Comments on finite temperature/density in holographic QCD

(Review + Unpublished analyses + Speculation)

Shigeki Sugimoto
(YITP)
Introduction

Top-down holographic QCD

Gauge/String duality predicts the following equivalence:

4 dim U(Nc) QCD with Nf massless quarks + massive adjoint matters (realized in a D-brane system) = Type IIA string theory in a 10 dim curved background with Nf probe D8-branes “holographic QCD”

- They are conjectured to be equivalent even at finite Nc.
- The left side flows to real (massless) QCD at low energies.
- The details are not important in this talk.

Let’s just call it

“QCD” = “String theory”
More on the correspondence (1)

“QCD” \(\equiv\) “String theory”

- glueballs ↔ closed strings \(\circ\) graviton
- mesons ↔ open strings \(\circ\) gauge field
- baryons ↔ D-brane \(\bullet\) soliton

Manifestation of the quark-hadron duality!

If you are not familiar with terminologies in string theory, just use this dictionary to translate them in terms of hadrons.
More on the correspondence (2)

“QCD”  =  “String theory”

parameters

\[
\begin{align*}
M_{kk} & : \text{mass scale of the extra-fields (cut-off scale for real QCD)} \\
\lambda = g^2 N_c & : \text{'t Hooft coupling at } M_{kk}
\end{align*}
\]

\[
\begin{align*}
1/N_c \text{ expansion} & \leftrightarrow \text{Loop expansion} \\
1/\lambda \text{ expansion} & \leftrightarrow \text{Derivative (}\alpha'\text{) expansion}
\end{align*}
\]

“QCD” will become real QCD in the limit: $M_{kk} \to \infty$, fixing $\Lambda_{QCD}$.
But, this is unfortunately difficult in the “String theory” side, because $\lambda \to 0$.
For this reason, we keep $M_{kk}$ finite ($\sim 1\text{GeV}$).

→ We shouldn’t expect good results for $T>>\Lambda_{QCD}$, $\mu_q>>\Lambda_{QCD}$.
We start with low temp/density and try to see what happens for larger values, hoping that it captures some qualitative features of real QCD.
plan

1 ✓ Introduction
2 Review of key properties
3 Temperature
4 Density
5 Summary
We just focus on two key properties of the “String theory”.

1. The space-time is **5 dim**.

   (It is actually 10 dim, but we can reduce it to 5 dim, if we are interested in quantities for real 4 dim QCD.)

2. It is a theory of **Strings**.
The space-time is 5 dim. (cf. bottom-up model [Son-Stephanov 2003])

The low energy effective theory of open strings (mesons) is a 5 dim U(Nf) gauge theory

\[ S_{5\text{dim}} \sim N_c \int_{5\text{dim}} d^4 x dz \text{Tr}(F^2) + N_c \int_{5\text{dim}} \text{Tr}(A \wedge F \wedge F + \cdots) \]

Yang-Mills action

Chern-Simons 5 form

\[ A_\mu(x^\mu, z) = \sum_{n=1}^{\infty} B^{(n)}_\mu(x^\mu) \psi_n(z) \]

basis of functions of \( z \)

interpreted as \( \rho, \ a_1, \ \rho', \ \cdots \)

\[ A_z(x^\mu, z) = \varphi(x^\mu) \phi_0(z) + \sum_{n=1}^{\infty} \varphi^{(n)}(x^\mu) \phi_n(z) \]

basis of functions of \( z \)

interpreted as \( \pi \) eaten by \( B^{(n)}_\mu \)

→ generalization of hidden local sym [Bando-Kugo-Uehara-Yamawaki-Yanagida 1985]

External gauge fields associated with \( U(Nf)_L \times U(Nf)_R \) sym can be introduced by

\[ A^L_\mu(x^\mu) = \lim_{z \to +\infty} A_\mu(x^\mu, z), \ A^R_\mu(x^\mu) = \lim_{z \to -\infty} A_\mu(x^\mu, z) \]

→ Couplings to the electromagnetic gauge field can also be calculated.
“Evidence of extra dimensions” (1)

- Existence of $a_1$ meson with $m_\rho < m_{a_1}$
  
  *(prediction: $m_{a_1}/m_\rho \simeq 1.53$  experiment: $m_{a_1}/m_\rho \simeq 1.59$)*

