Integrable One-point Functions in AdS/dCFT

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Based on:

- M. de Leeuw, C.K., G. Linardopoulos, ArXiv:1802.01258[hep-th], Phys.Lett. B781 (2018) 238
- I. Buhl Mortensen, M. de Leeuw, A. Ipsen, C.K., M. Wilhelm, ArXiv: 1704.07386 [hep-th], Phys.Rev.Lett. 119 (26) (2017) 261604
- Previous work involving the same authors plus G. Semenoff & K. Zarembo as well as ongoing work involving A.G. Grau & M.Volk

New Frontiers in String Theory

Yukawa Institute, Kyoto, July 5 , 2018

One-point functions --- Motivation

Possibly the simplest observables beyond the spectrum

Strong signs of integrability, closed determinant formulas
(Similarities with three-point functions)

Lead to a positive test of AdS/CFT in a situation where conformal symmetry is partially broken and susy symmetry is partially or fully broken.

Provide input for the boundary conformal bootstrap program
1+2=3

Reveal interesting connections to statistical physics
(Quenched action approach to non-equilibrium stat. phys.)

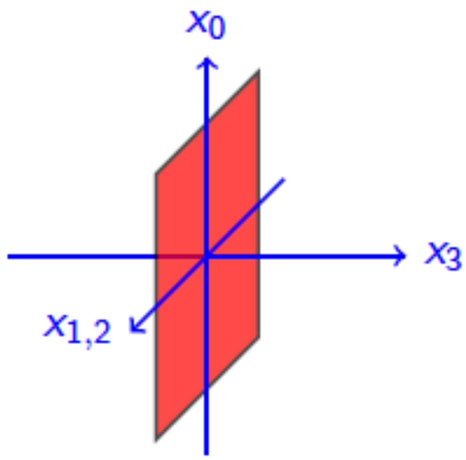
One-point functions in AdS/CFT

Consider a defect CFT ---- with string theory dual

Consider $\mathcal{N}=4$ SYM with a co-dimension one defect

$$S = \underbrace{S_{\mathcal{N}=4}}_{\text{bulk}} + \underbrace{S_{D=3}}_{\text{defect}},$$

Certain b.c. on bulk fields



Novel features

1. One-point functions

Cardy ´84

McAvity & Osborn '95

$$\langle \mathcal{O}^{\text{bulk}}_{\Delta}(x)\rangle = \frac{C}{|x_3|^{\Delta}}$$

Normalization given by: $\lim_{x_3\to\infty} \langle \mathcal{O}^{\text{bulk}}_{\Delta}(y+x)\mathcal{O}^{\text{bulk}}_{\Delta'}(z+x)\rangle = \frac{\delta_{\Delta\Delta'}}{|y-z|^{2\Delta}}$

2. Two-point functions between op's with different conf. dims.

3. Mixed correlators involving bulk and defect fields

Plan of the talk

I. The AdS/dCFT set-up and its extra parameter k

II. Gauge theory results on one-point functions, featuring integrability.

III. String theory predictions for one-point functions, involving double scaling limit.

IV. Confirming predictions (conf. symmetry is partially broken and susy partially or completely broken.)

V. Integrability at finite g ?

VI. Summary/Open problems

Part I.

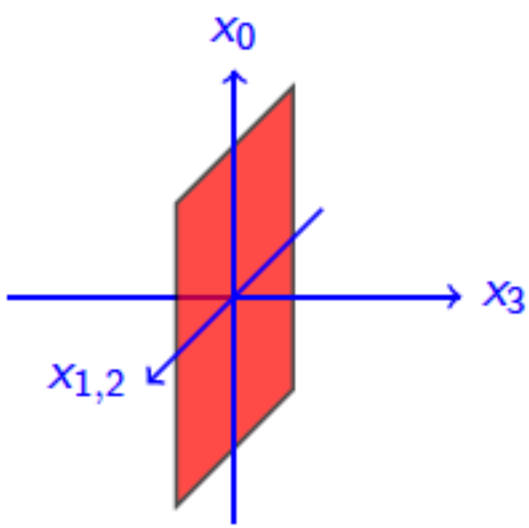
The AdS/dCFT set-up and its parameters

AdS/dCFT (D3-D5 or D3-D7 case)

String theory:

			x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
	N	$D3$	$\times$	$\times$	$\times$	$\times$						
	Probe	$D5$	$\times$	$\times$	$\times$		$\times$	$\times$	$\times$			
or	Probe	$D7$	$\times$	$\times$	$\times$		$\times$	$\times$	$\times$	$\times$	$\times$	

