Towards Entanglement of Purification for Conformal Field Theories

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Entanglement of Purification (EoP)



Optimized solution is hard to find...

Optimized purification ?



Object having the information of EoP in CFT ?

Our proposal for holographic CFT₂ : $E_P = - \left. \frac{\partial}{\partial n} \mathcal{F}_{\sigma_n} \right|_{n \to 1}$, where \mathcal{F}_{σ_n} is conformal block of twist operator.

This proposal is based on "**some heuristic arguments using the tensor network model of holography**". We did **not** drive this formula from the def of EoP.

This formula agree with the **EoP=EWC (Entanglement Wedge Cross-section)** conjecture in holographic CFT₂. [Umemoto, Takayanagi '17] [Nguyen, Devakul, Halbasch, Zaletel, Swingle '17]

"Holographic code model"

[Pastawski, Yoshida, Harlow, Preskill '15]





duality map = Isometry map

$$\mathcal{H}_{bulk}
ightarrow \mathcal{H}_{bdry}$$

"Ryu-Takayanagi formula" [Ryu, Takayanagi '06]





W "Bulk (HKLL) reconstruction" [Hamilton, Kabat, Lifschytz, Lowe '06]



Holographic code model captures the entanglement structure in AdS/CFT duality.



[Nguyen, Devakul, Halbasch, Zaletel, Swingle '17]

EoP by "replica trick" with twist op on geodesic

Assume followings (abstractly)

duality with the deformed boundary,

=bdry of EW of A and B

• optimized purification $|\Psi_{opt}\rangle$ defined on the bdry of the EW of A and B. (c.f. Miyaji, Takayanagi '15)

 ϕ_n , $\overline{\phi}_n$: "(anti-)twist operator" acting on this purified state.

$$E_P(\rho_{AB}) = -\frac{\partial}{\partial n} \left\langle \Psi_{opt} | \phi_n \overline{\phi}_n | \Psi_{opt} \right\rangle \Big|_{n \to 1}$$





 $\mathsf{Boundary} = A \cup B \cup \overline{AB}$

 $\rho_{AB} = \frac{\mathrm{Tr}}{AB} |0\rangle \langle 0|$

Boundary $= A \cup B$ $\rho_{AB} = |0\rangle \langle 0|$:pure state



"Bulk (HKLL) reconstruction"

[Hamilton, Kabat, Lifschytz, Lowe '06]



$$E_P(\rho_{AB}) = -\frac{\partial}{\partial n} \langle \phi_n \bar{\phi}_n \rangle \Big|_{n \to 1} = -\frac{\partial}{\partial n} \mathcal{F}_{\sigma_n} \Big|_{n \to 1}$$



"bulk-bulk (twist) propagator" on geodesic

[Hijano, Kraus, Perlmutter, and Snively '15]

Conformal Block in large-c CFT

in which twist op appears as internal line.

Summary & Discussion

Our proposal for holographic CFT₂:

$$E_P(A:B) = -\frac{\partial}{\partial n} \mathcal{F}_{\sigma_n}\Big|_{n \to 1}$$
,where \mathcal{F}_{σ_n} :conformal block of twist operator.
This formula agree with EoP=EWC conjecture.

• We also studied the conformal blocks which give the EWC in static BTZ geometry in large-c limit.

Summary & Discussion

Our *proposal* for holographic CFT₂ :

$$E_P(A:B) = -rac{\partial}{\partial n} {\mathcal F}_{\sigma_n} \Big|_{n o 1}$$

,where \mathcal{F}_{σ_n} :conformal block of twist operator.

This formula agree with **EoP=EWC** conjecture.

- **Direct computation of EoP on cMERA ?**
- In higer dimensional CFT ?
- For more generic QFT ?