Testing AdS$_6$/CFT$_5$ with strings

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New Frontiers in String Theory 2018
Yukawa Institute for Theoretical Physics, Kyoto

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Outline

1. Introduction
2. 5d SCFT from 5-branes
3. AdS_6 solutions in Type IIB
4. AdS/CFT with strings
5. Conclusions and future directions

1. Introduction

1. Introduction

QFTs in d>4 have become an important ingredient in our current understanding of (supersymmetric) QFT in general.

By compactification they reduce to QFTs in d≤4, and explain many non-obvious properties such as dualities and enhanced symmetries.

But defining QFTs in d>4 is challenging from the point of view of a classical Lagrangian.

In particular YM gauge theories in d>4 are non-renormalizable, and appear to require additional degrees of freedom, perhaps string theory, for UV-completeness.

1. Introduction

However, as is well known now, string theory itself provides indirect evidence that 5d and 6d QFT's can be UV-complete, in the sense that they are deformations of 5d or 6d superconformal field theories (SCFTs).

The existence of 5d and 6d SCFTs is perhaps one of the most interesting results of modern string theory.

This is yet another example where a beautiful mathematical result, the existence of superconformal algebras in $d \leq 6$, turns out to have a physical realization.

1. Introduction

5d theories: 5d gauge theories are non-renormalizable since $g^2 \sim m^{-1}$.

Seiberg (1996) argued that with 8 supersymmetries there can be a UV fixed point corresponding to an interacting non-Lagrangian SCFT.

A necessary condition is the convexity of the effective prepotential on the Coulomb branch:

$$\mathcal{F} = \frac{1}{2g_0} h_{ij}\phi^i\phi^j + \frac{c_0}{6} d_{ijk}\phi^i\phi^j\phi^k + \frac{1}{12}\left(\sum_{\vec{R}} |\vec{R}\cdot\vec{\phi}|^3 - \sum_f \sum_{\vec{w}\in W_f} |\vec{w}\cdot\vec{\phi} + m_f|^3\right)$$

$$\frac{1}{g_{eff}^2(\phi)} = \frac{\partial^2 \mathcal{F}}{\partial \phi^2} = \frac{1}{g_0^2} + c|\phi|$$

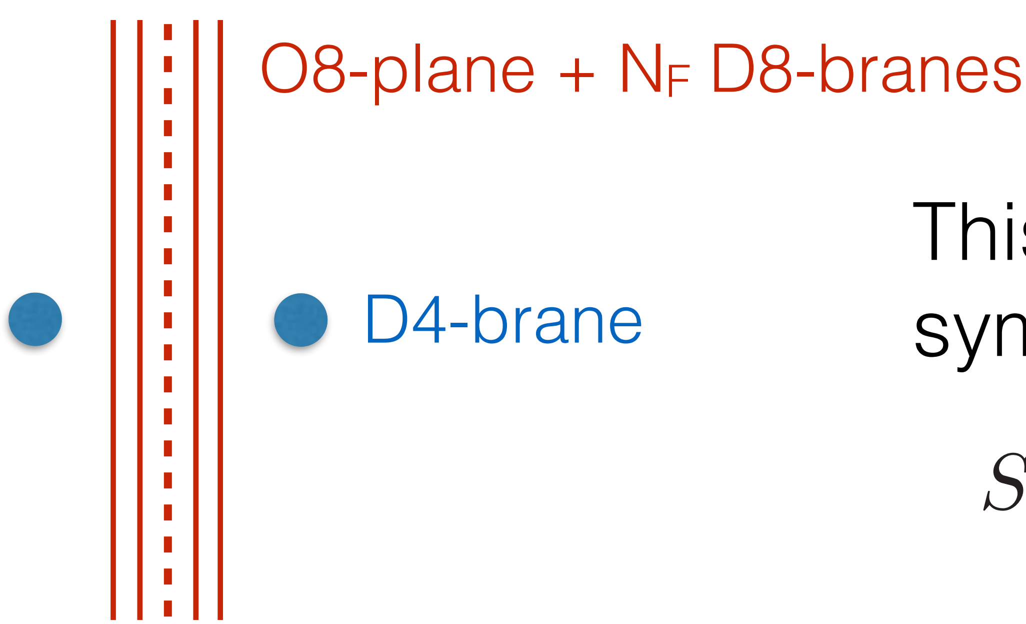
UV fixed point ⇨ $c > 0$

1. Introduction

Intriligator, Morrison and Seiberg (IMS) classified all 5d SUSY gauge theories (gauge groups, matter content) satisfying this condition.

Simplest set of examples is SU(2) with N_F flavors, for which a UV fixed point exists provided $N_F \leq 7$.

Supported by a brane construction in Type I' string theory:



This also shows that the global symmetry is enhanced:

$$SO(2N_F) \times U(1)_T \rightarrow E_{N_F+1}$$

1. Introduction

Do these 5d SCFTs have 6d AdS duals?

Problem is harder than in other dimensions, since the unique 5d superconformal algebra *F(4)* has only 8 supersymmetries and only $SU(2)_R$ R-symmetry.

Until recently only one solution was known, corresponding to the large N near horizon limit of the Type I' brane configuration, which is a warped AdS_6 x S^4 in Massive Type IIA SUGRA. Brandhuber + Oz

This is dual to a large N version of the field theory, Sp(N) with N_F flavors and an antisymmetric.

Generalized to orbifolds AdS_6 x S^4/Z_k and their dual quiver gauge theories. Bergman + Rodriguez-Gomez

1. Introduction

But there are many more 5d SUSY gauge theories with UV fixed points in the IMS classification, and even more from Type IIB 5-brane constructions, which I will review.

Recent work of D'Hoker, Gutperle, Karch and Uhlemann has uncovered a new class of AdS_6 solutions in Type IIB SUGRA.

These solutions involve 5-brane charges and appear to be related to the Type IIB 5-brane constructions, and therefore to specific 5d SCFTs, although a Maldacena-like derivation is not available.

I will report on a first set of quantitative tests of this conjecture, by comparing a class of states in the bulk to their presumed dual operators in the 5d SCFT.

