Complex Langevin analysis of the finite deinsity QCD

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In collaboration with

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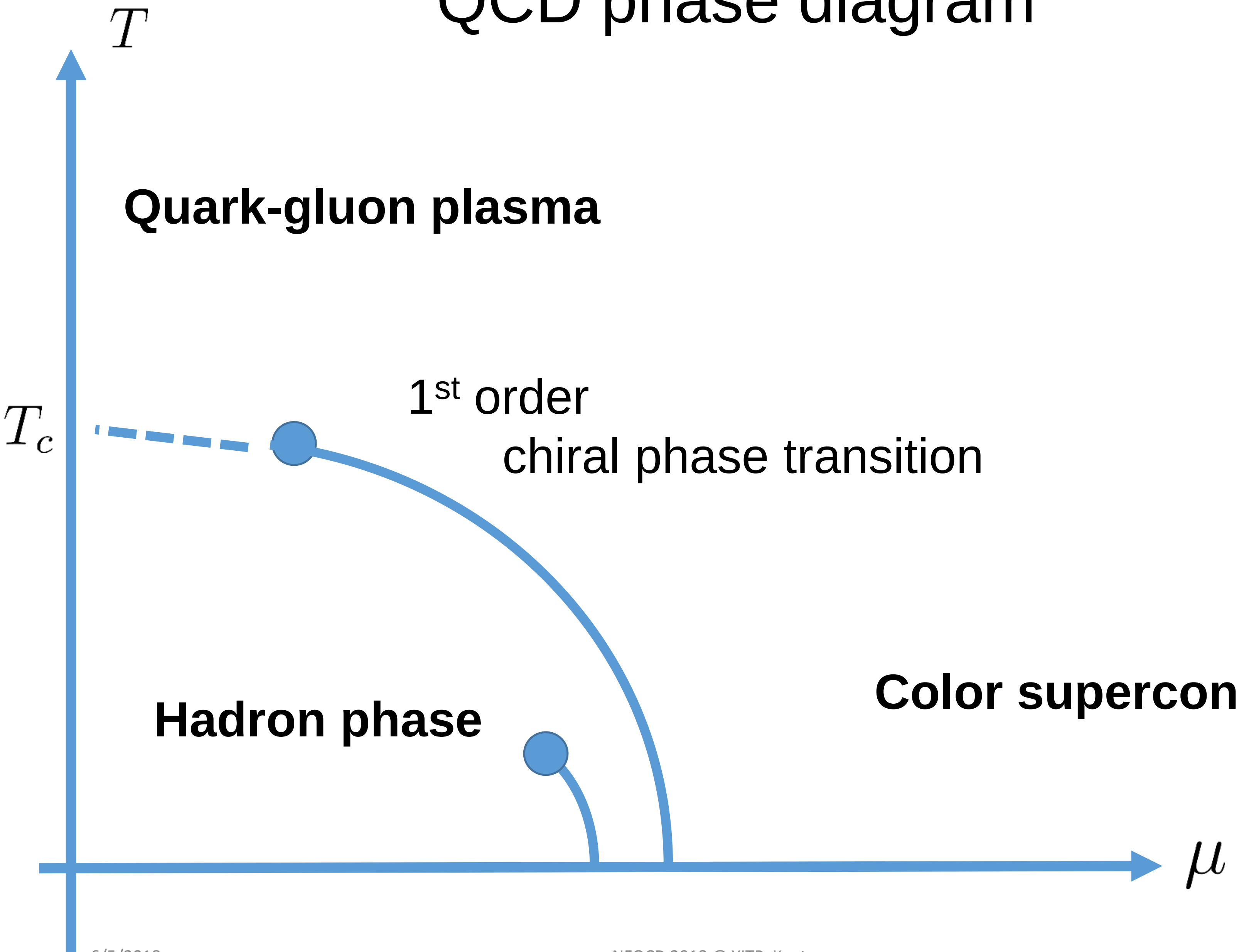
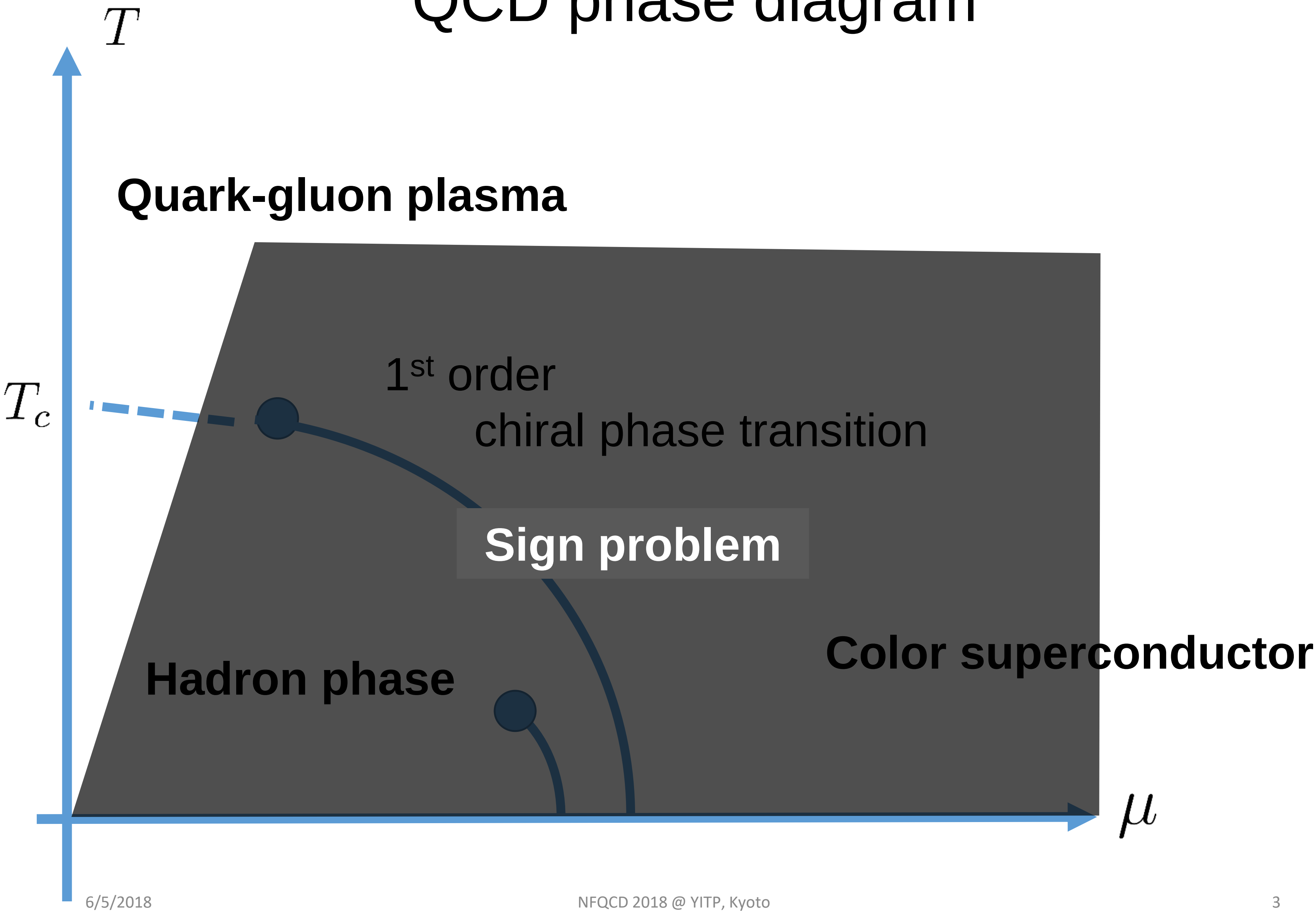
QCD phase diagram



QCD phase diagram



Finite density QCD

QCD partition function

$$Z = \int dU \det M[U, \mu] \mathrm{e}^{-S_g[U]}$$

$$M = D + m$$

The origin of the sign problem

$\det M$ is complex when $\mu > 0$

A way to resolve the sign problem:

complex Langevin approach

Basic idea of complex Langevin method

$$Z=\int_{-\infty}^{\infty} dx e^{-S(x)}, \quad S(x)\in\mathbb{C}$$

[Parisi 83], [Klauder 84]
[Aarts, Seiler, Stamatescu 09]
[Aarts, James, Seiler, Stamatescu 11]
[Seiler, Sexty, Stamatescu 13]
[Sexty 14] [Fodor, Katz, Sexty, Torok 15]
[Nishimura, Shimasaki 15]
[Nagata, Nishimura, Shimasaki 15]

Complexification

$$x\in\mathbb{R}\to z\in\mathbb{C} \quad S(x)\to S(z)$$

Complex Langevin equation

$$\frac{dz}{dt}=-\frac{\partial S(z)}{\partial z}+\eta(t)$$

$$\langle \eta(t)\rangle = 0$$

$$\langle \eta(t)\eta(t')\rangle = 2\delta(t-t')$$

$\langle \ldots \rangle$:noise average

We identify the noise effect as a *quantum fluctuation.*

Complex Langevin eq. for QCD

$$Z = \int dU \det M[U, \mu] \mathrm{e}^{-S_g[U]}$$

Complexification

$$U_{x\mu} \in SU(3) \to \mathcal{U}_{x\mu} \in SL(3, \mathbb{C}) \qquad S(U) \to S(\mathcal{U})$$

Gauge transformation

$$\mathcal{U}_{x\mu} \to g_x \mathcal{U}_{x\mu} g_{x+\hat{\mu}}^{-1}, \quad g_x \in SL(3, \mathbb{C})$$

The complex Langevin eq. of QCD

$$\mathcal{U}_{x\mu}(t+\epsilon) = \exp\left[i\left(-\epsilon \boxed{\mathcal{D}_{x\mu} S[\mathcal{U}]} + \sqrt{\epsilon}\eta_{x\mu}\right)\right] \mathcal{U}_{x\mu}(t)$$

Drift term

Criterion of correctness

Exponential falloff of the drift distribution

Complex Langevin is reliable

Power-law falloff of the drift distribution

Complex Langevin converges,
but gives incorrect answer

The main causes of the power-law fall

Excursion problem: large deviation of the link variable from SU(3)
$\rightarrow$ gauge cooling

Singular drift problem: small eigenvalue of the fermion matrix
$\rightarrow$ deformation of the Dirac operator

Numerical studies

We perform numerical simulation in two different temperature regions.

(1) "Low temperature" region:

→ silver blaze phenomenon? transition to the quark matter?

Nagata, Nishimura, Shimasaki [1805.03964]
Ito, Matsufuru, Moritake, Nishimura, Shimasaki, Tsuchiya, ST (preliminary)

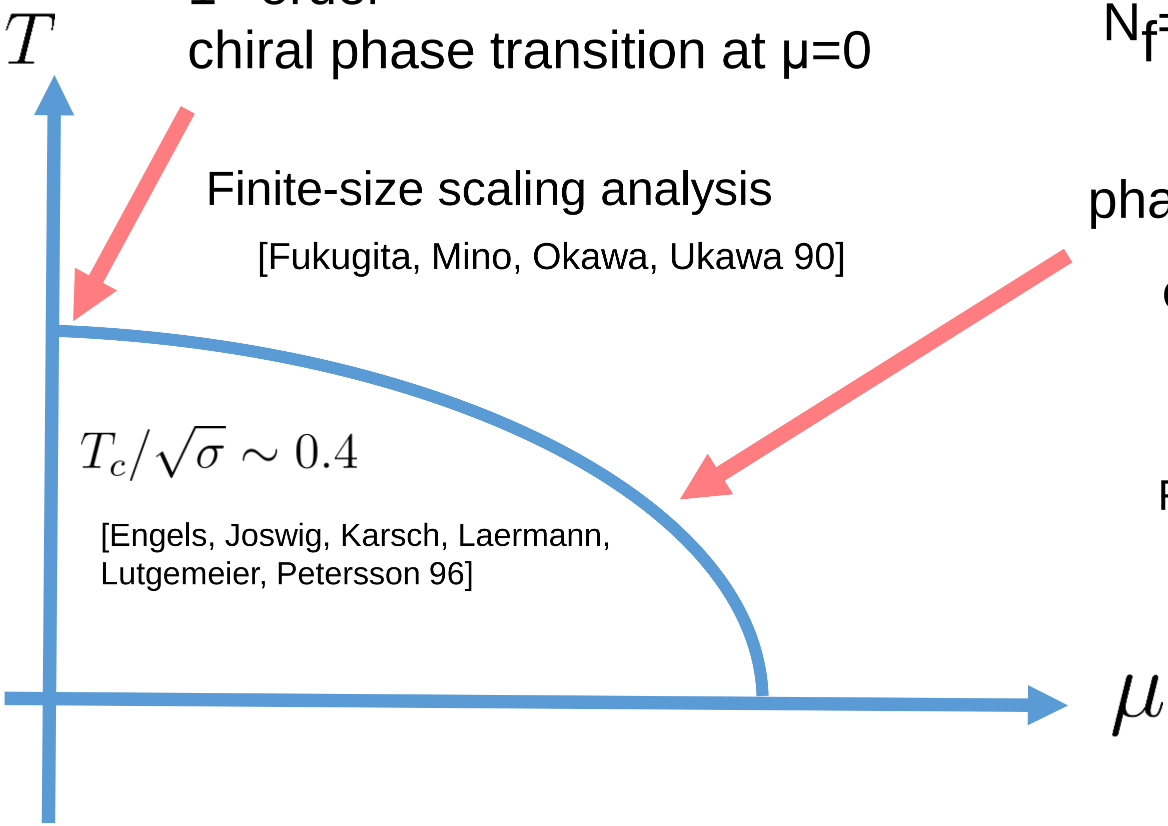
(2) "High temperature" region:

→ 1^{st} order phase transition?

