How to gauge-fix superstring field theory in the large Hilbert space

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Today's topic

- Berkovits' WZW-like theory

  - string field based on ξηϕ-system

    $\eta \varphi \neq 0 \quad \text{where} \quad \eta \equiv \eta_0$

  - large gauge invariance

    $\delta\varphi = Q\Lambda + \eta\Omega + \cdots$

  - various gauge conditions

- Unsolved gauge-fixing problem

  - how to gauge-fix it precisely?

$$S = \int_0^1 dt \left\langle A_t[\varphi] \, , Q \, A_\eta[\varphi] \right\rangle$$

Q : BRST operator
< , > : BPZ inner product
$A_\eta[\varphi]$: functional of string field s.t.

$$\eta \, A_\eta[\varphi] - A_\eta[\varphi] * A_\eta[\varphi] = 0$$

N.Berkovits gave the following solution

$$A_\eta[\varphi] = \left(\eta \, e^{t\varphi}\right) e^{-t\varphi}$$

$$A_t[\varphi] = \left(\partial_t \, e^{t\varphi}\right) e^{-t\varphi}$$

But, why large space?

- Why ξηϕ-system (not βγ-system)?

  - String field theory in small space is easily gauge-fixable

  - βγ-system is rather geometrical

  - usual (geometrical) propagator

Siegel gauge

$$b_0 \Psi_n = 0$$

Propagator

$$P_n = \frac{b_0}{L_0}$$

- Large gauge invariance enable us to take various gauge conditions

  - unusual propagators available (don't have to be geometrical unlike βγ)

Kroyter-Okawa-Schnabl-Torii-Zwiebach 2012

Gauge condition

$$\begin{aligned} b_0 \Phi_{(-n,0)} &= 0 \quad (n \geq 0)\,, \\ d_0 \Phi_{(-n,m)} + \alpha b_0 \Phi_{(-n,m+1)} &= 0 \quad (0 \leq m \leq n-1)\,, \\ d_0 \Phi_{(-n,n)} &= 0 \quad (n \geq 0)\,, \end{aligned}$$

$$\begin{aligned} b_0 d_0 \Phi_{(n+1,-1)} &= 0 \quad (n \geq 1)\,, \\ \alpha b_0 \Phi_{(n+1,-m)} + d_0 \Phi_{(n+1,-(m+1))} &= 0 \quad (1 \leq m \leq n-1)\,. \end{aligned}$$

Propagator

$$\mathcal{P}_{n+1,n+2} = \begin{pmatrix} P_b & \frac{d_0}{(\alpha+1)L_0} & 0 & \cdots & 0 & 0 & 0 \\ 0 & \frac{\alpha b_0}{(\alpha+1)L_0} & \frac{d_0}{(\alpha+1)L_0} & \ddots & \vdots & \vdots & \vdots \\ \vdots & 0 & \frac{\alpha b_0}{(\alpha+1)L_0} & \ddots & 0 & \vdots & \vdots \\ \vdots & \vdots & 0 & \ddots & \frac{d_0}{(\alpha+1)L_0} & 0 & \vdots \\ \vdots & \vdots & \vdots & \ddots & \frac{\alpha b_0}{(\alpha+1)L_0} & \frac{d_0}{(\alpha+1)L_0} & 0 \\ 0 & 0 & 0 & \cdots & 0 & \frac{\alpha b_0}{(\alpha+1)L_0} & P_d \end{pmatrix}$$

$$P_b = \left(\alpha + \frac{\eta_0 d_0}{L_0}\right)\frac{b_0}{(\alpha+1)L_0}\,, \quad P_d = \left(1 + \alpha\frac{Q b_0}{L_0}\right)\frac{d_0}{(\alpha+1)L_0}\,. \qquad d = [Q, b\xi]$$