- Vector meson dominance
  
  This 5 dim theory predicts complete vector meson dominance:

  Pion form factor
  
  $$F_\pi(k^2) = \sum_{n \geq 1} \left( \frac{g_\rho^n g_\rho^n \pi \pi}{k^2 + m_\rho^n} \right) \simeq \frac{g_\rho g_\rho \pi \pi}{k^2 + m_\rho^2}$$

  consistent with experiment

  $$\langle r^2 \rangle_{\pi^\pm} = -6 F'_\pi|_{k^2=0} \simeq (0.690 \text{ fm})^2 \quad \text{(prediction)}$$

  $$\simeq (0.672 \text{ fm})^2 \quad \text{(experiment)}$$

- Omega meson decay $\omega \to \pi^0 \gamma$ and $\omega \to \pi^0 \pi^+ \pi^-$

  agrees with GSW model! [Gell-Mann -Sharp-Wagner 1962]

  (※ Consistent with complete VMD, because there is an infinite tower of vector mesons)
“Evidence of extra dimensions” (2)

Nucleon electromagnetic form factor

\[ G_E^p(Q^2) \approx \frac{1}{(1 + Q^2/\Lambda^2)^2} \] (from experiment)

\[ G_E^p(Q^2) \approx \frac{m^2_\rho}{Q^2 + m^2_\rho} \] \( \times \)

\( \Rightarrow \) Not consistent with VMD?

\( \Rightarrow \) The infinite tower of vector mesons helps!

[Hashimoto-Sakai-S.S. 2008]
[See also, Hong-Rho-Yee-Yi 2007, Kim-Zahed 2008, Panico-Wulzer 2008]
It is a theory of **Strings**.

So far, we have only considered the 5 dim gauge field. There are also a lot of stringy massive states.

"Evidence of strings" (1)

[(Isovector) mesons in PDG]

\[ J = \alpha_0 + \alpha' m^2 \]

\[ \alpha_0 \approx 0.53 \quad \alpha' \approx 0.88 \text{ GeV}^{-2} \]

Meson spectrum in holographic QCD reproduces this behavior!
In the old days, string theory failed to be a realistic theory of hadrons:

- Need extra dimensions
- \( \exists \) massless particles with \( J=1 \) (open) and \( J=2 \) (closed)

\[ \Rightarrow \] These problems are solved by the idea of holography!

To compare with real QCD, we should get rid of the non-QCD artifacts:

\[
\begin{align*}
q-\bar{q} & \quad q-g-\bar{q} & \quad q-g-g-\bar{q} & \quad q-g-\cdots-g-\bar{q} & : \text{QCD mesons} \\
q-h-\bar{q} & \quad q-h-g-\bar{q} & \quad q-h-h-\bar{q} & \text{etc.} & : \text{non-QCD artifacts}
\end{align*}
\]

\((q : \text{quark}, \ \bar{q} : \text{anti-quark}, \ g : \text{gluon}, \ h : \text{heavy non-QCD particles})\)

\(\text{massless} \ \text{mass} \propto M_{KK} \sim 1 \text{ GeV}\)

- We can distinguish many of the non-QCD states by using extra SO(5) sym.
- The lightest state in a sector with given quantum number of a meson should be identified with a QCD meson.
“Evidence of strings” (2)

1st excited states

- $a_2(1320)$, $b_1(1235)$, $\pi(1300)$, $a_0(1450)$, …
- $J^{PC} = 2^{++}$, $1^{+-}$, $0^{-+}$, $0^{++}$

2nd excited states

- $\rho_3(1690)$, $\pi_2(1670)$, $\pi_1(1600)$, …
- $3^{--}$, $2^{-+}$, $1^{++}$

States with $J^{PC} = 2^{--}$, $1^{+-}$, $0^{++}$ are also predicted.

