Quark distribution functions of the pion and kaon in the chiral constituent quark model

Akira Watanabe
(Institute of High Energy Physics, Chinese Academy of Sciences)

Based on:
AW, Chung Wen Kao, Katsuhiko Suzuki, PRD **94**, 114008 (2016)
AW, Takahiro Sawada, Chung Wen Kao, PRD **97**, 074015 (2018)

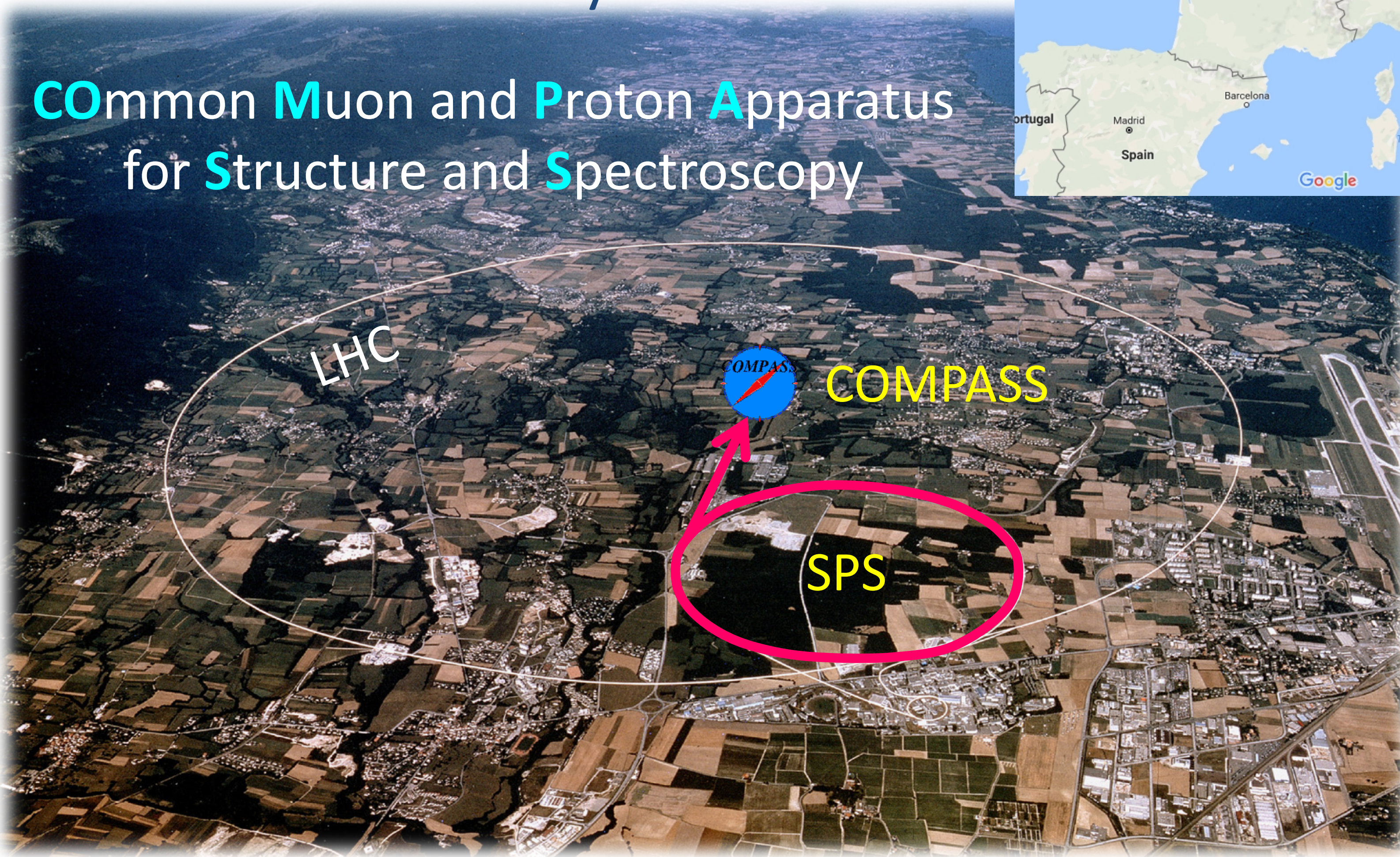
New Frontiers in QCD 2018 (NFQCD2018)

June 11, 2018 @ Yukawa Institute for Theoretical Physics

Talk plan

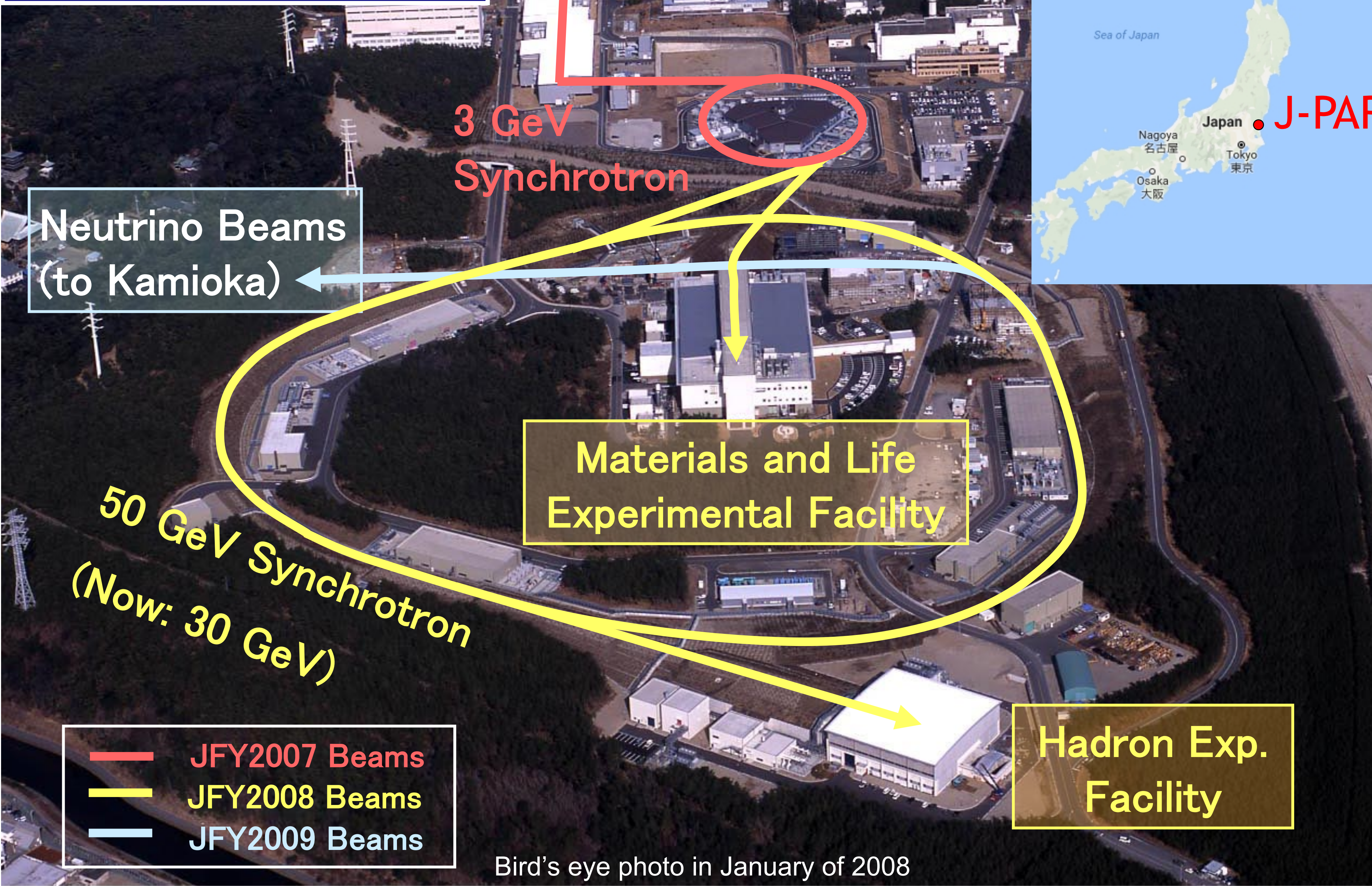
- Introduction
- Dressing corrections to the constituent quarks
- Numerical results for the pion quark distribution function
- Results for the kaon
- Summary

