New Directions with Democratic Theories

Michael Kroyter

0911.2962, 1010.1662, in progress with S. Giaccari

Holon Institute of Technology

New Frontiers in String Theory 2018 – YITP, Kyoto
19 – July – 2018

Outline

1. Introduction
2. Formulating the Cohomology Problem
3. Constructing Democratic Theories
4. Outlook

Outline

1. Introduction
2. Formulating the Cohomology Problem
3. Constructing Democratic Theories
4. Outlook

Introduction

Several superstring field theories were formulated in recent years.
Very impressive progress!

Somewhat earlier another formulation appeared:
"Democratic" (all pictures are present) open SSFT.

Introduction

Several superstring field theories were formulated in recent years.
Very impressive progress!

Somewhat earlier another formulation appeared:
"Democratic" (all pictures are present) open SSFT.

The democratic theory

- All pictures; cubic; large Hilbert space; single mid-point insertion.
- Includes (unifies) the Ramond sector.
- BV (classical) master equation is formally straightforward.
- Generalization to general D-brane system is straightforward.
- Partial gauge fixing of NS sector gives the modified theory.

Introduction

Several superstring field theories were formulated in recent years.
Very impressive progress!

Somewhat earlier another formulation appeared:
"Democratic" (all pictures are present) open SSFT.

The democratic theory

- All pictures; cubic; large Hilbert space; single mid-point insertion.
- Includes (unifies) the Ramond sector.
- BV (classical) master equation is formally straightforward.
- Generalization to general D-brane system is straightforward.
- Partial gauge fixing of NS sector gives the modified theory.

Did not play any role in recent developments. Why?

Possible Criticism of the Democratic Theory

- The space of string fields?
- Mid-point problems?
- $\mathcal{O}_p$ and X_p operators known only implicitly.
- Witten's theory at $pic = -1$?
- Gauge fixing to a non-fixed picture?
- Scattering amplitudes?
- Operators of arbitrarily negative conformal weight?
- Symplectic form?

Outline

1 Introduction

2 Formulating the Cohomology Problem

3 Constructing Democratic Theories

4 Outlook

Formulating String Field Theory

- Identify the worldsheet cohomology problem.
- Extend vertex operatoes off-shell to string fields.
- Reinterpret cohomology problem as e.o.m and gauge symmetry.
- Derive from an action.
- Add interaction terms: Non-linear e.o.m and gauge symmetry.
- Verify that a proper covering of moduli space is obtained.

Formulating String Field Theory

- Identify the worldsheet cohomology problem.
- Extend vertex operatoes off-shell to string fields.
- Reinterpret cohomology problem as e.o.m and gauge symmetry.
- Derive from an action.
- Add interaction terms: Non-linear e.o.m and gauge symmetry.
- Verify that a proper covering of moduli space is obtained.

Various Formulations of the Cohomology Problem

The cohomology problem for the open RNS string

- In the small space $\Psi \in H_S$ $(\eta\Psi = 0)$ at a fixed picture number:
  $Q\Psi = 0, \qquad \delta\Psi = Q\Lambda, \qquad pic(\Psi) = p, \qquad gh(\Psi) = 1.$
  $\eta\Psi = 0 \quad \Rightarrow \quad \eta\Lambda = 0.$
- In the large space $\Psi \in H_L$ at a fixed picture number:
  $Q\eta\Psi = 0, \qquad \delta\Psi = Q\Lambda_1 + \eta\Lambda_2, \qquad pic(\Psi) = p, \qquad gh(\Psi) = 0.$

These two formulations are behind most of the SSFT formulations.

Various Formulations of the Cohomology Problem

The cohomology problem for the open RNS string

- In the small space $\Psi \in H_S$ $(\eta\Psi = 0)$ at a fixed picture number:
  $Q\Psi = 0, \qquad \delta\Psi = Q\Lambda, \qquad pic(\Psi) = p, \qquad gh(\Psi) = 1.$
  $\eta\Psi = 0 \quad \Rightarrow \quad \eta\Lambda = 0.$
- In the large space $\Psi \in H_L$ at a fixed picture number:
  $Q\eta\Psi = 0, \qquad \delta\Psi = Q\Lambda_1 + \eta\Lambda_2, \qquad pic(\Psi) = p, \qquad gh(\Psi) = 0.$

These two formulations are behind most of the SSFT formulations.

