On Stochastic Quantisation and Quantum Gravity

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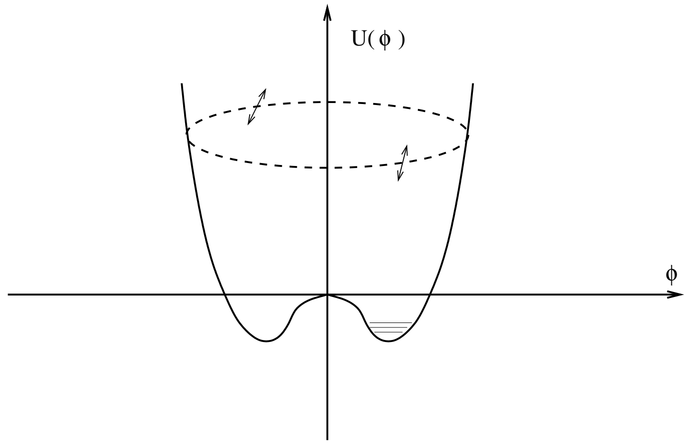

Figure 1: The dashed line represents one among the oscillating vacuum solutions and the double arrows represent symbolically the pair creations and annihilations that occur to periodically compensate for the conservation of the total energy when τ varies. So the "inertial force" due to the second order terms maintains the vacuum away from the absolute minima of the potential for a while and the processes is reminiscent of a turbulent phenomenon. Because of the friction terms in the second order Langevin equation, the angular velocity decreases as $\tau \to \infty$ and the vacuum will fall in one of the absolute minima of the potential. Then, either the ordinary quantisation or the classical behaviour will prevail with the damping of the τ dependence.

To be shown later

1)There is some logics for the existence of the strong fluctuation that triggers the exit of inflation (EI).

- Before EI, gravity is in its quantum phase (QG), with Euclidean correlations functions that presumably cannot be Wick rotated, so there is perhaps no Minkowski time and no particles. There is only the stochastic time to order the phenomena in this primordial phase with no particles, no scattering, and no clock that can possibly exist.

-The regime is like having a discrete time : a strong coherent state of dark energy oscillates with creations and absorptions of bound states by Schwinger (black holes). Physics is governed by fluctuations. (Cargèse 2016).

- After EI, gravity is in its classical phase (CG), meaning that the Wick rotation is possible, and the space is big enough to contain particles, which can be used to build clocks.

- So, in QG there is no Minkowski time and the analyticity properties in the Euclidean correlators of $g_{\mu\nu}(\tau, x)$ are very different before and after the exit of inflation.

There must be a phase transition between the two phases QG and CG, and thus a microscopic theory with new parameters.

To explain the exit of the universe inflation the suggestive picture of 2016 was : at very early periods the time variable is discrete instead of smooth.

The idea was that having discrete time is formally like an embedding the early universe in a non-singular and intense gravitational coherent state with very fast oscillations (like a gravitational laser beam bouncing back and forth between the boundary of the universe), giving a new time scale ΔT.

What triggers inflation ($\equiv$ sharp and fast decrease of the cosmological constant) is an abundance of pair creations and annihilations of black holes, by a generalisation to gravity of the Schwinger mechanism, in presence of the rapidly oscillating gravitational field (ie the vacuum). In this phase, the cosmological constant oscillates very quickly.

This introduces a time scale $\sim 10^{-15}\tau_{Planck} \sim 10^{-60}$ seconds that is a physical UV cutoff, via a Markov process with a discrete time equation (without invoking stochastic quantisation)

$$\frac{\Delta g_{\mu\nu}(T,x)}{\Delta T} = f(g_{\mu\nu}(T,x)) + \hbar\eta(T,x)$$

The drift force of the metrics is a combination of the metrics $g_{\mu\nu}$, the Riemann curvature $R_{\mu\nu}$ plus maybe the energy momentum tensor $T_{\mu\nu} \sim F^2$ of p-form gauge fields. This was a 2016 attempt.

2) A close inspection of Stochastic quantisation suggests a unifying picture, where QG effects can only exist before EI . After EI, gravity is in its classical phase.

