Light-cone gauge string field theory and dimensional regularization - Computation of FI D terms

N. Ishibashi

University of Tsukuba, Tomonaga center for the history of the universe

18 July 2018

New frontiers in string theory 2018

String field theory (SFT)

String field

$$x^\mu(\sigma) \longrightarrow \Phi[x(\sigma)]$$

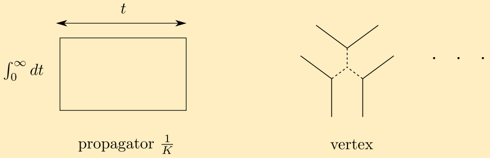
String field action

$$S = \Phi K \Phi + \Phi^3 + \cdots$$

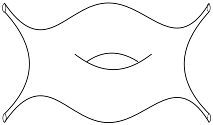
- We would like to discuss dynamical problems of superstring theory using SFT.

Feynman amplitudes of superstring field theory

$$S = \Phi K \Phi + \Phi^3 + \cdots$$

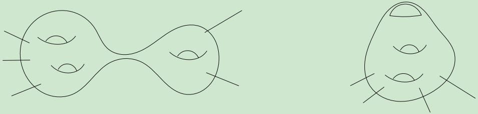


- Amplitudes can be calculated perturbatively.
- The results coincide with those from the first quantized theory.

$$A = \sum_{\text{worldsheet}}$$ 

Divergences of the Feynman amplitudes for superstrings

1. Infrared divergences (physical)



2. Spurious singularities (unphysical)

- The amplitudes of a valid superstring field theory should be free of the divergences of the second kind.

In the case of LC gauge superstring field theory

These divergences can be regularized by formulating the theory in noncritical dimensions.

In this talk, I would like to explain

- what the spurious singularities are
- how the regularization works
- computation of Fayet-Iliopoulos D terms using the formulation

Based on collaborations with Baba and Murakami and N. I. in progress

Outline

1. Divergences of Feynman amplitudes for superstrings

2. Light-cone gauge superstring field theory

3. Computation of Fayet-Iliopoulos D terms

4. Conclusions

§1 Divergences of Feynman amplitudes for superstrings

$$A = \int_{\mathcal{M}} \prod_K dt_K \left[\left\langle V_1 \cdots V_N \prod_\alpha \int_{C_\alpha} b \prod_i X(z_i) \right\rangle + \cdots \right]$$

M : moduli space of the Riemann surface

The integrand becomes singular at

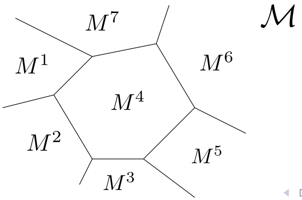
1. $t = t_0 \in \partial\mathcal{M}$: infrared divergences
2. $t = t_0 \notin \partial\mathcal{M}$: spurious singularities

- In the 1-st quantized formalism, this expression is derived by fixing the local symmetries on the worldsheet. GO
- The integrand may diverge at the point where the gauge slice is not transverse to the gauge orbit. $\rightarrow$ spurious singularities GO

Spurious singularities in 1-st quantized formalism

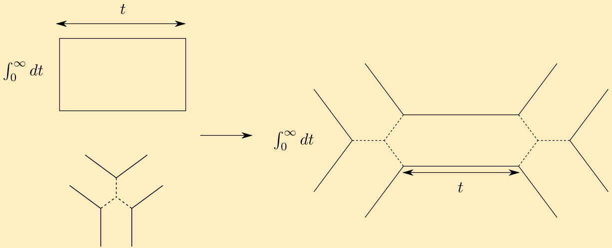
$$A = \int_{\mathcal{M}} \prod_K dt_K \left[\left\langle V_1 \cdots V_N \prod_\alpha \int_{C_\alpha} b \prod_i X\left(z_i\left(t\right)\right) \right\rangle + \cdots \right]$$

- It is difficult to find a goood gauge slice globally on $\mathcal{M}$. A possible way to go is to divide $\mathcal{M}$ into patches. (Sen-Witten)
  - It is possible to find a good slice in each patch.
  - One can get and expression of A with contributions from the boundaries of the patches.