COMPASS facility at CERN



- Fixed target experiment at the end of M2 SPS beam line
- Nearly 220 physicists from 13 countries and 24 institutions

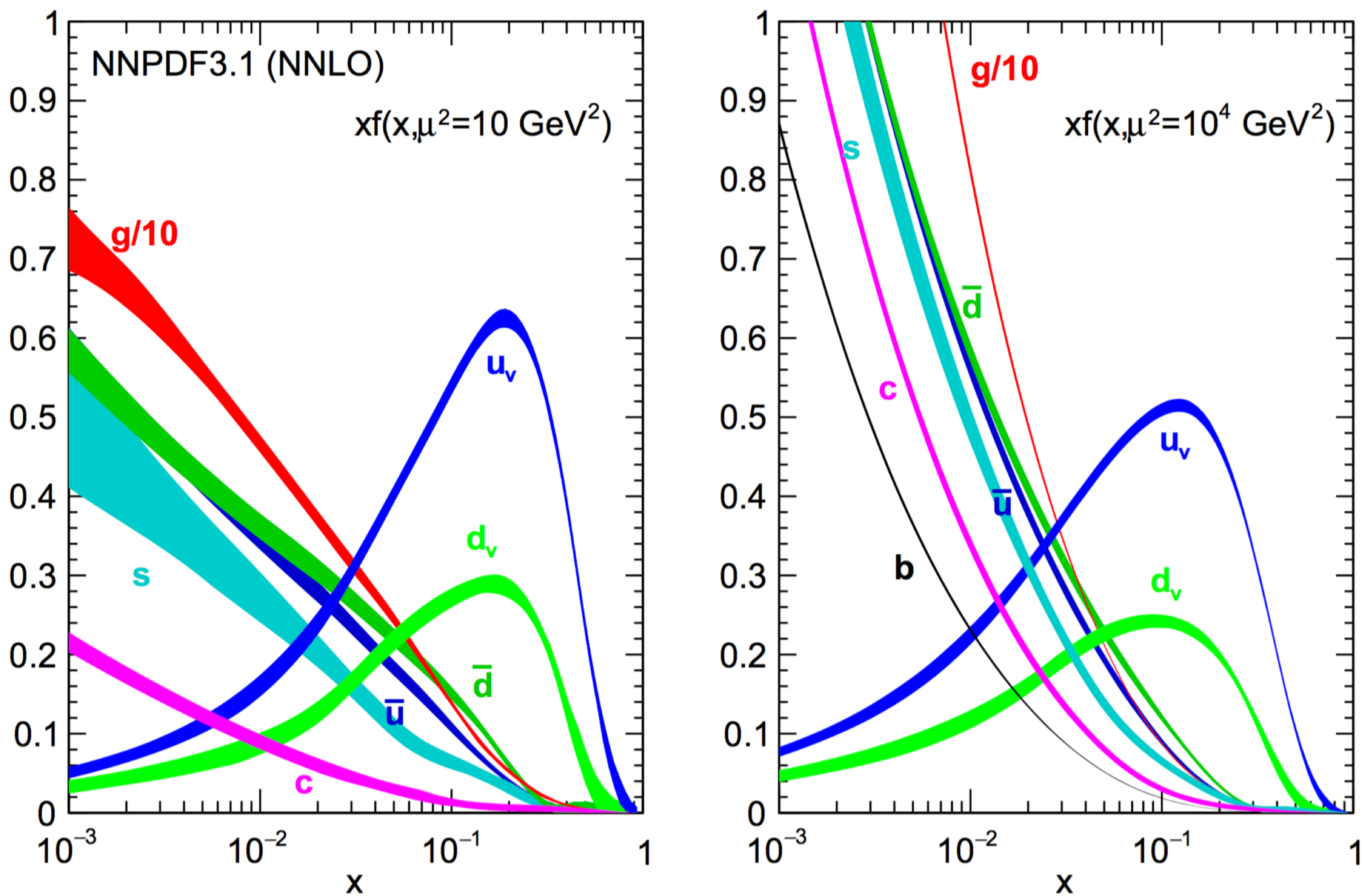
J–PARC Facility
(KEK/JAEA)



Bird's eye photo in January of 2008

Proton parton distribution functions (PDFs)

NNPDF Collaboration (2017)



Pionic parton distributions revisited

M. Glück, E. Reya, I. Schienbein

Institut für Physik, Universität Dortmund, D-44221 Dortmund, Germany

Received: 3 March 1999 / Revised version: 3 May 1999 / Published online: 15 July 1999

Abstract. Using constituent quark model constraints we calculate the gluon and sea–quark content of pions solely in terms of their valence density (fixed by πN Drell–Yan data) and the known sea and gluon distributions of the nucleon, using the most recent updated valence–like input parton densities of the nucleon. The resulting small–x dynamical QCD predictions for $g^{\pi}(x,Q^2)$ and $\bar{q}^{\pi}(x,Q^2)$ are unique and parameter free. Simple analytic parametrizations of the resulting parton distributions of the pion are presented in LO and NLO. These results and parametrizations will be important, among other things, for updated formulations of the parton distributions of real and virtual photons.