Ito, Matsufuru, Moritake, Nishimura, Shimasaki, Tsuchiya, ST (preliminary)

Phase diagram of 4 flavor QCD

1st order
chiral phase transition at μ=0



Finite-size scaling analysis

[Fukugita, Mino, Okawa, Ukawa 90]

$T_c/\sqrt{\sigma} \sim 0.4$

[Engels, Joswig, Karsch, Laermann, Lutgemeier, Petersson 96]

We use
N_f=4, staggered fermion

phase transition at finite μ

Canonical method

[de Forcrand, Kratochvila 06]
[Li, Alexandru, Liu, Meng 10]

Reweighting

[Fodor, Katz, Sexty, Torok 15]

Setup: low temperature region

- $N_f = 4$, staggered fermion
- Lattice size: $8^3 \times 16$ cf) $4^3 \times 8$ results in J. Nishimura's talk
- $\beta = 5.7$
- $\mu a = 0.0 – 0.5$
- Quark mass: $m_q a = 0.01, 0.05$
- Langevin steps = $10^5 – 10^6$
- Computational resource: K computer

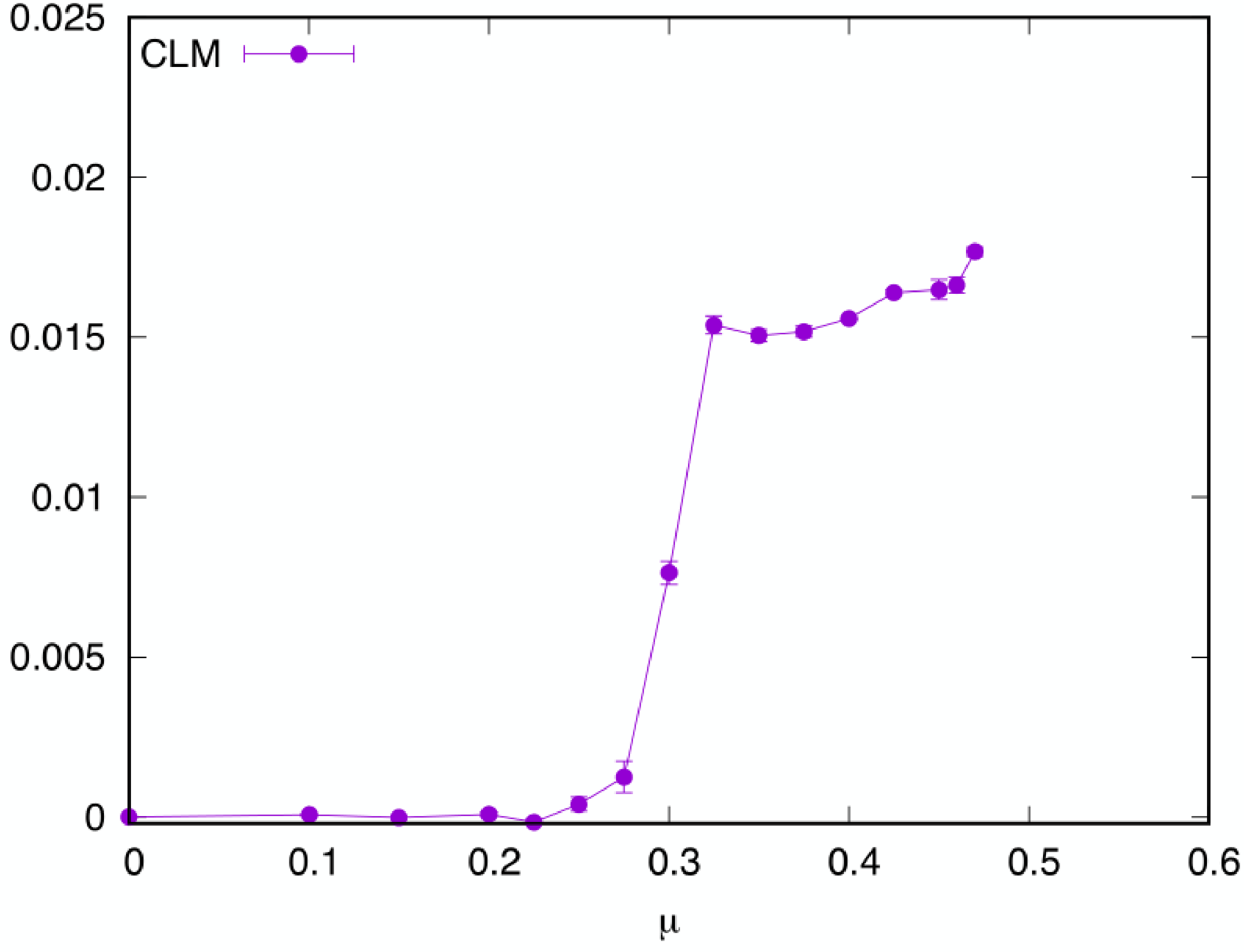
We compare the results with the RHMC results of the phase quenched (PQ) simulation.