Spurious singularities in SFT

SFT amplitudes coincide with those from the 1-st quantized approach.



- An SFT corresponds to a specific choice of the gauge slice.
- The Feynman rule of SFT should yield a good gauge slice for any Riemann surface.

Sen's SFT for closed superstrings

master action in BV formalism

$$S = \frac{1}{g_s^2}\left[-\frac{1}{2}\left\langle\tilde{\Psi}\middle| c_0^- Q_B \mathcal{G} \middle|\tilde{\Psi}\right\rangle + \left\langle\tilde{\Psi}\middle| c_0^- Q_B \middle|\Psi\right\rangle + \sum_{n=1}^{\infty}\{\{\Psi^n\}\}\right]$$

- infinitely many interaction terms of order $\hbar^k$ $(k = 0, 1, 2, \cdots)$
- There are a lot of freedom in choosing them (adding stubs, etc.)
- One can arrange these interaction terms order by order in $\hbar$ so that the amplitudes are free of spurious singularities.

§2 Light-cone gauge superstring field theory

LC gauge → string field

$$\begin{cases} X^+ = t \\ \psi^+ = 0 \end{cases} \longrightarrow \Phi\left[t, \alpha, X^i(\sigma), \psi^i(\sigma), \bar{\lambda}^A(\sigma)\right]$$

- Only physical degrees of freedom
  - Lorentz invariance, supersymmetry, etc. are not manifest
- Simple SFT action GO
- Tractable spurious singularities GO
  - All we should deal with are the contact term divergences

Dimensional regularization

LC SFT can be formulated in any spacetime dimensions.

$$S = \int \left[\frac{1}{2} \Phi \cdot (i\alpha\partial_t - H) \Phi + \frac{g_s}{3} \Phi \cdot (\Phi * \Phi) \right]$$

$$A = \sum_{\text{spin structure}} \int \prod_K dt_K \left\langle V_1^{\text{LC}} \cdots V_N^{\text{LC}} \prod_{I=1}^{2g-2+N} \left[(\partial^2 \rho)^{-\frac{3}{4}} T_F^{\text{LC}}(z_I) \right] \right\rangle_{\text{LC}} e^{-\frac{d-2}{8}\Gamma}$$

- $e^{-\Gamma}$ diverges when the LC diagram becomes singular. GO
- Taking d to be large and negative, divergences are regularized. GO
  - $i\alpha\partial_t - H \sim p^2 - m^2 - \frac{10-d}{8}$: $\frac{10-d}{8}$ works as an infrared regulator
  - Chiral fermions are dealt with by considering a linear dilaton background.

$Q^2 \sim \frac{10-d}{8} \to 0$

$$A = \sum_{\text{spin structure}} \int \prod_K dt_K \left\langle V_1^{\mathrm{LC}} \cdots V_N^{\mathrm{LC}} \prod_{I=1}^{2g-2+N} \left[\left(\partial^2 \rho\right)^{-\frac{3}{4}} T_F^{\mathrm{LC}}(z_I) \right] \right\rangle_{\mathrm{LC}} e^{-(1-Q^2)\Gamma}$$
$$= \sum_{\text{spin structure}} \int_{\mathcal{M}} \prod_K dt_K \left\langle V_1 \cdots V_N \prod_\alpha \int_{C_K} b \prod_I X(z_I) \right\rangle$$

- The amplitudes can be expressed using a conformal gauge worldsheet theory with $Q_B^2 = 0$. GO
    - The regularization preserves the gauge invariance.
- The conformal gauge expression coincides with the one from the 1-st quantized formalism by Sen-Witten in the limit $Q \to 0$, if the latter is (absolutely) convergent.

