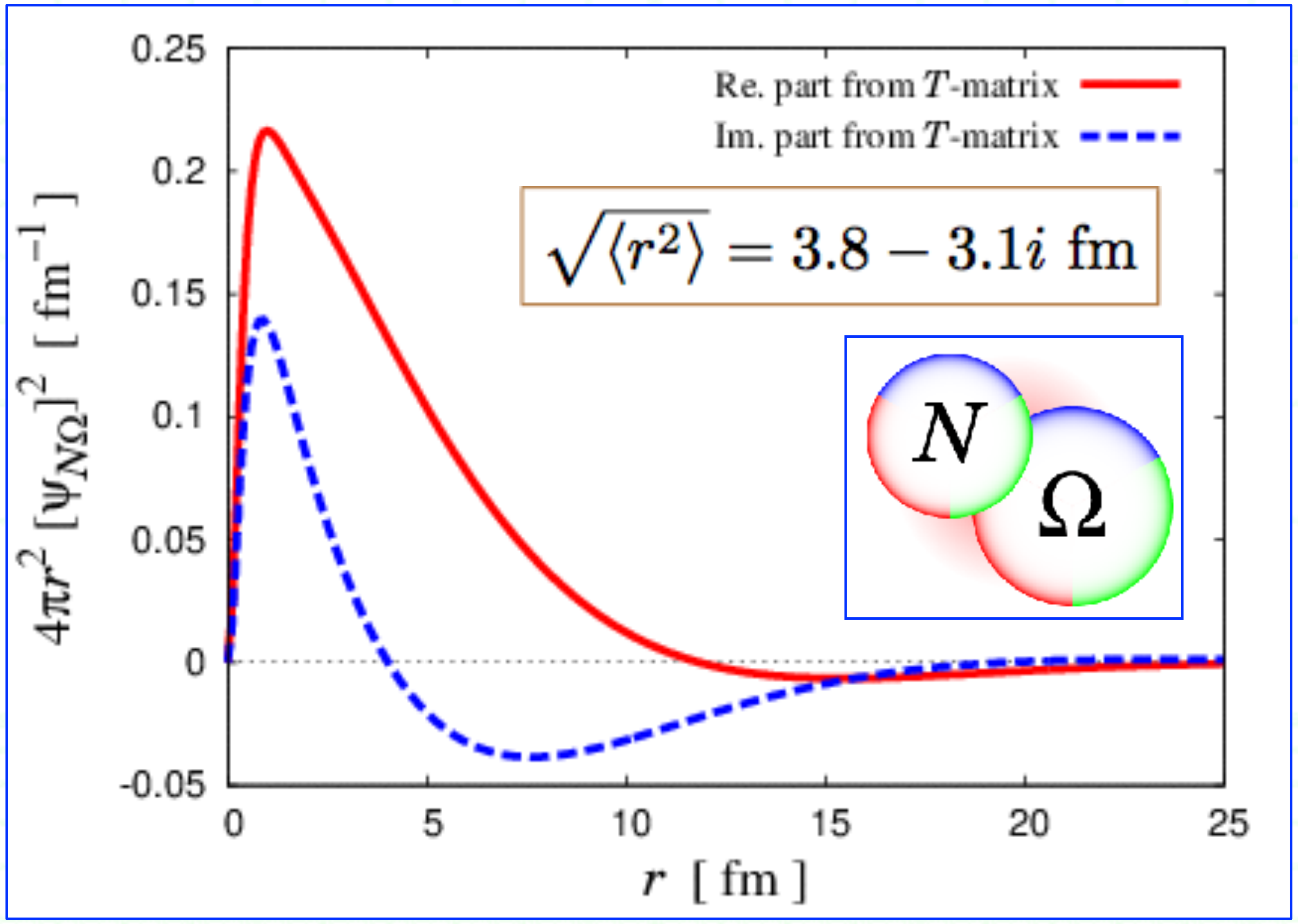
--- Almost real.

3. Properties of the $N\Omega$ interaction

++ $N\Omega(^5S_2)$ quasi-bound state ++

- Indeed, **the $N\Omega$ $(^5S_2)$ scattering amplitude contains a resonance pole** which **corresponds to the $N\Omega$ $(^5S_2)$ quasi-bound state** !





- **Pole** at $E_{pole} = 2611.3 - 0.7\,i$ MeV.

<--> $B_E = 0.1$ MeV, $\Gamma = 1.5$ MeV.

- From the residue at the pole, we can extract **the bound-state wave function $\psi_{N\Omega}$** (see figure).