How to extract reliable data

m = 0.05

Baryon number density

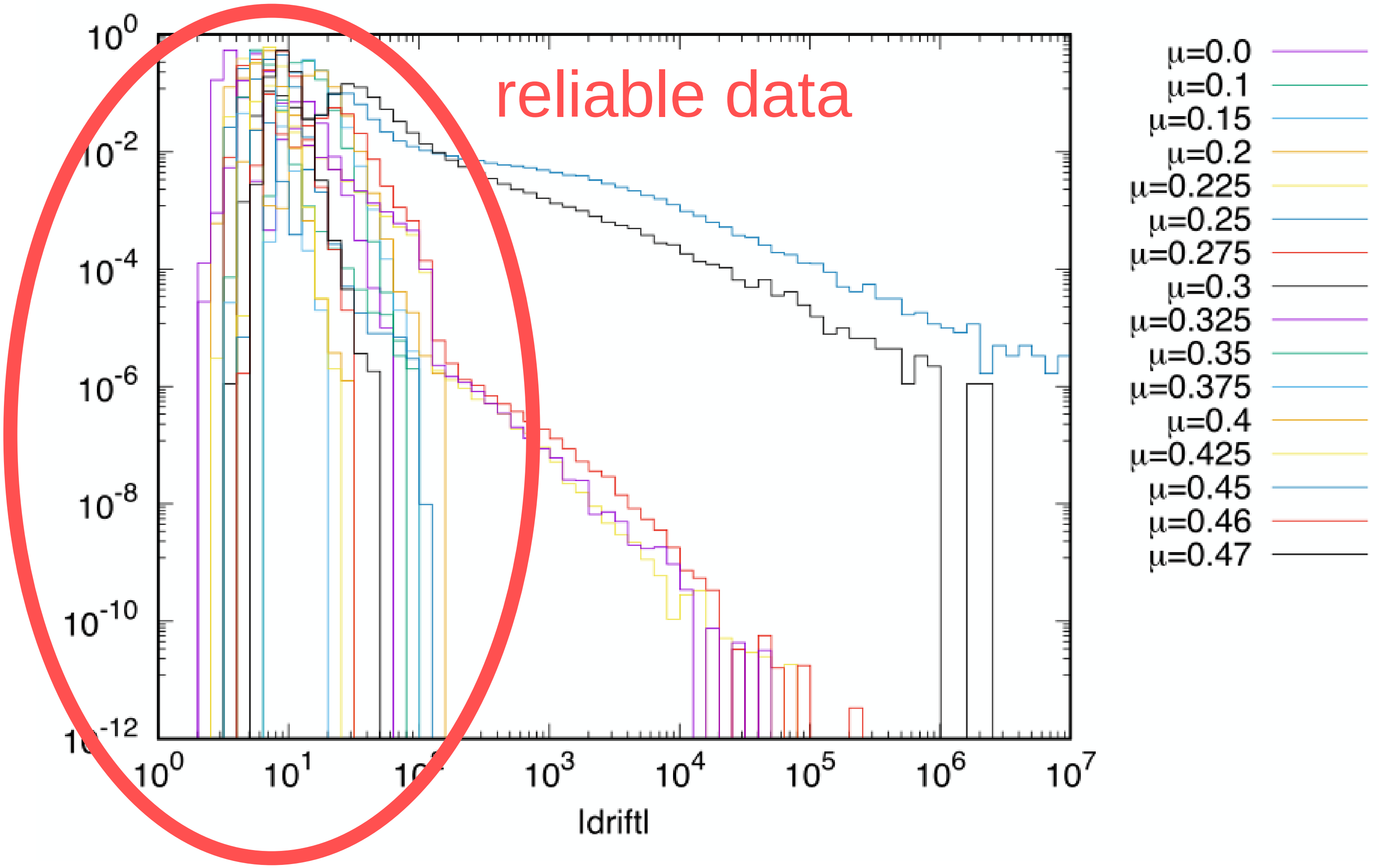
<n>



μ

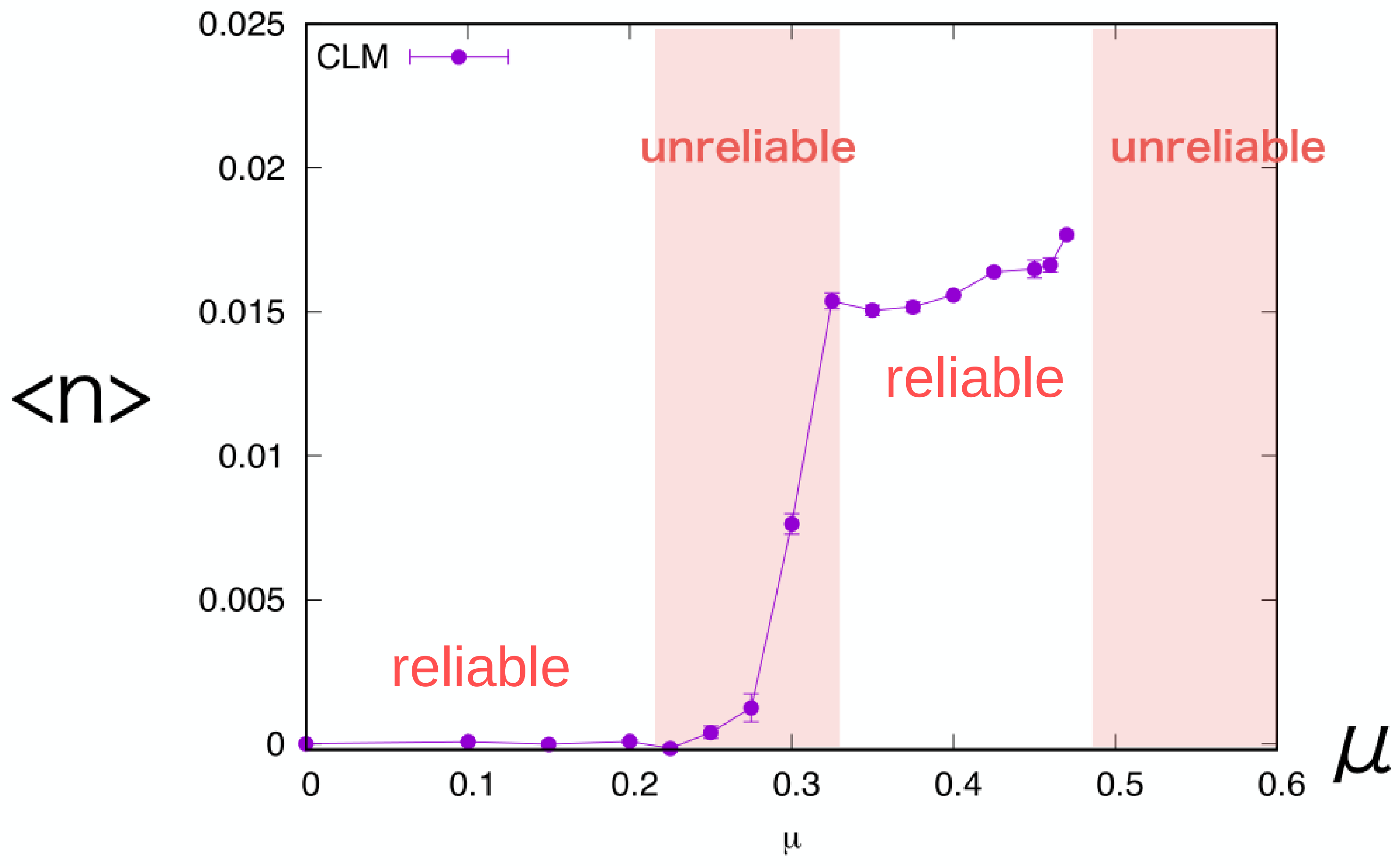
Histogram of the drift term

m = 0.05



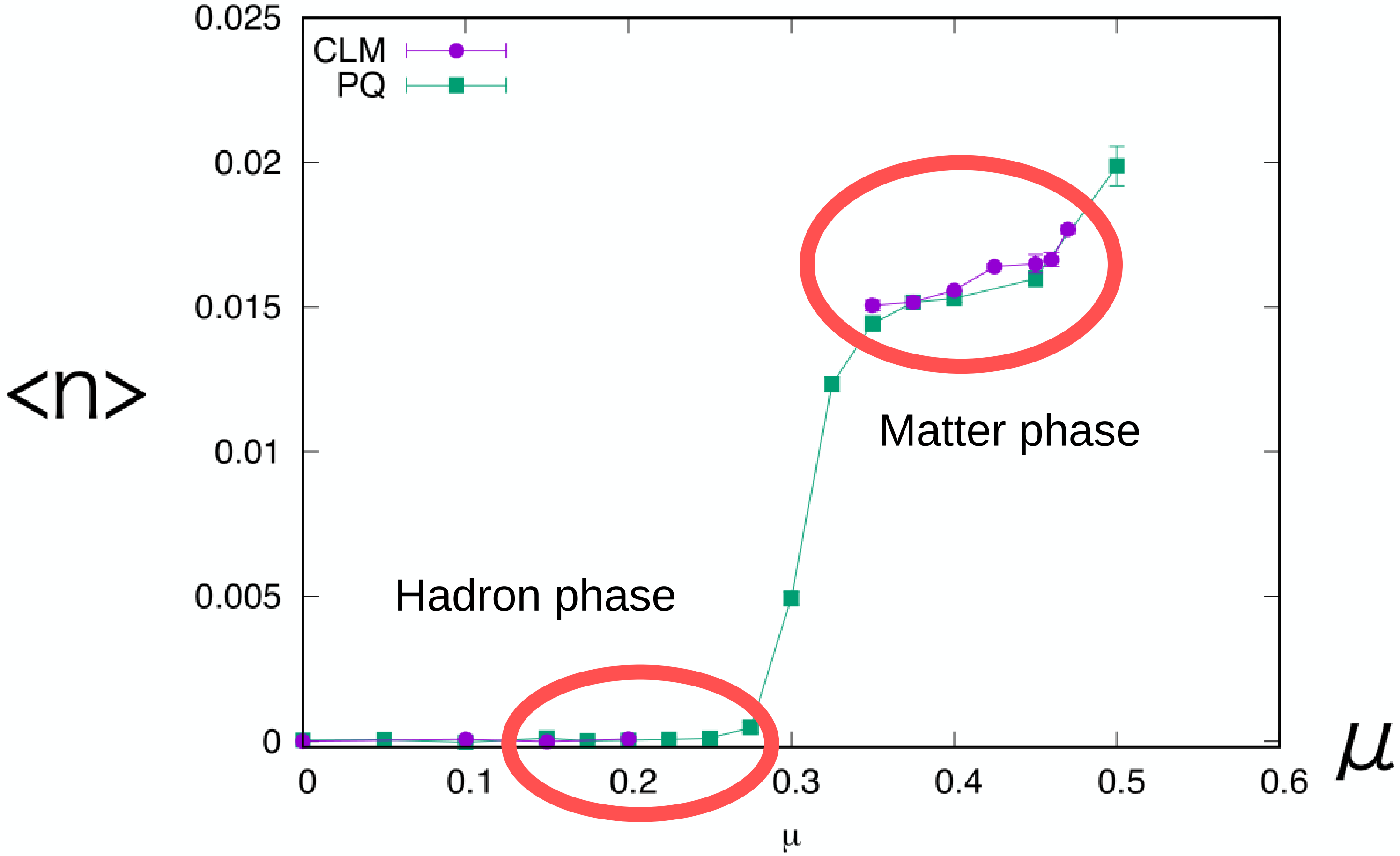
Baryon number density

m = 0.05



Comparison with PQ simulation

m = 0.05

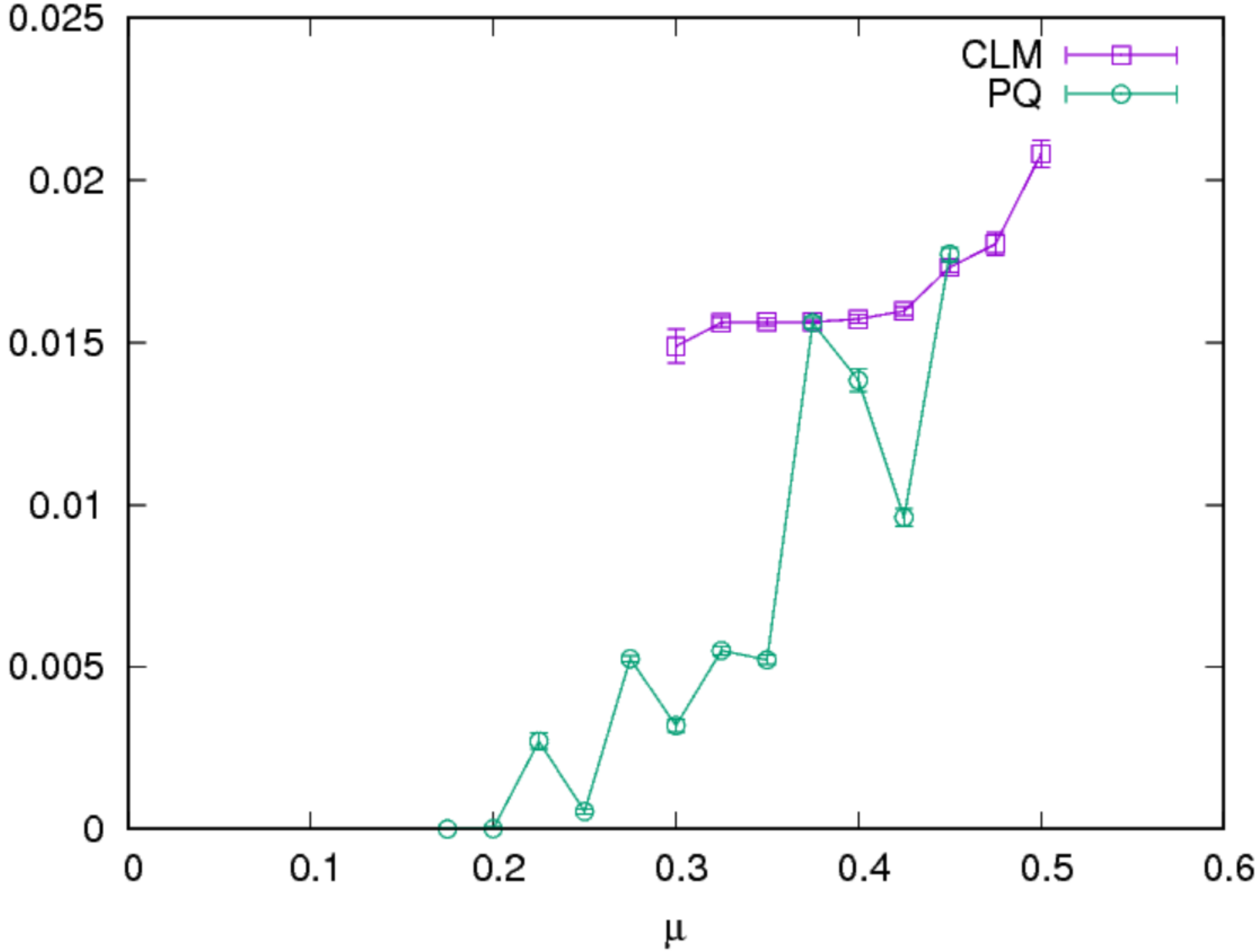


Qualitative difference is not observed → due to too heavy pion?

Comparison with PQ simulation

m = 0.01

<n>

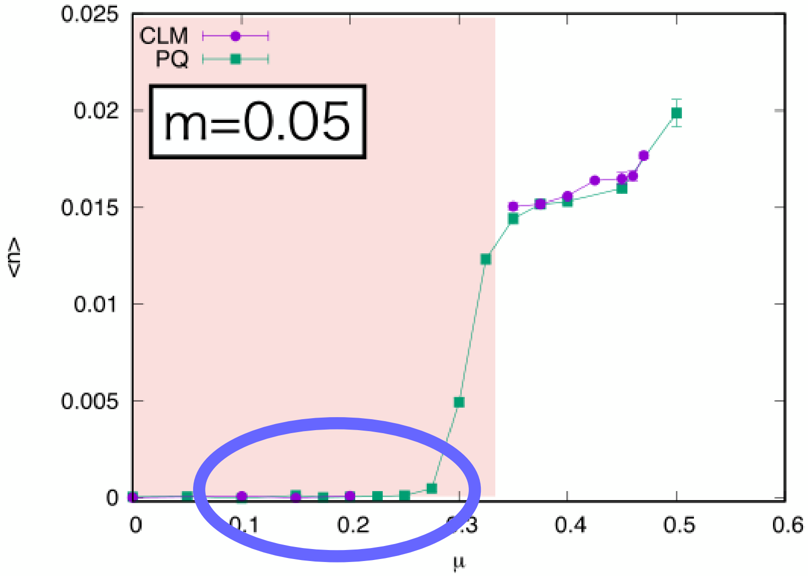


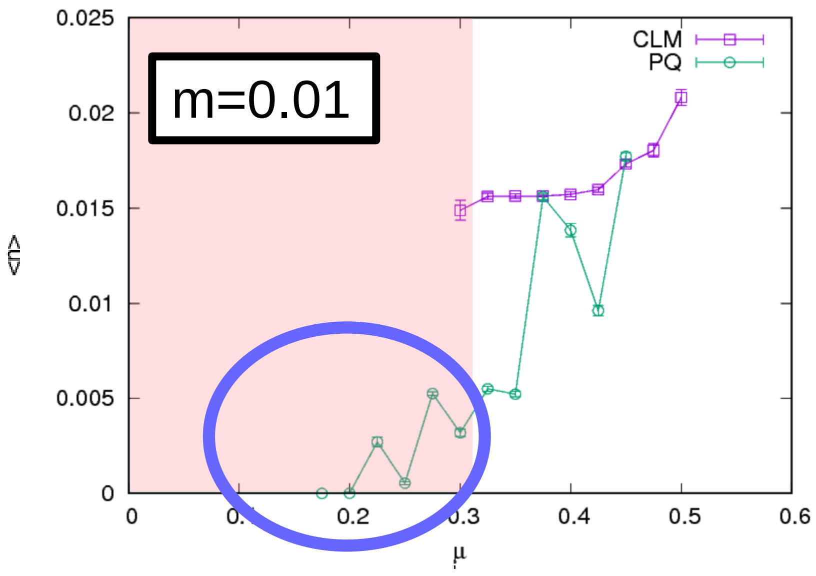
μ

m = 0.01 vs 0.05 (μ<0.3)

PQ

As m=0.05 → 0.01,
critical chemical potential
lowers.

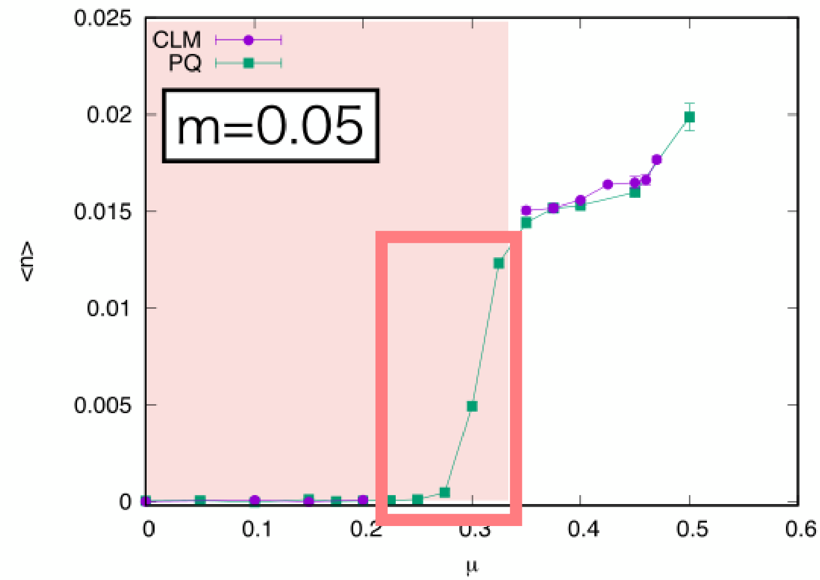




m = 0.01 vs 0.05 (μ<0.3)

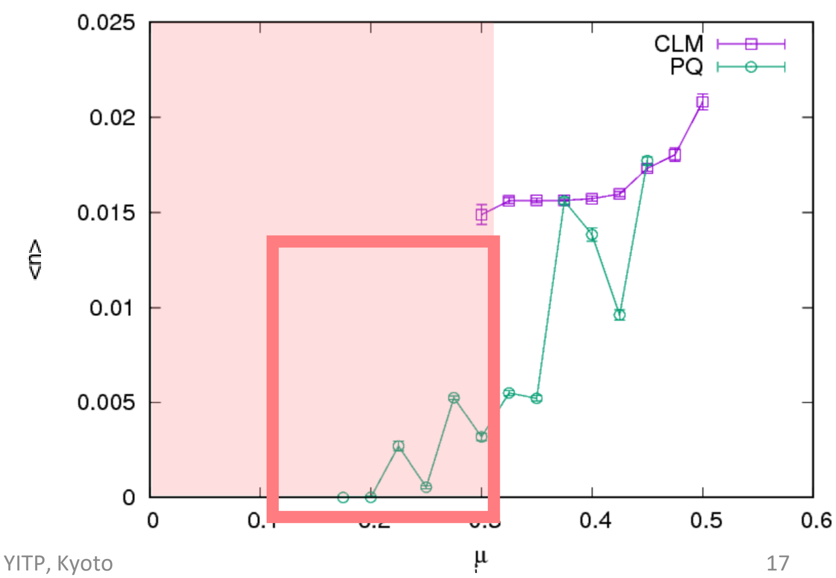
PQ

As m=0.05 → 0.01, critical chemical potential lowers.



