# *EFFECTIVE HOPPING IN HOLOGRAPHIC BOSE AND FERMI HUBBARD MODELS*

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References: MF-Harrison-Karch-Meyer-Paquette, JHEP04(2015)068

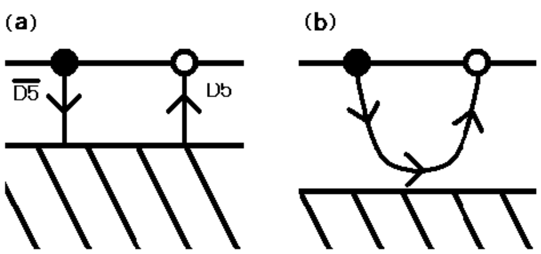
# *A holographic defect lattice in the probe limit*

- Dictionary of the holographic lattice in the probe limit

| Impurity in a lattice site | D5 branes on an $AdS_2$ slice |
|---|---|

  - An interesting lattice formulation such as the dimerization transition in the probe limit

    *Kachru-Karch-Yaida ``09, 10*




Picture taken from *Kachru-Karch-Yaida*

- A holographic Kondo model:

  A holographic $AdS_2$ superconductor coupled to an $AdS_3$ metallic state

  *Erdmenger-Hoyos-O'Bannon-Wu ``13*

  - A holographic Kondo model with two impurities (RKKY interactions) *O'Bannon-Papadimitriou-Probst ``15*

# *EFFECTIVE HOPPING IN HOLOGRAPHIC BOSE AND FERMI HUBBARD MODELS*

- **A holographic dual to the Bose-Hubbard model**
- **Effective hopping in a holographic Bose-Hubbard model on 2-site**
- **A top-down Hubbard model**
- **Effective hopping in the large N Bose-Hubbard model on 2-site**

# *A holographic dual to the Bose-Hubbard Model*

- Bose-Hubbard model as the effective theory on an optical lattice, including the hopping term + Short-range repulsive interactions $U$
  - The extension to the $SU(N)$ Bose-Hubbard model

Conjecture of *MF-Harrison-Karch-Meyer-Paquette, `14*

2d gravity on the $AdS_2$ hard wall

$SU(N)$ Bose-Hubbard model

$$H = -w\sum_{<ij>}(b_{ai}{}^{\dagger}b_{aj} + b_{aj}{}^{\dagger}b_{ai})$$

$$-\mu\sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1),\ n_i = b_{ai}^{\dagger}b_{ai}$$

| Dual Gravity Side | Large $N$ Bose-Hubbard model |
|---|---|
| $A_{t,i}$ | $\mu$ (chemical potential) & $b_i^{a\dagger}b_{ia}$ (occupation number) |
| $\phi_{i,j}$ | $t_{\rm hop}$ (hopping amplitude) & $b_i^{a\dagger}b_{ja}$ (hopping operator) |
| hard wall cut-off $u_h$ | $U$ (on-site Coulomb interaction) |

- Realization of the lobe-shaped phase structure, zero modes, etc..

# *A motivation: computation of the effective hopping*

- To compute the VEV of the hopping term in both sides of the duality concretely and compare them on 2-site
- A top-down Holographic Hubbard model and the effective hopping?
- The effective hopping and the Jordan-Wigner transformation

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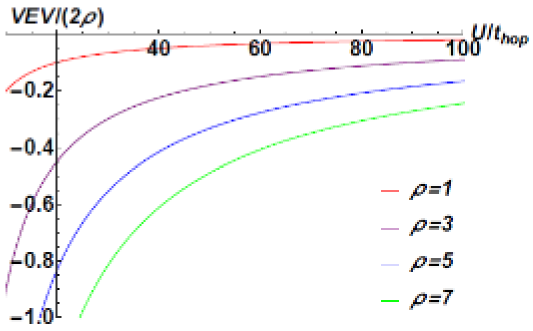
# *Effective hopping in the homogeneous phase*

- The effective hopping VEV= $\frac{dF}{dt_{hop}}$ ($F$ is free energy) with the IR boundary condition (Dirichlet b.c. on $\phi$)
  - fixed even occupation numbers & fixed IR parameters

