QCD critical point and Hydro+

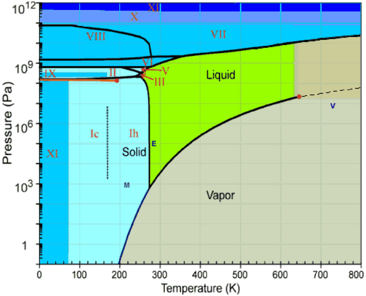
M. Stephanov

with Y. Yin (MIT), 1712.10305

Substance[13][14]	Critical temperature	Critical pressure (absolute)
Argon	−122.4 °C (150.8 K)	48.1 atm (4,870 kPa)
Ammonia[15]	132.4 °C (405.5 K)	111.3 atm (11,280 kPa)
Bromine	310.8 °C (584.0 K)	102 atm (10,300 kPa)
Caesium	1,664.85 °C (1,938.00 K)	94 atm (9,500 kPa)
Chlorine	143.8 °C (416.9 K)	76.0 atm (7,700 kPa)
Ethanol	241 °C (514 K)	62.18 atm (6,300 kPa)
Fluorine	−128.85 °C (144.30 K)	51.5 atm (5,220 kPa)
Helium	−267.96 °C (5.19 K)	2.24 atm (227 kPa)
Hydrogen	−239.95 °C (33.20 K)	12.8 atm (1,300 kPa)
Krypton	−63.8 °C (209.3 K)	54.3 atm (5,500 kPa)
CH_4 (methane)	−82.3 °C (190.8 K)	45.79 atm (4,640 kPa)
Neon	−228.75 °C (44.40 K)	27.2 atm (2,760 kPa)
Nitrogen	−146.9 °C (126.2 K)	33.5 atm (3,390 kPa)
Oxygen	−118.6 °C (154.6 K)	49.8 atm (5,050 kPa)
CO_2	31.04 °C (304.19 K)	72.8 atm (7,380 kPa)
N_2O	36.4 °C (309.5 K)	71.5 atm (7,240 kPa)
H_2SO_4	654 °C (927 K)	45.4 atm (4,600 kPa)
Xenon	16.6 °C (289.8 K)	57.6 atm (5,840 kPa)
Lithium	2,950 °C (3,220 K)	652 atm (66,100 kPa)
Mercury	1,476.9 °C (1,750.1 K)	1,720 atm (174,000 kPa)
Sulfur	1,040.85 °C (1,314.00 K)	207 atm (21,000 kPa)
Iron	8,227 °C (8,500 K)	
Gold	6,977 °C (7,250 K)	5,000 atm (510,000 kPa)
Water[2][16]	373.946 °C (647.096 K)	217.7 atm (22.06 MPa)

Critical point
– end of phase coexistence –
is a ubiquitous phenomenon

Water:



Is there one in QCD?

QCD critical point

- QCD is a *relativistic* QFT of a fundamental force,
  not quite like non-relativistic fluids.
- But a critical point is a very universal phenomenon –
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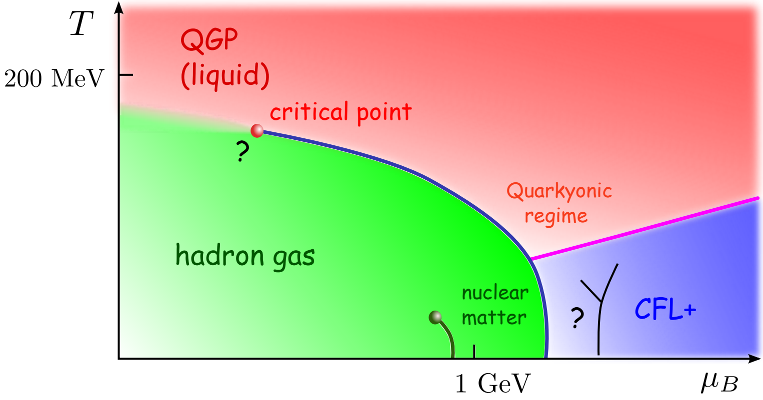
In QCD:

- The two phases: quark-gluon plasma and hadron gas.

  Experiments: QGP has liquid properties – almost perfect fluidity.
- If the phases are separated by a first-order phase transition, there must also be a critical point!