- In the large space $\Psi \in H_L$ at a fixed picture number:
  $(Q - \eta)\Psi = 0, \qquad \delta\Psi = (Q - \eta)\Lambda, \qquad pic(\Psi) = p, \qquad gh(\Psi) = 1.$
- In the large space $\Psi \in H_L$ at an arbitrary picture range:
  $(Q - \eta)\Psi = 0, \quad \delta\Psi = (Q - \eta)\Lambda, \quad p_1 < pic(\Psi) < p_2, \quad gh(\Psi) = 1.$
  In particular one can take $p_1 = -\infty$ and/or $p_2 = \infty$.

Various Formulations of the Cohomology Problem

The cohomology problem for the open RNS string

- In the small space $\Psi \in H_S$ $(\eta\Psi = 0)$ at a fixed picture number:
  $Q\Psi = 0, \qquad \delta\Psi = Q\Lambda, \qquad pic(\Psi) = p, \qquad gh(\Psi) = 1.$
  $\eta\Psi = 0 \quad \Rightarrow \quad \eta\Lambda = 0.$
- In the large space $\Psi \in H_L$ at a fixed picture number:
  $Q\eta\Psi = 0, \qquad \delta\Psi = Q\Lambda_1 + \eta\Lambda_2, \qquad pic(\Psi) = p, \qquad gh(\Psi) = 0.$

These two formulations are behind most of the SSFT formulations.

- In the large space $\Psi \in H_L$ at a fixed picture number:
  $(Q - \eta)\Psi = 0, \qquad \delta\Psi = (Q - \eta)\Lambda, \qquad pic(\Psi) = p, \qquad gh(\Psi) = 1.$
- In the large space $\Psi \in H_L$ at an arbitrary picture range:
  $(Q - \eta)\Psi = 0, \quad \delta\Psi = (Q - \eta)\Lambda, \quad p_1 < pic(\Psi) < p_2, \quad gh(\Psi) = 1.$
  In particular one can take $p_1 = -\infty$ and/or $p_2 = \infty$.
  But there is a subtlety here.

The Cohomology Problem of $(Q-\eta)$ for $\Psi \in H_L$ (Berkovits 01)

If $pic(\Psi) = p$ is fixed, the equation $(Q-\eta)\Psi = 0$ gives components at pictures p, $p-1$, which should vanish independently.
Then, $Q\Psi = \eta\Psi = 0$, i.e. $\Psi \in H_S$ and obeys the standard equation.

The gauge transformation $\delta\Psi = (Q-\eta)\Lambda$ implies $\Lambda = \Lambda_1 + \Lambda_2$ with
$pic(\Lambda_1) = p, \quad pic(\Lambda_2) = p+1, \quad \eta\Lambda_1 = Q\Lambda_2 = 0.$ Then,
$\Lambda_1 = \eta\tilde{\Lambda}_1, \quad \Lambda_2 = Q\tilde{\Lambda}_2.$ All in all: $\delta\Psi = Q\eta\tilde{\Lambda}.$

The Cohomology Problem of $(Q-\eta)$ for $\Psi \in H_L$ (Berkovits 01)

If $pic(\Psi) = p$ is fixed, the equation $(Q-\eta)\Psi = 0$ gives components at pictures p, $p-1$, which should vanish independently.
Then, $Q\Psi = \eta\Psi = 0$, i.e. $\Psi \in H_S$ and obeys the standard equation.

The gauge transformation $\delta\Psi = (Q-\eta)\Lambda$ implies $\Lambda = \Lambda_1 + \Lambda_2$ with
$pic(\Lambda_1) = p, \quad pic(\Lambda_2) = p+1, \quad \eta\Lambda_1 = Q\Lambda_2 = 0$. Then,
$\Lambda_1 = \eta\tilde{\Lambda}_1, \quad \Lambda_2 = Q\tilde{\Lambda}_2$. All in all: $\delta\Psi = Q\eta\tilde{\Lambda}$.

The equation $(Q-\eta)\Psi = 0$ and gauge transformation $\delta\Psi = (Q-\eta)\Lambda$ define the standard cohomology without restricting the picture number.

Changing the picture is a gauge transformation: Let $pic(\Psi) = p$. Then,
$(Q-\eta)\Psi = 0 \quad \Rightarrow \quad Q\Psi = \eta\Psi = 0 \quad \Rightarrow \quad \Psi = \eta\Phi = \eta(\xi\Psi)$.
Define $\Lambda = \xi\Psi$. Then $\delta\Psi = (Q-\eta)(\xi\Psi) = X\Psi - \Psi$. So $p \to p+1$.

