Mirrored K3 automorphisms and non-geometric compactifications

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Based on:

D.I. and Vincent Thiéry, arXiv:1310.4116

D.I., arXiv:1503.01552

Chris Hull, D.I. and Alessandra Sarti, arXiv:1710.00853

Introduction

Non-geometric superstring compactifications

- Generic SUSY compactifications ➥ non-geometric?
- Non-geometric compactifications ➥ very few massless moduli
- Only sporadic classes known ➥ T-folds,...

Three view-points on non-geometry

- Worldsheet : asymmetric 2d superconformal field theories
- Non-geometric symmetries (*e.g.* T-duality) ➥ quotient of geometric solutions
- Four-dimensional low-energy SUGRA ➥ gauging by "non-geometric" fluxes

Scope of this presentation

- Asymmetric $K3 \times T^2$ Gepner models
- Mathematical understanding ➥ Mirrored K3 automorphisms
- Corresponding 4d gauged supergravities

A simple example

T^2 compactification

- $\mathrm{d}s^2 = \frac{\rho_2}{\tau_2}|\mathrm{d}x_1 + \tau\,\mathrm{d}x_2|^2$, $\rho_1 = B_{12}$
- Factorized moduli space:

$$\underbrace{\frac{SL(2,\mathbb{R})}{SL(2,\mathbb{Z})_\tau \times U(1)}}_{\text{complex structure } \tau} \times \underbrace{\frac{SL(2,\mathbb{R})}{SL(2,\mathbb{Z})_\rho \times U(1)}}_{\text{Kähler } \rho}$$

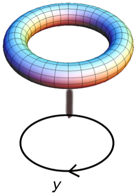
- Exchanged by T-duality along x_1

Order 4 symmetry

- $\sigma_4 : \begin{cases} x^1 & \mapsto -x^2 \\ x^2 & \mapsto x^1 \end{cases}$
- Induced $SL(2,\mathbb{Z})_\tau$ action on complex structure moduli space: $\tau \mapsto -1/\tau$
- Fixed point $\tau = i \leftrightarrow$ square torus

Generalized Scherk-Schwarz (monodrofolds) *(Dabholkar, Hull)*

- σ_4 monodromy twist: $(x^i, y) \sim (\sigma_4 x^i, y + 2\pi R)$
  - ➥ breaks all SUSY
- Double T-fold $(x^i_{\rm L}, y) \sim (-x^i_{\rm L}, y + 2\pi R)$
  - ➥ corresponds to $\begin{cases} \tau & \mapsto -1/\tau \\ \rho & \mapsto -1/\rho \end{cases}$, $y \mapsto y + 2\pi R$
- SUSY from right-movers only *(Hellerman, Walcher)*



Asymmetric Landau-Ginzburg/Gepner orbifolds

Gepner models/LG orbifolds for K3 surfaces

Landau-Ginzburg models

- $\mathcal{N} = (2,2)$ QFTs in two dimensions, chiral multiplets Z_ℓ

$$L = \int \mathrm{d}^4\theta\, K(Z_\ell, \bar{Z}_\ell) + \int \mathrm{d}^2\theta\, W(Z_\ell) + h.c.$$

- Quasi-homogeneous polynomial with an isolated critical point:

$$\boxed{W(\lambda^{w_\ell} Z_\ell) = \lambda^d W(Z_\ell)}$$

- Flows to a $(2,2)$ SCFT in the IR

Some LG orbifold models for K3 surfaces

- Quantum non-linear sigma-model for a K3 surface with "$-\infty$ volume":
- LG model $\boxed{W = Z_1^{p_1} + \cdots + Z_4^{p_4}}$
- Quotient by $j_W : Z_\ell \mapsto e^{2i\pi/p_\ell} Z_\ell$, of order $K = \text{lcm}\,(p_1, \cdots, p_4)$ (*"GSO"*)

  ➥ fields in twisted sectors $\gamma = 0, \ldots, K-1$
- IR fixed point: $\mathcal{N} = (2,2)$ SCFT with $c = \bar{c} = 6$ and $Q_R, \bar{Q}_R \in \mathbb{Z}$