CL

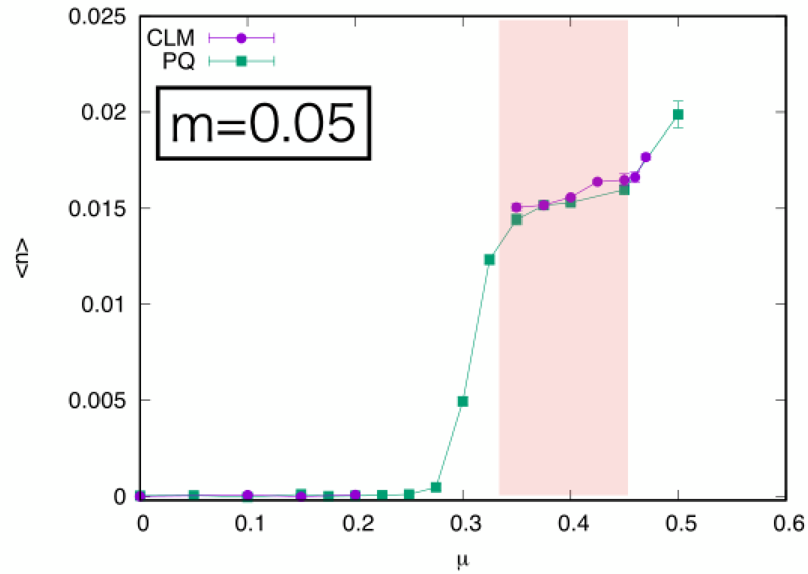
Region where the singular drift problem occurs depends on mass.



m = 0.01 vs 0.05 (μ>0.3)

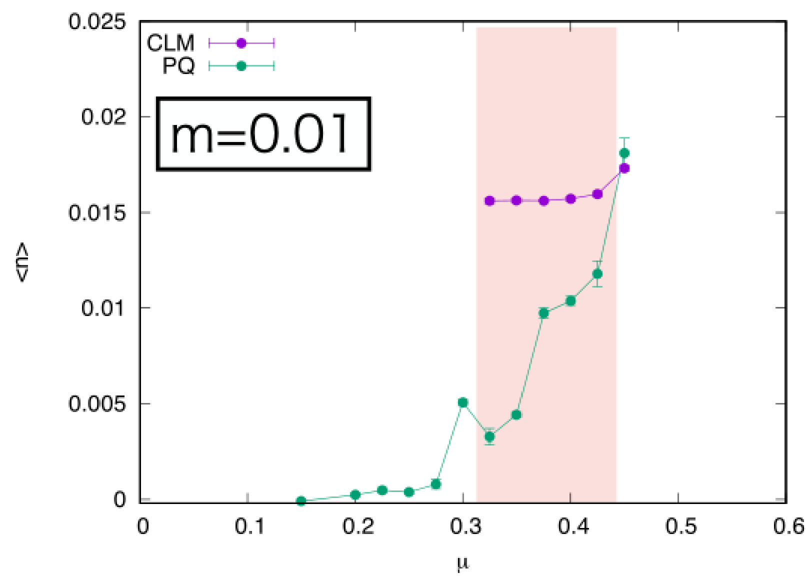
PQ

<n> (~ pion density) is sensitive to the change of mass. (finite size effect may arise)



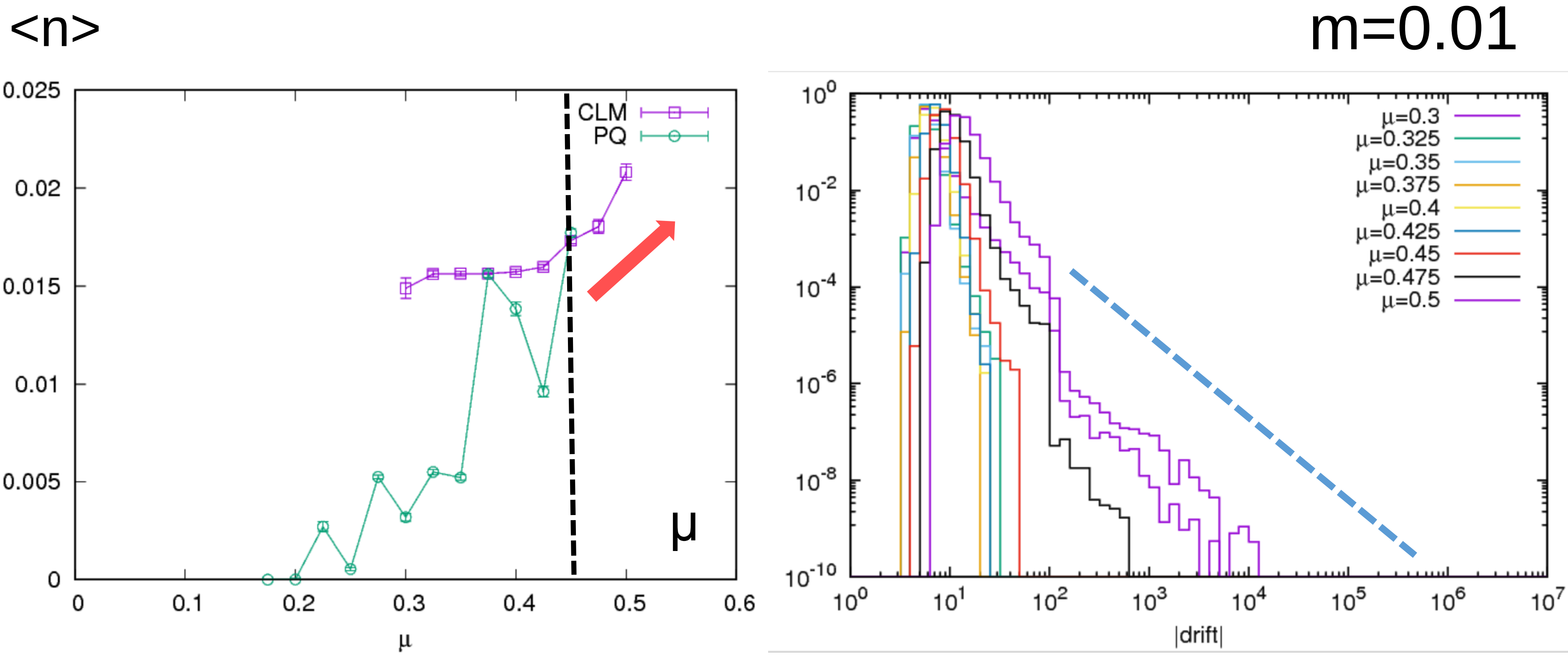
CL

<n> (= baryon number density) is not sensitive.



CLM succeeded to take into account the complex phase of the fermion determinant.

Transition to quark matter phase?



For μ<0.45, CLM is manifestly reliable without the deformation technique.

For μ>0.475, clear power-law tail is not formed yet. Further statistics is needed to make a conclusion.

Setup: high temperature region

- N_f = 4, staggered fermion
- Lattice size: $16^3 \times 12$, $20^3 \times 12$, $24^3 \times 12$
- β = 5.0 - 5.5
- μ/T = 1.2, 2.4
- Quark mass: $m_q a$ = 0.01
- Langevin steps = 10^4 – 10^5
- Computational resource: K computer

Physical scales

$$a \simeq 0.11\text{fm} \quad m_\pi \simeq 530\text{MeV}$$ (β=5.2)

$$150\text{MeV} \leq T$$

$$0 \leq \mu \lesssim 455\text{MeV}$$

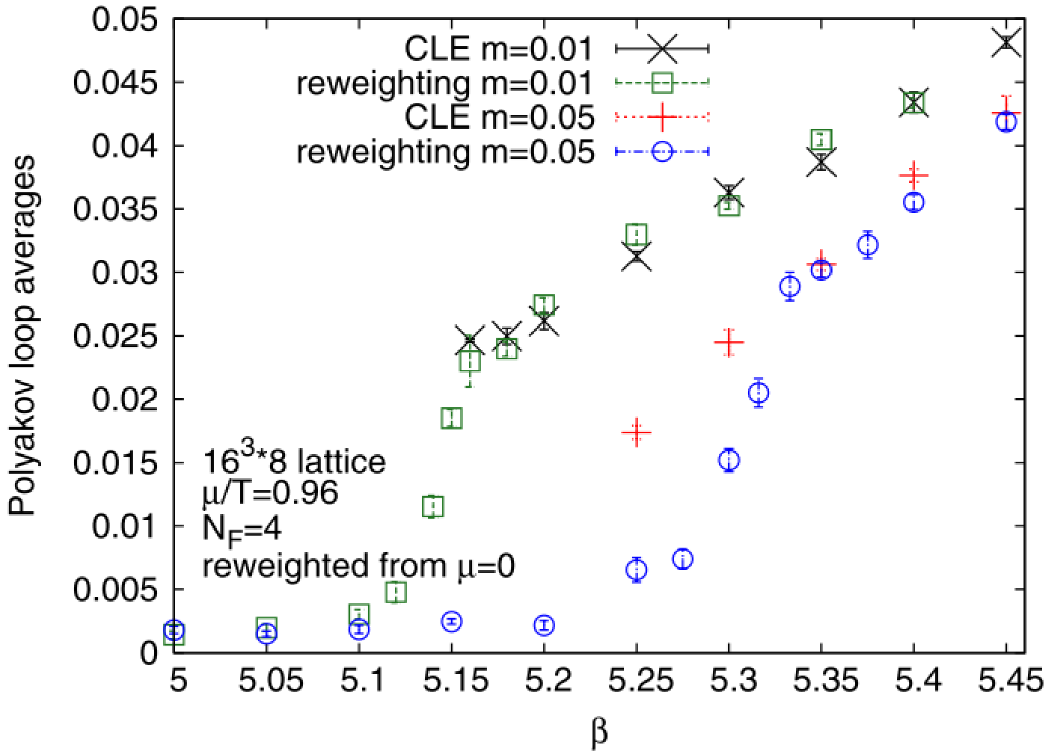
Table of lattice spacing

[Fodor, Katz, Sexty, Torok 15]

Previous study

[Fodor, Katz, Sexty, Torok 15]

Previous studies of N_f = 4 high density QCD:



Lattice size: $16^3 \times 8$

Complex Langevin is valid only when β > 5.15

Phase transition is found at β ~ 5.15 by **reweighting**

Our claim

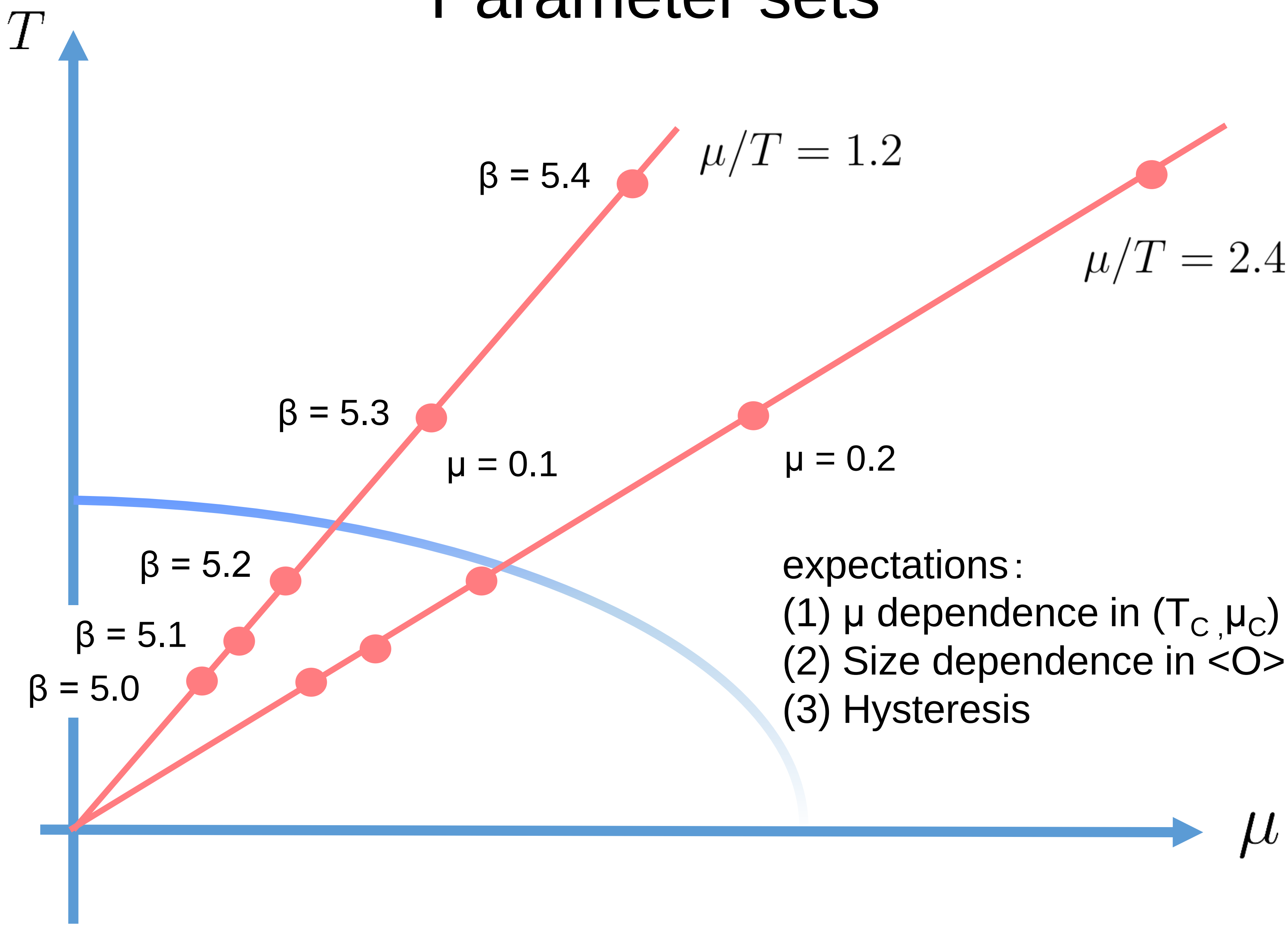
If the temporal lattice size is large enough,
complex Langevin may be able to detect the phase transition.

