Comments on finite temperature/density in holographic QCD

(Review + Unpublished analyses + Speculation)

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1 Introduction

- Top-down holographic QCD

[Witten 1998]
[Sakai-SS 2004, 2005]

Gauge/String duality predicts the following equivalence:

4 dim U(Nc) QCD with Nf massless quarks + massive adjoint matters (realized in a D-brane system)	=	Type IIA string theory in a 10 dim curved background with Nf probe D8-branes "holographic QCD"

- They are conjectured to be equivalent even at finite Nc.
- The left side flows to real (massless) QCD at low energies.
- The details are not important in this talk.

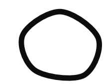
Let's just call it

"QCD" = "String theory"

More on the correspondence (1)

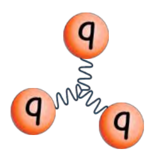
"QCD" = "String theory"

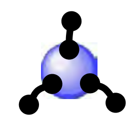
glueballs 

↔ closed strings  graviton

mesons 

↔ open strings  gauge field

baryons 

↔ D-brane  soliton

Manifestation of the quark-hadron duality!

If you are not familiar with terminologies in string theory,
just use this dictionary to translate them in terms of hadrons.

More on the correspondence (2)

"QCD" = "String theory"

parameters

- M_{KK} : mass scale of the extra-fields (cut-off scale for real QCD)
- $\lambda = g^2 N_c$: 't Hooft coupling at M_{KK}

$1/N_c$ expansion	$\leftrightarrow$	Loop expansion
$1/\lambda$ expansion	$\leftrightarrow$	Derivative (α') expansion

"QCD" will become real QCD in the limit: $M_{KK} \to \infty$, fixing Λ_{QCD}.

But, this is unfortunately difficult in the "String theory" side, because $\lambda \to 0$.

For this reason, we keep M_{KK} finite ($\sim$ 1GeV) .

$\to$ We shouldn't expect good results for $T \gg \Lambda_{QCD}$, $\mu_q \gg \Lambda_{QCD}$.
We start with low temp/density and try to see what happens for larger values, hoping that it captures some qualitative features of real QCD.

plan

✔ 1 Introduction

2 Review of key properties

3 Temperature

4 Density

5 Summary

② Review of key properties

- We just focus on two key properties of the “String theory”.

① The space-time is 5 dim.

(It is actually 10 dim, but we can reduce it to 5 dim,
if we are interested in quantities for real 4 dim QCD.)

② It is a theory of Strings.

① The space-time is 5 dim. (cf. bottom-up model [Son-Stephanov 2003])

- The low energy effective theory of open strings (mesons) is a 5 dim U(Nf) gauge theory

(μ=0,1,2,3) 5th coordinate

open strings  → 5 dim gauge field $A_\mu(x^\mu, z),\ A_z(x^\mu, z)$

$$S_{\text{5dim}} \sim N_c \int_{\text{5dim}} d^4x dz\, \text{Tr}(F^2) + N_c \int_{\text{5dim}} \text{Tr}(A \wedge F \wedge F + \cdots)$$

Yang-Mills action   Chern-Simons 5 form

- mode expansion

basis of functions of z   basis of functions of z

$$A_\mu(x^\mu, z) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x^\mu)\psi_n(z) \qquad A_z(x^\mu, z) = \varphi(x^\mu)\phi_0(z) + \sum_{n=1}^{\infty} \varphi^{(n)}(x^\mu)\phi_n(z)$$

interpreted as $\rho,\ a_1,\ \rho',\ \cdots$ interpreted as π eaten by $B_\mu^{(n)}$

→ generalization of hidden local sym [Bando-Kugo-Uehara-Yamawaki-Yanagida 1985]

- External gauge fields associated with $U(Nf)_L$ x $U(Nf)_R$ sym can be introduced by

$$A_\mu^L(x^\mu) = \lim_{z\to+\infty} A_\mu(x^\mu, z),\ \ A_\mu^R(x^\mu) = \lim_{z\to-\infty} A_\mu(x^\mu, z)$$

→ Couplings to the electromagnetic gauge field can also be calculated.

"Evidence of extra dimensions" (1)

- Existence of a_1 meson with $m_\rho < m_{a_1}$

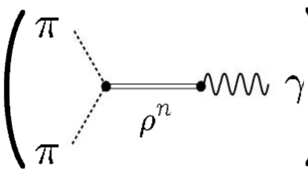
  (prediction: $m_{a_1}/m_\rho \simeq 1.53$ experiment: $m_{a_1}/m_\rho \simeq 1.59$)

- Vector meson dominance

  This 5 dim theory predicts complete vector meson dominance:

Pion form factor

$$F_\pi(k^2) = \sum_{n\geq 1} \left(\text{diagram} \right) = \sum_{n\geq 1} \frac{g_{\rho^n} g_{\rho^n \pi\pi}}{k^2 + m_{\rho^n}^2} \simeq \frac{g_\rho g_{\rho\pi\pi}}{k^2 + m_\rho^2}$$

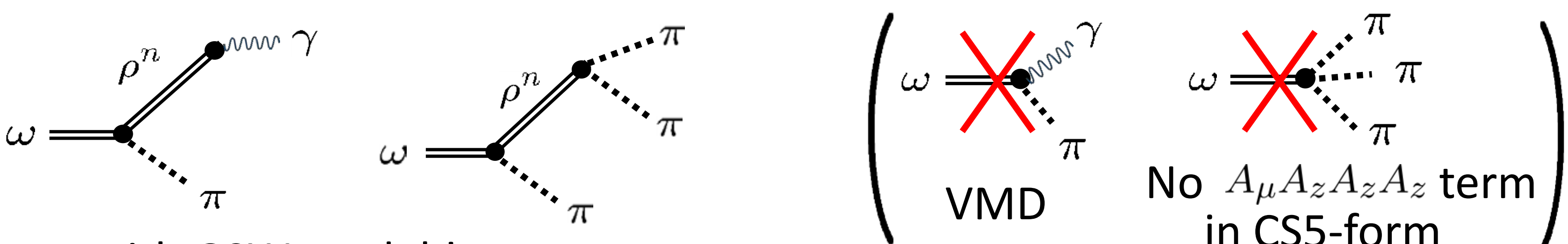


consistent with experiment

$$\langle r^2 \rangle^{\pi^\pm} = -6F'_\pi|_{k^2=0} \simeq (0.690\ \text{fm})^2 \quad \text{(prediction)}$$

$$\simeq (0.672\ \text{fm})^2 \quad \text{(experiment)}$$

- Omega meson decay $\omega \to \pi^0\gamma$ and $\omega \to \pi^0\pi^+\pi^-$



agrees with GSW model ! [Gell-Mann -Sharp-Wagner 1962]

