AdS_3 at the String Scale

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Based on work with Lorenz Eberhardt, Kevin Ferreira, Rajesh Gopakumar, Chris Hull, Juan Jottar, and Wei Li.

Overview

I. The CFT dual of $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$

[Eberhardt, MRG, Gopakumar, Li ’17]
[Eberhardt, MRG, Li ’17]

II. Higher Spin Symmetry from Worldsheet

[MRG, Gopakumar, Hull ’17]
[Ferreira, MRG, Jottar ’17]
[MRG, Gopakumar ’18]

Symmetries

The dual CFT of string theory on

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

$$\mathrm{Vir} \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$$

with 4 supercharges

is believed to have a large $\mathcal{N} = 4$ superconformal symmetry.

[Boonstra, Peeters, Skenderis '98; Elitzur, Feinerman, Giveon, Tsabar '99; de Boer, Pasquinucci, Skenderis '99; Gukov, Martinec, Moore, Strominger '04; ...]

Dual CFT

Despite the fact that this is, in some sense, a bigger symmetry than the familiar small N=4 algebra

small $\mathcal{N} = 4$

$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ $\longleftrightarrow$ symmetric orbifold

string theory

$\mathrm{Sym}_{N+1}(\mathbb{T}^4) \equiv (\mathbb{T}^4)^{\otimes(N+1)}/S_{N+1}$

the dual CFT is not known in this case.

[Gukov, Martinec, Moore, Strominger ’04]

Large $\mathcal{N}=4$ mysteries

Part of the reason why this dual has not yet been determined, is due to the complicated structure of the BPS bounds of

$$D(2,1|\alpha) \subset A_\gamma$$

Symmetry algebra of supergravity = wedge subalgebra ($D(2,1|\alpha)$)

Large N=4 superconformal algebra (A_γ)

Large $\mathcal{N} = 4$

Since the large N=4 algebra contains two current algebras, the algebra is characterised by two parameters: in addition to the central charge

$$c = \frac{6k^+k^-}{k^+ + k^-}$$

[Sevrin, Troost, Van Proeyen, Schoutens, Spindel, Theodoridis '88-'90; Goddard, Schwimmer '88]

have parameter

$$\gamma = \frac{k^-}{k^+ + k^-} \, , \qquad \alpha = \frac{k^-}{k^+} = \frac{\gamma}{1-\gamma} \, .$$

($k^{\pm}$:size of the two S3s.)

BPS bound

Highest weight representations are parametrised by

$$(h;\, j^+, j^-, u)$$

conformal dimension

spins w.r.t the two su(2)s

u(1) eigenvalue

BPS bound

[Gunaydin, Petersen, Taormina, van Proeyen '89; Petersen, Taormina '90]

$$h_{A_\gamma} \geq \frac{1}{k^+ + k^-}\left[k^+ j^- + k^- j^+ + u^2 + (j^+ - j^-)^2\right] .$$

BPS bound

This is to be compared with BPS bound of $D(2,1|\alpha)$, i.e. the subalgebra generated by the wedge modes

$$L_0, L_{\pm 1} \ ; \quad G^a_{\pm\frac{1}{2}} \ ; \quad A_0^{\pm,i}$$

The relevant highest weight representations are parametrised by

$$(h; j^+, j^-)$$

since there is no u(1) charge.

BPS bound

The BPS bound of $D(2,1|\alpha)$ is [de Boer, Pasquinucci, Skenderis '99]

$$h_{D(2,1|\alpha)} \geq \frac{1}{k^+ + k^-}\left[k^+ j^- + k^- j^+\right] .$$

This differs from the BPS bound of A_γ from above

$$h_{A_\gamma} \geq \frac{1}{k^+ + k^-}\left[k^+ j^- + k^- j^+ + u^2 + \boxed{(j^+ - j^-)^2}\right] .$$

u(1) charge

BPS bound

So even if we restrict to u=0, the stringy BPS bound is stronger than the supergravity BPS bound

$$h \geq h_{A_\gamma} \geq h_{D(2,1|\alpha)}$$

with equality only if $j^+ = j^-$.

This leads to the strange phenomenon that any sugra BPS state with $j^+ \neq j^-$ **has** to acquire non-trivial quantum corrections, even just to satisfy the stringy BPS bound!

[de Boer, Pasquinucci, Skenderis '99]
[Gukov, Martinec, Moore, Strominger '04]

BPS spectrum

Furthermore, according to the analysis of
[de Boer, Pasquinucci, Skenderis ’99], the
sugra BPS spectrum does contain such states.