For instance, when $\beta = 5.2$, $m_q a = 0.01$,
the temperature becomes…

$N_T = 6$	$T \sim 300$ MeV	[Fodor, Katz, Sexty, Torok 15]
$N_T = 8$	$T \sim 220$ MeV	
$N_T = 12$	$T \sim 150$ MeV	Our study

…

Parameter sets



expectations :

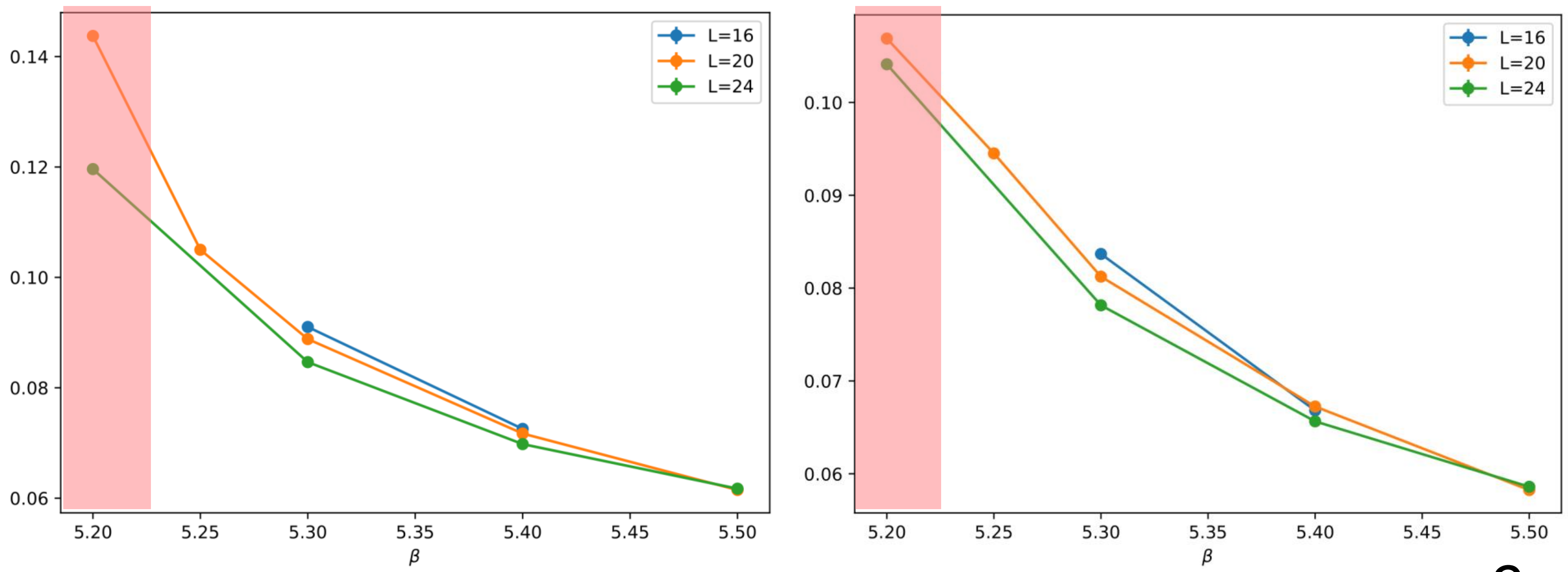
(1) μ dependence in (T_C ,μ_C)

(2) Size dependence in <O>

(3) Hysteresis

Chiral condensate

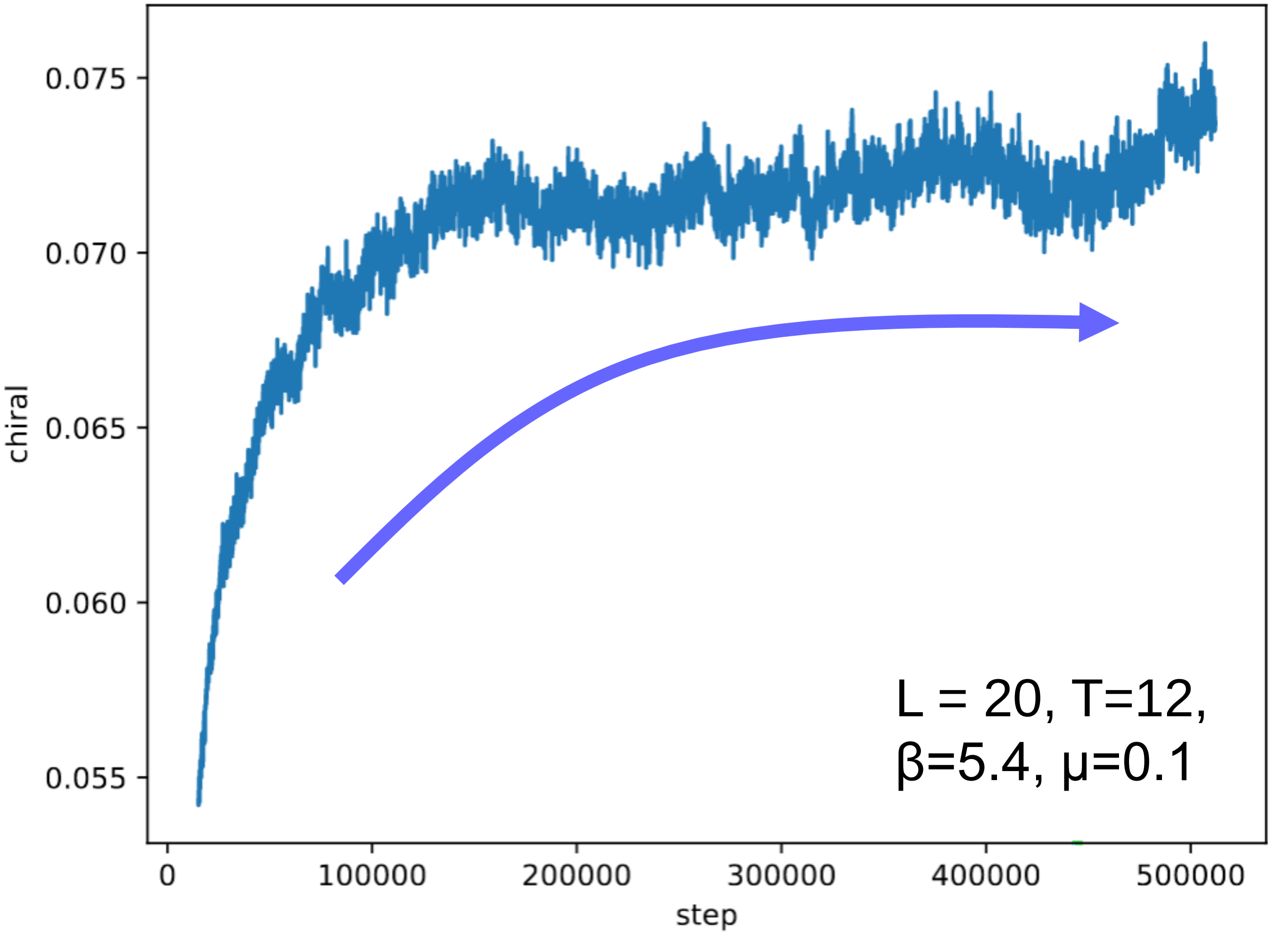
$\mu/T = 1.2$ $\mu/T = 2.4$



β

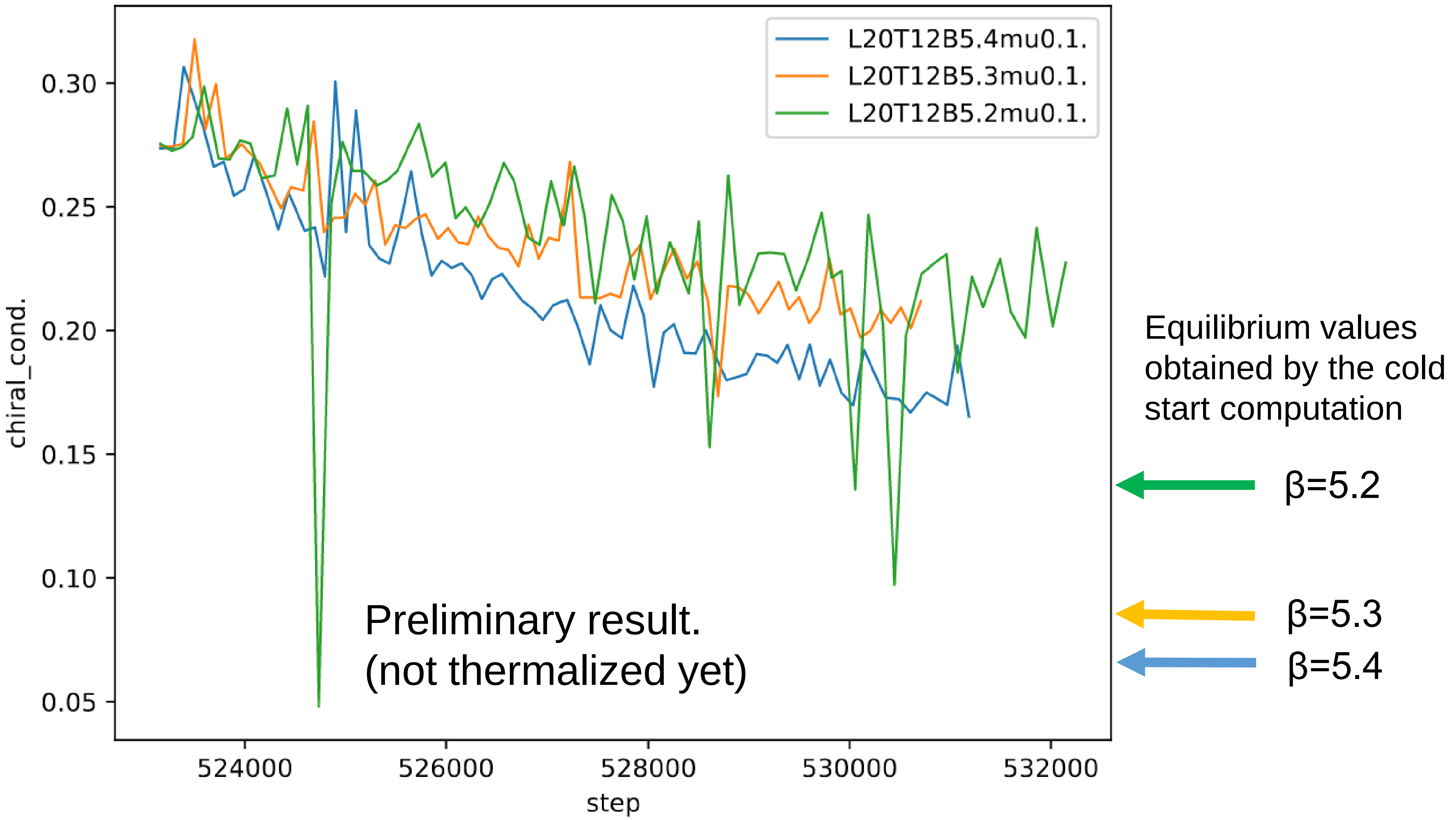
Size dependence and μ/T dependence are not observed yet.

History of chiral condensate

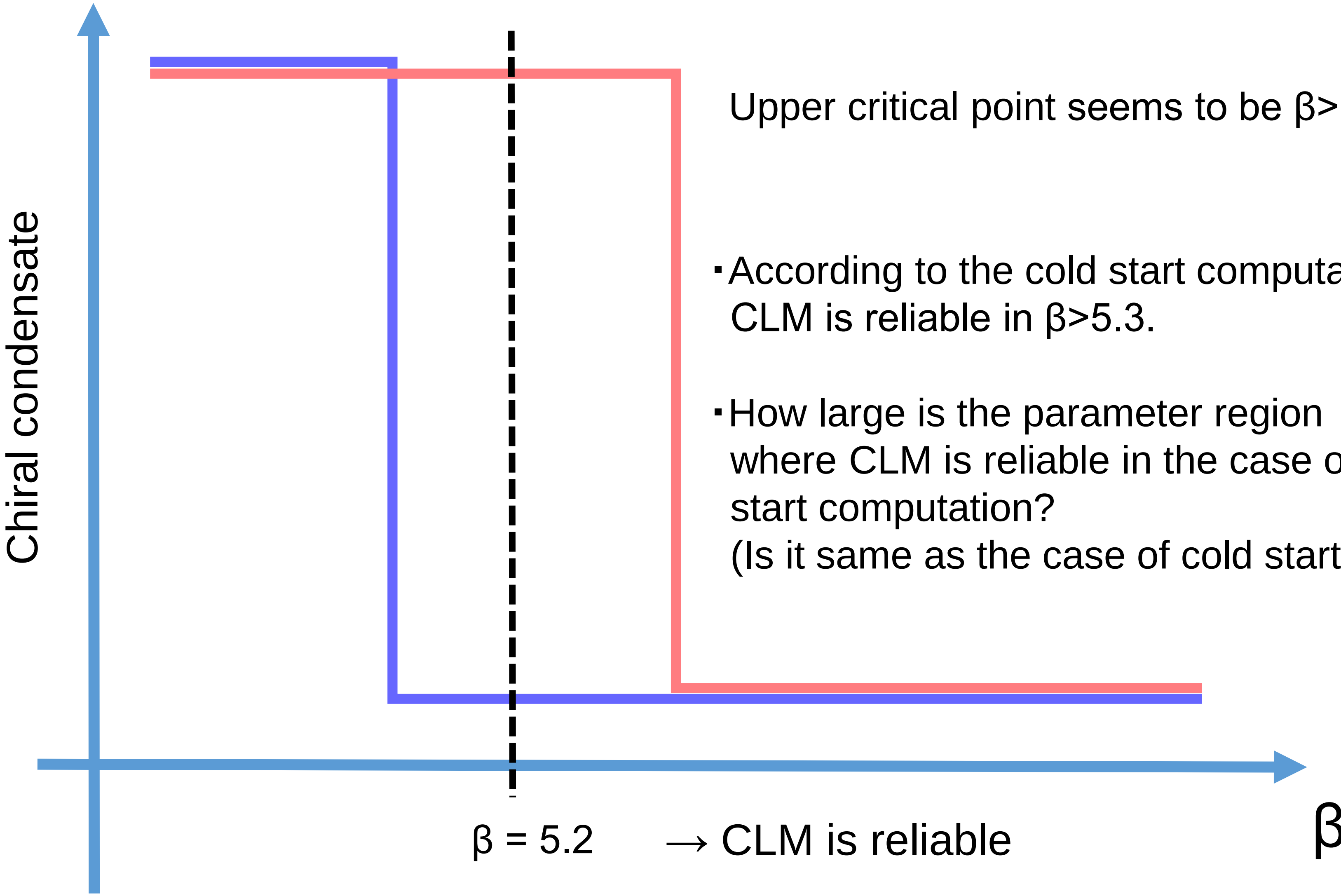


Our simulation is performed by the “**cold start**”.
History may depend on the initial condition around the 1st order transition line.

