Composite operator and condensate in the $SU(N)$ Yang-Mills theory with $U(N-1)$ stability group

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in collaboration with
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Motivation

- dual superconductivity picture is a promising candidate for explaining confinement
- low energy region of QCD as "Abelian" theory + monopoles
- how do the "non-Abelian" d.o.f decouple? acquire a IR mass!
- evidence on the lattice (e.g. exponential fall-off of the corresponding gluon propagator)
- analytical approach: mass term through condensation of dimension-two operator
- mass term would furthermore imply e.g. stabilization of Savvidy vacuum, quark confinement at low temperatures...

1 Reformulated Yang-Mills Theory

- reformulation of $G = SU(N)$ Yang-Mills theory based on Cho-Faddeev-Niemi decomposition with respect to the stability group H

$$\mathcal{A}_\mu = \mathcal{X}_\mu + \mathcal{V}_\mu \;\in \mathrm{Lie}(G/H) \oplus \mathrm{Lie}(H) \tag{1}$$

- the residual field $\mathcal{V}_\mu$ transforms inhomogeneously and is identified as the IR dominant mode
- the remaining or coset field $\mathcal{X}_\mu$ transforms homogeneously and decouples in the IR
- decomposition defined in terms of the normalized color field $\mathfrak{n}$
- additional symmetry under rotations along $\mathfrak{n}$
- novel viewpoint: non-linear change of variables [*Kondo et al. '08*]

$$\{\mathcal{A}_\mu\} \Longrightarrow \{\mathcal{X}_\mu; \mathcal{V}_\mu; \mathfrak{n}\} \tag{2}$$

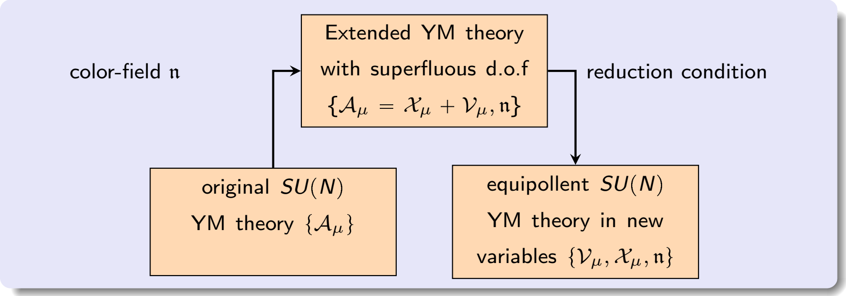
- problem: mismatch between degrees of freedom
- removal of the additional d.o.f through *reduction condition*

$$[\mathfrak{n}, \mathcal{D}_\mu[\mathcal{A}]\mathcal{D}^\mu[\mathcal{A}]\mathfrak{n}] = 0 \iff \mathcal{D}_\mu[\mathcal{V}]\mathcal{X}^\mu = 0 \qquad (3)$$

- in addition, this constraint reduces the enlarged gauge symmetry to the original $SU(N)$ symmetry
- incorporate reduction condition in a gauge fixing manner in the path integral

Reformulated $G = SU(N)$ Yang-Mills theory

[*Kondo et al. '08*]



reduction condition:

$$\mathcal{D}_\mu[\mathcal{V}]\mathcal{X}^\mu = 0 \tag{4}$$

- we adopt the "minimal" choice $H = U(N-1)$, which requires the introduction of only a single color field (other popular choice: $H = U(1)^{N-1}$ related to the maximal Abelian gauge)
- difficult to handle $\mathfrak{n}$ analytically $\Longrightarrow$ fixed choice: $\mathfrak{n} = T^{N^2-1}$ (last Cartan generator)