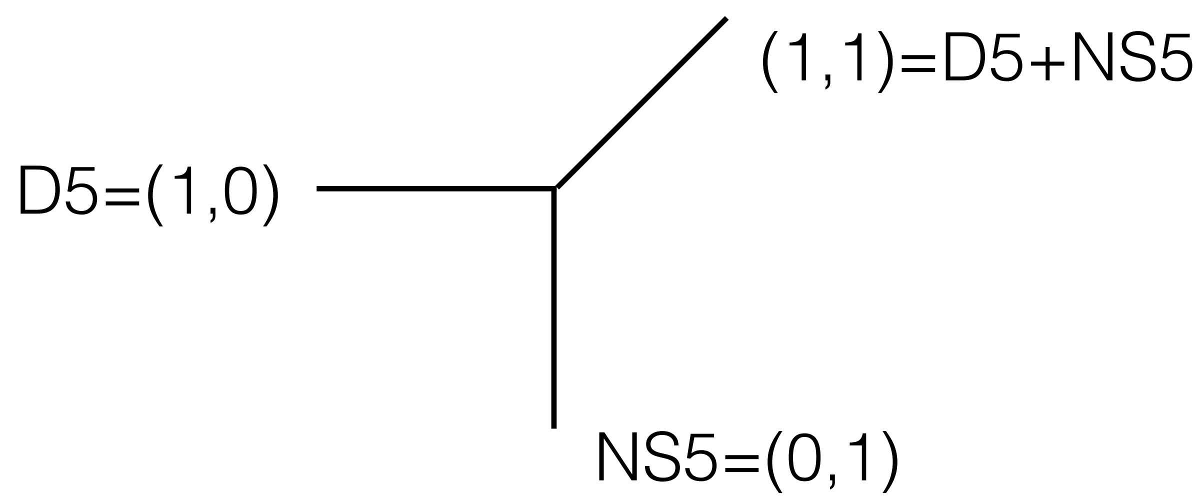
2. 5d SCFTs from 5-branes

2. 5d SCFTs from 5-branes

The most general construction of 5d SCFTs is in terms of configurations of (p,q) 5-branes in Type IIB string theory.

Aharony, Hanany, Kol

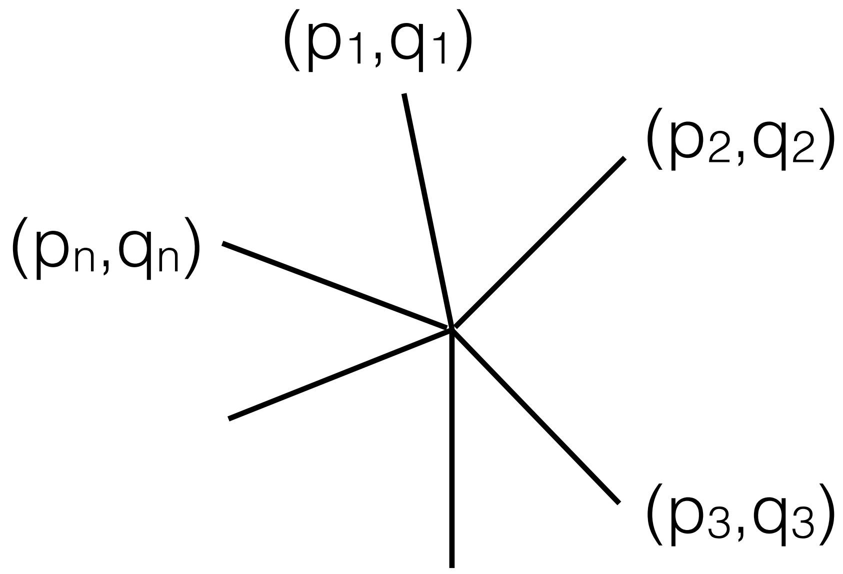
These configurations involve 5-brane junctions like:



A 5-brane configuration preserves 8 SUSYs if it's planar, and the relative angles are fixed by the values of (p,q).

2. 5d SCFTs from 5-branes

Every multi-5-brane junction describes a a 5d SCFT.



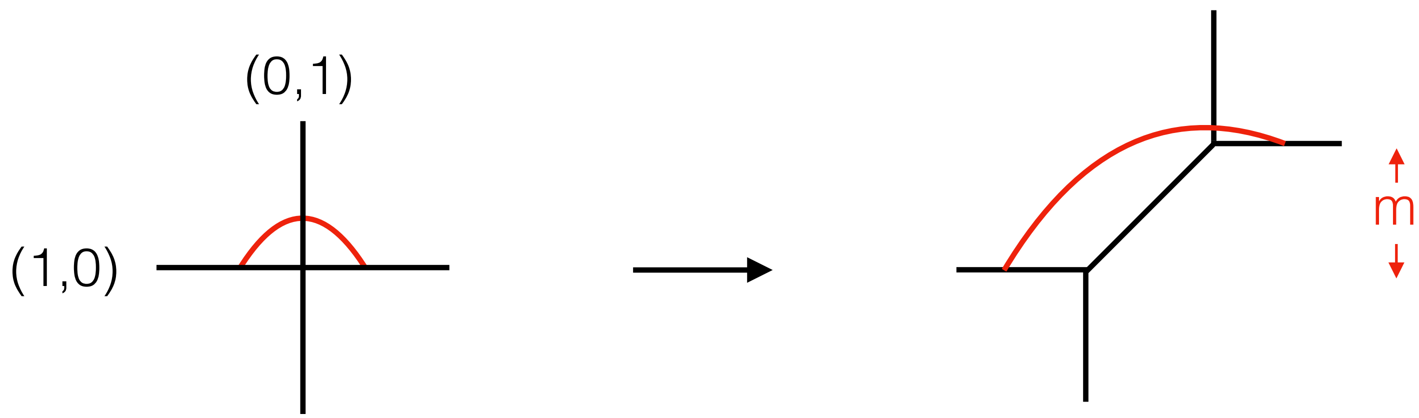
$$\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} q_i = 0$$

Why do we think this is true?

In many cases there is a geometric mass deformation that gives an ordinary 5d SUSY gauge theory which has a UV fixed point.

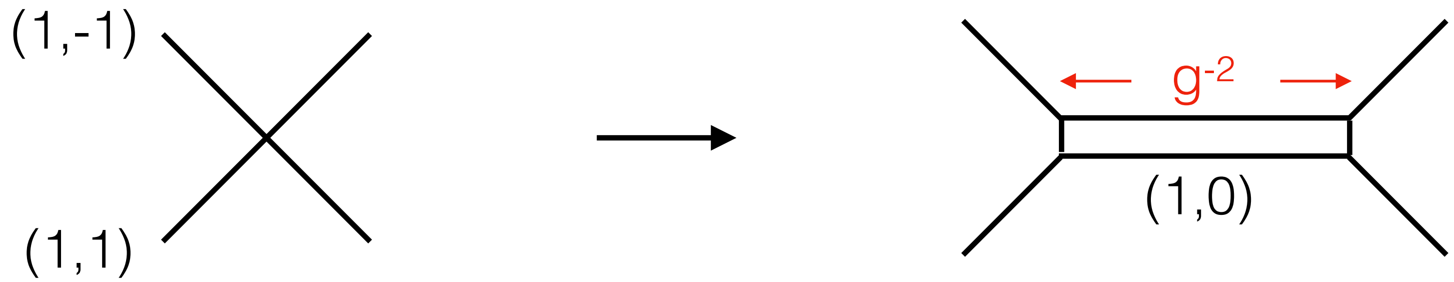
2. 5d SCFTs from 5-branes

Really simple example:



Free massless hypermultiplet   Free massive hypermultiplet

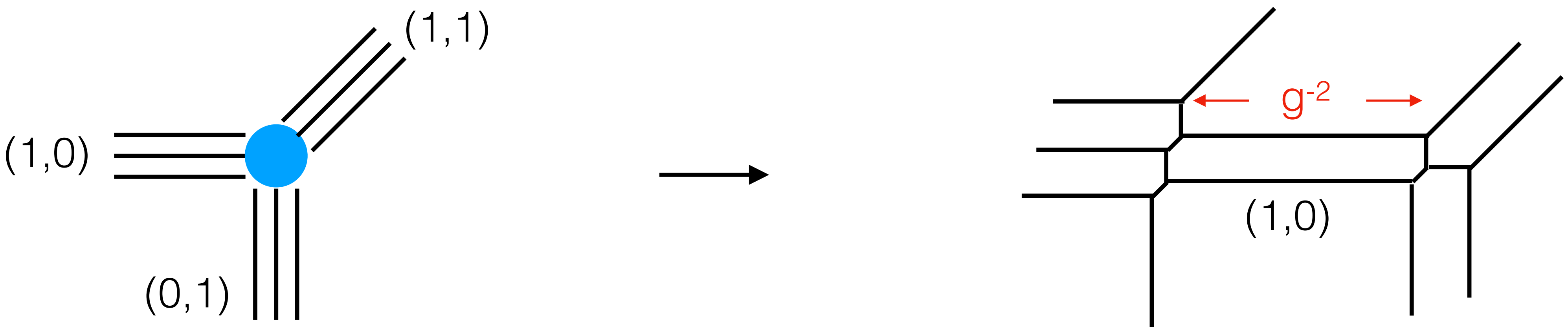
Simple interacting theory example:



E_1 theory   SU(2) SYM with coupling g

2. 5d SCFTs from 5-branes

Fancier example:



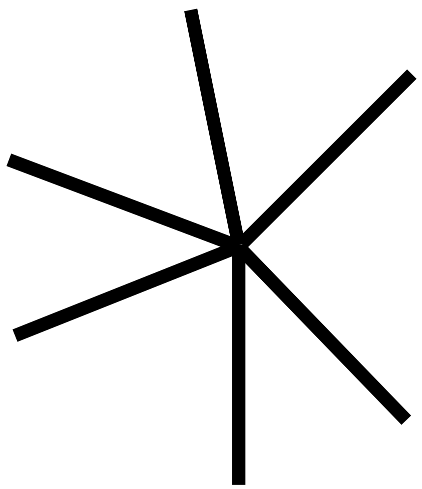
T_3 theory = E_6 theory

SU(2) SYM + 5 flavor hypermultiplets

To see all the flavors explicitly requires some brane-gymnastics.

2. 5d SCFTs from 5-branes

More generally:



Describes a 5d SCFT with $\sum_{i=1}^{n} N_i - 3$ mass parameters, which in some cases can be used to deform the theory to an IR gauge theory.

The mass parameters can be thought of as VEVs of scalars in background vector multiplets of global symmetries.

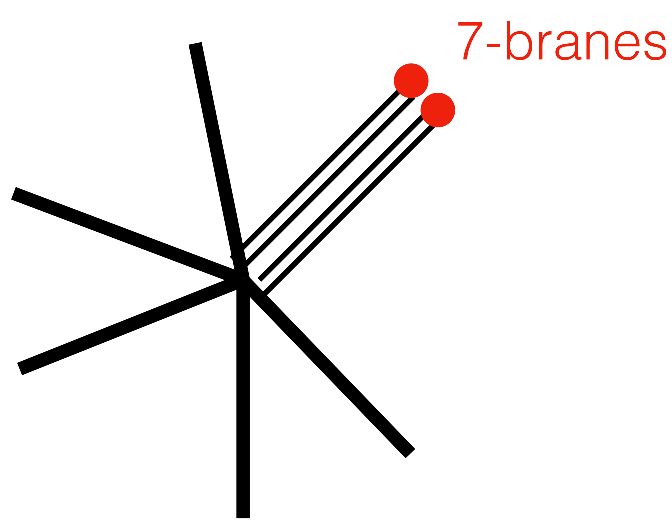
The global symmetry of the (unconstrained) multi-5-brane junction is:

$$\prod_{i}^{n} SU(N_i) \times U(1)^{n-3}$$

2. 5d SCFTs from 5-branes

All the theories in the IMS classification are reproduced by multi-5-brane junctions, if we also include constrained junctions:

Benini, Benvenutti, Tachikawa



These describe limits on the Higgs branch of the unconstrained junction theories.

But the 5-brane construction predicts many more theories with UV fixed points beyond the IMS classification: quiver gauge theories, more matter fields, and higher matter representations.

3. AdS_6 solutions in Type IIB

3. AdS_6 solutions in Type IIB

A non-technical overview of the Type IIB SUGRA solutions found by D'Hoker, Gutperle, Karch and Uhlemann last year.

Type IIB BPS equations were previously studied by others (Apruzzi et. al. and H. Kim et. al.), but no explicit solutions.

The geometry is a fibration of AdS_6 x S^2 over a Riemann surface Σ:

$$ds^2 = f_6^2(w) ds^2_{AdS_6} + f_2^2(w) ds^2_{S^2} + 4\rho^2(w)|dw|^2$$

SO(2,5) SU(2)$_R$

with a complex 2-form potential:

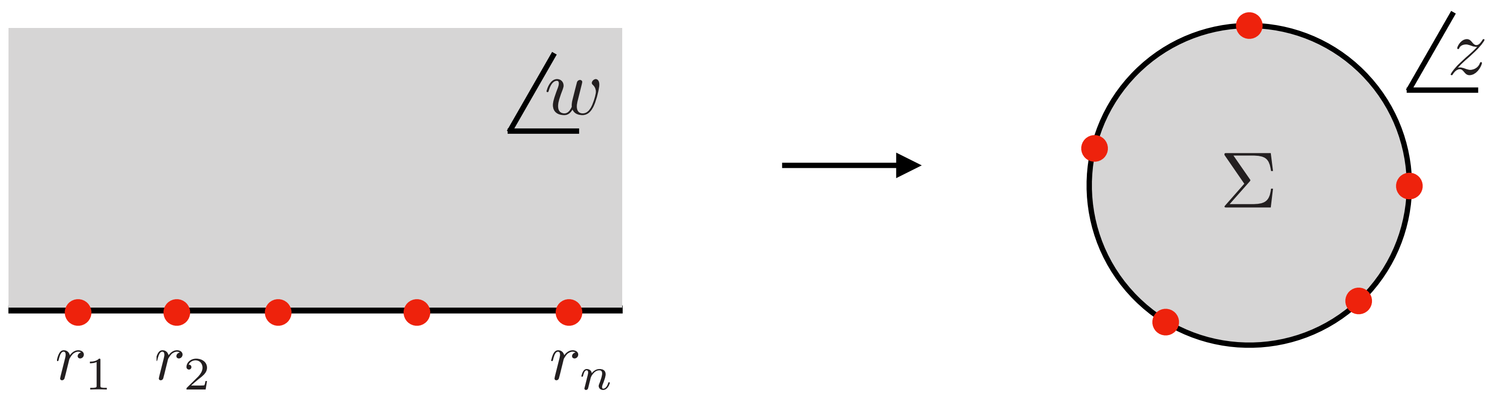
$$C_{(2)} = B_2^{NSNS} + iC_2^{RR} = C(w) \cdot vol_{S^2}$$

and an axion-dilaton: $\tau(w) = \chi(w) + ie^{-\phi(w)}$

3. AdS_6 solutions in Type IIB

The solution is specified by a locally holomorphic function $\mathcal{A}(w)$ on Σ, subject to regularity conditions.