- For the $p\Omega^-$ state, **the Coulomb interaction will assist**:

$\Delta B_{\mathrm{Coulomb}} \sim 1$ MeV $\quad \Delta\Gamma_{\mathrm{Coulomb}} \sim 1$ MeV

3. Properties of the $N\Omega$ interaction

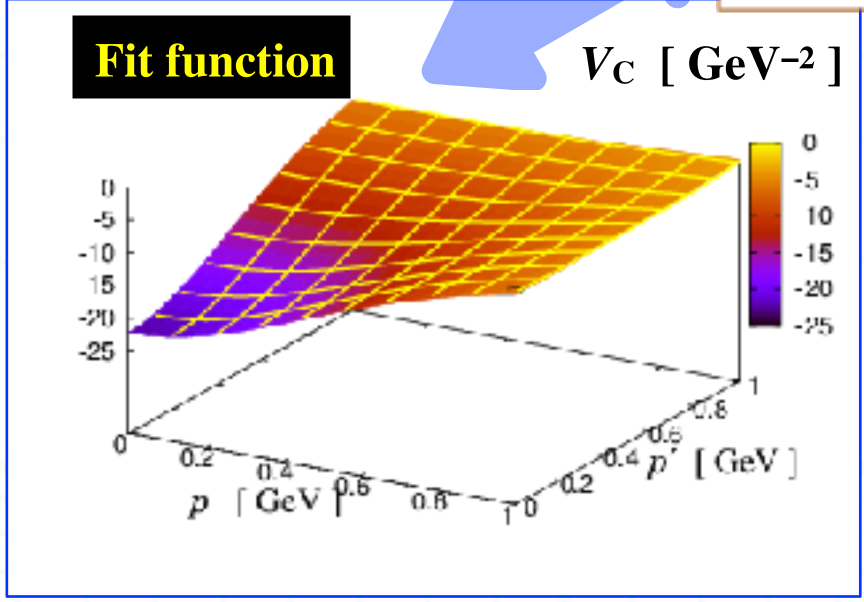
++ Equivalent local potential ++

- **Our $N\Omega$ ($^5S_2$) interaction is non-local.**

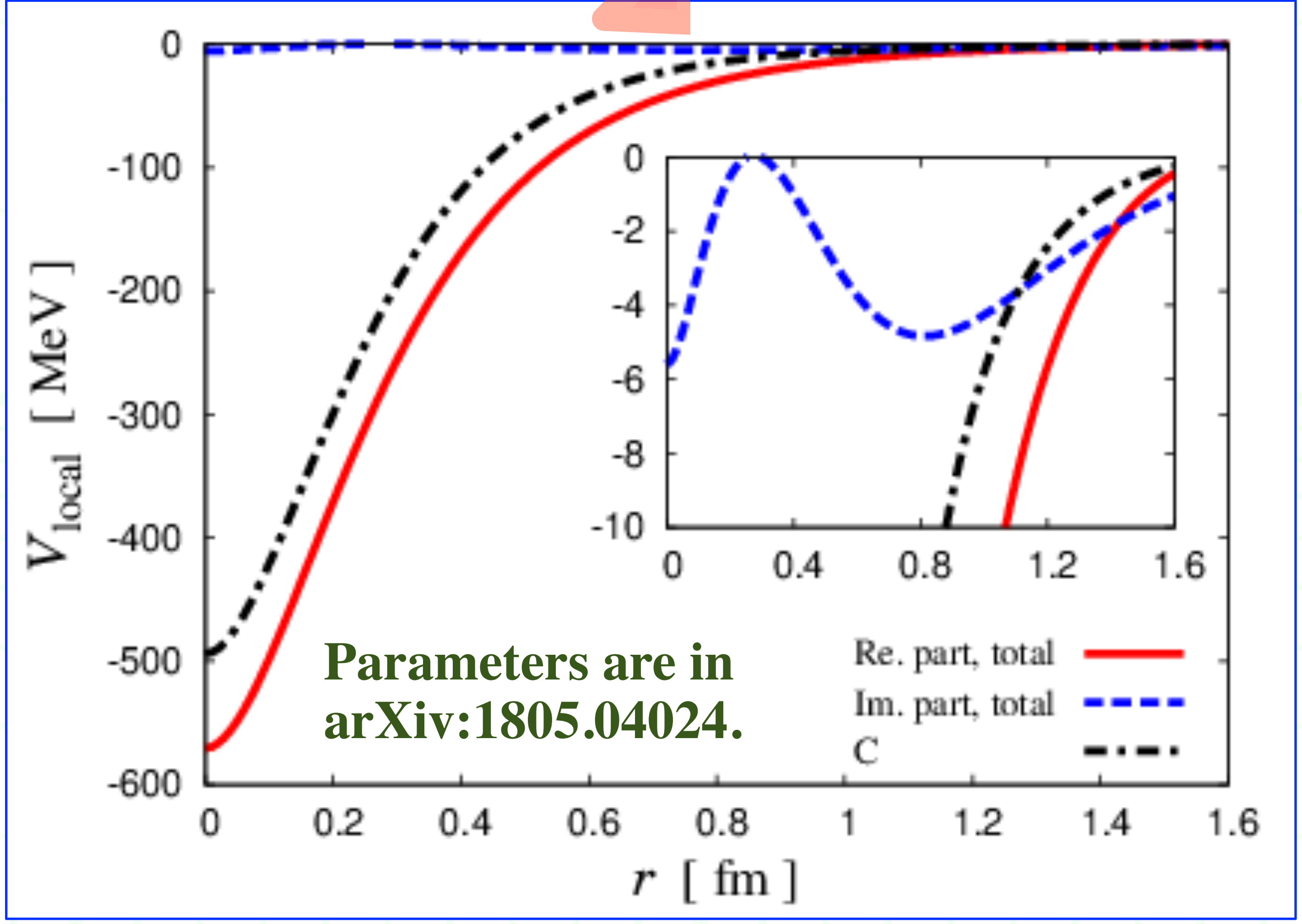
--> We construct a local potential as the sum of Yukawa potentials which is fitted to our $N\Omega$ ($^5S_2$) interaction.

