Heavy-ion collisions at high baryon densities: Modelling the QCD phase transition

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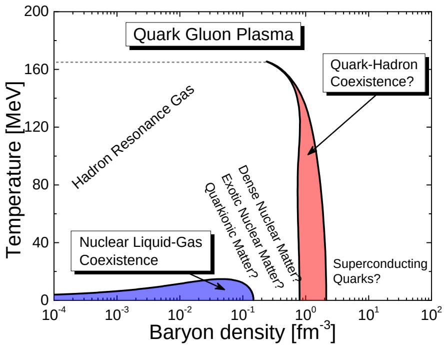
Phase Transitions

Important features of the thermodynamic properties of materials can be depicted in a so called Phase Diagram.



Different Types of Phase Transitions

- Crossover: Smooth transition where the first derivatives of the free energy are always continuous. Butter.
- 1st order: Phase transition where the first derivatives of the free energy show a discontinuity and the latent heat is finite.
- 2nd order: Derivatives of the free energy are also continuous, but higher order derivatives diverge. Critical behavior.

Motivation

This talk: Only the phase transition! Nothing about the CeP.

- How do we find the first order phase transition?

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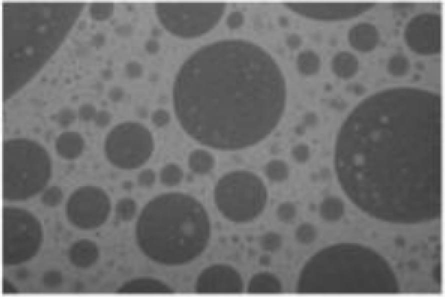
If it is a phase transition

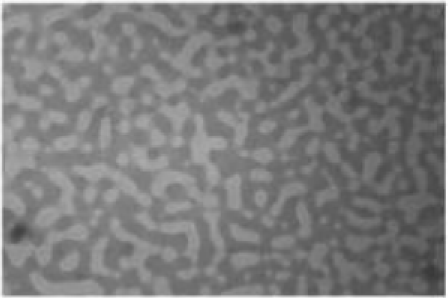
Do we have the models that can tell us what to look for?
E.g. fluid-dynamics.

Phase separation

Nucleation vs. spinodal decomposition

- Nucleation: Thermal fluctuations serve as seeds for bubble formation. (e.g. ice in water). SLOW!
- Spinodal decomposition: System is quenched below separation temperature. Instabilities occur (e.g. hot oil + water). FAST!





Importance of models

Why dynamical models?

- Finite number of particles and volume
- Conservation laws
- Finite Lifetime
- Can separate out different physics effects
- ...

Important ingredients

1. A dynamic model: In this case fluid dynamics.

Solving Fluid-Dynamics

The equations for relativistic fluid-dynamics are solved numerically on a grid of cell size $\Delta x = 0.2$ fm:

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{and} \quad \partial_\mu N^\mu = 0$$

$T^{\mu\nu}$ is the relativistic energy momentum tensor and N^μ the baryon four-current. In ideal fluid-dynamics these can be written as:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} \quad \text{and} \quad N^\mu = nu^\mu$$

To close this system of equations one needs the equation of state, of the form $p = p(\epsilon, n)$, as an additional input.

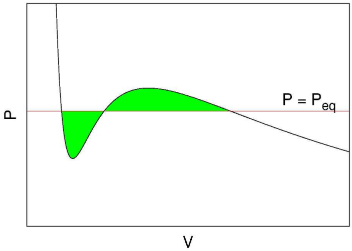
Important ingredients

1. A dynamic model: In this case fluid dynamics.
2. A "proper" equation of state.

The Maxwell Construction

Problem

Most model calculations, including a 'phase transition', are based on a Maxwell constructed EoS!



Van der Waals equation isotherm. [1]

"In thermodynamic equilibrium, a necessary condition for stability is that pressure P does not increase with volume V."

" The Maxwell construction is a way of correcting this deficiency."

[1] http://www.wikipedia.com

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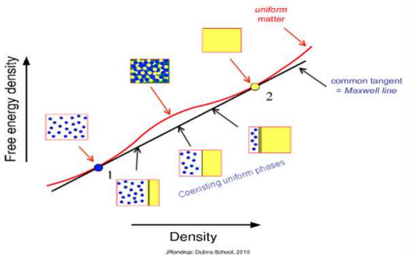


[1]

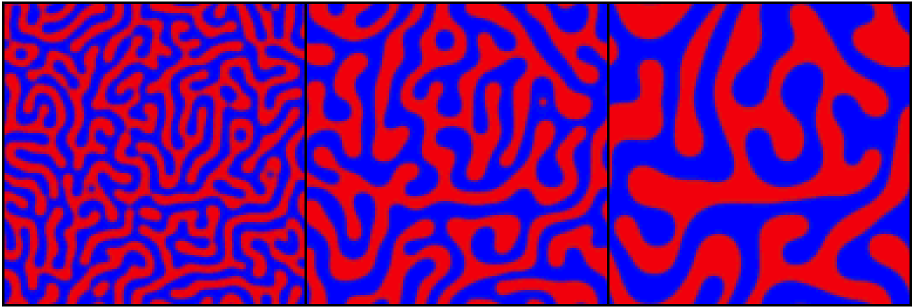
"In thermodynamic equilibrium, a necessary condition for stability is that pressure P does not increase with volume V."
" The Maxwell construction is a way of correcting this deficiency."

- In a dynamical scenario, locally the system is not necessarily stable.
- Phase separation occurs.

[1] http://www.wikipedia.com

Non-Equilibrium Phase Transition

"Spinodal decomposition is essentially a mechanism for the rapid unmixing of a mixture of liquids or solids from one thermodynamic phase, to form two coexisting phases."

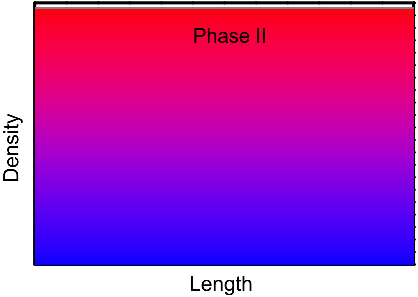


I takes place for example when one quenches a mixture of two substances rapidly below the demixing temperature. Then, the two substances separate locally, giving rise to the complicated structures which can be seen on the leftmost picture.