History from the hot configuration



Possible hysteresis



Upper critical point seems to be β>5.2

- According to the cold start computation, CLM is reliable in β>5.3.
- How large is the parameter region where CLM is reliable in the case of hot start computation?
  (Is it same as the case of cold start ?)

Summary

- Complex Langevin method (CLM) is applied to explore 4-flavor QCD phase diagram in finite density region.

- CLM is reliable in the nuclear matter phase. We confirm the qualitative difference from the phase quenched simulation.
- Transition to the quark matter phase is suggested.

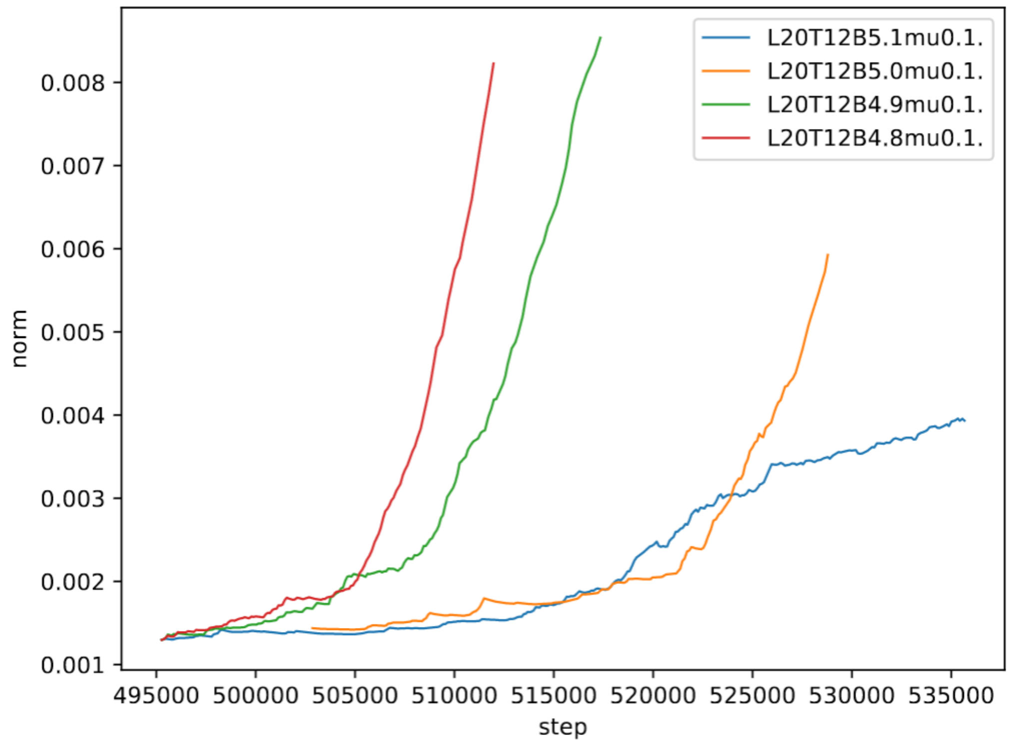
- 1^{st} order phase transition in high temperature region is under investigation.
- Hysteresis of the chiral condensate is explored.

Appendix

How to make “hot” configuration

In CLM, extremely small β (say β=0) is not allowed to ensure the stable computation.

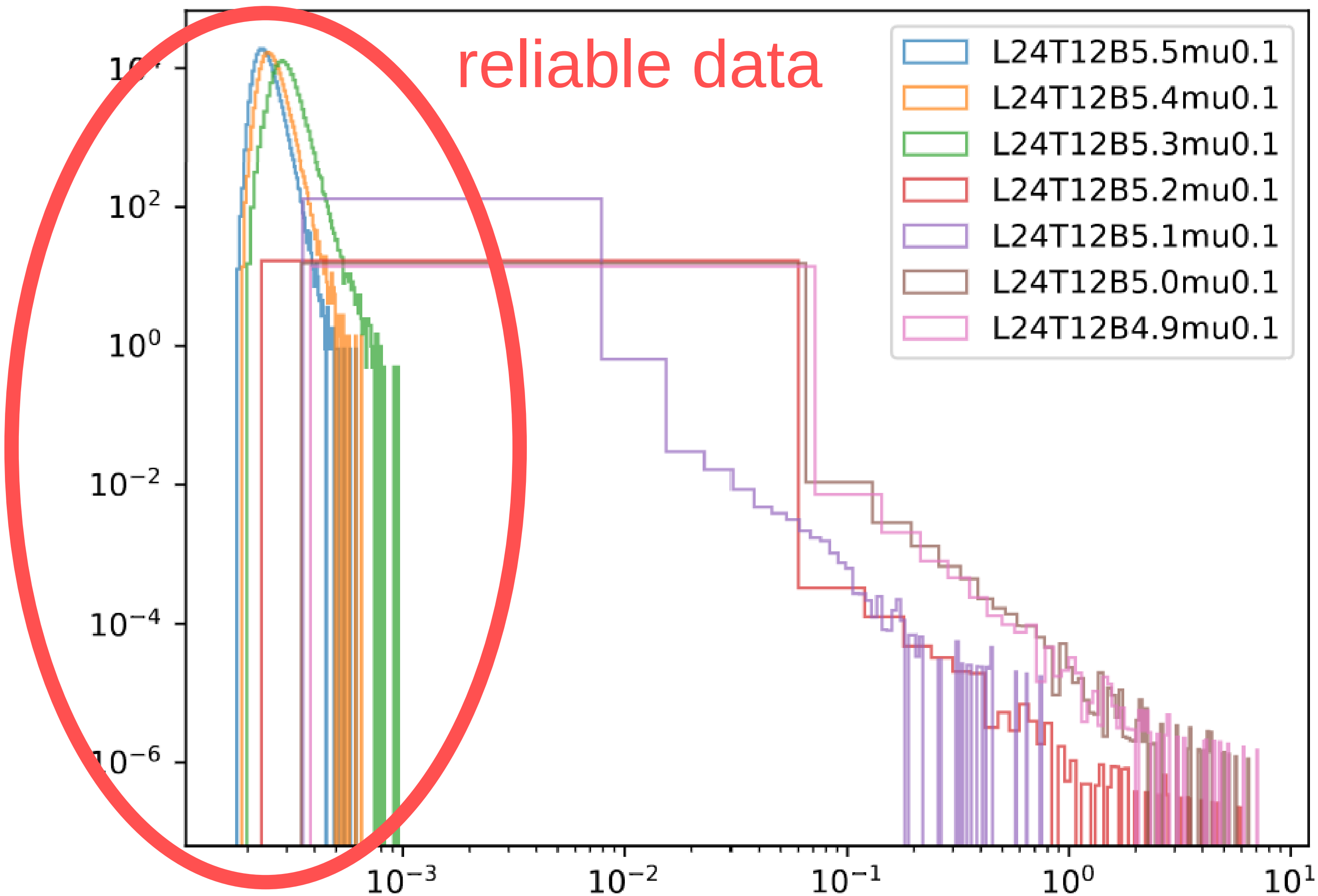
unitarity norm



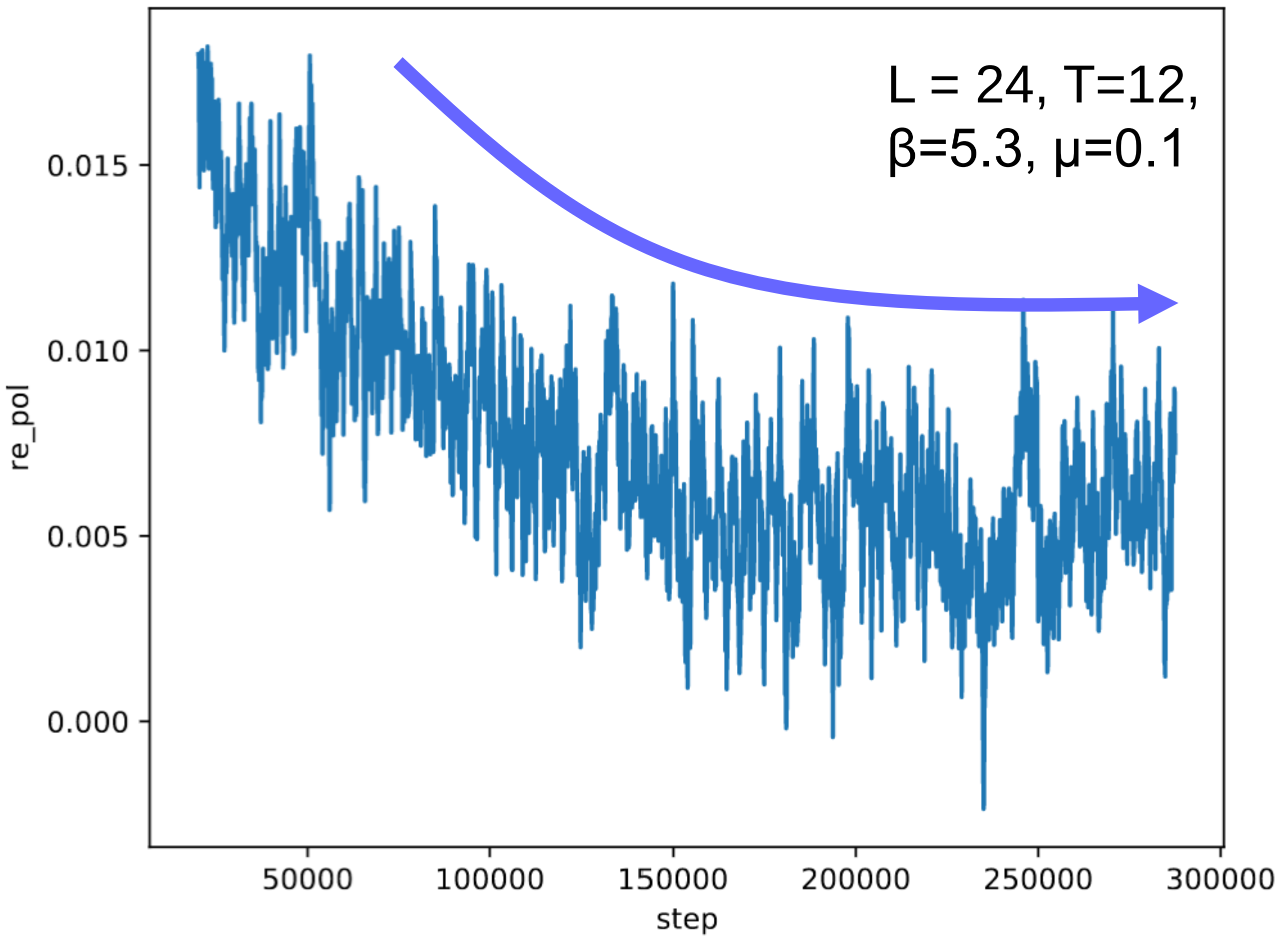
We prepare time-series data as … → β=5.0 → β=5.2 ~ 5.4

Histogram of the drift term

L=24 μ=0.1



History of Polyakov loop

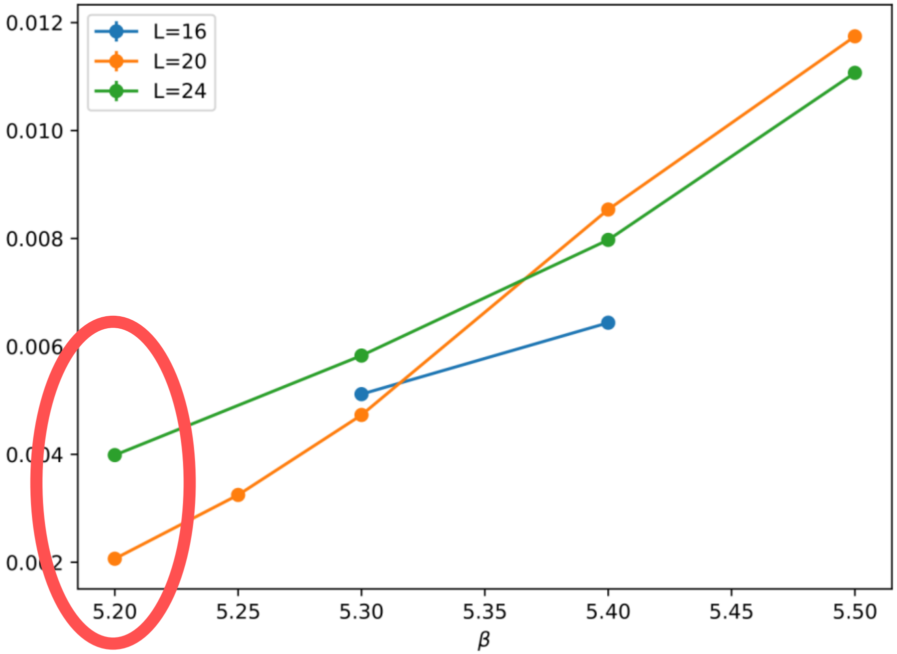


Our simulation is performed by the “**cold start**”.