$$\tilde{V}_{\rm local}(q) = \sum_{n=1}^{9} \frac{C_n}{q^2+m_n^2}\left(\frac{\Lambda^2}{\Lambda^2+q^2}\right)^2$$

S-wave projection & parameter fitting.



Coordinate space.



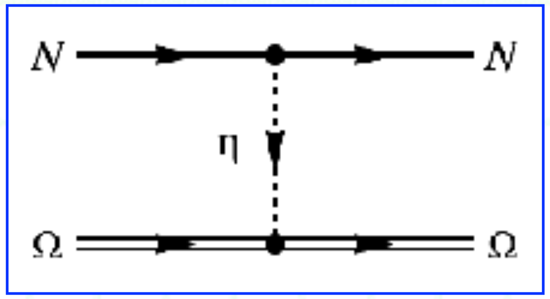
- **The local Pot. reproduces $N\Omega$ ($^5S_2$) properties very well.**

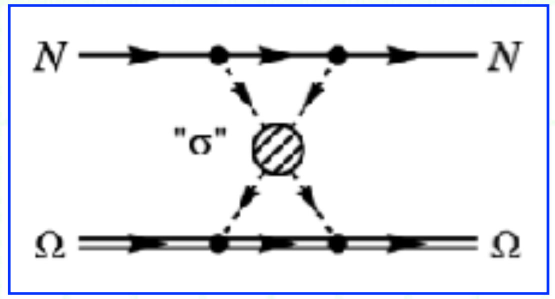
--- Why don't you use to calculate Ω-nucleon(s) systems ?

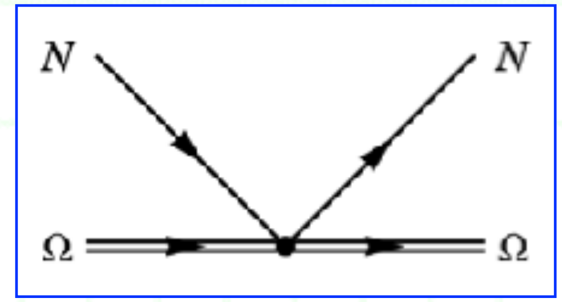
4. Summary and outlook

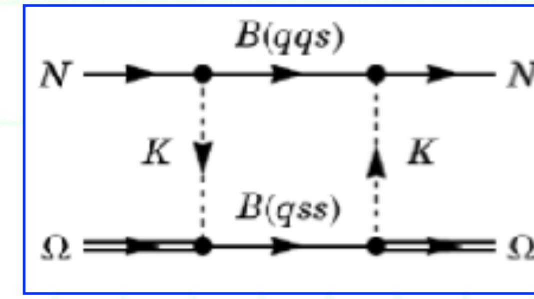
4. Summary and outlook

- We constructed the $N\Omega$ ($^5S_2$) interaction according to the diagrams:









  - The conventional exchanges of the η, "σ", and K (in terms of box) mesons do not provide sufficient attraction.
  - Most of the attraction indicated in recent lattice QCD simulations is attributed to the short-range contact interaction.