(※ Consistent with complete VMD, because there is an infinite tower of vector mesons)

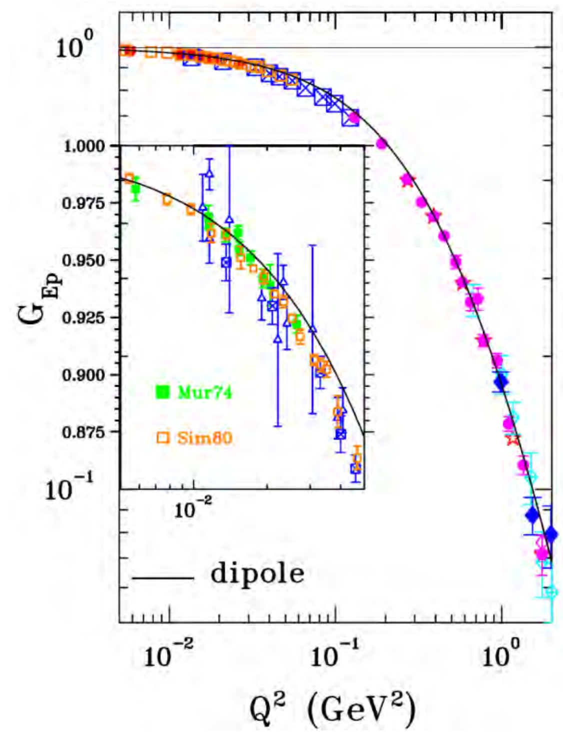
“Evidence of extra dimensions” (2)

(taken from Perdrisat et. al. hep-ph/0612014)

- Nucleon electromagnetic form factor

$$G_E^p(Q^2) \simeq \frac{1}{(1+Q^2/\Lambda^2)^2}$$ (from experiment)

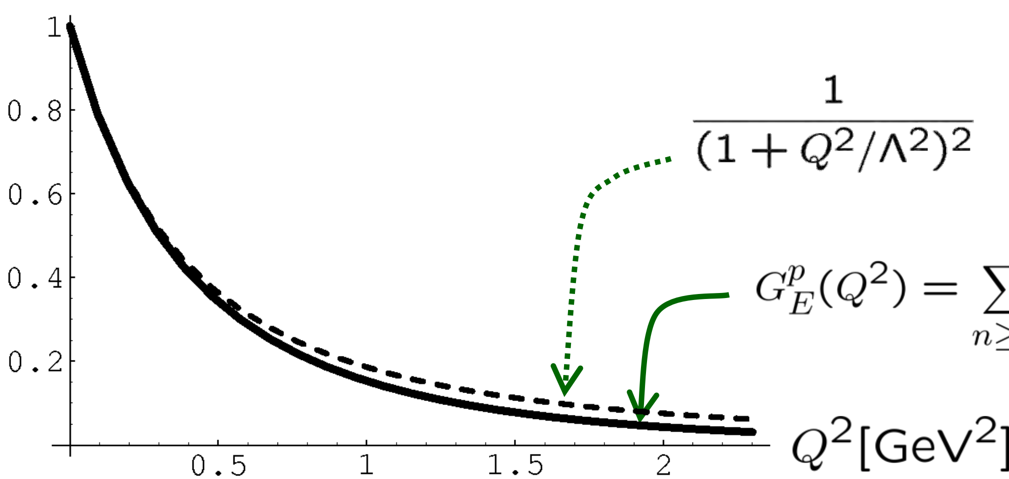
dipole



→ Not consistent with VMD?

$$G_E^p(Q^2) \simeq \frac{m_\rho^2}{Q^2+m_\rho^2}$$ ✗

⇒ The infinite tower of vector mesons helps!



$$\frac{1}{(1+Q^2/\Lambda^2)^2}$$ dipole

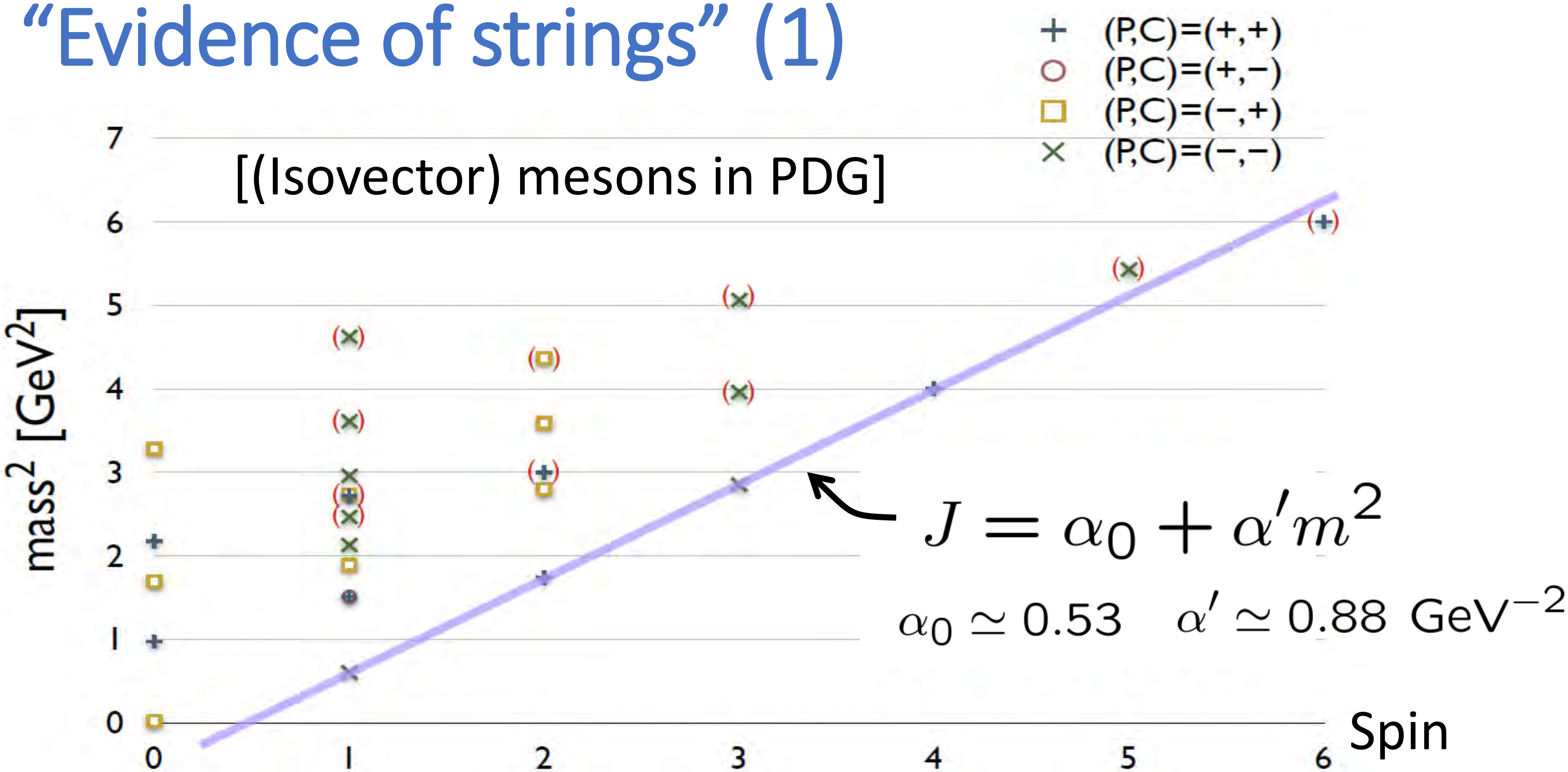
$$G_E^p(Q^2) = \sum_{n\geq 1} \frac{g_{\rho^n} g_{\rho^n NN}}{Q^2+m_n^2}$$ our result