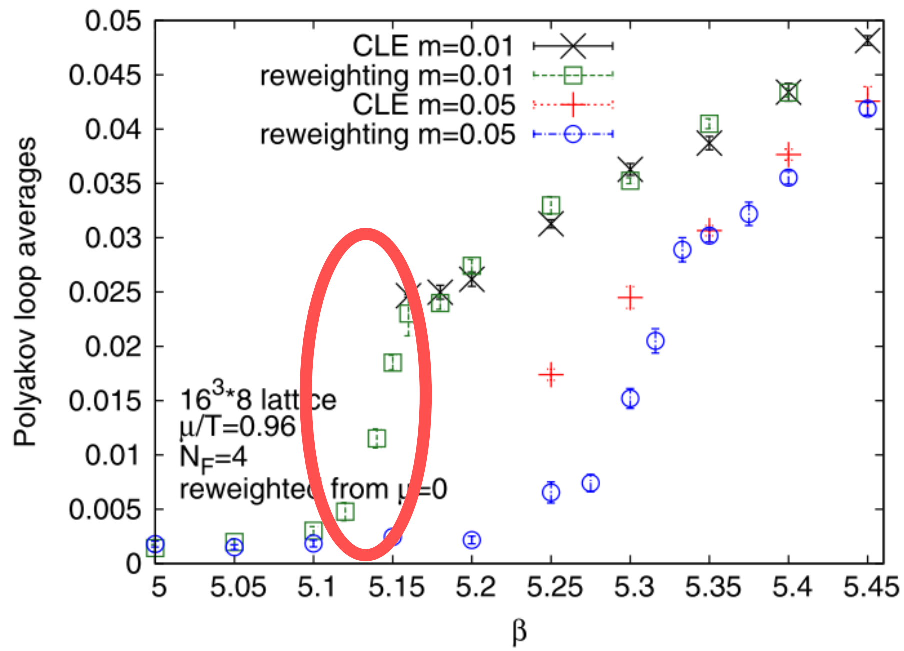
History may depend on the initial condition around the 1^{st} order transition line.

Comparison with previous study

$$\mu/T = 1.2$$



Fodor, Katz, Sexty, Torok (2015)

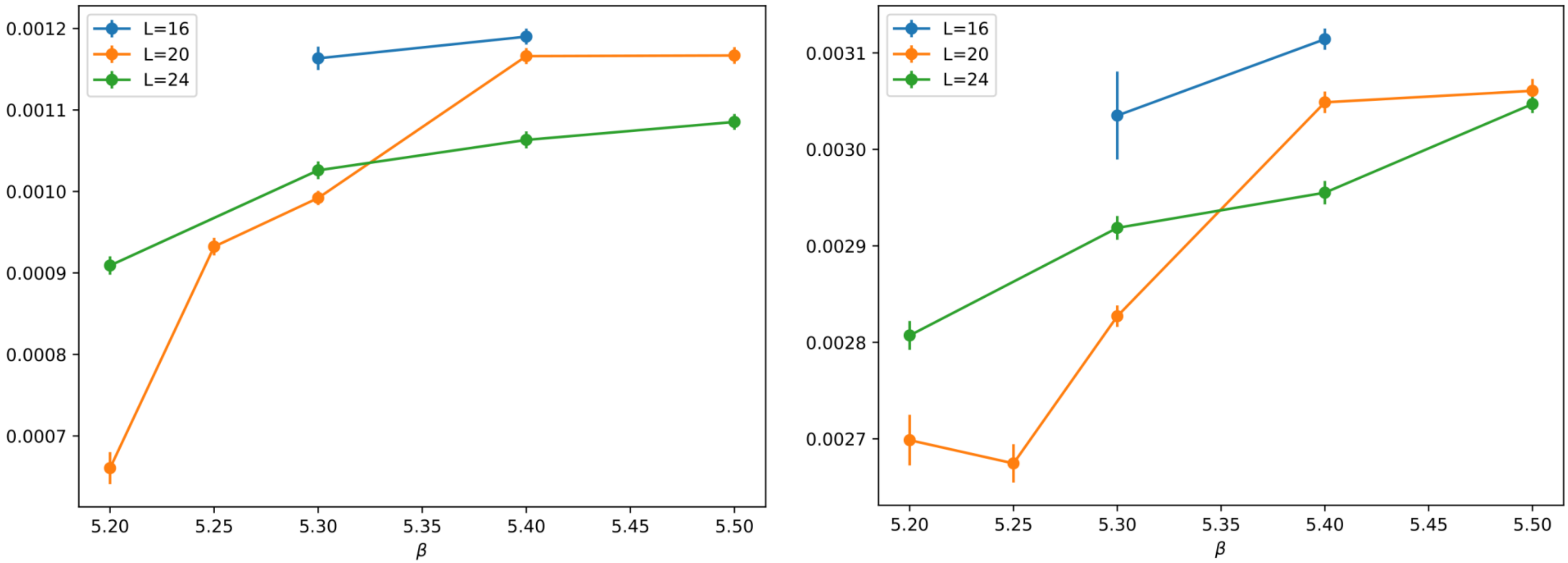


expectation： phase transition around β=5.2

transition temperature becomes lower as μ/T is larger

Baryon number density

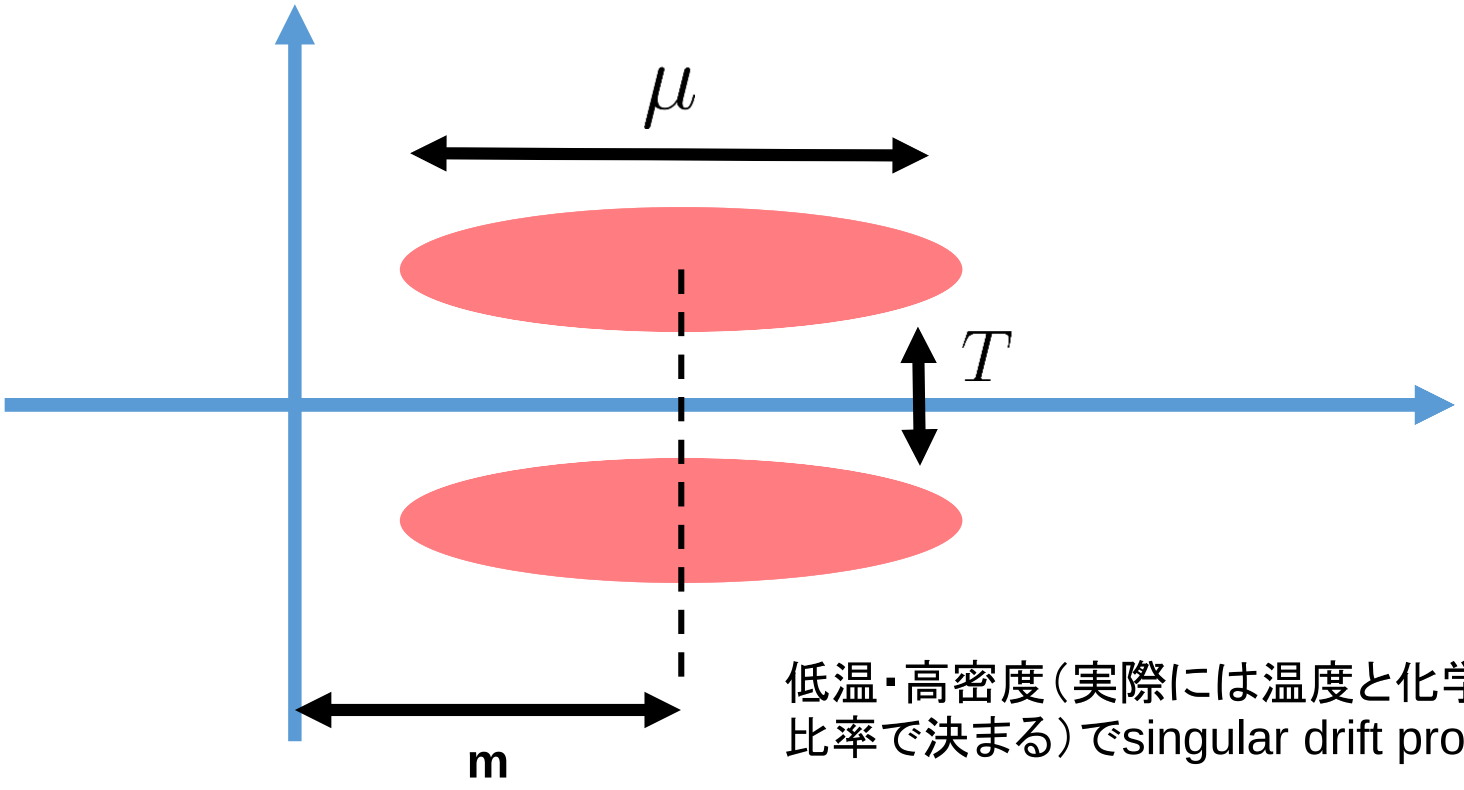
$\mu/T = 1.2$ $\mu/T = 2.4$



expectation : phase transition around β=5.2

transition temperature becomes lower as μ/T is larger

(D+m) の固有値分布の典型的なふるまい



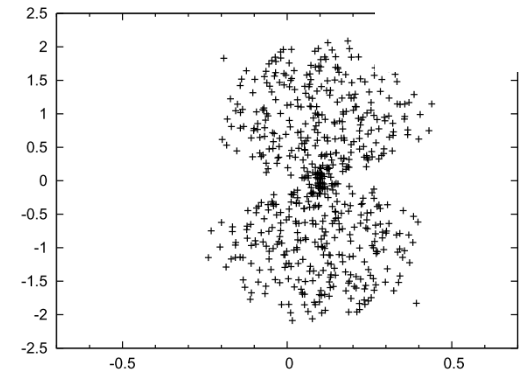
低温・高密度（実際には温度と化学ポテンシャルの比率で決まる）でsingular drift problemが発生する