Gauge theory



Defect of co-dimension one

$$S = \underbrace{S_{\mathcal{N}=4}}_{\text{D3-D3 strings}} + \underbrace{S_{D=3}}_{\substack{\text{D3-D5 strings or} \\ \text{D3-D7 strings}}}$$

Parameters: λ, N

Karch & Randall '01,

Freedman & Ooguri '01,

Brane Geometry

Geometry of D5 brane: $AdS_4 \times S^2$ (1/2 susy)

Geometry of D7 brane: $AdS_4 \times S^2 \times S^2$ or $AdS_4 \times S^4$ (No susy)

Precise embedding of D-brane determined by $S = S_{DBI} + S_{WZ}$

Issues

One-point functions at leading order in QFT involve a high number of loops

The D3-D7 probe brane set-up is unstable to fluctuations

AdS/dCFT a strong-weak coupling duality (eternal problem)

Consider a non-trivial vacuum

$$\phi_i^{\rm cl}(x) = \phi_i^{\rm cl}(x_3), \qquad i = 1, \ldots, 6, \qquad \psi_\alpha^{\rm cl} = 0, \quad A_\mu^{\rm cl} = 0,$$

$$\frac{d^2 \phi_i^{\rm cl}}{dx_3^2} = [\phi_j^{\rm cl}, [\phi_j^{\rm cl}, \phi_i^{\rm cl}]]$$

"D3-D5 brane solution:" $\phi_i^{\rm cl} = 0, \; i = 4, 5, 6$

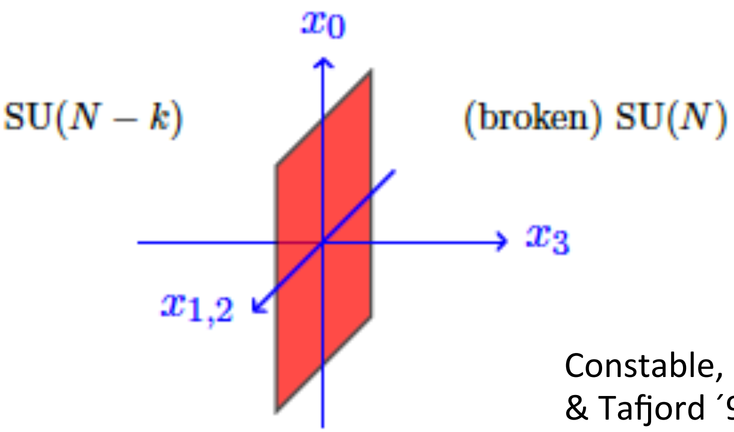
For $x_3 > 0$

$$\phi_i^{\rm cl} = \frac{1}{x_3} \begin{pmatrix} (t_i)_{k \times k} & 0 \\ 0 & 0 \end{pmatrix}, \; i = 1, 2, 3$$

where t_i constitute a k-dimensional irreducible representation of $SU(2)$, in particular $[t_i, t_j] = \epsilon_{ijk} t_k$,

For $x_3 < 0$

$$\phi_i^{\rm cl} = 0 \in \mathfrak{su}(N-k)$$



Constable, Myers & Tafjord ´99

k is an additional (tunable) parameter

``D3-D7 brane solutions''

$SO(3) \times SO(3)$ symmetric solution

For $x_3 > 0$

$$\phi_i^{\rm cl} = \frac{1}{x_3}\begin{pmatrix} (t_i)_{k_1\times k_1} \otimes 1_{k_2\times k_2} & 0 \\ 0 & 0 \end{pmatrix}, \;\; i = 1,2,3$$

$$\phi_i^{\rm cl} = \frac{1}{x_3}\begin{pmatrix} 1_{k_1\times k_1} \otimes (t_i)_{k_2\times k_2} & 0 \\ 0 & 0 \end{pmatrix}, \;\; i = 4,5,6$$

Broken $SU(N)$ invariance

For $x_3 < 0$

$$\phi_i^{\rm cl} = 0 \in \mathfrak{su}(N - k_1 k_2), \;\; i = 1,\ldots,6$$

$SU(N - k_1 k_2)$ invariance

SO(5) symmetric solution

For $x_3 > 0$

$$\phi_i^{\rm cl} = \frac{1}{\sqrt{8}x_3}\begin{pmatrix} (G_i)_{d_G\times d_G} & 0 \\ 0 & 0 \end{pmatrix}, \;\; i = 1,2,3,4,5$$

$$\phi_6^{\rm cl} = 0,$$

where $G_{ij} = [G_i, G_j]$ generate a d_G-dimensional irreducible representation of $SO(5)$

