Deformations of 4d SCFTs and infrared supersymmetry enhancement

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based on collaborations with

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"New Frontiers in String Theory 2018", YITP, July 5th, 2018

- KM and J. Song, "Enhancement of Supersymmetry via Renormalization Group Flow and the Superconformal Index", **1606.05632**

- KM and J. Song, "N=1 Deformations and RG Flows of N=2 SCFTs", **1607.04281**

- P. Agarwal, KM and J. Song, "N=1 Deformations and RG Flows of N=2 SCFTs, part II", **1610.05311**

- E. Nardoni, KM and J. Song, "Landscape of Simple Superconformal Field Theories in 4d", **1806.08353**

Introduction

Conformal field theory (CFT) is one of the important objects in theoretical physics, displaying the physics in the fixed point in QFT; in critical phenomena in condensed matter theory;
still a latest **new frontier in string theory**.

We focus here on **CFTs in 4d**. These are characterized by

- Central charges, a and c
- Global symmetry (flavor central charge)
- Operator spectrum, OPE coefficients, 3-point functions, defects, …

Q: What is the simplest CFT in 4d?

“Simple” ?

A measure of it is **the central charge**

- counts “degrees of freedom”

- **a satisfies (weak) a-theorem** [Cardy, Komargodsky-Schwimmer]

$$a_{\mathrm{UV}} > a_{\mathrm{IR}}$$

Q’: What is the CFT with the smallest central charge (except for free theories) ?

With supersymmetry

The situation becomes easier in superconformal field theory (SCFT).

- superconformal group highly restricted

- Localization computation of partition functions

[Kapustin-Willet-Yaakov, Hama-Hosomichi-Lee, Pestun, ...]

- Hofman-Maldacena bound: $\frac{1}{2} \leq \frac{a}{c} \leq \frac{3}{2}$ (tighter for N≧2)

Q: What is the simplest SCFT?

N=2 bound on c

There is a lower bound on c for interacting N=2 SCFTs

[Liendo-Ramirez-Seo]

$$c \geq \frac{11}{30}$$

This bound is saturated by **the Argyres-Douglas theory**

[Aharony-Tachikawa]
[Shapere-Tachikawa]

$$a = \frac{43}{120}, \quad c = \frac{11}{30}$$

This is a candidate for the minimal N=2 SCFT.
(a is minimal among the known N=2 theories.)

Argyres-Douglas theory

- was originally found at a special point on the Coulomb branch of N=2 SU(3) pure SYM with mutually non-local massless particles **[Argyres-Douglas, Argyres-Plesser-Seiberg-Witten]**
- with the Coulomb branch operator of scaling dimension 6/5

Strongly coupled; no known Lagrangian description....

- chiral algebra **[Beem-Lemos-Liendo-Peelears-Rastelli-van Rees, Beem-Rastelli]**
- conformal bootstrap **[Cornagliotto-Lemos-Liendo]**
- superconformal index in some limits **[Buican-Nishinaka, Cordova-Shao, Song]**

Can we engineer the AD theory as a fixed point of an RG flow?

N=1 SU(2) adjoint SQCD w/ N_f=1
coupled to a gauge-singlet **[KM-Song]**

Some UV theory

↓

N=2 Argyres-Douglas theory

The main point, which has not been fully studied, is the deformation coupling with free (gauge-singlet) chiral fields. **[Seiberg, Leigh-Strassler]**

This example shows that coupling singlets leads to an interesting IR fixed point. Other example… **[Kim-Razamat-Vafa-Zafair]**

What is the simplest N=1 SCFT?

By numerical conformal bootstrap, the lower bound for the central charge of the N=1 SCFTs with one chiral operator $\mathcal{O}$ with the particular chiral ring relation, $\mathcal{O}^2 \sim 0$: **[Poland-Stergiou]**

$$\Delta(\mathcal{O}) > 1.41, \quad c > 0.111$$

the SCFT which saturates the bound?

No known SCFT…

We search **all the IR fixed points of adjoint SU(2) SQCD with N_f=1**, allowing the deformations by relevant operators and/or coupling with free sectors.