consequences of this symmetry breaking

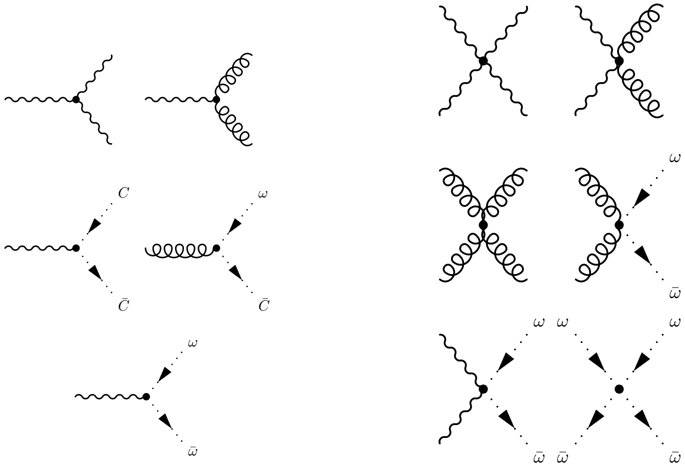
- reduction condition appears as a "gauge fixing" term for the coset gluon $\mathcal{X}_\mu \Longrightarrow$ breaking $SU(N) \to U(N-1)$
- despite the symmetry breaking: at least a (on-shell) BRST-invariant dimension-2 operator can be introduced

$$\mathcal{O} = \mathrm{Tr}_{G/H}\left(\mathcal{X}_\mu \mathcal{X}^\mu - 2i\xi \mathcal{C}\bar{\mathcal{C}}\right) \tag{5}$$

$$\mathcal{L}_{YM} + i\delta_B\bar{\delta}_B\,\mathrm{Tr}_{G/H}\left(\mathcal{X}_\mu\mathcal{X}^\mu - i\xi\mathcal{C}\bar{\mathcal{C}}\right) - i\delta_B\,\mathrm{Tr}_{U(1)}\left[\bar{\mathcal{C}}\left(2\partial_\mu\mathcal{V}^\mu + \alpha\mathcal{N}\right)\right]$$
$$-i\delta_B\,\mathrm{Tr}_{SU(N-1)}\left[\bar{\mathcal{C}}\left(2\partial_\mu\mathcal{V}^\mu + \lambda\mathcal{N}\right)\right]$$

2 One-loop analysis and multiplicative renormalizability

Based on the set of vertices...



...the one-loop RG functions were obtained ✓

For $N = 2$ (MAG): agreement with [*e.g. Shinohara et al. '03*]

$$\gamma_X = \frac{g^2}{(4\pi)^2}\frac{N}{2}\left(\frac{17}{6} - \frac{\xi}{2} - \frac{\alpha + (N-2)\lambda}{N-1}\right) \qquad \beta_g = -\frac{g^3}{(4\pi)^2}\frac{11}{3}N,$$

$$\gamma_V = \frac{g^2}{(4\pi)^2}\left(\frac{13N+9}{6} - \frac{\lambda}{2}(N-1)\right), \qquad \tilde{\gamma}_V = \frac{g^2}{(4\pi)^2}\frac{11}{3}N,$$

$$\gamma_\omega = \gamma_{\bar{\omega}} = \frac{g^2}{(4\pi)^2}\frac{N}{2}\left(3 - \frac{\alpha + (N-2)\lambda}{N-1}\right), \qquad \tilde{\gamma}_C = -\tilde{\gamma}_{\bar{C}} = \frac{g^2}{(4\pi)^2}\frac{N}{2}(3+\xi),$$

$$\gamma_C = \frac{g^2}{(4\pi)^2}\left(\frac{(\xi+3)(N-3)}{4} + (N-1)\lambda\right), \qquad \gamma_{\bar{C}} = \frac{g^2}{(4\pi)^2}\frac{3(\xi+1+2\lambda) - N(\xi-3+6\lambda)}{4},$$

$$\gamma_\lambda = \frac{g^2}{(4\pi)^2}\left(\frac{13N+9}{3} - \lambda(N-1)\right), \qquad \gamma_\alpha = \frac{g^2}{(4\pi)^2}\frac{22}{3}N,$$

$$\mu\partial_\mu\xi = \xi\gamma_\xi = \frac{g^2}{(4\pi)^2}\left(\frac{4}{3}\xi - \xi^2 - 3\right)N \qquad (6)$$

