

# Chiral EFT approach to **Nuclear Many-Body Systems**

- with implications for **Neutron Stars** -



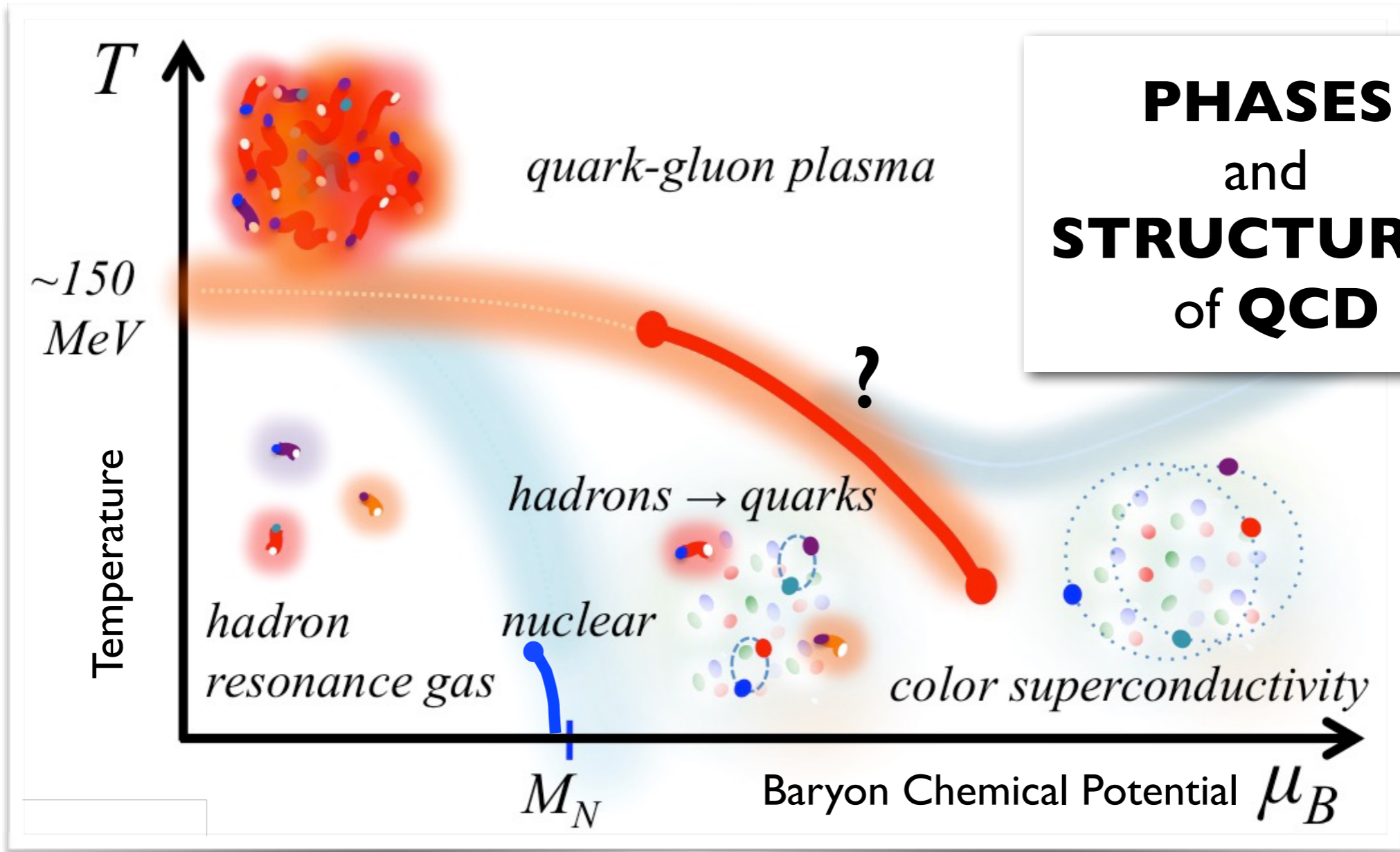
Wolfram Weise  
Technische Universität München



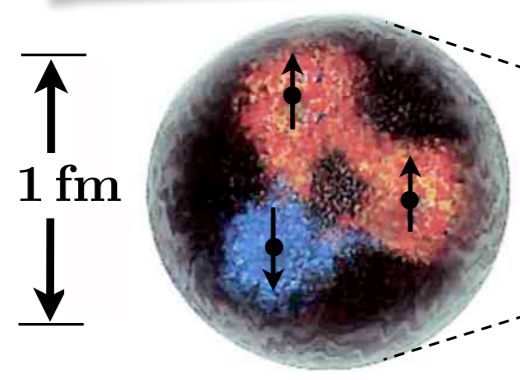
PHYSIK  
DEPARTMENT

- ★ **Introductory reminder of relevant scales**
- ★ **From nuclear to dense baryonic matter**
  - Nuclear chiral thermodynamics
  - In-medium chiral EFT and renormalisation group methods
  - Compressed baryonic matter and massive neutron stars
- ★ **Chiral SU(3) EFT: strangeness and baryonic matter**
  - Hyperon puzzle in neutron stars ?

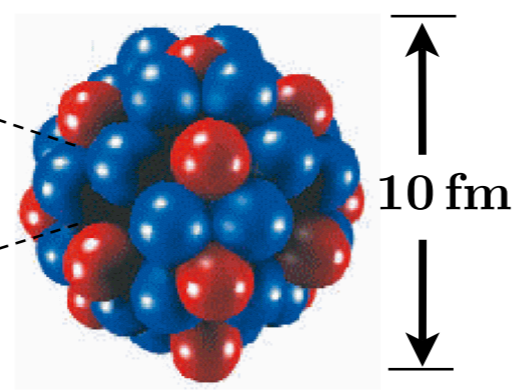
# PHASES and STRUCTURES of QCD



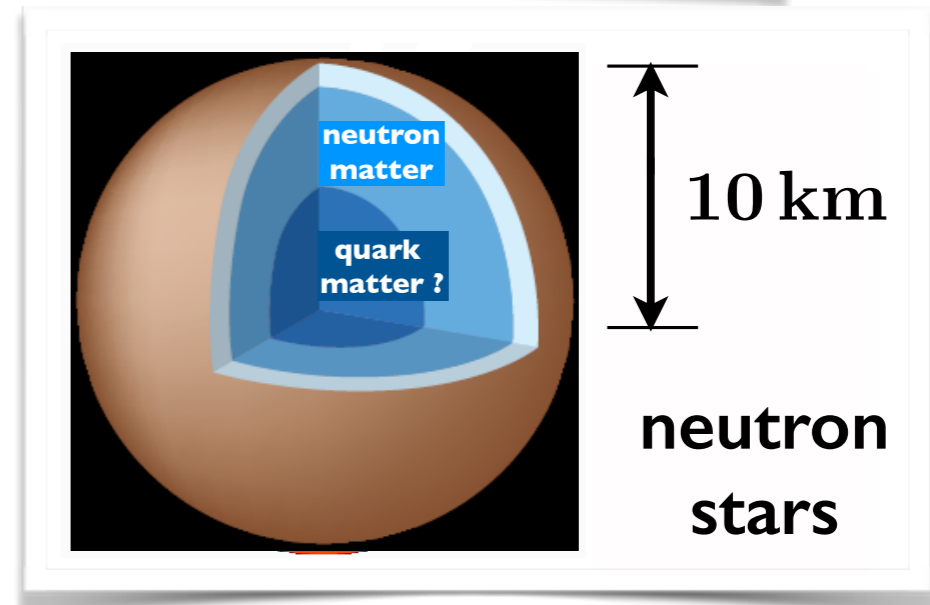
from:  
G. Baym,  
T. Hatsuda,  
et al.  
arXiv:1707.04966



nucleons



nuclei



# 1.

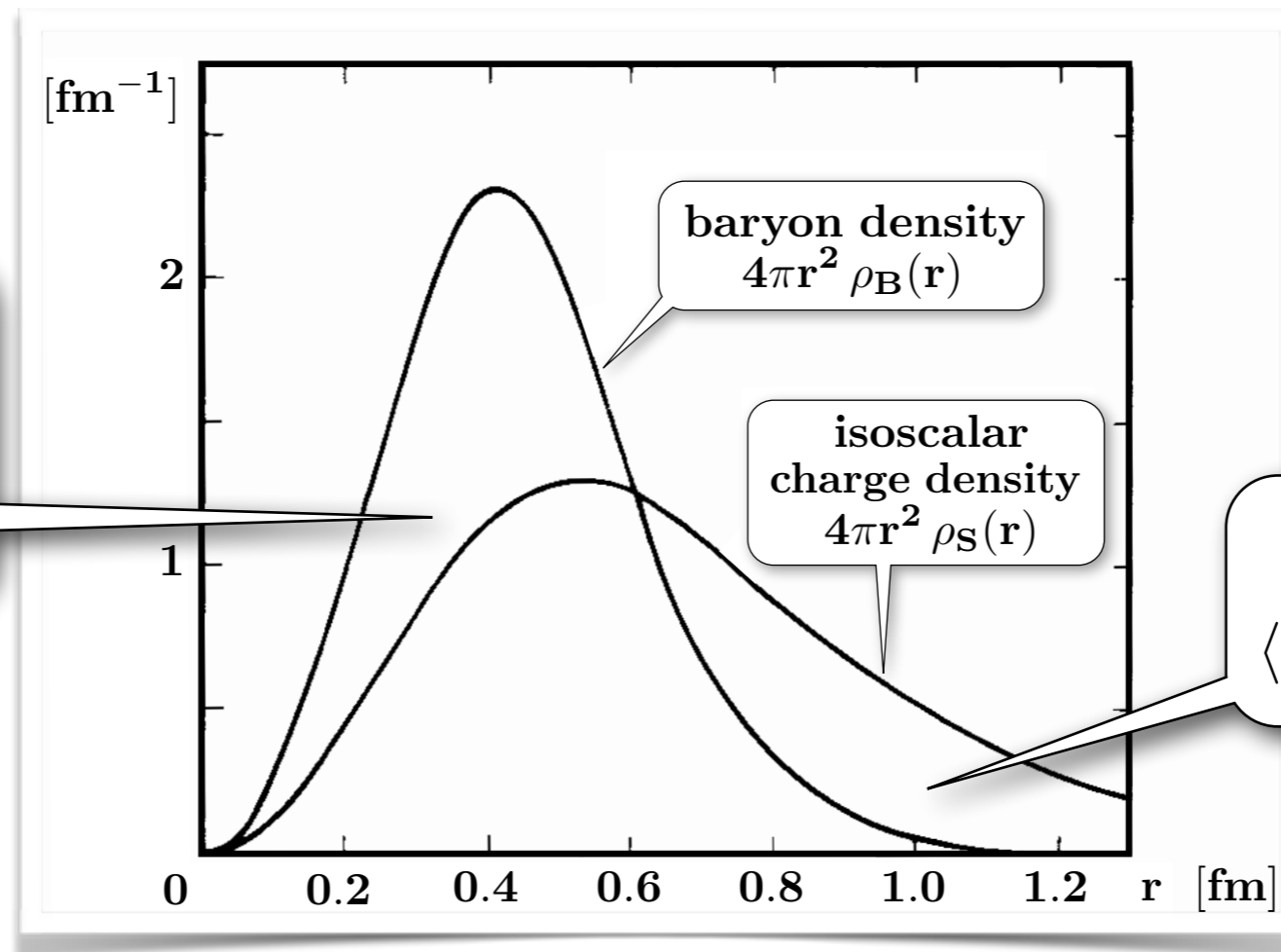
## *Introductory Notes about Scales*

- **Nucleon structure and its low energy realisation: valence quark core and  $q\bar{q}$  (mesonic) cloud**
- **Distance scales in dense baryonic matter**
- **Repulsive NN core from Lattice QCD**



# Historical Reminder: **CHIRAL SOLITON MODEL** of the **NUCLEON**

- ... based on non-linear sigma model + 4th order term
- Compact valence quark core + meson cloud



N. Kaiser,  
U.-G. Meißner, W.W.

Nucl. Phys.  
A466 (1987) 685

compact  
baryonic core  
 $\langle r^2 \rangle_B^{1/2} \simeq 0.5 \text{ fm}$

mesonic cloud  
 $\langle r^2 \rangle_{E, \text{isoscalar}}^{1/2} \simeq 0.8 \text{ fm}$

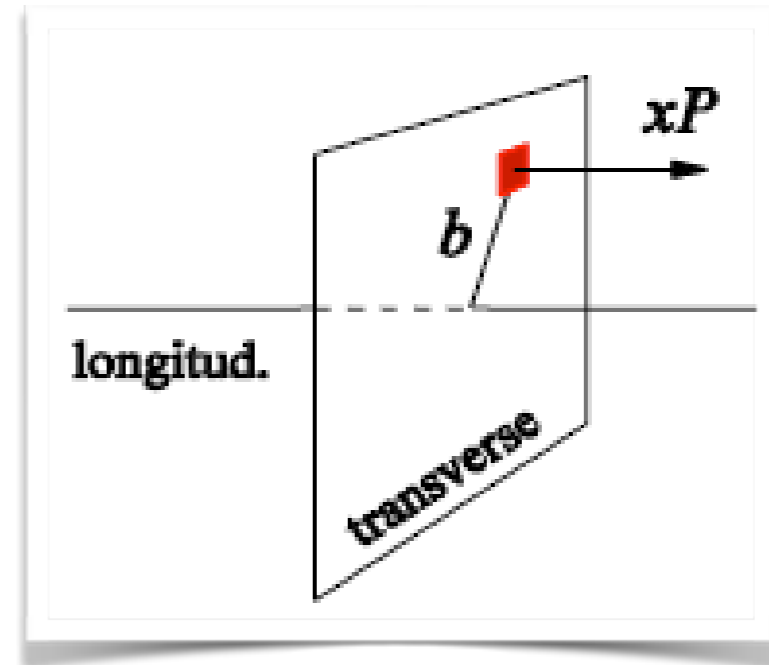
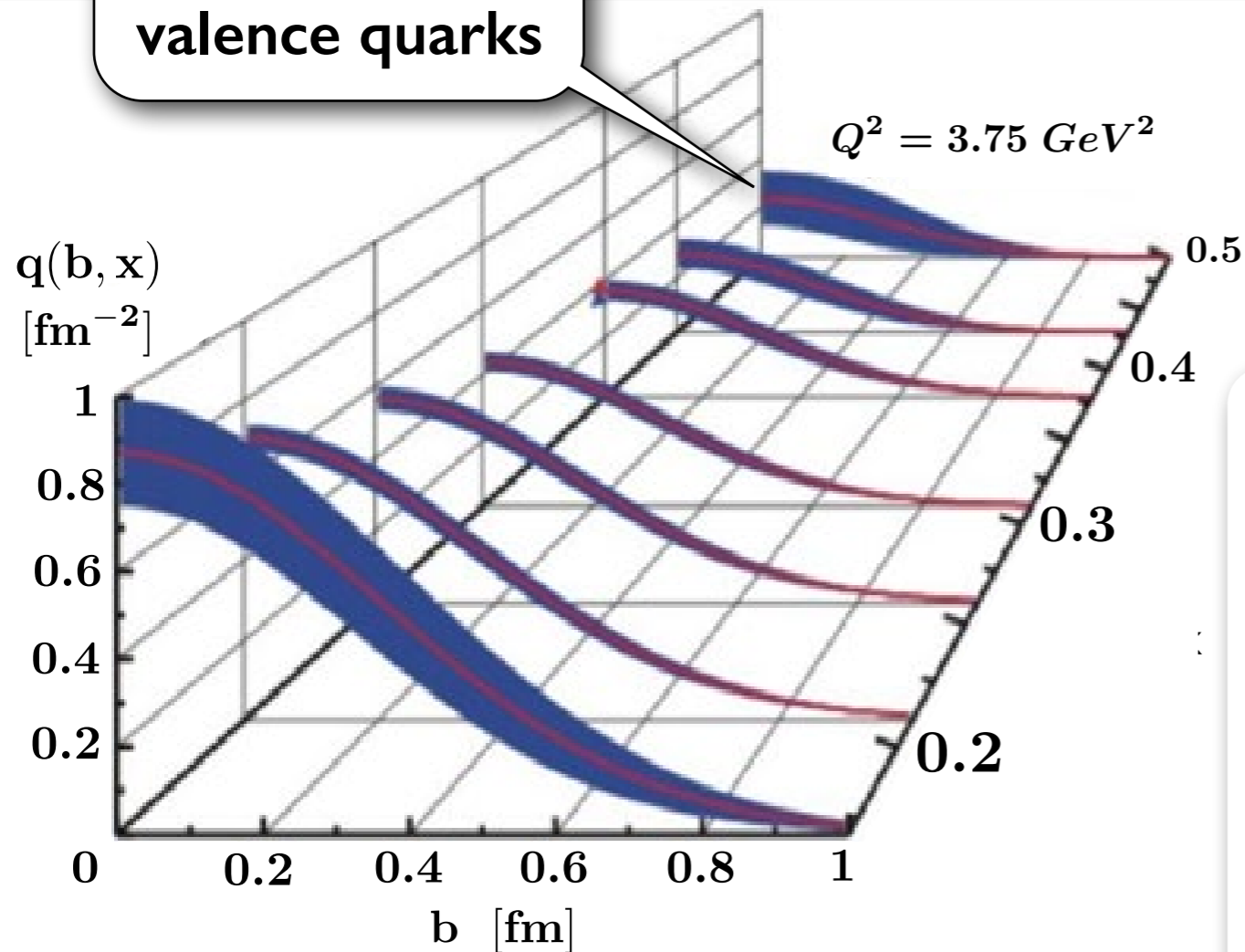
- Necessary condition for **Chiral EFT** approach to nuclear systems :  
**Separation of scales between compact baryonic core and (multi-)pion cloud**