Stochastic time τ is the universal parameter for ordering all phenomena before the exit of inflation. with the Euclidean $g_{\mu\nu}(x) \to g_{\mu\nu}(x,\tau)$ solution of the Langevin equation with noise $\eta(\tau,x)$:

$$\Delta T^2 \frac{\partial^2 g_{\mu\nu}(\tau,x)}{\partial^2 \tau} + \alpha \frac{\partial g_{\mu\nu}(\tau,x)}{\partial \tau} = \frac{\delta S[g_{\rho\sigma}]}{\delta g_{\mu\nu}(\tau,x)} + \hbar \eta(\tau,x)$$

$$\text{with} \quad << f(\eta) >>= \int [d\eta] f(\eta) \exp -\frac{1}{\alpha} \int \mathrm{d}\tau \mathrm{d}x \ \eta(\tau,x)^2 \qquad ===> \quad << \mathcal{O}(g_{\mu\nu}(x,\tau) >>= \mathrm{well-defined}$$

Minkowski time can emerge by analytic continuation. It is just a convenient parameter to order phenomena when gravity is classical.

In CG $g_{\mu\nu}(x) = \lim_{\tau\to\infty} g(\mu\nu,\tau)$ satisfies the Einstein equation of motion.

In QG $g_{\mu\nu}(x,\tau)$ oscillates in τ and never reaches an equilibrium because $\int [dg_{\mu\nu}] \exp -\frac{1}{2\hbar}\int R$ is not defined.

After EI, all experiments are such that effectively $\tau=\infty$. All fields $A_\mu(x,\tau), \ldots$ are effectively $A_\mu(x,\infty), \ldots$, with the equilibrium distribution $\exp -S[A]$ in the path integral. Clocks for x^0 only exist at $\tau=\infty$.

Changing the small distance behaviour of gravity in SQ in a controllable way :

$$\Delta T^2 \frac{\partial^2 g_{\mu\nu}(\tau,x)}{\partial^2 \tau} + \alpha \frac{\partial g_{\mu\nu}(\tau,x)}{\partial \tau} = \frac{\delta S[g_{\rho\sigma}]}{\delta g_{\mu\nu}(\tau,x)} + \hbar \eta(\tau,x)$$

α is a positive real number. η is a Gaussian noise. For $\Delta T = 0$, one has the standard stochastic quantisation. With $\hbar \neq 0$, the process cannot converge to a stationary Fokker-Planck distribution at $\tau \to \infty$. For $\alpha = 0$, oscillatory solutions occur in function of τ with no possible relaxation when $\tau \to \infty$.

For $\Delta T \neq 0$, the standard first order equation is recovered when the action of $\Delta T^2 \frac{\partial^2}{\partial^2 \tau}$ is much smaller than that of $\alpha \frac{\partial}{\partial \tau}$.

So $\Delta T \neq 0$ is like a physical UV cutoff for gravity, which makes a difference for the early stochastic time effects. For standard theories, but not gravity, having $\Delta T = 0$ or $\Delta T \neq 0$ makes no difference, when one reaches the limit $\tau = \infty$.

Notice that even for $\hbar = 0$ one has a complicated flow for

$$\frac{\delta S[g_{\rho\sigma}]}{\delta g_{\mu\nu}(\tau,x)} = R_{\mu\nu}(\tau,x) - g_{\mu\nu}(\tau,x) R + \kappa g_{\mu\nu}(\tau,x)$$

At any given value of τ, $g_{\mu\nu}(x,\tau)$ can be decomposed into a classical $g^{cl}_{\mu\nu}(x,\tau)$ and a quantum $g^{Q}_{\mu\nu}(x,\tau)$ proportional to $\hbar$.

Both may oscillate in function of τ.

There must be a compensating mechanism for the quantum oscillations depending on τ around a certain value of $g^{cl}_{\mu\nu}(x,\tau)$ that also oscillates.

The stronger $g^{cl}_{\mu\nu}$ is, the stronger can be the quantum effect that this vacuum emits and absorbs pairs of bound states $g^{Q}_{\mu\nu}$ through the "Schwinger type effects", that is creations and annihilations of black holes. The analogy is the Schwinger effect for electron-positron pairs in an electromagnetic laser beam.

This phase can perdure until one has a fluctuation where the energy carried by $g^{cl}_{\mu\nu}$ becomes so low that gravity becomes purely classical, very diluted, and one relaxes to the standard model in a classical gravitational background. The microscopic scale of time is ΔT that can as small as one wishes.