In addition, none of the dual CFT candidates had a matching BPS spectrum… It was therefore argued that only the index of [Gukov, Martinec, Moore, Strominger ’04] had to agree.

Constraint from index is however quite weak — as a consequence, no clear conclusion could be reached…

Problem revisited

Decided to revisit this problem by studying the world-sheet description of string theory on this background.

[Eberhardt, MRG, Gopakumar, Li '17]

For pure NS-NS flux, can describe the background in terms of WZW models

[Elitzur, Feinerman, Giveon, Tsabar '99]

$$\mathfrak{sl}(2,\mathbb{R})^{(1)}_{k} \oplus \mathfrak{su}(2)^{(1)}_{k^+} \oplus \mathfrak{su}(2)^{(1)}_{k^-} \oplus \mathfrak{u}(1)^{(1)}$$

Criticality:

$$\frac{1}{k} = \frac{1}{k^+} + \frac{1}{k^-} \quad \Rightarrow \quad k = \frac{k^+ k^-}{k^+ + k^-} \,.$$

Problem revisited

Impose physical state condition in covariant formulation (in NS-NS sector)

$$L_n \Phi = 0 \ , \quad n > 0 \ , \qquad G_r \Phi = 0 \ , \quad r > 0 \ , \qquad \left(L_0 - \frac{1}{2} \right) \Phi = 0$$

mass-shell condition

$$N = \frac{1}{2} + \frac{j_0(j_0 - 1)}{k} - \frac{j_0^+(j_0^+ + 1)}{k^+} - \frac{j_0^-(j_0^- + 1)}{k^-} \ .$$

spins of ground state representation

Problem revisited

Spacetime spectrum has A_γ symmetry at levels $k^\pm$, and the spacetime conformal dimension is to be identified with

[Elitzur, Feinerman, Giveon, Tsabar '99]
see also [Giveon, Kutasov, Seiberg '98]

$$L_0^{\text{spacetime}} = \mathcal{J}_0^{3\,\mathfrak{sl}(2,\mathbb{R})} \ ,$$

while the spins with respect to the two su(2)'s are directly the same.

With this dictionary in hand, we can then look for the spacetime BPS states using the worldsheet description.

Spacetime BPS spectrum

We have looked systematically for the states with smallest spacetime conformal dimension for a given choice of spins.

[Eberhardt, MRG, Gopakumar, Li '17]

For these states $N = \frac{1}{2}$ and $j = j_0 - 1$, and the smallest value of j turns to be

$$h = j = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{k\,j^+(j^+ + 1)}{k^+} + \frac{k\,j^-(j^- + 1)}{k^-}}$$
$$= -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{k^-\,j^+(j^+ + 1)}{k^+ + k^-} + \frac{k^+\,j^-(j^- + 1)}{k^+ + k^-}}\ .$$

[This is the analysis in unflowed sector for u=0; similar for flowed sectors.]

Spacetime BPS spectrum

Using the Maldacena-Ooguri (unitarity) bound

$$j_0 \leq \frac{k+1}{2}$$

[Hwang '91]
[Evans, MRG, Perry '98]
[Maldacena, Ooguri '00]

where $j = j_0 - 1$, we have checked that these states where

$$h = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{k^- j^+ (j^+ + 1)}{k^+ + k^-} + \frac{k^+ j^- (j^- + 1)}{k^+ + k^-}}$$

satisfy the spacetime A_γ BPS bound,

$$h_{A_\gamma} \geq \frac{1}{k^+ + k^-} \left[k^+ j^- + k^- j^+ + (j^+ - j^-)^2 \right] .$$

Spacetime BPS spectrum

$$h = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{k^- \, j^+(j^+ + 1)}{k^+ + k^-} + \frac{k^+ \, j^-(j^- + 1)}{k^+ + k^-}}$$

satisfy the spacetime A_γ BPS bound,

$$h_{A_\gamma} \geq \frac{1}{k^+ + k^-}\left[k^+ j^- + k^- j^+ + (j^+ - j^-)^2\right] .$$

but only saturate it for

$$\boxed{j^+ = j^-}$$

[Eberhardt, MRG, Gopakumar, Li ’17]

Sugra interpretation

Given the usual relation between string and sugra considerations, this suggests that the same conclusion should also hold in supergravity!