For Σ = disc (upper half plane): $\mathcal{A}(w) = \mathcal{A}_0 + \sum_{i=1}^{n} Z_i \ln(w - r_i)$



The residues Z_i depend on the positions of the poles r_i on ∂Σ and on other parameters. Regularity conditions and SL(2,R) reduces the number of real parameters to 2n-2.

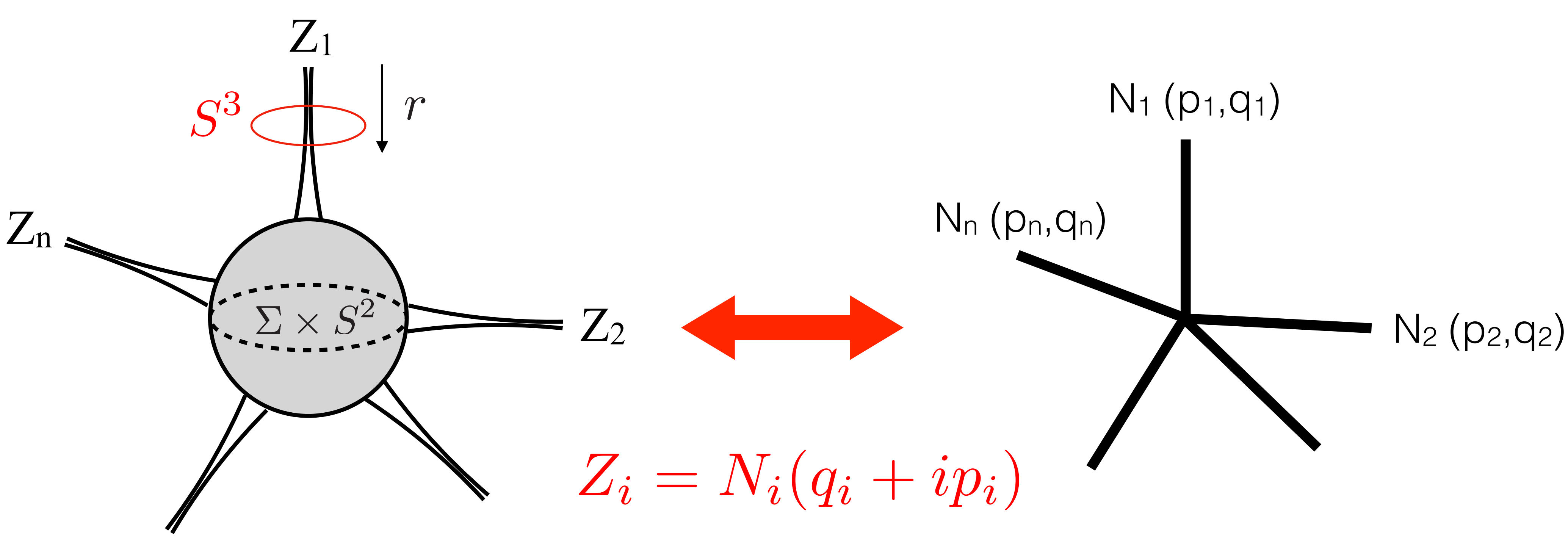
In particular we get the condition: $\sum_{i=1}^{n} Z_i = 0$

3. AdS_6 solutions in Type IIB

Key properties of the disc solution:

- The S^2 shrinks on the boundary except at the poles $w=r_i$.
- The near-pole geometry is $R_+ \times S^3$, with a flux on S^3: $\int_{S_i^3} dC_{(2)} = Z_i$

This suggests that this solution is in fact the `near horizon' limit of the multi-5-brane junction!



$$Z_i = N_i(q_i + i p_i)$$

3. AdS$_6$ solutions in Type IIB

Axion-dilaton near the poles

For Re(Z_i)$\neq 0$, i.e. $q_i \neq 0$, we find:

$$\chi_i \approx \frac{p_i}{q_i} \qquad\qquad e^{-\phi_i} \approx \frac{1}{(N_i q_i)^2} \frac{r}{\sqrt{\ln r}}$$

Taking $p_i = 0$ (NS5-brane) and using S-duality we find the behavior of the axion-dilaton near a pole with $q_i = 0$ (D5-brane):

$$\chi_i \approx 0 \qquad\qquad e^{\phi_i} \approx \frac{1}{(N_i p_i)^2} \frac{r}{\sqrt{\ln r}}$$

Both in agreement with the expected behavior for 5-branes.

3. AdS_6 solutions in Type IIB

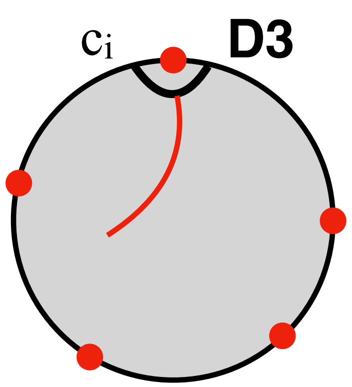
U(1) gauge fields

The reduction of the RR 4-form on the 3-cycles gives vector fields:

$$A_i = \int_{c_i} C_4^{RR}$$

However the 2-form flux gives rise to a mass-gap in general.

This can be seen by looking at a charged particle described by a 3-brane wrapping a 3-cycle:



$$\int_{c_i} dC_{(2)} = N_i(q_i + ip_i)$$

Must attach N_i (p_i,q_i) strings to the 3-brane. The strings go off to infinity.

3. AdS_6 solutions in Type IIB

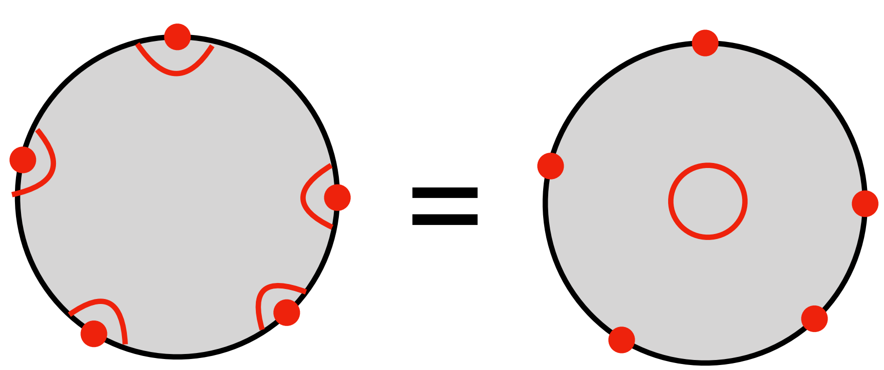
U(1) gauge fields

Massless U(1) gauge fields correspond to flux-free 3-cycles:

$$\sum_{i=1}^{n} a_i \, c_i \quad \text{with} \quad a_i \in \mathbb{Z}_+ \quad \text{such that} \quad \sum_{i=1}^{n} a_i (q_i + i p_i) = 0$$

There are n-2 linearly independent solutions.

