# Towards holography for quantum mechanics

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Despite immense progress some questions which we would like to ask using holography still seem to remain beyond reach

Try to construct a holographic duality by going to an extremely simple setting where everything would be under control on both sides of the duality

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# Outline

Motivation
Questions from holography
Tensor network constructions

Requirements for a holographic description
Partition function
Correlation functions
The “gravity” subsector

Holographic description for a quantum-mechanical free particle

Conclusions and outlook

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## The original AdS/CFT correspondence

$\boxed{\mathcal{N} = 4 \text{ Super Yang-Mills theory}} \quad \equiv \quad \boxed{\text{Superstrings on } AdS_5 \times S^5}$

**Two main parameters**

- tHooft coupling $\lambda = g_{YM}^2 N_c$,
  - governs string scale effects

$$\alpha'_{eff} \propto 1/\sqrt{\lambda}$$

  - The $\lambda \to 0$ limit is accessible on the perturbative gauge theory side
  - For a long time it seemed to be impossible to access this regime on the string side until huge progress using integrability
- The number of colors $N_c$
  - planar limit – roughly classical (gravity+)
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   $\equiv$ Anomalous dimensions in the planar limit
   $\equiv$ energy levels of a single string in $AdS_5 \times S^5$

   Most complete solution: Quantum Spectral Curve

   Gromov, Kazakov, Leurent, Volin

2. **OPE coefficients and three string interactions:**

   Most advanced framework: Hexagon approach

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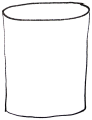

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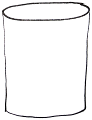

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   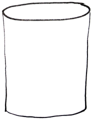

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   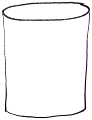

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   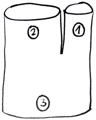
   

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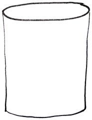

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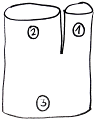


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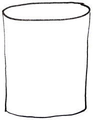

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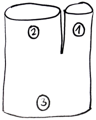


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## Even with all this knowledge there are still open problems at large $N_c$

- The dual description of thermal plasma ($\mathcal{N} = 4$ SYM at nonzero temperature) at large $N_c$, strong coupling is given by a planar black hole solution
- What is the dual description of thermal plasma still at large $N_c$ but for $\lambda \to 0$?
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Klebanov, Polyakov

- The singlet sector of free scalar $O(N)$ vector model in 3D – dual to 4D Vasiliev gravity
- Very nontrivial check of 3-point correlation functions Giombi, Yin
- Very intriguing – first time no strings directly involved
- The boundary field theory is completely under control
- On the bulk side the situation is less clear – action for Vasiliev gravity is not really known (although some proposals exist)
- In particular unfortunately it is not known how to quantize Vasiliev gravity...

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**It would be very interesting to construct a holographic model where the bulk action would be completely known...**

## Tensor networks

- Consider a 1D spin chain system of length $L$ ($L$ is large, perhaps infinite) with some hamiltonian. One is interested in finding the ground state wavefunction
- The wave function $|\Psi\rangle = \Psi_{s_1 s_2 \ldots s_L} |s_1 s_2 \ldots s_L\rangle$ has exponentially many components. These components can be understood as defining a rank $L$ tensor, which can be pictorially represented as

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- Tensor networks provide variational ansatzae with less components e.g. Matrix Product State (MPS) is of the form

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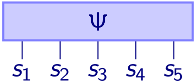
$\Psi_{s_1 s_2 \ldots s_5} =$ 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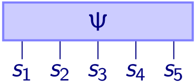


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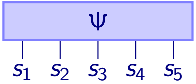
$\Psi_{s_1 s_2 \dots s_5} =$ 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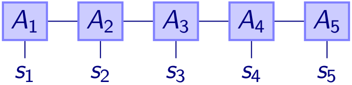
$\Psi_{s_1 s_2 \dots s_5} =$ 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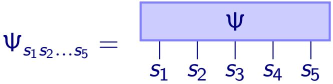


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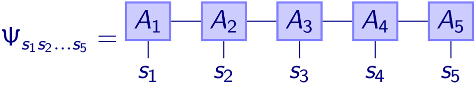


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# Tensor network constructions and holography

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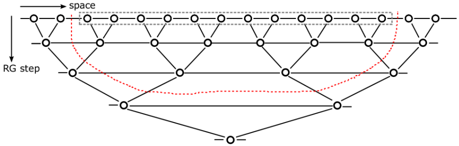