  ➥ Gepner model: solvable $(2,2)$ SCFT

K3 Gepner/Landau-Ginzurg orbifolds

Some symmetries of Gepner models

- $W = Z_1^{p_1} + \cdots + Z_4^{p_4}$ ➥ discrete symmetry group $\big(\mathbb{Z}_{p_1} \times \cdots \times \mathbb{Z}_{p_4}\big)/J_W$
- $Z_\ell \mapsto e^{\frac{2i\pi r_\ell}{p_\ell}} Z_\ell$ with $\sum_\ell \frac{r_\ell}{p_\ell} \in \mathbb{Z}$ ➥ group SL_W of SUSY-preserving symmetries
- Quantum symmetry of LG orbifold $\boxed{\sigma_K^{\mathfrak{Q}} : \phi_\gamma \mapsto e^{2i\pi\gamma/K}\phi_\gamma,\ \gamma = 0, \ldots, K-1}$

Orbifolds of Gepner models

- Supersymmetric orbifold of a K3 Gepner model by $G \subset SL_W$
  ➥ other point in K3 NLSM moduli space
- Quotient by $\langle \sigma_{p_\ell} \rangle$, with $\boxed{\sigma_{p_\ell} : Z_\ell \mapsto e^{2i\pi/p_\ell} Z_\ell}$ for given ℓ
  ➥ breaks all space-time SUSY

★Latter case: space-time SUSY can be partially restored using discrete torsion

Asymmetric K3 Gepner models

Previous works

- Asymmetric models from simple currents *(Schellekens & Yankielowicz 90)*
- LG orbifolds *(Intriligator & Vafa 90)*
- Asymmetric $K3 \times T^2$ models in type II *(DI & Thiéry 13)*
- Asym. CY_3 models from simple currents & fractional mirror sym. *(DI 15)*
- More general CY_3 models *(Blumenhagen, Fuchs & Plauschinn 16)*

A simple class of asymmetric $K3$ LG orbifolds

- Quotient of $W = Z_1^{p_1} + \cdots + Z_4^{p_4}$ by $\boxed{\sigma_{p_1} \,:\, Z_1 \mapsto e^{2i\pi/p_1} Z_1}$
  - ➥ twisted sectors $r = 0, \ldots, p_1 - 1$
- Order p subgroup of the quantum sym. group generated by $\sigma^{\Omega}_{p_1} := (\sigma^{\Omega})^{K/p_1}$
  - ➥ charge $Q^{\Omega}_{p_1} := \frac{\gamma}{p_1}$

★ Modified $\mathbb{Z}_{p_1}$ orbifold charge: $\hat{Q}_{p_1} = Q_{p_1} + \frac{\gamma}{p_1} \mod 1$

★ Modified $\mathbb{Z}_K$ orbifold charge: $\hat{Q}_K = Q_K - \frac{r}{p_1} \mod 1$

} discrete torsion

- Orbifold theory: $Q_R \in \mathbb{Z}$ but $\bar{Q}_R \notin \mathbb{Z}$ ➥ WS chiral space-time SUSY

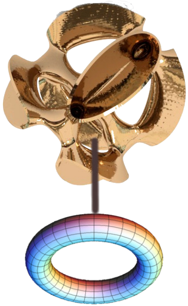
K3 fibrations with non-geometric monodromies

Asymmetric $K3 \times T^2$ Gepner models in type IIA/IIB *(DI, Thiéry)*

- $K3$ Gepner model $W = Z_1^{p_1} + Z_2^{p_2} + Z_3^{p_3} + Z_4^{p_4}$ times $\mathbb{R}^2$ (x, y) in type IIA/B
- Freely-acting $\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2}$ quotient

$$\begin{cases} Z_1 & \mapsto e^{2i\pi/p_1} Z_1 \\ x & \mapsto x + 2\pi R_1 \end{cases} \qquad \begin{cases} Z_2 & \mapsto e^{2i\pi/p_2} Z_2 \\ y & \mapsto y + 2\pi R_2 \end{cases}$$

- Each quotient modified by the discrete torsion discussed before



Main features

Supersymmetry breaking

- All space-time supercharges from left-movers ➥ non-geometric
- Breaking $\mathcal{N}=4 \ \rightarrow \ \mathcal{N}=2$ in 4d, gravitini masses: $M^2 = \frac{\rho_2}{\tau_2} + \frac{(\rho_1 \pm 1)^2}{\rho_2 \tau_2}$
- SUSY-breaking can be achieved much below string scale
  ➥ can be analyzed reliably as spontaneous breaking in SUGRA