However there are n-3 nontrivial solutions, since $\sum_{i=1}^{n} c_i = 0$:



In agreement with the U(1) factors in the global symmetry:

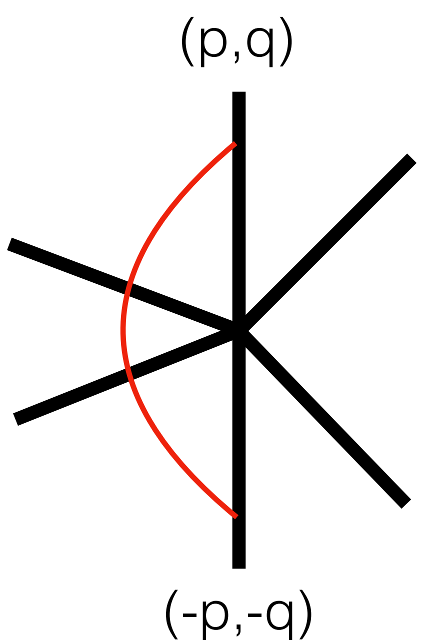
$$\prod_i^n SU(N_i) \times U(1)^{n-3}$$

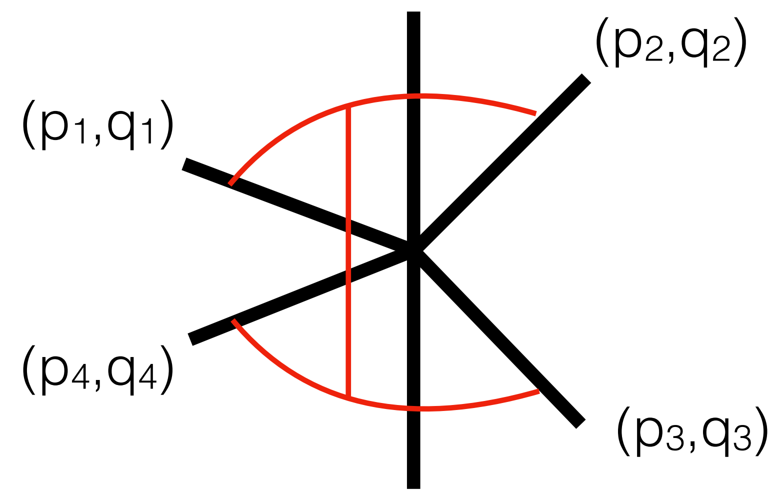
4. AdS_6/CFT_5 with strings

4. AdS_6/CFT_5 with strings

The Type IIB 5-brane construction of 5d SCFTs suggests that the spectrum of operators includes a class of BPS operators that correspond to strings with endpoints on the external 5-branes.

These can be open strings with two endpoints, or multi-string junctions (string-webs) with more endpoints:



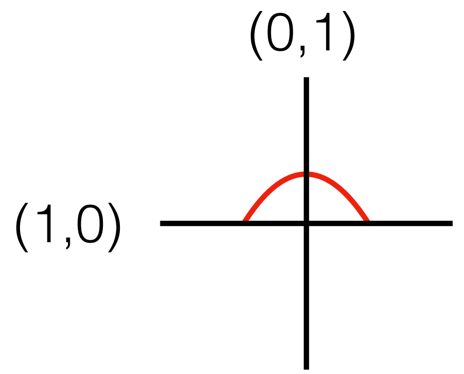


$$\sum_{i=1}^{4} p_i = \sum_{i=1}^{4} q_i = 0$$

4. AdS_6/CFT_5 with strings

From the point of view of the IR gauge theory, when available, these are gauge-invariant operators.

For example in the really simple example:



The open string is just the free massless hypermultiplet.

We claim that at large N_i these operators have O(N) scaling dimensions, and are dual to bulk states described by open or multi strings embedded in the Type IIB solution.

I will present two specific examples shortly.

4. AdS_6/CFT_5 with strings

Embedding strings in the Type IIB solution

The string extends along a 1d subspace between two poles in Σ described by the embedding *w(σ)*.

Induced metric: $$ds^2 = -f_6^2(w)dt^2 + 4\rho^2(w)|w'(\sigma)|^2 d\sigma^2$$

The action of a (p,q) string reduces to:

$$S_{(p,q)} = -\frac{1}{2\pi\alpha'}\int d^2\sigma\, f_6(w)\rho(w)|w'|\sqrt{\mathfrak{q}^T\mathcal{M}\mathfrak{q}} \qquad \mathfrak{q} = \begin{pmatrix} p \\ q \end{pmatrix} \quad \mathcal{M} = \begin{pmatrix} 1 & -\chi \\ -\chi & \chi^2 + e^{-2\phi} \end{pmatrix}$$

Gives the EOM for the embedding:

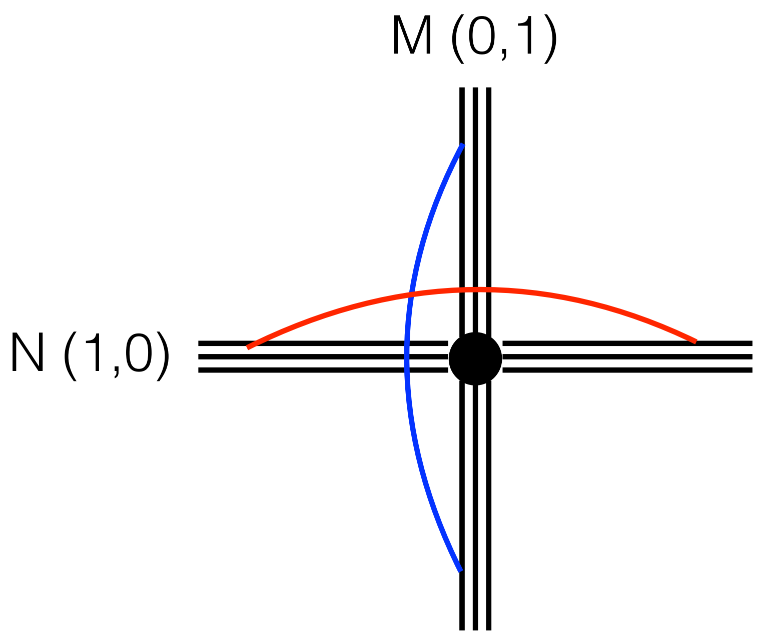
$$\bar{w}''/\bar{w}' - w''/w' + (\bar{w}'\partial_{\bar{w}} - w'\partial_w)\ln\left(f_6^2\rho^2\mathfrak{q}^T\mathcal{M}\mathfrak{q}\right) = 0$$

With solution in hand, the mass to be compared with the scaling dimension is given by:

$$m_{(p,q)} = -\int d\sigma\,\mathcal{L}_{(p,q)}$$

4. AdS_6/CFT_5 with strings

A. The $+_{N,M}$ theory



Generalization of the free hypermultiplet theory $+_{1,1}$.