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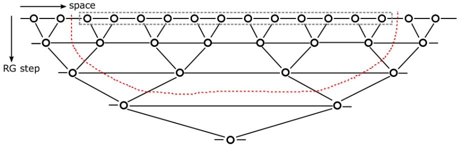


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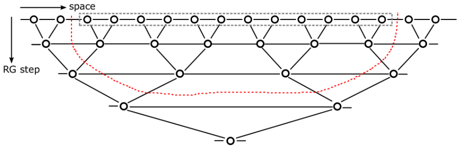


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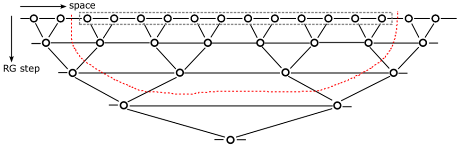


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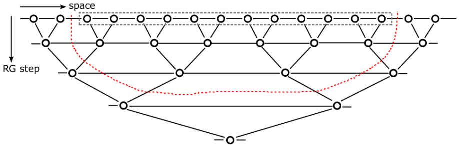


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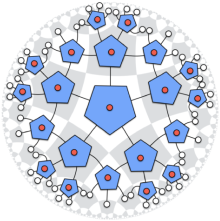

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- Attempt a holographic description for the simplest possible theory that one could think of...
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## Requirements for a holographic description

Suppose that the field theory is defined on some fixed $d$-dimensional spacetime geometry $\Sigma$

I **Equality of partition functions**

- The dual holographic theory should be defined on a higher dimensional manifold $M$, having $\Sigma$ as a boundary.
- We should have equality of partition functions

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- E.g this would provide a bulk interpretation of the thermodynamics of the theory...

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- We should be able to compute correlation functions for operators in the boundary theory from the bulk theory

**IIb The generating function for correlation functions**

Gubser, Klebanov, Polyakov; Witten

- Observables/operators in the boundary theory should be associated to fields in the bulk theory
- Boundary values of the bulk fields (up to a possible rescaling by $z^{\#}$) should give sources for the corresponding operator in the generating function of correlators

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- The boundary theory is defined on a manifold $\Sigma$ with fixed metric
- There should be a bulk field associated with the energy-momentum tensor and the boundary metric on $\Sigma$
- This would define a gravitational subsector in the bulk theory
- Standard example: Fefferman-Graham expansion of the bulk metric

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z)dx^\mu dx^\nu + dz^2}{z^2} + \dots$$

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Try to satisfy the above requirements **I-III** for one of the simplest systems possible, the quantum mechanical free particle in one dimension.

- Direct (but much simpler) analog of the massless free boson (abelian WZW/CS)
- Extremely simplified system – no spatial direction – no complications coming from error correcting code arguments etc.
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## The system

$$S = \int dt \, \frac{1}{2}\dot{q}^2$$

- Consider the bulk spacetime to be of the form

$$M = \{(t, z) : z \geq 0\}$$

- Since in the 2D massless boson case we have dual abelian Chern-Simons, here we expect to have a 2D abelian BF topological theory

$$S_{BF} = \int_M B \, dA = \int B \, (\partial_t A_z - \partial_z A_t) dtdz$$

- As the action vanishes on the constraint manifold $dA = 0$, we need to impose appropriate boundary conditions and boundary action
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## Step I – partition functions

- We will impose the following boundary conditions for the BF theory

$$B = -A_t \,|_{z=0} \qquad\qquad A_t = 0 \,|_{z\to\infty}$$

- Again in analogy to WZW/CS, we have to supplant the BF action with a boundary term so that the variation at the boundary vanishes

$$S^I_{bulk} = S_{BF} + \frac{1}{2}\int_{\{z=0\}} B^2 dt$$

- The Lagrange multiplier field $B$ imposes the constraint $dA = 0$, hence we may set

$$A_z = -\partial_z \Phi \qquad\qquad A_t = -\partial_t \Phi$$

- The boundary values of $\Phi(t,z)|_{z=0}$ will be identified with $q(t)$ hence the partition functions coincide as on the constraint surface

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## Step II – bulk fields for sources

- Consider generating functions of all correlators of $q(t)$

$$\int dt\, \frac{1}{2}\dot{q}^2 + \int dt\, j(t)q(t)$$

- We would like to introduce a new bulk field associated with the source $j(t)$
- In terms of the BF theory gauge field, the particle position $q(t)$ can be understood essentially as a Wilson line

$$\int_{z=0}^{\infty} A_z\, dz = -\int_{z=0}^{\infty} \partial_z \Phi(t,z) = \Phi(t,0) - \Phi(t,\infty) \to \Phi(t,0)$$

- So we have

$$q(t) = \int_L A$$

where the line $L$ is attached to the boundary at time $t$ and goes to infinity in the bulk.