QCD phase diagram (sketch)



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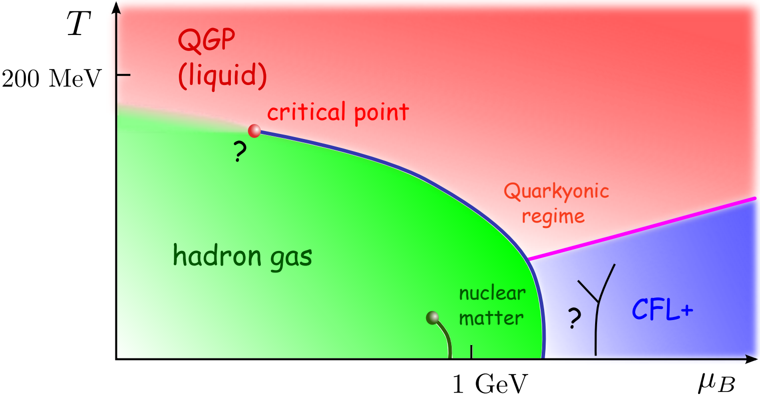

Lattice QCD at $\mu_B \lesssim 2T$ – a crossover

Therefore, if at larger μ_B $\exists$ first-order transition $\Rightarrow$ $\exists$ critical point

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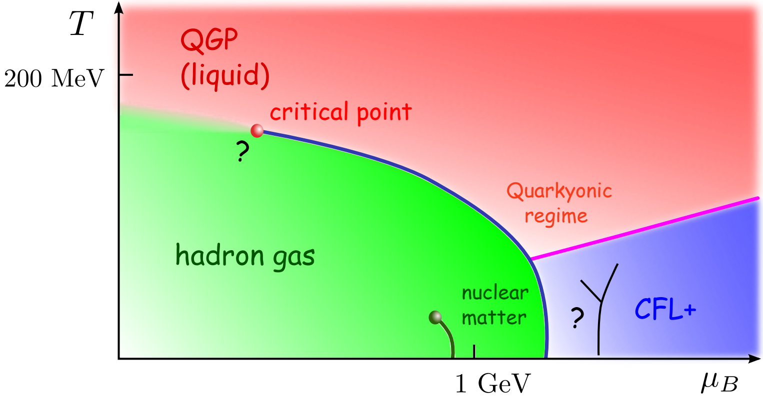

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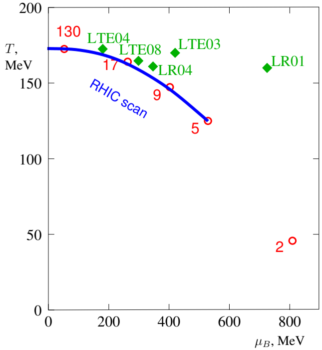
Critical point discovery challenges

Essentially two approaches to discovering the QCD critical point.

Each with its own challenges.

- Lattice simulations. *Sign problem.*
- Heavy-ion collisions.

  Encouraging progress
  and intriguing new results.



Challenge in connecting the two: *non-equilibrium dynamics.*

Fluctuations as signatures of the critical point

Fluctuations are observables on the lattice and in heavy-ion collisions.

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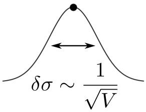

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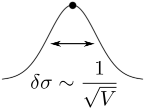
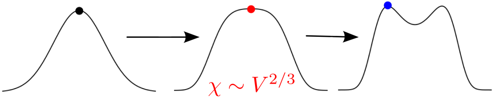


- At the critical point $S(\sigma)$ “flattens”. And $\chi \equiv \langle \delta\sigma^2 \rangle V \to \infty$.



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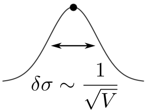

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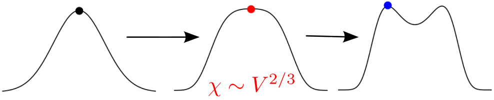
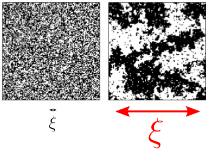




CLT?