Motivation of the “large” space

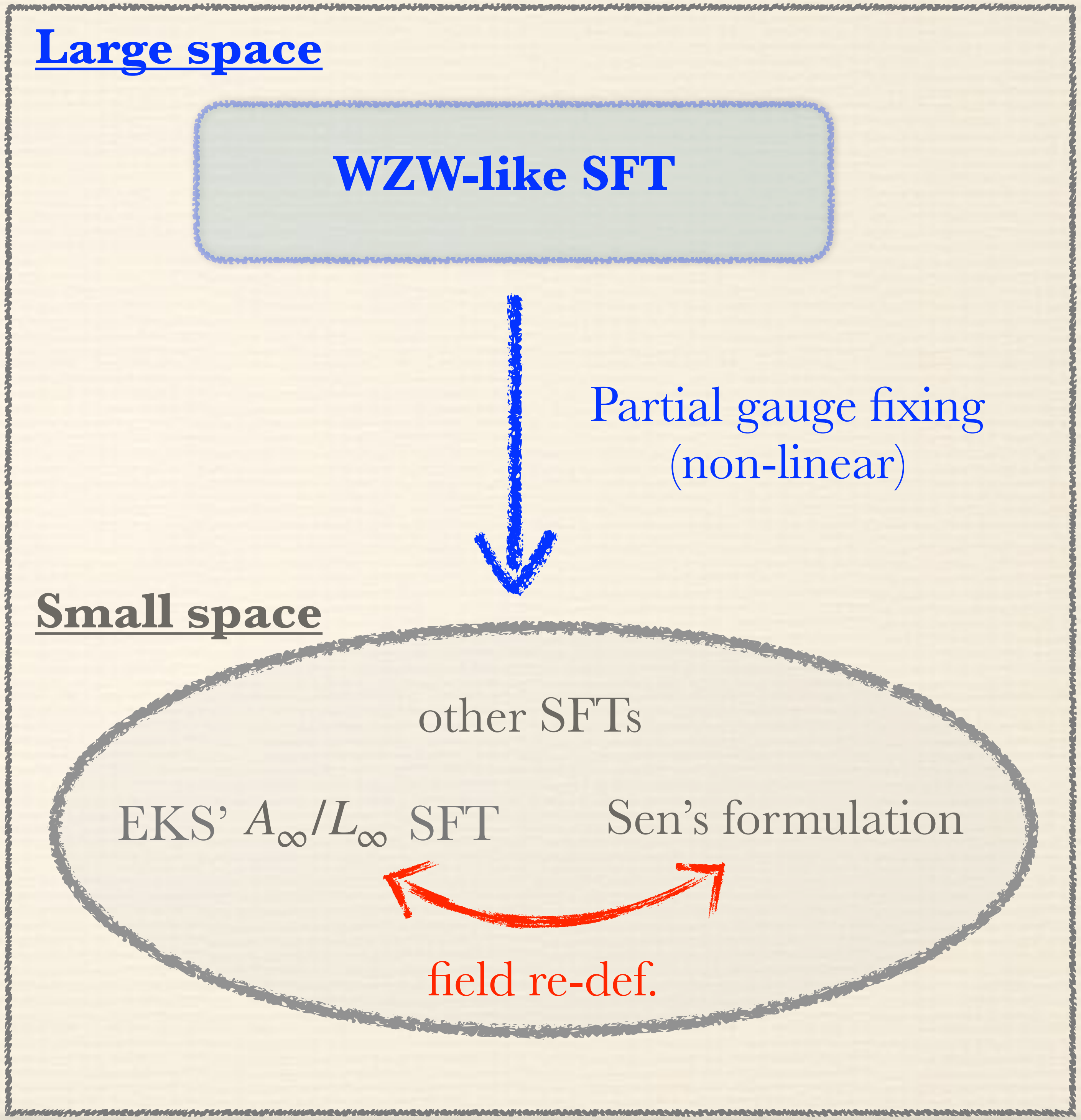
- Partial gauge-fixing

  - yields other SFTs

- Large string field is interesting

  - unusual (non-geo.) propagators

  - to understand SFT itself



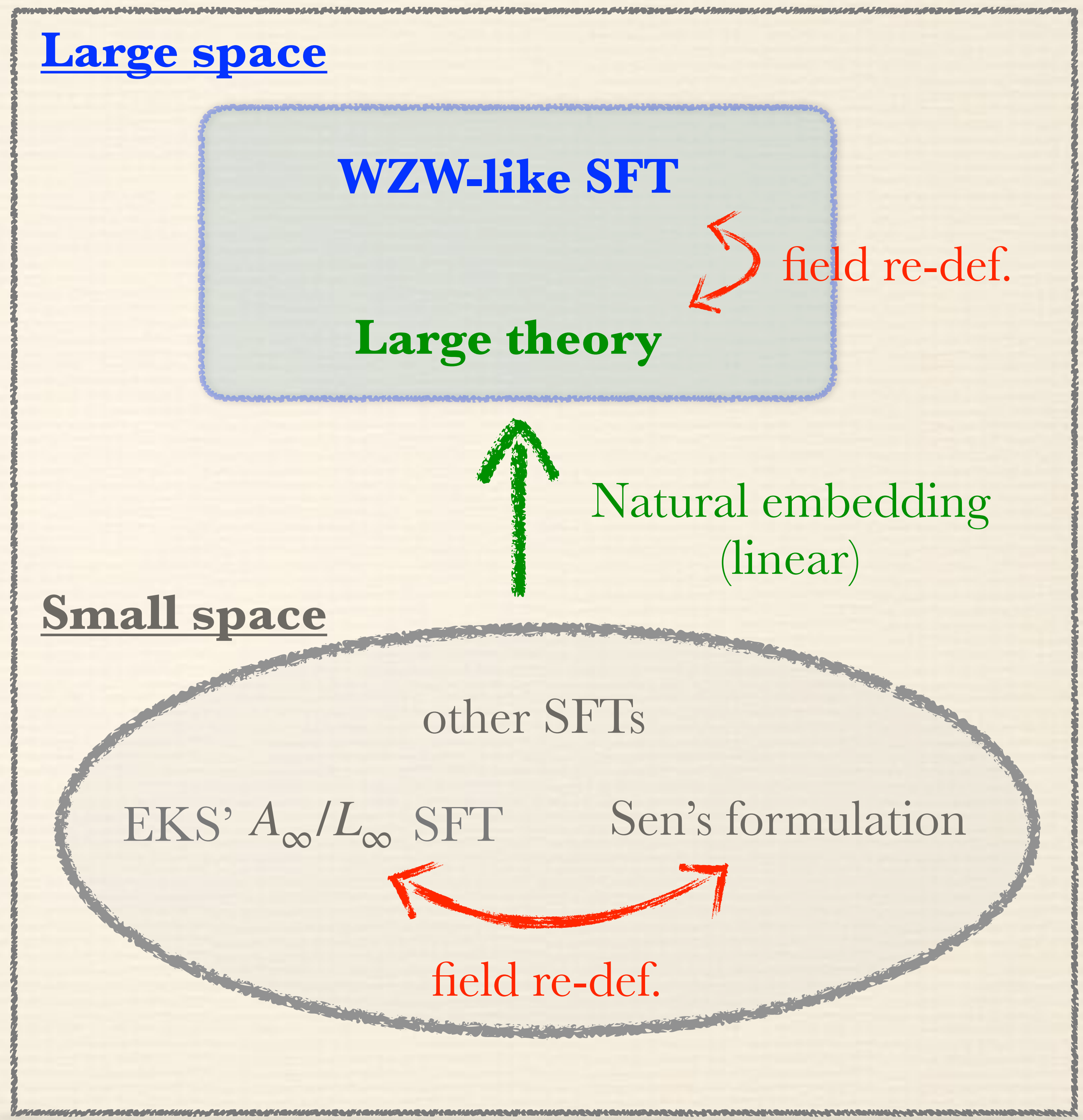
Motivation of the "large" space

- Natural embedding

  - every SFT can be large

Large-space technique is applicable

to SFT defined in *small space* !!



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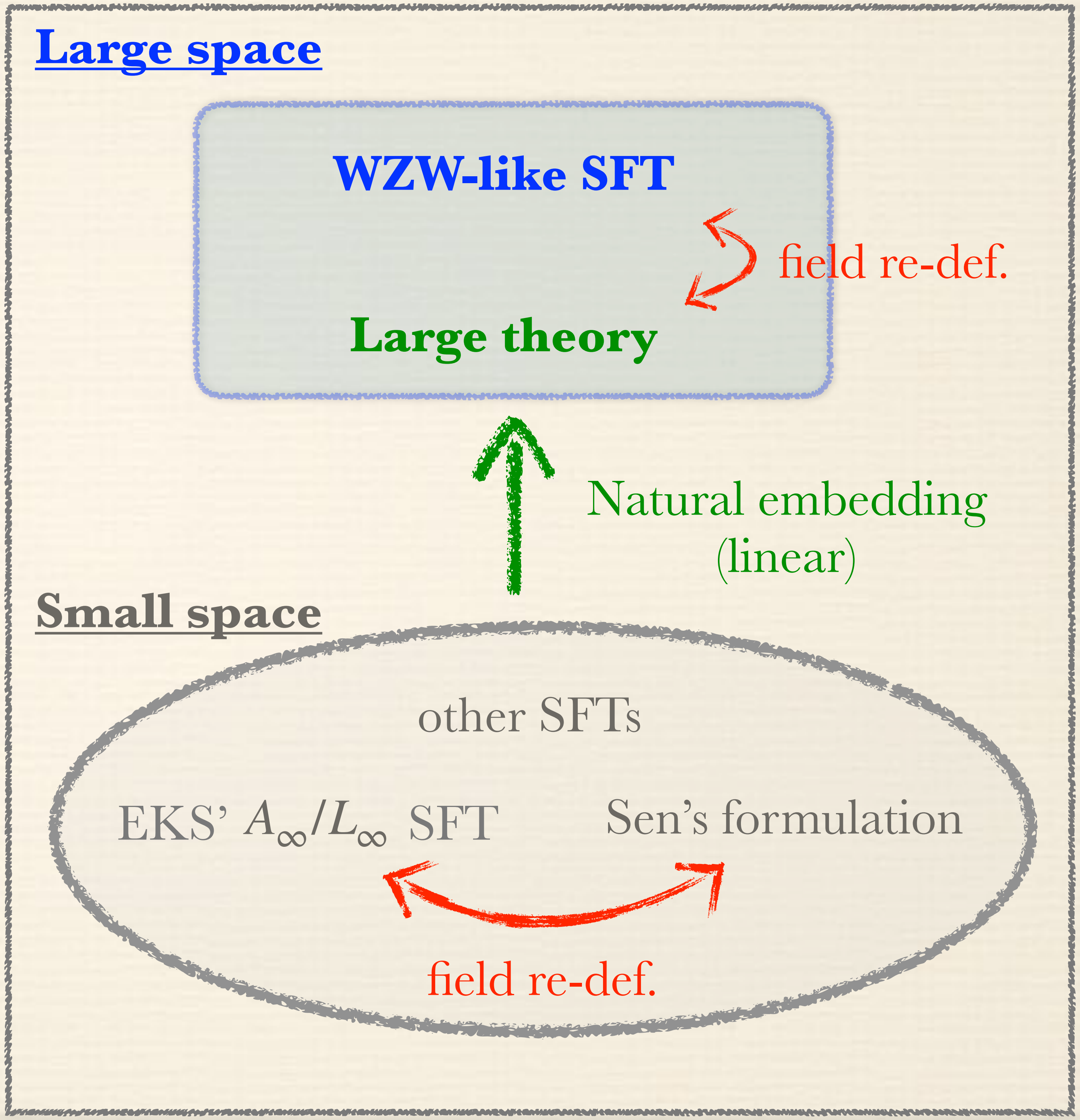
BV string fields-antifields

don't have to be small



Motivation: Natural embedding

- Gauge parameters can be ***large***.