# Transverse distributions of quarks in the proton

## Deeply Virtual Compton Scattering @ JLab

R. Dupré, M. Guidal, M. Vanderhaeghen  
Phys. Rev. D95 (2017) 011501

compact core:  
valence quarks

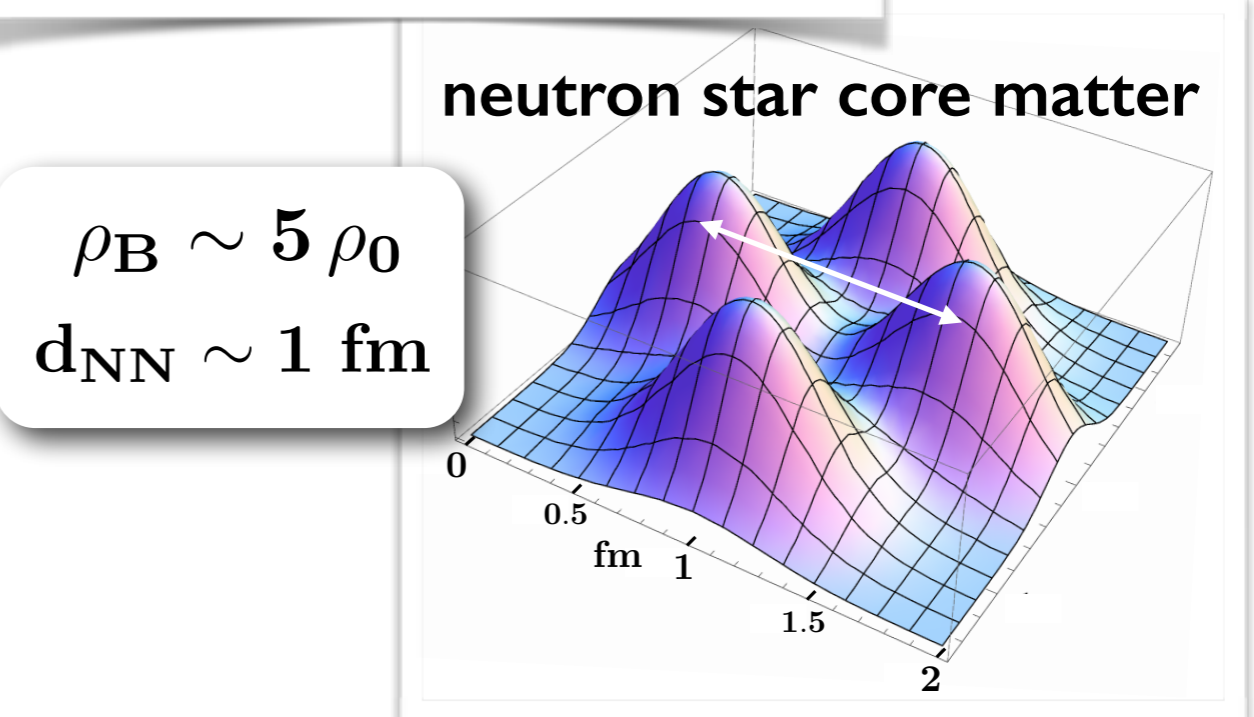
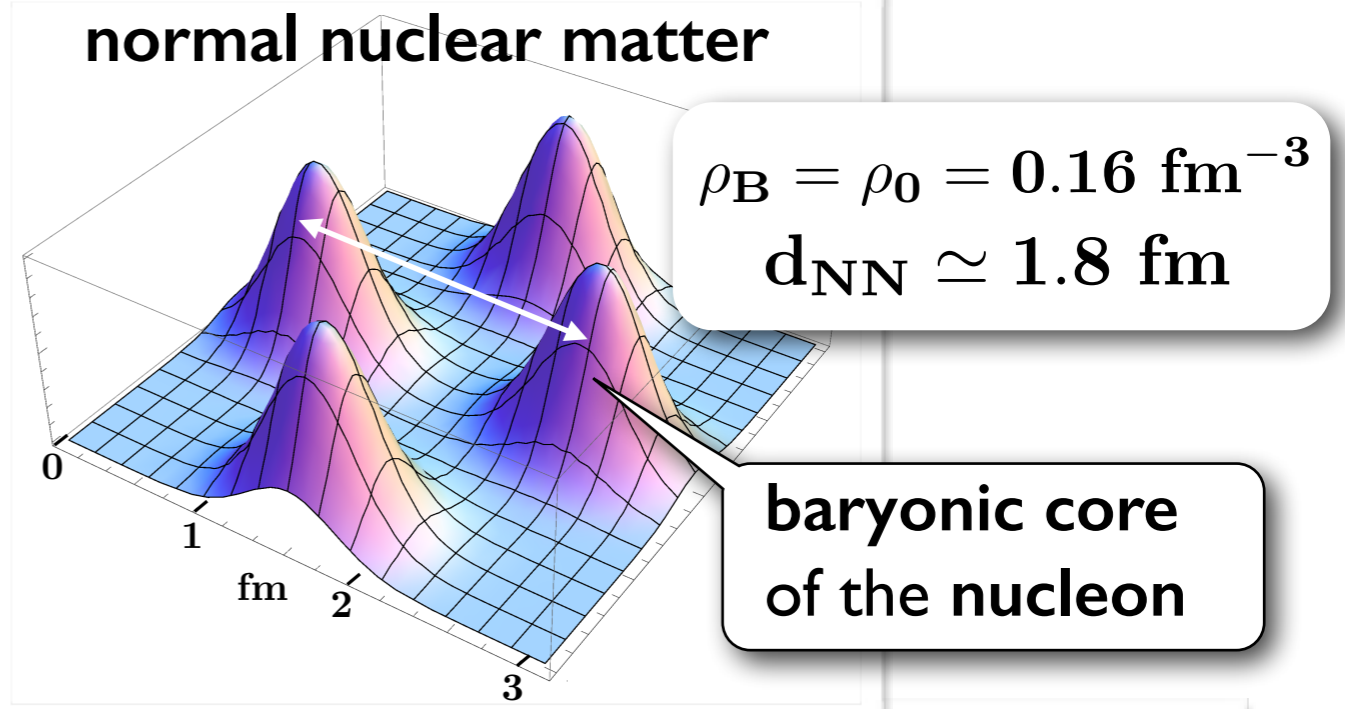
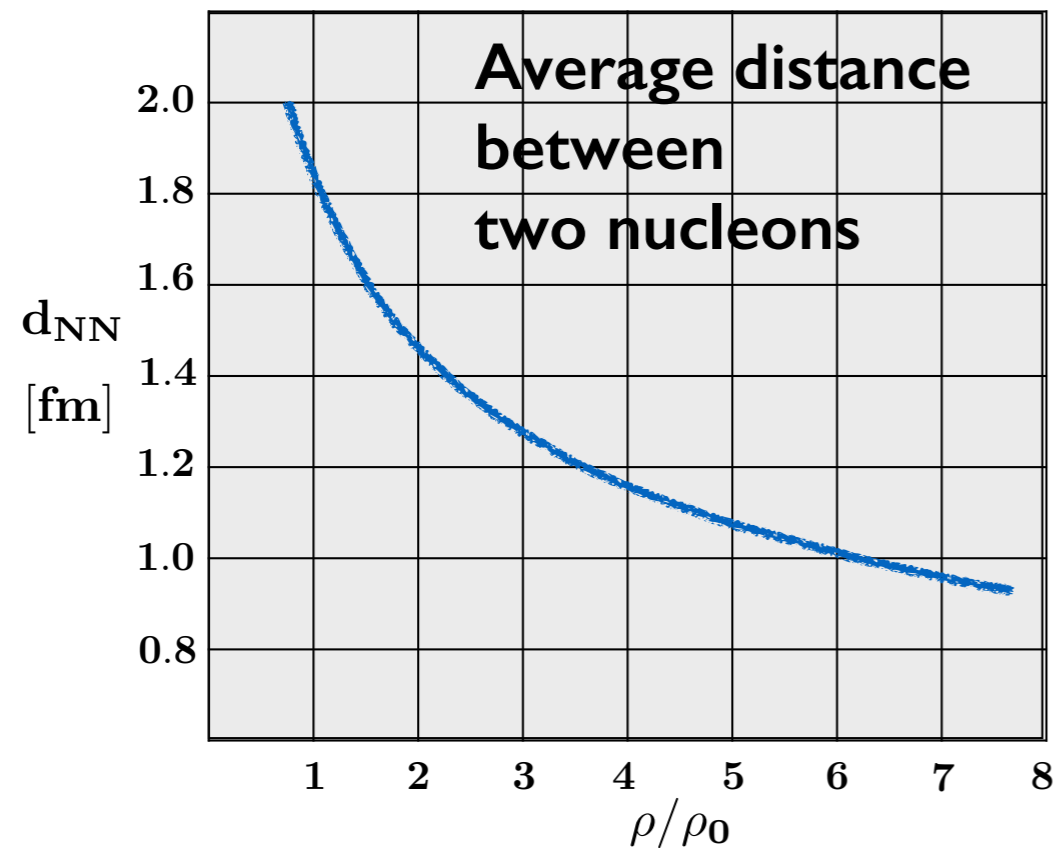


$$\langle b^2 \rangle \simeq 0.16 \text{ fm}^2 \cdot \ln(1/x)$$

- Valence quark region:  
 $0.3 < x < 0.5$
- Core size:

$$R_{core} = \sqrt{\frac{3}{2} \langle b^2 \rangle} \sim 0.4 - 0.5 \text{ fm}$$

# Densities and Distance Scales in Baryonic Matter



- (Multi-)pion fields in space between baryonic sources (ChEFT)
- Quark cores of nucleons overlap (percolate) at baryon densities

$$\rho_B > 5 \rho_0$$

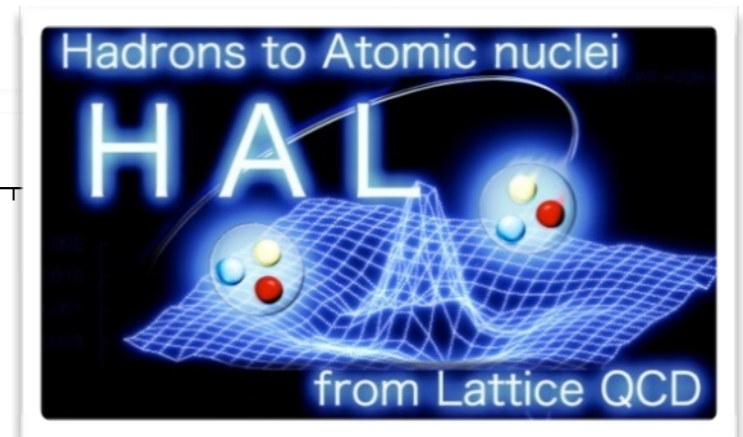
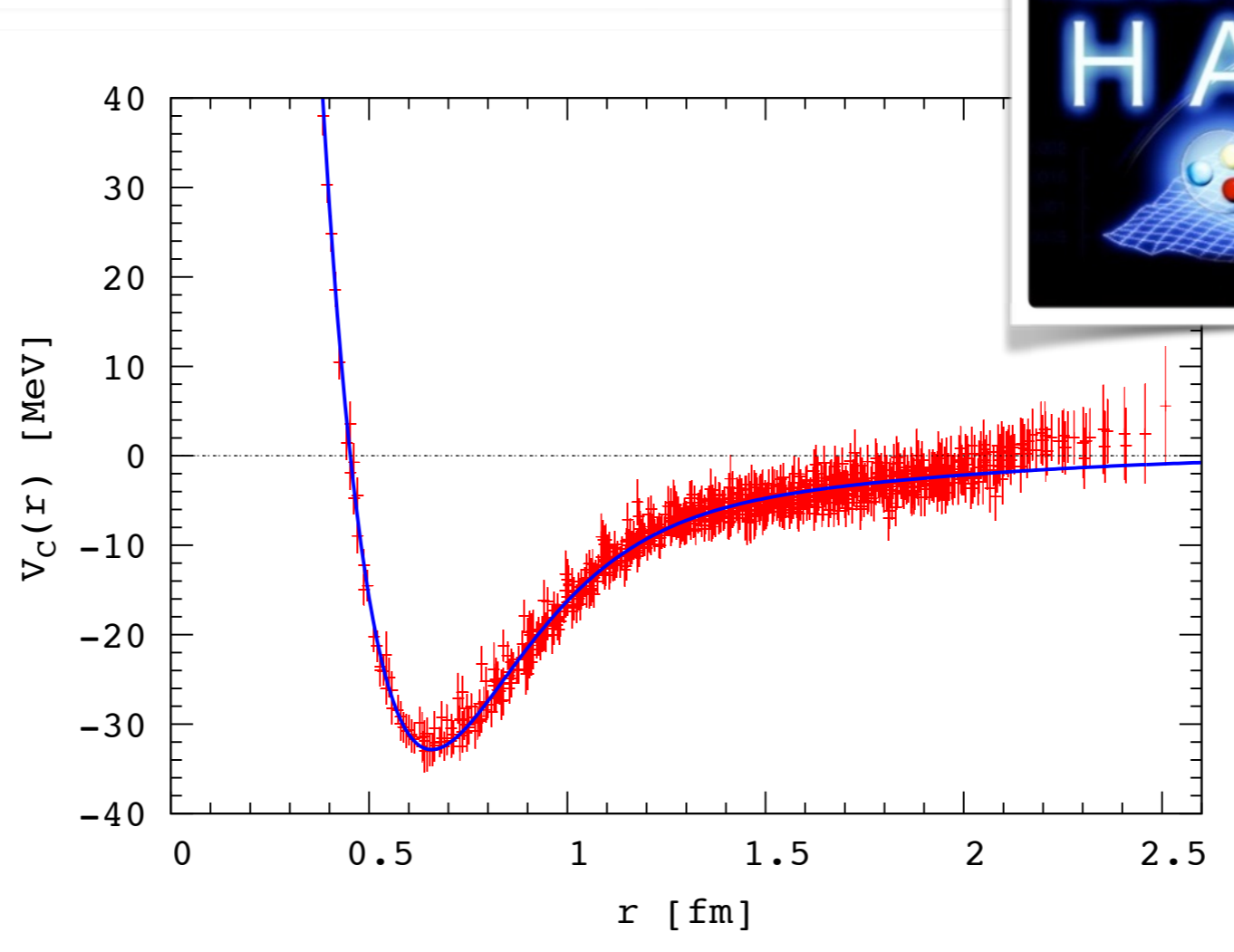
# NUCLEAR FORCES from LATTICE QCD

NN Central Potential ( $S = 0, l = 1$ )

deduced from LQCD two-nucleon (6-quark) correlation function

previously:  
unphysically large  
u- and d-quark  
masses, but :

stable 0.5 Fermi  
repulsive core



now:

towards  
physical  
quark  
masses

S.Aoki, T. Hatsuda, N. Ishii  
Prog. Theor. Phys. 123 (2010) 89

S.Aoki  
Eur. Phys. J. A49 (2013) 81

# 2.

## *Chiral Effective Field Theory and related approaches to Nuclear Many-Body Problems*

- **Chiral Nuclear Forces**
- **In-medium Chiral Perturbation Theory**
- **Non-perturbative methods :**
  - Energy Density Functional**
  - Functional Renormalization Group**