- Defining a second order Langevin equation with a drift force that is the Einstein equation is a new idea. It is a natural generalisation (as Schrodinger → Klein–Gordon equation).

$$\Delta T^2 \frac{\partial^2 g_{\mu\nu}(\tau, x)}{\partial^2 \tau} + \alpha \frac{\partial g_{\mu\nu}(\tau, x)}{\partial \tau} = \frac{\delta S[g_{\rho\sigma}]}{\delta g_{\mu\nu}(\tau, x)} + \hbar \eta(\tau, x)$$

Not surprisingly, it gives more room for finite stochastic time phenomenon.

In fact, if one looks at the stochastic time Hamiltonian, one finds a double Hilbert space structure, one for the vacuum, and one for the quantum oscillations

It is in fact a very stimulating exercise to examine the meaning of a second order Langevin equation in zero dimensions, to define precisely what is second order stochastic quantisation in a soluble case. (A forthcoming paper by L.B. and S. Wu.)

Once this example is understood, it gives more precise ideas about the further information one can learn by computing the correlators of a given QFT at finite stochastic time.

The zero dimensional case

Consider the zero dimensional case QFT of two real variables x and y with an action

$$S(x,y) = \frac{1}{2}M^2x^2 + \frac{1}{2}m^2y^2 + \frac{\lambda}{2}xy^2$$

What's the behaviour of the stochastic correlators

$$<< x^p(\tau)y^q(\tau) >>= ?????$$

and what is their limits for $\tau = \infty$, with both α and ΔT non zero ?

It has to be the simple integrals

$$\lim_{\tau=\infty} << x^p(\tau)y^q(\tau) >>= \int dxdy x^p y^q \exp -(\frac{1}{2}M^2x^2 + \frac{1}{2}m^2y^2 + \frac{\lambda}{2}xy^2)$$

modulo some renormalisations, which can be computed in perturbation theory of λ. The approach to equilibrium exponentially damps the dependence in the initial conditions of the coupled Langevin equation that define the τ evolution, with oscillations when $\Delta T \neq 0$.

Second order stochastic quantisation implies two noises η_x and η_y

$$\left(a^2\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}+2b\,\frac{\mathrm{d}}{\mathrm{d}\tau}\right)x(\tau)+M^2x(\tau)+\frac{\lambda}{2}\,y^2(\tau)=\sqrt{\hbar}\,\eta_x(\tau),$$

$$\left(a^2\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}+2b\,\frac{\mathrm{d}}{\mathrm{d}\tau}\right)y(\tau)+m^2y(\tau)+\lambda\,x(\tau)y(\tau)=\sqrt{\hbar}\,\eta_y(\tau)$$

$$\langle\,\eta_x(\tau)\,\rangle=\langle\,\eta_y(\tau)\,\rangle=0,\qquad \langle\,\eta_x(\tau)\eta_x(\tau')\,\rangle=\langle\,\eta_y(\tau)\eta_y(\tau')\,\rangle=2b\delta(\tau-\tau')$$

For $a\neq 0$, we need to complete each Langevin equation with two boundary conditions, instead of one condition in the first order case $a=0$.

We consider the approximation that $x(\tau)$, is a "coherent state', a state that minimises maximally the quantum fluctuations.

$x(\tau)$ is a state that is as close as possible to a solution where one neglects everywhere η_x when y behaves quantumly around it (η_y fluctuates randomly).

This situation was roughly advocated to in 2016, to define the primordial cosmology, by separating eg $g_{\mu\nu}$ in classical and quantum parts and defining their balanced evolutions.

The above coupled Langevin equations, for the case where effectively $\eta_x = 0$, are

$$
\begin{aligned}
a^2\ddot{x} + 2b\,\dot{x} + M^2(x - x_{\rm cl}) + \frac{\lambda}{2}\,y^2 \quad &= 0,\\
a^2\ddot{y} + 2b\,\dot{y} + m^2 y + \lambda\,xy &= \sqrt{\hbar}\,\eta_y
\end{aligned}
$$

A perturbative resolution in λ is clearly possible, giving an exponential damping depending on b, m, M and oscillations functions of a, M, m that define physics at finite τ and are worth to explore.

The propagators G_M and G_m have a double pole structure instead of being of parabolic type for $a = 0$.

The Green's function $G^M(\tau)$ of the operator $D^M_\tau = a^2\partial^2_\tau + 2b\partial_\tau + M^2$ satisfying $D^M_\tau G^M = \delta(\tau)$ can be computed by a Laplace or Fourier transform.