No good candidate states for $a_0(980)$, $\pi_1(1400)$

- Suggesting that they are 4 quark states.
3 Temperature

No phase transition, but a rapid raise around $T = 150\sim 200$ MeV

(Lattice result from Borsanyi et. al. PLB730, 99 (2014)
Figure taken from Vovchenko et al. arXiv:1412.5478)
Large $N_c$ QCD (large $N_c$ limit)

(Schematic picture)

$\varepsilon/T^4$ vs. $T$

- $\sigma_{SB} \sim O(N_c^2)$: free gas of quarks & gluons
- Large $N_c$ limit

Holographic QCD:
- Background becomes black hole (brane) $\rightarrow$ deconfinement
  - [Witten 1998]
- Chiral sym restoration
  - [Sakai-SS 2004, Aharony-Sonnenschein-Yankielowicz 2007]

Phase transition at $T_c \sim O(N_c^0)$
Can we see this behavior?

(Lattice result from Borsanyi et. al. PLB730, 99 (2014)
Figure taken from Vovchenko et al. arXiv:1412.5478)
**Hadron Resonance Gas (HRG) model**

- HRG model = Free gas of hadrons listed in PDG tables
- Lattice result compared with HRG

Taken from Bazavov et. al. (HotQCD) arXiv:1407.6387
HRG of mesons in holographic QCD

Our mesons ($N_f = 2$) up to 3rd excited states

$\varepsilon/T^4$

PDG mesons with $s=0$

$(\pi, \eta$ are massless)

$T$ (MeV)

$\rho/T^4$

$T$ (MeV)

$(\pi, \eta$ are massless)

$\varepsilon/T^4$

$(\pi, \eta$ masses adjusted to exp. values)

$T$ (MeV)

$p/T^4$

$(\pi, \eta$ masses adjusted to exp. values)

$T$ (MeV)
What happens if we increase $T$?

String theory predicts:

$$\rho(E) \sim e^{E/T_H}$$

Hagedorn Temperature

$$T_H \sim 200\text{MeV}$$
Transition to Blackhole

The backreaction becomes important when

$$G_N \varepsilon \sim \mathcal{O}(1)$$

Schwarzschild radius for D dim Blackhole

$$R_{\text{BH}}^{D-3} \sim G_N M$$

$$G_N \sim \mathcal{O}(N_c^{-2})$$

$$S_{\text{gravity}} \sim \frac{1}{G_N} \int d^D x \sqrt{-g} (R + \cdots)$$

The previous analysis breaks down when

$$\varepsilon \sim \mathcal{O}(N_c^2)$$

consistent with the previous picture

Entropy

$$S_{\text{BH}} \sim \frac{\text{Area}}{G_N} \sim MR_{\text{BH}}$$

$$S_{\text{string}} \sim \log(\#\text{states}) \sim \frac{M}{\sqrt{\sigma}}$$

$$\sigma \sim \mathcal{O}(1) : \text{string tension}$$

They match when

$$R_{\text{BH}} \sim \mathcal{O}(1)$$
This suggests a phase transition at a point denoted by $T_c$, which is close to the Hagedorn temperature $T_H$. The entropy density $\sigma / T^4$ shows a behavior proportional to $O(N_c^0)$ near $T_c$, transitioning to $O(N_c^2)$ in the Hagedorn regime.
Baryon = soliton in 5dim U(N_f) gauge theory

Baryon # : $N_B = \frac{1}{8\pi^2} \int \text{Tr}(F \wedge F)$
Size of the soliton is small when $\lambda >> 1$
charged under U(1) via CS-term
They want to live near $z=0$

Repulsive at short distance mainly because of U(1) charge.

$\nabla$ U(1) part $\ni \omega$-meson

5 dim $\rightarrow V(r) \propto r^{-2}$

(cf. field theoretical argument [Aoki-Balog-Weisz 2010])
Finite density

Possible interpretation:
oscillation modes in z direction $\rightarrow N(940), N(1535), N(1710), \cdots$
transformed to a linear combination of these states (?)

higher density

[Hata-Sakai-SS-Yamato 2007]
much higher density

We can put more baryons in the extra dimension!

[baryonic popcorn]

Doraemon’s 4 dim pocket

Analysis in a toy model

Treat solitons as charged fermion in 5 dim.
Only consider U(1) part of the U(Nf) gauge field.

→ Blackboard
(If I have time.)
Summary

“QCD” = “String theory”

These properties seem to be crucial in understanding hot/dense QCD using hadrons.

Questions:

Can we predict the order of phase transitions (for finite Nc)?
Can we really put things in the extra dimensions?
Thank you