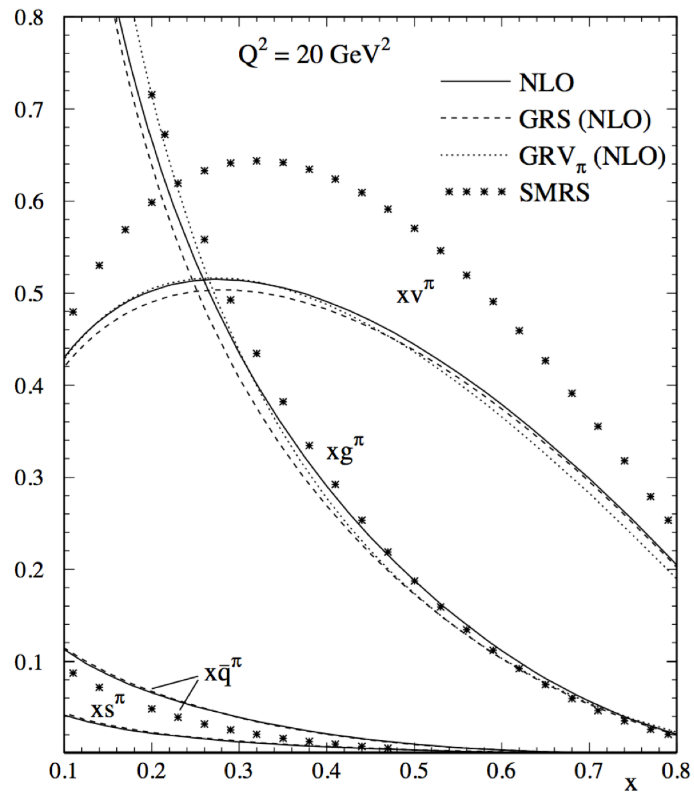


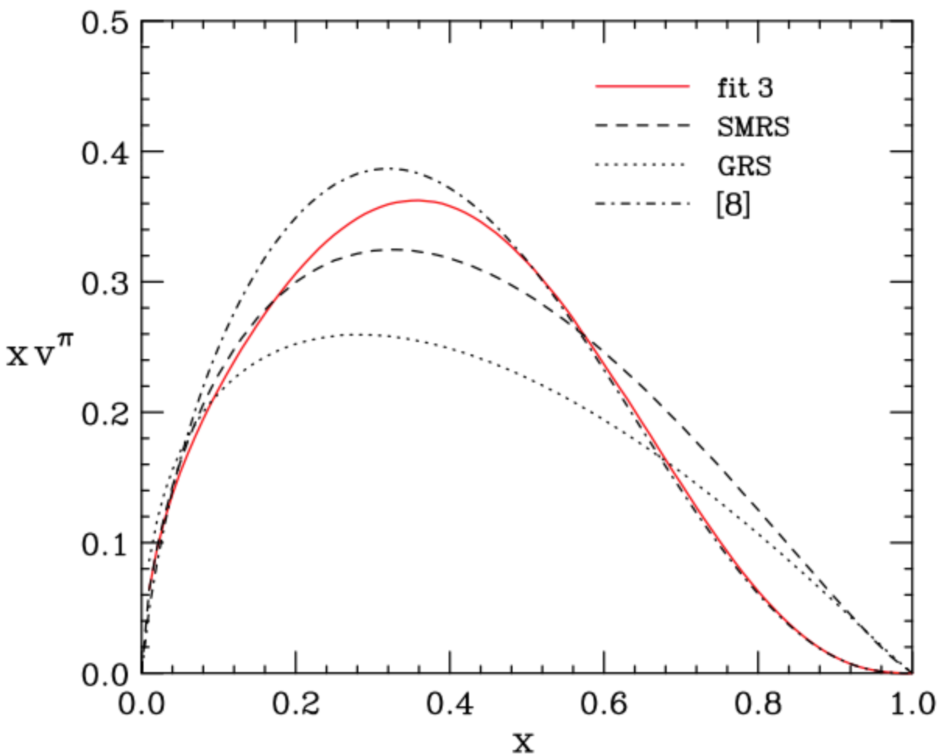
Fig. 2. Comparison of our NLO valence distribution at $Q^2 = 20$ GeV^2 with the one of GRV$_\pi$ [5] and GRS [6]. This density plays the dominant role for describing presently available πN Drell–Yan dimuon production data. For illustration, the gluon and sea densities are shown as well. The SU(3)$_{\text{flavor}}$ symmetric GRV$_\pi$ sea $\bar{q}^{\pi} = s^{\pi}$ is not shown, since it is similar to s^{π} of our present analysis and of GRS which are all generated from a vanishing input at $Q^2 = \mu^2$, cf. (3). The SMRS [3] results refer also to a SU(3)$_{\text{flavor}}$ symmetric sea $\bar{q}^{\pi} \equiv \bar{u}^{\pi^+} = d^{\pi^+} = s^{\pi} = \bar{s}^{\pi}$

PRL 105, 252003 (2010) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2010

Soft-Gluon Resummation and the Valence Parton Distribution Function of the Pion

Matthias Aicher,[1] Andreas Schäfer,[1] and Werner Vogelsang[2]
[1]*Institute for Theoretical Physics, University of Regensburg, D-93040 Regensburg, Germany*
[2]*Institute for Theoretical Physics, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*
(Received 15 September 2010; published 16 December 2010)

We determine the valence parton distribution function of the pion by performing a new analysis of data for the Drell-Yan process $\pi^- N \to \mu^+ \mu^- X$. Compared to previous analyses, we include next-to-leading-logarithmic threshold resummation effects in the calculation of the Drell-Yan cross section. As a result of these, we find a considerably softer valence distribution at high momentum fractions x than obtained in previous next-to-leading-order analyses, in line with expectations based on perturbative-QCD counting rules or Dyson-Schwinger equations.



[8]: Hecht-Roberts-Schmidt (2001) based on DSE

- Their result is quite different from GRS, and much softer at large x.
- It may be important to check this new result with a different approach.

Chiral constituent quark model (CCQM)

Effective interaction Lagrangian:

$$\mathcal{L}_{\text{int}} = -\frac{g_A}{f}\bar{\psi}\gamma^\mu\gamma^5\left(\partial_\mu\Pi\right)\psi$$

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$\Pi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

Relationships between the dressed and bare states:

$$|U\rangle = \sqrt{Z_u}|u_0\rangle + a_\pi|d\pi^+\rangle + \frac{a_\pi}{2}|u\pi^0\rangle + a_K|sK^+\rangle + \frac{a_\eta}{6}|u\eta\rangle$$

$$|D\rangle = \sqrt{Z_d}|d_0\rangle + b_\pi|u\pi^-\rangle + \frac{b_\pi}{2}|d\pi^0\rangle + b_K|sK^0\rangle + \frac{b_\eta}{6}|d\eta\rangle$$

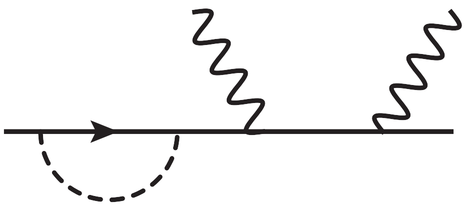
$$|S\rangle = \sqrt{Z_s}|s_0\rangle + c_K|uK^-\rangle + c_K|d\bar{K}^0\rangle + \frac{2c_\eta}{3}|s\eta\rangle$$

Constituent quark = Dressed current quark

Dressing corrections to constituent quarks

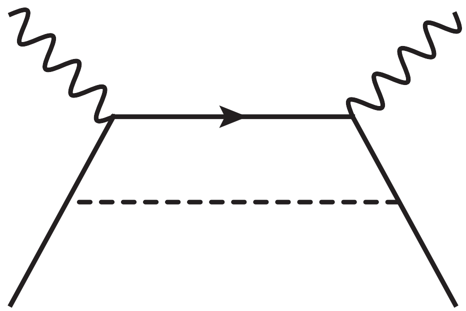
Diagrams contributing to the constituent quark structure:

(a)



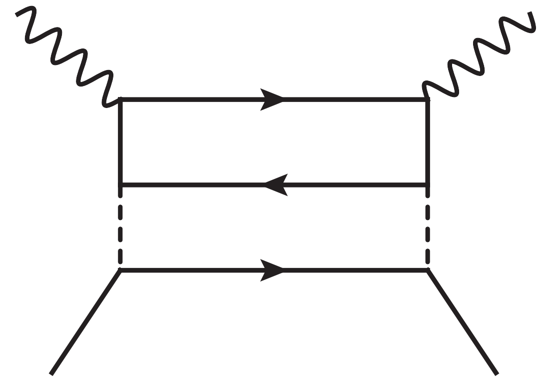
Via this diagram the renormalization constant is calculated.

(b)



This shows the Goldstone boson spectator process.

(c)



This diagram probes the structure of the Goldstone boson itself with the constituent quark being spectator.

Formalism in infinite momentum frame

The infinite momentum frame is utilized to calculate the contributions. Since all the particles are on-shell in this frame, one-dimensional convolution formalism can be used throughout the following calculations.