Summary of results

We find **35 "good" fixed points** by possible deformations of adjoint SU(2) w/ N_f=1. These includes

- **Argyres-Douglas theories H_0 and H_1**
- **the theory with minimal a, H_0***

$$a_{H_0^*} = \frac{263}{768} \simeq 0.3424, \quad c_{H_0^*} = \frac{271}{768} \simeq 0.3529.$$

- **the theory with minimal c, T_0**

$$a_{T_0} \simeq 0.3451, \quad c_{T_0} \simeq 0.3488$$

In addition, there are 30 "ugly" or "bad" fixed points with accidental U(1) symmetry.

Plan of talk

- **W=0 fixed point**
- **Lagrangian for Argyres-Doulgas theory H_0**
- **All deformations of W=0 point**

W=0 fixed point

W=0 fixed point

We consider N=1 supersymmetric SU(2) gauge theory with one adjoint and two fundamental chiral multiples.

- The global (anomaly-free) symmetry is **SU(2) x U(1)$_F$ x U(1)$_{R0}$**

	(q, q')	ϕ
SU(2)	**2**	**1**
U(1)$_{R0}$	1/2	1/4
U(1)$_F$	-2	1

- **The theory is asymptotically free**
- We expect the theory is in conformal phase in the origin

Central charges

The central charges of the SCFT are determined from the anomaly coefficients of the IR R-symmetry: **[Anselmi-Freedman-Grisaru-Johansen]**

$$a = \frac{3}{32}(3\mathrm{Tr}R_{\mathrm{IR}}^3 - \mathrm{Tr}R_{\mathrm{IR}}), \quad c = \frac{1}{32}(9\mathrm{Tr}R_{\mathrm{IR}}^3 - 5\mathrm{Tr}R_{\mathrm{IR}})$$

In our case, the IR R-symmetry is a combination of two U(1)'s. Thus consider the following

$$R_{\mathrm{IR}}(\epsilon) = R_0 + \epsilon\mathcal{F}$$

The true R symmetry is determined by maximizing trial central charge [Intriligator-Wecht]

$$a(\epsilon) = \frac{3}{32}(3\mathrm{Tr}R_{\mathrm{IR}}(\epsilon)^3 - \mathrm{Tr}R_{\mathrm{IR}}(\epsilon))$$

decoupling issue

The trϕ² operator hits the unitarity bound (Δ<1). We interpret this as being decoupled.

[cf. Seiberg, Kutasov-Parnachev-Sahakyan]

After subtracting this, we get

$$a = \frac{15012 + 601\sqrt{601}}{65712} \simeq 0.452668, \quad c = \frac{5841 + 430\sqrt{601}}{32856} \simeq 0.498618$$

The R-charges of the UV fields are

$$R(q) = R(q') = \frac{1}{111}(105 - 2\sqrt{601}) \simeq 0.504, \quad R(\phi) = \frac{1}{111}(3 + \sqrt{601}) \simeq 0.248$$

A way to pick up the interacting part is by introducing a chiral multiplet X to set trϕ²=0: $\delta W = X \mathrm{tr}\phi^2$

$$a_{\text{chiral}}(r) = -a_{\text{chiral}}(2-r)$$

Conformal?

While we don't have any proof that the W=0 theory is an SCFT, we list some clues:

- a-maximization gives a reasonable answer
- superconformal index gives a reasonable answer

  [Kinney-Maldacena-Minwalla-Raju, Romelsberger]

  The index
  - does not vanish
  - has contribution of one stress-energy tensor
  - does not have a non-unitary operator

- **Coulomb branch:**

  Let us deform the theory by adding $W = \lambda \mathrm{tr} q q' \phi$ (without ϕ^2 term). This is in N=1 Coulomb phase parametrized by $\mathrm{tr}\phi^2$ **[Intriligator-Seiberg]** Then, the moduli space has singularities which collapse to the origin when $\lambda \to 0$. This is an N=1 analog of Argyres-Douglas theory.

A Lagrangian for

Argyres-Douglas theory H_0

Lagrangian for H_0

Let us now consider the deformation by

$$W = X \mathrm{tr}\phi^2 + \mathrm{tr}\phi q^2 + M \mathrm{tr}\phi q'^2$$

	q	q'	ϕ	M
$U(1)_{R0}$	1/2	-5/2	1	6
$U(1)_{\mathcal{F}}$	1/2	7/2	-1	-6

By a-maximization, we get the central charges

$$a = \frac{43}{120}, \quad c = \frac{11}{30}$$

which are the same as those of **the Argyres-Douglas theory H_0**.

