The $N\Omega$ Interaction:
Meson Exchanges, Inelastic Channels, and Quasi-Bound State

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1. Introduction
1. Introduction

++ What is dibaryon? ++

- **Dibaryons**: States of baryons number $B = 2$
  generated by strong interactions.

--- Regardless of their structure.

- There is only single **well-established**
  dibaryon, the deuteron, which is
  a proton-neutron molecule.

Weinberg (1965).

---

Field renormalization constant! (1965).

---

Compact hexa-quarks

Hadronic two-body molecules

Meson-assisted dibaryon
1. Introduction

++ Why dibaryons ? ++

- Motivations to study dibaryons:
  - First of all, **does such “exotic” states exist or not ?**
    - New forms of hadrons / nuclei.

- Compact hexa-quarks:
  - How the quark-confinement mechanism work ?
  - Compare with typical hadrons.
    - Properties of constituent quarks (such as mass $M_q \sim 300$ Me) are different from those in typical hadrons ?

- Hadronic molecules (including meson-assisted dibaryons):
  - Information on the hadron-hadron interaction.
  - New few-body systems.
1. Introduction

++ Theor. predictions / Exp. implications ++

- Many Theor. predictions / Exp. implications on the dibaryons.
  - Implications by recent HAL QCD method.
    - K. Sasaki et al. [HAL QCD], PoS LATTICE 2016; ...
    - Seen near the $N\Xi$ threshold ???
  - $\bar K N N$.
    - Bound by the strongly attractive $\bar K N$ interaction.
      Akaishi-Yamazaki (2003); Dote-Hyodo-Weise (2008); ...
    - “Peak” seen in FINUDA, J-PARC E27 & E15 etc.
      Agnello et al. (2005); Ichikawa et al. (2015); Sada et al. (2016); ...
    - ??
  - $d^*(2380)$ [ $I (J^P) = 0 (3^+) $ ].
    - Found in the $p n \rightarrow d \pi \pi$ reaction. WASA-at-COSY (2011).
    - $\Delta\Delta$ molecules ??? Dyson-Xuong (1964).
  - ...

Recent Developments in Quark-Hadron Sciences @ Yukawa Inst., Kyoto Univ. (Jun. 11 - 15, 2018)
1. Introduction

++ Theor. predictions / Exp. implications ++

- We are now in a very good time to discuss dibaryons.
  - Recent remarkable progress in hadron Exp. enables us to examine “traditional” ideas of dibaryons.

- Further information is available from numerical simulations of lattice QCD, especially with the physical quark masses.

- More hadron-hadron pairs!
- More binding energy to be “stable”!
1. Introduction

++ Predictions of the \( N\Omega \) bound state ++

- \( N\Omega \) dibaryon system.
  - Combination of \( N(uud / udd) [\text{octet}] + \Omega(sss) [\text{decuplet}] \).
    No same flavor. --> No repulsive core !?

- Calculations in quark models.
  Huang, Ping and Wang, *Phys. Rev.* C92 (2015) 065202; ...

--- Although the details are different, these calculations indicate the existence \( N\Omega \) bound state.
1. Introduction

++ The $N\Omega$ system from lattice QCD ++

- The $S$-wave $N\Omega$ interaction ($J^P = 2^+$) in HAL QCD analysis of lattice QCD data.

Etminan et al. [HAL QCD], *Nucl. Phys.* A928 (2014) 89.

Doi et al. [HAL QCD], *EPJ Web of Conf.* 175 (2018) 05009.

- $m_\pi = 875$ MeV.
- **Bound:** $B_E \sim 19$ MeV.
- **No repulsive core!**
- Implying $N\Omega$ bound state.

- $m_\pi = 146$ MeV.
- **Bound, but almost in the unitary limit.**
1. Introduction

++ Motivation ++

- We want to understand the $N\Omega (^{5}S_{2})$ interaction.
  - What is the origin of the attraction? <-- Physics behind it.
  - Connect lattice-QCD quark masses and physical quark masses.
  - Discuss decay modes.

- We construct a baryon-baryon interaction model with meson exchanges.
  - We expect that the meson exchange will play an important rule to generate the attraction.