The Green's function for x is (with $aM > b > 0$), is (analogous for y with $M \to m$)

$$G^M(\tau) = \theta(\tau)\frac{\exp(-E^M_+\tau)-\exp(-E^M_-\tau)}{a^2(E^M_- - E^M_+)}`\big(\exp(-E^M_+\tau) - \exp(-E^M_-\tau)\big),$$

$$E^M_\pm = \frac{1}{a^2}\big(b \pm i\sqrt{a^2M^2 - b^2}\,\big)$$

For $a, b \neq 0$, the free propagator has an exponential damping factor, with a characteristic time that is proportional to b^{-1} (when τ is counted in units of a^2) τ-oscillations that can be of a very high frequency (in units of a) if the mass M is large enough.

In the QFT generalisation, one replaces M^2 by $M^2 + \vec{k}^2$, where $\vec{k}$ stands for the momentum of the particle. Care must be given to the possible UV divergencies when $\vec{k}^2$ becomes very large.

There is a forward propagation of modes with positive and negative energy in the τ evolution, and one has insertions of the field $x(\tau)$ in addition to the insertions of η_x on the propagators of the $y(\tau)$.

Computational techniques :

- The Feynman diagrams that one can draw to describe perturbation theory involve closed loops, which can be computed at a any given finite order of the perturbative expansion in λ

- On the propagators of the $y(\tau)$ in the diagrams, one has insertions of the field $x(\tau)$ of η_x.

- The Feynman diagrams are finite integrals, since they are over a one dimensional momentum space, with neither infra-red nor ultra-violet divergences, because $m \neq 0$ and $M \neq 0$.

- The loops can be interpreted as creations of a virtual pairs created by the vertex $\lambda x y \overline{y}$ of particle and antiparticle y and $\overline{y}$ at a given value of the stochastic time, each one propagating forwardly in τ, until they annihilate at a further stochastic time, with possible interactions with the "classical field" $x(\tau)$. This is a bit unfamiliar, but one gets used to it. The elementary quantum processes that build the perturbation theory are the possible decay, annihilation and diffusion reactions

$$x_{\text{cl}} \to y + \overline{y}, \qquad y + \overline{y} \to x_{\text{cl}}, \qquad y + x_{\text{cl}} \to y + x_{\text{cl}}, \qquad \overline{y} + x_{\text{cl}} \to \overline{y} + x_{\text{cl}}$$

whose strength is proportionally to λ.

This suggests that a double Fock space must be constructed. It is made of all possible states that can occur for the τ evolution, one for the vacuum field x, for its possible oscillations, and the other one for the ordinary possible quanta emissions of the field y. The friction term proportional to b makes disappear the phenomena that occur during the τ evolution in the limit $\tau \to \infty$, if the limit exists.

The way to go is to diagrammatically express $x(\tau)$ and $y(\tau)$ in function of η at a given order of perturbation and then to compute at the same order of perturbation theory $<< x^p(\tau)y^q(\tau) >>$ by averaging over the η_y, wherever they are inserted, using their Gaussian distribution.

What can be shown to all order in λ is that the dependance in the initials conditions and in a, b is damped by terms of order $\mathcal{O}(\exp -\tau) \sim \exp - \sim b\tau + i \sim a\tau$ .The approach to equilibrium is like wise.

We can check this explicitly for the two point functions $\langle\, x(\tau)x(\tau')\,\rangle$ and $\langle\, y(\tau)y(\tau')\,\rangle$ to first non trivial order in λ. To do so, one solves the coupled Langevin equations perturbatively, using their Green functions.

$$x(\tau) = x_0(\tau) + \lambda x_1(\tau) + o(\lambda^2), \qquad y(\tau) = y_0(\tau) + \lambda y_1(\tau) + o(\lambda^2)$$

The 0th and first order terms in λ give

$$D_\tau^M(x_0 - x_{\rm cl}) = 0, \qquad D_\tau^m y_0 = \eta$$

$$D_\tau^M x_1 + \tfrac{1}{2} y_0^2 = 0, \qquad D_\tau^m y_1 + x_0 y_0 = 0$$

One finds

$$x_0(\tau) = x_{\rm cl} + c_+^M \exp(-E_+^M \tau) + c_-^M \exp(-E_-^M \tau) = x_{\rm cl} + o(e^{-\tau})$$

where $c_\pm^M$ are constants that are determined by the chosen values of x_0 at some τ_1 and τ_2. $o(e^{-\tau})$ stands for terms dominated by $e^{-\epsilon\tau}$ for some $\epsilon > 0$ as $\tau \to \infty$, times some oscillations. Thus

$$\langle\, x_0(\tau)\,\rangle = x_{\rm cl} + o(e^{-\tau})$$

$$y_0(\tau) = (G^m * \eta)(\tau) + c_+^m \exp(-E_+^m \tau) + c_-^m \exp(-E_-^m \tau) = (G^m * \eta)(\tau) + o(e^{-\tau})$$