Chiral ring of H_0

We had the following chiral operators

$$\bcancel{\mathrm{tr}\phi q^2}, \quad \bcancel{\mathrm{tr}\phi q q'}, \quad \mathrm{tr} q q', \quad \bcancel{\mathrm{tr}\phi q'^2}, \quad X, \quad M$$

The F-term conditions are

$$0 = qq + Mq'^2 + 2X\phi, \quad 0 = \mathrm{tr}\phi q'^2, \quad 0 = \phi q, \quad 0 = M\phi q', \quad 0 = \mathrm{tr}\phi^2.$$

Thus, **the generators in the chiral ring** are only

$$\mathrm{tr} q q', \quad M$$

(moduli space of X is uplifted quantum mechanically)

dim = 11/5, 6/5 → **form N=2 Coulomb branch operator multiplet**

Localization computations

- One can get the superconformal index in full generality which agrees with the results in some limits **[KM-Song, Agarwal-KM-Song]**

- Other partition functions **[Fredrickson-Pei-Yan-Ye, Gukov, Fluder-Song]**

- 3d reduction and mirror quiver **[Benvenuti-Giacomelli]**

N=1 deformation

Suppose we have an N=2 SCFT ***T*** with **non-Abelian flavor symmetry F.**

[Gadde-KM-Tachikawa-Yan, Agarwal-Bah-KM-Song]
[Agarwal-Intriligator-Song]
cf. [Heckman-Tachikawa-Vafa-Wecht]

Then let us

- **couple N=1 chiral multiplet M in the adjoint rep of F by the superpotential**

$$W = \mathrm{tr}\mu M$$

- **give a nilpotent vev to M (which is specified by the embedding ρ: SU(2)→F), which breaks F.** $W = \sum_j \mu_{j,j} M_{j,-j}$

(For F=SU(N), this is classified by a partition of N or Young diagram.)

This gives IR theory $T_{IR}[T, \rho]$.

T = SU(2) w/ 4 flavors

In this case, F = SO(8)

We consider the principal embedding of SO(8), the vev which breaks SO(8) completely.

The adjoint rep decomposes as

28 → 3, 7, 7, 11

$$M_{1,-1}, M_{3,-3}, M'_{3,-3}, M_{5,-5}$$

→ after integrating out the massive fields,
we get the superpotential

$$W = \phi qq + M_1 \phi^2 qq' + M_3 qq' + M_5 \phi q' q' + M'_3 \phi^3 q' q',$$

T = SU(2) w/ 4 flavors

Other choices of embeddings:

- **[5,1³], [4,4]** (with SU(2)) → **H_1 theory** (SU(2) flavor symmetry)

$$a = \frac{11}{24}, \quad c = \frac{1}{2}$$

- **[3²,1²]** (with U(1)xU(1)) → **H_2 theory** (SU(3) flavor symmetry)

$$a = \frac{7}{12}, \quad c = \frac{2}{3}$$

- other embeddings → N=1 SCFTs

Chiral algebra?

For principal embedding: we conjecture that the condition for T to have enhancement of supersymmetry in the IR is as follows:

- F is of ADE type
- 2d chiral algebra satisfies the Sugawara condition:

[Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees]

$$\frac{\dim F}{c} = \frac{24h^\vee}{k_F} - 12$$

- **rank-one theories H_1, H_2, D_4, E_6, E_7, E_8 → H_0**
- **SU(N) SQCD with 2N flavors → (A_1, A_{2N})**
- **Sp(N) SQCD with 2N+2 flavors → (A_1, A_{2N+1})**
- **(A_1, D_k) theory [Cecotti-Neitzke-Vafa] → (A_1, A_{k-1})**
- **some quiver gauge theories → (A_N, A_L)**

[Agarwal-Sciarappa-Song]

All deformations of W=0 point

Relevant deformations

There are **5 relevant operators** (Δ<3):

$$
\begin{aligned}
\mathrm{tr}\phi q q: &\quad R \simeq 1.256, \quad \Delta \simeq 1.885\\
\mathrm{tr} q q': &\quad R \simeq 1.008, \quad \Delta \simeq 1.513\\
\mathrm{tr}\phi q q': &\quad R \simeq 1.256, \quad \Delta \simeq 1.885\\
\mathrm{tr}\phi q' q': &\quad R \simeq 1.256, \quad \Delta \simeq 1.885\\
X: &\quad R \simeq 1.504, \quad \Delta \simeq 2.256
\end{aligned}
$$