Non-Equilibrium Phase Transition

Equilibrium Phase Transition (Maxwell construction)

As the system dilutes, the phases are always well separated



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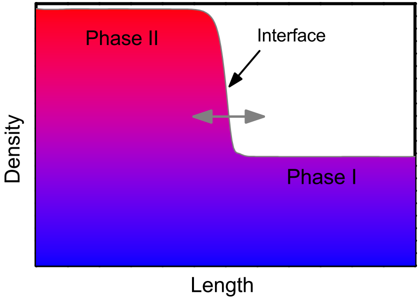

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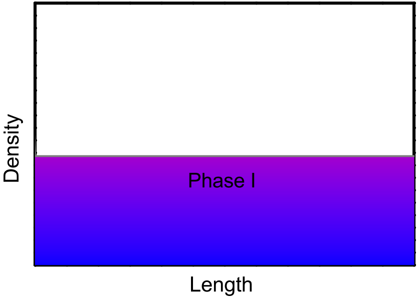

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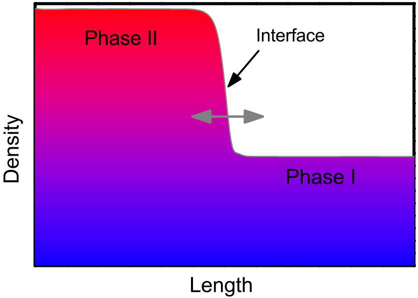
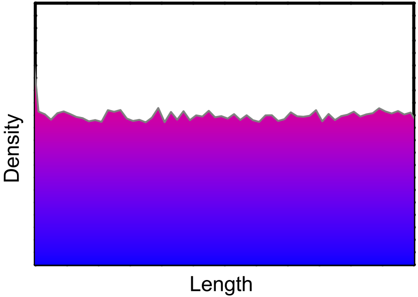


Non-Equilibrium Phase Transition

Phase separation is a dynamical process.



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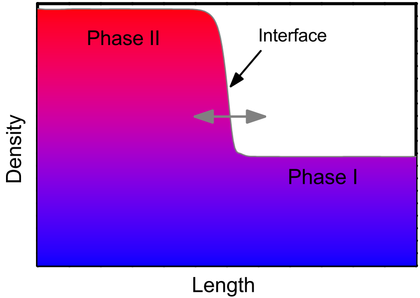

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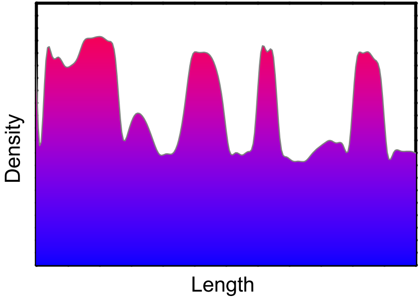

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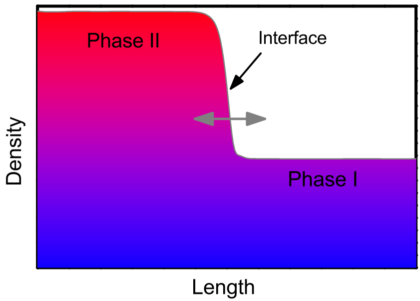
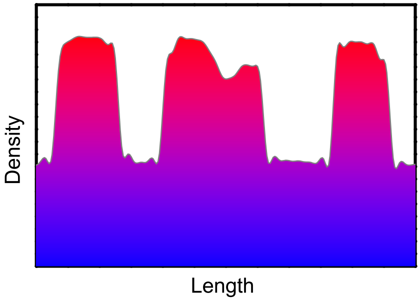
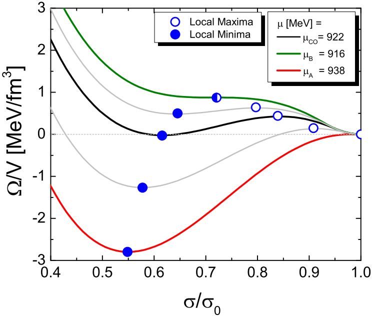




A better example: A Model for the Liquid-Gas P.T.

Take a mean field model: Walecka or hadronic σ-ω model.

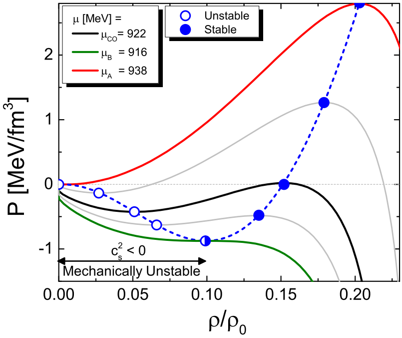
- σ serves as the order parameter.
- Calculate the Grand Canonical potential Ω/V for fixed T and μ as function of σ



- At $T = 0$ and $\mu = \mu_{CO}$, Ω/V has two, equally deep, minima.
- Within the spinodal region always two minima!
- Each corresponding to a meta-stable phase.
- Maxima: unstable!

A Model for the Liquid-Gas P.T.

$P = -\Omega/V$ as a function of density at $T = 0$

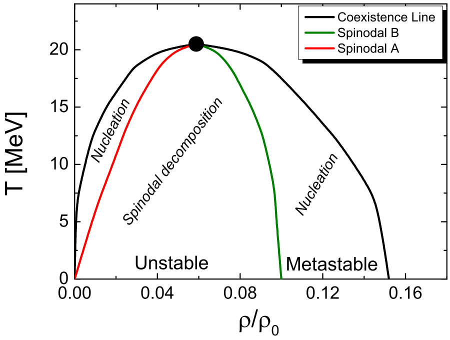


- Curve through the points gives the nuclear EoS.
- Mechanically unstable region defined by imaginary speed of sound

$$\frac{\partial P}{\partial \epsilon}\Big|_{T=0} = c_s^2 < 0$$

The Phase Diagram in Density

A More convenient representation of the phase diagram is in T and ρ.



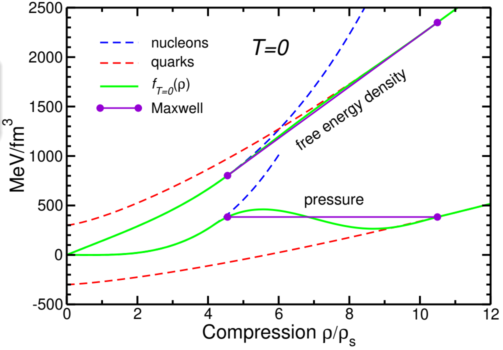
- Metastable and Unstable regions visible.
- Note: Order of Spinodals is reversed
- Supercooled and Overheated phases.