- problematic: "gauge-fixing" parameter corresponding to the reduction condition has no fixed point

Multiplicative renormalizability of the composite operator

- mixing with other operators of same quantum number has to be taken into account

$$\begin{pmatrix} \left[\frac{1}{2}X_\mu^a X_a^\mu\right] \\ \left[\frac{1}{2}V_\mu^j V_j^\mu\right] \\ \left[i\omega^a\bar{\omega}^a\right] \\ \left[iC^j\bar{C}^j\right] \\ \left[\frac{1}{2}V_\mu^\gamma V_\gamma^\mu\right] \\ \left[iC^\gamma\bar{C}^\gamma\right] \end{pmatrix} = \begin{pmatrix} 1-Z_1^{(1)} & 0 & -Z_3^{(1)} & 0 & 0 & 0 \\ -Z_7^{(1)} & 1-Z_8^{(1)} & -Z_9^{(1)} & 0 & 0 & 0 \\ -Z_{13}^{(1)} & 0 & 1-Z_{15}^{(1)} & 0 & 0 & 0 \\ 0 & -Z_{20}^{(1)} & 0 & 1 & 0 & 0 \\ -Z_{25}^{(1)} & 0 & -Z_{27}^{(1)} & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \left[\frac{1}{2}X_\mu^a X_a^\mu\right]_R \\ \left[\frac{1}{2}V_\mu^j V_j^\mu\right]_R \\ \left[i\omega^a\bar{\omega}^a\right]_R \\ \left[iC^j\bar{C}^j\right]_R \\ \left[\frac{1}{2}V_\mu^\gamma V_\gamma^\mu\right]_R \\ \left[iC^\gamma\bar{C}^\gamma\right]_R \end{pmatrix}$$

The requirement of multiplicative renormalizability translates into

$$-Z_1^{(1)} + Z_X^{(1)} + \xi Z_{13}^{(1)} \overset{!}{=} Z_\xi^{(1)} - Z_{15}^{(1)} + Z_\omega^{(1)} + \frac{1}{\xi} Z_3^{(1)} \tag{7}$$

Indeed, we find

$$-Z_1^{(1)} + Z_X^{(1)} + \xi Z_{13}^{(1)} = \frac{g^2 \mu^{-2\epsilon}}{(4\pi)^2 \epsilon} \left[\frac{N}{6}(13 - 3\xi) \right] \tag{8}$$

$$Z_\xi^{(1)} - Z_{15}^{(1)} + Z_\omega^{(1)} + \frac{1}{\xi} Z_3^{(1)} = \frac{g^2 \mu^{-2\epsilon}}{(4\pi)^2 \epsilon} \left[\frac{N}{6}(13 - 3\xi) \right] \tag{9}$$

The composite operator is multiplicatively renormalizable ✓

$$\gamma_{\mathcal{O}} = \frac{g^2}{(4\pi)^2} \frac{N}{3}(13 - 3\xi) \tag{10}$$

For N=2 (MAG): agreement with [*Dudal et al. '03*]

3 LCO formalism and existence of the condensate

- developed by [Knecht, Verschelde; '95, '01] to tame new divergences coming from composite operator source term $\sim J\mathcal{O}$
- starting point: add additional piece to bare Lagrangian

$$\mathcal{L}_{LCO} = \frac{1}{2}(\kappa + \delta\kappa)J^2 \tag{11}$$

- requiring RG invariance of this additional term and assuming that $\kappa[g^2(\mu), \xi(\mu)]$ yields an ODE for the parameter κ[3],

$$\left[2\epsilon + 2\gamma_{\mathcal{O}} - \beta_{g^2}\frac{\partial}{\partial g^2} - \xi\gamma_\xi\frac{\partial}{\partial\xi}\right](\kappa + \delta\kappa) = 0 \tag{12}$$

- this can, in principle, be solved order by order

$$\kappa(g^2, \xi) = \frac{\kappa_0}{g^2} + \hbar\kappa_1(\xi) + \hbar^2 g^2\kappa_2(\xi) + \ldots \tag{13}$$

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[3]From now on, $\alpha = \lambda = 0$ is adopted as they are fixed points.