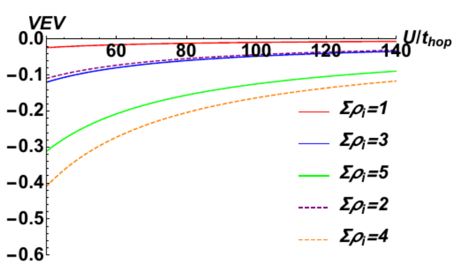


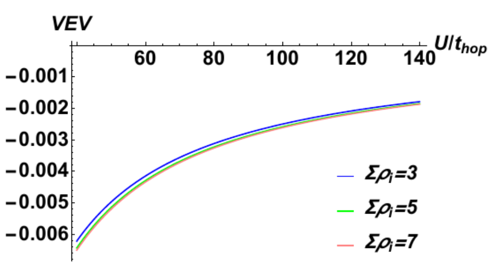

- Results and features:
  - Bi-local's VEV approaches zero as $\sim -4\rho^2\, t_{hop}/U$ at large $U/t_{hop}$
    $\rho$: charge density
  - The VEVs do not coincide at large $t_{hop}/U$ (the transition to superfluidity?)
  - $M$ corresponds to the anomalous dimension. The hopping term becomes relevant or irrelevant depending on $M$.

# *Effective hopping in the non-homogeneous phase*

- The more relevant VEV in the non-homogeneous phase
  - Fixed odd occupation numbers
  - Dirichlet (or Neumann) boundary condition

- Results and features: Smaller VEV in the non-homogeneous phase & Neumann b.c.
  - Dirichlet b.c. suits the actual Bose-Hubbard model




- Left: (non)-homogeneous phases (Dirichlet b.c.)

Right: non-homogeneous phase (Neumann b.c.)

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# A top-down holographic Hubbard model on 2-site

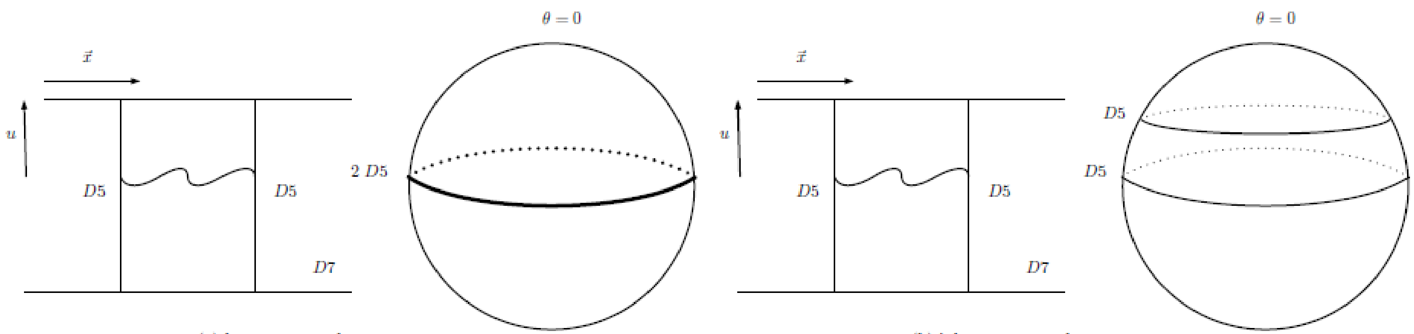
- A previous approach of the holographic Hubbard model: the D3 and non-Abelian D5 system *Kachru-Karch-Yaida `10*

- A D3-D7 system ⇔ $AdS_5$ soliton×$S^5$ and D7-brane at the tip

- Inserting defect $n_F = 2$ non-Abelian D5 branes in this D3-D7 system
  - ✧ The induced hard wall metric arised in the solitonic geometry
  - ✧ No D5-anti D5 unlike a previous approach

| | $x^0$ | $x^1$ | $x^2$ | $x^3=\tau$ | $x^4=u$ | $x^5$ | $x^6$ | $x^7$ | $x^8$ | $x^9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $N_c$ D3 | × | × | × | × | · | · | · | · | · | · |
| $k$ D7 | × | × | × | · | · | × | × | × | × | × |
| $n_F$ D5 | × | · | · | · | × | × | × | × | × | · |