§3 Computation of Fayet-Iliopoulos D terms

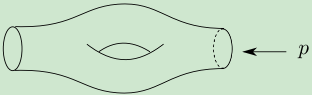
SO(32) heterotic string theory compactified on a CY-manifold with $A_i = \omega_i$

With anomalous $U(1)$'s, FI D terms appear at one loop

$$\begin{aligned} V &= -\frac{1}{2}D^2 + D\left(cg_s^2 - |\phi|^2\right) + \cdots \\ &\to \frac{1}{2}\left(cg_s^2 - |\phi|^2\right)^2 + \cdots \end{aligned}$$

- The supersymmetric vacuum is at $|\phi|^2 = cg_s^2$
- $c > 0$ can be obtained by calculating the tachyonic mass $m^2 = -cg_s^2$ of ϕ at the classical vacuum $\phi = 0$. (Dine-Seiberg-Witten, Dine-Ichinose-Seiberg, Atick-Dixon-Sen, Green-Seiberg, ..., Witten, Sen)

Computation of the m^2

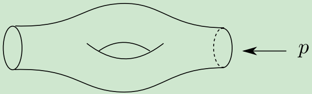


- One loop mass correction

$$
\begin{aligned}
\Sigma\left(p^2\right)\Big|_{p^2=0} &\sim \int d^2\tau d^2 z \left\langle V^{(0)}(z,\bar{z})\, V^{(0)}(0,0)\right\rangle\Big|_{p^2=0} \\
&\sim \int d^2\tau d^2 z \left[p^2 |z|^{-2-2p^2} \langle V_D(0,0)\rangle\right]\Big|_{p^2=0} \\
&\sim \int d^2\tau \langle V_D(0,0)\rangle
\end{aligned}
$$

- Sen's SFT reproduces this result. Sen went further and described the supersymmetric vacuum using the SFT.

Computation of the m^2 by LC SFT



- With the infrared regulator $Q^2 \sim \frac{10-d}{8} \to 0$

$$\begin{aligned}\Sigma\left(p^2\right)\Big|_{p^2=0} &\sim \int d^2\tau d^2 z \left\langle V^{(0)}(z,\bar{z})\, V^{(0)}(0,0)\right\rangle\Big|_{p^2=0} \\ &\sim \int d^2\tau d^2 z \left[|z|^{-3Q^2}\,\bar{z}^{-1}\left\langle \psi^{-}(z)\,\psi^{-}(0)\right\rangle \langle V_D(0,0)\rangle\right]\Big|_{p^2=0} \\ &\sim \int d^2\tau\, \langle V_D(0,0)\rangle\, f(\tau,\bar{\tau})\end{aligned}$$

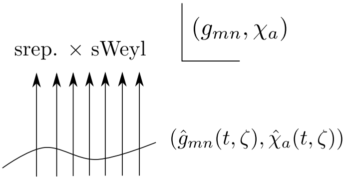
- We have not been able to check if this agrees with the known result.

§4 Conclusions

- In order to regularize the divergences of the Feynman amplitudes, we formulate light-cone gauge super string field theory in noncritical dimensions.
- Taking $d \to 10$, we obtain the amplitudes which coincide with those from the first quantized approach.
- FI D terms may be calculated using the formalism.

1-st quantized amplitudes ▸ BACK

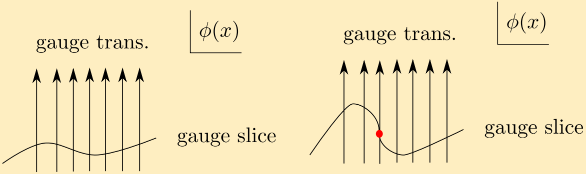
$$
\begin{aligned}
A &= \int \frac{[dg_{mn} d\chi_a dX^\mu d\psi^\mu]}{\text{superrep.} \times \text{superWeyl}} e^{-I} V_1 \cdots V_N \\
&= \int \prod_K dt_K \, [dX^\mu dbdcd\beta d\gamma] \, e^{-I_{\text{g.f.}}} \left[V_1 \cdots V_N \prod_K \int_{C_K} b \prod_i X(z_i) + \cdots \right] \\
&= \int_{\mathcal{M}} \prod_K dt_K \left[\left\langle V_1 \cdots V_N \prod_\alpha \int_{C_K} b \prod_i X(z_i) \right\rangle + \cdots \right]
\end{aligned}
$$