Broken $SU(N)$ invariance

For $x_3 < 0$

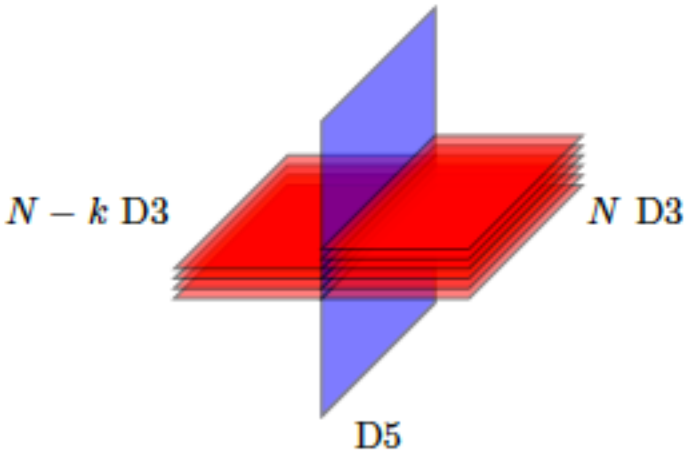
$$\phi_i^{\rm cl} = 0 \in \mathfrak{su}(N - d_G), \;\; i = 1,\ldots,6$$

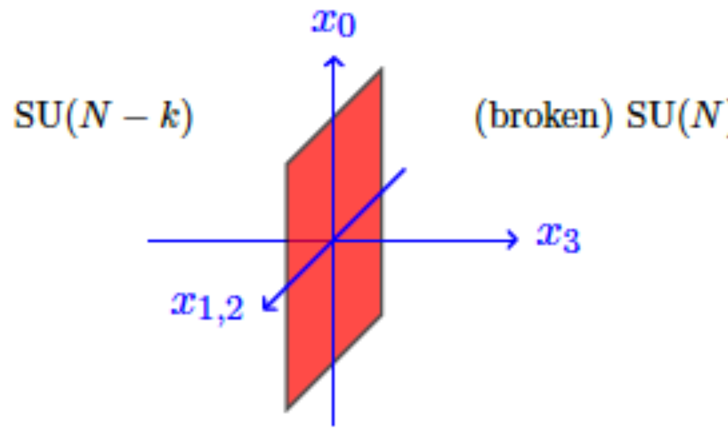
$SU(N - d_G)$ invariance

OBS: AdS/CFT with additional parameters --- The dimension of the representation, i.e.

k or (k_1, k_2) or d_G

The extra parameter on the string theory side





D5 brane: Geometry $AdS_4 \times S^2$

Fuzzy funnel solution
Constable, Myers
& Tafjord ´99, '01

Background gauge field F: k units of magnetic flux on S^2

$$\int_{S^2} \mathcal{F} = 2\pi k$$

D7 brane: Geometry $AdS_4 \times S^2 \times \tilde{S}^2$ or $AdS_4 \times S^4$

Background gauge field F: $\int_{S^2} \mathcal{F} = 2\pi k_1, \quad \int_{\tilde{S}^2} \mathcal{F} = 2\pi k_2$ or $d_G = \frac{1}{8\pi} \int_{S^4} F \wedge F$

Precise embedding of D-brane determined by $S = S_{DBI} + S_{WZ}$

Part II.

One-point functions in gauge theory

One-point functions in gauge theory

Scalar operators can have non-zero 1-pt fcts already at tree-level

Wish: A Systematic approach to the computation of 1-pt functions of *conformal* operators using the tools of integrability

Conformal scalar operators=Eigenstates of integrable SO(6) spin chain

Consider first SU(2) subsector for D3-D5 case:

$$Z = \Phi_1 + i\Phi_4, \; W = \Phi_2 + i\Phi_5$$

$$\mathcal{O} = \mathrm{Tr}(\underbrace{Z\,Z\,ZW\,W\,Z\,Z\ldots W}_{\text{L fields}}) \sim |\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow\ldots\downarrow\rangle_L$$

Eigenstates (of length L= Δ with M flipped spins):

$$\mathcal{O}_\Delta \sim |\{u_i\}\rangle = \hat{B}(u_M)\ldots\hat{B}(u_1)|0\rangle_L, \qquad |0\rangle_L = |\uparrow\uparrow\ldots\uparrow\rangle_L$$

$$\langle\mathcal{O}_\Delta(x)\rangle = (\mathrm{Tr}(ZZZWWZZ\ldots W) + \ldots)\,|_{Z\to\frac{t_1}{x_3},W\to\frac{t_2}{x_3}} \equiv \frac{C(\{u_i\})}{x_3^\Delta}$$

The matrix product state

Matrix Product State associated with the defect for given k:

deLeeuw, C.K. & Zarembo '15,

$$\langle MPS_k| = \mathrm{tr}_a \prod_{l=1}^{L} \left(\langle \uparrow_l | \otimes (t_1^{(k)})^a + \langle \downarrow_l | \otimes (t_2^{(k)})^a \right)$$