$\delta\sigma$ is not an average of ∞ many *uncorrelated* contributions: $\xi \to \infty$

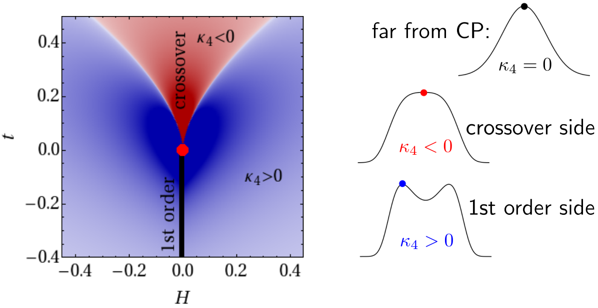
In fact, $\langle \delta\sigma^2 \rangle \sim \xi^2/V$.

Higher order cumulants

- $n > 2$ cumulants (shape of $P(\sigma)$) depend stronger on ξ.

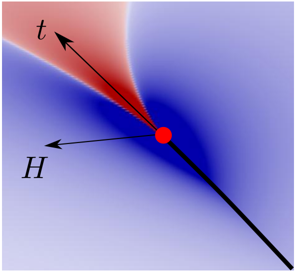
  E.g., $\langle\sigma^2\rangle \sim \xi^2$ while $\kappa_4 = \langle\sigma^4\rangle_c \sim \xi^7$ *[PRL102(2009)032301]*
- For $n > 2$, sign depends on which side of the CP we are.

  This dependence is also universal. *[PRL107(2011)052301]*
- Using Ising model variables:



Mapping Ising to QCD phase diagram

T vs μ_B:

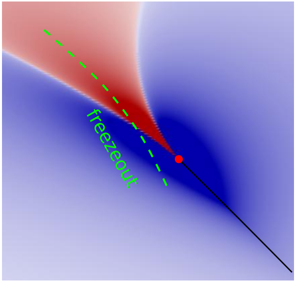


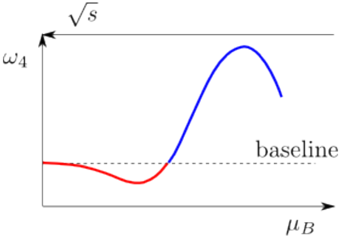
- In QCD $(t, H) \to (\mu - \mu_{\text{CP}}, T - T_{\text{CP}})$

- $\kappa_n(N) = N + \mathcal{O}(\kappa_n(\sigma))$

Beam Energy Scan I: intriguing hints

Equilibrium κ_4 vs T and μ_B:





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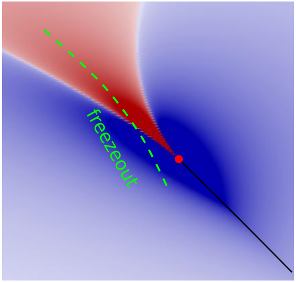

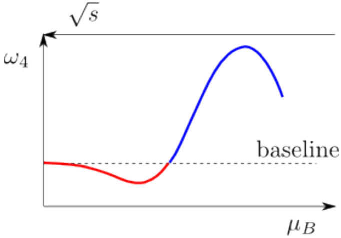

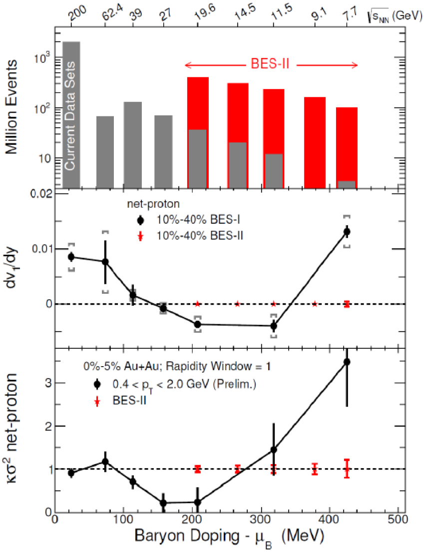

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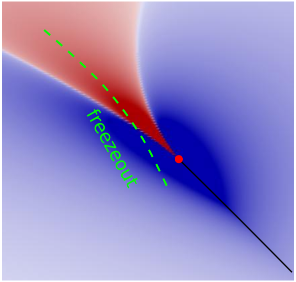