# **PIONS** and **NUCLEI** in the context of **LOW-ENERGY QCD**

- **CONFINEMENT** of quarks and gluons in hadrons
- Spontaneously broken **CHIRAL SYMMETRY**



**LOW-ENERGY QCD**  
at (energy and momentum) scales  
 $Q < 4\pi f_\pi \sim 1 \text{ GeV}$   
is realised as an  
**E**ffective **F**ield **T**heory of  
Nambu-Goldstone Bosons (**PIONS**)

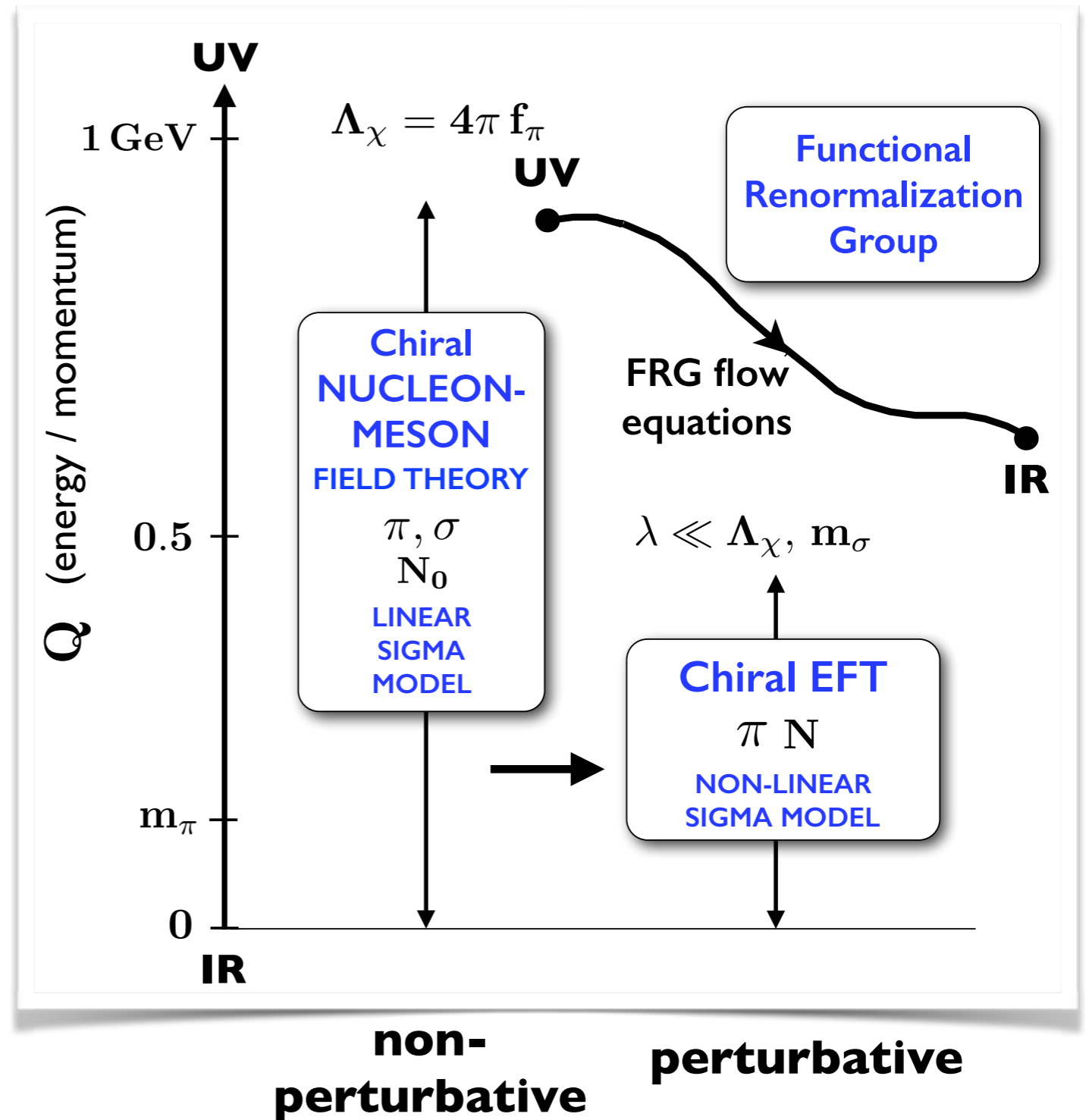


# Theoretical FRAMEWORKS and METHODS

Chiral  
Effective Field  
Theory

and

Functional  
Renormalization  
Group



# CHIRAL EFFECTIVE FIELD THEORY

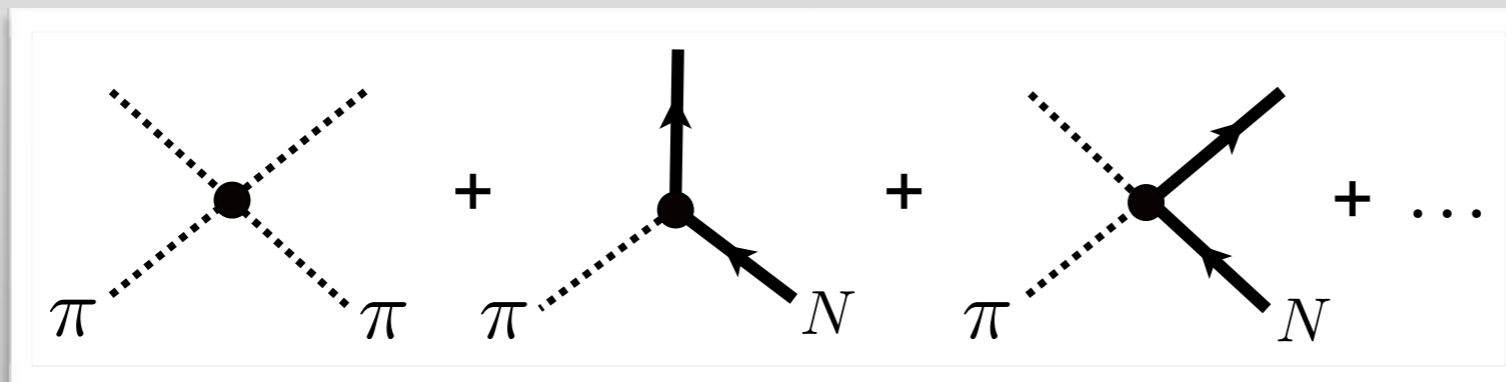
## Interface of QCD and Nuclear Physics

- Interacting systems of **PIONS** (light / fast) and **NUCLEONS** (heavy / slow):

$$\mathcal{L}_{eff} = \mathcal{L}_\pi(U, \partial U) + \mathcal{L}_N(\Psi_N, U, \dots)$$

$$U(x) = \exp[i\tau_a \pi_a(x) / f_\pi]$$

- Construction of Effective Lagrangian: **Symmetries**



short  
distance  
dynamics:  
contact terms

- Nambu-Goldstone Pions** : derivative couplings  $\propto \partial^\mu \pi(x)$


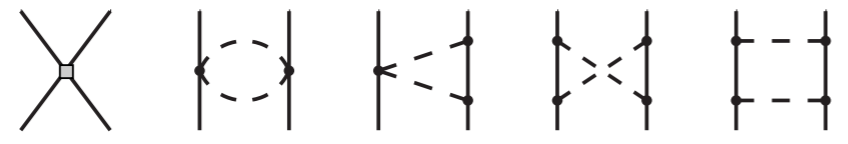
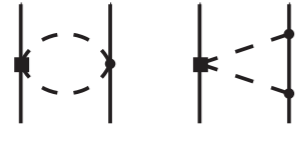
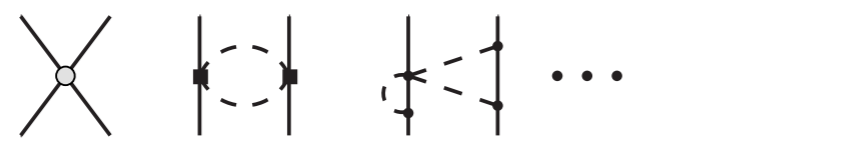
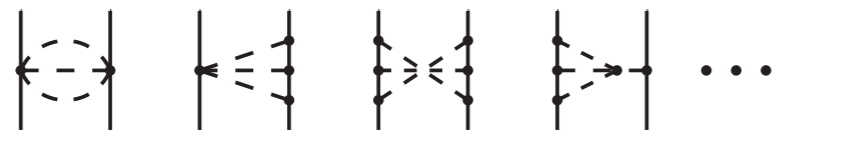
# NUCLEON-NUCLEON INTERACTION

## from CHIRAL EFFECTIVE FIELD THEORY

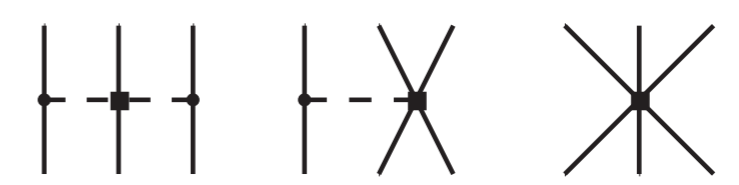
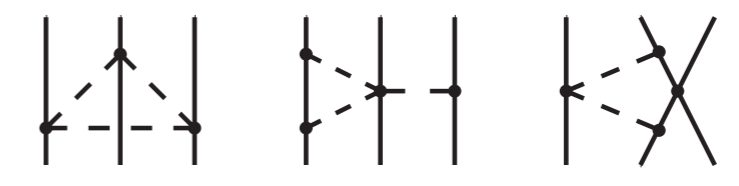
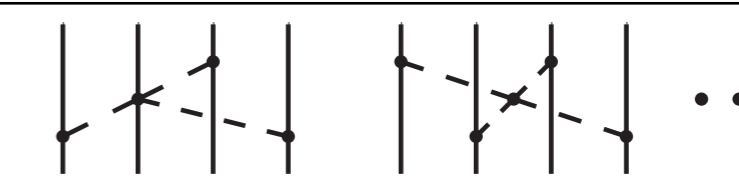
Weinberg

Bedaque & van Kolck

Bernard, Epelbaum, Kaiser, Meißner ; ...

	NN interaction
LO	
NLO	
N <sup>2</sup> LO	
N <sup>3</sup> LO	 

- Systematically organized hierarchy in powers of  $\frac{Q}{\Lambda}$  (Q: momentum, energy, pion mass)

3 – body forces	
N <sup>2</sup> LO	
N <sup>3</sup> LO	
4 – body forces	
N <sup>3</sup> LO	

- NN interaction state-of-the-art: N<sup>4</sup>LO plus convergence tests at N<sup>5</sup>LO



# Nuclear Many-Body Problem

## IN-MEDIUM CHIRAL PERTURBATION THEORY

- **Small scales:** energy, momentum,  $m_\pi$ ,  $k_F \ll 4\pi f_\pi \sim 1 \text{ GeV}$

- “Medium insertion” in the nucleon propagators:

$$(\gamma_\mu p^\mu + M_N) \left[ \frac{i}{p^2 - M_N^2 + i\epsilon} - 2\pi \delta(p^2 - M_N^2) \theta(p^0) \theta(k_F - |\vec{p}|) \right]$$



Loop expansion of (In-Medium) Chiral Perturbation Theory

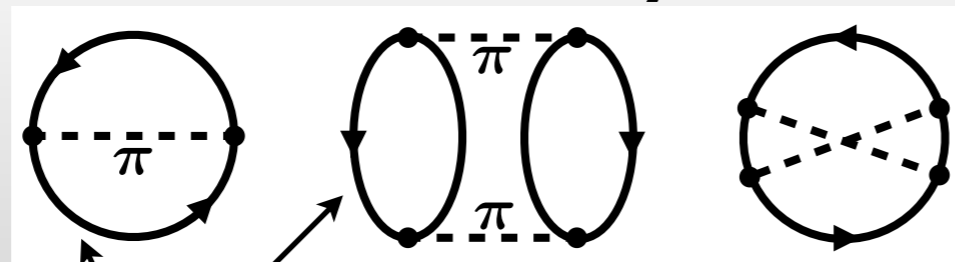
↔ Expansion of **ENERGY DENSITY**  $\mathcal{E}(k_F)$  in powers of Fermi momentum [modulo functions  $f_n(k_F/m_\pi)$ ]



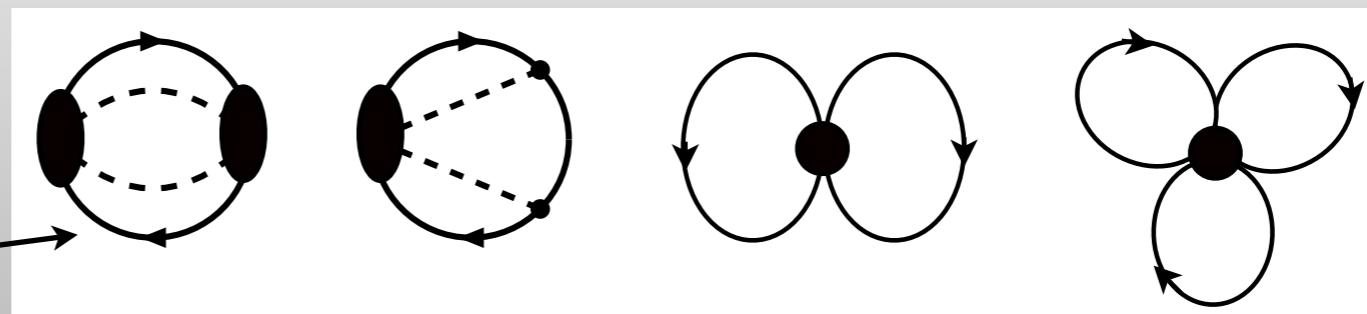
Nuclear thermodynamics: free energy density

(3-loop order)

N. Kaiser, S. Fritsch, W.W. (2002-2005)

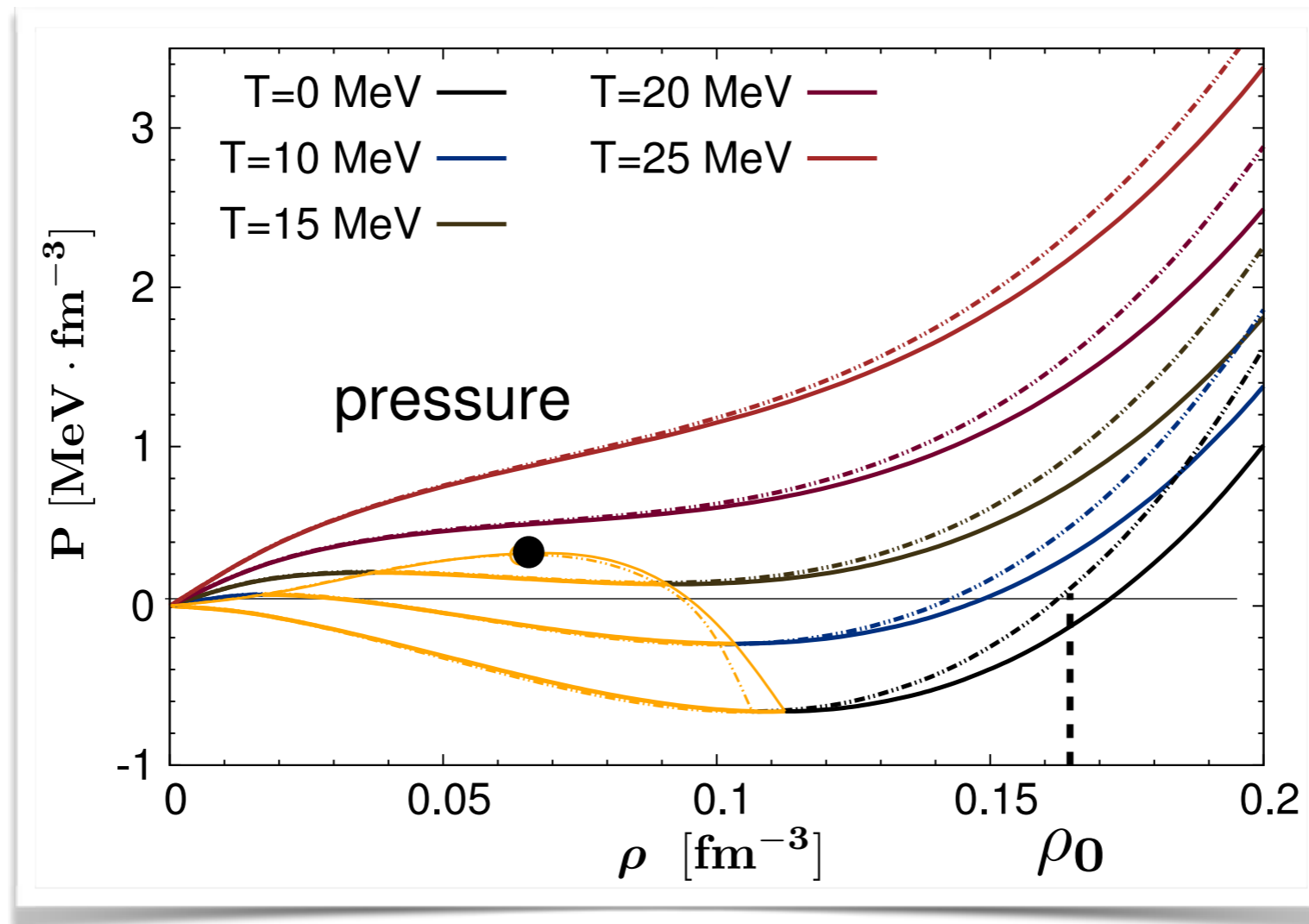


in-medium nucleon propagators incl. Pauli blocking



# NUCLEAR THERMODYNAMICS from CHIRAL EFT

- Symmetric nuclear matter : 1st order **liquid-gas phase transition**
- N3LO chiral NN interactions + N2LO 3-body forces