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- In order to construct a bulk action which reduces to

$$\int dt\, j(t) q(t)$$

we will need two ingredients
- We will introduce another two-dimensional abelian BF theory

$$\int C\, d\alpha$$

- We use the global 1-form $dt$ (this will be modified later)
- Introduce a constraint term in the action

$$D\, \alpha \wedge dt$$

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- Now the flatness condition $d\alpha = 0$ ensures $\alpha = j(t)dt$, so we can generate the wanted term from a simple bulk interaction between $\alpha$ and $A$:

$$\int_M \alpha \wedge A = \int_M j(t)dt \wedge (A_t dt + A_z dz) = \int j(t) \int_0^\infty A_z dz dt = \int j(t) q(t) dt$$

- At this stage the overall bulk action is

$$S^{II}_{bulk} = \int_M (B\, dA + C\, d\alpha + \alpha \wedge A + D\, \alpha \wedge dt) + \frac{1}{2}\int_{\partial M} B^2 dt$$

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$$S^{II}_{bulk} = \int_M (B\,dA + C\,d\alpha + \alpha \wedge A + D\,\alpha \wedge dt) + \frac{1}{2}\int_{\partial M} B^2 dt$$

- The appearance of an explicit $dt$ is not very pleasing – but we will get rid of it shortly

## Step III – the "gravity" subsector

- Since the quantum mechanical path integral is essentially just a QFT on a 1-dimensional worldline, one can introduce a fixed 1-dimensional metric $g_{tt}(t)$ and write the action as

$$\frac{1}{2}\int \sqrt{g}\, g^{tt}(\partial_t q)^2 = \frac{1}{2}\int \frac{1}{e}\dot{q}^2$$

  and the einbein $e = e(t)$ is a given function of time...
- We would like to introduce a natural bulk field which goes over to the einbein at the boundary.
- At the same time we will replace the 1-form $dt$ (which is necessarily closed)
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$$\int E\, d\eta$$

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- We will modify the boundary conditions

$$A_t + \eta_t B = 0|_{z=0}$$

and fix the boundary value of $\eta_t$

- Accordingly we need to modify the additional boundary action

$$\frac{1}{2}\int_{\{z=0\}} B^2 dt \longrightarrow \frac{1}{2}\int_{\partial M} B^2\,\eta$$

(this works as $\delta\eta_t = 0|_{z=0}$)

- Now the resulting action will take the form

$$\frac{1}{2}\int_{\partial M} B^2\,\eta = \frac{1}{2}\int \frac{1}{\eta_t} A_t^2 dt = \frac{1}{2}\int \frac{1}{\eta_t}\dot{q}^2$$

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$$A_t + \eta_t B = 0|_{z=0} \qquad \alpha_t = j(t)|_{z=0} \qquad \eta_t = e(t)|_{z=0}$$

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## Step IV – integrate out boundary degrees of freedom

- Ultimately we should integrate out $B$ and $A$ to obtain the final bulk action involving only the bulk fields corresponding to sources for $q(t)$ and the energy-momentum tensor $T_{tt}$

$$e^{iS^{eff}_{bulk}[C,D,E,\alpha,\eta]} = \int DB\, DA\, e^{iS^{III}_{bulk}[B,A,C,D,E,\alpha,\eta]}$$

- Unfortunately this seems to be quite nonlocal...
- One can speculate whether this is a generic situation and a local holographic bulk action in this sense occurs only in special circumstances??? (like large $N$ and/or strong coupling?)

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## Conclusions

- We have constructed a dual description of a quantum mechanical free particle which realizes formally some basic requirements for holography
- The bulk fields include a source for the field $q(t)$
- ... and a field reducing to the einbein at the boundary
- $N$ components/singlet? relation to 2D Vasiliev
- Symmetries?
- How to incorporate $V(q)$ for the quantum mechanical system?
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