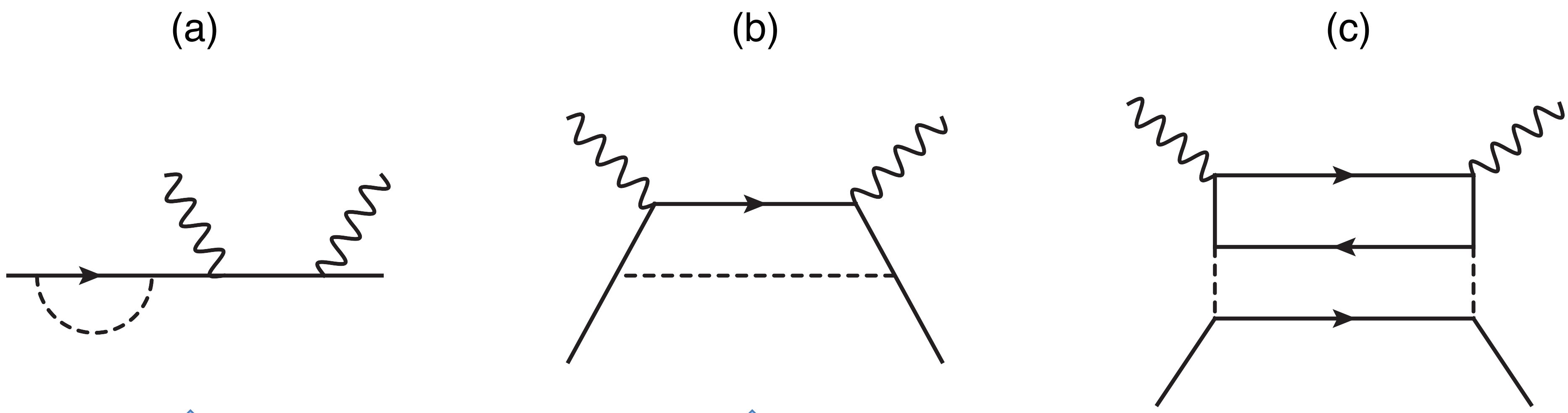
Splitting function

$$P_{j\alpha/i}(y) = \frac{1}{8\pi^2}\left(\frac{g_A \bar{m}}{f}\right)^2 \int dk_T^2 \frac{\left(m_j - m_i y\right)^2 + k_T^2}{y^2(1-y)\left(m_i^2 - M_{j\alpha}^2\right)^2}$$

This function gives the probability to find a constituent quark *j* carrying the light-cone momentum fraction y together with a spectator Goldstone boson *(a = π, K)*, both of which coming from a parent constituent quark *i*.

$$P_{\alpha j/i}(x) = P_{j\alpha/i}(1-x)$$

Dressing corrections



Can be evaluated by moments of the splitting function

$$q_j(x) = \int_x^1 \frac{dy}{y} P_{j\alpha/i}(y)\, q_i\left(\frac{x}{y}\right)$$

$$q_k(x) = \int \frac{dy_1}{y_1}\frac{dy_2}{y_2} V_{k/\alpha}\left(\frac{x}{y_1}\right) P_{\alpha j/i}\left(\frac{y_1}{y_2}\right) q_i(y_2)$$

Moments and renormalization constant

We define the moments of the splitting functions,

$$\left\langle x^{n-1} P_{j\alpha/i} \right\rangle \equiv \int_0^1 dx x^{n-1} P_{j\alpha/i}(x)$$

As for the first moments,

$$\left\langle P_{j\alpha/i} \right\rangle = \left\langle P_{\alpha j/i} \right\rangle \equiv \left\langle P_{\alpha} \right\rangle$$

In terms of those, the renormalization constant *Z* is then given by

$$Z = 1 - \frac{3}{2}\left\langle P_{\pi} \right\rangle - \left\langle P_{K} \right\rangle$$

We find *Z ~ 0.7* using the standard parameter set.
Numerically, about 75% of the deviation from *Z = 1* comes from the pion dressing and the rest from the kaon.

Quark distribution functions of π^+

We explicitly write down the quark distribution functions of π^+ using the splitting functions. Here $u_0(x)$ and $d_0bar(x)$ are the bare quark distribution functions.

$$u(x) = Zu_0(x) + \frac{1}{2}P_{u\pi/u} \otimes u_0 + V_{u/\pi} \otimes P_{\pi d/u} \otimes u_0 + V_{u/\pi} \otimes P_{\pi\bar{u}/\bar{d}} \otimes \bar{d}_0$$
$$+ \frac{1}{4}V_{u/\pi} \otimes P_{\pi u/u} \otimes u_0 + \frac{1}{4}V_{u/\pi} \otimes P_{\pi\bar{d}/\bar{d}} \otimes \bar{d}_0 + V_{u/K} \otimes P_{Ks/u} \otimes u_0$$

$$\bar{u}(x) = P_{\bar{u}\pi/\bar{d}} \otimes \bar{d}_0 + \frac{1}{4}V_{\bar{u}/\pi} \otimes P_{\pi u/u} \otimes u_0 + \frac{1}{4}V_{\bar{u}/\pi} \otimes P_{\pi\bar{d}/\bar{d}} \otimes \bar{d}_0$$

where

$$P \otimes q \equiv \int_x^1 \frac{dy}{y} P(y) q\left(\frac{x}{y}\right)$$

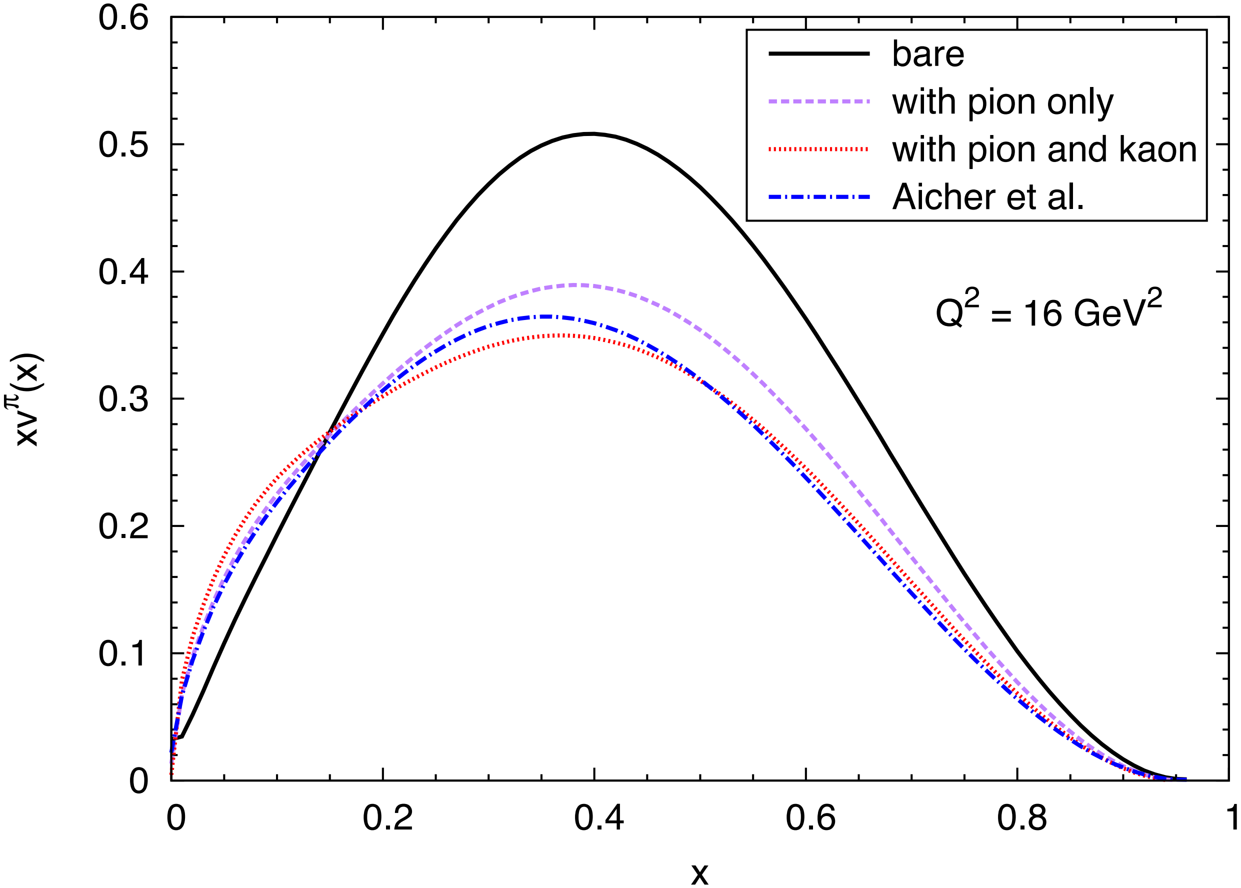
$$\int_0^1 dx u_0(x) = \int_0^1 dx \bar{d}_0(x) = 1$$