For $0 < \tau_1 \le \tau_2$:

$$<< y_0(\tau_1) y_0(\tau_2) >> = \int_0^{\tau_1} {\rm d}\tau_1' \int_0^{\tau_2} {\rm d}\tau_2'\, G^m(\tau_1 - \tau_1') G^m(\tau_2 - \tau_2') \,\langle\, \eta(\tau_1')\eta(\tau_2')\,\rangle + o(e^{-\tau_1})$$

For $\tau_2 = \tau_1 = \tau$, $<< y_0(\tau)^2 >> = \dfrac{b}{a^4(E_+^m + E_-^m) E_-^m E_+^m} + o(e^{-\tau}) = \dfrac{1}{2m^2} + o(e^{-\tau}).$

We now go to the the next order

$$x_1(\tau) = -\frac{1}{2}\int_0^\tau \mathrm{d}\tau'\, G^M(\tau-\tau') y_0(\tau')^2 + o(e^{-\tau}).$$

$$\begin{aligned} << x_1(\tau) >> &= -\frac{1}{4m^2}\int_0^\tau \mathrm{d}\tau'\, G^M(\tau-\tau') + o(e^{-\tau}) \\ &= -\frac{1}{4m^2}\cdot\frac{1}{a^2(E_-^M - E_+^M)}\left(\frac{1}{E_+^M} - \frac{1}{E_-^M}\right) + o(e^{-\tau}) = -\frac{1}{4m^2M^2} + o(e^{-\tau}) \end{aligned}$$

Finally

$$\lim_{\tau\to\infty} << x(\tau) >> = x_{\rm cl} - \frac{\lambda}{4m^2M^2} + o(\lambda^2)$$

$$\begin{aligned} \lim_{\tau\to\infty} << y(\tau)^2 >> &= \lim_{\tau\to\infty} << y_0(\tau)^2 \; + 2\lambda y_0(\tau) y_1(\tau) >> + o(\lambda^2) \\ &= \frac{1}{2m^2} + 2\lambda\cdot\left(-\frac{x_{\rm cl}}{4m^4}\right) + o(\lambda^2) \\ &= \frac{1}{2(m^2+\lambda x_{\rm cl})} + o(\lambda^2) \end{aligned}$$

This is a check of the general result for computing $<< x^p y^q >>$ and its limit.

The final result is in fact

$$
\begin{aligned}
\lim_{T\to\infty} \langle\, x^p(T) y^q(T)\,\rangle \quad &= \\
\int \mathrm{d}x\mathrm{d}y\, x^p y^q\, \delta(x - x_{\mathrm{cl}} + o(\lambda)) \exp - \Big((M^2(x - x_{cl})^2 + m^2 y^2 + \tfrac{1}{2}\lambda x y^2)(1 + o(\lambda^2)) \Big) \\
&= \quad x_{\mathrm{cl}}^p \int \mathrm{d}y\, y^q \; \exp\big(- m^2 y + \tfrac{1}{2}\lambda x_{\mathrm{cl}}\, y^2 \big)(1 + o(\lambda^2))
\end{aligned}
$$

The figure at the beginning of these slides explains the process, with the oscillating damping

Supersymmetric representation

For the second order stochastic evolution, we have a "Fokker-Planck" Hamiltonian H^{FP} for the τ evolution. It is defined form the supersymmetric Lagrangian $\mathcal{L}_{susy}$ associated to the Langevin equation. The path integral of $\exp - \int d\tau \mathcal{L}_{susy}$ computes all correlators of the $q(\tau)$:

$$\int d\tau \mathcal{L}_{susy} = \int d\tau s_{\text{top}} \left(\overline{\psi}\left(a^2 \ddot{q} + 2b\,\dot{q} + \frac{\partial S}{\partial q} - \frac{1}{2}\eta\right)\right)$$
$$= - \int d\tau \frac{1}{2}\eta^2 + \eta\left(a^2\ddot{q} + 2b\,\dot{q} + \frac{\partial S}{\partial q}\right) - \overline{\psi}\left(a^2\ddot{\psi} + 2b\,\dot{\psi} + \frac{\partial^2 S}{\partial q^2}\psi\right)$$

$$\int d\tau \mathcal{L}_{susy} \sim \int d\tau \frac{1}{2}\left(a^2\ddot{q} + 2b\,\dot{q} + \frac{\partial S}{\partial q}\right)^2 - \overline{\psi}\left(a^2\ddot{\psi} + 2b\,\dot{\psi} + \frac{\partial^2 S}{\partial q^2}\psi\right)$$

This gives H^{FP} by the appropriate Legendre transform, with the (for once good) property that it depends on higher order derivatives.

($s_{\text{top}}\, q = \Psi, s_{\text{top}}\, \overline{\Psi} = b$ is the nilpotent topological BRST symmetry operator for stochastic quantisation introduced in 1989 in L.B. and B. Grossman.)

The double Hilbert space structure

Ostrogradsky solved the general question of writing a Lagrangian/Hamiltonian formalism for theories with higher order derivatives, eg, $L(q, \dot{q}, \ddot{q})$.

A general variation of L, for arbitrary variations δq, one has $\delta \dot{q}$ is

$$\delta L = \left(\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial L}{\partial \dot{q}} + \frac{\mathrm{d}^2}{\mathrm{d}\tau^2}\frac{\partial L}{\partial \dot{q}} \right) \delta q + \frac{\mathrm{d}}{\mathrm{d}\tau}\left(p_0 \delta q_0 + p_1 \delta \dot{q}_1 \right)$$

So, the Euler-Lagrange equation of motion are generalised into

$$\frac{\partial L}{\partial q} = \frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial L}{\partial \dot{q}} - \frac{\mathrm{d}^2}{\mathrm{d}\tau^2}\frac{\partial L}{\partial \ddot{q}}$$

The last term identifies the independent momenta as

$$q_0 \equiv q, \qquad q_1 \equiv \dot{q}, \qquad p_0 \equiv \frac{\partial L}{\partial \dot{q}} - \frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial L}{\partial \ddot{q}}, \qquad p_1 \equiv \frac{\partial L}{\partial \ddot{q}}$$

The phase space is parametrised by the conjugate coordinates (q_0, q_1, p_0, p_1), so it is doubled . (as can be simply understood because twice as many initial conditions are needed as in the standard case).

q and $\dot{q}$ can be measured simultaneously in the quantum mechanics defined by replacing the (odd)coordinates by operators and the Poisson brackets by (anti)commutators.) The uncertainty relation holds only between q_0 and p_0 (for $a \neq 0$), and between q_1 and p_1

As a consequence we have the non trivial oscillations of the vacuum in the τ evolution. This tells us that we have this system of double oscillations with oscillations of the vacuum and creations and absorptions of quantum.

It may happen that there is no possibility of getting an Euclidean time in the Euclidean QFT one gets from enhanced stochastic quantisation. The unitarity of the theory at finite stochastic time is in fact not needed. It will occur only in the limit, if it exists.

The construction of the phase space must be completed for the entire supersymmetric stochastic Lagrangian $\mathcal{L}_{susy}$ giving a graded symplectic structure and a supersymmetric Hamiltonian.

Both the Ψ and $\overline{\Psi}$ have their own independent momenta for $a \neq 0$ and there is also a doubling for the fermionic part of the phase space as compared to the case $a = 0$.

Eventually one gets a Fokker-Planck Hamiltonian that defines the time evolution by first order Hamiltonian equations.

Conclusion

Let us summarise the rough mechanism we have imagined. When the Universe is at a scale of the order of the Planck length or maybe much smaller, it may function as a resonant cavity for dark energy, filled with an oscillating gravitational coherent state that defines its geometry.

The dynamics of QG is governed by the stochastic time evolution, and the limit $\tau = \infty$ cannot be reached in QG. This vacuum state oscillates at a frequency of the order of magnitude of τ_{Planck}^{-1} or much higher. Such high frequency powerful gravitational coherent states can trigger locally strong enough fluctuations of the vacuum, creating and destroying abundant amounts of black holes by gravitational Schwinger giving a transition that reduces drastically the value of the cosmological constant, so the phase of QG is sharply changed into that of CG. This gives a microscopic scenario for the sharp exit of the inflation. It depends on a new time scale, which an adjustable parameter of the Langevin equation.