Similarly, one can decrease the picture.

Starting from a bounded picture range we can send Ψ to any given picture.

The Cohomology Problem of $(Q-\eta)$ for $\Psi \in H_L$ (Berkovits 01)

If $pic(\Psi) = p$ is fixed, the equation $(Q-\eta)\Psi = 0$ gives components at pictures p, $p-1$, which should vanish independently.
Then, $Q\Psi = \eta\Psi = 0$, i.e. $\Psi \in H_S$ and obeys the standard equation.

The gauge transformation $\delta\Psi = (Q-\eta)\Lambda$ implies $\Lambda = \Lambda_1 + \Lambda_2$ with
$pic(\Lambda_1) = p, \quad pic(\Lambda_2) = p+1, \quad \eta\Lambda_1 = Q\Lambda_2 = 0.$ Then,
$\Lambda_1 = \eta\tilde{\Lambda}_1, \quad \Lambda_2 = Q\tilde{\Lambda}_2.$ All in all: $\delta\Psi = Q\eta\tilde{\Lambda}$.

The equation $(Q-\eta)\Psi = 0$ and gauge transformation $\delta\Psi = (Q-\eta)\Lambda$ define the standard cohomology without restricting the picture number.

Changing the picture is a gauge transformation: Let $pic(\Psi) = p$. Then,
$(Q-\eta)\Psi = 0 \quad \Rightarrow \quad Q\Psi = \eta\Psi = 0 \quad \Rightarrow \quad \Psi = \eta\Phi = \eta(\xi\Psi)$.
Define $\Lambda = \xi\Psi$. Then $\delta\Psi = (Q-\eta)(\xi\Psi) = X\Psi - \Psi$. So $p \to p+1$.

Similarly, one can decrease the picture.

Starting from a bounded picture range we can send Ψ to any given picture.

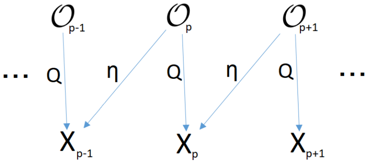
What if it is unbounded?

Multi-Picture Changing Operators and Their Potentials

Q and η have trivial cohomology in H_L since $\mathcal{O}_0 \equiv -c\xi\partial\xi e^{-2\phi}$, $\mathcal{O}_1 \equiv \xi$ are their contracting homotopy operators: $Q\mathcal{O}_0 = 1, \quad \eta\mathcal{O}_1 = 1.$

One can obtain from these operators picture changing operators:
$Q\mathcal{O}_1 = X \equiv X_1, \quad \eta\mathcal{O}_0 = Y \equiv X_{-1}.$

This structure can be extended to arbitrary pinture:



In this infinite chain $X_0 \equiv 1$ and all picture changing operators X_p and their potentials $\mathcal{O}_p$ are weight zero primaries.

The $\mathcal{O}_p$ Operators

Most of the $\mathcal{O}_p$ are:

- Not uniquely defined.
- Defined in terms of complicated expressions.
- Are only implicitly know.

Examples $(G_m \equiv i\psi_\mu \partial X^\mu)$

- $\mathcal{O}_{-1} = \frac{1}{5} c\,\xi\partial\xi e^{-3\phi} G_m - \xi e^{-2\phi}$.
- $\mathcal{O}_0 \equiv -c\xi\partial\xi e^{-2\phi}$.
- $\mathcal{O}_1 \equiv \xi$.
- $\mathcal{O}_2 = -c\xi\xi' + \xi e^{\phi} G_m + \left(2b\eta\xi\phi' + \eta\xi b' - 2b\xi\eta' - \frac{29b'' + 51b'\phi' + 2b\phi'^2}{86}\right) e^{2\phi}$.

The $\mathcal{O}_p$ Operators

Most of the $\mathcal{O}_p$ are:

- Not uniquely defined.
- Defined in terms of complicated expressions.
- Are only implicitly know.

Examples ($G_m \equiv i\psi_\mu \partial X^\mu$)

- $\mathcal{O}_{-1} = \frac{1}{5}\, c\, \xi\partial\xi e^{-3\phi} G_m - \xi e^{-2\phi}$.
- $\mathcal{O}_0 \equiv -c\xi\partial\xi e^{-2\phi}$.
- $\mathcal{O}_1 \equiv \xi$.
- $\mathcal{O}_2 = -c\xi\xi' + \xi e^{\phi} G_m + \left(2b\eta\xi\phi' + \eta\xi b' - 2b\xi\eta' - \frac{29b'' + 51b'\phi' + 2b\phi'^2}{86}\right) e^{2\phi}$.