[Hashimoto-Sakai-S.S. 2008]
[See also, Hong-Rho-Yee-Yi 2007, Kim-Zahed 2008, Panico-Wulzer 2008]

② It is a theory of Strings.

So far, we have only considered the 5 dim gauge field.
There are also a lot of stringy massive states.

"Evidence of strings" (1)



Meson spectrum in holographic QCD reproduces this behavior!

★ Comments

- In the old days, string theory failed to be a realistic theory of hadrons:
  - Need extra dimensions
  - ∃ massless particles with J=1 (open) and J=2 (closed)

  ⇒ These problems are solved by the idea of holography!

- To compere with real QCD, we should get rid of the non-QCD artifacts:

q-$\bar{q}$ $\quad$ q-g-$\bar{q}$ $\quad$ q-g-g-$\bar{q}$ $\quad$ q-g-$\cdots$-g-$\bar{q}$: QCD mesons

q-h-$\bar{q}$ $\quad$ q-h-g-$\bar{q}$ $\quad$ q-h-h-$\bar{q}$ $\quad$ etc. : non-QCD artifacts

$$(\underbrace{q : \text{quark},\ \bar{q} : \text{anti-quark},\ g : \text{gluon}}_{\text{massless}},\ h : \text{heavy non-QCD particles})$$

h → $\text{mass} \propto M_{\text{KK}} \sim 1\ \text{GeV}$

- We can distinguish many of the non-QCD states by using extra SO(5) sym.
- The lightest state in a sector with given quantum number of a meson should be identified with a QCD meson.

"Evidence of strings" (2)

[Imoto-Sakai-S.S. 2010]