Constructing an Effective EoS: Another way

If we don't have an effective model that describes both phases

Obtain the free energy density $f_T(\rho) = \epsilon_T(\rho) - T s_T(\rho)$ by a spline between a Gas of int. nucleons+pions and a QGP.

$\partial^2_\rho f_T(\rho) > 0$: (Meta-)Stable
$\partial^2_\rho f_T(\rho) < 0$: Unstable



Alternatively: Do a Maxwell construction.

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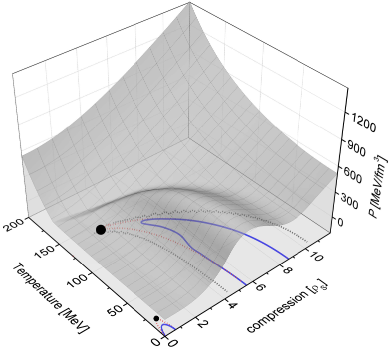
$\partial_\rho^2 f_T(\rho) > 0$: (Meta-)Stable
$\partial_\rho^2 f_T(\rho) < 0$: Unstable

Fluid evolution: $T \neq$ const. but $S/A =$ const.

$c_s^2\big|_T < 0$: Isothermal spinodal
$c_s^2\big|_{S/A} < 0$: Isentropic spinodal



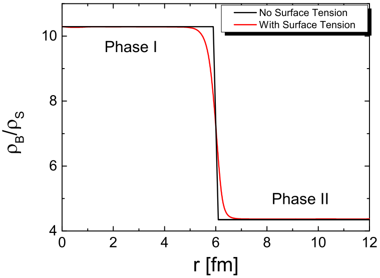
Alternatively: Do a Maxwell construction.

Important ingredients

1. A dynamic model: In this case fluid dynamics.
2. A "proper" equation of state.
3. Long range interactions. Lead to a surface energy.

The Gradient Term

A proper description of spinodal decomposition requires that finite-range effects be incorporated.



We rewrite the local pressure as

$$p(\boldsymbol{r}) = p_0(\varepsilon(\boldsymbol{r}), \rho(\boldsymbol{r})) - a^2 \frac{\varepsilon_s}{\rho_s^2} \rho(\boldsymbol{r}) \boldsymbol{\nabla}^2 \rho(\boldsymbol{r})$$

The gradient term will cause a diffuse interface to develop when matter of two coexisting phases are brought into physical contact.

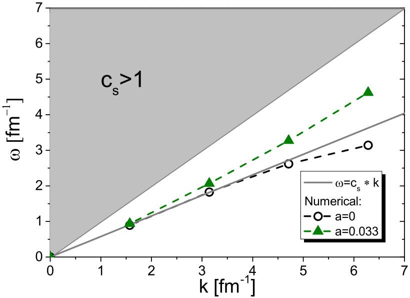
The dispersion relation in Numerical Fluid-Dynamics

Calculation in a box with periodic boundaries

Ideal fluid dynamics: $\omega^2 = c_s^2 k^2$

$+$ Gradient term: $\omega^2 = c_s^2 k^2 + a^2 \frac{(\varepsilon_s/h)}{(\rho/\rho_s)^2} k^4$

$+$ Shear and bulk viscosity: $\omega^2 = c_s^2 k^2 + a^2 \frac{(\varepsilon_s/h)}{(\rho/\rho_s)^2} k^4 - \zeta \frac{\omega}{h_0} k^2$



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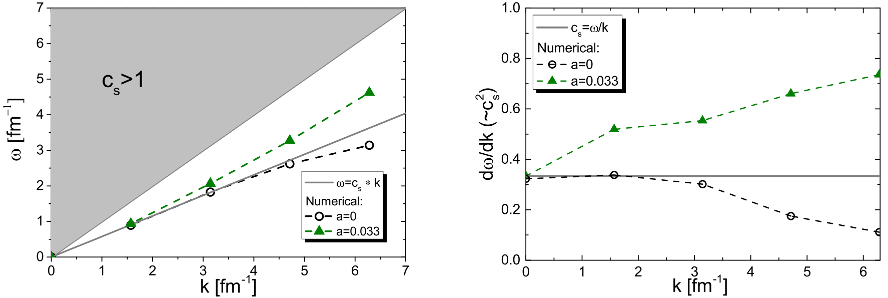
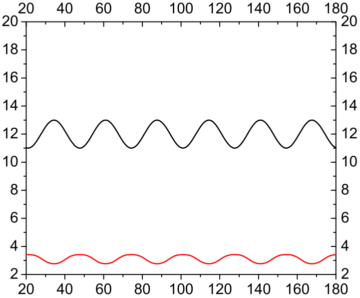


Growth Rates in the unstable phase

Calculation in a box with periodic boundaries

The amplitude of a density undulation should grow exponentially within the unstable region

$$A(t) = A_{t=0}(e^{\gamma_k t} + e^{-\gamma_k t})$$



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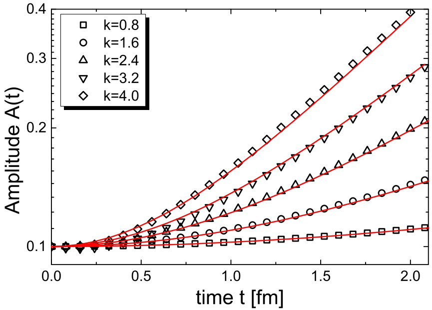

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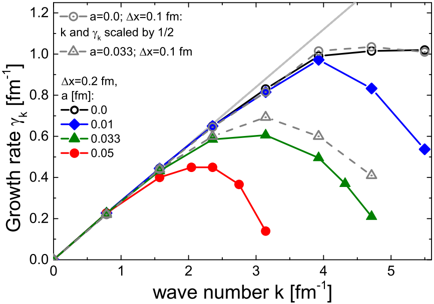


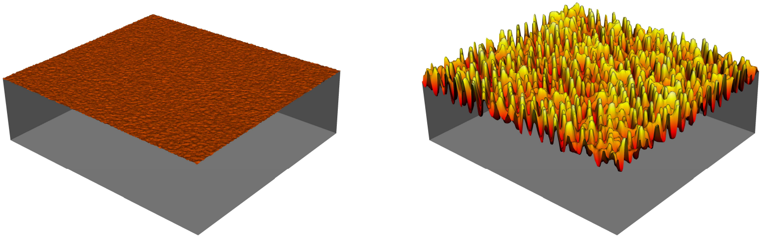
- Numerical viscosity cuts off large wave number growth
- The gradient term modifies the growth rates:

$$\gamma_k^2 = |v_s|^2 k^2 - a^2(\varepsilon_s/h)(\rho/\rho_s)^2 k^4$$

- Depending on a certain wave numbers are favored

Show Animation I

Initialize Random noise in the unstable region and let it evolve.