$$\int_0^1 dx u_{\rm val}(x) = \int_0^1 dx \{u(x) - \bar{u}(x)\}$$
$$= Z + \frac{3}{2}\langle P_\pi \rangle + \langle P_K \rangle$$
$$= 1$$

Procedure

1. Assume the functional form of VQDF at $Q=Q_0$: $v_{bare}(x)=Nx^{\alpha}(1-x)^{\alpha}$
2. Calculate the dressing corrections
3. Assume the values of α and Q_0, and perform the QCD evolution from $Q=Q_0$ to 4 GeV
4. Compare the result with that of Aicher et al.
5. Vary Q_0 to fit ours to their result
6. If cannot fit, vary α

$v_{dressed}(x)$ of π with original input set



- The suppression at large *x* is well reproduced by evaluating the dressing corrections.
- The first moments are quite similar to each other.

input: $v_0(x) = Nx^{1.8}(1-x)^{1.8}$

VQDFs of kaon

- The resulting $v_{dressed}(x)$ depends on $v_{bare}(x)$ at $Q=Q_0$.
- Once $v_{dressed}(x)$ at certain Q^2 is assumed, one can investigate $v_{bare}(x)$ at $Q=Q_0$ via the presented framework.
- There are available experimental data for the valence u quark ratio u_K/u_π.
- Evaluating the dressing corrections and performing the parameter fit with the ratio, one can study the VQDFs of the kaon.

Procedure

1. Multiply $v^{\pi}{}_{dressed}(x)$ at Q by the ratio u_K/u_π and obtain the kaon VQDF (result 1)
2. Assume the functional form of the bare distribution in the kaon at $Q=Q_0$: $v^{K(u)}{}_{bare}(x)=Nx^{\alpha}(1-x)^{\beta}$
3. Calculate the dressing corrections
4. Perform the QCD evolution from $Q=Q_0$ to higher Q and obtain the kaon VQDF (result 2)
5. Compare the result 2 with the result 1
6. Vary α and β to obtain the best fit
7. VQDF of the sbar quark can be also calculated via the assumption: $v^{K(sbar)}{}_{bare}(x)=Nx^{\beta}(1-x)^{\alpha}$

Valence u quark distribution of K^+

$$Z_u = 1 - \frac{3}{2}\left\langle P_\pi \right\rangle - \left\langle P_{K(i=u)} \right\rangle - \frac{1}{6}\left\langle P_{\eta(i=u)} \right\rangle$$

$$\begin{aligned}
u(x) &= Z_u u_0(x) + \frac{1}{2} P_{u\pi/u} \otimes u_0 + V_{u/\pi} \otimes P_{\pi d/u} \otimes u_0 + \frac{1}{4} V_{u/\pi} \otimes P_{\pi u/u} \otimes u_0 \\
&\quad + V_{u/K} \otimes P_{Ks/u} \otimes u_0 + V_{u/K} \otimes P_{K\bar{u}/\bar{s}} \otimes \bar{s}_0 \\
&\quad + \frac{1}{6} P_{u\eta/u} \otimes u_0 + \frac{1}{36} V_{u/\eta} \otimes P_{\eta u/u} \otimes u_0 + \frac{1}{9} V_{u/\eta} \otimes P_{\eta\bar{s}/\bar{s}} \otimes \bar{s}_0 \\
\bar{u}(x) &= P_{\bar{u}K/\bar{s}} \otimes \bar{s}_0 + \frac{1}{4} V_{\bar{u}/\pi} \otimes P_{\pi u/u} \otimes u_0 \\
&\quad + \frac{1}{36} V_{\bar{u}/\eta} \otimes P_{\eta u/u} \otimes u_0 + \frac{1}{9} V_{\bar{u}/\eta} \otimes P_{\eta\bar{s}/\bar{s}} \otimes \bar{s}_0
\end{aligned}$$

$$\begin{aligned}
\int_0^1 dx u_{\text{val}}(x) &= \int_0^1 dx \left\{ u(x) - \bar{u}(x) \right\} \\
&= Z_u + \frac{3}{2}\left\langle P_\pi \right\rangle + \left\langle P_{K(i=u)} \right\rangle + \frac{1}{6}\left\langle P_{\eta(i=u)} \right\rangle \\
&= 1
\end{aligned}$$

Valence *sbar* quark distribution of K^+

$$Z_s = 1 - 2\left\langle P_{K(i=\bar{s})}\right\rangle - \frac{2}{3}\left\langle P_{\eta(i=\bar{s})}\right\rangle$$

$$\begin{aligned}\bar{s}\left(x\right) =& Z_s\bar{s}_0\left(x\right) + V_{\bar{s}/K}\otimes P_{Ks/u}\otimes u_0 + V_{\bar{s}/K}\otimes P_{K\bar{u}/\bar{s}}\otimes\bar{s}_0 + V_{\bar{s}/K}\otimes P_{K\bar{d}/\bar{s}}\otimes\bar{s}_0\\ &+\frac{2}{3}P_{\bar{s}\eta/\bar{s}}\otimes\bar{s}_0 + \frac{1}{9}V_{\bar{s}/\eta}\otimes P_{\eta u/u}\otimes u_0 + \frac{4}{9}V_{\bar{s}/\eta}\otimes P_{\eta\bar{s}/\bar{s}}\otimes\bar{s}_0,\\ s\left(x\right) =& P_{sK/u}\otimes u_0 + \frac{1}{9}V_{s/\eta}\otimes P_{\eta u/u}\otimes u_0 + \frac{4}{9}V_{s/\eta}\otimes P_{\eta\bar{s}/\bar{s}}\otimes\bar{s}_0,\end{aligned}$$

$$\begin{aligned}\int_0^1 dx\bar{s}_{\mathrm{val}}(x) &= \int_0^1 dx\left\{\bar{s}(x) - s(x)\right\}\\ &= Z_s + 2\left\langle P_{K(i=\bar{s})}\right\rangle + \frac{2}{3}\left\langle P_{\eta(i=\bar{s})}\right\rangle\\ &= 1\end{aligned}$$

