Towards a cubic closed string field theory

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Outline: 1. Introduction

Introduction

Hubbard–Stratonovich action

IBL_∞ action

Conclusion

Motivations

String field theory:

- QFT with infinite number of spacetime fields
- split string amplitudes into Feynman diagrams
- motivations:
  - unified framework: off-shell amplitudes, renormalisation
  - classical solutions
  - consistency of string theory
  - path integral $\rightarrow$ non-perturbative effects

String field theory – definitions

- CFT Hilbert space $\mathcal{H}$, string field $\Psi \in \mathcal{H}$
- (bosonic) classical action

$$S = \sum_{n=2}^{\infty} \frac{g_s^{n-2}}{n!} \mathcal{V}_n(\Psi^n)$$

g_s : coupling constant

- string vertex $\mathcal{V}_n : \mathcal{H}^{\otimes n} \to \mathbb{C}$

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- BRST operator: $\ell_1 = Q_B$

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- string vertex $\mathcal{V}_n : \mathcal{H}^{\otimes n} \to \mathbb{C}$, string product $\ell_n : \mathcal{H}^{\otimes n} \to \mathcal{H}$
- BRST operator: $\ell_1 = Q_B$
- eom and gauge invariance (L_∞ structure)

$$\sum_{n=1}^{\infty} \frac{g_s^{n-1}}{n!} \ell_n(\Psi^n) = 0, \qquad \delta\Psi = \sum_{n=1}^{\infty} \frac{g_s^{n-1}}{n!} \ell_n(\Lambda, \Psi^{n-1})$$

String field

- Fourier expansion

$$|\Psi\rangle = \sum_r \int \frac{\mathrm{d}^D k}{(2\pi)^D} \, \psi_r(k) \, |k, r\rangle$$

  - $\psi_r(k)$: spacetime fields
  - $\{|k, r\rangle\}$: basis of $\mathcal{H}$
  - k : spacetime momentum (non-compact dim.)
  - r : discrete modes (Lorentz indices, compact dim., etc.)

- classical closed string

$$N_{\text{gh}}(\Psi) = 2, \qquad L_0^- \, |\Psi\rangle = b_0^- \, |\Psi\rangle = 0$$

String field theory – difficulties

- infinite number of interactions $\rightarrow$ non-polynomial action
- definition of vertex $\mathcal{V}_n$:
  - n holomorphic function $\{f_{n,i}\}$ (with constraints)
  - subspace of moduli space $\bar{\mathcal{V}}_n \subset \mathcal{M}_{0,n}$
  - gluing with propagators and summing
    $=$ single covering of $\mathcal{M}_{0,n}$

$\rightarrow$ no explicit representation for $n > 5$

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But:

- general properties understood, used to prove consistency of string theory [Sen-Zwiebach '94, Sen '15-18, ...]
- expect progress from [1806.00449, Headrick-Zwiebach][1806.00450, Headrick-Zwiebach]

The open string miracle

- open string: $\exists$ truncation to cubic interactions [Witten '86]

$$\ell_2(A, B) \sim A * B, \qquad \forall n > 2 : \quad \ell_n = 0$$

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- many explicit computations:
  - analytic and level-truncation solutions (tachyon condensation, brane decay, marginal deformations, defects, time-dependent backgrounds. . .)
  - effective actions
  - etc.

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Way out: use auxiliary fields to decompose interactions

1. Hubbard–Stratonovich transformation
2. IBL_∞ structure

Outline: 2. Hubbard–Stratonovich action

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Hubbard–Stratonovich transformation

Hubbard–Stratonovich (intermediate field) representation:

- originally: many-body physics (condensed matter and nuclear physics)
- decompose interactions to lower-order interactions through auxiliary fields
- optimal representation: cubic interactions, quadratic in the physical field, linear in the auxiliary fields

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- (vector) ϕ^4, ϕ^{2n} [1003.1037, Rivasseau-Wang][1601.02805, Lionni-Rivasseau], matrix and tensor models [1609.05018, Lionni-Rivasseau]

Scalar ϕ^4 model

- ϕ^4 action

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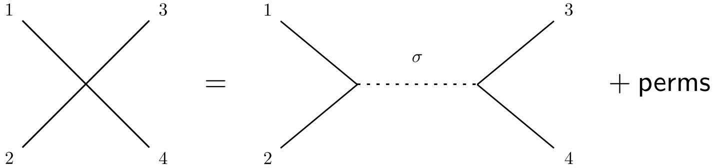
$$S_{\mathrm{HS}} = -\int \mathrm{d}^d x \left(\frac{1}{2}\,\phi K \phi - 2\lambda\,\frac{\sigma^2}{2} + 2\lambda\,\sigma\phi^2 \right)$$

- Feynman graphs



Scalar ϕ^4 vector model

- vector ϕ^4 [Weinberg vol. 2]

$$S = \int \mathrm{d}k_1 \cdots \mathrm{d}k_4 \, V_{i_1 i_2 i_3 i_4}(k_1, \ldots, k_4) \phi_{i_1}(k_1) \cdots \phi_{i_4}(k_4) + \frac{1}{2} \int \mathrm{d}k \, \phi_i(k) K_{ij}(k) \phi_j(-k)$$

