The path optimization method to avoid the sign problem

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Path optimization method

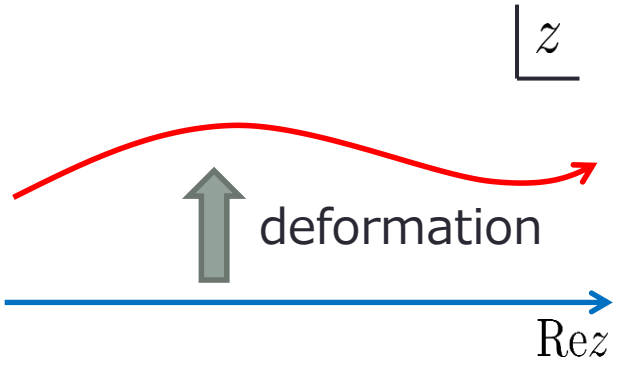
The sign problem

$$\langle \mathcal{O} \rangle = \frac{\int dx \mathcal{O} e^{-S}}{\int dx e^{-S}} = \frac{\langle e^{i\theta} \mathcal{O} \rangle_{pq}}{\langle e^{i\theta} \rangle_{pq}}$$ (pq:phase-quench)

average phase factor $\langle e^{i\theta} \rangle_{pq} = \frac{\int dx \frac{e^{-S}}{|e^{-S}|} |e^{-S}|}{\int dx |e^{-S}|}$

Path optimizaiton

- deform the integral path in the complex plane to maximize average phase factor.



regard sign problem
as optimization problem

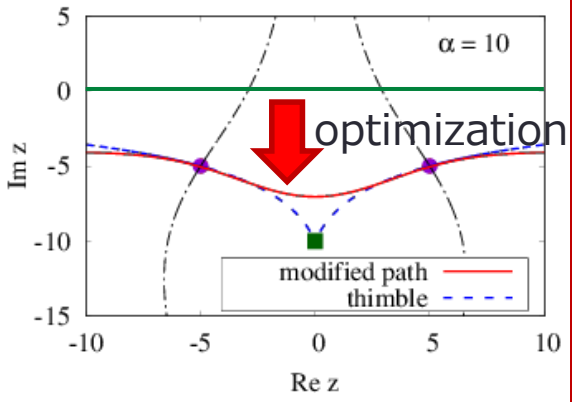
Results

- 1-dimensional model

$$\mathcal{Z} = \int dx (x + i\alpha)^{50} \exp(-x^2/2)$$

$$z(t) = t + i\left(c_1 \exp(-\frac{c_2^2 t^2}{2}) + c_3\right)$$

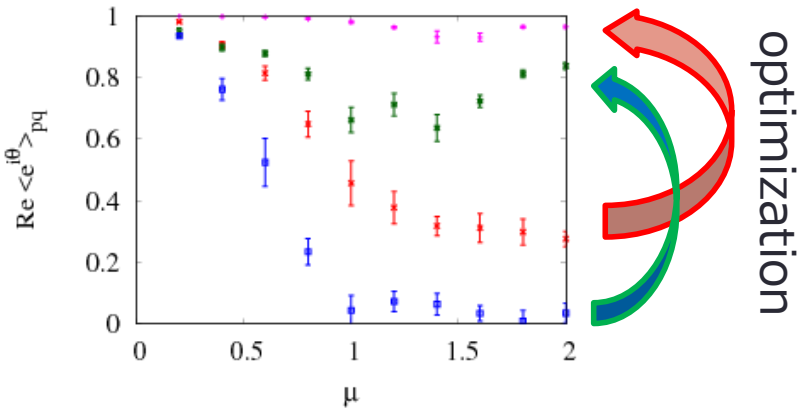
- Path



- 2D complex ϕ^4

$$S = \sum_x \left[(4+m^2)\phi_x^* \phi_x + \lambda(\phi_x^* \phi_x)^2 - \sum_{\nu=0}^{1} (\phi_x^* e^{-\mu\delta_{\nu,0}} \phi_{x+\hat{\nu}} + \phi_{x+\hat{\nu}}^* e^{\mu\delta_{\nu,0}} \phi_x) \right]$$

- average phase factor



- Next Step: gauge theory

$$f : \mathrm{SU}(N) \to \mathrm{SL}(N, \mathbb{C})$$

$$\mathcal{Z} = \int_{\mathrm{SU}(N)} dU e^{-S(U)}$$

$$= \int_{f(\mathrm{SU}(N))} d\mathcal{U} e^{-S(\mathcal{U})}$$