Global symmetry:

$$SU(N)^2 \times SU(M)^2 \times U(1)$$

Open (1,0) string (fundamental string):

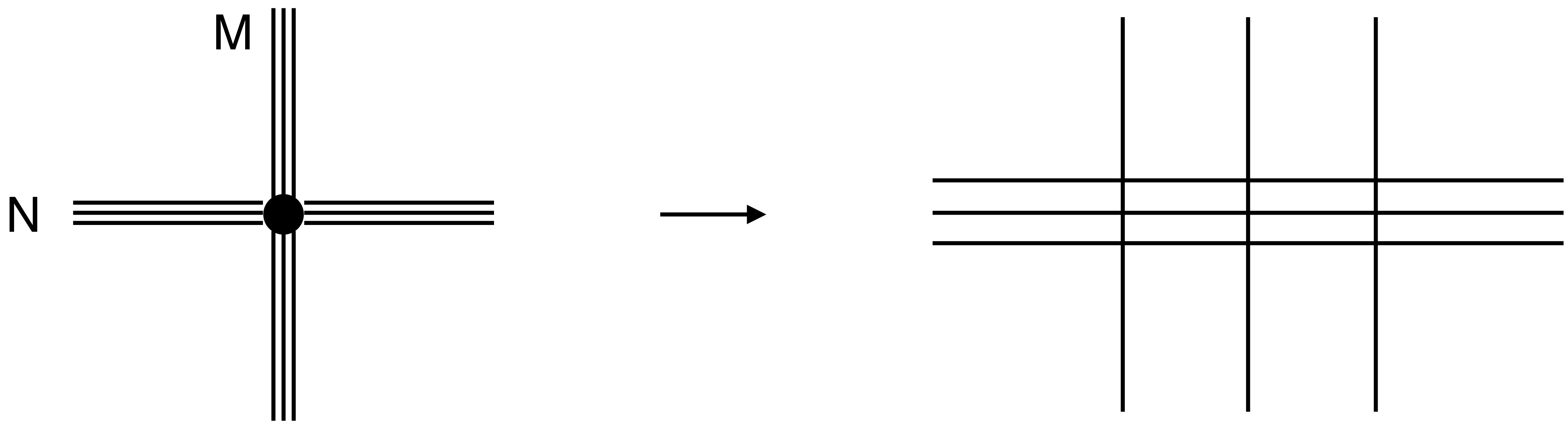
$$\mathcal{O}_{F1} \in (\mathbf{N}, \bar{\mathbf{N}}, \mathbf{1}, \mathbf{1})_{Q_{F1}} \qquad \Delta_{F1} = \frac{3}{2}M \qquad Q_{F1} = M$$

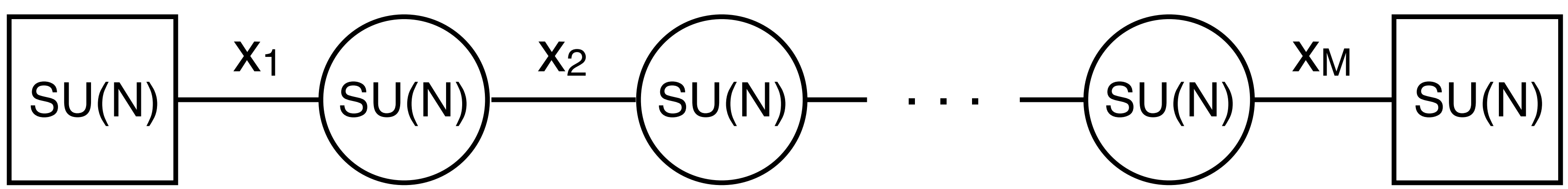
Open (0,1) string (D1-brane):

$$\mathcal{O}_{D1} \in (\mathbf{1}, \mathbf{1}, \mathbf{M}, \bar{\mathbf{M}})_{Q_{D1}} \qquad \Delta_{D1} = \frac{3}{2}N \qquad Q_{D1} = N$$

4. AdS_6/CFT_5 with strings

Look at the IR gauge theory obtained by deformation:





IR global symmetry = $SU(N)^2 \times U(1)_x^M \times U(1)_T^{M-1}$

$\subset$ UV global symmetry

4. AdS$_6$/CFT$_5$ with strings

Some of the string operators can be identified directly in the IR gauge theory, from which we can compute their dimension and U(1) charge.

F1 operator: $\mathcal{O}_{F1} = \mathcal{O}^a_b = [x_1 x_2 \cdots x_M]^a_b \in (\mathbf{N}, \bar{\mathbf{N}})$

$$\Delta_{F1} = \frac{3}{2}M \qquad Q_{F1} = \sum_{i=1}^{M} Q_{x_i} = M$$

Some components of D1 operator: $\mathcal{O}_{D1} \supset \mathcal{O}_i = \det x_i$

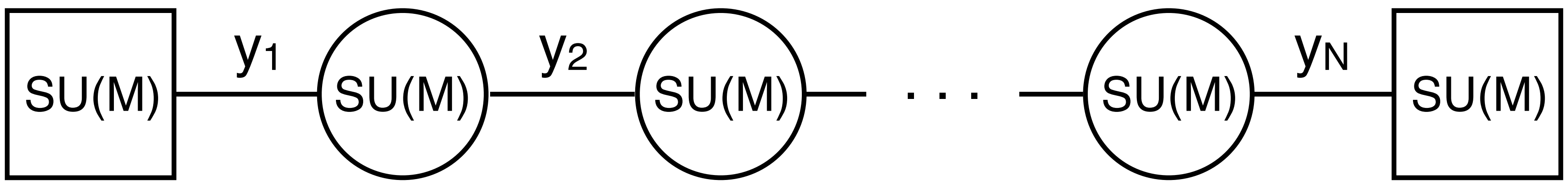
$$\Delta_{D1} = \frac{3}{2}N \qquad Q_{D1} = N$$

This also shows a chiral-ring relation between the operators:

$$\prod_{i=1}^{M} \mathcal{O}_i = \det \mathcal{O}^a_b \quad \Rightarrow \quad (\mathcal{O}_{D1})^M \sim (\mathcal{O}_{F1})^N$$

4. AdS_6/CFT_5 with strings

We get a complementary point of view from the S-dual IR gauge theory:



IR global symmetry = $SU(M)^2 \times U(1)_y^N \times U(1)_T^{N-1}$

$$\mathcal{O}_{D1} = \widetilde{\mathcal{O}}^a_b = [y_1 y_2 \cdots y_N]^a_b \in (\mathbf{M}, \bar{\mathbf{M}}) \quad \Delta_{D1} = \frac{3}{2}N \qquad Q_{D1} = N$$

$$\mathcal{O}_{F1} \supset \widetilde{\mathcal{O}}_i = \det y_i \qquad \Delta_{F1} = \frac{3}{2}M \qquad Q_{F1} = M$$

Again there is a chiral-ring relation:

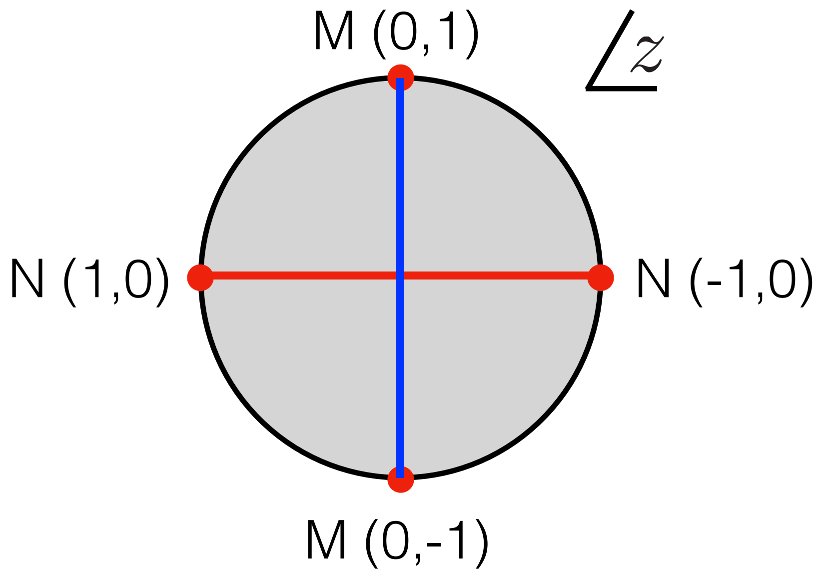
$$\prod_{i=1}^{M} \widetilde{\mathcal{O}}_i = \det \widetilde{\mathcal{O}}^a_b \quad \Longrightarrow \quad (\mathcal{O}_{D1})^M \sim (\mathcal{O}_{F1})^N$$

4. AdS_6/CFT_5 with strings

Dual Type IIB supergravity viewpoint

(1,0) string embedding: $z = \sigma$

$$m_{(1,0)} = \frac{3}{2}M = \Delta_{F1}$$



In agreement with the field theory for O(M) dimension operator.

(0,1) string embedding: $z = i\sigma$

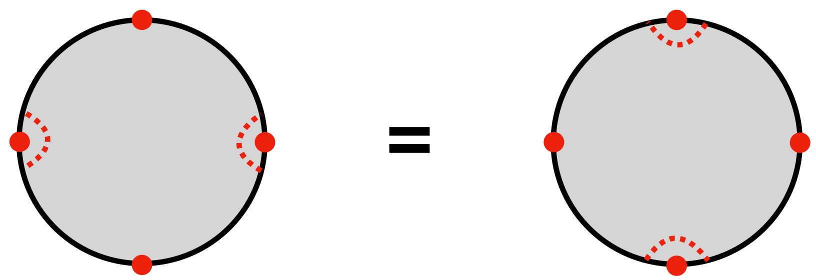
$$m_{(0,1)} = \frac{3}{2}N = \Delta_{D1}$$

In agreement with the field theory for O(N) dimension operator.

4. AdS$_6$/CFT$_5$ with strings

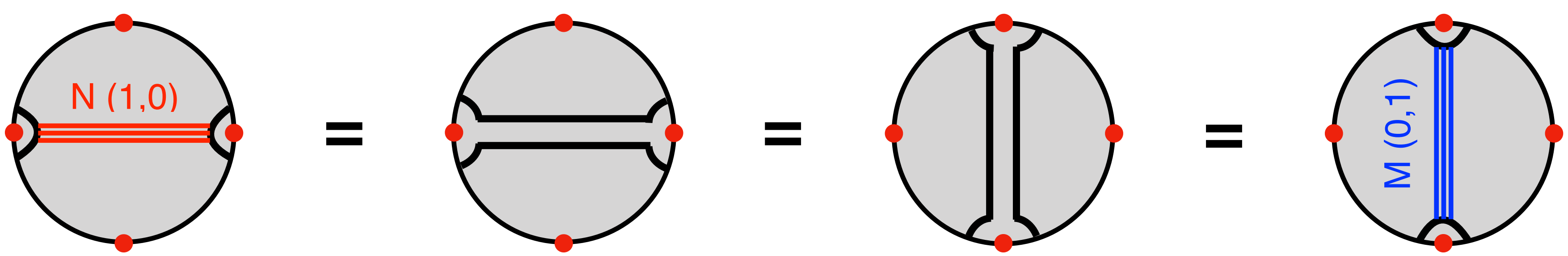
U(1) charges and chiral ring relation

Recall that the U(1)'s are associated to flux-free 3-cycles:



single U(1)

A 3-brane on this cycle describes a charged particle:



This is the chiral ring relation! $(\mathcal{O}_{D1})^M \sim (\mathcal{O}_{F1})^N$

It also gives the U(1) charges: $Q_{F1} = M \quad Q_{D1} = N$

4. AdS_6/CFT_5 with strings

B. The T_N theory

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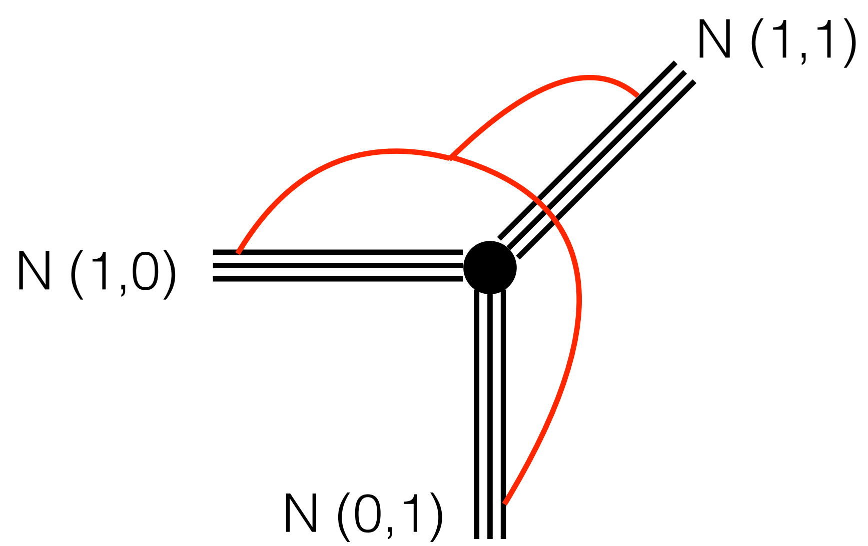


Generalization of the $E_6=T_3$ theory.

Global symmetry $SU(N)^3$

We cannot connect the 5-branes with open strings, but we can with an open 3-string, a (1,0)-(0,1)-(1,1) string junction:

$$\mathcal{O}_{3string} \in (\mathbf{N}, \mathbf{N}, \mathbf{N}) \qquad \Delta_{3string} = \frac{3}{2}(N-1)$$

This operator is known in the 4d version of the theory (Gaiotto-Maldacena).