  - e.o.m. of SFT in small space : $Q\Psi = 0$ where $\eta\Psi = 0$.

  - gauge variation must be small : $\eta(\delta\Psi) = 0$

$$\delta\Psi = Q\lambda_1 \quad \text{with small gauge parameter} \quad \eta\,\lambda_1 = 0$$

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$$\delta\lambda_1 = Q\lambda_2$$

$$\vdots$$

$$\delta\lambda_n = Q\lambda_{n+1}$$

$$\vdots$$

$\longrightarrow$

Field & Gauge parameters

$$\{\Psi\,;\,\lambda_1\,,\lambda_2\,,\ldots\,,\lambda_n\,,\ldots\}$$

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$$\vdots$$
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→ **Field & Gauge parameters** $\{\Psi\,;\,\lambda_1\,,\lambda_2\,,\,\dots\,,\lambda_n\,,\dots\}$

→ ***Fields :*** $\{\Psi\,;\,\Psi_1\,,\Psi_2\,,\,\dots\,,\Psi_n\,,\dots\}$ ***Anti-fields :*** $\{\Psi_0^*\,;\,\Psi_1^*\,,\Psi_2^*\,,\,\dots\,\Psi_n^*\,,\dots\}$

Ready-made procedure : $S_{BV} \sim \langle\, \psi, Q\psi\,\rangle$ where $\psi = \Psi + \sum \Psi_n + \sum \Psi_n^*$.

Motivation: Natural embedding

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We don't have to use small λ itself.

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  - gauge variation must be small : $\eta(\delta\Psi) = 0$

$\delta\Psi = Q(\eta\Lambda_1)$ with ***large*** gauge parameter $\lambda_1 \equiv \eta\Lambda_1 \neq 0$

We don't have to use small λ itself.

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$$\delta\Psi = Q(\eta\Lambda_1) \quad \text{with } \textbf{\textit{large}} \text{ gauge parameter } \lambda_1 \equiv \eta\Lambda_1 \neq 0$$

$$\delta\Lambda_1 = Q\Lambda_{2,0} + \eta\Lambda_{2,1}$$

$$\vdots$$

$$\delta\Lambda_{g,p} = Q\Lambda_{g+1,p} + \eta\Lambda_{g+1,p+1}$$

$$\vdots$$

Very different gauge reducibility

Motivation: Natural embedding

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$$\vdots$$

$\longrightarrow$

Gauge parameters

$$\Lambda_1 \ , \ \Lambda_{2,0} \ , \ \Lambda_{3,0} \ , \ \dots$$
$$\Lambda_{2,1} \ , \ \Lambda_{3,1} \ , \ \dots$$
$$\Lambda_{3,2} \ , \ \dots$$

Although string field is small, BV fields-antifields can be large !!

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$$\vdots$$

→

Gauge parameters

$$\Lambda_1 \,,\ \Lambda_{2,0} \,,\ \Lambda_{3,0} \,,\ \dots$$
$$\Lambda_{2,1} \,,\ \Lambda_{3,1} \,,\ \dots$$
$$\Lambda_{3,2} \,,\ \dots$$

Although string field is small, BV fields-antifields can be large !!

Likewise, one can consider a ***large*** *string field :* $\Psi \equiv \eta\Phi$

Motivation: "Large" theory

- Embedding into "large" theory

  - every SFTs can be large

  - enlarged gauge symmetry

$$\delta\Phi = Q\Lambda_{1,0} + \eta\Lambda_{1,1} + \cdots$$

SFT in small space

$$S[\Psi] = \frac{1}{2}\big\langle \Psi , Q\Psi \big\rangle_{\mathrm{Ker}[\eta]} + \cdots$$

Embedding $\Psi \equiv \eta\Phi$

Large theory

$$S[\Phi] = \frac{1}{2}\big\langle \Phi , Q\,\eta\,\Phi \big\rangle + \cdots$$

Even for very trivial embeddings, "WZW-like str." arises.

Is this "large" theory gauge-fixable ??

Today's topic : "large A_∞ / L_∞" theory

- Focus on "***large*** A_∞ ***theory***"

- Motivation 1 -

It is the simplest WZW-like theory.

<u>***SFT in small space***</u>

$$S[\Psi] = \frac{1}{2}\big\langle \Psi , Q\Psi \big\rangle_{\mathrm{Ker}[\eta]} + \cdots$$

Embedding

$$\Psi \equiv \eta \Phi$$

<u>***Large theory***</u>

$$S[\Phi] = \frac{1}{2}\big\langle \Phi , Q\, \eta\, \Phi \big\rangle + \cdots$$

Today's topic : "large A_∞/L_∞" theory

- Focus on "***large*** A_∞ ***theory***"

- Motivation 1 -

It is the simplest WZW-like theory.

- Motivation 2 -

If how to gauge-fix it is clarified, you can use some techniques of the large Hilbert space for your small theory.

SFT in small space

$$S[\Psi] = \frac{1}{2}\langle \Psi , Q\Psi \rangle_{\mathrm{Ker}[\eta]} + \cdots$$

Embedding $\Psi \equiv \eta\Phi$

Large theory

$$S[\Phi] = \frac{1}{2}\langle \Phi , Q\,\eta\,\Phi \rangle + \cdots$$

Today's topic : "large A_∞/L_∞" theory

- Focus on "***large*** A_∞ ***theory***"

<u>*SFT in small space*</u>

$$S[\Psi] = \frac{1}{2}\big\langle \Psi \, , Q\Psi \big\rangle_{\mathrm{Ker}[\eta]} + \cdots$$

Embedding

$$\Psi \equiv \eta\Phi$$

<u>*Large theory*</u>

$$S[\Phi] = \frac{1}{2}\big\langle \Phi \, , Q \, \eta \, \Phi \big\rangle + \cdots$$

- Motivation 1 -

It is the simplest WZW-like theory.

- Motivation 2 -

If how to gauge-fix it is clarified, you can use "large" merits and techniques of the large Hilbert space for your small theory.

- Motivation 3 -

It gives ***another representation*** of Berkovits' WZW-like theory.
(the same kinetic term and gauge reducibility)

Today's topic : "large A_∞/L_∞" theory

You can rewrite…

Large A_∞ gives the following solution

$$A_\eta[\Phi] = \pi_1 \widehat{\mathbf{G}} \frac{1}{1 - t\eta\Phi}$$

$$A_t[\Phi] = \pi_1 \widehat{\mathbf{G}} \frac{1}{1 - t\eta\Phi} \otimes \Phi \otimes \frac{1}{1 - t\eta\Phi}$$

WZW-like SFT

$$S = \int_0^1 dt \left\langle A_t[\varphi] \,, Q\, A_\eta[\varphi] \right\rangle$$

Q : BRST operator

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- Motivation 3 -

It gives ***another representation*** of Berkovits' WZW-like theory.