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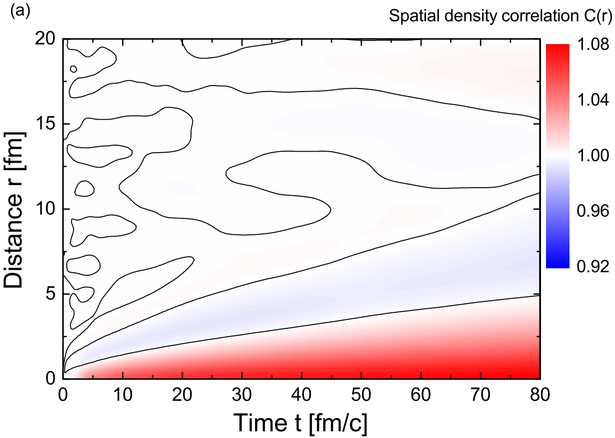
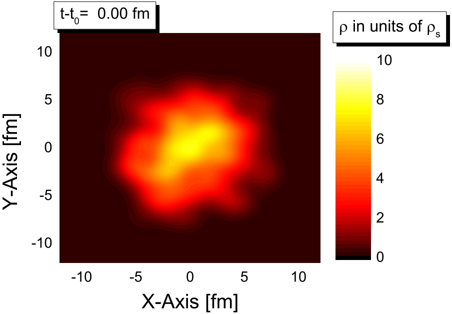


Initial State And Setup

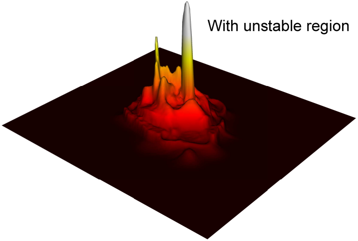
We apply the UrQMD transport model for the initial, non-equilibrium, part of the collision.

- When contracted nuclei have passed through each other,
- energy-, momentum- and baryon densities are mapped onto the computational grid.



Evolution in Fluid-Dynamics

EoS with unstable phase:



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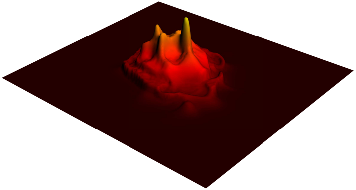

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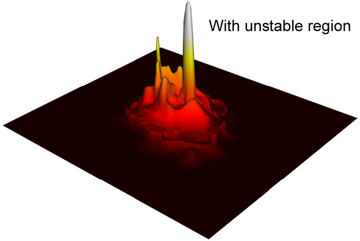

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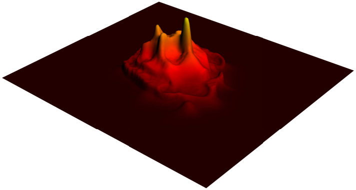

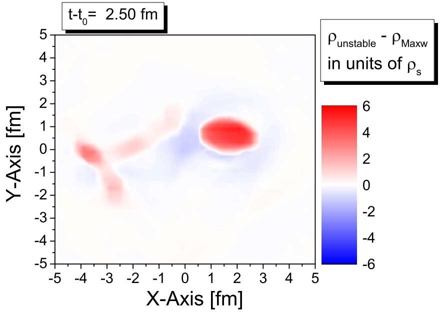

Notable difference observed.
Unstable phase leads to clustering of baryonnumber!

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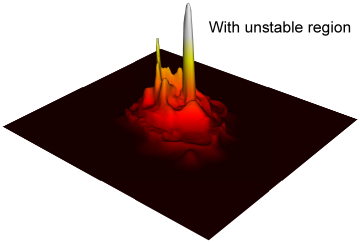

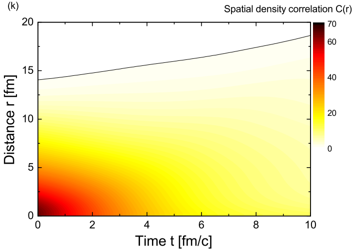

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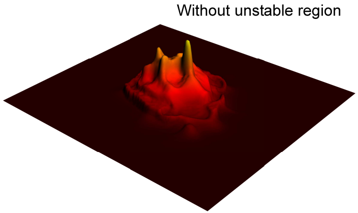
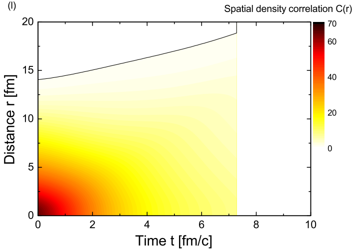
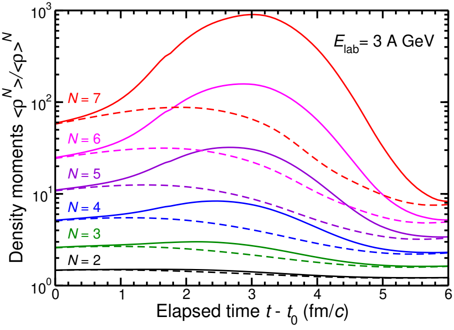




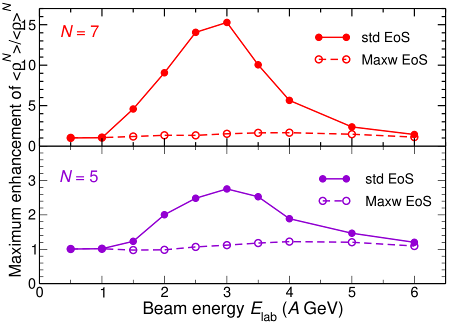
Let's be more quantitative

Define Moments of the net baryon density distribution:

$$\langle \rho^N \rangle \equiv \frac{1}{A} \int \rho(\boldsymbol{r})^N \rho(\mathbf{r})\, d^3\boldsymbol{r}$$



As a function of time



As a function of beam energy

How to make particles from densities

Typically done by use of the Cooper-Frye equation

$$E\frac{dN}{d^3p} = \int_\sigma f(x,p)p^\mu d\sigma_\mu \quad (1)$$

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Important

Since the volume elements (cells) are usually small $\Delta x < 1fm$, the particle number in a cell is also $N_p << 1$ and so one assumes it is Poisson distributed!