To confirm this, we have performed the KK reduction of 9d sugra, compactified on

$$\mathrm{S}^3 \times \mathrm{S}^3$$

adjusting the techniques of [Deger, Kaya, Sezgin, Sundell '98] to the present case.

[Restricted our analysis to the scalar NS-NS fields around a pure NS-NS background; note that this analysis had not been done by de Boer et.al. who had **assumed** that all harmonics would be BPS, and had only organised them in short multiplets using group theory.]

Sugra calculation

The calculation is a real tour de force, but the end result is simple: it confirms precisely the stringy prediction, and in particular shows that also in supergravity the only BPS states arise for

$$j^+ = j^- \ .$$

Furthermore, all supergravity states satisfy already automatically the A_γ bound, without the need for any miraculous quantum correction.

[Eberhardt, MRG, Gopakumar, Li ’17]

Consequences

This resolves this rather mysterious problem.

It also implies that in the search for the CFT dual, one may try again to match directly the BPS spectrum — without any need to invoke index arguments.

cf. also [Baggio, et.al. ’17]

In fact, there is a rather natural proposal for the dual CFT (at least for certain combinations of charges).

[Eberhardt, MRG, Li ’17]

Dual CFT

The $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$ background arises as the near horizon limit of

[Gukov, Martinec, Moore, Strominger '04]

$$Q_5^+ = k^+ \text{ D5-branes wrapping } \mathrm{S}^3 \times \mathrm{S}^1$$

$$Q_5^- = k^- \text{ flux units}$$

This suggests that dual CFT should be symmetric orbifold of

see also [Elitzur, Feinerman, Giveon, Tsabar '99]

$$\mathrm{S}^3 \times \mathrm{S}^1 \;\cong\; \mathfrak{su}(2)_k^{(1)} \oplus \mathfrak{u}(1)^{(1)}$$

Symmetric orbifold

$$\mathrm{S}^3 \times \mathrm{S}^1 \;\cong\; \mathfrak{su}(2)^{(1)}_k \oplus \mathfrak{u}(1)^{(1)}$$

is generated by $(\kappa = k-2)$

- 4 free fermions + 1 free boson
- $\mathfrak{su}(2)_\kappa$ current algebra

$\Big\} \cong \mathcal{S}_\kappa$ theory

has A_γ symmetry

We have analysed in detail the single particle BPS spectrum of this symmetric orbifold, ….

[Eberhardt, MRG, Li '17]

Symmetric orbifold

[Eberhardt, MRG, Li '17]

… and it matches exactly that of sugra or world-sheet analysis with the parameters $(\text{for } Q_5^- \geq Q_5^+)$

$$\left(\mathcal{S}_{(Q_5^-/Q_5^+)-1}\right)^{Q_1 Q_5^+} / S_{Q_1 Q_5^+}$$

- only makes sense if Q_5^-/Q_5^+ is integer (anomaly?)
- for $Q_5^+ = 1$ it agrees with instanton moduli space prediction
- for $Q_5^- \to \infty$ it leads to symmetric orbifold of $\mathbb{T}^4$

Symmetric orbifold

In fact, agreement of BPS spectra works as well as for the familiar case of $\mathbb{T}^4$:

- there are gaps in the worldsheet spectrum
- the agreement continues up to $h = \frac{c}{12}$

Incidentally, all BPS states are N=2 chiral primaries; in particular also moduli agree.

Other proposals

It would be very interesting to understand to which extent this fits together with the proposal of Tong '14 that takes a different brane configuration as the starting point.

It would also be very interesting to understand the CFT dual for the cases that are not covered by this proposal (i.e. if Q_5^- / Q_5^+ is not an integer.)

Overview

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II. Higher Spin Symmetry from Worldsheet

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Motivation

At the tensionless point in moduli space, string theory on AdS is dual to a (nearly) free conformal field theory.

The conserved currents of the free CFT correspond to massless higher spin fields in AdS, and the tensionless string theory contains a Vasiliev higher spin theory as a (closed) subsector.

[Fradkin & Vasiliev, '87]
[Vasiliev, '99…]

[Sundborg, '01], [Witten, '01], [Mikhailov, '02],
[Klebanov & Polyakov, '02], [Sezgin & Sundell, '03..]