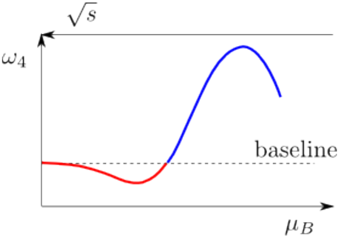

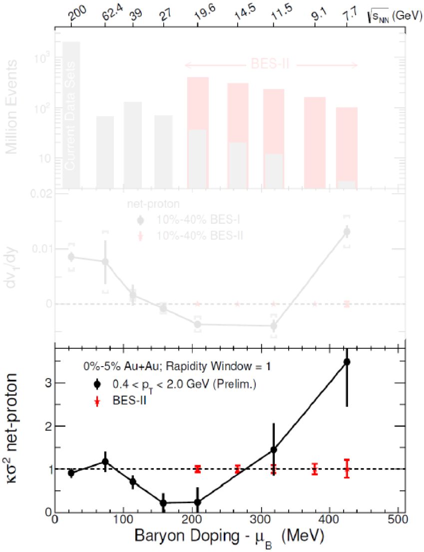

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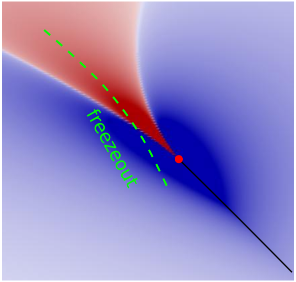
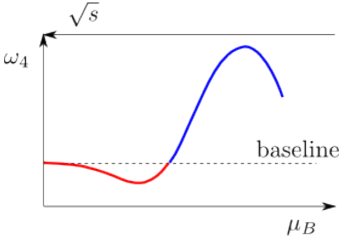
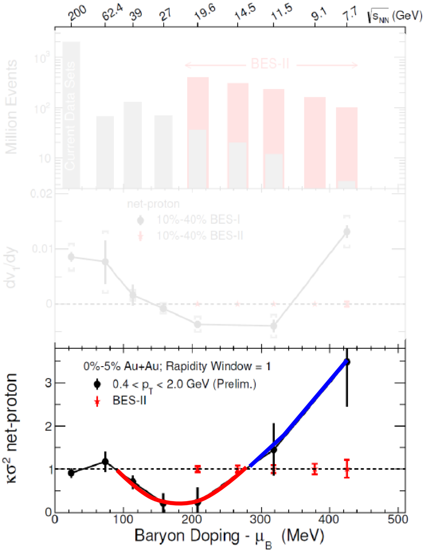






"intriguing hint" (2015 LRPNS)

Non-equilibrium physics is essential near the critical point.

The challenge taken on by BEST COLLABORATION

- Goal: build a *quantitative* theoretical framework describing critical point signatures for comparison with experiment.
- Strategy:
  - Parameterize QCD equation of state with unknown $T_{\rm CP}$ and $\mu_{\rm CP}$ as variable parameters.
  - Use it in a hydrodynamic simulation and compare with experiment to determine or constrain $T_{\rm CP}$ and $\mu_{\rm CP}$.

Parameterized EOS for hydro simulations

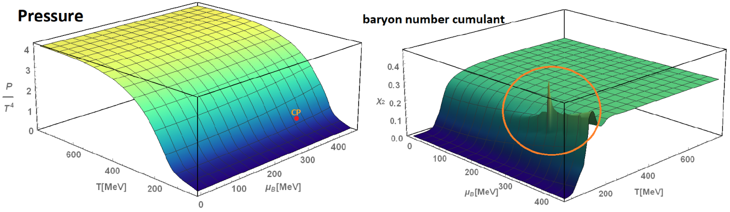
Parotto *et al*, 1805.05249

- Variable parameters ($T_{\rm CP}$, $\mu_{\rm CP}$, slopes, etc.) control Ising-QCD mapping near the QCD critical point: $P = P^{\rm Non\text{-}Ising} + P^{\rm Ising}$.
- Lattice data at $\mu_B = 0$ is matched:

Decomposition: Taylor coefficients from Lattice QCD contain an **"Ising"** contribution and a **"Non-Ising"** one:

$$\boxed{T^4 c_n^{LAT}(T) = T^4 c_n^{\text{Non-Ising}}(T) + T_C^4 c_n^{\text{Ising}}(T)} \qquad (\bigstar)$$

(parametrization of continuum extrapolated WB Lattice data from S. Borsanyi et al., JHEP 1011 (2010) 077, R. Bellwied et al., Phys.Rev. D92 (2015) no.11, 114505)



This EOS is ready to be used in a hydrodynamic simulation.