Moduli space

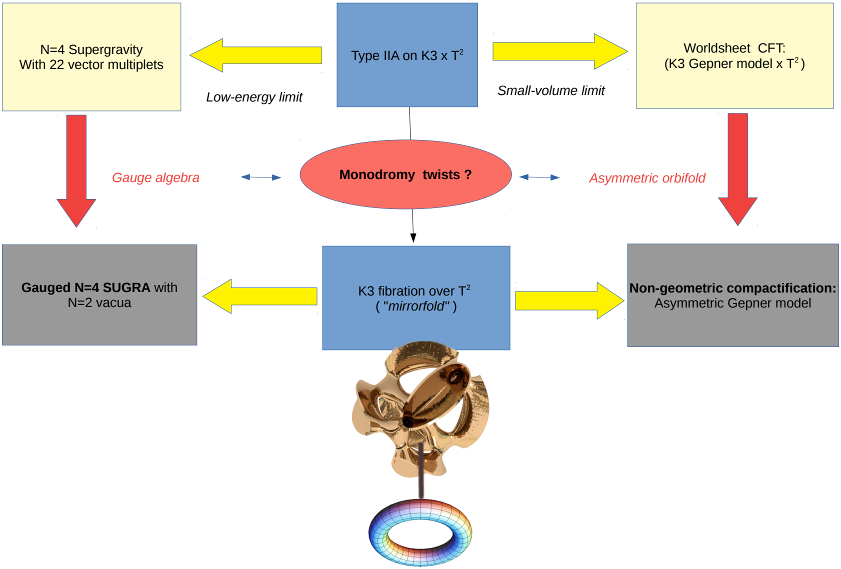
- ρ and τ moduli of the T^2 and axio-dilaton S always massless
- For about 50% of the models: all K3 moduli become massive

Low-energy 4d theory

- Axio-dilaton and torus moduli in vector multiplets ➥ $\mathcal{N}=2$ STU SUGRA
- Surviving K3 moduli (if any): hypermultiplets

★ $\mathcal{N}=2$ vacua of a gauged $\mathcal{N}=4$ supergravity?

Roadmap


N=4 Supergravity
With 22 vector multiplets
Type IIA on K3 x T^2
Worldsheet CFT:
(K3 Gepner model x T^2)
Low-energy limit
Small-volume limit
Gauge algebra
Monodromy twists ?
Asymmetric orbifold
Gauged N=4 SUGRA with
N=2 vacua
K3 fibration over T^2
("mirrorfold")
Non-geometric compactification:
Asymmetric Gepner model

Non-linear sigma models on K3 and mirrored automorphisms

K3 surfaces: elementary facts

K3-surfaces

- K3 surface X: Kähler 2-fold with a nowhere vanishing holomorphic 2-form Ω
- Hodge diamond: $\begin{matrix} & & h^{0,0} & & \\ & h^{1,0} & & h^{0,1} & \\ h^{2,0} & & h^{1,1} & & h^{0,2} \\ & h^{2,1} & & h^{1,2} & \\ & & h^{2,2} & & \end{matrix} \quad = \quad \begin{matrix} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1 & & \end{matrix}$
- Inner product: $(\alpha, \beta) \in H^2(X,\mathbb{Z}) \times H^2(X,\mathbb{Z}) \mapsto \langle \alpha, \beta \rangle = \int \alpha \wedge \beta \in \mathbb{Z}$
- $H^2(X,\mathbb{Z})$ isomorphic to unique even, unimodular lattice of signature $(3,19)$:
$$\Gamma_{3,19} \cong E_8 \oplus E_8 \oplus U \oplus U \oplus U \; , \qquad U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
- Lattice of total cohomology $H^\star(X,\mathbb{Z})$: $\boxed{\Gamma_{4,20} \cong E_8 \oplus E_8 \oplus U \oplus U \oplus U \oplus U}$

Moduli space of Ricci-flat metrics on K3

- Ricci-flat metric on X $\leftrightarrow$ space-like oriented 3-plane $\Sigma = (\mathrm{Re}(\Omega), \mathrm{Im}(\Omega), J) \subset \mathbb{R}^{3,19} \cong H^2(X,\mathbb{R})$, modulo large diffeomorphisms
- $\boxed{\mathcal{M}_{\text{KE}} \cong {}_{O(\Gamma_{3,19})}\backslash\, O(3,19) \,/\, O(3) \times O(19) \;\times \mathbb{R}_+}$