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String field theory

- string field action to $O(g_s^2)$

$$S = \frac{1}{2}\,\mathcal{V}_2(\Psi^2) + \frac{g_s}{3!}\,\mathcal{V}_3(\Psi^3) + \frac{g_s^2}{4!}\,\mathcal{V}_4(\Psi^4)$$

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- introduce auxiliary field $\Sigma \in \mathcal{H}^{\otimes 2}$

$$\delta S = -\frac{g_s^2}{4!}\,\mathcal{V}_4(\Sigma - \Psi \otimes \Psi, \Sigma - \Psi \otimes \Psi)$$

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Properties

- new product $m_2 : \mathcal{H}^{\otimes 2} \to \mathcal{H}^{\otimes 2}$

$$\mathcal{V}_4(A_1, \ldots, A_4) = \langle A_1 \otimes A_2 | m_2(A_3, A_4) \rangle$$

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$$\ell_1(\Psi) + \frac{g_s}{2} \, \ell_2(\Psi^2) + \frac{g_s^2}{3!} \, \langle \Psi | \, m_2(\Sigma) = 0$$
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- Fourier expansion $\to$ entangled states

$$|\Sigma\rangle = \sum_{r,s} \int \frac{\mathrm{d}^D k_1}{(2\pi)^D} \frac{\mathrm{d}^D k_2}{(2\pi)^D} \, \sigma_{rs}(k_1, k_2) \, |\phi_r(k_1)\rangle \otimes |\phi_s(k_2)\rangle$$

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- action still not exactly cubic:
  - cubic in the field
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  - same properties as $\mathcal{V}'_3$ from [Sen-Zwiebach '94]
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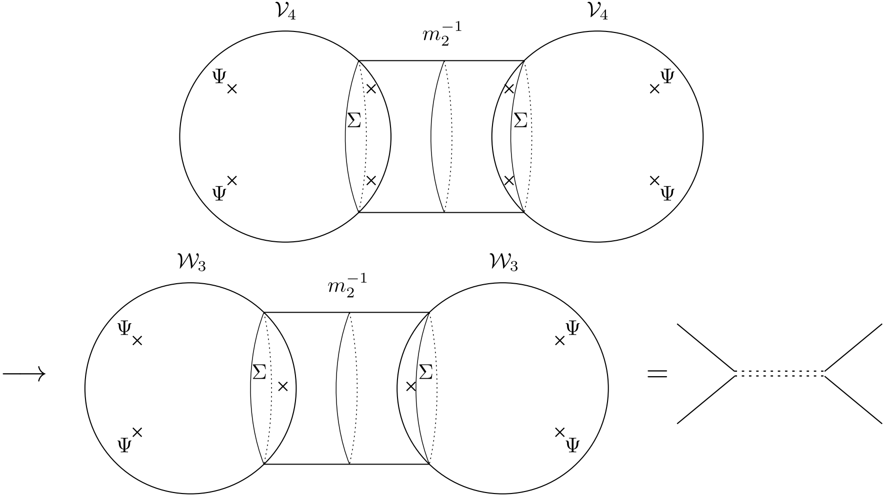
  - twist sewing of special puncture $\to$ reproduces $\partial\mathcal{V}_4$ (with $\mathrm{SL}(2, \mathbb{C})$ local coordinates)
- worldsheet: insert states of tensor product at different punctures $\to$ merge them together (no singularities)
  - $\mathcal{V}_4$: glue two punctures with two punctures
  - $\mathcal{W}_2$: glue one puncture with one puncture

Worldsheet interpretation (2)



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Questions:

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  $\rightarrow$ will encounter products with more outputs
- m_2: two inputs and two outputs
  $\rightarrow$ not a L_∞ product
- which structure will have the action?

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$\Longrightarrow$ IBL_∞ algebra

Outline: 3. IBL_∞ action

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IBL_∞ algebra

- symmetric tensor algebra (implicit symmetrization)

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- hat: embedding map from $\mathcal{H}^{\otimes n}$ to $S\mathcal{H}$

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- induces a BV algebra [1511.01591, Markl-Voronov]

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- note: generalize at quantum level (new index $g \geq 0$)

IBL_∞ string field theory

Conjecture 1

The non-polynomial L_∞ closed string field theory is equivalent to a IBL_∞ string field theory on-shell.

IBL_∞ string field theory

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Conjecture 2

There is a parametrization such that the action is cubic in the fields.

Comments

- the multi-output products are built from the L_∞ products

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- worldsheet interpretation?

Outline: 4. Conclusion

Introduction

Hubbard–Stratonovich action

IBL_∞ action

Conclusion

Outlook

Summary: path for constructing a cubic closed string field theory

- SFT with IBL_∞ algebra
- algebraic description of SFT with auxiliary fields

If correct:

- classical solutions (consistent truncation of auxiliary fields)
- thermal effects (σ saddle point = mean field theory)
- explicit computations of vertices and amplitudes
- non-perturbative effects

If not correct:

- learn more about the structure of SFT
- if holds up to some order, still useful for perturbative computations