Hydrodynamics breaks down near the critical point

- Hydrodynamics, as an EFT, relies on separation of scales:

  Evolution rate (e.g., expansion time, $\mathcal{O}(10)$fm) much slower than the local equilibration rate (typically, $\mathcal{O}(0.5-1)$fm).

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- In fact, magnitude of ξ, and thus fluctuations/cumulants $\kappa_n \sim \xi^p$, is estimated using $\xi \sim \tau_{\text{expansion}}^{1/z}$.

- To be more quantitative we need to describe the breakdown of hydro due to critical slowing down.

Hydro+

[MS-Yin,1712.10305]

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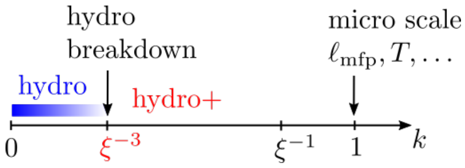
- Breakdown of *locality* is manifested in large gradient corrections to pressure due to $\zeta \sim \xi^3 \to \infty$.

$$p_{\text{hydro}} = p_{\text{equilibrium}} - \zeta\, \boldsymbol{\nabla} \cdot \boldsymbol{v}$$



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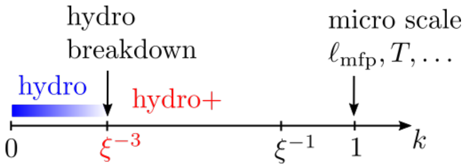

- Extending hydro by adding the critically slow modes $\to$ Hydro+

What are the additional slow modes?

- An *equilibrium* thermodynamic state is completely characterized by average values $\bar{\varepsilon}$, $\bar{n}$, $\ldots$.

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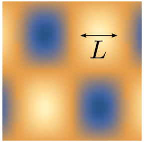
- Hydrodynamics describes *partial-equilibrium states*, i.e., equilibrium is only local, because equilibration time $\sim L^2$.

  Fluctuations in such states are not necessarily in equilibrium.



Nonequilibrium fluctuations

- Measures of fluctuations are *additional* variables needed to characterize the partial-equilibrium state.

  2-point (and n-point) functions of fluctuating hydro variables: $\langle \delta\varepsilon\delta\varepsilon \rangle$, $\langle \delta n \delta n \rangle$, $\langle \delta\varepsilon \delta n \rangle$, ... (Or probability functional).

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- Relaxation rates of 2pt functions is of the same order as that of corresponding 1pt functions.

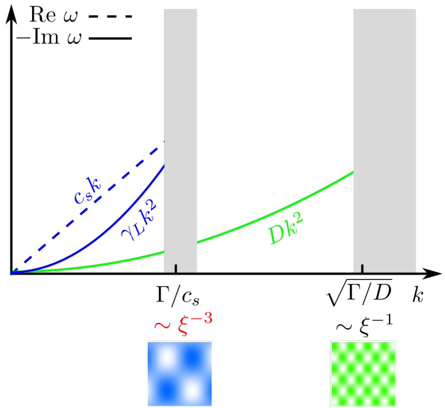
  But effects of fluctuations are *usually suppressed* due to averaging out: $\sqrt{\xi^3/V} \sim (k\xi)^{3/2} \ll 1$ by CLT.

  This is why 1st-order hydrodynamics exists (for $d > 2$).

Critical fluctuations

- Near CP there is *parametric* separation of relaxation time scales.

  The slowest and thus most out-of-equilibrium mode is charge diffusion at const p: $s/n \equiv m$.