- 1st excited states

→ $a_2(1320),\ b_1(1235),\ \pi(1300),\ a_0(1450),\ \cdots$

$J^{PC} = 2^{++}$ $\quad 1^{+-}$ $\quad 0^{-+}$ $\quad 0^{++}$

- 2nd excited states

→ $\rho_3(1690),\ \pi_2(1670),\ \pi_1(1600),\ \cdots$

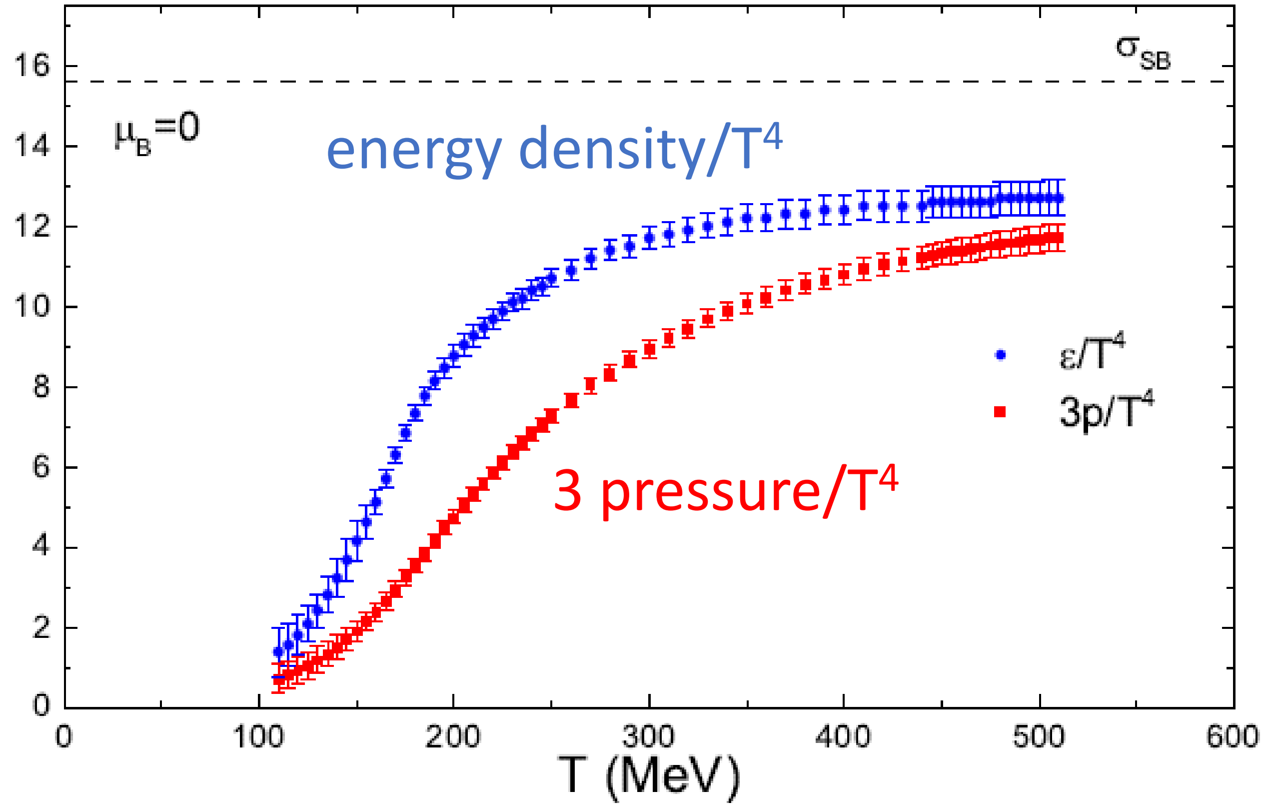
3^{--} $\quad 2^{-+}$ $\quad 1^{-+}$

States with $J^{PC} = 2^{--},\ 1^{+-},\ 0^{++}$ are also predicted.

- No good candidate states for $a_0(980),\ \pi_1(1400)$

→ Suggesting that they are 4 quark states.

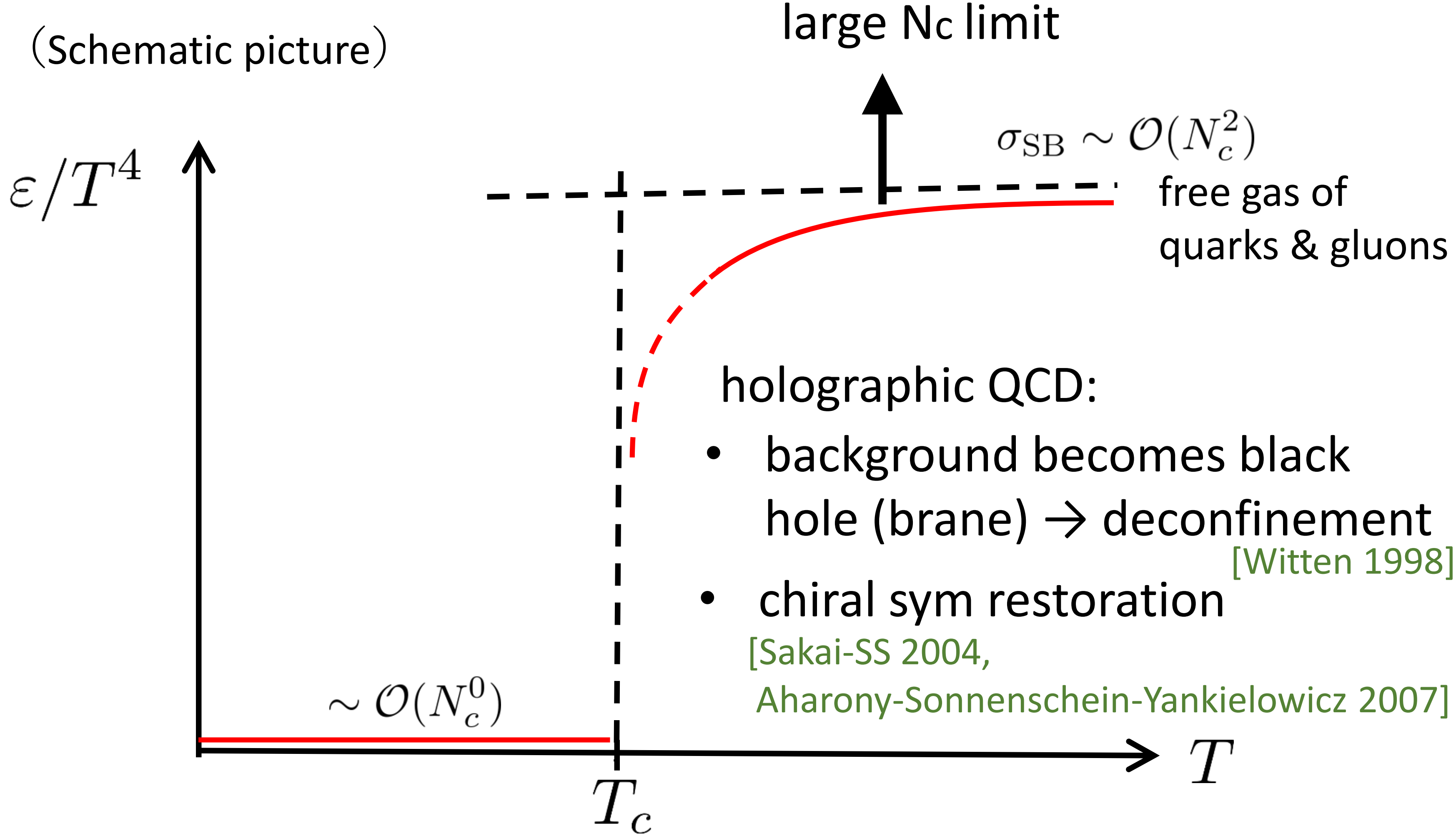
3 Temperature



(Lattice result from Borsanyi et. al. PLB730, 99 (2014)
Figure taken from Vovchenko et al. arXiv:1412.5478)

No phase transition, but a rapid raise around T = 150～200 MeV

★ Large Nc QCD (large Nc limit)