4. AdS_6/CFT_5 with strings

This is consistent with known special cases:

N=2: $\Delta_{(\mathbf{2},\mathbf{2},\mathbf{2})} = \frac{3}{2}$

T_2 = theory of 4 free hypermutiplets

N=3: $\Delta_{(\mathbf{3},\mathbf{3},\mathbf{3})} = 3$

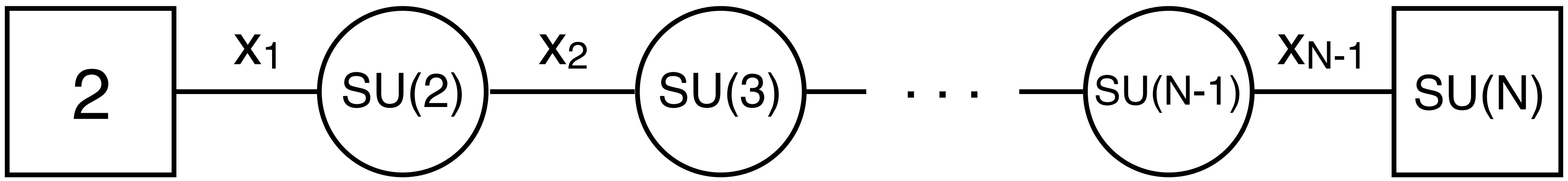
T_3 = E_6 theory

The 3-string operator is the scalar component of a conserved current multiplet which gives the expected enhanced global symmetry:

$$SU(3)^3 + (\mathbf{3}, \mathbf{3}, \mathbf{3}) + \mathrm{c.c} \to E_6$$

4. AdS_6/CFT_5 with strings

Some components of 3-string operator can also be identified explicitly in the IR gauge theory: Bergman + Zafrir



IR global symmetry = $SU(N) \times SO(4) \times U(1)_x^{N-2} \times U(1)_T^{N-2}$

$\subset$ UV global symmetry

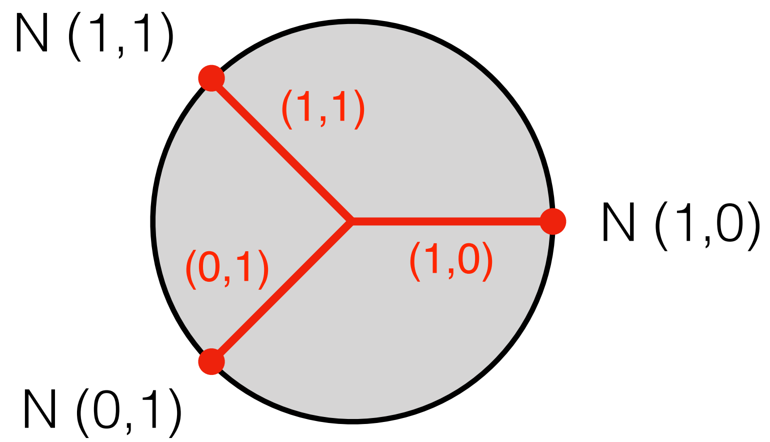
$$\left.\begin{array}{l} \mathcal{O}^a_{\tilde{b}} = [x_1 \cdots x_{N-1}]^a_{\tilde{b}} \\ \mathcal{O}_{a\tilde{b}} = \epsilon_{\alpha\beta}\, [\tilde{x}_1]^\alpha_a\, [x_2 \cdots x_{N-1}]^\beta_{\tilde{b}} \end{array}\right\} \in (\mathbf{N}, \mathbf{4}) \qquad \Delta = \frac{3}{2}(N-1)$$

4. AdS_6/CFT_5 with strings

Dual Type IIB supergravity viewpoint

(1,0) string embedding (0,1) string embedding (1,1) string embedding

$$z = \sigma \qquad z = e^{\pi i/3}\sigma \qquad z = e^{2\pi i/3}\sigma$$



$$m = m_{(1,0)} + m_{(0,1)} + m_{(1,1)} = \frac{3}{2}N$$

In agreement with the operator dimension for large N.

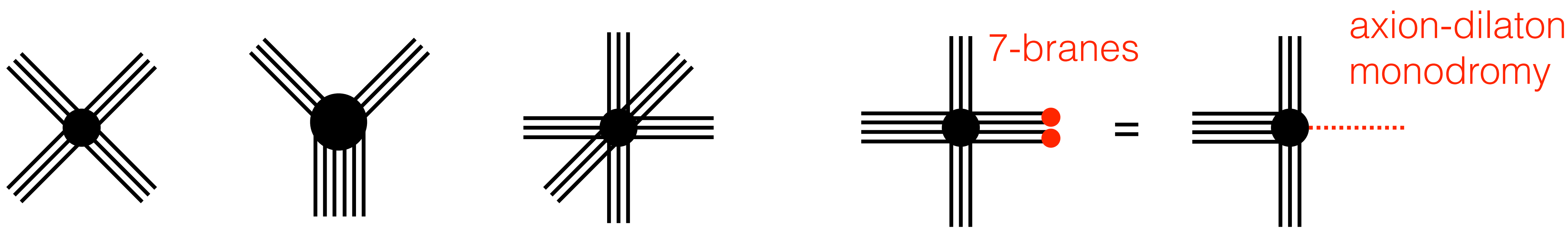
5. Conclusions and future directions

5. Conclusions and future directions

- A new class of Type IIB SUGRA solutions with a warped $AdS_6 \times S^2 \times \Sigma$ geometry and (p,q) 5-brane fluxes.
- Conjectured to be dual to 5d SCFTs described by the corresponding multi-5-brane junctions in flat space.
- This represents major progress in our understanding of AdS/CFT in 5d (previously only one example).
- I presented the first series of quantitative tests of this conjecture: a class of O(N) scaling-dimension operators dual to open strings and multi-strings embedded in Σ.

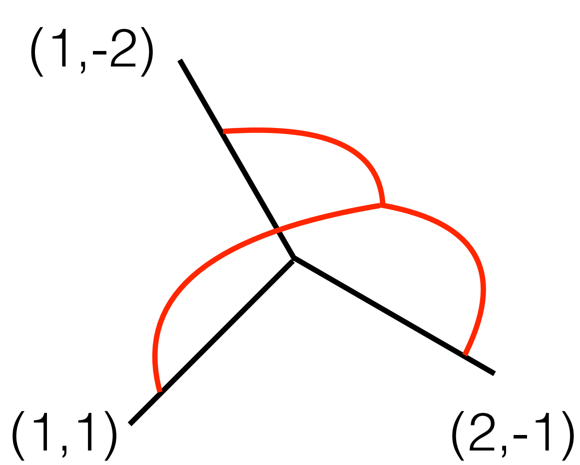
5. Conclusions and future directions

- More examples with gauge theory deformations:



Dual to Σ with branch points.

- There are also 5d SCFTs without any gauge theory deformations, e.g. the “E_0 theory”:



SUGRA dual of 3-string gives:

$$\Delta_{3string} = \frac{9}{2} N$$

Prediction for the SCFT …

5. Conclusions and future directions

More tests:

- Details of O(1) dimension operators - KK SUGRA spectrum.
- Other large dimension operators from wrapped 3-branes?
- S^5 free energy: Type IIB SUGRA result scales like N^4 (Gutperle, Marasinou, Trivella, Uhlemann), in contrast to Massive Type IIA SUGRA solution that scales like $N^{5/2}$.

  Very recent numerical analysis of SUSY localization agrees (Fluder, Uhlemann).

Thank you