AdS_3 example

[MRG, Gopakumar, '14]

Concrete realisation of this idea in context of AdS_3 :
CFT dual of string theory on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ at tensionless point is

$$\mathrm{Sym}(\mathbb{T}^4) \equiv (\mathbb{T}^4)^{\otimes(N+1)}/S_{N+1}$$

$$\cup$$

$$\mathcal{W}_\infty^{(\mathcal{N}=4)}[0]$$

CFT dual of Vasiliev higher spin theory on AdS_3

HS theories vs string theory

[MRG, Gopakumar '13]

This example arose as a particular limit of the duality between Higher Spin theories and dual CFTs with large $\mathcal{N}=4$ superconformal symmetry.

hs theory based on $\mathrm{shs}_2[\lambda]$ $\longleftrightarrow$ $$\frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2} \oplus \mathfrak{u}(1)_\kappa} \oplus \mathfrak{u}(1)_\kappa \ .$$

Wolf space cosets

[Sevrin, Troost, Van Proeyen, Schoutens, Spindel, .. '88/'89]

in 't Hooft limit with $\lambda = \dfrac{N+1}{N+k+2}$.

Direct understanding

The identification between higher spin theories and string theory is, so far, however rather indirect, i.e. can only see the higher spin symmetry via the dual CFT.

Try to find more direct description of it. This requires a world-sheet approach since the higher spin symmetry is only expected to emerge in the tensionless (stringy) limit — far away from usual supergravity regime.

Dual CFT

To start with, let us consider bosonic case, i.e. WZW model based on sl(2,R). [Maldacena, Ooguri '00]

The dual (`spacetime') CFT lives on the boundary of AdS3, and we have, as before, the identifications

$$L_0^{\rm CFT} = J_0^3 \ , \qquad L_1^{\rm CFT} = J_0^- \ , \qquad L_{-1}^{\rm CFT} = J_0^+ \ ,$$

with a similar relation for the right-movers.

The spacetime energy and spin are then given as

$$E = h + \bar{h} \ , \qquad s = h - \bar{h} \ .$$

spacetime conformal dimension of left- and right-movers

Massless higher spins

On the other hand, the AdS mass is

$$m^2_{\mathrm{AdS}_3} = (E - |s|)(E + |s| - 2)$$

Given that spacetime conformal dimensions are non-negative, massless higher spin fields only arise for

$$E = \pm s \qquad h = 0 \text{ or } \bar{h} = 0.$$

chiral fields of spacetime CFT

Physical states

This description is again covariant, i.e. we need to impose physical state condition

$$L_n^{\text{tot}}\Phi = 0 \quad n > 0$$
$$(L_0^{\text{tot}} - 1)\Phi = 0 \ .$$

In particular, the second condition (mass-shell) condition implies that

$$\frac{C}{k-2} + h_0 + N = 1 \ .$$

Casimir of sl(2,R)

World-sheet conformal dim. of internal CFT

Representations I

The sl(2,R) ground state representations that appear in the world-sheet spectrum are the

Discrete lowest weight reps:

$$\mathcal{D}^+_j : \qquad C = -j(j-1) \ , \quad J^-_0 |j,j\rangle = 0$$

↑

quasi-primary from spacetime CFT perspective!

Continuous reps:

$$\mathcal{C}(p,\alpha) : \qquad C = \tfrac{1}{4} + p^2 \ , \quad |j,m\rangle \text{ with } m \in \alpha + \mathbb{Z}$$

No-ghost theorem

Because of the Maldacena-Ooguri (unitarity) bound,

$$\text{MO-bound:} \qquad \frac{1}{2} < j < \frac{k-1}{2}$$

[Hwang '91]
[Evans, MRG, Perry '98]
[Maldacena, Ooguri '00]

the spectrum is bounded from above. Additional states are spectrally flowed images of these two classes of representations

[Maldacena, Ooguri '00]
see also [Henningson et.al. '91]

They are not Virasoro highest weight, and are therefore best described in terms of the spectral flow w.

Long Strings

Here the interpretation is that w is the winding number of the string around the boundary of AdS.

In particular, the w=1 continuous representation describes the long string running near the boundary of AdS. It is stable since

tension is compensated by the NS flux of the AdS space.

Physical spectrum

With these preparations at hand, we can now study the physical spectrum of the theory.

In particular, we can look systematically for massless (higher spin) fields, i.e., physical states with h=0, say.

Let us begin by analysing the unflowed discrete representations.

Unflowed discrete reps

Let us begin by analysing the unflowed discrete representations.