Object to calculate: $C_k\left(\{u_j\}\right) = \dfrac{\left\langle MPS_k \middle| \{u_j\}_{j=1}^{M} \right\rangle_L}{\langle \{u_j\} | \{u_j\} \rangle^{\frac{1}{2}}}$

Notice: 3 parameters: k,L,M

Non-vanishing result only if

- L and M both even
- Bethe state has total momentum zero
- The rapidities come in pairs, i.e. $\{u_i\} = \{-u_i\}$
  Follows from the fact that $Q_{2n+1}|MPS\rangle = 0$ for all $n \in N_0$

Result for k=2

I. Of determinant form

deLeeuw, C.K. & Zarembo '15,

$$C_2\left(\{u_j\}\right) = \frac{\langle MPS_2 \mid \{u_j\}\rangle}{\langle \{u_j\}|\{u_j\}\rangle^{\frac{1}{2}}} = 2^{1-L}\sqrt{\frac{Q(\frac{i}{2})}{Q(0)}}\sqrt{\frac{\det G^+}{\det G^-}} \qquad (*)$$

$$Q(u) = \prod_{i=1}^{M}(u-u_i)$$ Baxter polynomial

OBS: well-known determinant (due to Gaudin)

$$\langle \{u_j\}|\{u_j\}\rangle = Q(\tfrac{i}{2})\ \det G = Q(\tfrac{i}{2})\ \det G^+ \det G^-$$

$$G_{ij} = \partial_{u_i}\Phi_j, \qquad \exp[i\Phi_k] \equiv \left(\frac{u_k - \frac{i}{2}}{u_k + \frac{i}{2}}\right)^L \prod_{\substack{j=1 \\ j\neq k}}^{M} \frac{u_k - u_j + i}{u_k - u_j - i}$$

(*) Follows from results involving overlaps of Bethe states and the Néel state plus its n-fold raised versions.

Poszgay '13, Brockmann, Nardis, Wouter & Caux, '14, Brockmann '14, Tsuchiya '98

Result for any k

II. Interesting factorization property

Buhl-Mortensen, de Leeuw, C.K. & Zarembo, 15.

$$C_k = i^L T_{k-1}(0) \sqrt{\frac{Q(\frac{i}{2})Q(0)}{Q^2(\frac{ik}{2})}} \sqrt{\frac{\det G_+}{\det G_-}}$$

Can be proven by recursion $|MPS_{k+2}\rangle = \hat{\mathbf{T}}\left(\frac{ik}{2}\right)|MPS_k\rangle - \left(\frac{k+1}{k-1}\right)^L |MPS_{k-2}\rangle$

T_n the transfer matrix

For the n+1 dimensional representation (in the auxiliary space)

$$T_n(u) = \sum_{a=-\frac{n}{2}}^{\frac{n}{2}} (u+ia)^L \frac{Q(u+\frac{n+1}{2}i)Q(u-\frac{n+1}{2}i)}{Q(u+(a-\frac{1}{2})i)Q(u+(a+\frac{1}{2})i)}$$

Plays an important role in

- Extension of the formula to one and higher loops
- Extension of the formula to the full scalar SO(6) sector

Extension to full scalar SO(6) sector (D3-D5 case)

Consider operators built from

$$X = \phi_1 + i\phi_4, \quad W = \phi_2 + i\phi_5, \quad Y = \phi_3 + i\phi_6$$
$$\overline{X} = \phi_1 - i\phi_4, \quad \overline{W} = \phi_2 - i\phi_5, \quad \overline{Y} = \phi_3 - i\phi_6$$

Conformal operators=Eigenstates of SO(6) integrable spin chain

SO(6) sector closed at one-loop only

Eigenstates of length L: $|u_i, v_j^+, v_k^-\rangle_L$

characterized by three sets of rapidities $\{u_i\}_{i=1}^{M}, \ \{v_j^+\}_{i=1}^{N^+}, \ \{v_j^-\}_{i=1}^{N^-}$

Corresponding operator:
$$\begin{aligned} L - M &= \#X - \#\overline{X}, \\ M - N_+ &= \# W + \#\overline{X} + \#\overline{Y}, \\ N_+ - N_- &= \#Y - \#\overline{Y}, \\ N_- &= \# \text{ barred fields} \end{aligned}$$

Extension to the full scalar SO(6) sector (D3-D5 case)

de Leeuw, C.K & Linardopoulos, 18.