- however, this implies that in order to obtain the n - loop part of κ, we need all other quantities to $n+1$ - loop order, e.g.

$$\left[\xi\gamma_{\xi,1}\partial_\xi - 2\gamma_{\mathcal{O},1} - \beta_{g^2,1}\right]\kappa_0 = 2\delta\kappa_0 \tag{14}$$

- $\delta\kappa_0$ is the coefficient of the one-loop divergent part quadratic in the sources, coming from

$$-\frac{i}{2}d_{G/H}\,\mathrm{Tr}\log\left(-p^2 g_{\mu\nu} + (1-\xi^{-1})p_\mu p_\nu + g_{\mu\nu}J\right) + i d_{G/H}\,\mathrm{Tr}\log(-p^2+\xi J) \tag{15}$$

$$\Longrightarrow \frac{\delta\kappa_0}{\epsilon} = \frac{d_{G/H}}{2}\frac{3-\xi^2}{(4\pi)^2\epsilon} \tag{16}$$

- the solution to the ODE for the tree-level part of κ is then obtained

$$\kappa_0 = \frac{2(N-1)}{N}\xi + C(4\xi - 3\xi^2 - 9) \qquad (17)$$

- a choice for the integration constant C is motivated later
- to discuss the existence of the condensate, the tree-level part will be enough
- one more problem to be tackled: the quadratic source term κJ^2 spoils the usual construction of the generating functional

- removal through Hubbard-Stratonovich transformation

$$1 = \int d\sigma \ \ \mathrm{Exp}\left[-i\frac{1}{2g^2\kappa}(\sigma - g\mathcal{O} - g\kappa J)^2\right] \tag{18}$$

- this yields the transition

$$\mathcal{L} + \mathcal{O}J + \frac{1}{2}\kappa J^2 + \mathcal{L}_{counter} \Longrightarrow \mathcal{L} - \frac{\sigma^2}{2g^2\kappa} + \frac{1}{\kappa}\frac{\sigma}{g}\mathcal{O} - \frac{1}{2\kappa}\mathcal{O}^2 + \frac{\sigma}{g}J + \mathcal{L}_{counter} \tag{19}$$

- nontrivial vev of σ induces tree-level mass term for the coset gluon

$$m_X^2 = \frac{g\langle\sigma\rangle}{\kappa_0} \tag{20}$$

$$\kappa_0 = \frac{2(N-1)}{N}\xi + C(4\xi - 3\xi^2 - 9)$$

- final step: effective potential for σ

$$V(\sigma) = \frac{\sigma^2}{2\kappa_0} - \frac{\kappa_1}{2\kappa_0^2} g^2\sigma^2 - \frac{3}{64\pi^2} 2(N-1)\frac{g^2\sigma^2}{\kappa_0^2}\left(\frac{5}{6} - \log\left[\frac{g\sigma}{\kappa_0\bar{\mu}^2}\right]\right) + \frac{1}{64\pi^2} 2(N-1)\frac{\xi^2 g^2\sigma^2}{\kappa_0^2}\left(\frac{3}{2} - \log\left[\frac{\xi g\sigma}{\kappa_0\bar{\mu}^2}\right]\right) \tag{21}$$