(the same kinetic term and gauge reducibility)

… how to gauge-fix ??

SFT is infinitely reducible and now gauge symmetry is enlarged…

"the Antifield formalism" enables us to treat such a gauge theory.

Find

"classical BV master action"

and

"gauge-fixing fermion"

Today's plan

1. Conventional BV approach

1.1) Minimal set, usual string field-antifield ⟶ ***breakdown***

1.2) + some remediations (+ trivial gauge transformations)

2. Gauge fixing fermions

3. Constrained BV approach (if I have time)

Berkovits' constraint BV ⟶ ***almost (but not precisely) correct***

+ improved constraints ⟶ ***precisely correct for large theory***

1. Conventional BV approach

~~1. Conventional BV approach~~

0. 4-slides review of Batalin-Vilkovisky's

Antifield formalism

4-slides review of BV (1/4)

Let us consider a Lagrangian $S[\phi^i] = \int d^D x \, \mathcal{L}(\phi^i, \partial_\mu \phi^i, \partial_{\mu_1 \mu_2} \phi^i, \dots, \partial_{\mu_1 \dots \mu_n} \phi^i)\,.$

Its e.o.m. is given by $\frac{\delta S}{\delta \phi^i} \equiv \frac{\partial S}{\partial \phi^i} - \partial_\mu \frac{\partial S}{\partial(\partial_\mu \phi^i)} + \cdots + (-)^n \partial_{\mu_1 \dots \mu_n} \frac{\partial S}{\partial(\partial_{\mu_1 \dots \mu_n} \phi^i)} = 0\,.$

When there is a gauge invariance

$$\delta_\epsilon \phi^i(x) = \mathcal{R}^i{}_a \epsilon^a \equiv \int dy \Big[R^i{}_a(x,y) \epsilon^a(y) + R^{i\,\mu}{}_a(x,y) \partial_\mu \epsilon^a + \cdots + R^{i\,\mu_1 \dots \mu_n}{}_a(x,y) \partial_{\mu_1 \dots \mu_n} \epsilon^a(y) \Big]$$

you find the Noether identities : $\delta_\epsilon S = \frac{\delta S}{\delta \phi^i} \delta_\epsilon \phi^i = \frac{\delta S}{\delta \phi^i} \mathcal{R}^i{}_a \epsilon^a = 0\,.$

When $\delta_\epsilon \phi^i(x) = \mathcal{R}^i{}_a \epsilon^a$ gives a generating set of the gauge transformations,

$$\frac{\delta S}{\delta \phi^i} \mathcal{R}^i{}_a = 0 \quad \Longleftrightarrow \quad \mathcal{R}^i{}_a \ : \ \text{null vectors exist}\,.$$

So, for gauge-fixing, we need ghosts

$$\mathcal{R}^i{}_a \, \epsilon^a \quad \Longrightarrow \quad \mathcal{R}^i{}_a \, c^a \ : \ \text{ghost fields } c^a \text{ appear}\,.$$

4-slides review of BV (2/4)

If null vectors of $\frac{\delta S}{\delta \phi^i} \mathcal{R}^i{}_a = 0$ are degenerate, its gauge symmetry is reducible.

$$\mathcal{R}^i{}_a \mathcal{U}^a{}_\alpha = 0 \qquad \Longleftrightarrow \qquad \mathcal{U}^a{}_\alpha \ : \ \text{further null vectors exist}\,.$$

This "gauge symmetry of gauge symmetry" requires "higher ghosts"

$$\delta_\lambda \epsilon^a = \mathcal{U}^a{}_\alpha \, \lambda^\alpha \qquad \Longrightarrow \qquad \mathcal{U}^a{}_\alpha \, C_{\text{2nd}}{}^a \ : \ \text{ghosts for ghosts } C_{\text{2nd}}{}^\alpha \text{ appear}\,.$$

Likewise, higher gauge symmetries need further higher ghosts.

The antifield formalism defines a BRST-like operation for these ghosts.

As BRST, the physical states are given by its cohomology.

4-slides review of BV (3/4)

In the above, we ignored a trivial but important symmetry.

$$\delta_\omega S = \frac{\delta S}{\delta \phi^i}\,\delta_\omega \phi^i = 0 \quad \text{with} \quad \delta_\omega \phi^i = \omega^{ij}\,\frac{\delta S}{\delta \phi^j}\,, \qquad \omega^{ij} = -\omega^{ji}\,.$$

Not only gauge theories, every theories have this gauge invariance.
It is called as "trivial gauge transformations" :

$$\big[\,\text{trivial}\,,\, {}^\forall\text{gauge transf.}\,\big] = \text{trivial}$$

However, it may not be factorised and the gauge algebra may be open :

$$\big[\,\text{non trivial}\,,\,\text{non trivial}\,\big] = \text{non trivial} + \text{trivial}$$

Namely, one may find the following gauge commutator

$$[\delta_a, \delta_b]\phi^i = \left[\frac{\delta \mathcal{R}^i{}_b}{\delta \phi^j}\,\mathcal{R}^j{}_a - \frac{\delta \mathcal{R}^i{}_a}{\delta \phi^j}\,\mathcal{R}^j{}_b\right]\lambda_b\,\lambda_a = \mathcal{R}^i{}_c\,\Lambda^c_{ab} + \frac{\delta S}{\delta \phi^j}\,\Omega^{ji}_{ab}\,.$$

On-shell vanishing terms make your BRST procedure terrible.

"Antifields" resolve it: $\delta_\omega \phi^i = \omega^{ij}\,\dfrac{\delta S}{\delta \phi^i} \quad \Longrightarrow \quad \text{Antifield } (\phi^i)^* \quad \text{s.t.} \quad \delta_{\text{BRST}}(\phi^i)^* = \omega^{ij}\,\dfrac{\delta S}{\delta \phi^i}$

4-slides review of BV (4/4)

a) Introduce appropriate (higher) ghosts, which we also call "fields" ϕ^i .

b) Introduce an "antifield" $(\phi^i)^*$ for each "field".

c) Define the antibracket $(\ ,\ )$ on the space of all fields and antifields

$$(F \, , \, G) \equiv \sum_i \left[\frac{\delta_r F}{\delta \phi^i} \frac{\delta_l G}{\delta (\phi^i)^*} - \frac{\delta_r F}{\delta (\phi^i)^*} \frac{\delta_r G}{\delta \phi^i} \right] .$$

d) Find a solution $S_{\mathsf{bv}} = S_{\mathsf{bv}}[\phi, \phi^*]$ of the master equation,

$$(S_{\mathsf{bv}} \, , \, S_{\mathsf{bv}}) = 0 \quad \text{with the initial condition} \quad S_{\mathsf{bv}}|_{\phi^*=0} = S$$

"the BV master action" S_{bv} is an intrinsic object of the gauge theory and gives the generator of BRST : $\delta_{\mathrm{BRST}} F = (S_{\mathsf{bv}} \, , \, F)$.

e) Fix your gauge by constructing appropriate gauge-fixing fermion F : $(\phi_i)^* \equiv \dfrac{\partial F[\phi]}{\partial \phi_i}$

1. Conventional BV approach

Free SFT in small space

- SFT in small space gives a good exercise of BV.