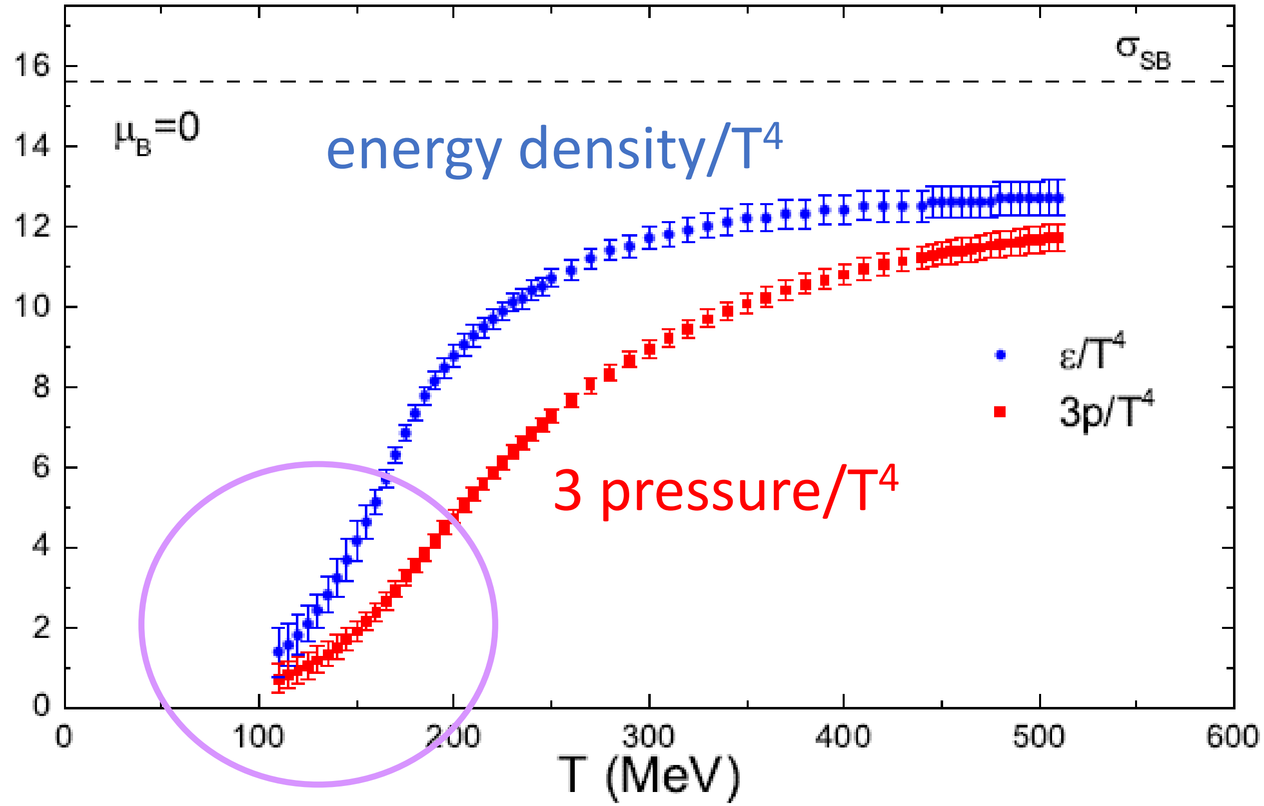


holographic QCD:

- background becomes black hole (brane) → deconfinement [Witten 1998]
- chiral sym restoration [Sakai-SS 2004, Aharony-Sonnenschein-Yankielowicz 2007]

phase transition at $T_c \sim \mathcal{O}(N_c^0)$

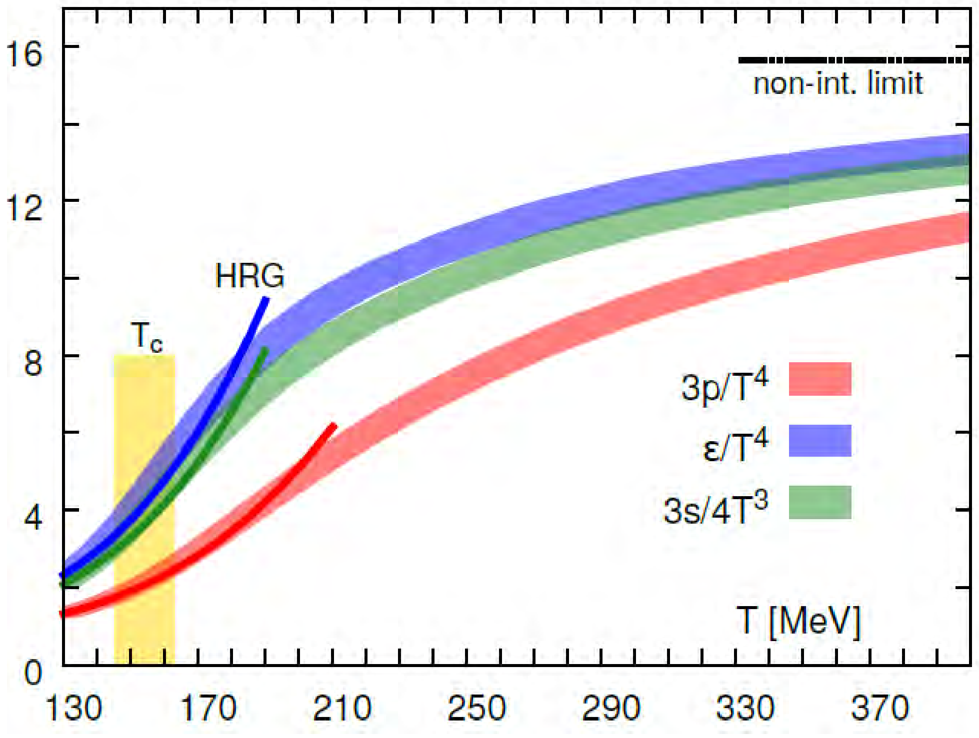
Can we see this behavior?



(Lattice result from Borsanyi et. al. PLB730, 99 (2014)
Figure taken from Vovchenko et al. arXiv:1412.5478)

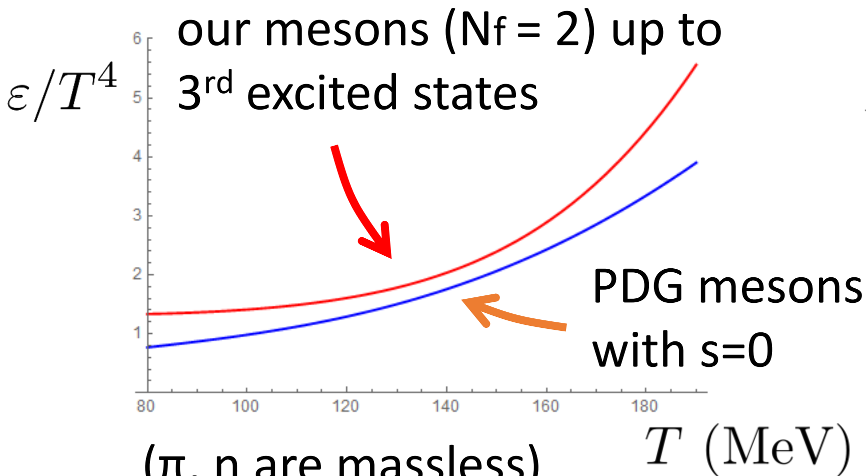
★ Hadron Resonance Gas (HRG) model

- HRG model = Free gas of hadrons listed in PDG tables
- Lattice result compared with HRG