# *Homogeneous and non-homogeneous phase (non-Abelian generalization)*

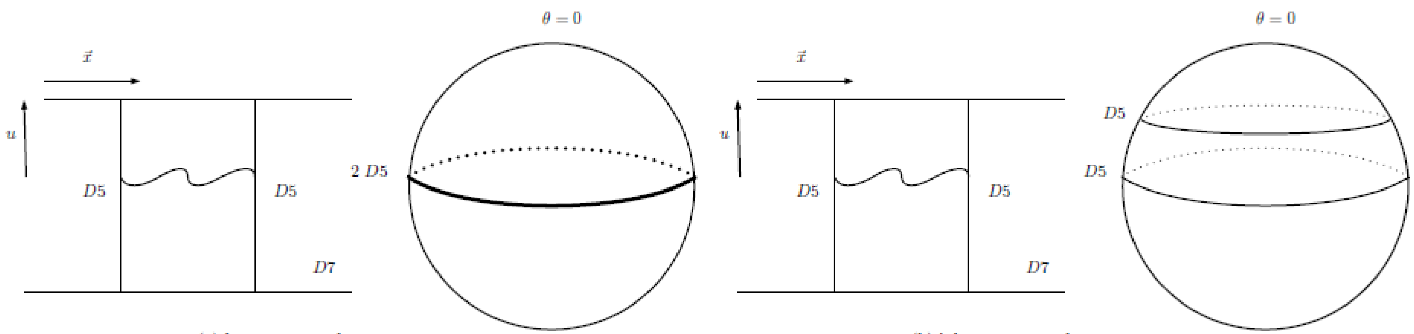



(a) homogeneous phase (b) inhomogeneous phase

$$A_b = \mathrm{diag}(a_b^{(1)}, a_b^{(2)}), \quad \theta = \mathrm{diag}(\theta_{11}, \theta_{22}), \quad \Phi^{x_1} = \begin{pmatrix} 0 & w^{x^1} \\ \bar{w}^{x_1} & 0 \end{pmatrix}$$

- The homogeneous phase: $a_b^{(1)} = a_b^{(2)}$, $\theta_{11} = \theta_{22}$
  - The eigenvalues of $\Phi^{x1}$ are $\pm|w^{x1}|$, which is the separation
- The inhomogeneous phase: $a_b^{(1)} \neq a_b^{(2)}$, $\theta_{11} \neq \theta_{22}$
  - The eigenvalues of $\Phi^{x1}$ are $\pm|w^{x1}|$

# *Homogeneous and non-homogeneous phase (non-Abelian generalization)*



(a) homogeneous phase (b) inhomogeneous phase

Symmetry breaking: $U(2) \rightarrow U(1)_b$

- The homogeneous phase: $a_b^{(1)} = a_b^{(2)}$, $\theta_{11} = \theta_{22}$
  - The eigenvalues of $\Phi^{x1}$ are $\pm|w^{x1}|$, which is the separation
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  - The eigenvalues of $\Phi^{x1}$ are $\pm|w^{x1}|$

# *Features of the top-down Fermi Hubbard model*

| **Top-Down Fermi Hubbard** |
|---|
| $n_F$ *U(1)* fields and bi-fundamentals $w_i$ |
| Bi-fundamental's mass *(ML)²=2* |
| Quantized number of F1 sting $n_i$, *Camino-Paredes-Ramallo `01* |
| Near half filling state ($n_i = N/2$) |
| Brane tension *O(N)* close to equator |

- Similar to a holographic large $N$ Bose-Hubbard model: In the large spin $N$ limit, statistics of bosons and fermions are the same.
- Free energy at the half filling realizes on-site interaction term with $U \sim u_h/N$.
  - Hopping term generated from the bi-fundamental $w_i$
  - Large $N$ scaling: $F \sim O(N), n_j \sim O(N), \langle c^{\dagger}_{1,\alpha} c_{2,\alpha} + c.c. \rangle \sim O(N)$