Selection rules:

- Momentum of Bethe state equal to zero
- M and $L + N_+ + N_-$ even
- The rapidities come in pairs, i.e. $\{u_i\}, \{v_j^+\}, \{v_l^-\} = \{-u_i\}, \{-v_j^+\}, \{-v_l^-\}$

Result for C_k:

- Exact formula valid for any, L, M, N^+, N^- and k

$$C_k^{SO(6)} = \sqrt{\frac{Q(0)Q(\frac{i}{2})Q(\frac{ik}{2})Q(\frac{ik}{2})}{\bar{Q}_+(0)\bar{Q}_+(\frac{i}{2})\bar{Q}_-(0)\bar{Q}_-(\frac{i}{2})}} \cdot \mathbb{T}_{k-1}(0) \cdot \sqrt{\frac{\det G_+}{\det G_-}}$$

$$\mathbb{T}_n(x) = \sum_{a=-\frac{n}{2}}^{\frac{n}{2}} (x + ia)^L \frac{Q_+(x + ia)Q_-(x + ia)}{Q(x + i(a + \frac{1}{2}))Q(x + i(a - \frac{1}{2}))}.$$

- Proof is lacking. Can be checked numerically up to $L = 8, k = 6$ for $SO(6)$ and $L = 16, k = 6$ for $SU(3)$. (Involves summing 10^{12} terms.)

Extension to SO(5) symmetric the D3-D7 brane set-up

de Leeuw, C.K & Linardopoulos, 17.

Selection rules:

- Momentum of Bethe state equal to zero
- The rapidities come in pairs, i.e. $\{u_i\}, \{v_j^+\}, \{v_l^-\} = \{-u_i\}, \{-v_j^+\}, \{-v_l^-\}$
- $(L, M, N_+, N_-) = (L, M, M/2, M/2)$

Result for C_n:

- Trivialises for $SU(2)$ sub-sector, $N_- = N_+ = 0$.
- Trivialises for $SU(3)$ sub-sector, $N_- = 0$
- No closed expression found for the full $SO(6)$ sector (yet)

Should we expect a closed expression?

Integrability of Matrix Product States

In statistical physics: $\langle MPS_k \left| \{u_j\}_{j=1}^M \right\rangle_L \sim \langle \text{Initial} \left| \{u_j\}_{j=1}^M \right\rangle_L$

Proposed criterion for integrability of MPS: $\hat{Q}_{2m+1}|MPS\rangle = 0, \quad m \geq 1$

Piroli, Pozsgay
Vernier '17

NB: Imply pairing of roots for Bethe states in order to have non-vanishing overlap with MPS

- D3-D5 MPS fulfills integrability criterion
- D3-D7 SO(5) symmetric MPS fulfills integrability criterion
- D3-D7 SO(3) x SO(3) symmetric MPS does not fulfills integrability criterion

Puzzle: No closed form for 1-pt functions for the SO(5) symmetric D3-D7 case (yet)

Possible key to progress: A scattering picture with a reflection matrix

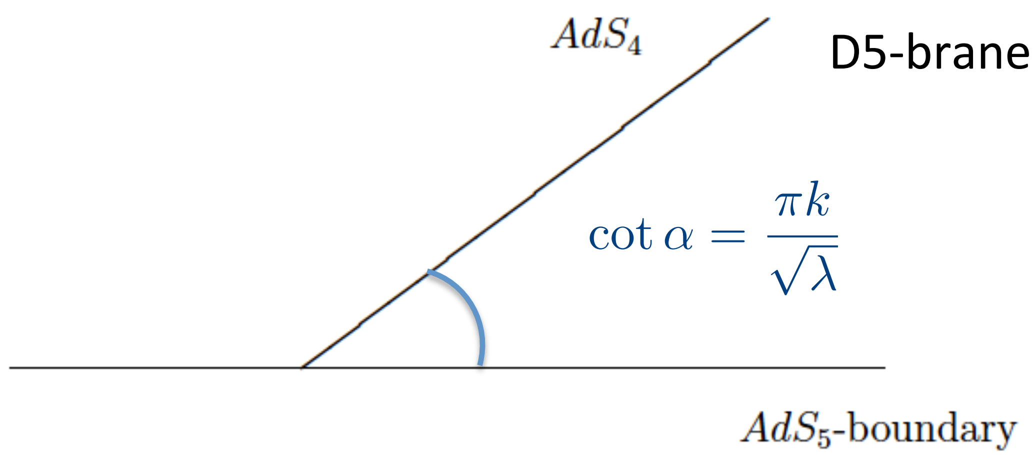
Part III.

One-point functions in string theory

The double scaling limit

D3-D5 probe brane system suggests a new double scaling limit

Nagasaki & Yamaguchi '12,



$$\cot\alpha = \frac{\pi k}{\sqrt{\lambda}}$$

$$\lambda \to \infty, \; k \to \infty, \; \frac{\lambda}{k^2} \text{ finite} \qquad (N \to \infty)$$

One can compare perturbative gauge theory to semi-classical string theory (or sugra).