In this case, the mass-shell condition becomes

$$-\frac{j(j-1)}{k-2}+N=1$$

where we have set $h_0 = 0$. This can be rewritten as

$$j^2 - j - (k-2)(N-1) = 0$$

Unflowed discrete reps

At level N the sl(2,R) spin is at least

$$h = j - N \quad \overset{h=0}{\Longrightarrow} \quad j = N \ .$$

Plugging into the above equation then leads to

$$N^2 - N - (k-2)(N-1) = 0$$

There is one obvious solutions:

$$N = j = 1 : \quad \text{graviton}$$

Unflowed discrete reps

The other solution of the quadratic equation arises for

$$N = k - 2\ .$$

However, since N=j, this implies

$$j = k - 2 \geq \frac{k-1}{2} \qquad \begin{array}{l}(\text{for } j = N = 2, 3, \ldots, \\ \text{ i.e., } k = 4, 5, \ldots)\end{array}$$

Not allowed by the MO-bound!

Thus there are no massless higher spin fields from discrete unflowed representations. The same conclusion also holds for the spectrally flowed discrete reps.

Flowed representations

For the spectrally flowed continuous representations, the mass-shell condition becomes

$$\frac{C}{k-2} - wm - \frac{k}{4}w^2 + N = 1 \quad \text{where} \quad C = \frac{1}{4} + p^2$$

is the Casimir of the ground state representation and m the magn. quantum number. Demanding h=0 with

$$h = m + \frac{k}{2}w = 0 \quad \Longrightarrow \quad m = -\frac{wk}{2} \ ,$$

we get

$$\boxed{\frac{\frac{1}{4} + p^2}{k-2} + \frac{k}{4}w^2 + N = 1}$$

Flowed representations

$$\boxed{\frac{\frac{1}{4}+p^2}{k-2}+\frac{k}{4}w^2+N=1}$$

For spectral flow w=1, the mass-shell condition becomes (for N=0)

$$p^2+\frac{1}{4}=-\frac{k^2}{4}+\frac{3}{2}k-2$$

which has the solution

$$k=3 \qquad \text{and} \qquad p=0\ .$$

Higher Spin Symmetry

In fact, an infinite set of higher spin fields becomes massless at this point: for the right-movers we have to solve the right-moving analogue of

$$\boxed{\frac{C}{k-2} - wm - \frac{k}{4}w^2 + N = 1} \qquad (k=3, p=0, w=1)$$

i.e.

$$\frac{1}{4} - \bar{m} - \frac{3}{4} + \bar{N} = 1$$

which is solved by

$$\bar{m} = -\frac{3}{2} + \bar{N} \ .$$

Thus get a massless higher spin field for every right-moving excitation (and similarly for left-movers)!

Supersymmetric version

The analysis of the supersymmetric version of this theory is similar. There are two interesting cases:

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4 \qquad\qquad \mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$\mathfrak{sl}(2)_k \oplus \mathfrak{su}(2)_{k'} \oplus \mathfrak{u}(1)^4 \qquad\qquad \mathfrak{sl}(2)_k \oplus \mathfrak{su}(2)_{k_+} \oplus \mathfrak{su}(2)_{k_-} \oplus \mathfrak{u}(1)^4$$

[N=1 susy WZW models]

Criticality: $k = k'$

$$\frac{1}{k} = \frac{1}{k_+} + \frac{1}{k_-}$$

Massless higher spins

The analogue of k=3 in the bosonic case is now

$$k = 1 \qquad \text{[corresponds to } k = 3 \text{ for bosonic } \mathfrak{sl}(2)\text{]}$$

For this value of the level, an infinite tower of massless higher spin fields appears in the w=1 spectrally flowed continuous representation with p=0.

In fact, a stronger statement is true: at k=3 and p=0, the susy mass-shell condition (in NS sector)

$$\frac{C}{k-2} - wm - \frac{k}{4}w^2 + N = \frac{1}{2} \qquad \text{where} \qquad C = \frac{1}{4} + p^2$$

Full spectrum

becomes for generic w

$$\frac{1}{4} - w\left(m + \frac{w}{4}\right) + N = \frac{1}{2} \ .$$

Solving for m and observing that the actual J_0^3 eigenvalue is

$$h = m + \frac{w}{2} = \frac{N}{w} + \frac{w^2-1}{4w} \ .$$

w-twisted modes

ground state energy in w-twisted sector

Symmetric orbifold formula for cycle length w!

Full symmetric orbifold

Thus we recover the full single-particle spectrum of the symmetric orbifold.