Valence u QDF ratio u_K/u_π

Volume 93B, number 3 PHYSICS LETTERS 16 June 1980

MEASUREMENT OF THE K^-/π^- STRUCTURE FUNCTION RATIO USING THE DRELL–YAN PROCESS

J. BADIER [d], J. BOUCROT [e], J. BOUROTTE [d], G. BURGUN [a], O. CALLOT [e], Ph. CHARPENTIER [a], M. CROZON [c], D. DÉCAMP [b], P. DELPIERRE [c], P. ESPIGAT [c], B. GANDOIS [a], R. HAGELBERG [b], M. HANSROUL [b], J. KARYOTAKIS [e], W. KIENZLE [b], P. Le DÛ [a], J. LEFRANÇOIS [e], Th. LERAY [c], J. MAILLARD [c], G. MATTHIAE [b], A. MICHELINI [b], Ph. MINÉ [d], G. RAHAL [a], O. RUNOLFSSON [b], P. SIEGRIST [a], A. TILQUIN [c], J. TIMMERMANS [b,1], J. VALENTIN [c], R. VANDERHAGHEN [d], S. WEISZ [b]

[a] *CEN-Saclay* – [b]*CERN, Geneva* – [c]*Collège de France, Paris* – [d]*Ecole Polytechnique, Palaiseau* – [e]*Laboratoire de l'Accélérateur Linéaire, Orsay*

Received 25 April 1980

The first measurement of the kaon to pion structure function ratio has been performed in a high integrated luminosity experiment studying the production of massive muon pairs. The ratio $\bar{u}_K/\bar{u}_\pi$ clearly deviates from 1 for values of $x_1 \geqslant 0.7$. A theoretical model comparison is also briefly discussed.

Valence u QDF ratio u_K/u_π

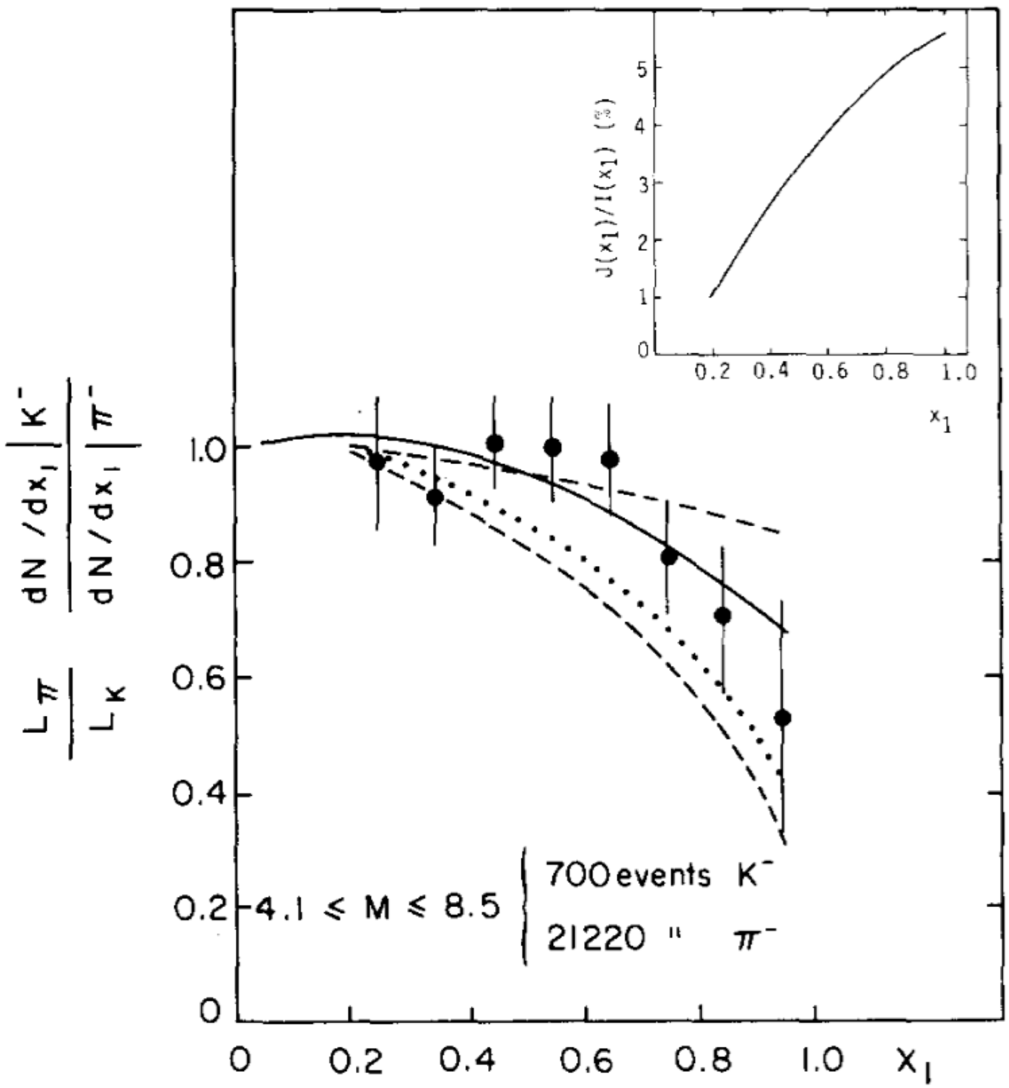
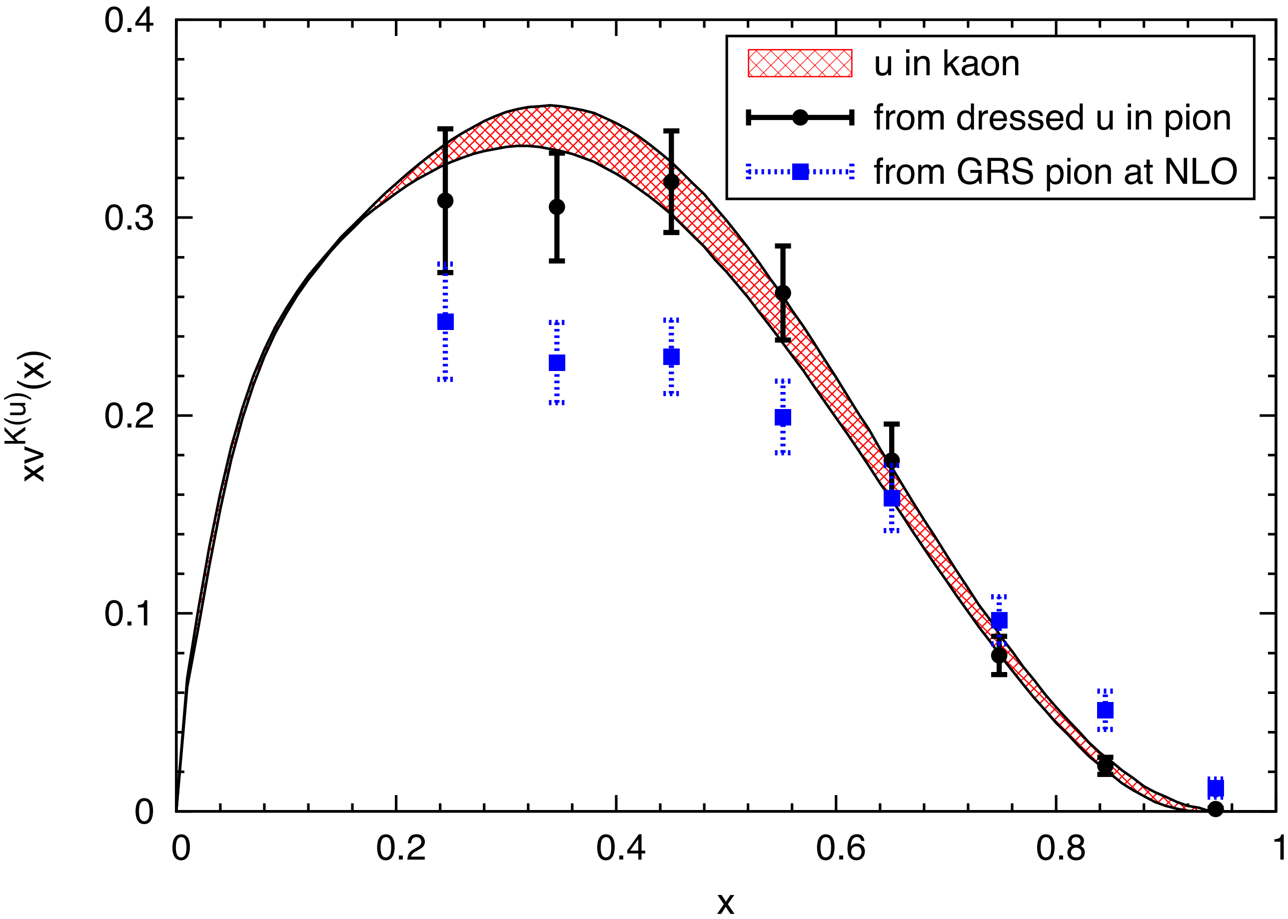