WHAT we calculate and WHEN?

Cumulants of net baryon number

$$
\begin{aligned}
K_1 &= M = \langle \mathrm{N} \rangle \\
K_2 &= \sigma^2 = \left\langle (\delta \mathrm{N})^2 \right\rangle \\
K_3 &= S\sigma^{3/2} = \left\langle (\delta \mathrm{N})^3 \right\rangle \\
K_4 &= \kappa\sigma^4 = \left\langle (\delta \mathrm{N})^4 \right\rangle - 3 \left\langle (\delta \mathrm{N})^2 \right\rangle^2
\end{aligned}
$$

WHAT we calculate and WHEN?

- In coordinate space.
- At a fixed time.

Cumulants of net baryon number

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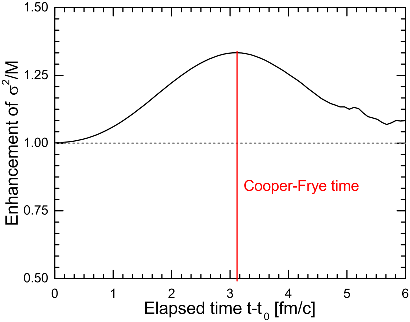
$$K_2 = \sigma^2 = \left\langle (\delta N)^2 \right\rangle$$
$$K_3 = S\sigma^{3/2} = \left\langle (\delta N)^3 \right\rangle$$
$$K_4 = \kappa\sigma^4 = \left\langle (\delta N)^4 \right\rangle - 3 \left\langle (\delta N)^2 \right\rangle^2$$

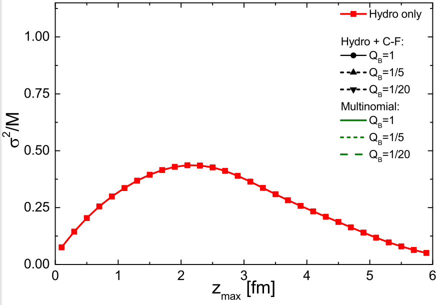


Effect on the cumulants - numerical results

Comparison with numerical results

- Take the hydro results with the spinodal clumping.

K_2/K_1



Effect on the cumulants - numerical results

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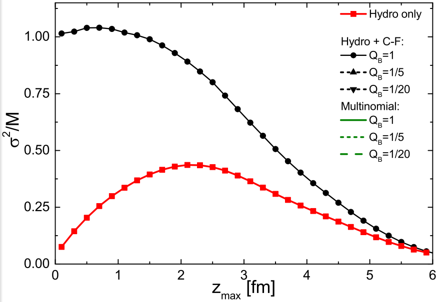

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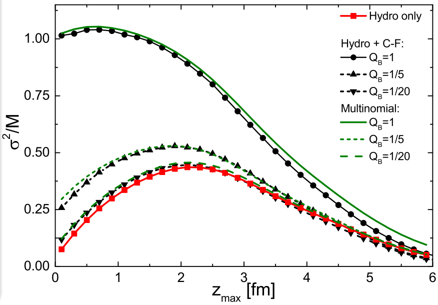

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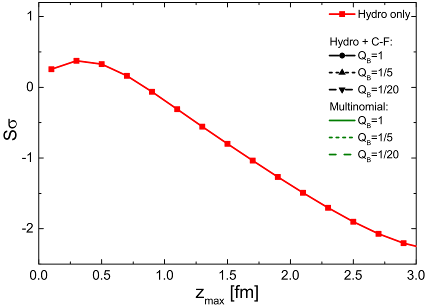

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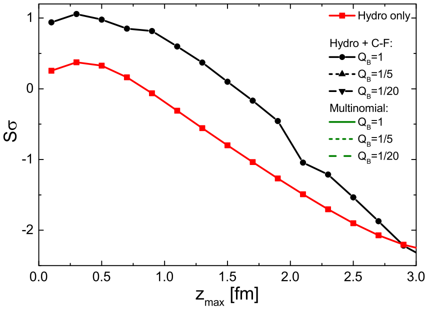
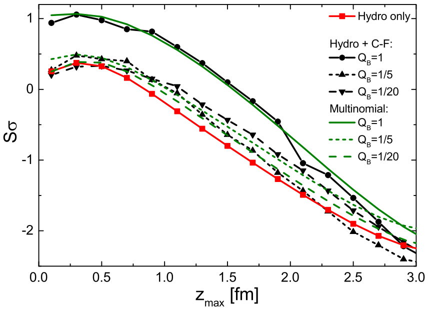


Effect on the cumulants - numerical results

Comparison with numerical results

- Take the hydro results with the spinodal clumping.
- Calculate the baryon number fluctuations in a spatial volume directly
- Use the C-F equation and sample baryons, conserving baryon number globally.
- Analytic and numerical results agree well.

K_3/K_2



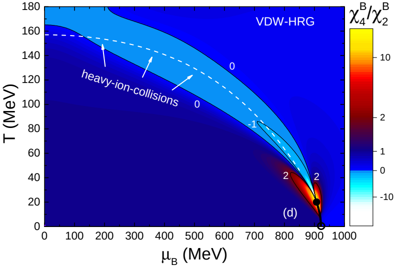
Effect on the cumulants - analytic - multinomial

In case you want to see the equations:

$$
\begin{aligned}
K_1^{B,CF,multi} &= \langle B \rangle = K_1^B \\
K_2^{B,CF,multi} &= K_2^B + Q_B \left(K_1^B - \frac{{K_1^B}^2 + K_2}{B_{tot}} \right) \\
K_3^{B,CF,multi} &= K_3^B + 3Q_B \left(K_2^B - \frac{2K_1^B K_2^B + K_3^B}{B_{tot}} \right) \\
&\quad + Q_B^2 \left(K_1^B - 3\frac{{K_1^B}^2 + K_2^B}{B_{tot}} \right. \\
&\quad \left. + 2\frac{{K_1^B}^3 + 3K_1^B K_2^B + K_3^B}{B_{tot}^2} \right)
\end{aligned} \tag{2}
$$

How important are nuclear potentials?