We first consider a master action for SFT in ***small*** space.

- there is a ready-made procedure.

After that, we consider a master action for ***large*** theory.

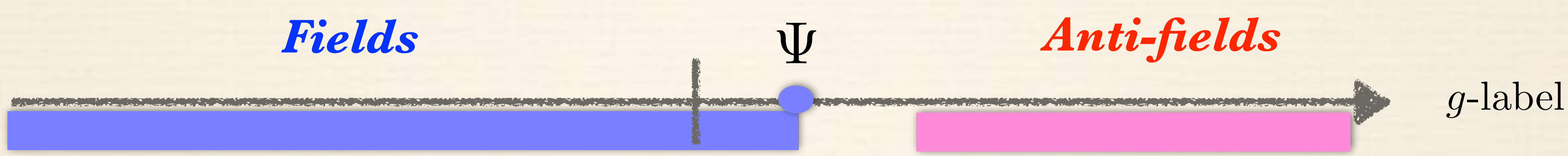
Free SFT in small space

Free action : $S[\Psi] = \frac{1}{2} \langle \, \Psi, \, Q \, \Psi \, \rangle_{\text{Ker}[\eta]}$ where $\eta \, \Psi = 0$.

It yields an infinite tower of gauge transformations.

$$\delta \Psi = Q \, \lambda_0 \qquad \delta \lambda_{-g} = Q \, \lambda_{-1-g} \qquad \Longrightarrow \qquad \delta(\delta \lambda_{-g}) = 0$$

We find the spectrum of "string fields-antifields" as



Then, the master action is given by just replacing Ψ with ψ :

$$S_{\mathsf{bv}} = \frac{1}{2} \langle \, \psi \, , \, Q \, \psi \, \rangle \quad \text{where} \quad \psi \equiv \Psi + \sum_{\text{ghost}} \Psi_{-g} + \sum_{\text{antifield}} (\Psi_{-g})^*$$

Free SFT in small space

Free action : $S[\Psi] = \frac{1}{2} \langle \Psi, Q \Psi \rangle_{\mathrm{Ker}[\eta]}$ where $\eta \Psi = 0$.

(linear) A_∞

$Q^2 = 0$

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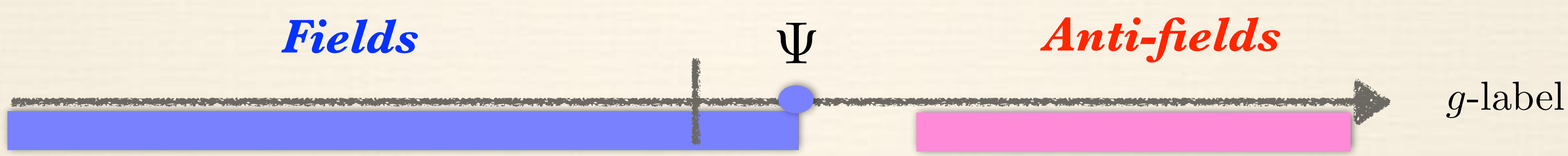

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Likewise, one can find interacting BV master action.

Interacting SFT in small space

Action : $S[\Psi] = \int_0^1 dt \left\langle \Psi , \mathbf{M} \frac{1}{1-t\Psi} \right\rangle_{\ker[\eta]} = \frac{1}{2} \langle \Psi, Q\Psi \rangle_{\ker[\eta]} + \frac{1}{3} \langle \Psi, M_2(\Psi^2) \rangle_{\ker[\eta]} + \cdots$.

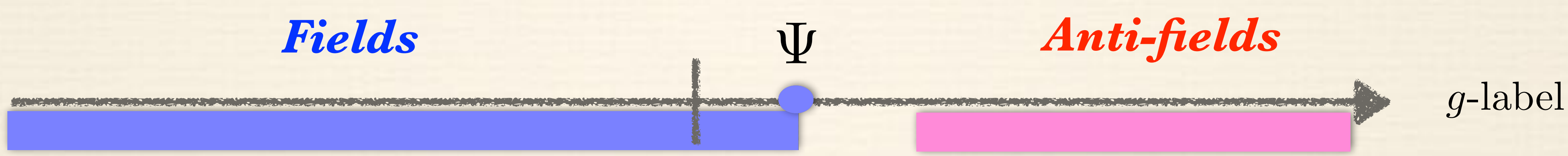
nonlinear A_∞

$\mathbf{M}^2 = 0$

An infinite tower of gauge transformations:

$$\delta\Psi = [\mathbf{M}, \lambda] \frac{1}{1-\Psi} \quad \delta\lambda_g = [\mathbf{M}, \lambda_{g+1}] \frac{1}{1-\Psi} \quad \Longrightarrow \quad \delta\left(\delta\lambda_g\right) = 0$$

We find the ***same*** spectrum of "string fields-antifields" as



Likewise, the master action is given by just replacing Ψ with ψ :

$$S_{BV}[\psi] = \int_0^1 dt \left\langle \psi , \mathbf{M} \frac{1}{1-t\psi} \right\rangle_{\ker[\eta]} \quad \text{where} \quad \psi = \Psi + \sum_{ghosts} \Psi_n + \sum_{antifields} \Psi_n^*$$

How about "large" theory…??

We want to find

BV master action in the large Hilbert space

recall natural embeddings

Recall natural embedding

EKS open SFT in the **small** Hilbert space :

$$S[\Psi] = \frac{1}{2}\langle \Psi, Q\,\Psi \rangle_{\mathrm{Ker}[\eta]} + \frac{1}{3}\langle \Psi, \mathbf{M}_2(\Psi)^2\rangle_{\mathrm{Ker}[\eta]} + \frac{1}{4}\langle \Psi, \mathbf{M}_3(\Psi)^3\rangle_{\mathrm{Ker}[\eta]} + \cdots .$$

$\eta\,\Psi = 0$

$\Psi = \eta\,\Phi$ where $\Phi \in$ large Hilbert space

We consider "**large SFT**" obtained by this trivial embedding :

$$S[\Phi] = \frac{1}{2}\langle \Phi, Q\,\eta\,\Phi \rangle + \frac{1}{3}\langle \Phi, \mathbf{M}_2(\eta\,\Phi)^2\rangle + \frac{1}{4}\langle \Phi, \mathbf{M}_3(\eta\,\Phi)^3\rangle + \cdots .$$

This embedded theory has "large gauge symmetries".

Although this replacement looks very trivial, gauge-fixing is highly complicated.