Singular drift problem:
(D+m) の固有値分布が原点近傍に集まってしまうことで、ドリフト項が大きくなってしまう問題

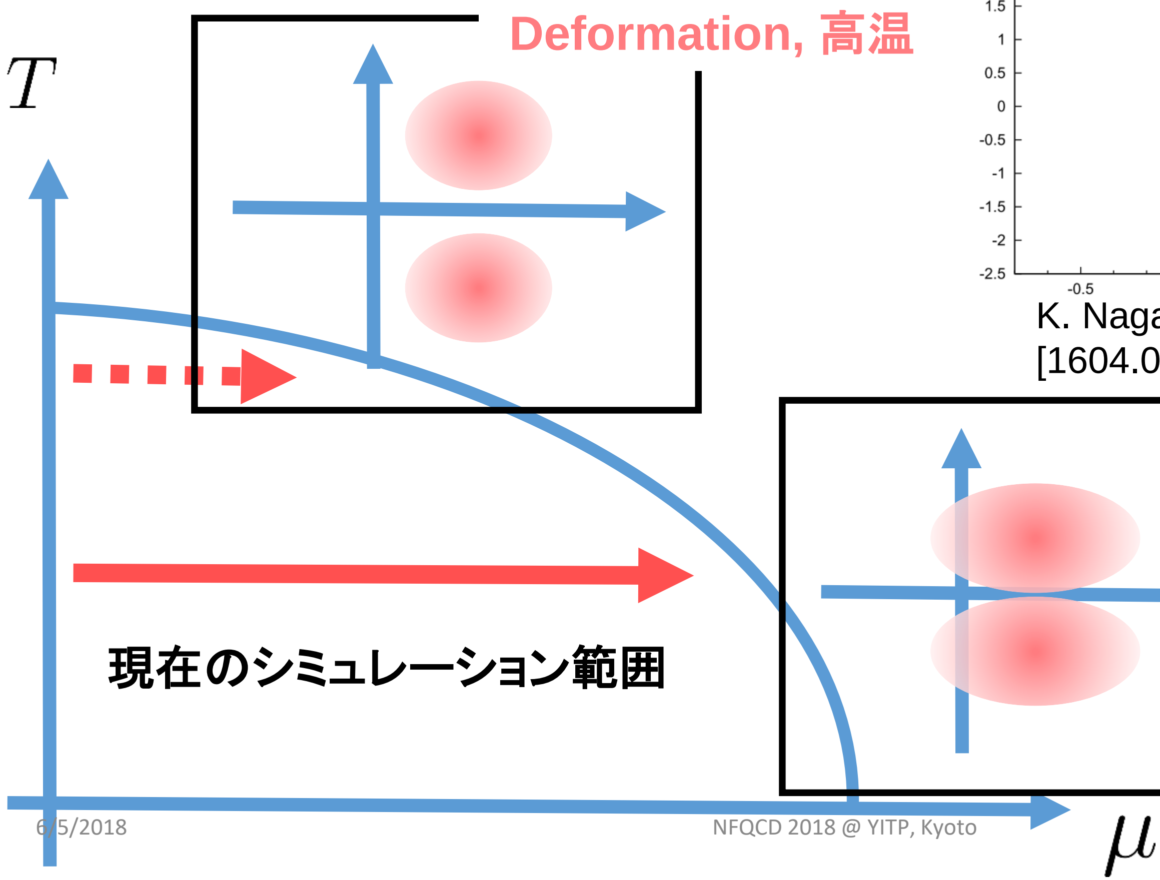
今後の展望

Singular drift を回避する方法

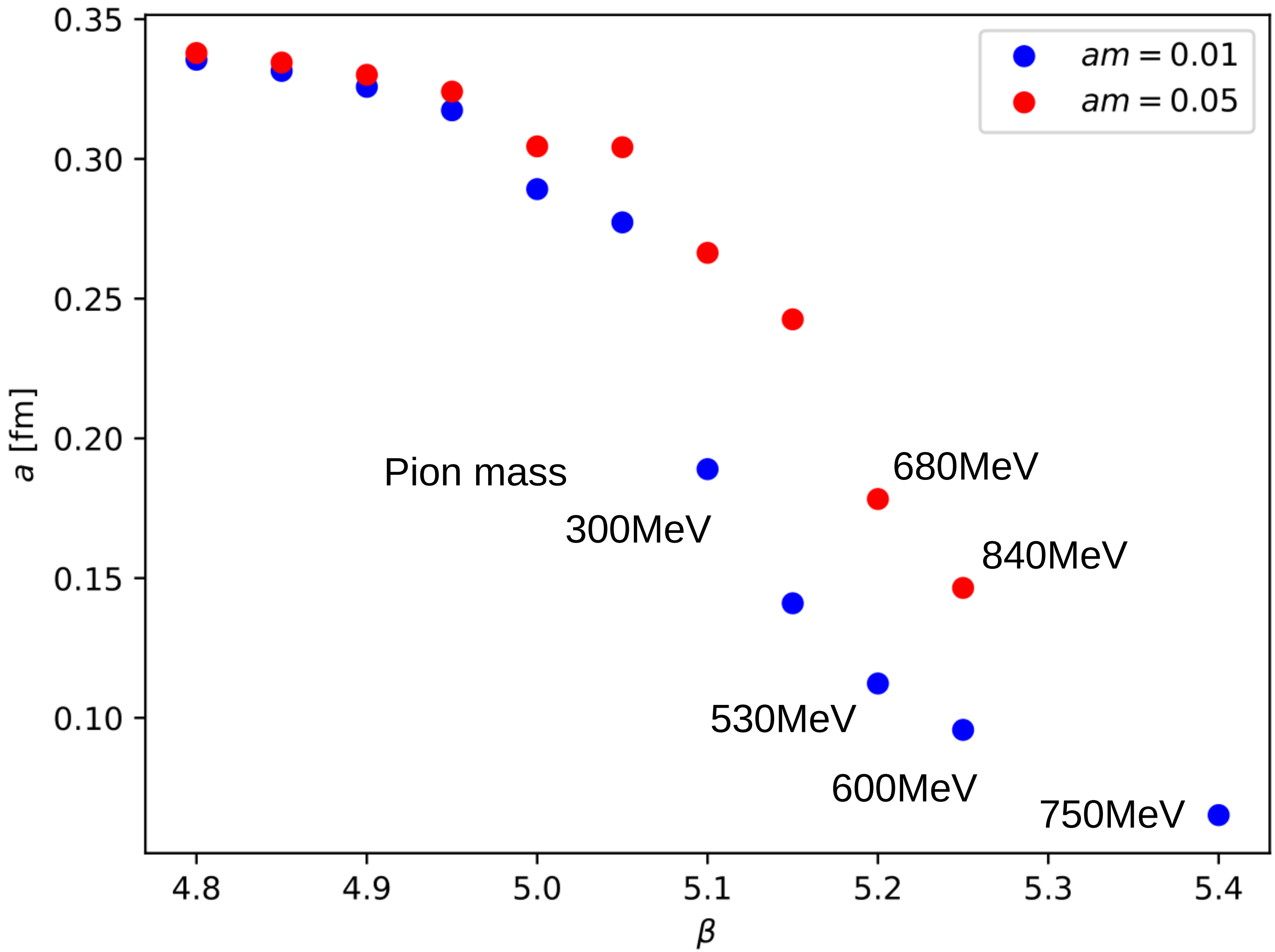
- Generalized gauge cooling
- Deformation
- High temperature



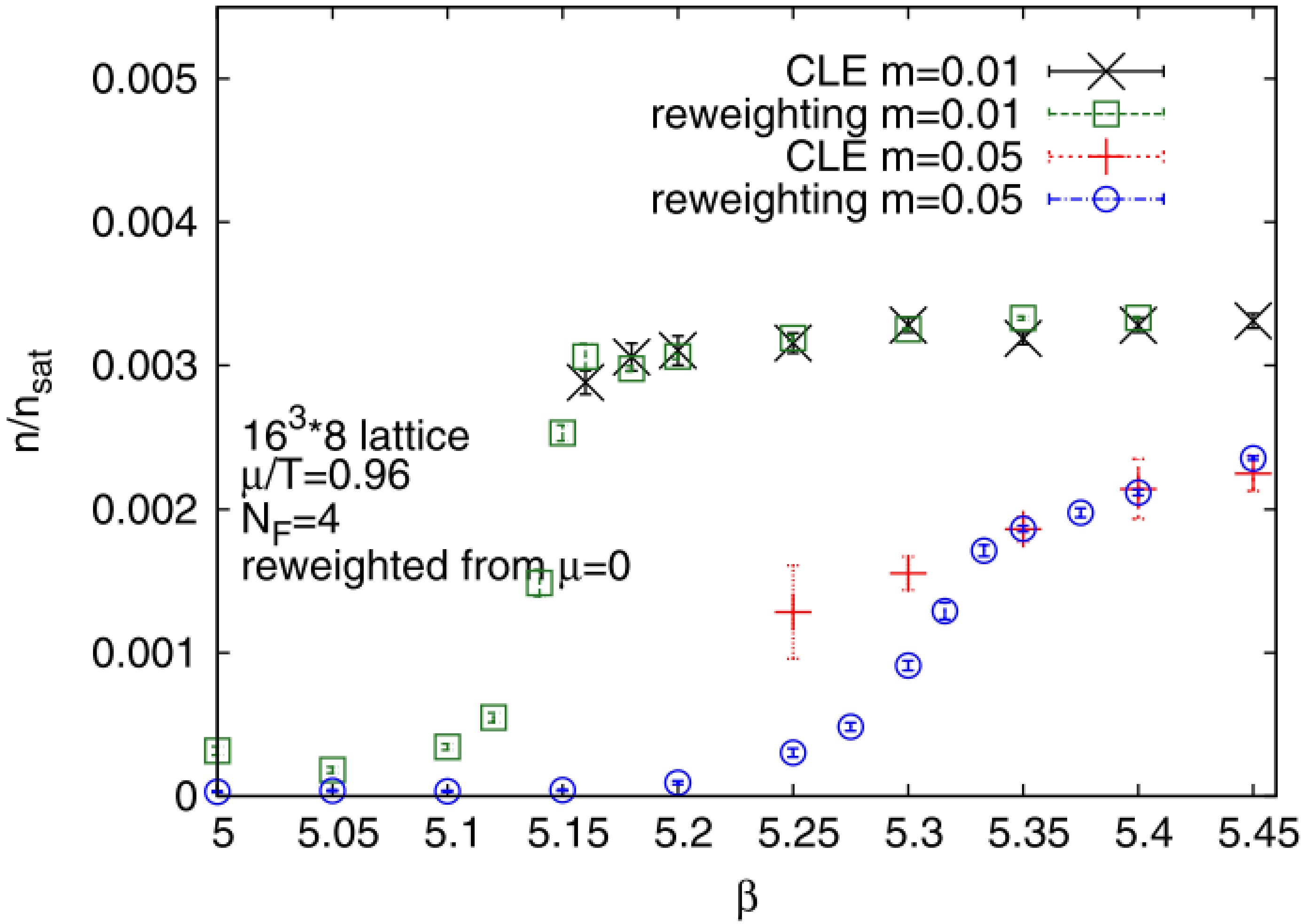
K. Nagata, J. Nishimura, S. Shimasaki (2016). [1604.07717]

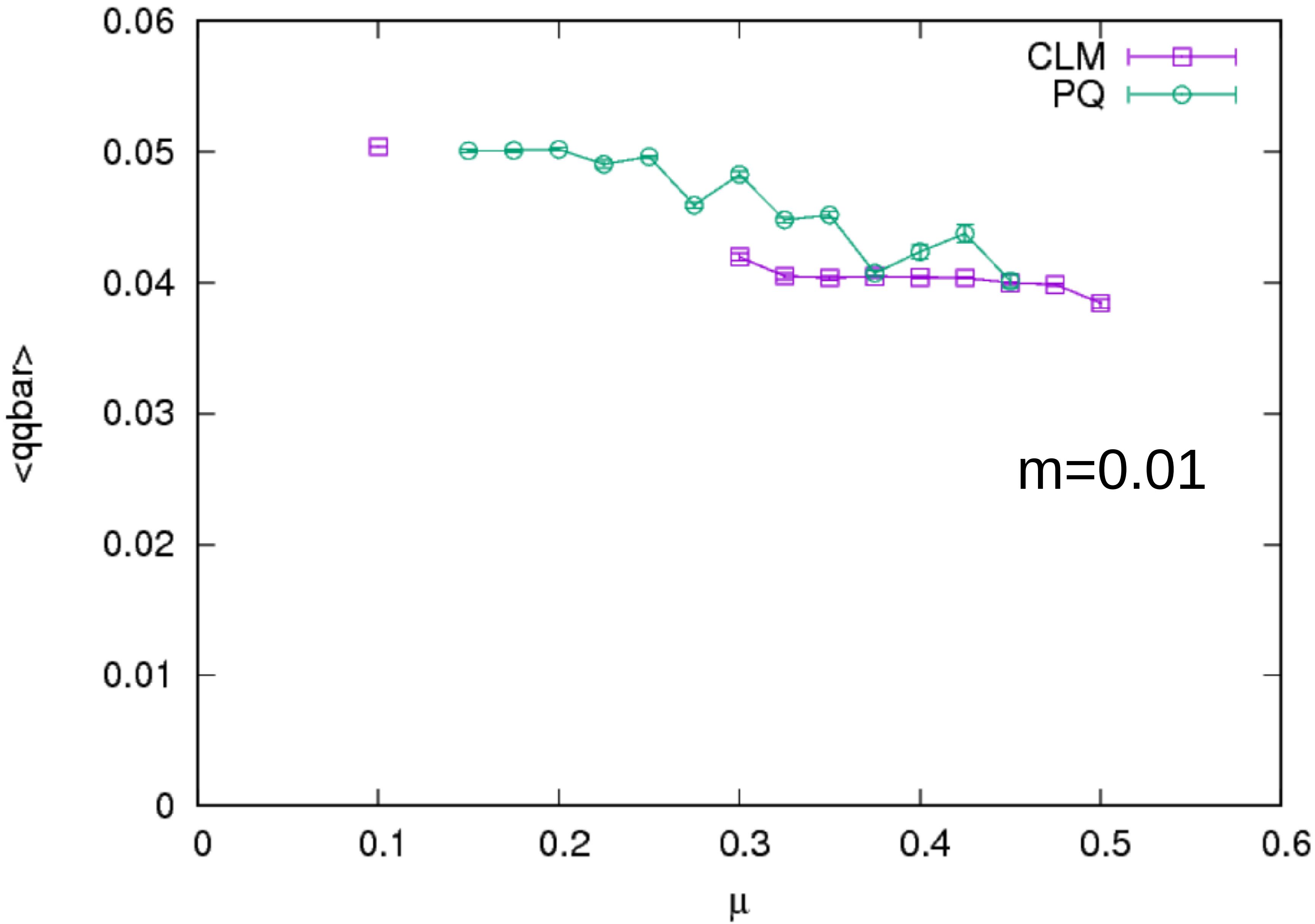


より一次相転移線に近い領域へアプローチできる可能性



0.35
0.30
0.25
0.20
0.15
0.10
a [fm]
4.8
4.9
5.0
5.1
5.2
5.3
5.4
β
am = 0.01
am = 0.05
Pion mass
300MeV
680MeV
840MeV
530MeV
600MeV
750MeV


CLE m=0.01
reweighting m=0.01
CLE m=0.05
reweighting m=0.05
16^3*8 lattice
μ/T=0.96
N_F=4
reweighted from μ=0
n/n_sat
β
0.005
0.004
0.003
0.002
0.001
0
5
5.05
5.1
5.15
5.2
5.25
5.3
5.35
5.4
5.45


0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.1
0.2
0.3
0.4
0.5
0.6
<qqbar>
μ
CLM
PQ
m=0.01

Justification of complex Langevin method

Associated Fokker-Planck-like equation becomes,

$$\frac{\partial}{\partial t}\Phi(x,y,t) = \left[\frac{\partial}{\partial x_i}\left\{\mathrm{Re}\left(\frac{\partial S}{\partial z_i}\right) + N_R\frac{\partial}{\partial x_i}\right\} + \frac{\partial}{\partial y_i}\left\{\mathrm{Im}\left(\frac{\partial S}{\partial z_i}\right) + N_I\frac{\partial}{\partial y_i}\right\}\right]\Phi(x,y,t)$$

Under *certain conditions*,

$$\int dxdyO(x+iy)\Phi(x,y) = \int dxO(x)P(x)$$

$$\frac{\partial}{\partial t}P(x,t) = \frac{\partial}{\partial x}\left(\frac{\partial S}{\partial x} + \frac{\partial}{\partial x}\right)P(x,t)$$

The stationary solution reads

$$P_{\mathrm{eq}}(x) \propto e^{-S(x)} \qquad \langle O(z(t))\rangle \to \frac{1}{Z}\int dxO(x)e^{-S(x)}, \quad t\to\infty$$

Criterion of correctness

A criterion for the correctness of the complex Langevin method

K. Nagata, J. Nishimura, S. Shimasaki [1508.02377, 1606.07627]

$$\mathcal{U}_{x\mu}(t+\epsilon) = \exp\left[i\left(-\epsilon\mathcal{D}_{x\mu}S[\mathcal{U}] + \sqrt{\epsilon}\eta_{x\mu}\right)\right]\mathcal{U}_{x\mu}(t)$$

Drift term $v_{x\mu}(\mathcal{U}) = -\mathcal{D}_{x\mu}S[\mathcal{U}]$

Probability distribution of the magnitude of the drift term plays a key role.

$$p(u) = \frac{1}{4N_V}\left\langle \sum_{x\mu} \delta(u - u_{x\mu}) \right\rangle \qquad u_{x\mu} = \sqrt{\frac{1}{N_c^2 - 1}\,\mathrm{tr}(v_{x\nu}v_{x\nu}^{\dagger})}$$