- Nuclear interaction do have an influence on the baryon number susceptibilities.



V. Vovchenko, M. I. Gorenstein and H. Stoecker, Phys. Rev. Lett. 118, no. 18, 182301 (2017)

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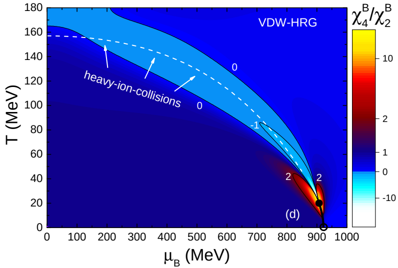
- Including an additional transition (even crossover) can make the whole scenario much more complicated.



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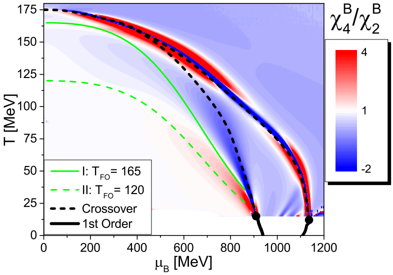

A. Mukherjee, JS and S. Schramm, Phys. Rev. C 96, no. 2, 025205

How important are nuclear potentials?

- Nuclear interaction do have an influence on the baryon number susceptibilities.
- Expected effects: increase of all cumulant ratios!

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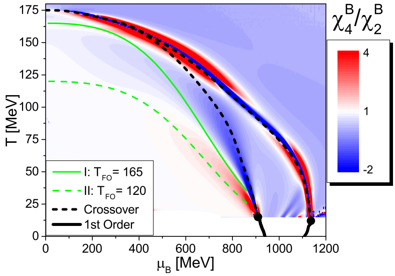

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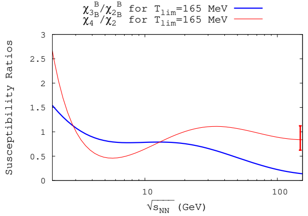

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A (qualitative) study of nuclear effects

- Use a microscopic transport model (UrQMD)
- Nuclear forces can be included via mean field approach.
- Has been used to describe particle spectra and collective flow at low beam energies ($E_{lab} < 10A$ GeV).

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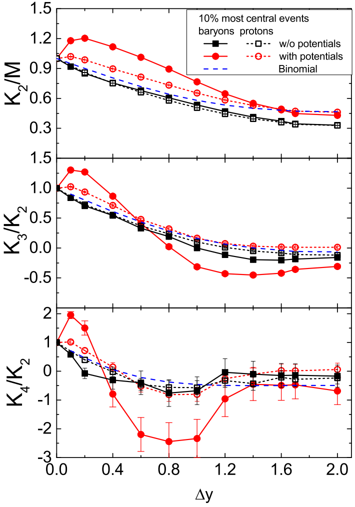
- They have enough statistics to actually measure baryon number fluctuations.

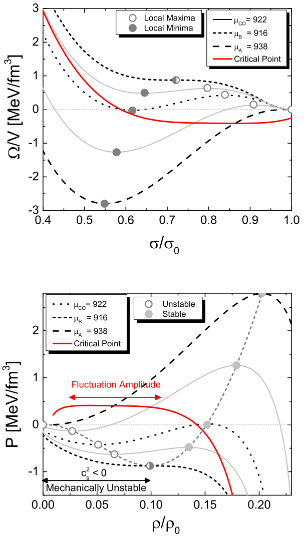
Simulation results

- The simulations show a trend similar to the grand canonical predictions.
- However, effects of finite size and baryon number conservation have a significant impact.
- Also: measuring only protons and not baryons is a problem!



JS, Y. Wang, A. Mukherjee, Y. Ye, C. Guo, Q. Li and H. Stoecker, arXiv:1804.08936 [nucl-th].

The Critical Point



Phase Transition:

- Density domains created dynamically inside the unstable and metastable region → Driven by pressure gradient: Timescales similar to expansion times.
- Amplitude of fluctuations favor certain values.
- Thermal fluctuations add but are not necessary

Critical Point:

- Density domains created only at the critical point → In equilibrium, the critical point is reached only by precise tuning.
- Amplitudes of fluctuation are not driven by pressure gradient: Timescales?
- Fluctuation on all possible length scales
- No creation of fluctuations by the EoS. Driven by thermal fluctuations and long range interactions (e.g. gradient term).
- requires $c_s = 0$ for some time.

Summary

- I have described a way to include effects of instabilities due to a first order phase transition in fluid dynamics.
- The associated instabilities may cause significant amplification of initial density irregularities.
- The present study demonstrates that the phase structure does affect the character of the density evolution.
- Hopefully, these results will stimulate efforts to develop analysis techniques for extracting the related observables from experimental data.

Summary

- I have described a way to include effects of instabilities due to a first order phase transition in fluid dynamics.
- The associated instabilities may cause significant amplification of initial density irregularities.
- The present study demonstrates that the phase structure does affect the character of the density evolution.
- Hopefully, these results will stimulate efforts to develop analysis techniques for extracting the related observables from experimental data.

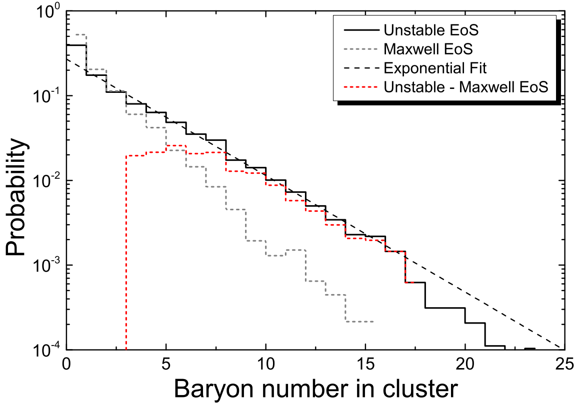
- So far we are still trying to identify observables sensitive on the enhanced fluctuations.
- (Average) Radial flow, nuclei production and angular correlations show no large signal of the unstable dynamics of the phase transition.