The IR fixed points under these deformations are as follows:

δW	(a, c)	R_q	IR theory	$SU(2)$	$U(1)_F$
$q^2\phi$	$(0.3451, 0.3488)$	0.9244	$\mathcal{T}_1$ SCFT	–	–
$q'^2\phi$	$(0.3451, 0.3488)$	0.9244	$\mathcal{T}_1$ SCFT	–	–
$qq'\phi$	$(\frac{3}{16}, \frac{1}{8})$		Coulomb	∘	–
qq'	$(\frac{3}{16}, \frac{1}{8})$		Coulomb	∘	–
X	$(\frac{3}{16}, \frac{1}{8})$		Coulomb	∘	–

(in this case $U(1)_F$ is broken, no need to do a-maximization.)

Other deformations

The following 4 operators are "super"-relevant (**Δ**<2):

$$\begin{aligned} \mathrm{tr}\phi qq: &\quad R \simeq 1.256, \quad \Delta \simeq 1.885 \\ \mathrm{tr}qq': &\quad R \simeq 1.008, \quad \Delta \simeq 1.513 \\ \mathrm{tr}\phi qq': &\quad R \simeq 1.256, \quad \Delta \simeq 1.885 \\ \mathrm{tr}\phi q'q': &\quad R \simeq 1.256, \quad \Delta \simeq 1.885 \end{aligned}$$

For such operator O, one can consider a deformation by adding a free chiral multiplet M, and the superpotential coupling W = M O.

By including this deformation, **we find 3 new fixed points**

	W	a	c	$SU(2)$	$U(1)_F$
$\mathcal{T}_1$	$X\phi^2 + q^2\phi$	0.3451	0.3488	$-$	$-$
$\mathcal{T}_2$	$X\phi^2 + Mq^2\phi$	0.4727	0.5351	$-$	$\circ$
$\mathcal{T}_3$	$X\phi^2 + Mqq'\phi$	0.4723	0.5327	$U(1)_d$	$\circ$
$\mathcal{T}_4 = H_1$	$X\phi^2 + Mqq'$	$\frac{11}{24} = 0.4583$	$\frac{1}{2} = 0.5$	$\circ$	$\circ$

Deformations (next level)

From these 4 fixed points we consider further deformations.
Note however that, the dimensions of UV fields are not same as those of W=0 fixed point. Therefore, the (super-)relevant deformations are different from those of W=0 point.

From T_1 fixed point,

	W	a	c
$\mathcal{T}_{1,1} (= H_0)$	$X\phi^2 + q^2\phi + Mq'^2\phi$	$\frac{43}{120} = 0.3583$	$\frac{11}{30} = 0.3667$

From T_2 fixed point,

	W	a	c
$\mathcal{T}_{2,1}$	$X\phi^2 + M_1q^2\phi + M_1qq'$	0.4416	0.4806
$\mathcal{T}_{2,2} = \mathcal{T}_3$	$X\phi^2 + M_1q^2\phi + M_1q'^2\phi$	0.4723	0.5327
$\mathcal{T}_{2,3} = \mathcal{T}_{1,1}$	$X\phi^2 + M_1q^2\phi + q'^2\phi$	$\frac{43}{120} = 0.358333$	$\frac{11}{30} = 0.3667$
$\mathcal{T}_{2,4}$	$X\phi^2 + M_1q^2\phi + M_1^2$	0.4332	0.4919
$\mathcal{T}_{2,5}$	$X\phi^2 + M_1q^2\phi + M_2qq'\phi$	0.4927	0.5714
$\mathcal{T}_{2,6}$	$X\phi^2 + M_1q^2\phi + M_2q'^2\phi$	0.4925	0.5698

all level computation

By applying these deformations to all levels, we found totally 35 “good” fixed points.