C.Wellenhofer,  
J.W.Holt,  
N.Kaiser, W.W.  
Phys. Rev.  
C89 (2014) 064009  
C92 (2015) 015801

**Critical  
temperature  
of  
liquid-gas  
first-order  
transition :**

$$T_c = 17.4 \text{ MeV}$$

► **Empirical position of liquid-gas critical point :** J. B. Elliot et al. : Phys. Rev. C87 (2013) 054622

$$T_c = 17.9 \pm 0.4 \text{ MeV} \quad P_c = 0.31 \pm 0.07 \text{ MeV} \cdot \text{fm}^{-3} \quad \rho_c = 0.06 \pm 0.01 \text{ fm}^{-3}$$

# ISOSPIN-ASYMMETRIC MATTER and SYMMETRY ENERGY

- Energy per particle :  $E(\rho, \eta) = E(\rho, \eta = 0) + S(\rho) \eta^2 + \dots$       $\eta = \frac{\rho_n - \rho_p}{\rho}$
- Symmetry energy :

$$S(\rho) = E_n(\rho) - E(\rho)$$

neutron  
matter

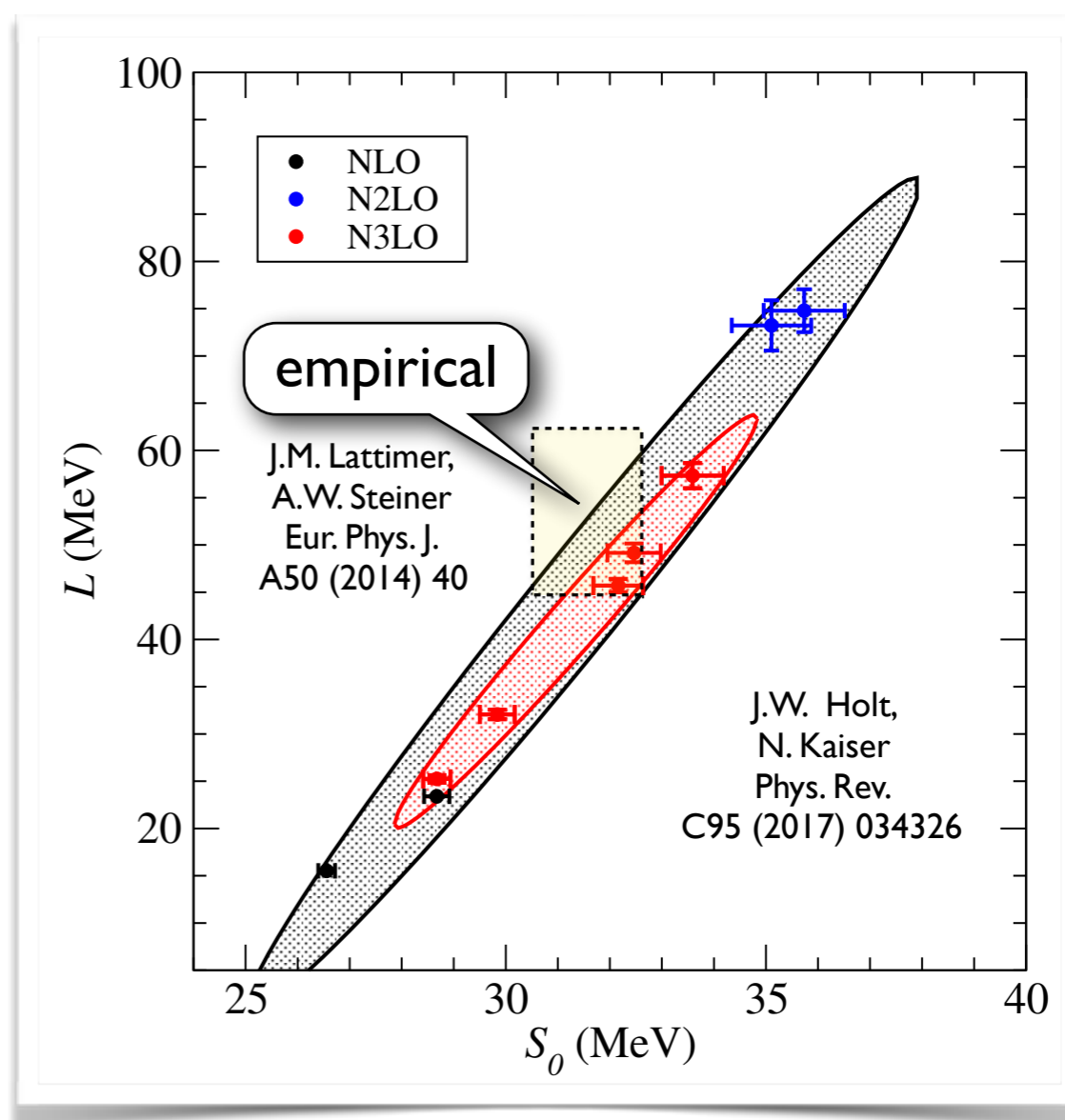
symmetric  
nuclear matter

$$S_0 \equiv S(\rho = \rho_0)$$

$$L = 3\rho \left. \frac{\partial S(\rho)}{\partial \rho} \right|_{\rho_0}$$

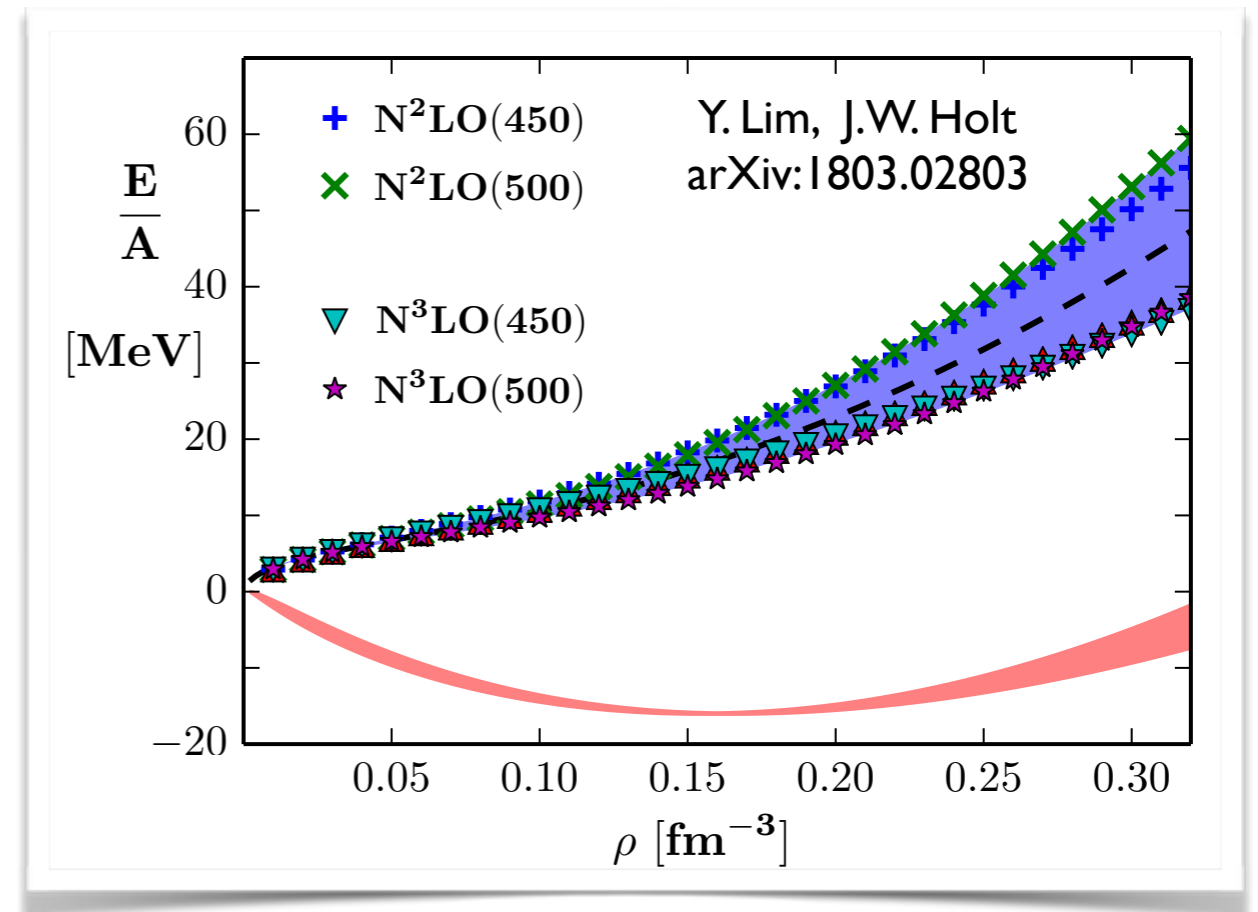
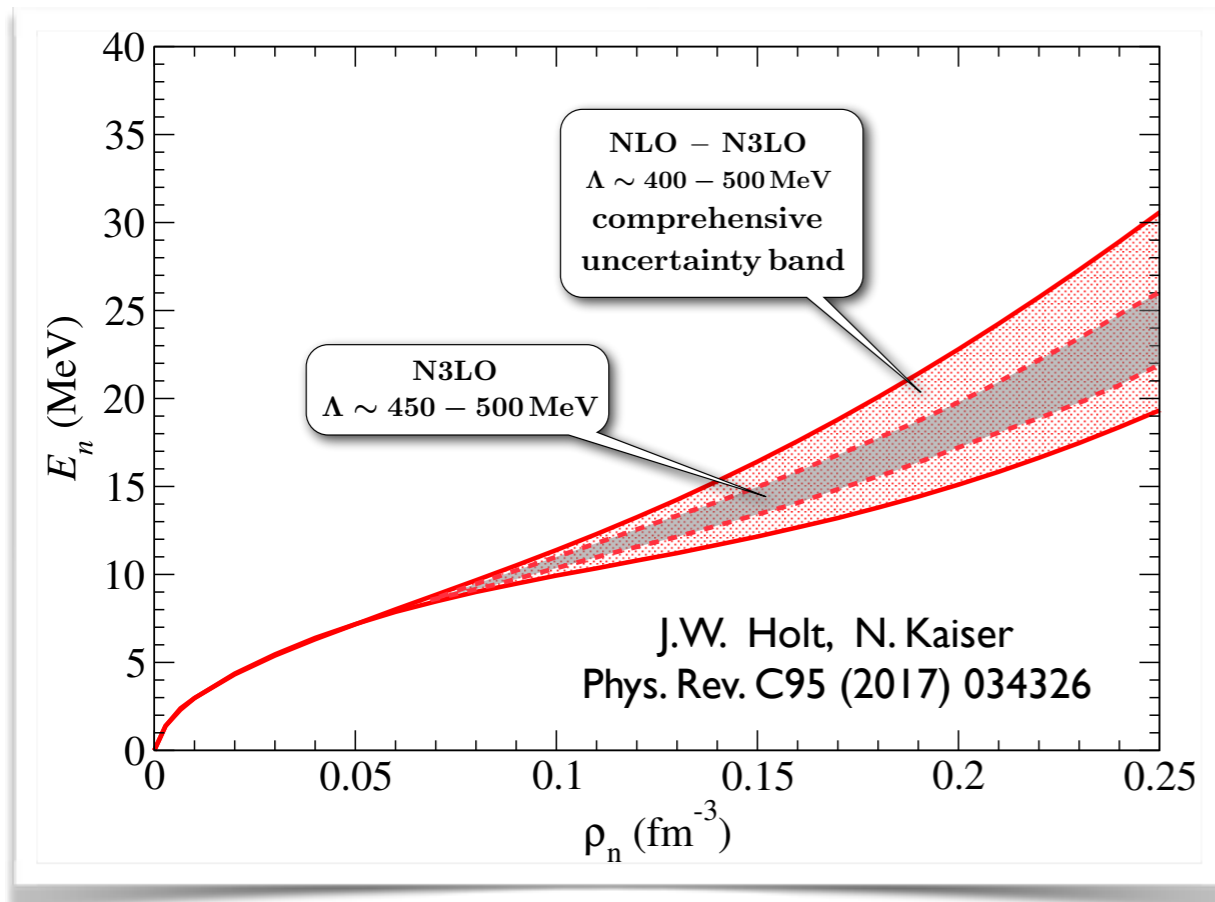
- Key quantities for neutron-rich nuclei and astrophysics

## N3LO chiral NN interactions + N2LO 3-body forces



# NEUTRON and NUCLEAR MATTER from CHIRAL EFT

- N3LO chiral NN interactions + N2LO 3-body forces
- Many-body perturbation theory (3rd order)



- Agreement with advanced many-body calculations (e.g. Quantum Monte Carlo computations - S. Gandolfi et al.: EPJ A50 (2014) 10)

C.Wellenhofer, J.W. Holt, N. Kaiser, W.W.: Phys. Rev. C89 (2014) 064009, C92 (2015) 015801