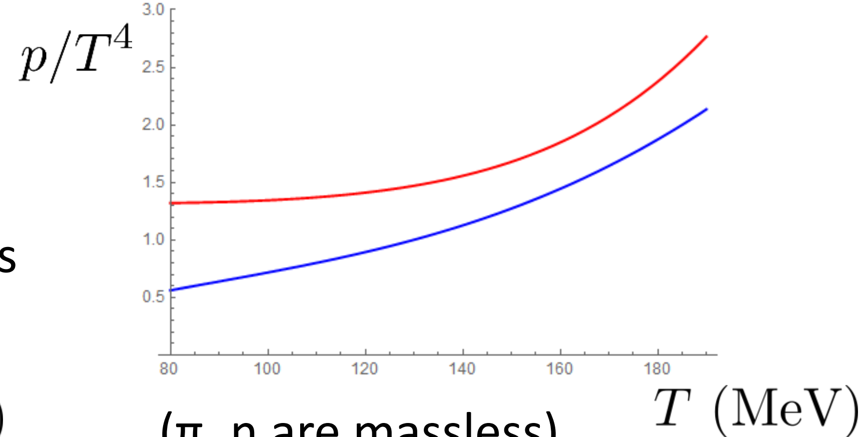


Taken from Bazavov et. al. (HotQCD) arXiv:1407.6387

★ HRG of mesons in holographic QCD



(π, η are massless)



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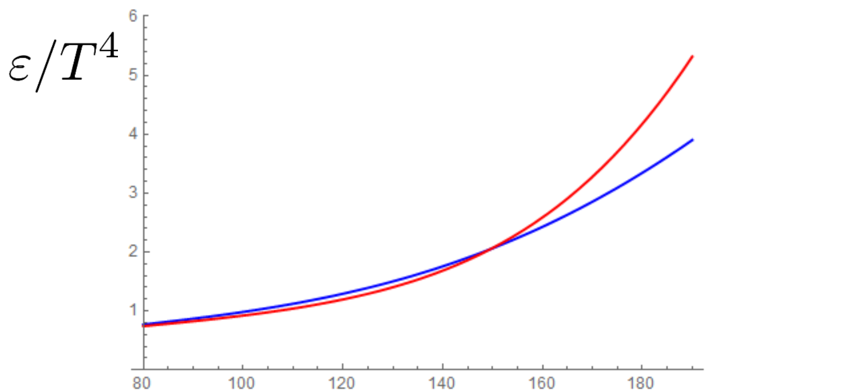
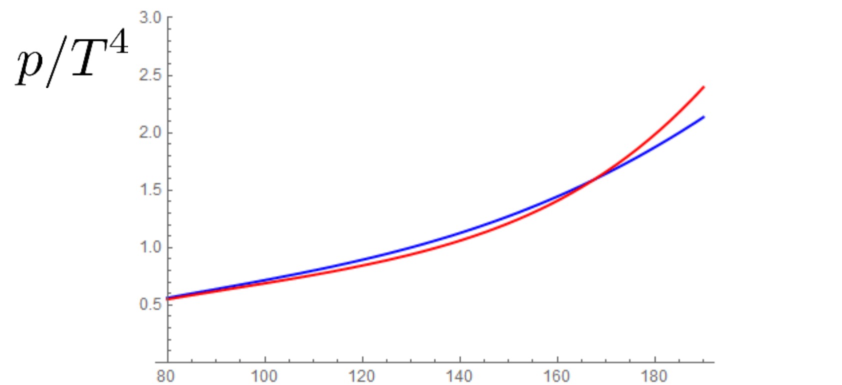


(π, η masses
adjusted to exp. values)



(π, η masses
adjusted to exp. values)

★ What happens if we increase T ?

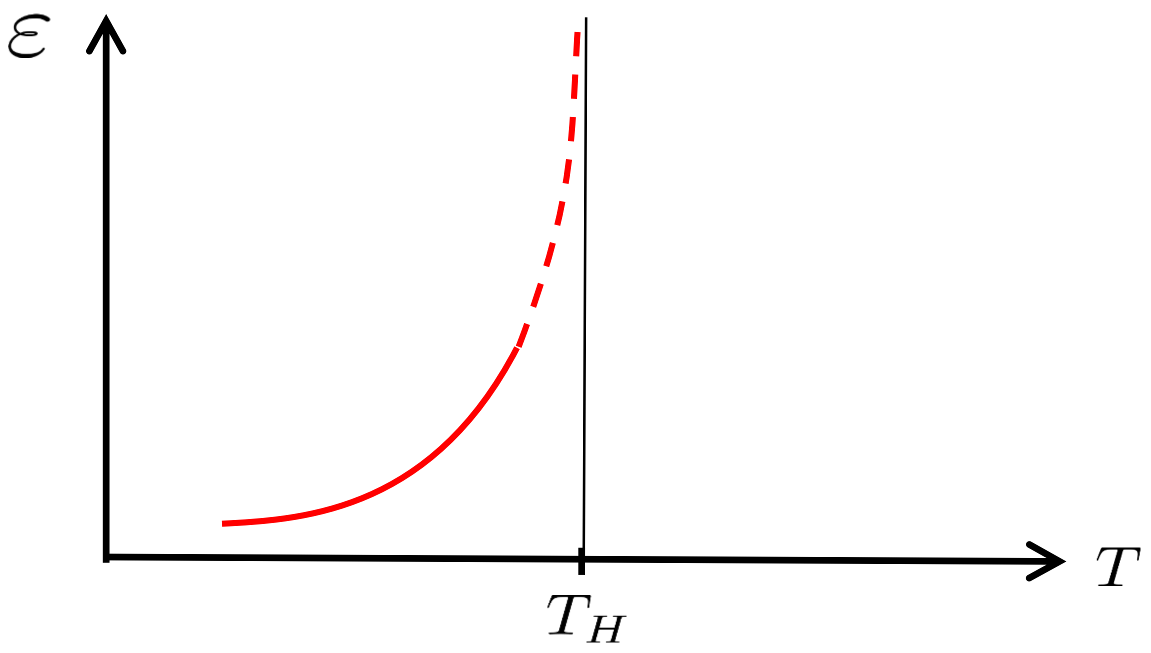
String theory predicts:

density of states

$$\rho(E) \sim e^{E/T_H}$$

Hagedorn Temperature

$T_H \sim 200\text{MeV}$



★ Transition to Blackhole

- The backreaction becomes important when

$$G_N \varepsilon \sim \mathcal{O}(1)$$

energy density

Newton constant

Schwarzschild radius for D dim Blackhole

$$(\text{cf. } R_{\text{BH}}^{D-3} \sim G_N M)$$

$$S_{\text{gravity}} \sim \frac{1}{G_N} \int d^D x \sqrt{-g}(R + \cdots) \qquad G_N \sim \mathcal{O}(N_c^{-2})$$