Cluster Distribution

Let's be more quantitative

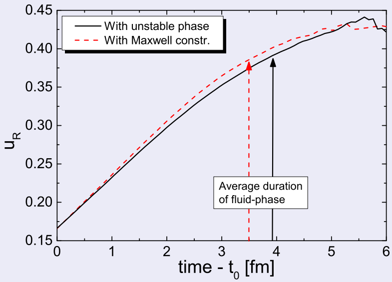
Define Clusters: A coherent chunk of baryon number with density above

$$\rho > 7\ \rho_S$$



- Probability distribution for cluster size is exponential (at time $t - t_0 \approx 3$ fm)
- Probability to find a large cluster drastically enhanced with unstable phase!

Radial Flow



The average radial velocity hardly shows a difference.

Nuclei Production

- Density fluctuations might lead to an enhanced production of (composite) nuclei:

$$E_A \frac{dN_A}{d^3 P_A} = B_A \left(E_\mathrm{p} \frac{dN_\mathrm{p}}{d^3 P_\mathrm{p}} \right)^Z \left(E_\mathrm{n} \frac{dN_\mathrm{n}}{d^3 P_\mathrm{n}} \right)^N \Bigg|_{P_\mathrm{p}=P_\mathrm{n}=P_A/A} .$$

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- Also the energy density is enhanced
- The increased excitation again counters the effect of the higher baryon density
- $\Rightarrow$ No observed enhancement

Angular Correlations

Initial state fluctuation can be observed in final state angular correlations.

$$V_{N\Delta}(p_\perp^a, p_\perp^b) = \left\langle \cos(n(\phi^a - \phi^b)) \right\rangle = v_n(p_\perp^a) \times v_n(p_\perp^b) + p_\perp \text{cons.} + \text{dec.} + ?$$ [a]

Can this be true also for fluctuations at the phase transition?

[a] K. Aamodt *et al.* [ALICE Collaboration], Phys. Lett. B **708**, 249 (2012)

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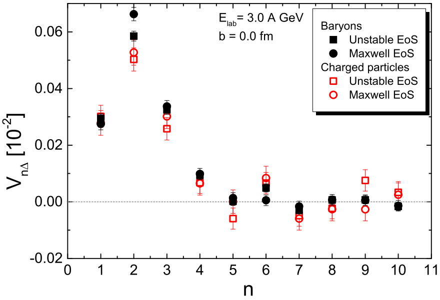

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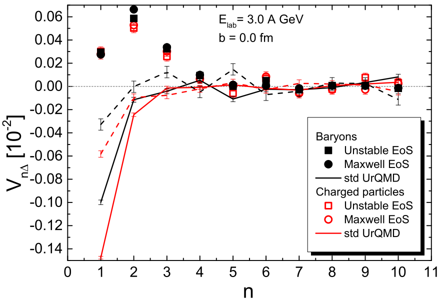
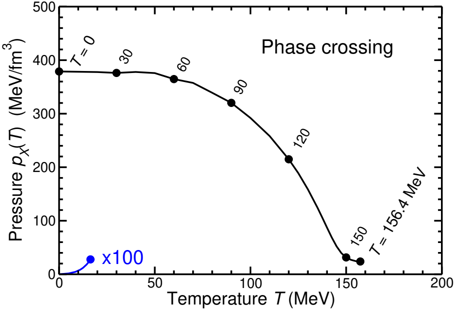


- No enhancement in the angular correlations is observed!
- A comparison with the microscopic model shows the importance of momentum conservation.

Differences to the Liquid-Gas P.T.



- Phase diagramm in pressure and temperature reveals that they are of different type.
- While the L.-G.- Phase transition is driven by the density, with no change of degrees of freedom
- The Hadron-Quark transition is driven by the entropy, with significant change of degrees of freedom
- Still, both can be a 'true' first order transition including a latent heat and a 'mixed phase region'.

Differences to the Liquid-Gas P.T.

The nuclear Liquid-Gas phase transition

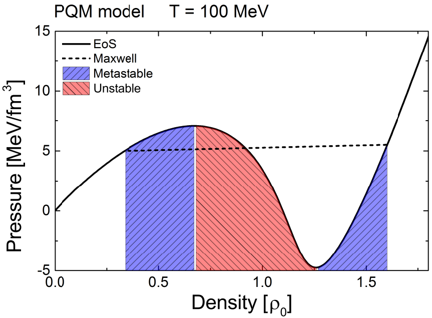
- $0 - 1$ times nuclear ground state density $\rho_0 \approx 0.15 - 0.16\ fm^{-3}$
- Nuclear ground state stable w.r.t. vacuum
- Several experimental constraints, e.g. binding energy, compressibility.
- Droplets/Fragments can be observed in detector

The hadron-quark phase transition

- $2 - 10?$ times nuclear ground state density
- Quark droplets/domains/clusters stable in dense hadronic medium but unstable w.r.t. vacuum
- Almost no theoretical constraints. Lattice QCD not conclusive (yet?).
- How to observe evidence for droplets/fragments in detector?

How Important is the specific EoS?

Use the PQM model, a constituent quark σ model with Polyakov Loop potential.



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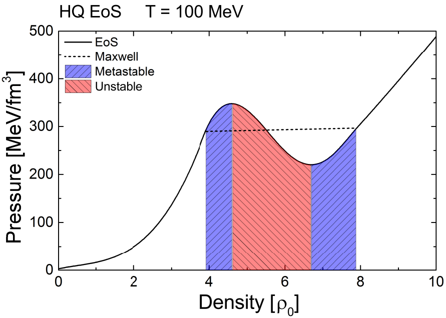

Stable Quark matter at $T = 0$? Is this physical?

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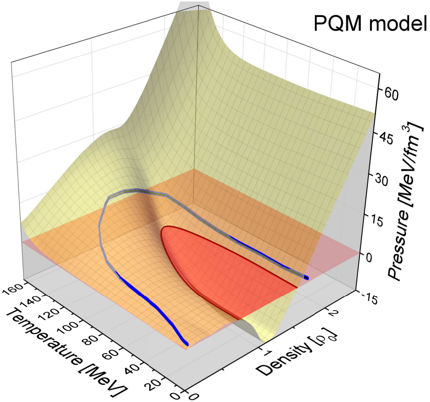