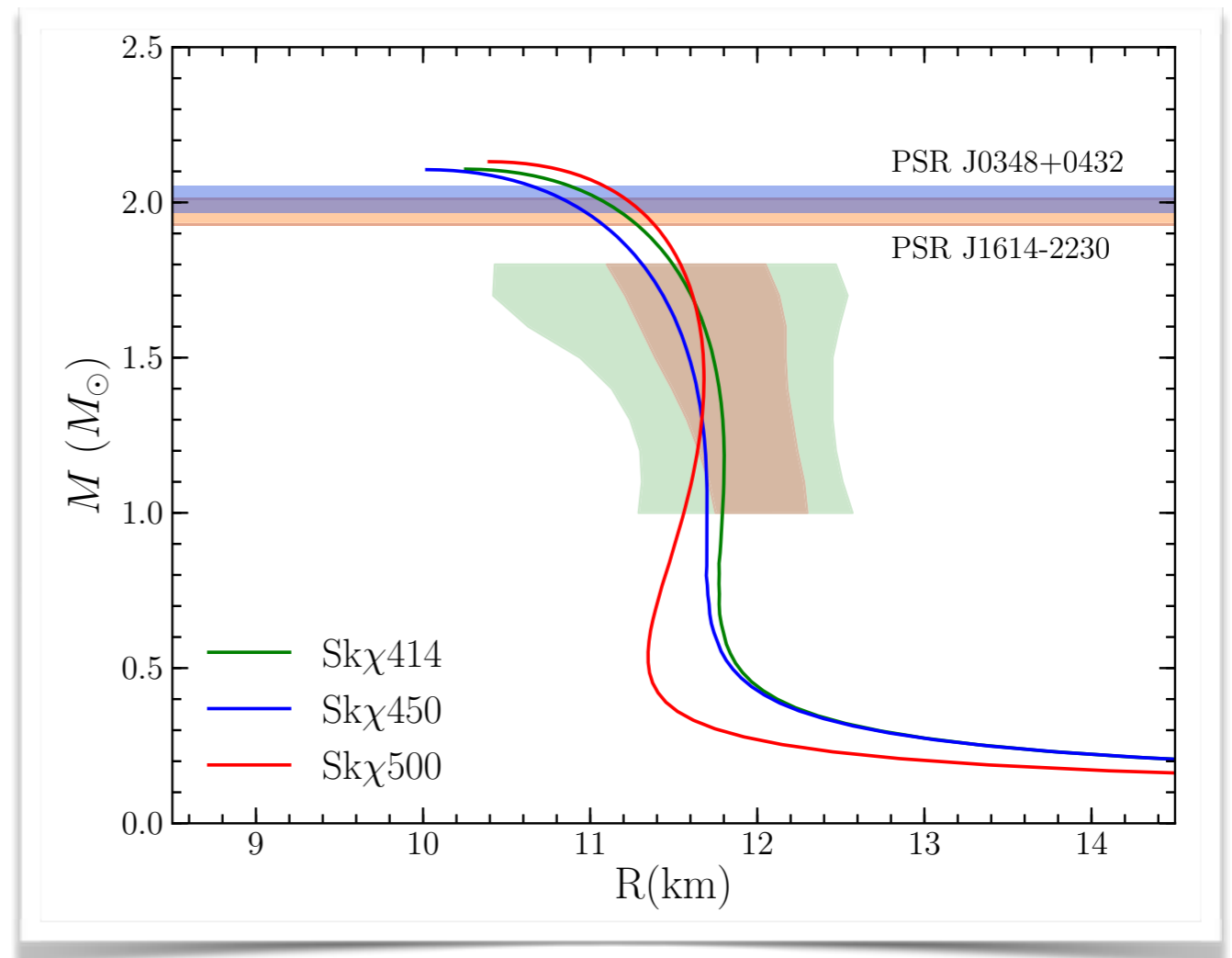
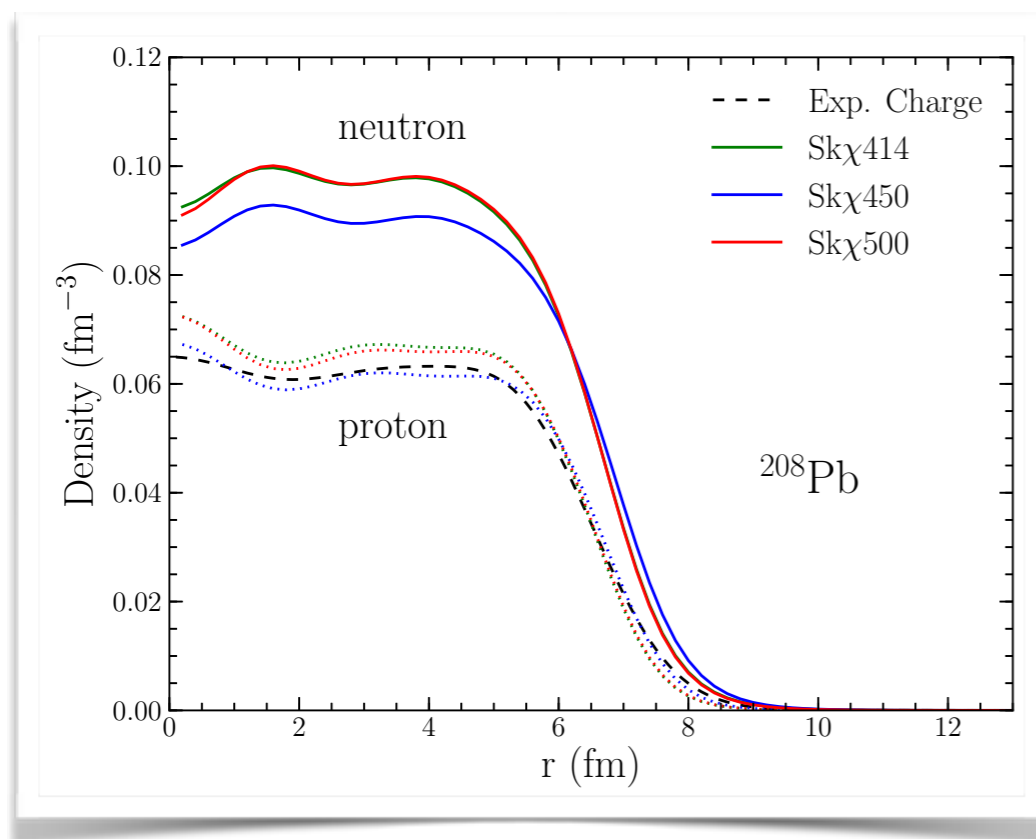


# NEUTRON STAR MATTER

- Energy density functional (Skyrme-Hartree-Fock) deduced from Chiral Effective Field Theory  
N3LO two-body interactions, N2LO three-body forces  
density dependence consistent with ChEFT expansion in powers of Fermi momenta

Y. Lim, J.W. Holt Phys. Rev. C95 (2017) 065805

- successfully reproduces properties of finite nuclei ...

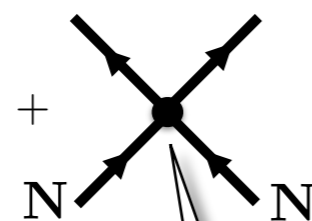


... and neutron star crust together with  $2 M_{\odot}$  constraint

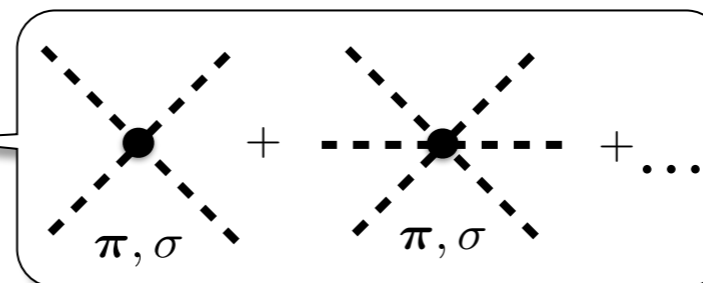
# Mesons, Nucleons, Nuclear Matter and Functional Renormalization Group

- Chiral nucleon - meson Lagrangian

$$\mathcal{L} = \bar{\mathbf{N}} i \gamma_\mu \partial^\mu \mathbf{N} + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}) + \dots$$



$-\mathcal{U}(\pi, \sigma)$



isoscalar & isovector  
current-current interactions

- Nambu-Goldstone boson  $\pi$  and “heavy”  $\sigma$
- Potential  $\mathcal{U}(\sigma, \pi)$  constructed to reproduce vacuum physics and nuclear matter at equilibrium
- Pionic fluctuations and nucleonic particle-hole excitations treated non-perturbatively using **FRG**

M. Drews, T. Hell, B. Klein, W.W. Phys. Rev. D88 (2013) 096011

M. Drews, W.W. Phys. Lett. B738 (2014) 187 Phys. Rev. C91 (2015) 035802

Review: M. Drews, W.W. Prog. Part. Nucl. Phys. 91 (2017) 347

# Renormalization Group strategies

k-dependent action

full propagator

## Wetterich's FRG flow equations

$$k \frac{\partial \Gamma_k[\Phi]}{\partial k} = \frac{1}{2} \text{Tr} \left[ k \frac{\partial R_k}{\partial k} \cdot \left( \Gamma_k^{(2)}[\Phi] + R_k \right)^{-1} \right] = \text{diagram}$$

$$\Gamma_{k=\Lambda}[\Phi] = S$$

$$\Gamma_k[\Phi]$$

scale regulator  $R_k$

$$\Gamma_{k=0}[\Phi] = \Gamma[\Phi]$$

C. Wetterich:  
Phys. Lett. B 301 (1993) 90

### ● Thermodynamics:

$$k \partial_k \bar{\Gamma}_k(T, \mu) = \left( \text{diagram}_1 + \text{diagram}_2 \right) \Big|_{T, \mu} - \left( \text{diagram}_3 + \text{diagram}_4 \right) \Big|_{T=0, \mu=\mu_c}$$

nucleons
pions

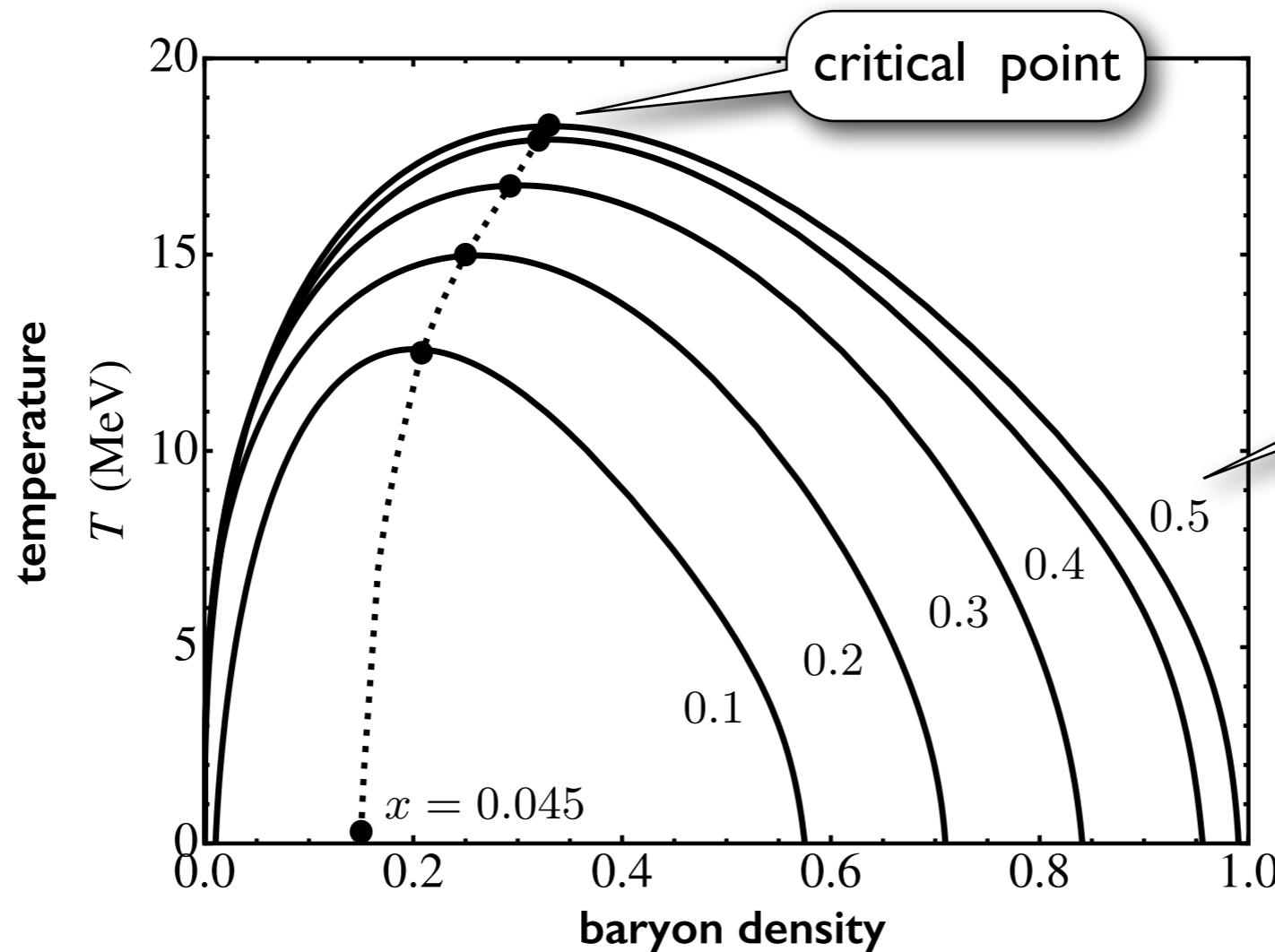
### Non-perturbative treatment of :

- multi-pion exchange processes
- nucleon-hole excitations
- multi-nucleon correlations

# PHASE DIAGRAM of NUCLEAR MATTER

- Trajectory of **CRITICAL POINT** of Liquid - Gas transition for asymmetric matter as function of proton fraction  $Z / A$

S. Fiorilla,  
N. Kaiser, W.W.  
Nucl. Phys.  
A 880 (2012) 65



M. Drews, T. Hell, B. Klein, W.W.  
Phys. Rev. D 88 (2013) 096011

M. Drews, W.W.  
Phys. Lett. B 738 (2014) 187  
Phys. Rev. C 91 (2015) 035802

... determined almost completely by isospin dependent (one- and two-) pion exchange dynamics

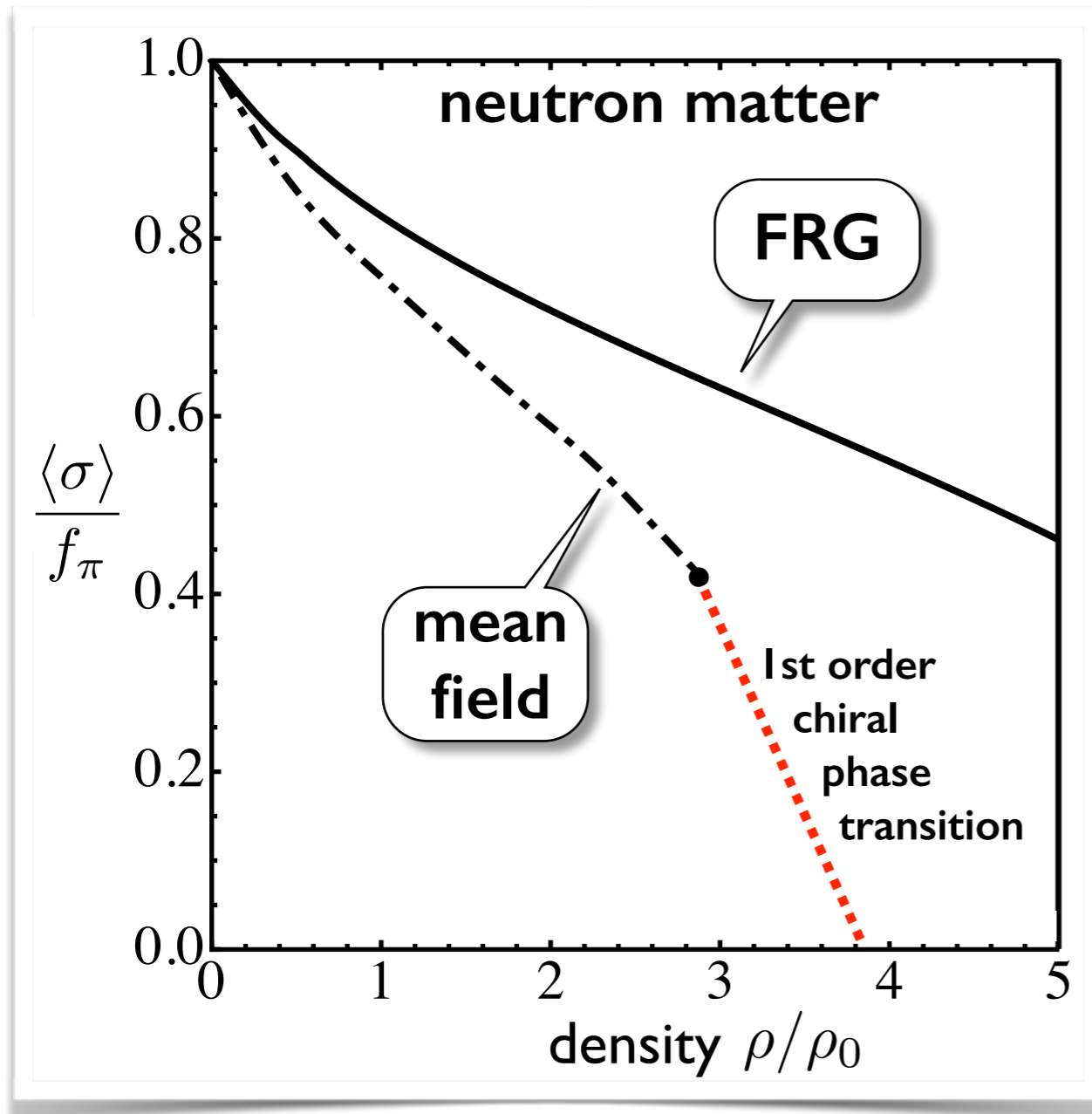
# CHIRAL ORDER PARAMETER in NEUTRON MATTER