- Fitting parameter (contact coupling constant only) to scattering length in HAL QCD, we obtain the $N\Omega$ ($^5S_2$) quasi-bound state.
  - $E_{\text{pole}} = 2611.3 - 0.7\,i$ MeV. --- $B_E = 0.1$ MeV, $\Gamma = 1.5$ MeV.
  - For the $p\Omega^-$ state, the Coulomb interaction will assist B_E and Γ.
  - $a = 5.3 - 4.3\,i$ fm, $r_{\text{eff}} = 0.74 + 0.04\,i$ fm.

- Can we find the $N\Omega$ bound state in heavy-ion collisions ... ?

Morita *et al.*, *Phys. Rev.* C94 (2016) 031901.

Thank you very much for your kind attention !

Appendix

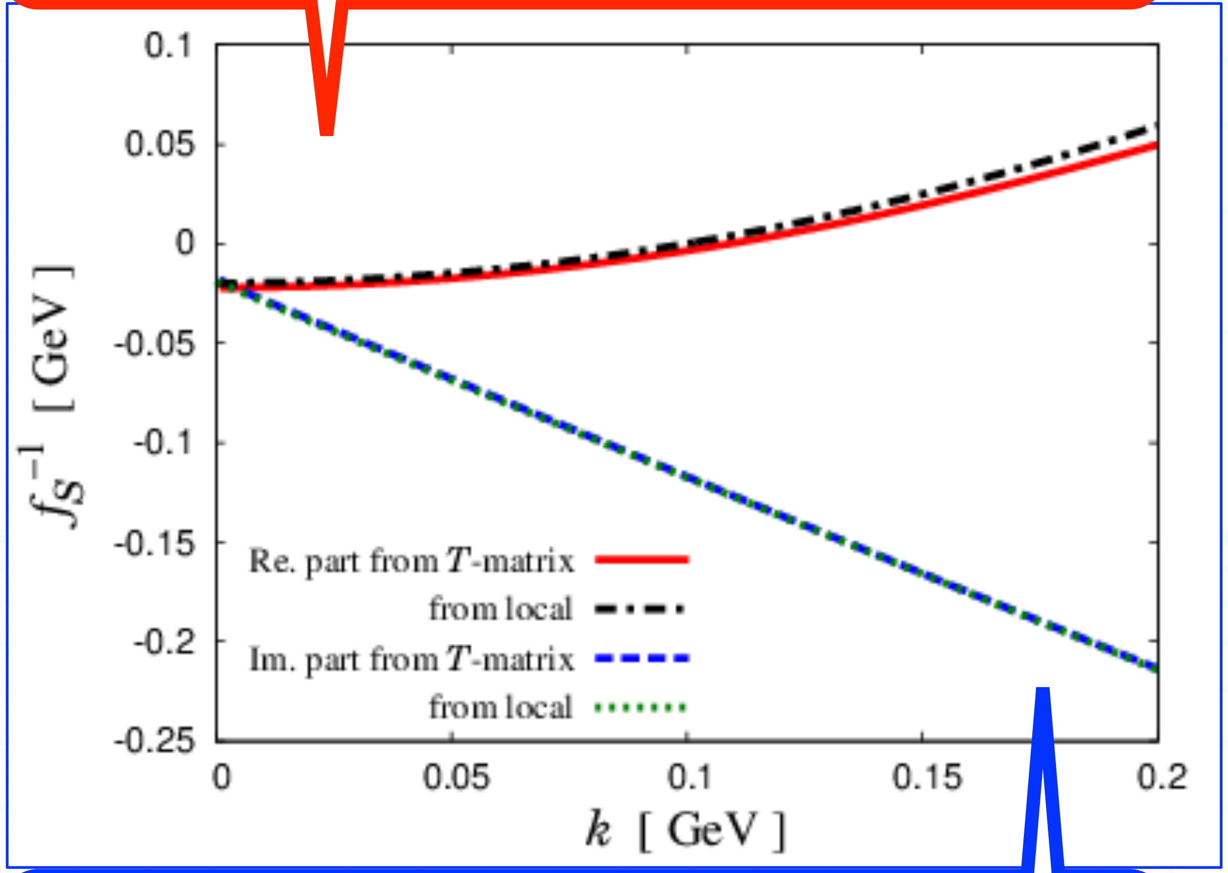
Appendix

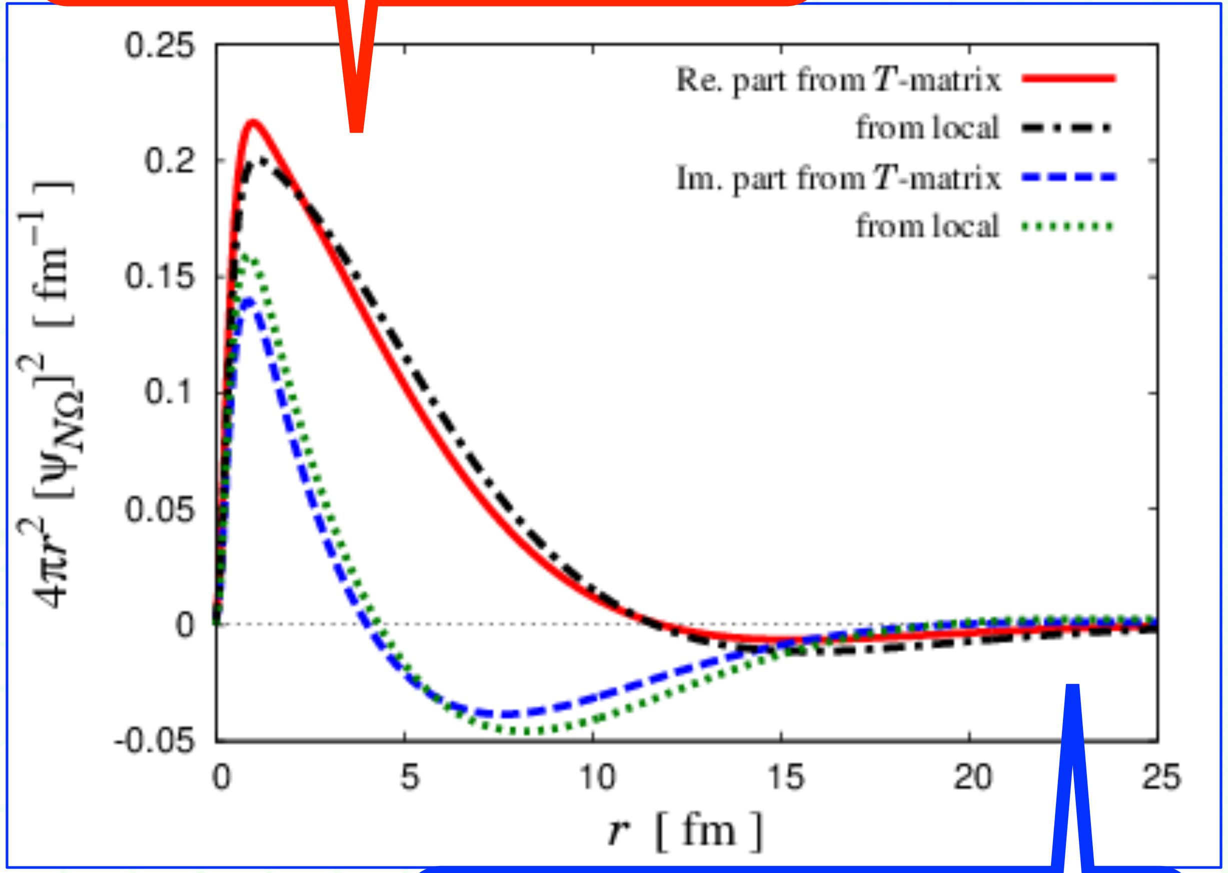
++ Properties of $N\Omega$ from the local potential ++

- We check that our local $N\Omega(^5S_2)$ potential reproduces the properties of the $N\Omega(^5S_2)$ system from the T-matrix.

From T-matrix:
$a = 5.3 - 4.3\,i$ fm, $r_{\rm eff} = 0.74 + 0.04\,i$ fm.

From T-matrix:
$E_{\rm pole} = 2611.3 - 0.7\,i$ MeV.





From local potential:
$a = 5.2 - 5.0\,i$ fm, $r_{\rm eff} = 0.78 + 0.06\,i$ fm.

From local potential:
$E_{\rm pole} = 2611.4 - 0.7\,i$ MeV.

Appendix

++ Comparison with the HAL QCD potential ++

- We compare **our local $N\Omega(^5S_2)$ potential** with **the HAL QCD potential of nearly physical quark masses**.