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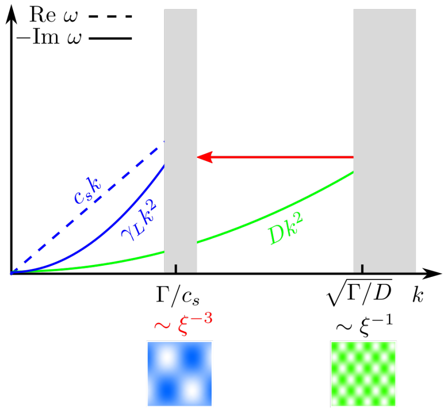

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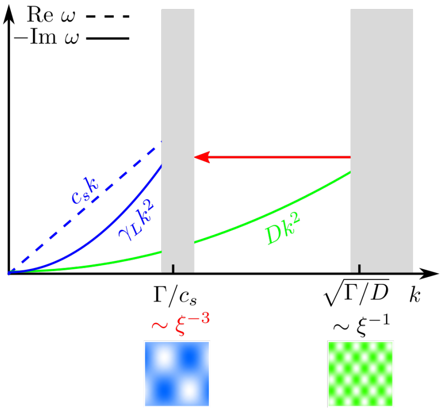

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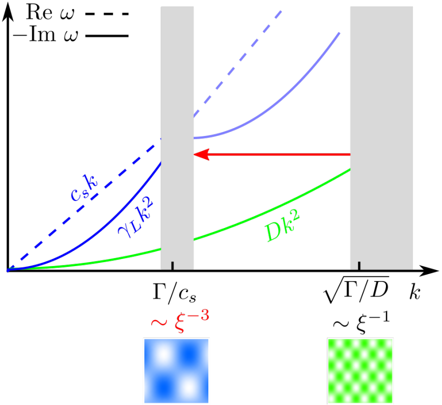
- Thus we need $\langle \delta m \delta m \rangle$ as the independent variable(s) in hydro+ equations.



New variables and their dynamics

- The new variable is 2-pt function $\langle \delta m \delta m \rangle$ (Wigner transform):

$$\phi_{\boldsymbol{Q}}(\boldsymbol{x}) = \int_{\Delta \boldsymbol{x}} \langle\, \delta m(\boldsymbol{x} + \Delta \boldsymbol{x}/2)\, \delta m(\boldsymbol{x} - \Delta \boldsymbol{x}/2) \,\rangle \;\; e^{i\boldsymbol{Q}\cdot\Delta\boldsymbol{x}}$$

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- Hydro(+) describes relaxation to eqlbrm, maximizing entropy.

  To ensure the 2nd law of thermodynamics is obeyed we need to know the entropy: $s_{(+)}(\varepsilon, n, \phi_{\boldsymbol{Q}})$, i.e., “EOS+”.

  Starting from the definition of S for a given ensemble of states

$$S = \sum_i p_i \log(1/p_i),$$ one arrives at …

Entropy of fluctuations

- …a result resembling 2-PI action: *(1712.10305)*

$$s_{(+)}(\varepsilon, n, \phi_{\boldsymbol{Q}}) = s(\varepsilon, n) + \frac{1}{2}\int_{\boldsymbol{Q}} \left(1 - \frac{\phi_{\boldsymbol{Q}}}{\bar{\phi}_{\boldsymbol{Q}}} + \log \frac{\phi_{\boldsymbol{Q}}}{\bar{\phi}_{\boldsymbol{Q}}}\right)$$

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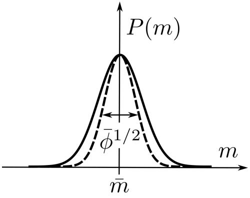
- Entropy = log # of states, depends on the width $\phi_{\boldsymbol{Q}}$:
  - Wider distribution – more microstates – more entropy: $\log(\phi/\bar{\phi})^{1/2}$;

    vs
  - Penalty for larger deviations from peak entropy (at $\delta m = 0$): $-(1/2)\phi/\bar{\phi}$.

Maximimum of $s_{(+)}$ is achieved at $\phi = \bar{\phi}$.



--- equilibrium (variance $\bar{\phi}$)

—- actual (variance ϕ)

Hydro+ mode kinetics

- The equation for $\phi_{\boldsymbol{Q}}$ is a relaxation equation:

$$(u\cdot\partial)\phi_{\boldsymbol{Q}} = -\gamma_\pi(\boldsymbol{Q})\pi_{\boldsymbol{Q}}, \quad \pi_{\boldsymbol{Q}} = -\left(\frac{\partial s_{(+)}}{\partial\phi_{\boldsymbol{Q}}}\right)_{\varepsilon,n}$$

$\gamma_\pi(\boldsymbol{Q})$ is known from mode-coupling calculation in model H (Kawasaki). It is universal. At $Q\sim\xi^{-1}$, $\gamma_\pi(Q)\sim\xi^{-3}$.