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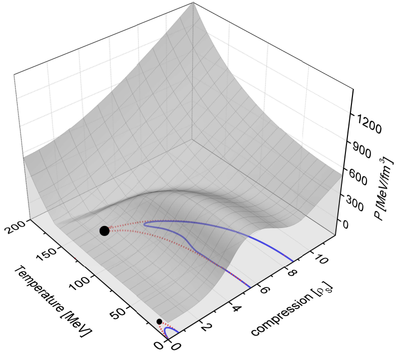
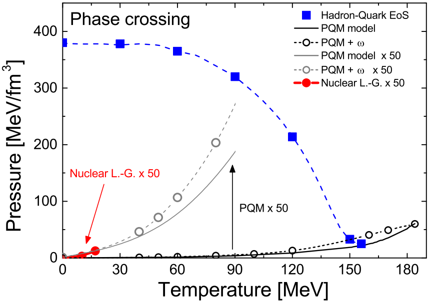
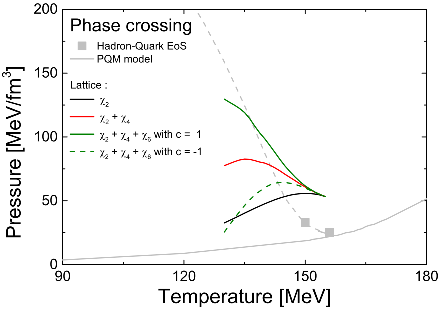


Stable Quark matter at $T = 0$? Is this physical?

Differences in the EoS

There are significant differences in the EoS along the transition curve, depending on the nature of the transition:

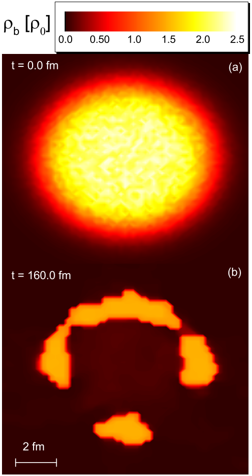


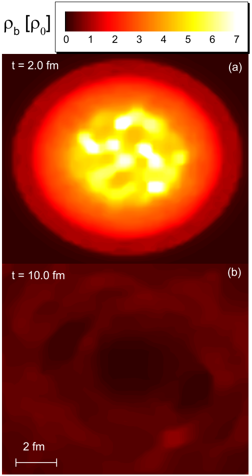


Is the PQM just a bad model for the Liquid gas transition (at finite density)?

Droplet Formation

Initialize a symmtric system + fluctuations and let it evolve:

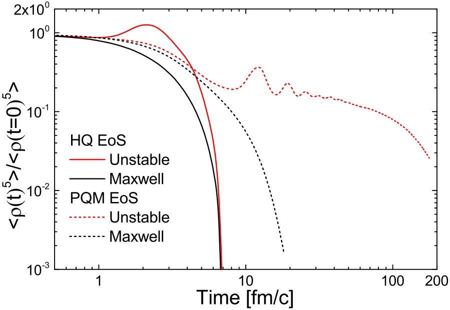


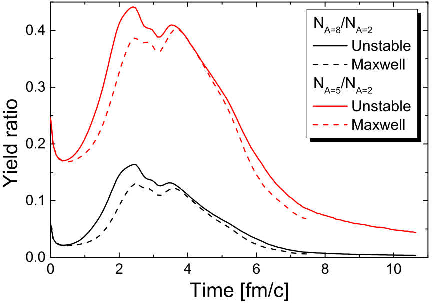


The PQM model produces almost stable dropletts of quark matter.
Physical???

Fluctuations Compared

Initialize a symmtric system + fluctuations and let it evolve:





The dropletts in the PQM model have a long lifetime, one can even see oscillation. Average 5th power of density is enhanced in both cases, however translation into cluster creation is small.

Effects of Viscosity

In actual systems there is some degree of physical dissipation which gives rise to both viscosity and heat conduction.

To leading order, the viscosity reduces the growth rate by

$$\approx \frac{1}{2}[\frac{4}{3}\eta + \zeta]k^2/h \quad (3)$$

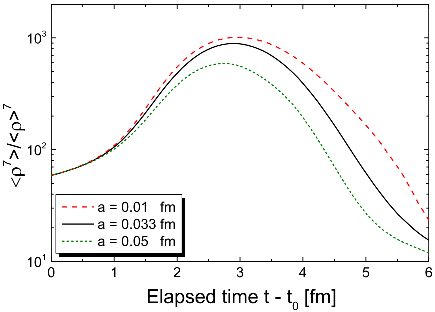
where η and ζ are the shear and bulk viscosity coefficients, respectively.

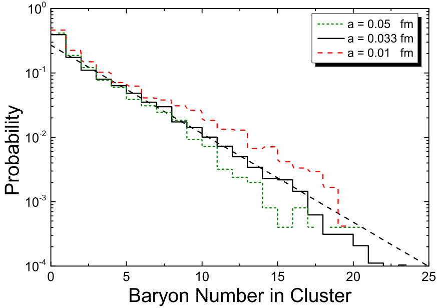
A heat conductivity generally increases the growth rate because the speed of sound will be closer to the iso-thermal v_T instead of the isentropic, increasing the size of the unstable region (and also decreasing the squared speed of sound).

Changing the Surface Tension

Parameter Dependencies

We change the value of the surface tension by varying a from 0.01 to 0.05.



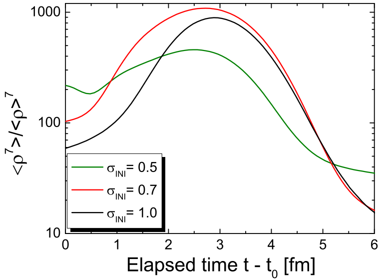


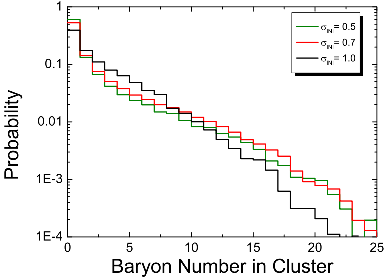
Quantitative results depend on choice of a.
Cluster formation is stronger for small values of a, as expected.

Changing the Initial Fluctuations

Parameter Dependencies

We change the value of the initial fluctuation width from $\sigma_{ini} = 0.5$ to 1.0 fm.





Strongest clustering observed for intermediate width!

Changing the Initial State

Model Dependencies?

For the initial state we can apply a version of the UrQMD model that includes nuclear interactions. [a]

[a] Q. -f. Li, Z. -x. Li, S. Soff, M. Bleicher and H. Stoecker, J. Phys. G **32**, 407 (2006)

The densities achieved in the UrQMD+Potentials calculations are considerably smaller than in those without (close to the geometrical overlap values).
This is mainly due to the repulsive interaction.