String theory compactifications on K3

Non-linear sigma-models on K3 surfaces

- $\int_\Sigma \mathrm{d}^2 z \left\{ g_{i\bar{\jmath}}\left(\partial_z \phi^i \partial_{\bar{z}} \phi^{\bar{\jmath}} + \partial_{\bar{z}} \phi^i \partial_z \phi^{\bar{\jmath}}\right) + b_{i\bar{\jmath}}\left(\partial_z \phi^i \partial_{\bar{z}} \phi^{\bar{\jmath}} - \partial_{\bar{z}} \phi^i \partial_z \phi^{\bar{\jmath}}\right) \right\}$
- $\int_{\phi(\Sigma)} b$ ➥ 22 real parameters
- Moduli space of NLSMs:
  Choice of metric & B-field ↔ choice of space-like oriented 4-plane $\Pi \subset \mathbb{R}^{4,20}$
- $\boxed{\mathcal{M}_\sigma \cong O(\Gamma_{4,20}) \backslash O(4,20) / O(4) \times O(20) \times \mathbb{R}_+}$ *(Seiberg, Aspinwall-Morrison)*
- K3 surfaces hyper-Kähler

➥ what does mirror symmetry mean?

Lattice-polarized K3 surfaces

Picard and transcendental lattices

- Picard lattice $S(X) = H^2(X,\mathbb{Z}) \cap H^{1,1}(X)$
  ➥ rank $\rho(X) \geqslant 1$ for an algebraic surface, signature $(1, \rho - 1)$
- Transcendental lattice $T(X) = H^2(X,\mathbb{Z}) \cap S(X)^\perp$, signature $(2, 20 - \rho)$

Moduli spaces of polarized K3 surfaces

- Lattice M of signature $(1, r - 1)$ with primitive embedding in $S(X)$
  ➥ M-polarized surface (X, M)
- Complex structure moduli space: $\mathcal{M}_M \cong {}_{O(M^\perp)}\backslash\, O(2, 20 - r) \,/_{O(2) \times O(20 - r)}$

Aspinwall-Morrison description of mirror symmetry

- Consider an algebraic K3 surface polarized by its Picard lattice $S(X)$

➥ Splitting of the NSLM moduli space (complex/Kähler)

$$\left({}_{O(T(X))}\backslash\, O(2, 20 - \rho) \,/_{O(2) \times O(\rho)} \right) \times \left({}_{O(S(X) \oplus U)}\backslash\, O(2, \rho) \,/_{O(2) \times O(20 - \rho)} \right)^{\times \mathbb{R}_+}$$

- A-M mirror symmetry: exchange of the two factors

Lattice-polarized vs. Berglund-Hübsch mirror symmetry

Lattice-polarized mirror symmetry *(Dolgachev)*

M-polarized surface (X, M) and $\tilde{M}$-polarized surface $(\tilde{X}, \tilde{M})$ LP-mirror if

$$\boxed{\Gamma^{3,19} \cap M^{\perp} = U \oplus \tilde{M}}$$

Berglund-Hübsch mirror symmetry (physicists' Greene-Plesser mirror)

- Hypersurface X_W in weighted projective space $\mathbb{P}_{[w_\ell]}$: $W = \sum_{i=1}^{4} \prod_{j=1}^{4} x_j{}^{a^i{}_j} = 0$
- Group of "SUSY" symmetries SL_W: $\{g_\ell, W(e^{2i\pi g_\ell} x_\ell) = W(x_\ell), \sum g_\ell \in \mathbb{Z}\}$
- $J_W \subseteq G \subseteq SL_W$ ➥ $\boxed{\text{K3 surface } (X_W, G)}$ (resolution of $X_W/(G/J_W)$)
- Berglund-Hübsch mirror surface: (X_{W^T}, G^T) with $W^T = \sum_{i=1}^{4} \prod_{j=1}^{4} x_j{}^{(a^T)^i{}_j}$ and $G^T = \{g \in G_{W^T},\ g(a^i{}_j)h^T \in \mathbb{Z},\ \forall h \in G\}$

Example

Self-mirror K3 surface

- Fermat hypersurface $\boxed{w^2 + x^3 + y^7 + z^{42} = 0 \text{ in } \mathbb{P}_{[21,14,6,1]}}$
- Picard lattice: $S(X) \cong E_8 \oplus U$
- Transcendental lattice $T(X) \cong E_8 \oplus U \oplus U$

  ➥ $S(X)$-polarized surface is LP self-mirror!
- Dual group for a Fermat surface: $G^T = SL_W$
- In this particular example, $|SL_W/J_W| = 1$

  ➥ This K3 surface is also its own Berglund-Hübsch mirror

★ In this example, LP mirror of the $S(X)$-polarized surface $\leftrightarrow$ BH mirror

★ Is is true in general? ➥ no!