Hydro+ mode kinetics

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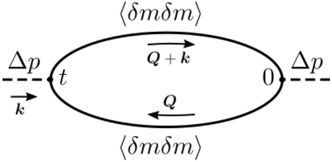
  $\gamma_\pi(\boldsymbol{Q})$ is known from mode-coupling calculation in model H (Kawasaki). It is universal. At $Q\sim\xi^{-1}$, $\gamma_\pi(Q)\sim\xi^{-3}$.

- The mode distribution function $\phi_{\boldsymbol{Q}}$ is similar to particle distribution function in kinetic theory (Wigner transform).

- In equilibrium, Hydro+ = 1-loop.
  Similar to kinetic theory vs HTL.
  Separation of scales: $Q\gg k\sim 1/L$.

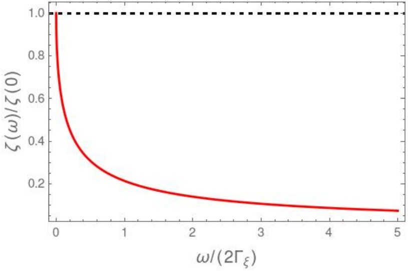


Hydro+ vs Hydro: real-time bulk response

Characteristic Hydro-to-Hydro+ crossover rate $\Gamma_\xi = D\xi^{-2} \sim \xi^{-3}$.

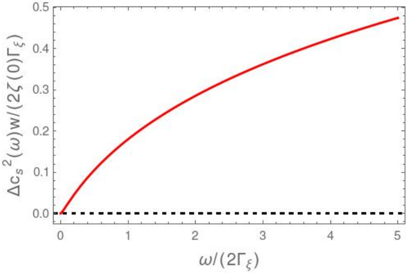
- Dissipation during expansion is overestimated in hydro (- - -):

  Only modes with $\omega \ll \Gamma_\xi$ experience large ζ.



- Stiffness of eos (sound speed) is underestimated in hydro (- - -):

  Only modes with $\omega \ll \Gamma_\xi$ are critically soft ($c_s \to 0$ at CP).



Summary

- A fundamental question about QCD:

  Is there a critical point on the QGP-HG boundary?

- Lattice: crossover for $\mu_B \lesssim 2T_c$.

  Thus first-order transition for larger $\mu_B \Leftrightarrow$ critical point.

- Intriguing results from experiments (BES-I).

  More to come from BES-II (also FAIR/CBM, NICA, J-PARC).

  Quantitative theoretical framework is needed $\Rightarrow$ BEST COLLABORATION .

- In H.I.C., the magnitude of the fluctuation signatures of CP is controlled by dynamical *non-equilibrium effects*.

  In turn, critical fluctuations affect hydrodynamics.

  The interplay of critical and dynamical phenomena: Hydro+.

More

Critical fluctuations and experimental observables

Observed fluctuations are related to fluctuations of σ.

[MS-Rajagopal-Shuryak PRD60(1999)114028; MS PRL102(2009)032301]

Think of a collective mode described by field σ such that $m = m(\sigma)$:

$$\delta n_{\boldsymbol{p}} = \delta n_{\boldsymbol{p}}^{\text{free}} + \frac{\partial \langle n_{\boldsymbol{p}} \rangle}{\partial \sigma} \times \delta\sigma$$

The cumulants of multiplicity $M \equiv \int_{\boldsymbol{p}} n_{\boldsymbol{p}}$:

- $\kappa_4[M] = \underbrace{\langle M \rangle}_{\text{baseline}} + \kappa_4[\sigma] \times g^4 \Big($  $\Big)^4 + \ldots,$ (the bracketed term $\sim M^4$)

 $= \int_{\boldsymbol{p}} \frac{n_{\boldsymbol{p}}}{\gamma_{\boldsymbol{p}}}$ $\leftarrow$ acceptance dependent