$$
\begin{aligned}
\epsilon^m &\longleftrightarrow b, c \ \text{(reparametrization)} \\
\epsilon^a &\longleftrightarrow \beta, \gamma \ \text{(supersymmetry)} \\
X(z) &= \delta(\beta) T_F + \cdots
\end{aligned}
$$

picture changing operator

Gauge slice



- When the gauge slice is not transverse to the gauge orbit at some point on the gauge slice,
  - $\triangle_{\rm FP} = 0$ if the relevant gauge parameter is Grassmann even
  - $\triangle_{\rm FP} = \infty$ if the relevant gauge parameter is Grassmann odd

- The integrand of a Feynman amplitude for superstrings may diverge when the gauge slice fails to be transverse.

Singularities ▸ BACK

$$A = \int_{\mathcal{M}} \prod_{\alpha} dt_{\alpha} \left[\left\langle V_1(Z_1) \cdots V_N(Z_N) \prod_{\alpha} \int_{C_\alpha} b \prod_i X(z_i) \right\rangle + \cdots \right]$$

$$\left\langle \prod_i \delta(\beta)(z_i) \prod_r \delta(\gamma)(Z_r) \right\rangle$$

$$\propto \frac{1}{\vartheta[\alpha]\left(\sum z_i - \sum Z_r - 2\triangle\right)} \cdot \frac{\prod_{i,r} E(z_i, Z_r)}{\prod_{i>j} E(z_i, z_j) \prod_{r>s} E(Z_r, Z_s)} \cdot \frac{\prod_r \sigma(Z_r)^2}{\prod_i \sigma(z_i)^2}$$

- Two kinds of singularities
  1. $z_i = z_j$: contact term divergence
  2. $\vartheta[\alpha]\left(\sum z_i - \sum Z_r - 2\triangle\right) = 0$
- The second one is harder to deal with.
  - global condition involving the positions of a lot of operators

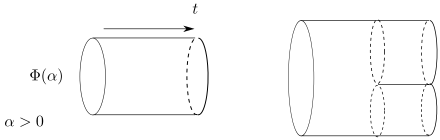
LC gauge SFT action ▸ BACK

$$S = \int \left[\frac{1}{2} \Phi \cdot (i\alpha\partial_t - H)\, \Phi + \frac{g_s}{3} \Phi \cdot (\Phi * \Phi) \right]$$

- String field $\Phi \left[t, \alpha, X^i(\sigma), \psi^i(\sigma), \bar{\lambda}^A(\sigma) \right]$

$$\begin{aligned} t &= x^+ \\ \alpha &= 2p^+ \end{aligned}$$

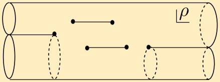
- propagator and vertex



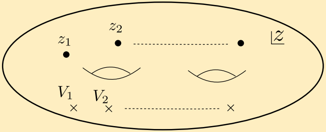
Feynman amplitudes for LC gauge SFT

$$
\begin{aligned}
A &= \sum_{\text{spin structure}} \int \prod_K dt_K \left\langle V_1^{\text{LC}} \cdots V_N^{\text{LC}} \prod_I \left[\left(\partial^2 \rho\right)^{-\frac{3}{4}} T_F^{\text{LC}}\left(z_I\right) \right] \right\rangle_{\text{LC}} e^{-\Gamma} \\
&= \sum_{\text{spin structure}} \int_{\mathcal{M}} \prod_K dt_K \left\langle V_1\left(Z_1\right) \cdots V_N\left(Z_N\right) \prod_K \int_{C_K} b \prod_I X\left(z_I\right) \right\rangle
\end{aligned}
$$

$\int \prod_K dt_K$



$\longrightarrow \int \prod_K dt_K$



- A naturally defined metric on LC diagram $ds^2 = d\rho d\bar{\rho}$
- $e^{-\Gamma}$: Weyl anomaly

Spurious singularities in LC SFT

BACK

$$A = \sum_{\text{spin structure}} \int \prod_K dt_K \left\langle V_1^{\text{LC}} \cdots V_N^{\text{LC}} \prod_I \left[\left(\partial^2 \rho\right)^{-\frac{3}{4}} T_F^{\text{LC}}(z_I) \right] \right\rangle_{\text{LC}} e^{-\Gamma}$$