- the tree part should remain positive, at least in the vicinity of $\xi = 0 \Longrightarrow$ restricts the choice of the integration constant in κ_0
- for later convenience:

$$C = -\frac{1}{11}\frac{N-1}{N} \tag{22}$$

- indeed, from the one-loop effective potential one finds a nontrivial extremum defined by

$$m_X^2 = \frac{g\sigma_*}{\kappa_0} = \bar{\mu}^2 \ \mathrm{Exp}\left[\frac{H_1}{g^2} + H_2\right] \tag{23}$$

$$V(\sigma_*) = -(3-\xi^2)\frac{2(N-1)}{128\pi^2} m_X^4 \tag{24}$$

with

$$H_1(\xi,\kappa_0) = -\frac{1}{(3-\xi^2)}\frac{32\pi^2}{2(N-1)}\kappa_0$$

$$H_2(\xi,\kappa_1) = \frac{1}{(3-\xi^2)}\left(\frac{32\pi^2}{2(N-1)}\kappa_1 + 1 + \frac{1}{2}\xi^2\log\xi^2 - \xi^2\right)$$

For N=2 (MAG): agreement with [*Dudal et al. '03*]

$$m_X^2 = \bar{\mu}^2 \ \text{Exp}\left[\frac{H_1}{g^2} + H_2\right]$$
$$H_1(\xi,\kappa_0) = -\frac{1}{(3-\xi^2)}\frac{32\pi^2}{2(N-1)}\kappa_0$$

- to obtain the correct UV limit $m_X^2 \xrightarrow{g^2\to 0} 0$, H_1 must be negative
- since $\kappa_0 > 0$ this restricts $\xi^2 < 3$, which is consistent with our interpretation of the reduction condition, namely setting $\xi = 0$ in the end
- then, in particular $V(\sigma_*) = -(3-\xi^2)\frac{2(N-1)}{128\pi^2} m_X^4 < 0$
- moreover, for $\xi = 0$ the function H_2 becomes an irrelevant constant and the gluon mass can be written as

$$m_X^2 = \text{const} \times \Lambda_{QCD}^2; \quad \Lambda_{QCD} = \bar{\mu}\ \text{Exp}\left[-\int^g \frac{dg'}{\beta_g(g')}\right]$$

The condensate shows up at the confinement scale ✓

4 Conclusion and Outlook

- one-loop analysis of $SU(N)$ Yang-Mills theory with $U(N-1)$ stability group has been performed
- in particular, the multiplicative renormalizability of a certain BRST invariant dimension-2 composite operator has been shown
- based on these results, the existence of the corresponding condensate was discussed within the LCO formalism
- for the "physical" choice $\xi = 0$, the condensate shows up at the QCD scale, generating a tree level mass for the coset gluon $\mathcal{X}_\mu$
- this can be seen as evidence for the "Abelian dominance" as one of the key properties of the dual superconductivity picture

However,

- the treatment of the "gauge parameter" ξ is still unsatisfying, as we set it to zero despite having no fixed point
- the question whether the coset gluon can obtain a mass or not is generically nonperturbative
- indeed, the problem is currently reinvestigated within the framework of the functional renormalization group

- naive input for the flowing effective average action

$$\Gamma_k = \Gamma_{YM} + \Gamma^{res}_{GF+FP} + \Gamma^{red}_{GF+FP} + U_k[\sigma] + \frac{Z_{\sigma,k}}{2}\partial_\mu\sigma\partial_\mu\sigma + \frac{h_{X,k}}{2}\sigma X^a_\mu X^a_\mu - ih_{\omega,k}\xi\sigma\omega^a\bar{\omega}^a + \frac{\ell_k}{8}X^a_\mu X^a_\mu X^b_\nu X^b_\nu - i\xi\frac{n_k}{2}X^a_\mu X^a_\mu \omega^b\bar{\omega}^b - \frac{y_k}{2}\xi^2\bar{\omega}^a\omega^a\bar{\omega}^b\omega^b \tag{25}$$

- calculating the flow according to the Wetterich equation

$$\partial_t\Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\,(\Gamma^{(2)}_k + R_k)^{-1}\partial_t R_k\right], t = \log(k/\Lambda). \tag{26}$$

- does $U_k[\sigma]$ develop a non-trivial minimum as well?

Thank you for your attention.