Doi et al. [HAL QCD], EPJ Web of Conf. 175 (2018) 05009.
2. Model construction
2. Model construction

++ \( N\Omega \) and coupled channels ++

- Consider the \( S \)-wave \( N\Omega \) channel of \( J^P = 2^+ \) and coupled channels.
  - Baryon-baryon systems in \( S = -3 \) & \( I = 1/2 \):

<table>
<thead>
<tr>
<th>Channel</th>
<th>Threshold [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda \Xi )</td>
<td>2434</td>
</tr>
<tr>
<td>( \Sigma \Xi )</td>
<td>2511</td>
</tr>
<tr>
<td>( N\Omega )</td>
<td>2611</td>
</tr>
<tr>
<td>( \Lambda \Xi(1530) )</td>
<td>2649</td>
</tr>
<tr>
<td>( \Sigma(1385) \Xi )</td>
<td>2703</td>
</tr>
<tr>
<td>( \Sigma \Xi(1530) )</td>
<td>2727</td>
</tr>
<tr>
<td>( \Sigma(1385) \Xi(1530) )</td>
<td>2918</td>
</tr>
</tbody>
</table>

- We take into account the decay channels \( (\Lambda \Xi, \Sigma \Xi) \) and one nearest closed channel \([\Lambda \Xi(1530)]]\) in addition to the elastic channel.
  - In particular, the \( N\Omega \) \((J^P = 2^+)\) couples to the decay modes \( \Lambda \Xi \) and \( \Sigma \Xi \) only in the \( D \) wave.  --> Expect small decay width.
2. Model construction

++ Elastic $N\Omega$ interaction ++

- For the elastic $N\Omega$ interaction, possible mediating mesons are only those with quantum numbers $I = 0$ and Charge = 0:
  - **Pseudoscalar**: the $\eta$ meson.
  - **Scalar**: the “$\sigma$” meson, which should be treated as the correlated two pseudoscalar mesons (cf. $NN$ force).
  - **Vector**: NO light vector mesons. Both $\omega$ and $\phi$ cannot mediate owing to OZI rule.

- As a consequence, we have the following diagrams in the conventional meson exchange:
2. Model construction

++ Elastic $N\Omega$ interaction ++

- Besides, we may consider further contributions at short ranges:
  - Exchanges of heavier mesons.
  - Color magnetic interactions at quark-gluon level.
  - ...

--> They are treated as a contact term:
2. Model construction

++ Inelastic $N\Omega$ interaction ++

- In addition, we take into account the inelastic channels: $\Lambda\Xi, \Sigma\Xi$, and $\Lambda\Xi^*$. 
- We consider the simplest coupling: the $K$ meson exchange.
- To concentrate on the $N\Omega$ interaction around its threshold, we neglect the transitions between inelastic channels such as $\Lambda\Xi \rightarrow \Lambda\Xi$, which will be subdominant contributions.

--> Our $N\Omega$ interaction contains the inelastic-channel contributions as a box diagram:
2. Model construction

++ Summary of the $N\Omega$ interaction ++

- We evaluate the $N\Omega$ ($^5S_2$) interaction with the above four diagrams.

$$V(E; p', p) = V_A(p', p) + V_B(p', p) + V_C(p', p) + \sum_{j=2}^{6} V_{\text{box}(j)}(E; p', p)$$

--- Note: Non-local interaction.

The interaction has energy dependence coming from the box terms, where the inelastic channels are integrated out.
2. Model construction

++ Effective Lagrangians ++

- Each vertex is governed by effective Lagrangians.

- **MBB coupling:**
  \[ \mathcal{L}_{MBB} = - \frac{F}{\sqrt{2} f} \left \langle \bar{B} \gamma^\mu \gamma_5 \{ \partial_\mu \Phi, \mathcal{B} \} \right \rangle - \frac{D}{\sqrt{2} f} \left \langle \bar{B} \gamma^\mu \gamma_5 \{ \partial_\mu \Phi, \mathcal{B} \} \right \rangle \]

  --- Parameter fixed by the octet-baryon decay.

- **MBD coupling:**
  \[ \mathcal{L}_{MBD} = - \frac{f_{MBD}}{m_\pi} \left \langle (\bar{\Delta}_\mu \cdot \partial^\mu \Phi) \mathcal{B} + h.c. \right \rangle \]

  --- Parameter fixed by the decuplet-baryon decay.

- **MDD coupling:**
  \[ \mathcal{L}_{MDD} = - \frac{f_{MDD}}{m_\pi} \left \langle (\bar{\Delta}_\mu \cdot \gamma^\nu \gamma_5 \Delta_\mu) \partial_\nu \Phi \right \rangle \]

  --- Parameter fixed by the SU(6) quark model.

- **Contact coupling:**
  \[ \mathcal{L}_{\text{contact}} = c \left \langle \bar{\Omega} \Omega \right \rangle (\bar{p}p + \bar{n}n) \]

  --- Model parameter \( c \).
2. Model construction

++ Details of the interaction ++

- We introduce a form factor for every diagram:

\[ F(q)^2 = \left( \frac{\Lambda^2}{\Lambda^2 + q^2} \right)^2 \]

--- Cut-off is fixed to be a hadronic scale: \( \Lambda = 1 \text{ GeV} \).