Independently of this proposition , an interesting physics shows up for the correlators at finite stochastic time, especially in the (existing) cases such that the limit $\tau \to \infty$ is not defined for $\hbar \neq 0$. Because of oscillations, instabilities can occur on the way, and the phase of the system can change. Our Standard Physics, with a Minkowski time, with its limitations, occurs in the limit $\tau \to \infty$.

A summary, for separating the questions

- Standard quantisation of Quantum Gravity tells us that one cannot handle Minkowski time consistently. In the canonical quantisation of gravity, the Wheeler–deWitt equation implies that the action of the Hamiltonian on physical states is zero, and therefore there cannot be evolution in time.

- Standard QFT methodology is by defining first the Euclidean theory, and then one computes all Euclidean correlators of fields and prove that there is an analytic continuation of e.g. x^4.

- Then one computes S matrix elements (doing something quite refined for the IR questions), and understand what is a physical clock. In this way $t \equiv iX^0$ can be used to describe the evolution of possible experiments for standard theories coupled to classical gravity. Causality follows. The question is : can we always do the Wick rotation ? (the answer is no).

- Simple QM examples exist such that he path integral cannot be defined : 1d conformal quantum mechanics with $S = \int dt(\dot{x}^2 + g/x^2)$, t etc....

-I The same with Euclidean quantum gravity : t cannot be defined in this way for d=4 because the Euclidean weight $\int d^4 x R\sqrt{g}$ is not positive, and its Euclidean path is generally meaningless.

- A non-trivial question is : do we really need a time when the Universe is so small that it cannot contain a single elementary particle ? (Only quantum gravity prevails, particles cannot be generated from the vacuum).

- That the Minkowski time t is an emerging quantity is not an absurd idea.

- The Wick rotation of $x^4 \to it$, with t for ordering phenomena (scattering and decays) could be relevant only in the phase where the Universe is very dilute, when gravity can be treated classically. Then one ca compute and check all interactions with renormalisable actions coupled to the gravitational background.

- QG describes a phase where only gravity is manifest, with none of the x^μ 's as a parameter for defining the evolution. Minkowski time can only emerge by a transition where gravity becomes classical, and for all processes we can let $\tau \to \infty$.

- Is their a microscopic theory with a mechanism that describes the transition between both phases, QG and CG? It is tempting enough to say that the transition is marked by the exit of the inflation.

Before the exit, particles and time may not exist, as free particles don't exist in solid Helium.

What's the general idea ?

The stochastic time τ, that is often thought of as a formal entity, could makes it's way as the the right parameter to describe universally the evolution of the correlators that describe the Universes.

The scale for the damping in τ is so small in mass^{-1} units that it cannot be observed in our phase.

In primordial cosmology none of the Euclidian coordinates can be Wick rotated, no clock can be constructed, etc... it may happen that one will never reach an equilibrium at $\tau = \infty$.

The universe cannot then be described as made of of particles, till one gets till a a huge fluctuation, materialised by the exit of the inflation.

After the exit of the inflation, we are effectively always in the shell $\tau = \infty$. Quantum gravity is "in the stochastic time bulk", and we cannot explore it with our experiments.

The intuition is that when quantum gravity prevails, we have strong oscillations in τ. We assume that the (physical) scale of their period, is say $10^{-15} T_{Planck} \sim 10^{-60s}$.

The idea of SQ is that for all fields ($\phi = g_{\mu\nu}, A_\mu, ...$), the correlators are defined by equilibrium distribution, are the consequence of a random process with an extra (stochastic) time parameter τ

$$< \phi(x_1, ...\phi(x_n,) > \int d[\phi]_x \phi(x_1)....\phi(x_n) exp - \frac{1}{\hbar} S[\phi] = lim_{\tau_p \to \infty} << \phi(x_1, \tau_1)....\phi(x_n, \tau_n) >>$$

Analogous to statistical physics, where the Boltzmann partition function uses the weight $\exp -\beta E$, and is the result of microscopic interactions.

The approach to the equilibrium is described by a Langevin equation with an exponential pace in function of a (not detectable) time.

We are talking of a suggestive generalisation of the Brownian motion to QFT, suggested by Parisi and Wu in 1981.