For D3-D7 with SO(3) x SO(3) symmetry: $$\cot\alpha = \frac{\pi\sqrt{k_1^2 + k_2^2}}{\sqrt{\lambda}}$$

C.K., Semenoff & Young '12,

NB: D3-D5 Both susy and conformal symmetry are partially broken

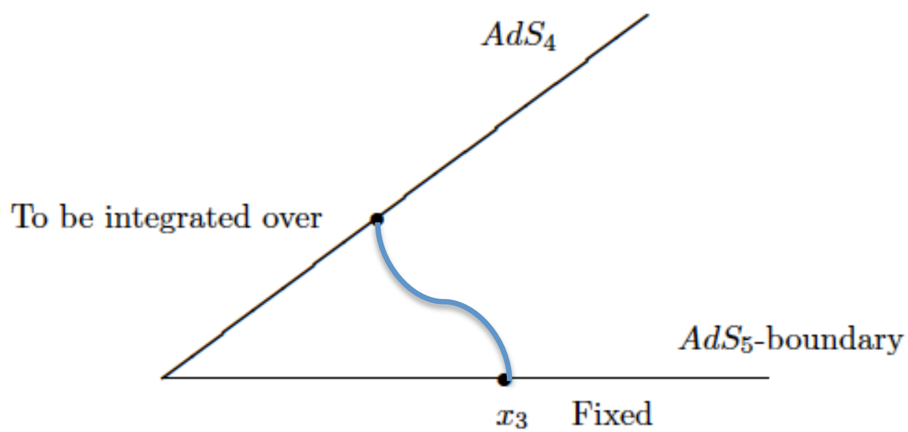
D3-D7 Susy completely broken

One-point functions of chiral primaries --- GKPW method

- Only one chiral primary with the appropriate symmetry for each (even) conf.dim. Δ
- Find the variation $\delta S_E = \delta(S_{DBI} + S_{WZ})$
- Expand fluctuations in terms of spherical harmonics on S^5

$$\delta S_E(X,\Omega) = \sum_{\Delta}\sum_{I} s_{\Delta I}(X)\, Y_{\Delta I}(\Omega)$$

- Pick $Y_{\Delta I}(\Omega)$ the wanted unique chiral primary
- Replace $s_{\Delta I}(X)$ with a bulk-to-boundary propagator reaching from a point z on the brane to x_3 at the boundary. Integrate over z.



Nagasaki & Yamaguchi '12,

Results in d.s.l. for chiral primary of length L

Match at leading order in d.s.l. both for D3-D5 and D3-D7

Nagasaki & Yamaguchi '12, C.K. , Semenoff, Young '12,

Next to leading order predictions

D3-D5 set-up:

Nagasaki & Yamaguchi '12,

$$\left.\frac{\langle Y_L(\lambda)\rangle}{\langle Y_L(0)\rangle}\right|_{sugra} = 1 + \frac{\lambda}{4\pi^2 k^2}\frac{L(L+1)}{4(L-1)} + \mathcal{O}\left(\left(\frac{\lambda}{4\pi^2 k^2}\right)^2\right)$$

D3-D7 set-up with SO(3) x SO(3) symmetry

Grau, C.K, Volk & Wilhelm, to appear

$$\left.\frac{\langle Y_L(\lambda)\rangle}{\langle Y_L(0)\rangle}\right|_{sugra} = 1 + \frac{\lambda}{4\pi^2(k_1^2+k_2^2)}\frac{1}{\left[k_1^2+k_2^2\right]^3(L-1)\sin(L+2)\phi}\Big[$$
$$+ 4Lk_1k_2\left[(k_1)^4+(k_2)^4+(k_1k_2)^2(L+1)\right]\cos L\phi$$
$$+\left[(k_2)^2-(k_1)^2\right]\left[4(k_1k_2)^2(L^2+L-1)+((k_1)^4+(k_2)^4)(L^2+3L-2)\right]\sin L\phi\Big]$$
$$+\mathcal{O}\left(\left(\frac{\lambda}{4\pi^2(k_1^2+k_2^2)}\right)^2\right), \qquad \phi = \arctan\left(\frac{k_1}{k_2}\right)$$

Part IV.