Fig. 2. The data points represent $(L_\pi/L_K)(dN/dx_1)_K/(dN/dx_1)_\pi$ as defined by eq. (4). The dashed curves represent the limits of the ratio $[\bar{u}_K(x_1)/\bar{u}_\pi(x_1)]C(x_1)^{-1}$ where $C(x_1)$ is defined in eq. (3), $\bar{u}_K/\bar{u}_\pi$ and $s_K/\bar{u}_K$ are taken from ref. [5], and the ratio $J(x_1)/I(x_1)$ is shown in the insert. The upper (lower) curve corresponds to $A = 1/8$ ($A = 1/2$). The dotted and solid curves represent the ratio $\bar{u}_K/\bar{u}_\pi$ from refs. [6] and [7], respectively.

Valence u QDF of K^+



Q^2=27GeV^2

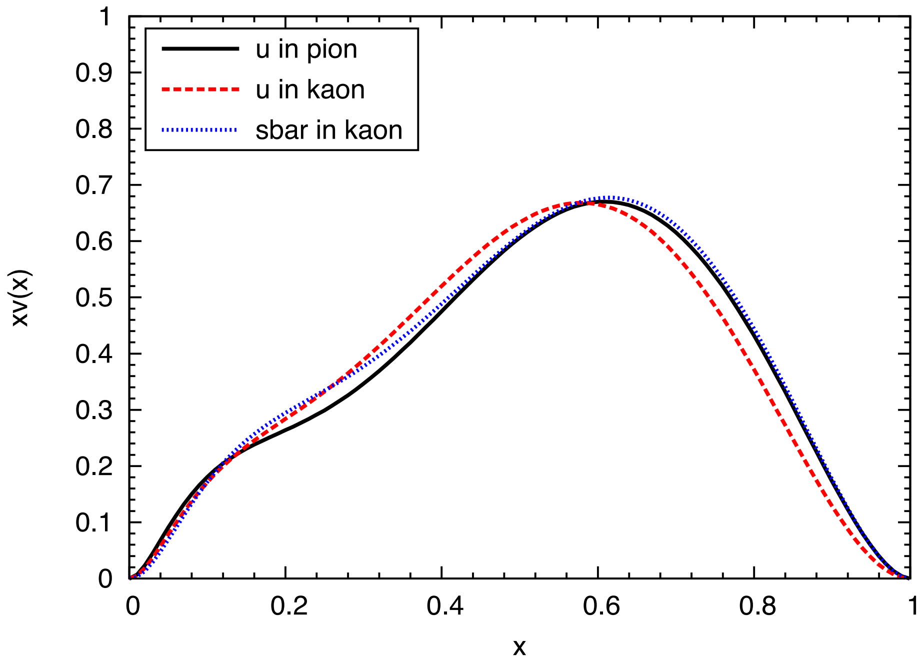
Red: $v^{K(u)}{}_{dressed}(x)$

Black: $v^{\pi}{}_{dressed}(x) * u_K/u_\pi$

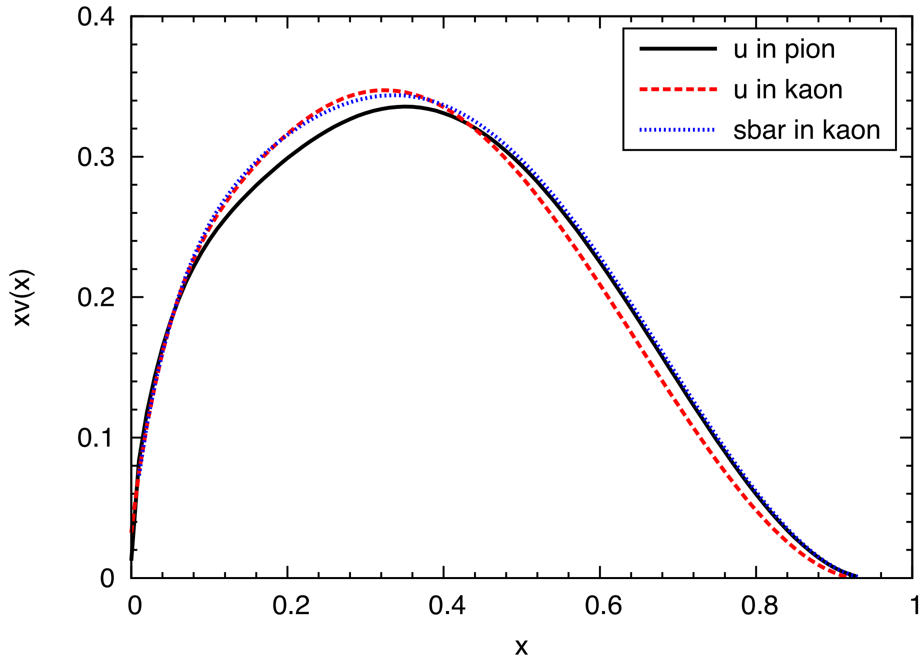
Blue: $GRS * u_K/u_\pi$

Resulting “dressed” VQDFs

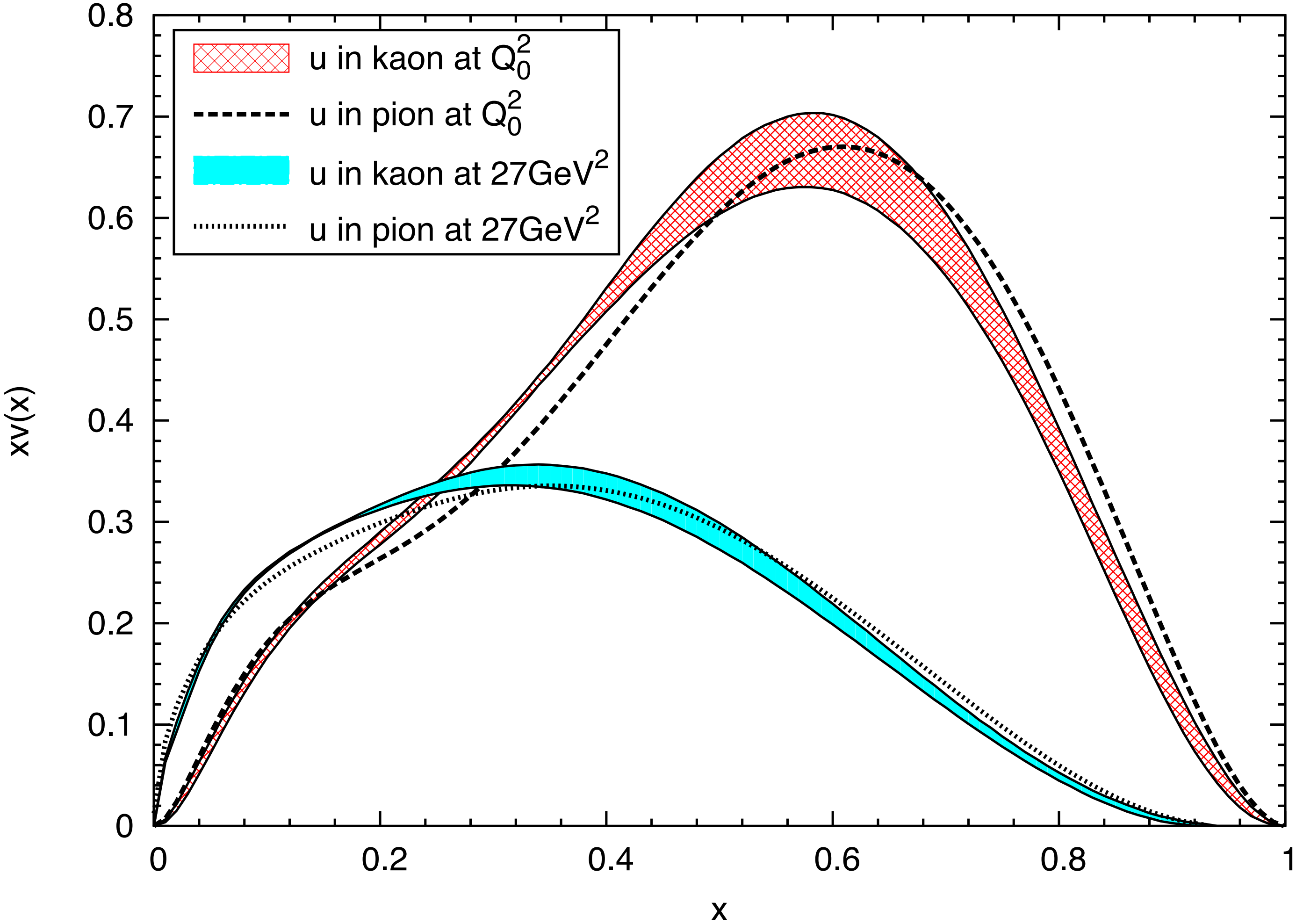
$Q^2 = Q_0^2$



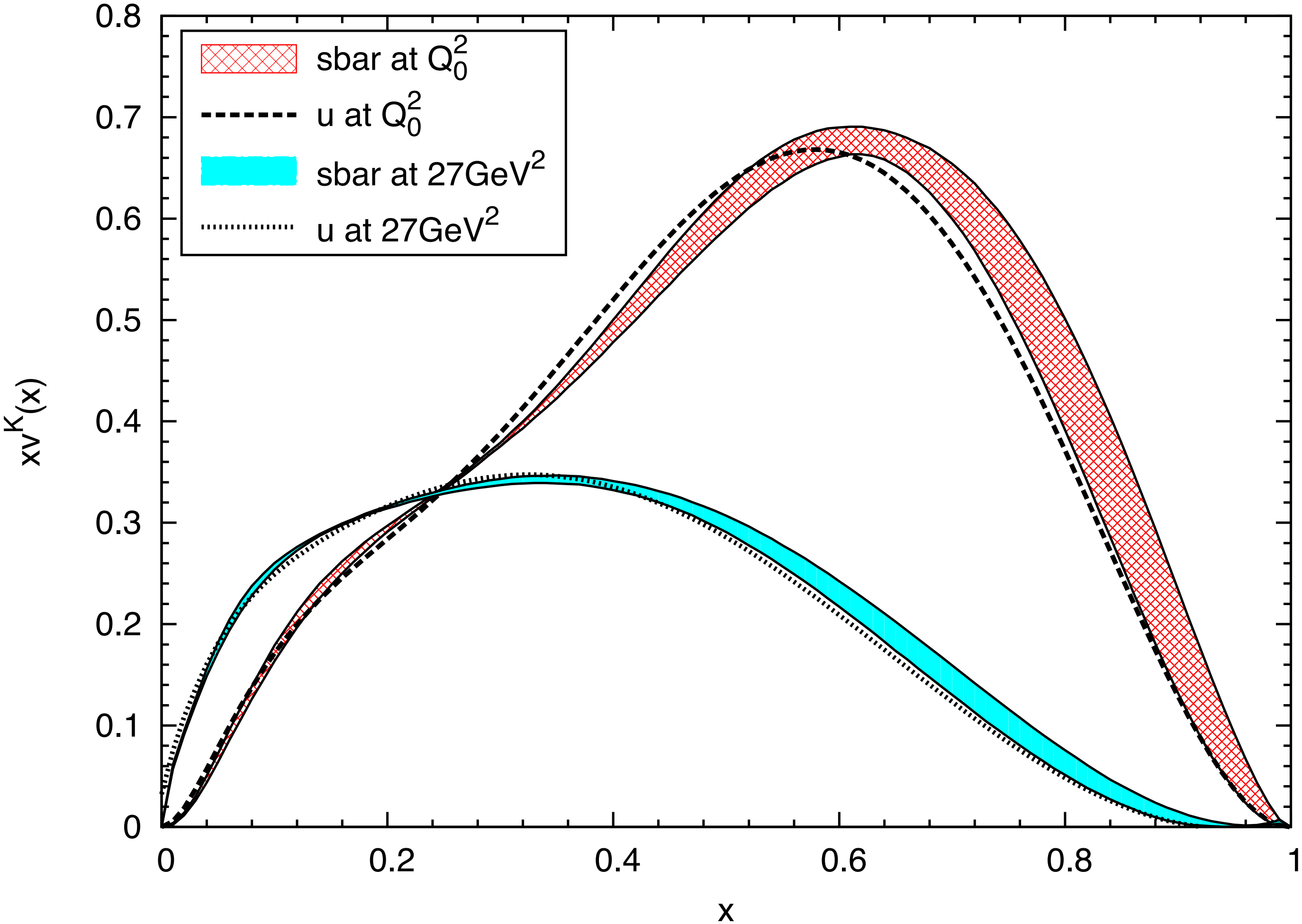
$Q^2 = 27GeV^2$



Resulting “dressed” valence *u* QDFs



Resulting "dressed" VQDFs of K^+



First three moments at Q^2=27GeV^2

q		$\langle x \rangle_q$	$\langle x^2 \rangle_q$	$\langle x^3 \rangle_q$
v^π	this work	0.23	0.094	0.048
	[24]	0.26	0.11	0.052
$v^{K(u)}$	this work	0.23	0.091	0.045
	[24]	0.28	0.11	0.048
$v^{K(\bar{s})}$	this work	0.24	0.096	0.049
	[24]	0.36	0.17	0.092

[24]: Chen-Chang-Roberts-Wan-Zong (2016)

Future plans

- Analysis with new data for the pion
  - ➢ New data of the pion-induced Drell-Yan process at COMPASS will be available soon
  - ➢ New pion PDF set
- Analysis for the kaon
  - ➢ Kaon-induced Drell-Yan experiments are planned at COMPASS and J-PARC
  - ➢ Kaon PDF set

Summary

- We have investigated VQDFs of the pion and kaon by evaluating the dressing corrections to the constituent quarks in the framework of the CCQM.
- We have shown that the suppression at large x for $v^{\pi}(x)$, which was found in the preceding works, can be well reproduced.
- Via analysis on the kaon, we have found a smaller SU(3) flavor symmetry breaking compared with results of the preceding studies.
- The discussed dressing corrections include corrections from very soft gluons which cannot be resummed in pQCD approaches.