Kinetic term is the same as Berkovits' one

Kroyter-Okawa-Schnabl-Torii-Zwiebach 2012
JHEP 03 (2012) 030

→ ***free BV master action is know.***

Free SFT in large space

Embedded free action : $S[\Phi] = \frac{1}{2}\langle\, \Phi,\, Q\,\eta\,\Phi\,\rangle$

Kroyter-Okawa-Schnabl-Torii-Zwiebach 2012
JHEP 03 (2012) 030

Large gauge invariances : $\delta\Phi = \eta\,\Lambda_{-1,1} + Q\,\Lambda_{-1,0}$

(linear) A_∞ ***pair***

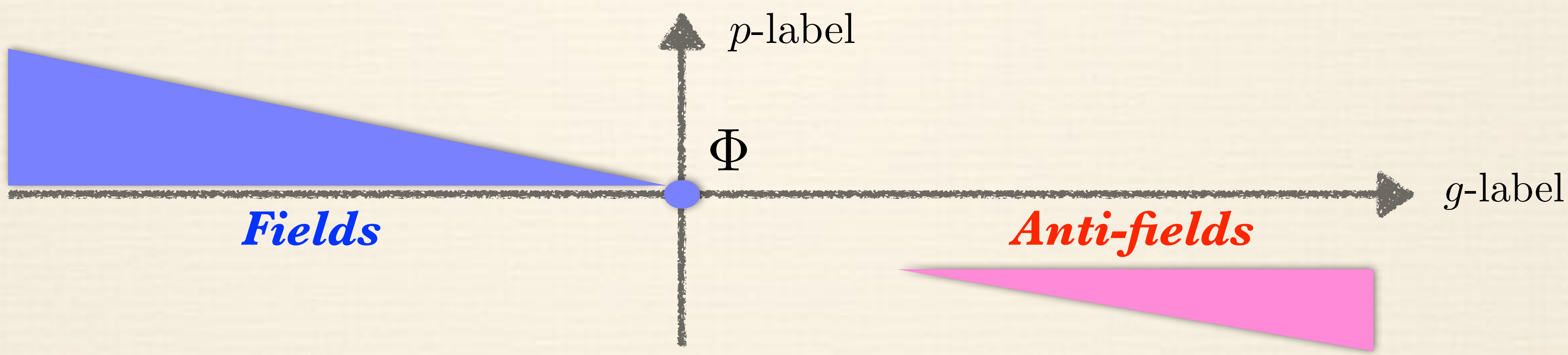
$Q^2 = 0 \qquad \eta^2 = 0$

$[Q, \eta] = 0$

An infinite tower of large gauge transformations.

$$\delta(\delta\Lambda_{-g,p}) = 0 \quad \text{with} \quad \delta\Lambda_{-g,p} = \eta\,\Lambda_{-1-g,p+1} + Q\,\Lambda_{-1-g,p}$$

We find the spectrum of "string fields-antifields" as



Then, the master action is **NOT** given by just replacing Φ :

$$S_{\mathsf{bv}} = \frac{1}{2}\langle \Phi,\, Q\,\eta\,\Phi\rangle + \sum_{g\geq 0}\sum_{p=0}^{g}\sum \big\langle (\Phi_{-g,p})^*,\, Q\,\Phi_{-1-g,p} + \eta\,\Phi_{-1-g,p+1}\big\rangle$$

Interacting SFT in large space

We try to construct a master action for the large SFT :

$$S[\Phi] = \frac{1}{2}\langle \Phi,\, Q\,\eta\,\Phi \rangle + \frac{1}{3}\langle \Phi,\, \mathbf{M}_2(\eta\,\Phi)^2\rangle + \frac{1}{4}\langle \Phi,\, \mathbf{M}_3(\eta\,\Phi)^3\rangle + \cdots .$$

Large gauge symmetry : $\delta\Phi = \boldsymbol{\eta}\,\Lambda' + \boldsymbol{Q}\,\Lambda + \mathbf{M}_2(\eta\,\Phi,\Lambda) + \mathbf{M}_2(\Lambda,\eta\,\Phi) + \cdots$

Mutually commutative A_∞ ***pair :*** $\mathbf{M}^2 = 0 \quad \eta^2 = 0 \quad [\mathbf{M},\eta] = 0$

→ ***The same BV fields-antifields as the free theory***

As usual, we try to construct a BV master action under

(1) usual string fields-antifields

(2) usual gauge generators M and η

(3) no ξ , no other products, no other (non-minimal) fields

Conventional BV approach breaks down

One can perturbatively construct its BV master action :

$$S_{\mathsf{bv}} = S + S^{(1)} + S^{(2)} + \cdots$$

$$S^{(1)}[\Phi, \Phi^*] = \Big\langle \Phi^*_{2,-1},\, \eta\, \Phi_{-1,1} + \pi_1 \mathbf{M} \boldsymbol{\Phi}_{-1,0} \frac{1}{1-\eta\, \Phi_{0,0}} \Big\rangle$$

$$S^{(2)}[\Phi, \Phi^*] = \Big\langle \Phi^*_{3,-1},\, \eta\, \Phi_{-2,1} + \pi_1\, \mathbf{M} \Big[\frac{\boldsymbol{\Phi}_{-1,0}}{2} (\boldsymbol{\eta} \boldsymbol{\Phi}_{-1,0}) + \boldsymbol{\Phi}_{-2,0} \Big] \frac{1}{1-\eta \Phi_{0,0}} \Big\rangle + \Big\langle \Phi^*_{3,-2},\, \eta\, \Phi_{-2,2} + \pi_1\, \mathbf{M} \Big[\frac{\boldsymbol{\Phi}_{-1,0}}{2} \mathbf{M} \boldsymbol{\Phi}_{-1,0} \Big] \frac{1}{1-\eta \Phi_{0,0}} \Big\rangle$$

$$- \frac{1}{2} \Big\langle \Phi^*_{2,-1},\, \pi_1\, \mathbf{M} \Big[\boldsymbol{\Phi}^*_{2,-1} \frac{\boldsymbol{\Phi}_{-1,0}}{2} (\boldsymbol{\eta} \boldsymbol{\Phi}_{-1,0}) + \boldsymbol{\Phi}^*_{2,-1} \boldsymbol{\Phi}_{-2,0} \Big] \frac{1}{1-\eta\, \Phi_{0,0}} \Big\rangle$$

However, there is no solution for $S^{(3)}$ ***and higher parts !!***

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$$\begin{aligned} S^{(2)}[\Phi, \Phi^*] = & \Big\langle \Phi^*_{3,-1},\, \eta\, \Phi_{-2,1} + \pi_1\, \mathbf{M} \Big[\frac{\boldsymbol{\Phi}_{-1,0}}{2} (\boldsymbol{\eta} \boldsymbol{\Phi}_{-1,0}) + \boldsymbol{\Phi}_{-2,0} \Big] \frac{1}{1 - \eta \Phi_{0,0}} \Big\rangle \\ & + \Big\langle \Phi^*_{3,-2},\, \eta\, \Phi_{-2,2} + \pi_1\, \mathbf{M} \Big[\frac{\boldsymbol{\Phi}_{-1,0}}{2} \mathbf{M} \boldsymbol{\Phi}_{-1,0} \Big] \frac{1}{1 - \eta \Phi_{0,0}} \Big\rangle \\ & - \frac{1}{2} \Big\langle \Phi^*_{2,-1},\, \pi_1\, \mathbf{M} \Big[\boldsymbol{\Phi}^*_{2,-1} \frac{\boldsymbol{\Phi}_{-1,0}}{2} (\boldsymbol{\eta} \boldsymbol{\Phi}_{-1,0}) + \boldsymbol{\Phi}^*_{2,-1} \boldsymbol{\Phi}_{-2,0} \Big] \frac{1}{1 - \eta\, \Phi_{0,0}} \Big\rangle \end{aligned}$$

However, there is no solution for $S^{(3)}$ ***and higher parts !!***

Actually, one cannot construct the BV master action without ξ ***!!***

But, why?? — If gauge algebra is generated by M and η ***only,***

we could construct it without using ξ ***.***

Conventional BV revisited

- Revisit the gauge invariance of the free theory : $S[\Phi] = \frac{1}{2}\langle\, \Phi,\, Q\,\eta\,\Phi\,\rangle$