(glueball)

→ The previous analysis breaks down when $\varepsilon \sim \mathcal{O}(N_c^2)$

→ consistent with the previous picture

- Entropy [Horowitz-Polchinski 1996]

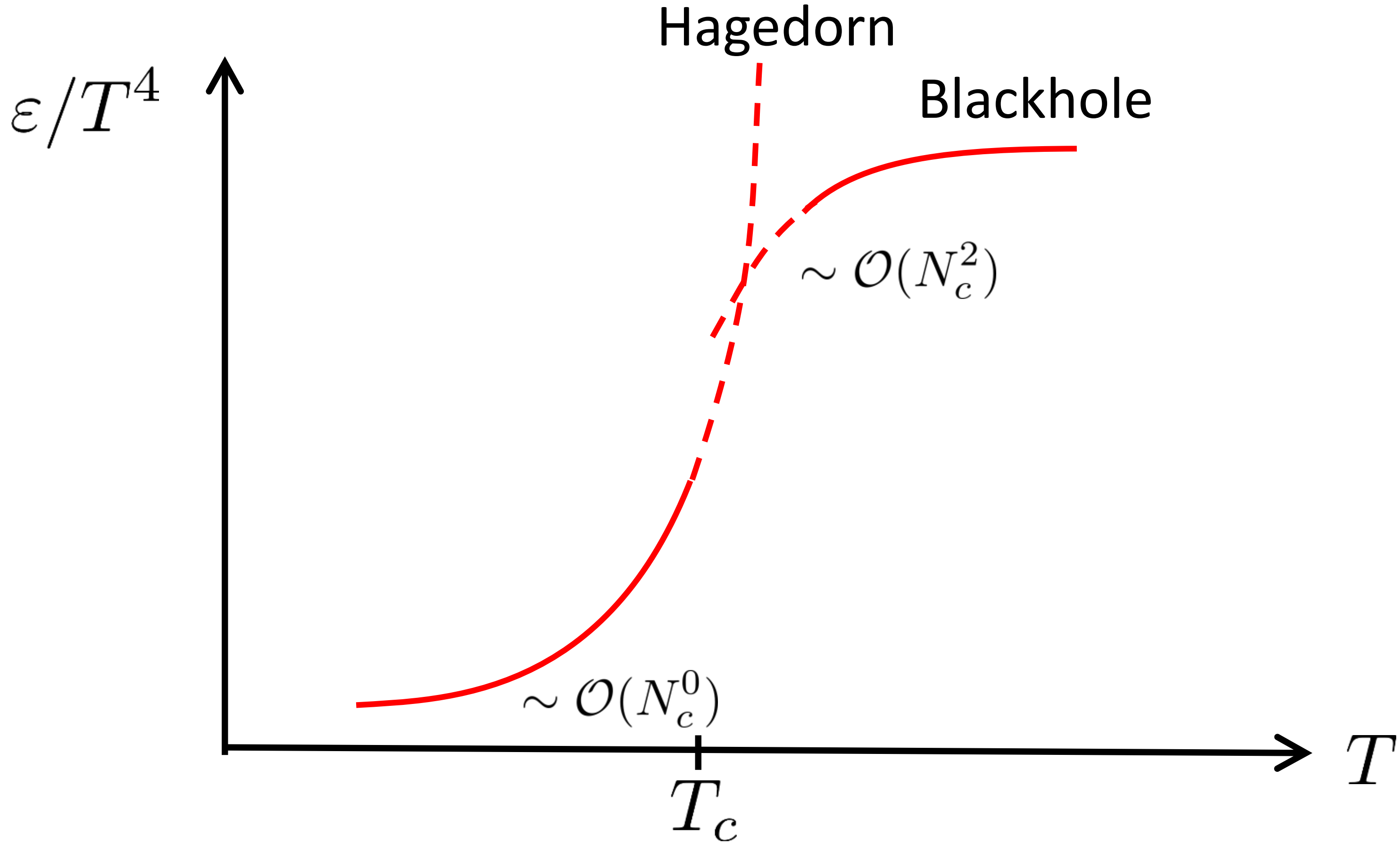
$$S_{\text{BH}} \sim \frac{\text{Area}}{G_N} \sim M R_{\text{BH}}$$

$$S_{\text{string}} \sim \log(\#\text{states}) \sim \frac{M}{\sqrt{\sigma}} \qquad \sigma \sim \mathcal{O}(1) : \text{string tension}$$

They match when $R_{\text{BH}} \sim \mathcal{O}(1)$!

★ This suggests

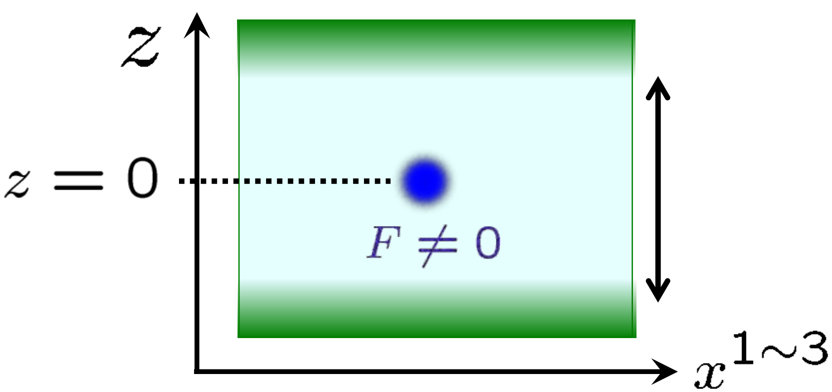
（Schematic picture）



phase transition at $T_c \sim T_H \sim \mathcal{O}(N_c^0)$

4 Density

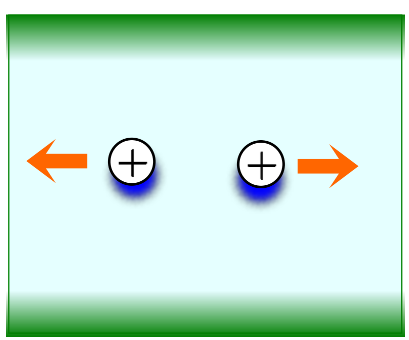
- Baryon = soliton in 5dim U(N_f) gauge theory