Automorphisms of K3 surfaces

Symplectic automorphisms of K3 surfaces

- A K3 automorphism σ is symplectic if it preserves the $(2,0)$ form: $\sigma^\star(\Omega) = \Omega$
- Central role in the Mathieu moonshine

Non-symplectic automorphisms

- Non-symplectic order p automorphism σ_p: $\sigma_p^\star(\Omega) = e^{\frac{2i\pi}{p}}\Omega$
- Invariant sublattice of $\Gamma_{3,19}$: $S(\sigma_p) \subseteq S(X)$
- Orthogonal complement $T(\sigma_p) = S(\sigma_p)^\perp \cap \Gamma_{3,19}$

Ex: self-mirror K3 surface $w^2 + x^3 + y^7 + z^{42} = 0$

- $\sigma_2 : w \mapsto e^{i\pi} w$
- $\sigma_3 : x \mapsto e^{2i\pi/3} x$
- $\sigma_7 : y \mapsto e^{2i\pi/7} y$

All cases:
$S(\sigma_p) \cong S(X)$
$T(\sigma_p) \cong T(X)$

Non-symplectic automorphisms and mirror symmetry

Non-symplectic automorphisms of prime order

- p-cyclic algebraic K3 surface $X_{W,G}$: $W = w^p + f(x,y,z) \circlearrowleft \sigma_p : w \mapsto e^{\frac{2i\pi}{p}} w$
- Berglund-Hübsch mirror $\tilde{X}_{W^T,G^T}$: $W^T = \tilde{w}^p + \tilde{f}(\tilde{x},\tilde{y},\tilde{z}) \circlearrowleft \tilde{\sigma}_p : \tilde{w} \mapsto e^{\frac{2i\pi}{p}} \tilde{w}$
- Theorem (Artebani et al., Comparin et al.):
  For prime $p \in \{2,3,5,7,13\}$, the $S(\sigma_p)$-polarized surface $X_{W,G}$ and the $S(\tilde{\sigma}_p)$-polarized surface $\tilde{X}_{W^T,G^T}$ are lattice-polarized mirrors.

Corollary: lattice decompositions *(Hull, DI, Sarti)*

- $\begin{cases} T(\sigma_p) := & S(\sigma_p)^\perp = & U \oplus S(\tilde{\sigma}_p) \\ T(\tilde{\sigma}_p) := & S(\tilde{\sigma}_p)^\perp = & U \oplus S(\sigma_p) \end{cases}$
- $T(\tilde{\sigma}_p)$ is the orthogonal complement of $T(\sigma_p)$ in $\Gamma_{4,20}$:

$$T(\tilde{\sigma}_p) \cong T(\sigma_p)^\perp \cap \Gamma_{4,20} \,.$$

  We obtain then the orthogonal decomposition over $\mathbb{R}$ (and $\mathbb{Q}$):

$$\Gamma_{4,20} \otimes \mathbb{R} \;\cong\; \Big(T(\tilde{\sigma}_p) \oplus T(\sigma_p) \Big) \otimes \mathbb{R} \,.$$

Mirrored K3 automorphisms

Abstract definition

(Hull, DI, Sarti)

- Let $X_{W,G}$ be a p-cyclic $S(\sigma_p)$-polarized K3 surface, and X_{W^T,G^T} its LP/BH mirror, regarded as an $S(\tilde{\sigma}_p)$-polarized K3 surface.
- By properties of K3 surfaces we can extend the diagonal action of $(\sigma_p, \tilde{\sigma}_p)$ from $T(\sigma_p) \oplus T(\tilde{\sigma}_p)$ to the whole lattice $\Gamma_{4,20}$.
- This defines an $O(\Gamma_{4,20})$ element associated with the action of a NLSM automorphism $\hat{\sigma}_p$, that we name *mirrored automorphism*.