  - Inv. under the gauge transf. $\delta\Phi = \eta\Lambda_{1,1} + Q\Lambda_{1,0}$

We should have used…

$$\begin{aligned}\delta\Phi &= \eta\Lambda_{1,1} + \eta\xi\, Q\Lambda_{1,0} + \xi\eta\, Q\Lambda_{1,0} \\ &= \eta\,(\Lambda_{1,1} + \xi\, Q\Lambda_{1,0}) + \xi\, Q\,(-\eta\Lambda_{1,0})\end{aligned}$$

If we define "new gauge parameters" as $\delta\Phi = \eta\,\Lambda^{\text{new}}_{1,1} + \xi\,Q\,(\eta\Lambda^{\text{new}}_{1,0})$ **,**

we find the following (factorised) gauge transformations :

$$\delta\Lambda^{\text{new}}_{g,0} = \eta\,\Lambda^{\text{new}}_{g+1,1} + \xi\,Q\,(\eta\Lambda^{\text{new}}_{g+1,0})$$

$$\delta\Lambda^{\text{new}}_{g,p} = \eta\,\Lambda^{\text{new}}_{g+1,p+1} \qquad (p>0)$$

Actually, vanishing "ξ-parts of p>0" generate "trivial transformations" !!

Trivial transformations appear !!

To see it explicitly, let us consider the interacting case :

$$\delta\Phi = \pi_1 [\![\mathbf{M},\, \boldsymbol{\Lambda}_{-1,0}]\!] \frac{1}{1-\eta\,\Phi} + \boldsymbol{\eta}\, \Lambda_{-1,1} \quad \Longrightarrow \quad \delta_g \Lambda_{-g,p} = \pi_1 [\![\mathbf{M},\, \boldsymbol{\Lambda}_{-g-1,p}]\!] \frac{1}{1-\eta\,\Phi} + \boldsymbol{\eta}\, \Lambda_{-g-1,p+1}$$

Re-definition : $\Lambda^{\rm new}_{-1,0} \equiv -\Lambda^{\rm old}_{-1,0}\,, \qquad \Lambda^{\rm new}_{-1,1} \equiv \pi_1\, \boldsymbol{\xi}\, [\![\mathbf{M},\, \boldsymbol{\Lambda}^{\rm old}_{-1,0}]\!] \frac{1}{1-\eta\,\Phi} + \Lambda^{\rm old}_{-1,1}$

Then, ξ-part of the gauge variation generates a trivial transformation !!

$$\begin{aligned} \delta\Lambda^{\rm new}_{-1,1} &= \pi_1\, \boldsymbol{\xi}\, [\![\mathbf{M},\, \delta\boldsymbol{\Lambda}^{\rm old}_{-1,0}]\!] \frac{1}{1-\eta\,\Phi} + \delta\Lambda^{\rm old}_{-1,1} \\ &= \pi_1\, \boldsymbol{\xi} \left[\!\!\left[\mathbf{M},\, \pi_1 [\![\mathbf{M}, \boldsymbol{\Lambda}^{\rm old}_{-2,1}]\!] \frac{1}{1-\eta\Phi} \right]\!\!\right] \frac{1}{1-\eta\,\Phi} + \boldsymbol{\eta} \left[\pi_1\, \boldsymbol{\xi}\, [\![\mathbf{M}, \boldsymbol{\Lambda}^{\rm old}_{-2,1}]\!] \frac{1}{1-\eta\Phi} + \Lambda^{\rm old}_{-2,2} \right] \\ &= \boldsymbol{\xi}\, \underline{T(\Lambda^{\rm new}_{-2,1})} + \boldsymbol{\eta}\, \Lambda^{\rm new}_{-2,2}\,, \end{aligned}$$

As a result, we obtain

$$\delta\Lambda^{\rm new}_{-g,0} = \pi_1\, \boldsymbol{\xi}\, [\![\mathbf{M}, \boldsymbol{\eta}\, \boldsymbol{\Lambda}^{\rm new}_{-g-1,0}]\!] \frac{1}{1-\eta\,\Phi} + \boldsymbol{\eta}\, \Lambda^{\rm new}_{-g-1,1}\,,$$

$$\delta\Lambda^{\rm new}_{-g,p} = \boldsymbol{\eta}\, \Lambda^{\rm new}_{-g-1,p+1}\,. \qquad \text{(p>0)}$$

BV master action in large space

- We consider the sum of fields carrying fixed picture number $\boldsymbol{p}$: $\varphi_p \equiv \sum_{g=p}^{\infty} \Phi_{-g,p}$
- Decompose it into η- and ξ-exacts : $\varphi_p = \varphi_p^{\xi} + \varphi_p^{\eta}$
- Introduce their antifields separately : $(\varphi_p^{\xi})^* = \sum_{g=p}^{\infty} (\Phi^{\,\xi}_{-g,p})^* \,, \qquad (\varphi_p^{\eta})^* = \sum_{g=p}^{\infty} (\Phi^{\,\eta}_{-g,p})^* \,.$
- BV master action is a functional of these : $S_{\mathsf{bv}} = S_{\mathsf{bv}}[\,\varphi, (\varphi^{\xi})^* (\varphi^{\eta})^*]$

$$S_{\mathsf{bv}} = \int_0^1 dt \left\langle \varphi_0 + \xi\, (\varphi_0^{\xi})^*, \, \mathbf{M} \frac{1}{1 - t\, \eta\, (\varphi_0 + \xi\, (\varphi_0^{\xi})^*)} \right\rangle + \sum_{p>0} \Big\langle (\varphi_{p-1}^{\eta})^*, \, \boldsymbol{\eta}\, \varphi_p \Big\rangle \,.$$

master eq. ⟹

$$\frac{1}{2} \big(S_{\mathsf{bv}}, \, S_{\mathsf{bv}} \big)_{\min} = \frac{\overleftarrow{\partial} S_{\mathsf{bv}}}{\partial \varphi^{\xi}} \cdot \frac{\overrightarrow{\partial} S_{\mathsf{bv}}}{\partial (\varphi^{\xi})^*} + \frac{\overleftarrow{\partial} S_{\mathsf{bv}}}{\partial \varphi^{\eta}} \cdot \frac{\overrightarrow{\partial} S_{\mathsf{bv}}}{\partial (\varphi^{\eta})^*} = 0 \,.$$

It generates appropriate BV-BRST transformations :

$$\delta \varphi_0 = \big(\varphi_0^{\xi} + \varphi_0^{\eta} \,, S_{\mathsf{bv}} \big)_{\min} = \pi_1\, \boldsymbol{\xi}\, \mathbf{M} \frac{1}{1 - \eta\, (\varphi_0 + \xi\, (\varphi_0)^*)} + \boldsymbol{\eta}\, \varphi_1 \,,$$

$$\delta \varphi_p = \big(\varphi_p^{\xi} + \varphi_p^{\eta} \,, S_{\mathsf{bv}} \big)_{\min} = \boldsymbol{\eta}\, \varphi_{p+1} \,,$$

$$\delta (\varphi_0)^* = \big((\varphi_0^{\xi})^* + (\varphi_0^{\eta})^* \,, S_{\mathsf{bv}} \big)_{\min} = \pi_1\, \mathbf{M} \frac{1}{1 - \eta\, (\varphi_0 + \xi\, (\varphi_0)^*)} \,,$$

$$\delta (\varphi_p)^* = \big((\varphi_p^{\xi})^* + (\varphi_p^{\eta})^* \,, S_{\mathsf{bv}} \big)_{\min} = \boldsymbol{\eta}\, (\varphi_{p-1}^{\eta})^* \,.$$

Summary

- Naive conventional approach works up to antifield number 2.
- Gauge algebra is generated by M and η, but ξ-parts generate "trivial transformations".
- Therefore, "ξ" must appear in the BV master action.