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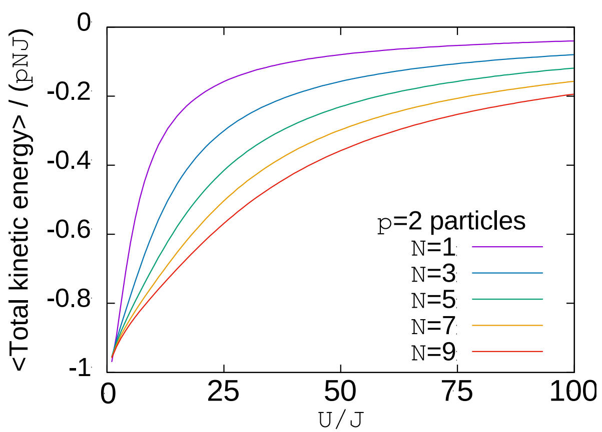
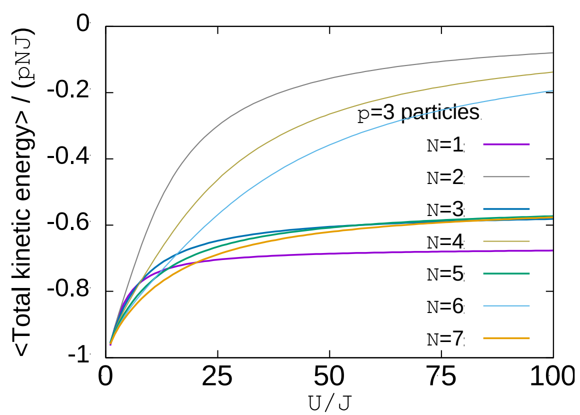
# *Effective hopping in the large N Bose-Hubbard model on 2-site*

Numerical result of VEV of hopping term fixing the particle number




Left ($2N$ particles): Mott insulator phase in even # of particles system at large $U$ (VEV$\sim 1/U$)

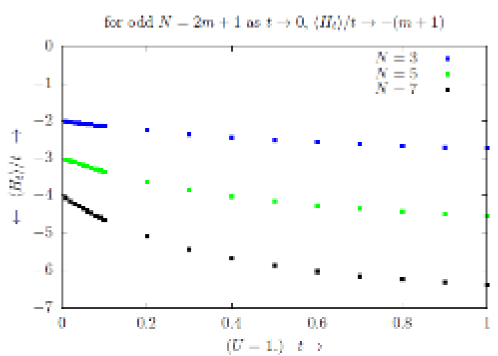
Right ($3N$ particles): VEV non-zero at large $U$ in the system of odd # of particles ⇔ Dirichlet b.c. gravity?

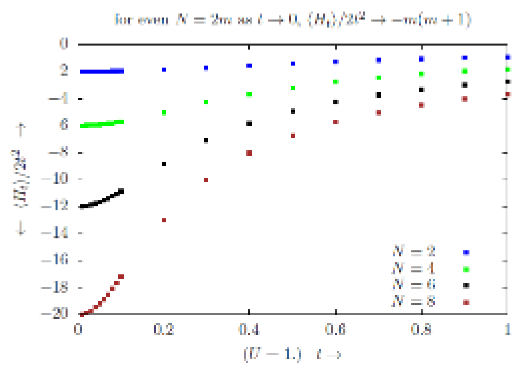
# *Quantum Mechanics for N particles*

- For *N* distinguished particles, the Hilbert space for the fermionic Hubbard model is same as the one of the Bose Hubbard model.

  - The Jordan-Wigner transformation: The hopping VEV for two cases are equal each other
  - The hopping VEV : $-(m+1)t$ for odd $N=2m+1$ and $t<<1$

    : $-m(m+1)t^2/U$ for even $N=2m$ and $t<<1$

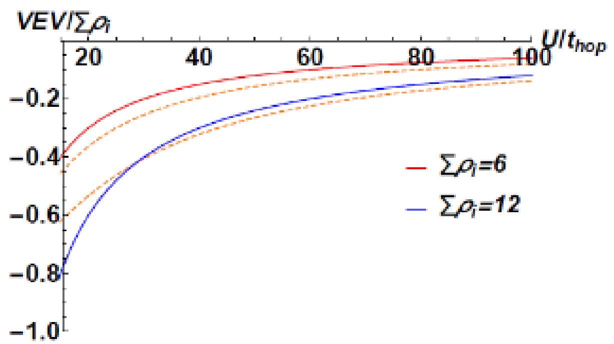


# *Discussion*

- Comparison of $\langle c^\dagger_{1,\alpha} c_{2,\alpha} + c_{1,\alpha} c^\dagger_{2,\alpha} \rangle$ :

  The gravity dual realizes the second order perturbation theory of the Bose-Hubbard model (large $U/t_{hop}$)




- Smaller VEV in the non-homogeneous phase & Neumann b.c.
  - A Dirichlet b.c. suits the actual Bose-Hubbard model

- A top down D3/D5/D7 system
  - Symmetry breaking: $U(2) \rightarrow U(1)_b$
  - Identification between Fermi-Hubbard at half filling and top-down model in the homogeneous phase

# *Thank you!*