One-loop one-point functions in gauge theory
Matching gauge and string theory

One-loop one-point functions

Motivation:

I. Setting up the quantum computation is non-trivial

II. Checking the one-loop prediction from string theory

Choose $\mathcal{O} = \mathrm{Tr} Z^L \in SU(2)$ sector

$$= c_1 Y_L + \ldots$$

$$\frac{\langle \mathrm{tr} Z^L(\lambda) \rangle}{\langle \mathrm{tr} Z^L(0) \rangle} = \frac{\langle Y_L(\lambda) \rangle}{\langle Y_L(0) \rangle}$$

III. Integrability and finite g

Setting up loop computations

1. Expand the $\mathcal{N}=4$ SYM action around ϕ_{cl}

Buhl-Mortensen, de-Leeuw, Ipsen, C.K., Wilhelm, '16

$$\phi_i = \phi_i^{\text{cl}} + \tilde{\phi}_i = \frac{t_i}{x_3} + \tilde{\phi}_i \qquad i = 1, 2, 3$$

➡ Complicated mass matrix

-Mixing involving both flavour and colour

-Mass terms involving x_3 dependence

➡ New cubic interaction terms

2. Gauge fix to kill a term of the type: $\frac{i}{2}[A_\mu, \phi_i^{cl}]\,\partial^\mu \tilde{\phi}_i$

3. Diagonalize mass matrix (using fuzzy sphere spherical harmonics)

$$S_{\text{mass}} = \frac{1}{x_3^2}\text{Tr}\Big([t_i,\tilde{\phi}_j][t_i,\tilde{\phi}_j] + [A_\mu, t_i][A^\mu, t_i] + 4i[A^3,\tilde{\phi}_i]t_i + [t_i,t_j][\tilde{\phi}_i,\tilde{\phi}_j]$$
$$+ [t_i,\tilde{\phi}_i][t_j,\tilde{\phi}_j] + [t_i,\tilde{\phi}_j][\tilde{\phi}_i, t_j] + \text{ghosts} + \text{fermions}\Big)$$

Setting up loop computations

4. Dealing with the space-time dependence of S_{mass}

Bosonic Propagator: $$\left(-\partial_\mu \partial^\mu + \frac{m^2}{x_3^2}\right) K(x,y) = \delta(x,y)$$

Define: $\tilde{K}(x,y) = x_3\, y_3\, K(x,y)$. Then

$\tilde{K}$ propagator with mass $\tilde{m}^2 = m^2 - 2$ in AdS_4 with metric $g_{\mu\nu} = \frac{1}{x_3^2}\delta_{\mu\nu}$

Special care needed for zero modes to keep ½ susy

Gaiotto & Witten ´08.

5. Regularization and renormalization needed

Consider propagators in $\mathrm{A}dS_{d+1}$ with $d = 3 - 2\epsilon$

Consider $3 + 2\epsilon$ fields of type $\tilde{\phi}_4, \tilde{\phi}_5, \tilde{\phi}_6$
and $3 - 2\epsilon$ fields of type A_0, A_1, A_2.

The one-loop computation

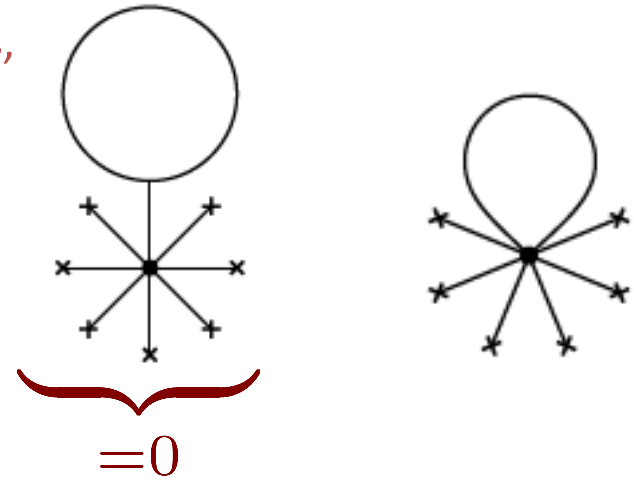
Start from SU(2) Bethe eigenstate

Two types of corrections

1. Loop contribution to one-point function
2. Correction of the eigenstate (not for chiral primaries)

1. Scalars, gauge fields, fermions, ghosts

Two planar diagrams



2. Can be found using $\Theta-\mathrm{morphism}$ Gromov & Vieira ´13, ‘14

Field theory result, chiral primaries

D3-D5 dCFT (1/2 susy):

$$\frac{\langle \mathrm{tr} Z^L \rangle_{\text{one-loop}}}{\langle \mathrm{tr} Z^L \rangle_{\text{tree level}}} = 1 + \frac{\lambda}{4\pi^2 k^2} \frac{L(L+1)}{4(L-1)}$$

Buhl-Mortensen, de-Leeuw, Ipsen, C.K., Wilhelm, '16

D3-D7 dCFT with SO(3) x SO(3) symmetry (no susy):