- Chiral Nucleon-Meson field theory and **F**unctional **R**enormalization **G**roup

M. Drews, W.W.

Phys. Rev. C91 (2015) 035802

Prog. Part. Nucl. Phys. 93 (2017) 69



- Chiral order parameter :  
sigma field  
 $\updownarrow$   
in-medium pion decay constant

$$\langle \sigma \rangle_\rho = f_\pi^*(\rho)$$

Important role of **fluctuations**  
(pionic and nucleon-hole)  
beyond mean-field approximation:

**DISAPPEARANCE** of  
**first-order**  
**chiral phase transition**

# NEUTRON STAR MATTER

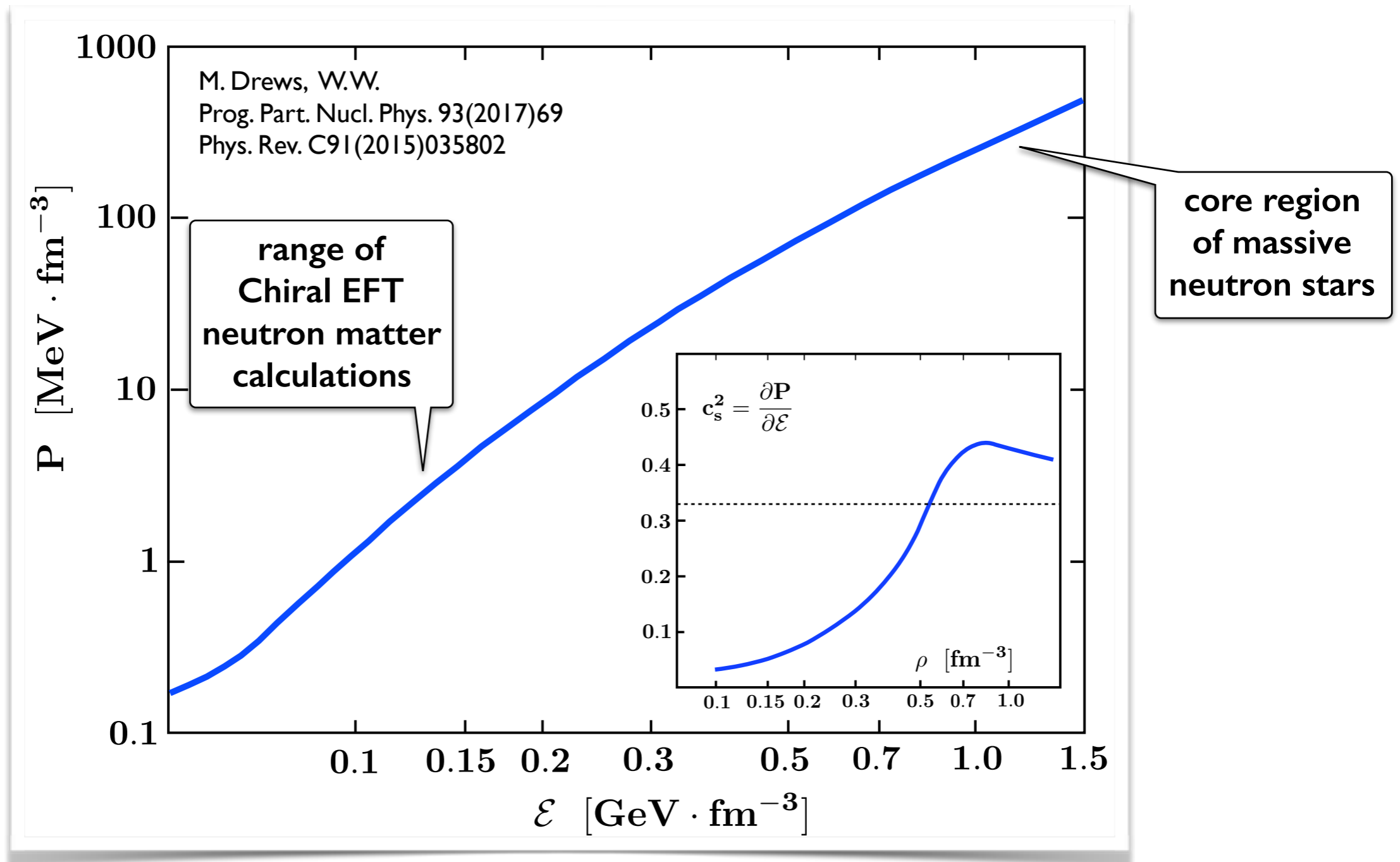
## Equation of State

- Chiral **FRG** calculations with inclusion of beta equilibrium

M. Drews, W.W.

Phys. Rev. C91 (2015) 035802

Prog. Part. Nucl. Phys. 93 (2017) 69



# NEUTRON STAR MATTER

## from **Chiral EFT** and **FRG**

● Symmetry energy range: 30 - 35 MeV

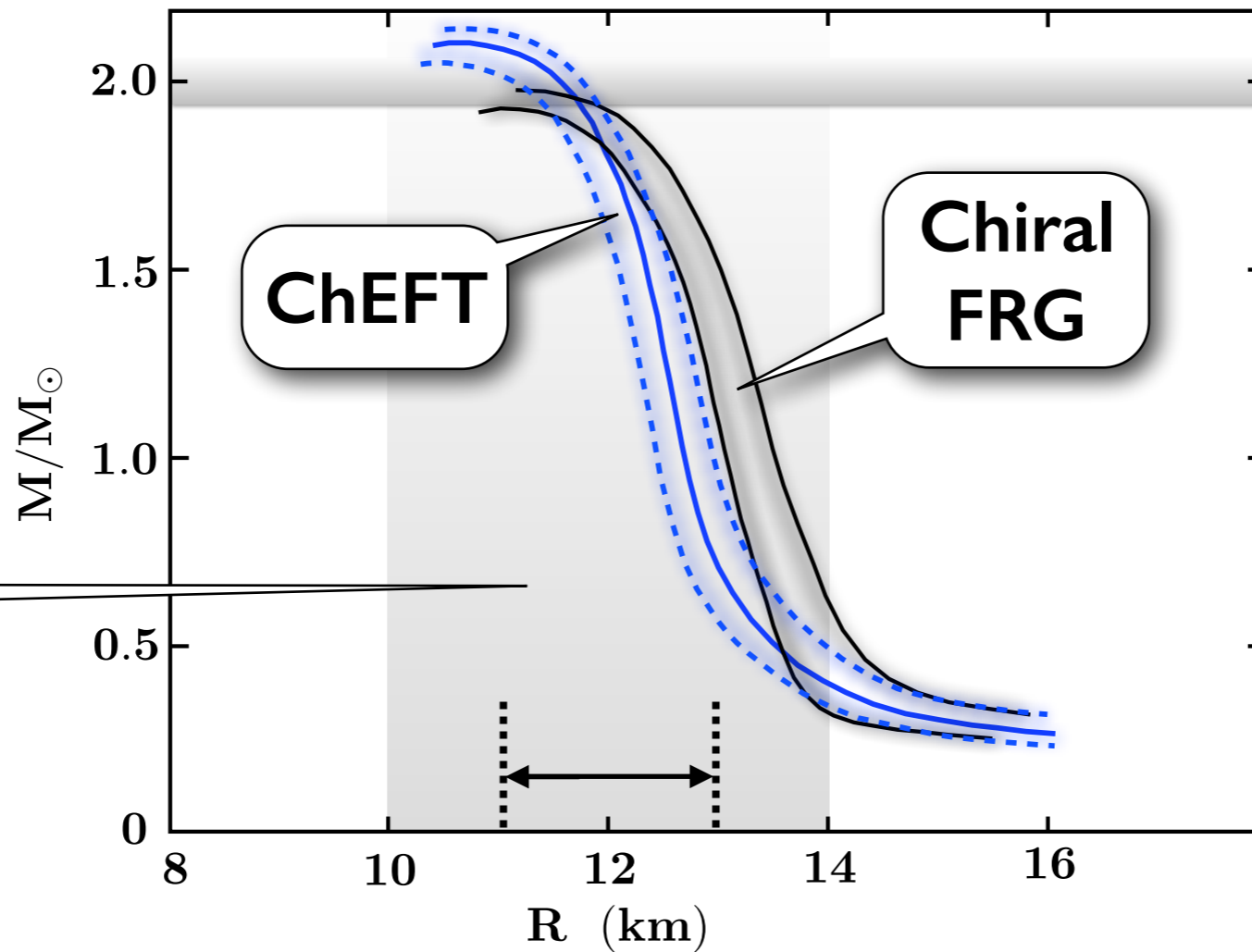
● Crust: SLy EoS

T. Hell, W.W.  
Phys. Rev.  
C90 (2014) 045801

M. Drews, W.W.

Phys. Rev.  
C91 (2015) 035802

Prog. Part. Nucl. Phys.  
93 (2017) 69



● **Central core density**

$$\rho_c \lesssim 5 \rho_0$$

**Radius window**

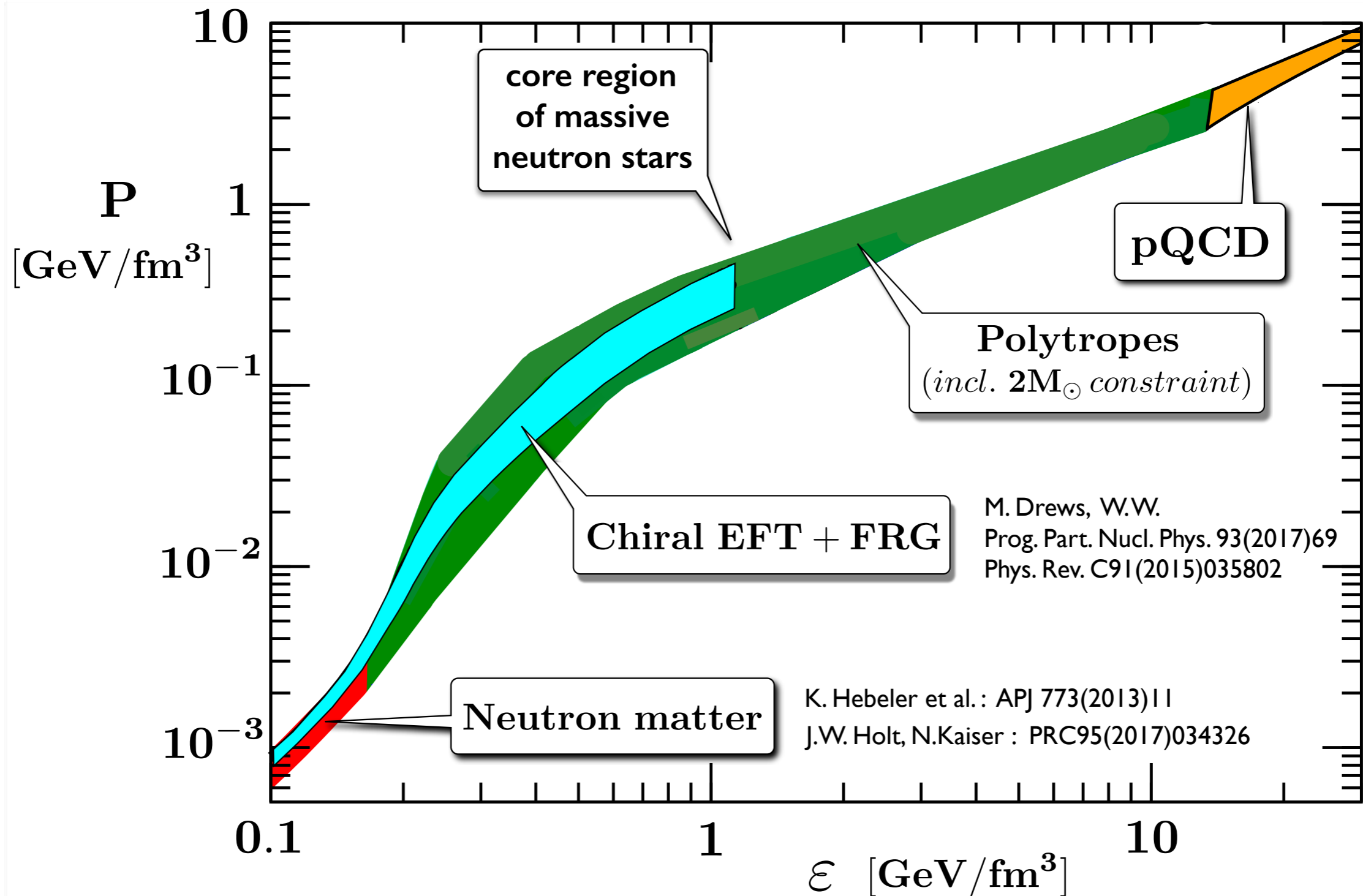
A.W. Steiner,  
J.M. Lattimer, E.F. Brown  
EPJ A52 (2016) 18

- Chiral many-body dynamics using “conventional” (pion & nucleon) degrees of freedom is consistent with neutron star constraints