- The \( \eta \) exchange and contact interactions are straightforwardly calculated.

- The “\( \sigma \)” exchange interaction is evaluated with the dispersion relation of \( NN\bar{N} \rightarrow \Omega\Omega \).

\[ \text{Im} T_{NN\bar{N} \rightarrow \Omega\Omega}(t) = \sum_{j=\pi,ka} \rho_j(t)T_{NN\bar{N} \rightarrow j}(t)T_{\Omega\Omega \rightarrow j}^*(t) \]

- The box interaction is:

\[ V_{\text{box}}(E; p', p) = \int_0^\infty \frac{dp''}{2\pi^2p''} \frac{V_{1j}(p', p'')V_{j1}(p'', p)}{E - \mathcal{E}_j(p'')} + i0 \]

\[ \mathcal{E}_j(p) = \sqrt{p^2 + m_j^2} + \sqrt{p''^2 + m_j'^2} \]
In our model, only the contact coupling constant $c$ is a free parameter.
--- Fixed by information from recent HAL QCD analysis!

We reproduce the scattering length of the HAL QCD analysis on $N\Omega\ (^{5}S_{2})$.

--- Scattering Amp. at the threshold:

$$T(E; p', p) = V(E; p', p) + \int_{0}^{\infty} \frac{dp''}{2\pi^2} {p''}^2 \frac{V(E; p', p'')T(E; p'', p)}{E - \mathcal{E}_{N\Omega}(p'')}$$

--- But with the nearly physical quark masses on the lattice.

--- Scattering length $a = 7.4 \pm 1.6 \text{ fm}$ at $t = 11$.

$log\delta_{0}(k/m_{\pi})$ vs $(k/m_{\pi})^2$

Doi et al. [HAL QCD], *EPJ Web of Conf.* 175 (2018) 05009.
* HAL QCD’s scattering length has opposite sign compared to ours.

--&gt; We fix $c = -22.1 \text{ GeV}^{-2}$ to reproduce $a = 7.4 \text{ fm}$ with lattice masses.
3. Properties of the $N\Omega$ interaction
3. Properties of the $N\Omega$ interaction

++ Elastic $N\Omega$ interaction ++

- First, we show the $N\Omega (^5S_2)$ interaction in elastic channels:

$$V(E; p', p) = V_A(p', p) + V_B(p', p) + V_C(p', p) + \sum_{j=2}^{6} V_{box(j)}(E; p', p)$$

- $V_A [ \text{GeV}^{-2} ]$ --- $\eta$ exchange.
- $V_B [ \text{GeV}^{-2} ]$ --- "$\sigma$" exchange.
++ Elastic $N\Omega$ interaction ++

- First, we show the $N\Omega$ ($^5S_2$) interaction in elastic channels:

$$V(E; p', p) = V_A(p', p) + V_B(p', p) + V_C(p', p) + \sum_{j=2}^{6} V_{box(j)}(E; p', p)$$

$V_A$ [GeV$^{-2}$] --- $\eta$ exchange.


$V_C$ [GeV$^{-2}$] --- contact.
3. Properties of the $N\Omega$ interaction

++ Elastic $N\Omega$ interaction ++

- Calculate $V$ with $p' = p$ : $V = V(p' = p, p)$.

- The contact term is dominant. --- This includes the parameter.

- The $\eta$ meson gives moderate attraction due to small $\eta NN$ coupling.

- The “$\sigma$” exchange is also moderate due to small “$\sigma$” $\Omega\Omega$ coupling.
3. Properties of the $N\Omega$ interaction

++ Inelastic $N\Omega$ interaction ++

- Next, we show the $N\Omega (^5S_2)$ interaction from inelastic channels:

$$V(E; p', p) = V_A(p', p) + V_B(p', p) + V_C(p', p) + \sum_{j=2}^{6} V_{\text{box}(j)}(E; p', p)$$

- Calculate $V_{\text{box}}$ with $p' = p$ and $E = m_N + m_\Omega$: $V = V_{\text{box}}(m_N+m_\Omega; p' = p, p)$.

- The attraction by inelastic channels is similar strength to the $\eta$ / “$\sigma$” exchange.