Grau, C.K, Volk & Wilhelm, to appear

$$\frac{\langle \mathrm{tr} Z^L \rangle_{\text{one loop}}}{\langle \mathrm{tr} Z^L \rangle_{\text{tree-level}}} = 1 + \frac{\lambda}{4\pi^2 (L-1)} \frac{1}{[(k_1)^2 + (k_2)^2]^4 \sin(L+2)\phi} \Big[+ 4L k_1 k_2 \left[(k_1)^4 + (k_2)^4 + (k_1 k_2)^2 (L+1)\right] \cos L\phi + \left[(k_2)^2 - (k_1)^2\right] \left[4(k_1 k_2)^2 (L^2 + L - 1) + ((k_1)^4 + (k_2)^4)(L^2 + 3L - 2)\right] \sin L\phi \Big]$$

$$\phi = \arctan\left(\frac{k_1}{k_2}\right)$$

Complete match with string theory in both cases

One point functions at higher loop orders

Buhl-Mortensen, de-Leeuw, Ipsen, C.K., Wilhelm ,17

D3-D5 dCFT: Can we guess the result based on integrability ?

Idea:

- Make the integrability characteristic replacements in the Bethe eqns.

$$u \pm \frac{i}{2} \longrightarrow x(u \pm \frac{i}{2}), \quad \Phi_k \longrightarrow \tilde{\Phi}_k, \quad u(x) = x + \frac{g^2}{x}, \quad g^2 = \frac{\lambda}{8\pi^2}$$

(plus dressing phase plus wrapping corrections)

- Make the same replacement in the transfer matrix, i. e.

$$T_n(u) \longrightarrow \tilde{T}_n(u) = g^L \sum_{a=-\frac{n}{2}}^{\frac{n}{2}} x(u+ia)^L \frac{Q(u+\frac{n+1}{2}i)Q(u-\frac{n+1}{2}i)}{Q(u+(a-\frac{1}{2})i)Q(u+(a+\frac{1}{2})i)}$$

- Possible asymptotic (i.e. so far without wrapping) formula

$$C_k = i^L \tilde{T}_{k-1}(0) \sqrt{\frac{Q(\frac{i}{2})Q(0)}{Q^2(\frac{ik}{2})}} \sqrt{\frac{\det \tilde{G}_+}{\det \tilde{G}_-}}$$

Outcome from the one-loop computation

Buhl-Mortensen, de-Leeuw, Ipsen, C.K., Wilhelm ,17

$$C_k = i^L \tilde{T}_{k-1}(0) \sqrt{\frac{Q(\frac{i}{2})Q(0)}{Q^2(\frac{ik}{2})}} \sqrt{\frac{\det \tilde{G}_+}{\det \tilde{G}_-}} \mathbb{F}_k \qquad (*)$$

$$\mathbb{F}_k = 1 + g^2 \left[\Psi(\tfrac{k+1}{2}) + \gamma_E - \log 2\right] \Delta^{(1)} + O(g^4)\,,$$

Checked for: L, M=2, any k (analytical derivation)
L=8, M=4, k=2,3,4,5,6 (numerically)

Exponentialization at higher loop orders ?

$$\mathbb{F}_k \stackrel{?}{=} 2^{L-\Delta} \exp\left[(\Delta - L)(\Psi(\tfrac{k+1}{2}) + \gamma_E)\right]$$

Two loop computation might clarify this (NB Two loops in AdS)

(*) reproduces string theory result in d.s.l. up to wrapping order!

Summary of results

- A closed expression for all tree-level scalar one-point functions in the D3-D5 case for any representation label k.
- Closed one-point function formula in SU(2) sub-sector at one-loop for D3-D5 set-up. (Proposal for asymptotic formula.)
- Match with string theory for D3-D5 to two leading orders in d.s.l. for the one-pt function of a chiral primary (conf. & susy partially broken.)
- Match with string theory for D3-D7 to two leading orders in double scaling parameter for the one-pt function of a chiral primary (susy completely broken).
- Ongoing investigations of the D3-D7 brane system. (Non-protected one-point functions vanish in SU(2) and SU(3) sector in the SO(5) symmetric case, no closed formula for SO(6) (yet) in either case)

Open questions:

- Understanding the integrability of D3-D5 from scattering theory

- Understanding if D3-D7 is integrable or not

- Working out the precise action of the 3D defect field theory
  (important for higher loop computations as well as the conf. bootstrap program)

- More elaborate data-mining using one and two-point functions
  by means of boundary conformal bootstrap eqns.

- More detailed comparisons with string theory for both one- and
  two-point functions :
  f.inst. involving spinning strings (i.e. non-protected operators)

- Moving on to higher loop orders (understand wrapping)

Thank you