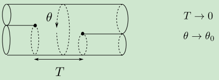
$$= \sum_{\text{spin structure}} \int_{\mathcal{M}} \prod_K dt_K \left[\left\langle V_1(Z_1) \cdots V_N(Z_N) \prod_K \int_{C_K} b \prod_I X(z_I) \right\rangle + \cdots \right]$$

1. $z_I = z_J$
2. $\vartheta[\alpha]\left(\sum z_I - \sum Z_r - 2\triangle\right) = 0$

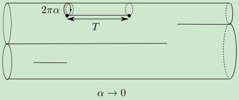
- No singularity of the second type.
  - No β, γ on the worldsheet (1-st line)
  - The ϑ is canceled by the one from the $\psi^{\pm}$ partition function (2-nd line)

Singular LC diagrams ▸ BACK

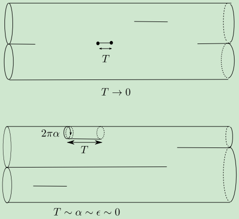
Contact term



Infinitely thin cylinder



Tiny neck



- $e^{-\Gamma}$ becomes singular when combinations of these phenomena happen.
- These correspond to contact term and infrared divergences.

Problems with chiral fermions ▸ BACK

- Naive dimensional regularization has problems with chiral fermions. We can avoid them by considering the theory in linear dilaton background $\Phi = -iQX^1$, instead of changing the spacetime dimensions

$$S = \frac{1}{16\pi} \int d^2z \sqrt{\hat{g}} \left(\hat{g}^{ab} \partial_a X^1 \partial_b X^1 - 2iQ\hat{R}X^1 + \cdots \right)$$

  - Doing so does not change the number of $\psi^\mu \sim \gamma^\mu$
  - $Q^2 \sim \frac{10-d}{8}$
  - We can change Q continuously.
  - This background breaks unitarity.

The worldsheet theory for $X^{\pm}, \psi^{\pm}$

$$S_{\pm} = -\frac{1}{2\pi}\int d^2z \partial X^+ \bar{\partial} X^- - \frac{d-10}{32\pi}\int d^2z \left(\partial\chi\bar{\partial}\chi + \hat{g}_{z\bar{z}}\hat{R}\chi\right) + \cdots$$
$$\chi \equiv \ln\left(-4\partial X^+ \bar{\partial} X^+\right) - \ln\left(2\hat{g}_{z\bar{z}}\right)$$

- This theory can be formulated in the case $\langle \partial_m X^+ \rangle \neq 0$.
  - In the case of the LC gauge amplitudes, we always have $\prod e^{-ip_r^+ X^-}$ $(p_r^+ \neq 0)$ and $\langle \partial_m X^+ \rangle \neq 0$.
- The interaction terms are made of $\partial X^+, \bar{\partial} X^+$ which have no singular OPE's among themselves.

The worldsheet theory for $X^{\pm}, \psi^{\pm}$ BACK

$$S_{\pm} = -\frac{1}{2\pi}\int d^2z \partial X^+ \bar{\partial} X^- - \frac{d-10}{32\pi}\int d^2z \left(\partial\chi\bar{\partial}\chi + \hat{g}_{z\bar{z}}\hat{R}\chi\right) + \cdots$$

$$T(z) = :\partial X^-(z)\partial X^+(z): - \frac{d-10}{8}\left[\frac{\partial^3 X^+}{\partial X^+} - \frac{3}{2}\left(\frac{\partial^2 X^+}{\partial X^+}\right)^2\right] + \cdots$$

- This theory is exactly solvable and turns out to be a superconformal field theory with $c = 3 + \frac{3}{2}(10-d)$.
  - The worldsheet theory has a nilpotent BRST charge

		$X^{\pm}$		X^i		ghosts		
c	$=$	$3 + \frac{3}{2}(10-d)$	$+$	$\frac{3}{2}(d-2)$	$-$	15	$=$	0