# NEUTRON STAR MATTER Equation of State

... and extrapolation  
to PQCD limit

A. Kurkela et al.: *Astroph. J.* 789 (2014) 127

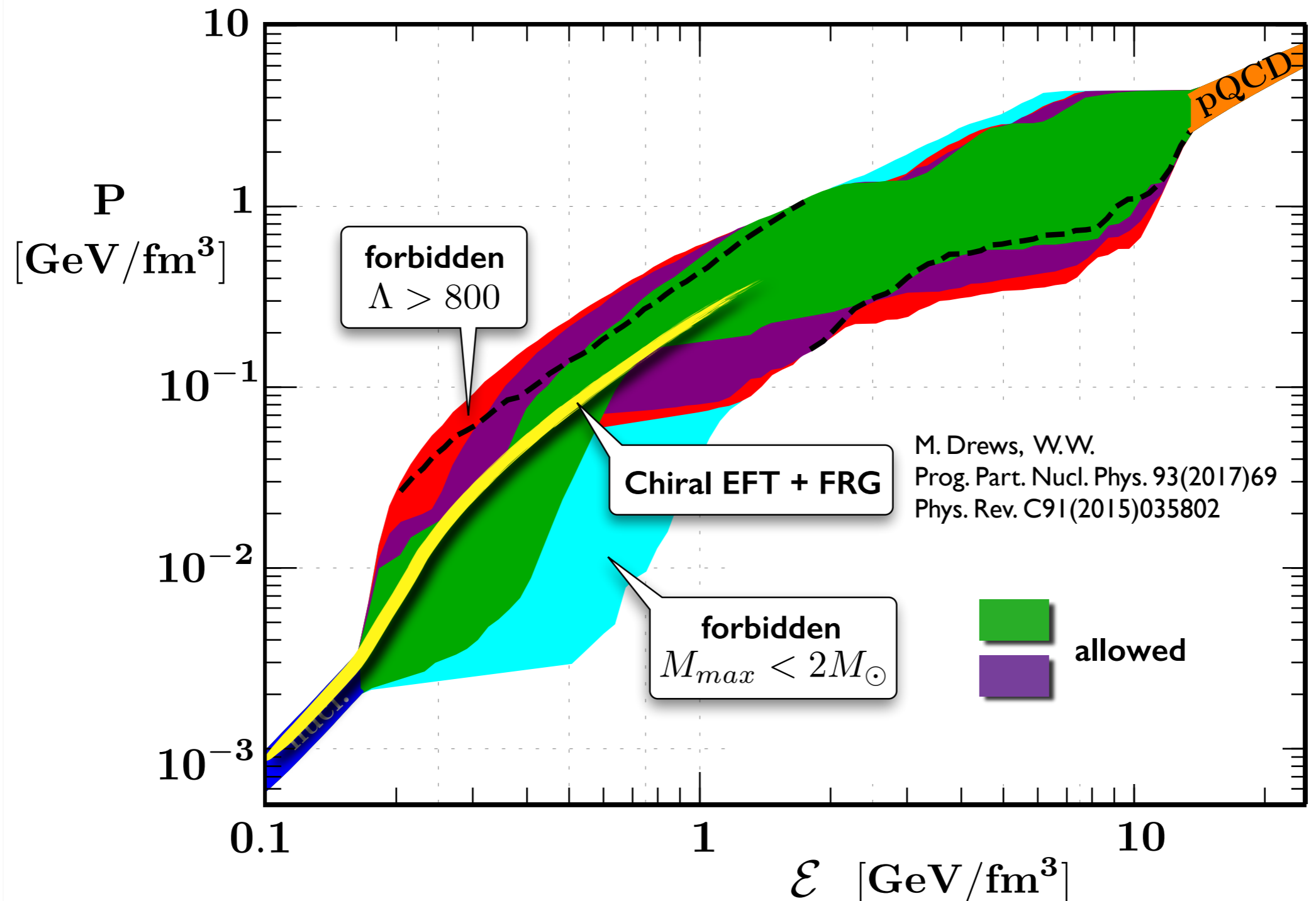




# NEUTRON STAR MATTER Equation of State

... now including  
GW constraints

E. Annala, T. Gorda, A. Kurkela, A. Vuorinen  
Phys. Rev. Lett. 120 (2018) 172703



# 3.

*Outlook:*

*Strangeness in Neutron Stars ?*

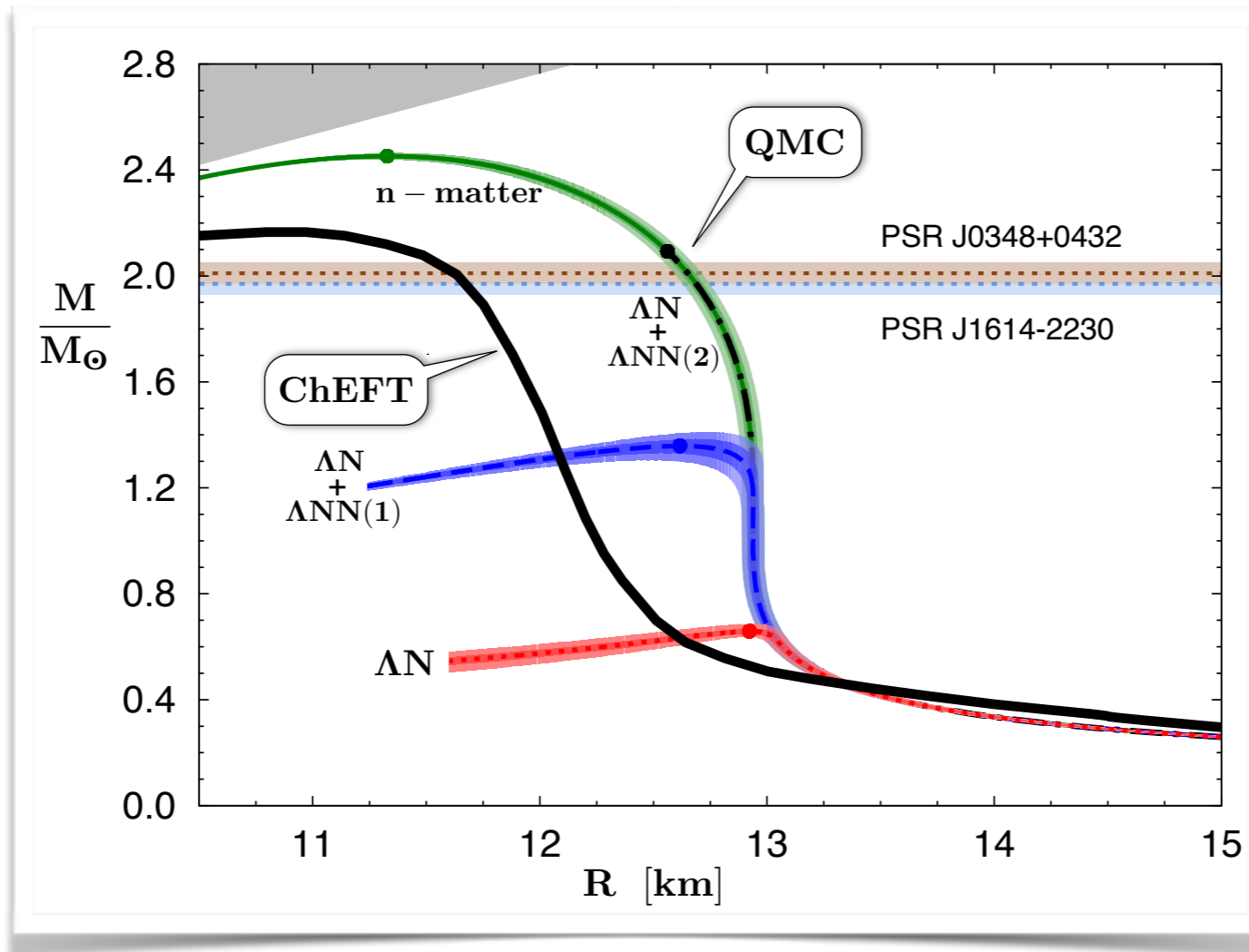
- **The Hyperon Puzzle**
- **Chiral SU(3) Effective Field Theory**
- **Hyperon - Nuclear Interactions**



# NEUTRON STAR MATTER including **HYPERONS**

Quantum Monte Carlo calculations using phenomenological hyperon-nucleon and hyperon-NN three-body interactions constrained by hypernuclei

**ChEFT**  
calculations  
“conventional”  
n-star matter  
—  
T. Hell, W.W.  
PRC90 (2014) 045801



**QMC**  
computations  
(hyper-neutron matter):

D. Lonardoni,  
A. Lovato,  
S. Gandolfi,  
F. Pederiva

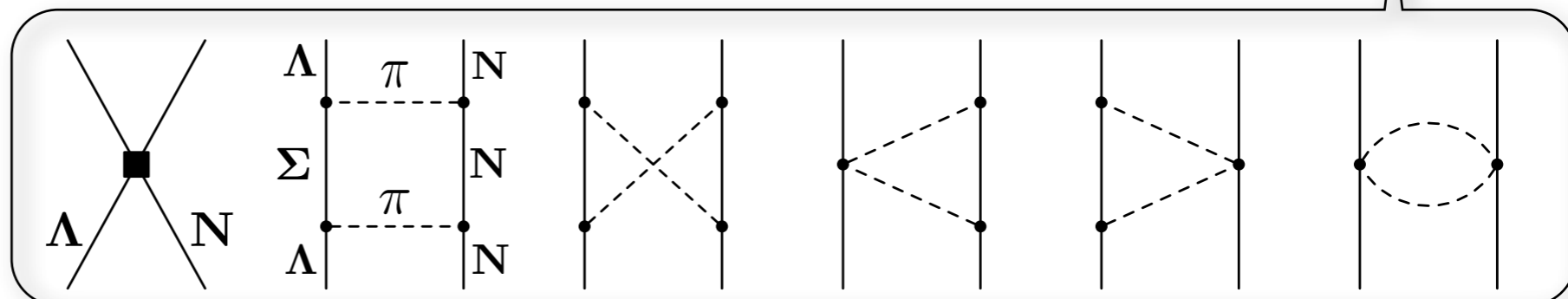
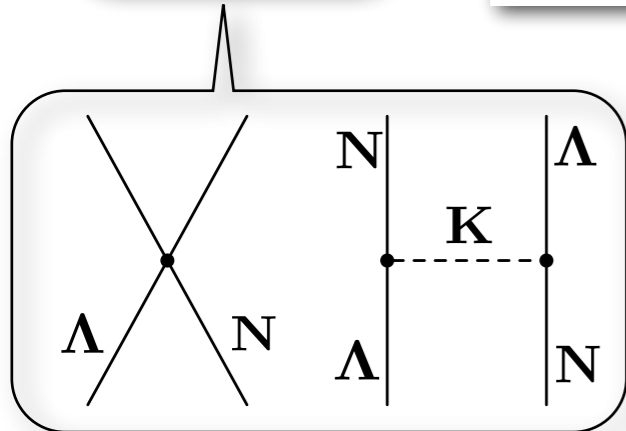
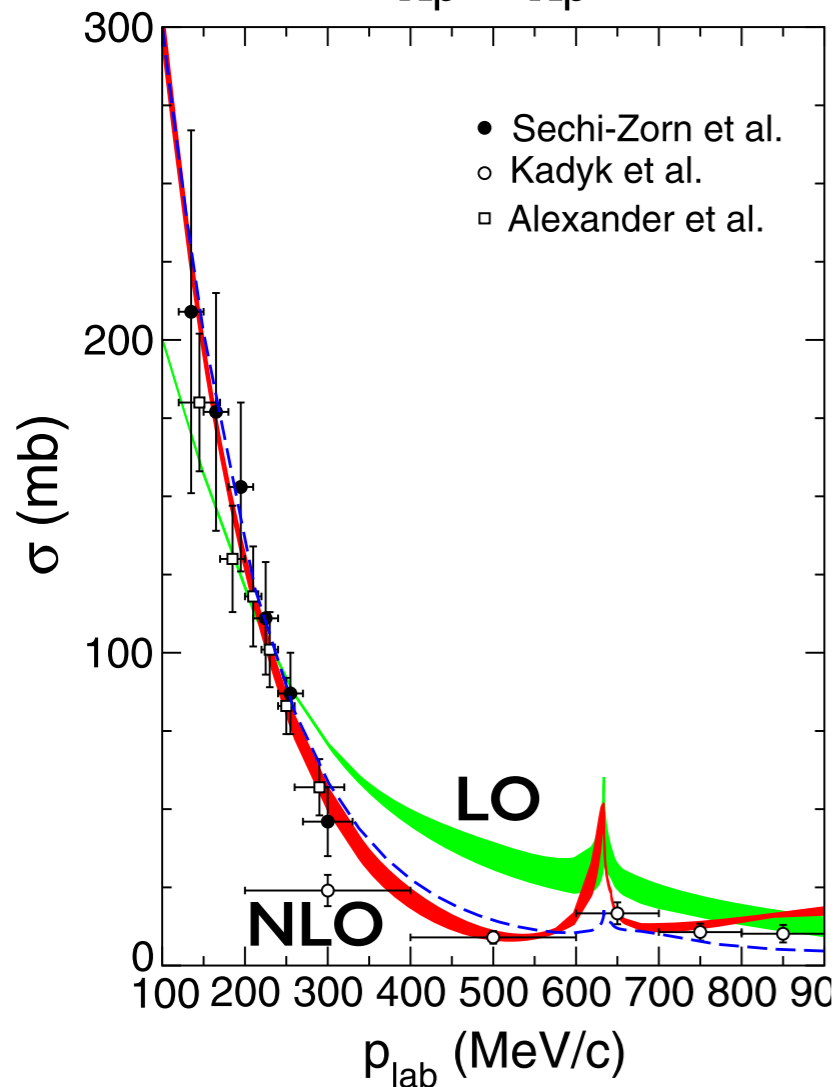
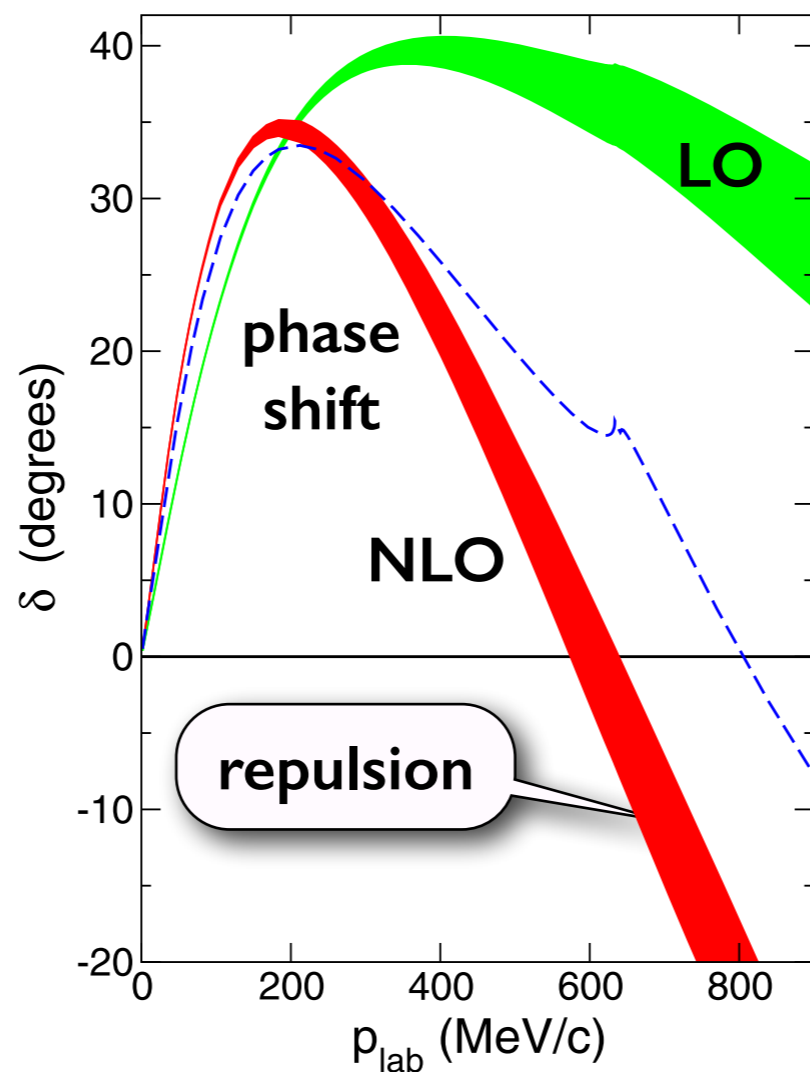
Phys. Rev. Lett.  
114 (2015) 092301

Inclusion of hyperons: EoS too soft to support 2-solar-mass n-stars  
unless: strong repulsion in **YN** and **YNN** ... interactions

**LO**

# Hyperon - Nucleon Interaction

from **CHIRAL SU(3) Effective Field Theory**

**NLO** $\Lambda p \rightarrow \Lambda p$  $\Lambda p \ ^1S_0$ 

- moderate attraction at low momenta  
→ relevant for hypernuclei
- strong repulsion at higher momenta  
→ relevant for dense baryonic matter

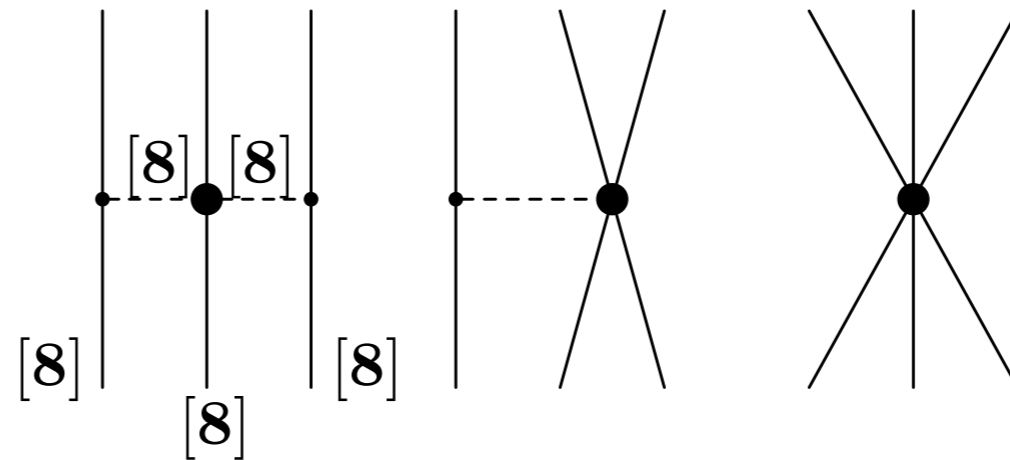
# HYPERON - NUCLEON - NUCLEON THREE-BODY FORCES from CHIRAL SU(3) EFT

S. Petschauer et al. Phys. Rev. C93 (2016) 014001

- **Chiral SU(3) Effective Field Theory:**  
interacting pseudoscalar meson & baryon octets + contact terms

3-baryon  
sector:

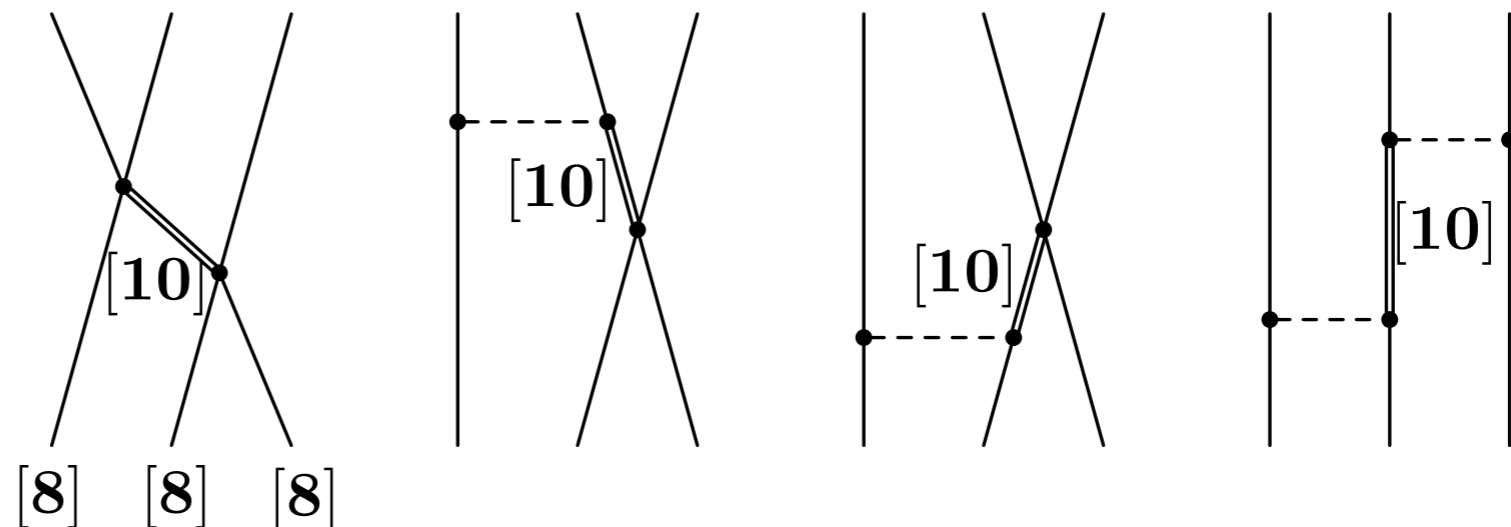
NNLO:



- **Chiral SU(3) Effective Field Theory with explicit decuplet baryons:**

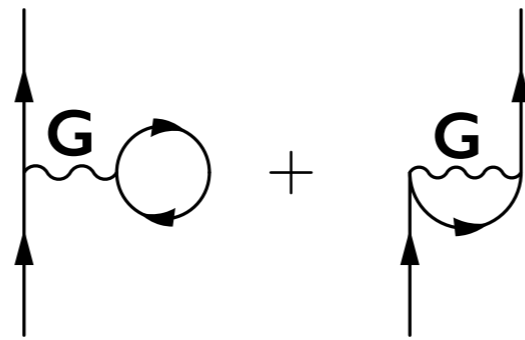
explicit treatment of  
baryon decuplet :

promotion to **NLO**

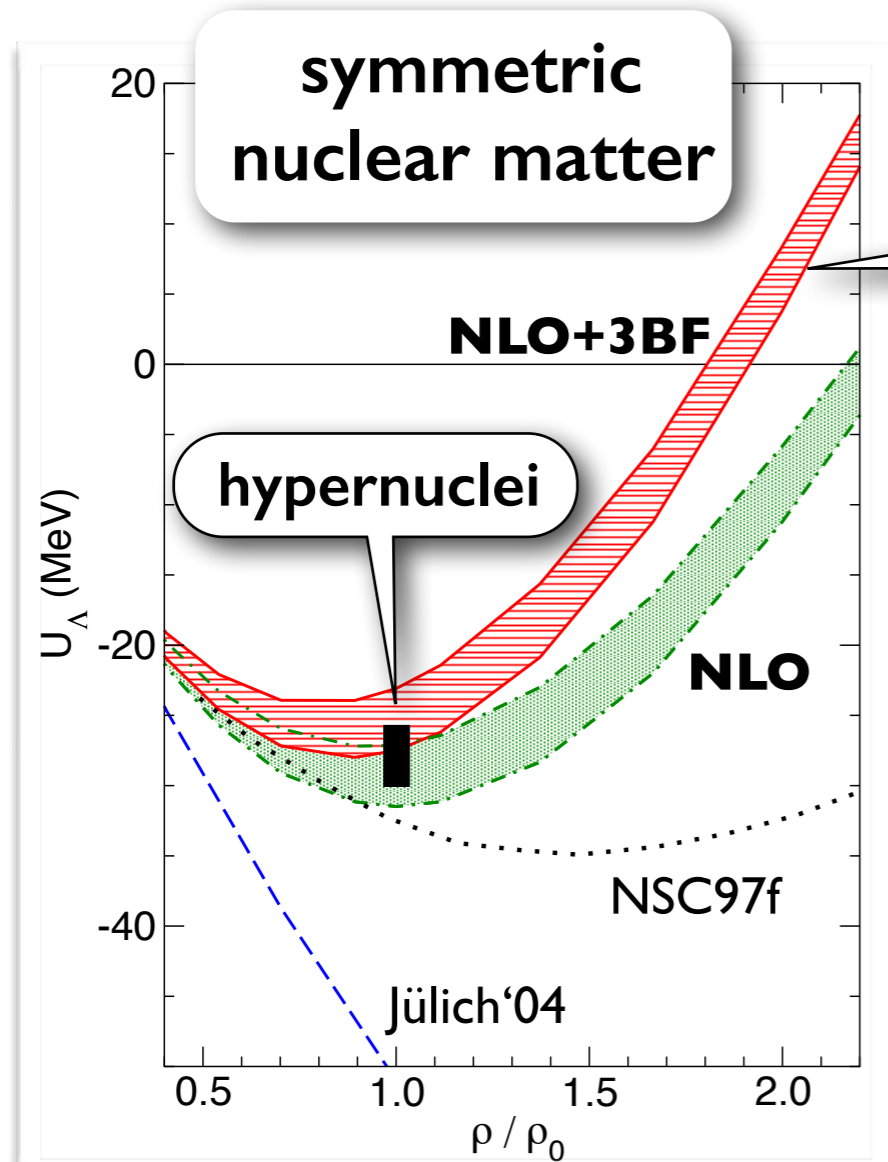


# Density dependence of $\Lambda$ single particle potential

- Brueckner calculations using chiral SU(3) interactions

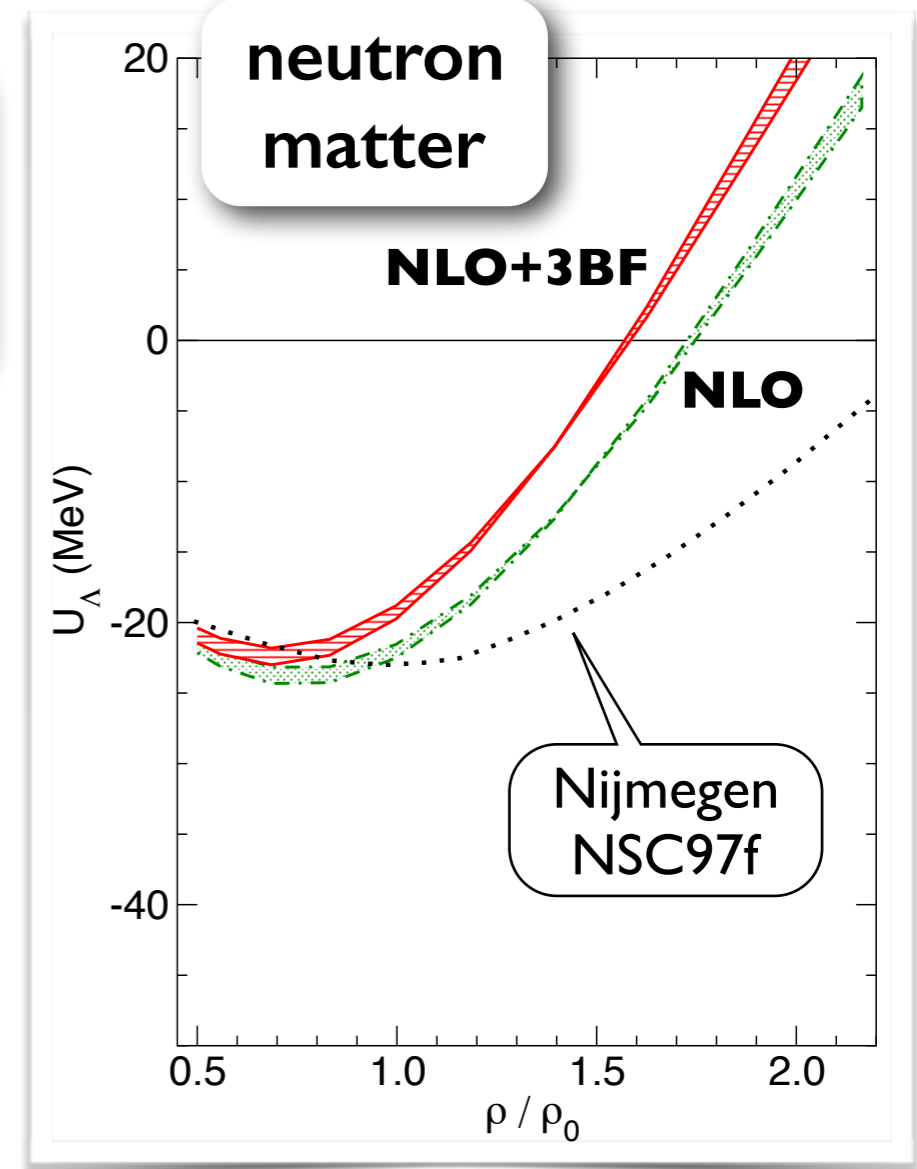


$$G(\omega) = V + V \frac{Q}{e(\omega) + i\epsilon} G(\omega)$$



Chiral SU(3)  
2- and 3-body  
forces

J. Haidenbauer,  
U.-G. Meißner,  
N. Kaiser,  
W.W.  
  
Eur. Phys. J.  
A53 (2017) 121



- ... towards a possible solution of the “hyperon puzzle” ?

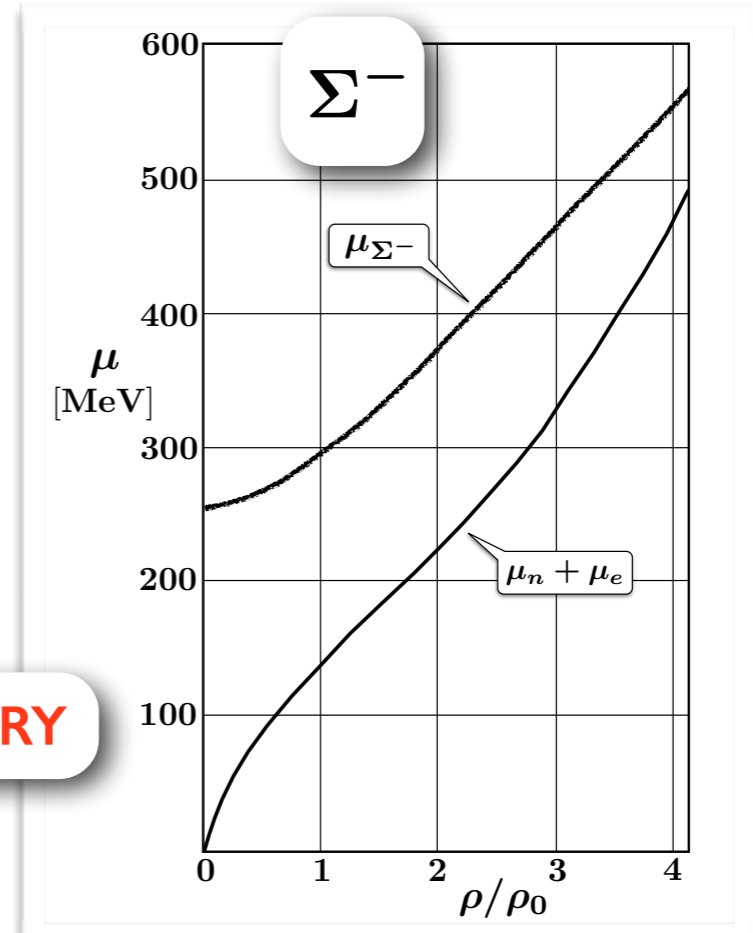
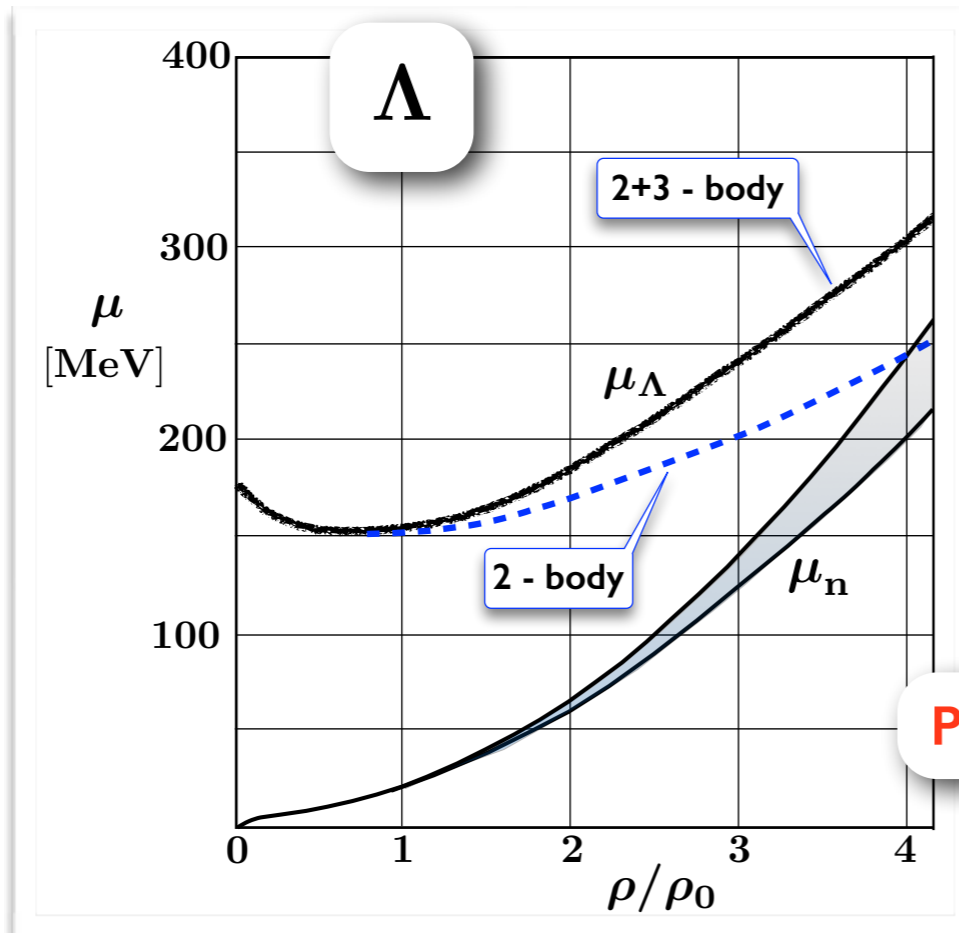
# Hyperons in Neutron Stars ?

- Onset conditions for appearance of hyperons in neutron stars :

Equalities for chemical potentials  $\mu_i = \frac{\partial \mathcal{E}}{\partial \rho_i}$

$$\mu_{\Lambda} = \mu_n$$

$$\mu_{\Sigma^-} = \mu_n + \mu_e = 2\mu_n - \mu_p$$



- Extrapolations using hyperon single particle potentials in neutron matter from Chiral SU(3) EFT interactions
- Extensive and more detailed calculations in progress (D. Gerstung, N. Kaiser, W.W.)

# SUMMARY

- Systematic framework at the interface of QCD (with light quarks) and physics of hadrons, nuclei and nuclear forces :

## Chiral Effective Field Theory combined with Functional Renormalization Group

- ChEFT + many-body perturbation theory works for  $\rho \lesssim 2 \rho_0$
- ChEFT + (non-perturbative) FRG may work for higher densities
  - ▶ **No** chiral phase transition in n-matter up to at least  $\rho > 5 \rho_0$
  - ▶ “Conventional” (non-exotic) **EoS** consistent with constraints from neutron stars ( $M_{max} \simeq 2 M_{\odot}$ , tidal deformability from GW)
  - ▶ **Strangeness in the neutron star core ?**  
New developments: hyperon-nuclear interactions from **Chiral SU(3) Effective Field Theory**