[KM-Nardoni-Song]

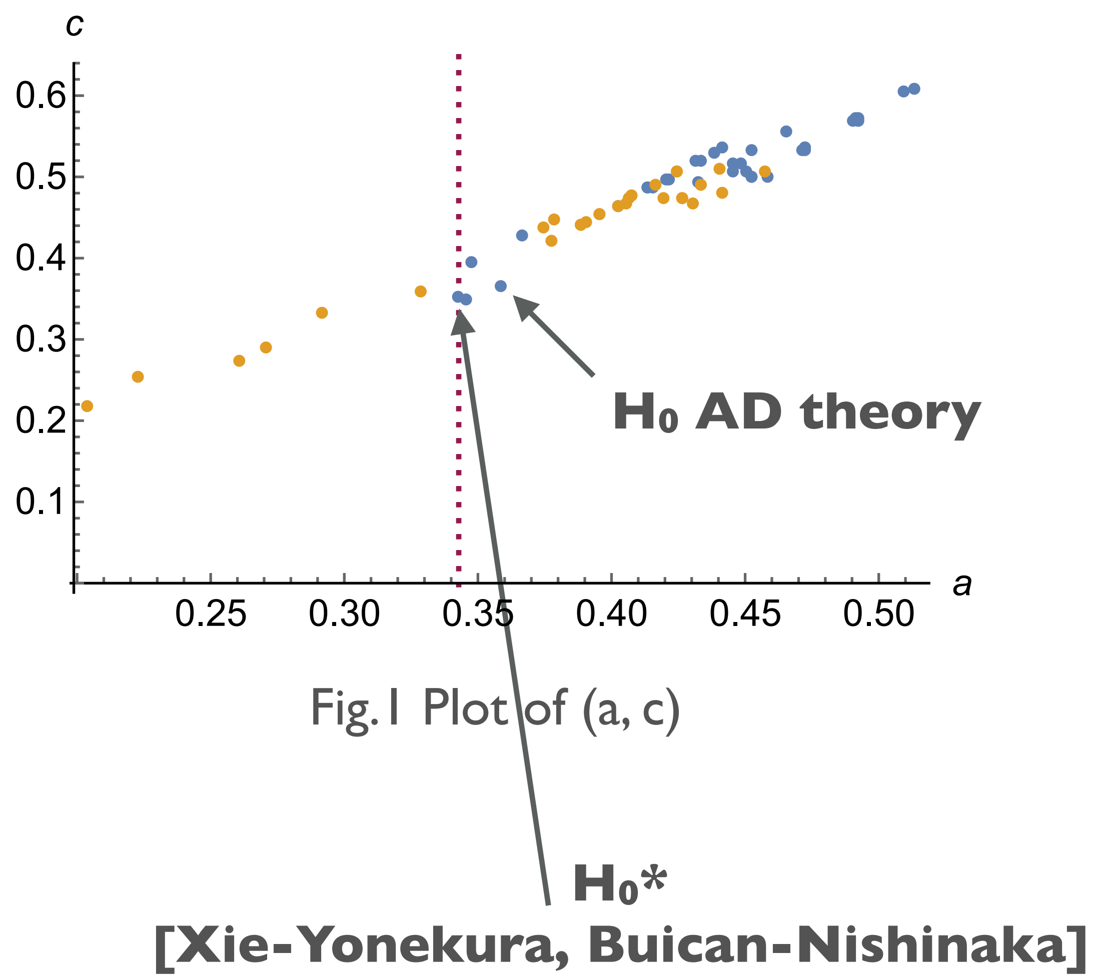

Fig.1 Plot of (a, c)