For a particular background an ansatz for $\mathcal{O}_3$ includes 371 free parameters and leads to a 94-parameter family of solutions. In a particularly simple case $\mathcal{O}_3$ is the sum of 336 terms.

If being primary is unnecessary, the expressions somewhat simplify.

The Space of String Fields

The $\mathcal{O}_p$ can be used to define contracting homotopy operators for $(Q-\eta)$:

$$\mathcal{O}_+ = -\sum_{p=1}^{\infty}\mathcal{O}_p, \qquad \mathcal{O}_- = \sum_{p=-\infty}^{0}\mathcal{O}_p$$

This implies that the cohomology at the large Hilbert space is empty.
A subtlety? A contradiction?

The Space of String Fields

The $\mathcal{O}_p$ can be used to define contracting homotopy operators for $(Q-\eta)$:

$$\mathcal{O}_+ = -\sum_{p=1}^{\infty} \mathcal{O}_p, \qquad \mathcal{O}_- = \sum_{p=-\infty}^{0} \mathcal{O}_p$$

This implies that the cohomology at the large Hilbert space is empty.
A subtlety? A contradiction?

What is the large Hilbert space? What is the small Hilbert space?
Not a Hilbert space, not an inner product space, not even a properly defined linear space... (e.g., is the state $\sum_n n!\alpha_n^\mu$ part of the space?)

The Space of String Fields

The $\mathcal{O}_p$ can be used to define contracting homotopy operators for $(Q-\eta)$:

$$\mathcal{O}_+ = -\sum_{p=1}^{\infty} \mathcal{O}_p, \qquad \mathcal{O}_- = \sum_{p=-\infty}^{0} \mathcal{O}_p$$

This implies that the cohomology at the large Hilbert space is empty.
A subtlety? A contradiction?

What is the large Hilbert space? What is the small Hilbert space?
Not a Hilbert space, not an inner product space, not even a properly defined linear space... (e.g., is the state $\sum_n n! \alpha_n^\mu$ part of the space?)

We believe that a proper definition for all these spaces exists.
In particular for a vertex operator V at any given picture, we would not accept $\mathcal{O}_\pm V$ as a legitimate state, since this is an infinite sum of gauge equivalent states.

The Huge Hilbert Space

We assume that proper definitions of H_S, H_L at a given picture exist. Similarly we should assume that H_L at an unbounded picture exists and that its $(Q - \eta)$ cohomology is the standard one.

Then, a space that includes $\mathcal{O}_\pm V$ for all $V \in H_L$ and that trivializes the $(Q - \eta)$ cohomology also exists.
This is the huge Hilbert space H_H.

The idea of embedding a space into a larger space which trivializes the cohomology of some operator can be very useful.
It was used in string field theory for defining various solutions as a formal gauge solutions, with the larger space including X^μ in a compactified theory, including ξ in SSFT formulations based on the small Hilbert space, and generally for "singular gauge parameters".
It is useful also in pure-spinor formulations and elsewhere.

Similarly, H_H might become useful for constructing solutions in a democratic SFT formulation.

Outline

1. Introduction
2. Formulating the Cohomology Problem
3. Constructing Democratic Theories
4. Outlook

A Linearized Democratic Theory

The string field Ψ lives in the large Hilbert space within any desirable range of picture numbers.

Find an action from which the linearized e.o.m could be derived:

$$(Q - \eta)\Psi = 0.$$

If $\delta\Psi$ can have arbitrary picture the action variation would not vanish:

$$S = \int \frac{1}{2}\Psi(Q - \eta)\Psi \quad ??$$

A Linearized Democratic Theory

The string field Ψ lives in the large Hilbert space within any desirable range of picture numbers.
Find an action from which the linearized e.o.m could be derived:

$$(Q - \eta)\Psi = 0.$$

If $\delta\Psi$ can have arbitrary picture the action variation would not vanish:

$$S = \int \frac{1}{2}\Psi(Q - \eta)\Psi \quad ??$$

No, the "integration" is in the large Hilbert space:
The ghost number and parity are wrong.