[MRG, Gopakumar '18]
see also [Giribet, et.al. '18]

However, there are three subtleties:

(1) Fermions and GSO
(2) Which orbifold do we actually get?
(3) Compatibility with OPE structure

Fermions

On the world-sheet, the fermions are GSO-projected and appear in both NS and R sector.

However, the dual CFT should **not** have a GSO projection, and only the perturbative (NS sector) should appear.

The relation is quite subtle since GSO projection depends on cardinality of flow, and structure of twisted sector on cardinality of the twist. However, everything comes out correctly in the end, using the abstruse identity.

[MRG, Gopakumar ’18]

Which orbifold

For $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$ the situation is cleanest: at k=1, criticality leads to

$$k_+ = k_- = 2$$

and thus the bosonic su(2) factors do not contribute at all. Then there are 4 bosons from

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

3 - - 1 = 4

which are reduced to 2 by physical state condition.

2 bos + 8 fer. = $(\mathcal{S}_0)^2$

[MRG, Gopakumar ’18]

Which orbifold

For $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ at k=1, criticality leads to $k' = 1$. Then the bosonic su(2) factor appears at level -1, and we can use

[Goddard, Olive, Waterson '87]

$$\mathfrak{su}(2)_{-1} \oplus \mathfrak{u}(1) = 4 \text{ symplectic bosons}$$

This leads to the boson counting

$$\mathrm{AdS}_3 \times \underbrace{\mathrm{S}^3 \times \mathrm{S}^1}_{} \times \mathbb{T}^3$$

3 4 sympl. 3 = 6 + 4 sympl.

i.e. to 4 real bosons (and 4 symplectic bosons) after the physical state condition is imposed.

Which orbifold

The 4 symplectic bosons behave as ghosts (at least for the partition function) and remove precisely 4 of the 8 fermions.

(They also lead automatically to the correct ground state energy in the twisted setor.)

Thus we end up with 4+4 free bosons and fermions, i.e. with the

symmetric orbifold of $\mathbb{T}^4$.

[MRG, Gopakumar '18]

OPE structure

The fusion rules of the world-sheet continuous representations are

[Maldacena, Ooguri '01]

$$[w_1] \otimes [w_2] = [w_1 + w_2 - 1] \oplus [w_1 + w_2] \oplus [w_1 + w_2 + 1]$$

whereas the single-particle states of the symmetric orbifold have OPEs

[Jevicki, Mihailescu, Ramgoolam '98]
[Pakman, Rastelli, Razamat '09]

$$[w_1] \otimes [w_2] = [w_1 + w_2 - 1] \oplus [w_1 + w_2 - 3] \oplus [w_1 + w_2 - 5] \oplus \cdots$$

see also [Giribet, et.al. '18]

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$$[w_1] \otimes [w_2] = \boxed{[w_1 + w_2 - 1]} \oplus [w_1 + w_2 - 3] \oplus [w_1 + w_2 - 5] \oplus \cdots$$

see also [Giribet, et.al. '18]

Chiral algebra

However, at least for the chiral fields in w=1 (untwisted sector), the symmetric orbifold OPE is trivial, and in the world-sheet theory the actual OPEs also trivialise since these fields have

$$\bar{h} = 0 \quad \Rightarrow \quad w = 1 \ , \quad p = 0 \ .$$

↑ states in trivial sl(2,R) rep

[Fusion respects tensor product rules of sl(2,R) reps.]

[MRG, Gopakumar, in progress]

Chiral algebra

Thus we can probably only conclude that the chiral symmetry algebra agrees….

Chiral algebra

Thus we can probably only conclude that the chiral symmetry algebra agrees….

… but this symmetry algebra is very large: Higher Spin Square.

In particular, this suggests that the presence of this extended higher spin symmetry fixes essentially the structure of the theory.

Conclusions I

- Shown that the BPS spectrum of string theory and supergravity on

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

agrees and contains only states with $j^+ = j^-$.

- Identified a natural candidate for dual CFT that reproduces correct BPS spectrum:

$$\left(\mathcal{S}_{(Q_5^-/Q_5^+)-1}\right)^{Q_1 Q_5^+} / S_{Q_1 Q_5^+}$$

Conclusions II

- Analysed whether string theory on AdS3 has massless higher spin fields, using the WZW world-sheet approach

  massless higher spin fields appear for k=1 from long strings

- In fact, the k=1 theory contains a sector that matches exactly the spectrum of the symmetric orbifold.

Thank you!