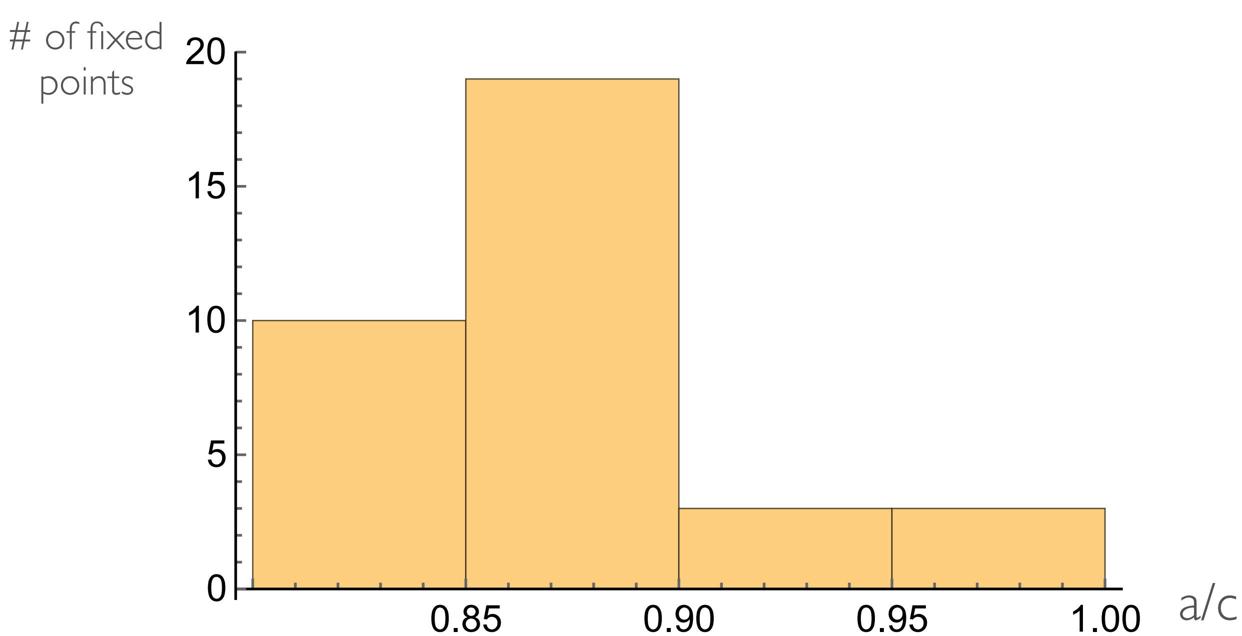

Fig. 2 Histogram of the ratio a/c of the 35 fixed points.

- **H_0^*, minimal a:** $W = X\mathrm{tr}\phi^2 + \mathrm{tr}\phi q^2 + M\mathrm{tr}\phi q'^2 + M^2$

  There is no global U(1) symmetry other than $U(1)_R$, the central charges

$$a_{H_0^*} = \frac{263}{768} \simeq 0.3422, \quad c_{H_0^*} = \frac{261}{768} \simeq 0.3529.$$

  which are the same as those studied by **[Xie-Yonekura, Buican-Nishinaka]**

- **T_0, minimal c:** $W = X\mathrm{tr}\phi^2 + \mathrm{tr}\phi q^2$

  There is a global U(1) symmetry and the central charges are

$$a_{T_0} = \frac{81108 + 1465\sqrt{1465}}{397488} \simeq 0.3451, \quad c_{T_0} = \frac{29088 + 1051\sqrt{1465}}{198744} \simeq 0.3488.$$

  Also, minimal a for SCFTs with global U(1). **[Benvenuti]**

Both theories have the scalar operator $\mathcal{O}$ with the lowest dimension **satisfying the relation $\mathcal{O}^2 \sim 0$**.

Minimal theory?

The minimal central charges are obtained by the deformation

$$W = X\mathrm{tr}\phi^2 + M\mathrm{tr}qq' + M^2 + \mathrm{tr}\phi q^2 + \hat{X}\mathrm{tr}\phi q'^2$$

The R-charges are determined to be

$$R(q) = \frac{7}{8}, \quad R(q') = \frac{1}{8}, \quad R(\phi) = \frac{1}{4}, \quad R(M) = 1, \quad R(X) = R(\hat{X}) = \frac{3}{2}$$

and the central charges:

$$a = \frac{417}{2048} \simeq 0.2036, \quad c = \frac{449}{2048} \simeq 0.2192,$$

cf. bootstrap bound: $\Delta(O) > 1.414, \quad c > 0.1111$

A subtlety is that the index computation shows that there is accidental U(1) global symmetry, as well as a unitarity-violating (fermionic) operator...

Conclusion and discussion

We found 35 fixed points obtained by the deformations of N=1 adjoint SU(2) SQCD w/ N_f=1. These includes two N=2 SCFTs and the minimal N=1 SCFTs.

- deformations of other UV Lagrangian theories? Smaller central charges?
- Why susy enhancement?? What is the condition?
- Holographic dual of the RG flow with the enhanced susy.
- The minimal theory which saturates the conformal bootstrap bound?
- string/M-theory realization?

Thank you!