instanton number

- Baryon # : $N_B = \frac{1}{8\pi^2}\int \mathrm{Tr}(F \wedge F)$
- Size of the soliton is small when λ>>1
- charged under U(1) via CS-term
- They want to live near z=0

- Repulsive at short distance mainly because of U(1) charge.



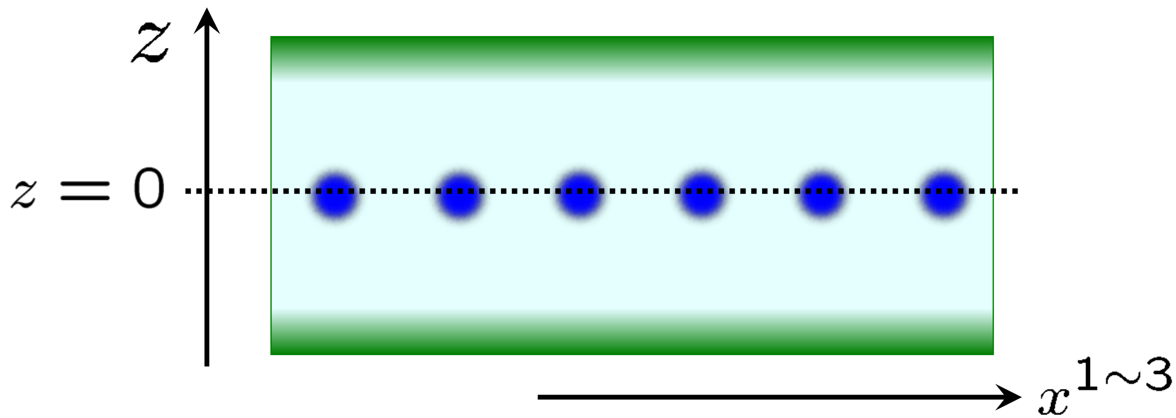
U(1) part $\ni \omega$-meson

5 dim → $V(r) \propto r^{-2}$

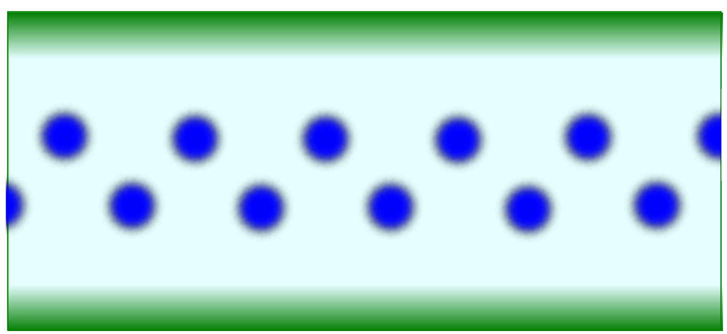
[Kim-Zahed 2009, Hashimoto-Sakai-SS 2009, Kim-Lee-Yi 2009,]

(cf. field theoretical argument [Aoki-Balog-Weisz 2010])

- Finite density



- higher density [Kaplunovsky-Melnikov -Sonnenschein 2012]



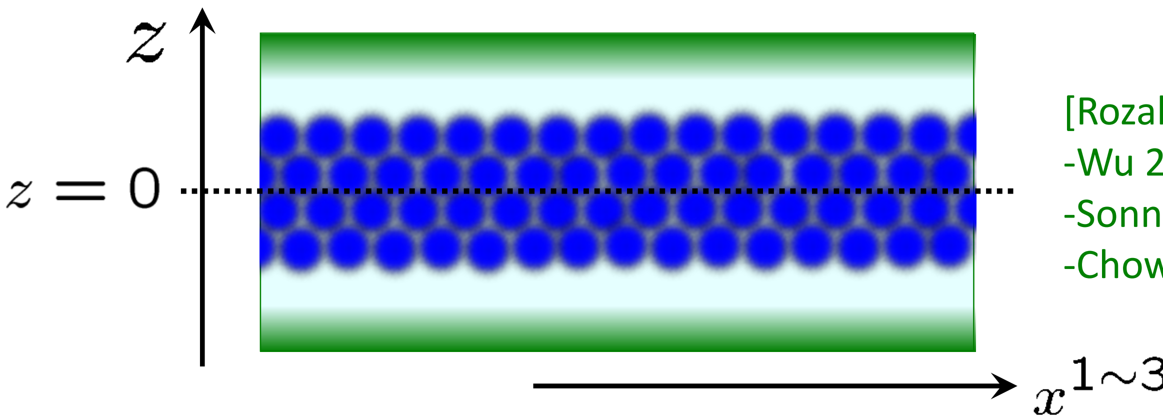
Possible interpretation:

[Hata-Sakai-SS-Yamato 2007]

oscillation modes in z direction → $N(940), N(1535), N(1710), \cdots$

transformed to a linear combination of these states (?)

- much higher density



[Rozali-Shieh-Van Raamsdonk -Wu 2007, Kaplunovsky-Melnikov -Sonnenschein 2012, de Boer -Chowdhury-Heller-Jankowski 2012,…]

We can put more baryons in the extra dimension!

"baryonic popcorn"

Doraemon's
4 dim pocket

★ Analysis in a toy model

Treat solitons as charged fermion in 5 dim.
Only consider U(1) part of the U(Nf) gauge field.

→ Blackboard

(If I have time.)

5 Summary

"QCD" = "String theory"

- 5 dim space-time
- Theory of strings

These properties seem to be crucial in understanding hot/dense QCD using hadrons.

Questions:

Can we predict the order of phase transitions (for finite Nc)?

Can we really put things in the extra dimensions?

Thank you