Properties

- $\hat{\sigma}_p^\star\big|_{T(\sigma_p)^{\mathbb{R}}} = \sigma_p^\star\,,\ \hat{\sigma}_p^\star\big|_{T(\tilde{\sigma}_p)^{\mathbb{R}}} = (\tilde{\sigma}_p)^\star$
- Denoting by μ the BH/LP mirror involution, $\boxed{\hat{\sigma}_p := \mu \circ \tilde{\sigma}_p \circ \mu \circ \sigma_p}$

Mirrored K3 automorphisms vs. asymmetric LG models

BH mirror symmetry and quantum symmetry of LG models

- In the Gepner model construction we have used:
  1. order p symmetry group of the superpotential $Z \mapsto e^{2i\pi/p} Z$
  2. order p subgroup of the quantum sym. group generated by $\sigma_p^{\Omega} := (\sigma^{\Omega})^{K/p}$
- These symmetries are exchanged by BH mirror symmetry $(\bar{Q}_R \mapsto -\bar{Q}_R)$

Non-geometric orbifolds from mirrored automorphisms

- $K3$ orbifold with discrete torsion ➥ projection $Q_p + Q_p^{\Omega} \in \mathbb{Z}$
- Corresponds to the diagonal action of $(\sigma_p, \tilde{\sigma}_p)$
- Therefore, a $K3$ bundle over T^2 with mirrored $K3$ automorphisms twists give at the fixed points an asymmetric $K3 \times T^2$ Gepner model

Non-geometric monodromies and gauged supergravity

Gaugings from twisted compactifications

$\mathcal{N} = 4$ SUGRA from $K3 \times T^2$ compactifications

- Type IIA/B on $K3 \times T^2$ ➥ $\mathcal{N} = 4$ SUGRA with 22 vector multiplets
- Field content: SUGRA multiplet $(g_{\mu\nu}, \psi^i_\mu, A^{1,\dots,6}_\mu, \chi^i, \tau)$
  22 vector multiplets $(A^a_\mu, \lambda^a_i, \mathcal{M})$
- Scalars $\mathcal{M}$, τ take value in the coset $\frac{O(6,22)}{O(6)\times O(22)} \times \frac{SL(2)}{O(2)}$
- Rigid $G = O(6,22) \times SL(2)$ symmetry

Gaugings from Scherk-Schwarz reductions

(Dabholkar, Hull 02; Ried-Edwards, Spanjaard 08)

- Promote subgroup $K \subset G$ to gauge symmetry ➥ structure constants t_{MNP}
- $K3 \times T^2$ with monodromy twists $e^{N_i} \in O(\Gamma_{4,20}) \subset O(6,22)$

  ➥ structure constants $\boxed{{t_{iI}}^J = {N_{iI}}^J}$
- Potential and SUSY breaking mass terms from t_{MNP}

Gauged SUGRA from mirrored K3 automorphisms

Gauged SUGRA from mirrored automorphisms

- Pair of mirrored K3 automorphisms $\hat{\sigma}_{p_i}$ ➥ twisted $K3 \times T^2$ compactification
- From the action of $\hat{\sigma}_{p_i}$ on $H^\star(X, \mathbb{Z})$ ➥ matrices $\hat{M}_{p_i} \in O(\Gamma_{4,20})$
- Provide structure constants of the corresponding $\mathcal{N} = 4$ gauged SUGRA

Vacua with spontaneous SUSY breaking $\mathcal{N} = 4 \ \to \mathcal{N} = 2$

- Gravitini in $(\mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{1})$ of $\{SU(2) \times SU(2) \cong SO(4)\} \times SO(20)$
- Half-SUSY vacua from monodromies $M_{p_i} \in \{SU(2) \times SO(20)\} \cap O(\Gamma_{4,20})$
- Ordinary non-symplectic $K3$ automorphism: space-like rotation $SO(2) \subset SO(4)$ (as $T(\sigma_p)$ signature $(2, \star)$) ➥ no SUSY
- Mirrored automorphism: $SO(2) \times SO(2) \subset SO(4)$ with same angle ➥ half-SUSY

Conclusions

- Non-geometric compactifications of superstring theory all likely the most generic ones yet poorly understood
- Large class of non-geometric compactifications based on Calabi-Yau rather than toroidal geometries
  ➥ first construction of "mirrorfolds"
- Analysis from 3 viewpoints:
  1. Worldsheet CFT
  2. Algebraic geometry
  3. Gauged SUGRA
- Involve new classes of symmetries of CY sigma-models: mirrored CY automorphisms
- Some open questions (work in progress):
  1. Fully explicit gauged SUGRA analysis
  2. How do they fit into $\mathcal{N} = 2$ heterotic/type II dualities in 4d
  3. Relation with the Mathieu moonshine
  4. Extension to CY_3-based constructions