Use a Lagrangian multiplier string field Φ of ghost number -1 and arbitrary picture?

$$S = \int \frac{1}{2}\Phi(Q - \eta)\Psi \quad ??$$

A Linearized Democratic Theory

The string field Ψ lives in the large Hilbert space within any desirable range of picture numbers.
Find an action from which the linearized e.o.m could be derived:

$$(Q-\eta)\Psi = 0.$$

If $\delta\Psi$ can have arbitrary picture the action variation would not vanish:

$$S = \int \frac{1}{2}\Psi(Q-\eta)\Psi \quad ??$$

No, the "integration" is in the large Hilbert space:
The ghost number and parity are wrong.

Use a Lagrangian multiplier string field Φ of ghost number -1 and arbitrary picture?

$$S = \int \frac{1}{2}\Phi(Q-\eta)\Psi \quad ??$$

No, Φ becomes dynamical: $(Q-\eta)\Phi = 0$. Not clear how to eliminate it.

The Democratic Theory

We must insert a non-dynamical operator to the action:

- This operator must commute with $(Q - \eta)$.
- Since the physical part of the vertex is in the small Hilbert space this operator should include ξ.
- It should carry no quantum numbers, e.g. be a zero weight primary.

Such an operator exists. Define: $\mathcal{O} \equiv \sum_{p=-\infty}^{\infty} \mathcal{O}_p = \mathcal{O}_- - \mathcal{O}_+$. Then,

$(Q - \eta)\mathcal{O} = 1 - 1 = 0$. The other properties also hold.

The Democratic Theory

We must insert a non-dynamical operator to the action:

- This operator must commute with $(Q-\eta)$.
- Since the physical part of the vertex is in the small Hilbert space this operator should include ξ.
- It should carry no quantum numbers, e.g. be a zero weight primary.

Such an operator exists. Define: $\mathcal{O} \equiv \sum_{p=-\infty}^{\infty} \mathcal{O}_p = \mathcal{O}_- - \mathcal{O}_+$. Then, $(Q-\eta)\mathcal{O} = 1 - 1 = 0$. The other properties also hold.

Can be extended to a non-linear theory:

Action: $S = \int \mathcal{O}(\frac{1}{2}\Psi(Q-\eta)\Psi + \frac{1}{3}\Psi^3)$.

E.O.M: $(Q-\eta)\Psi + \Psi^2 = 0$.

Gauge symmetry: $\delta\Psi = (Q-\eta)\Lambda + [\Psi,\Lambda]$.

A Mid-Point Insertion

For the described properties to hold, the interaction should be cyclic.
Can be achieved by inserting $\mathcal{O}$ at the string mid-point.
Classically all is well, but there are problems:

- The symplectic form is off-diagonal. Is it regular?
- Propagator?
- Scattering amplitudes?
- Partial gauge fixing the picture part of the gauge can lead to Witten's (inconsistent) theory.

Different gauge fixings led to other theories:
The modified cubic, Berkovits theory...

New Democratic Theories

Replace the kinetic term by: $S_0 = \frac{1}{2}\int \tilde{\mathcal{O}}\Psi(Q-\eta)\Psi.$

Here, $\tilde{\mathcal{O}} = \frac{1}{2\pi i}\oint \frac{dz\mathcal{O}(z)}{z}$. Look for an extension to an A_∞ theory.

New Democratic Theories

Replace the kinetic term by: $S_0 = \frac{1}{2}\int \tilde{\mathcal{O}}\Psi(Q-\eta)\Psi$.

Here, $\tilde{\mathcal{O}} = \frac{1}{2\pi i}\oint \frac{dz\mathcal{O}(z)}{z}$. Look for an extension to an A_∞ theory.

If possible:

- Inclusion of the Ramons sector would remain straightforward.
- The BV master equation would automatically hold, and not only formally.
- Presumable, such a version of the democratic theory would give a framework from which all the new theories could be derived.

Outline

1 Introduction

2 Formulating the Cohomology Problem

3 Constructing Democratic Theories

4 Outlook

Outlook

- Examine whether it is possible to obtain such a democratic theory.
- Consider new gauge fixings of the theory that would lead to new formulations.
- New expressions for scattering amplitudes?
- Extend to closed and to heterotic theories.
- Use the $\mathcal{O}_p$ and X_p operators in the study of moduli spaces.

Outlook

- Examine whether it is possible to obtain such a democratic theory.
- Consider new gauge fixings of the theory that would lead to new formulations.
- New expressions for scattering amplitudes?
- Extend to closed and to heterotic theories.
- Use the $\mathcal{O}_p$ and X_p operators in the study of moduli spaces.

THANK YOU!