$\Lambda\Xi$ is the largest among the box terms thanks to the large $K_{N\Lambda}$ coupling.

cf. Elastic $V$. 
Next, we show the $N\Omega$ ($^5S_2$) interaction from inelastic channels:

\[ V(E; p', p) = V_A(p', p) + V_B(p', p) + V_C(p', p) + \sum_{j=2}^{6} V_{\text{box}(j)}(E; p', p) \]

- We change the energy $E$.
- The energy dependence of the box interaction is not significant.
3. Properties of the $N\Omega$ interaction

++ $N\Omega(5S_2)$ scattering amplitude ++

- Information of the $N\Omega$ ($5S_2$) system is reflected in its scattering amplitude $f_S$ as a function of relative momentum $k$:

$$T(E; p', p) = V(E; p', p) + \int_0^\infty \frac{dp''}{2\pi^2 p''^2} \frac{V(E; p', p'')T(E; p'', p)}{E - E_{N\Omega}(p'')}$$

\[f_S(k) = (\text{kinematical factor}) \times T_{\text{on-shell}}\]

- Threshold parameters of the $N\Omega$ ($5S_2$) scattering:
  - **Scattering length $a$**: $a = -f_S(k = 0) = 5.3 - 4.3i$ fm
  - Complex due to decay Chan. Positive real part implies existence of a bound state.
  - **Effective range $r_{\text{eff}}$**: $r_{\text{eff}} = \left[ \frac{d^2 f_S^{-1}}{dk^2} \right]_{k=0} = 0.74 + 0.04i$ fm
  - --- Almost real.
3. Properties of the $N\Omega$ interaction

++ $N\Omega(^5S_2)$ quasi-bound state ++

- Indeed, the $N\Omega (^5S_2)$ scattering amplitude contains a resonance pole which corresponds to the $N\Omega (^5S_2)$ quasi-bound state!

- **Pole** at $E_{\text{pole}} = 2611.3 - 0.7 \imath \text{ MeV}$. 
  \[\leftrightarrow B_E = 0.1 \text{ MeV}, \Gamma = 1.5 \text{ MeV}.\]

- From the residue at the pole, we can extract the bound-state wave function $\psi_{N\Omega}$ (see figure).

- For the $p\Omega^-$ state, the Coulomb interaction will assist:
  \[\Delta B_{\text{Coulomb}} \sim 1 \text{ MeV} \quad \Delta \Gamma_{\text{Coulomb}} \sim 1 \text{ MeV}.\]
3. Properties of the $N\Omega$ interaction

++ Equivalent local potential ++

- Our $N\Omega ({}^5S_2)$ interaction is non-local.

--> We **construct a local potential** as the sum of Yukawa potentials which is **fitted to our $N\Omega ({}^5S_2)$ interaction**.

The local Pot. reproduces $N\Omega ({}^5S_2)$ properties very well.

--- Why don’t you use to **calculate $\Omega$-nucleon(s) systems**?
4. Summary and outlook
4. Summary and outlook

- We constructed the \( N\Omega (^5S_2) \) interaction according to the diagrams:

  ![Diagrams]

- The conventional exchanges of the \( \eta \), “\( \sigma \)”, and \( K \) (in terms of box) mesons do not provide sufficient attraction.

- Most of the attraction indicated in recent lattice QCD simulations is attributed to the short-range contact interaction.

- Fitting parameter (contact coupling constant only) to scattering length in HAL QCD, we obtain the \( N\Omega (^5S_2) \) quasi-bound state.
  - \( E_{\text{pole}} = 2611.3 - 0.7 i \text{ MeV} \). --- \( B_E = 0.1 \text{ MeV}, \Gamma = 1.5 \text{ MeV} \).
  - For the \( p\Omega^- \) state, the Coulomb interaction will assist \( B_E \) and \( \Gamma \).
  - \( a = 5.3 - 4.3 i \text{ fm}, r_{\text{eff}} = 0.74 + 0.04 i \text{ fm} \).

- Can we find the \( N\Omega \) bound state in heavy-ion collisions ... ?

Thank you very much for your kind attention!
Appendix
Appendix

++ Properties of $N\Omega$ from the local potential ++

- We check that our local $N\Omega(5S_2)$ potential reproduces the properties of the $N\Omega(5S_2)$ system from the $T$-matrix.

From $T$-matrix:

$a = 5.3 - 4.3 i \text{ fm}$, $r_{\text{eff}} = 0.74 + 0.04 i \text{ fm}$.

From local potential:

$a = 5.2 - 5.0 i \text{ fm}$, $r_{\text{eff}} = 0.78 + 0.06 i \text{ fm}$.

From $T$-matrix:

$E_{\text{pole}} = 2611.3 - 0.7 i \text{ MeV}$.

From local potential:

$E_{\text{pole}} = 2611.4 - 0.7 i \text{ MeV}$.
Appendix

++ Comparison with the HAL QCD potential ++

- We compare our local $N\Omega(5S_2)$ potential with the HAL QCD potential of nearly physical quark masses.