Comments

- Since string-field redefinitions connect different SFTs, other BV master actions are obtained via BV canonical transformation of this master action.
- Even for Berkovits' theory, "ξ" generates "trivial transformations".

(Thus, we need "ξ" even for the BV master action for Berkovits' theory)

2. Gauge-fixing fermions

Partially gauge-fixing fermion

- Our "large" BV master action reduces to known "small" BV master action.

  - Consider the following trivial pairs and (partially) gauge-fixing fermion :

$$S_{\text{trivial}} = \sum_{g,p} \Big[\big\langle \mathcal{N}^{\eta}_{1-g,p-1} , \, \xi_0 \, (\Psi_{2+g,-1-p})^* \big\rangle + \big\langle \mathcal{N}^{\xi}_{-1-g,1+p} , \, (C_{-1-g,1+p})^* \big\rangle \Big] \, .$$

$$F = \sum_{g,p} \Big[\big\langle \Phi_{-g,p} , \, \Psi_{2+g,-1-p} \big\rangle + \big\langle C_{-1-g,1+p} , \, \eta \, \Psi_{2+g,-1-p} \big\rangle \Big]$$

After some computations, we find

$$\big(S_{\mathsf{bv}} + S_{\text{trivial}} \big) \big|_F = S'_{\mathsf{bv}}[\psi] \equiv \int_0^1 dt \, \Big\langle \, \psi \, , \, \xi \, \mathbf{M} \frac{1}{1 - t \, \psi} \Big\rangle$$

where ψ is given by $\psi \equiv \sum_{g=0}^{\infty} \sum_{p=0}^{g} \delta_{p,0} \Big[\Psi_{1-g,p-1} + \Psi_{2+g,-1-p} \Big] \, .$

Gauge-fixing fermions

- Gauge conditions studied by S.Torii

$$\mathbb{B}_{-(n+2)} \begin{bmatrix} \Phi_{-n,0} \\ \vdots \\ \Phi_{-n,n} \end{bmatrix} = 0 \qquad \mathbb{B}_{-(n+2)} = \begin{bmatrix} b & 0 & \cdots & & 0 \\ y_n \zeta_0^n & x_n b_0 & & & \vdots \\ & y_n \zeta_0^n & x_n b_0 & & \\ & & \ddots & & 0 \\ & & & y_n \zeta_0^n & x_n b_0 \\ 0 & \cdots & & 0 & \zeta_0^n \end{bmatrix}$$

is given by the following trivial pairs and gauge-fixing fermion :

$$S_{\text{trivial}} = \sum_{n=0}^{\infty} \Big\langle \mathbb{B}_{-(n+2)} \mathcal{N}_{n+3} \, , \, (\Psi_{n+2})^* \Big\rangle + \big\langle (C_2)^* \, , \, \mathcal{N}_2 \big\rangle + \sum_{n=0}^{\infty} \big\langle (C_{-n})^* \, , \, \mathcal{N}_{-n} \big\rangle$$

$$F = \sum_{n=0}^{\infty} \big\langle \Phi_{-n} \, , \, \Psi_{n+2} \big\rangle + \big\langle C_2, \, b_0 \, \xi_0 \, \Psi_2 \big\rangle + \sum_{n=0}^{\infty} \big\langle C_{-n}, \, \mathbb{B}_{-(n+2)} \, \Psi_{n+3} \big\rangle$$

You can apply technique of large-space to your SFT in small-space !!

Conclusion

- BV master action in large space

We can gauge-fix SFT having “large gauge symmetries”.

You can apply large-space technique to SFT defined in small space via embeddings.

(Constrained BV gives elegant constructions of BV master actions.)

- Gauge-fixing fermion

Large theory indeed reduces to the original small theory by partial gauge fixing.

Gauge-fixing fermion imposing [KOSTZ]’s gauge-conditions was constructed.

Thank you for your attentions

3. Constrained BV approach

Re-assembling "string field-antifields"

Note that the BV formalism tells nothing about how to assemble "string antifields" unlike ghost string fields which are naturally determined from gauge parameter string fields: It just assigns an appropriate space-time antifield to each space-time ghost field which is a coefficient of given ghost string field.

$$\Phi_{-g,p} = \sum_r \phi^r_{g,p} \, |\mathcal{Z}^r_{-g,p}\rangle \quad \Longrightarrow \quad \{ (\phi^r_{g,p})^* | 0 \leq g, \, 0 \leq p \leq g ; r \in \mathbb{N} \}$$

As a simple resolution, we take the constrained BV approach and determine the string antifield assembly utilizing the constrained BV master equation itself. We write $\mathcal{A}_{\mathrm{min}} = \big\{ \Phi_{-g,p}, \, (\Phi_{-g,p})^* \big\}$ for the minimal set.

a) Introduce "extra" fields-antifields $\mathcal{A}_{\mathrm{ex}} = \big\{ \Phi^{\mathsf{ex}}_{g,-p}, \, (\Phi^{\mathsf{ex}}_{g,-p})^* \big\}$

b) Impose appropriate constraints $\Gamma[\phi] = 0$ where $\phi \in \mathcal{A}_{\mathrm{min}} \oplus \mathcal{A}_{\mathrm{ex}}$

Then, we consider $\big(S_{\mathsf{bv}} \, , \, S_{\mathsf{bv}} \big) = 0$ on $\dfrac{\mathcal{A}_{\mathrm{min}} \oplus \mathcal{A}_{\mathrm{ex}}}{\Gamma[\phi]}$, not $\big(S_{\mathsf{bv}} \, , \, S_{\mathsf{bv}} \big) = 0$ on $\mathcal{A}_{\mathrm{min}}$

Constrained BV master action

We found that Berkovits' proposal works well: We start with the action consists of string fields only

$$S_{\mathsf{bv}}[\varphi] = \frac{1}{2}\langle\, \varphi,\, Q\,\eta\,\varphi\,\rangle + \frac{1}{3}\langle\, \varphi,\, \mathbf{M}_2(\eta\,\varphi)^2\rangle + \frac{1}{4}\langle\, \varphi,\, \mathbf{M}_3(\eta\,\varphi)^3\rangle + \cdots .$$

$$\text{Sum of all siting fields :} \quad \varphi \equiv \Phi + \sum_{g>0}\sum_{p=0}^{g} \Phi_{-g,p} + \sum_{g\geq 0}\sum_{p=0}^{g} \Phi^{\mathsf{ex}}_{1+g,-p}\,.$$

and impose the constraint equations $\Gamma_{g,p} \equiv (\Phi_{-g,p})^* - \eta\, \Phi^{\mathsf{ex}}_{1+g,-p}$

They split into the first and second classes: Our action is invariant under the first class Γ, and the second class Γ defines the Dirac anti-bracket

$$(\,F\,,\,G\,)_{\Gamma} \equiv (\,F\,,\,G\,) - \sum_{a,b}(\,F\,,\,\Gamma_a\,)\,\big[(\Gamma,\Gamma)^{-1}\big]_{ab}\,(\,\Gamma_b\,,\,G\,)\,.$$

Then, anti-string fields are introduced in the action via constraints, and the master equation holds on the constrained subspace.

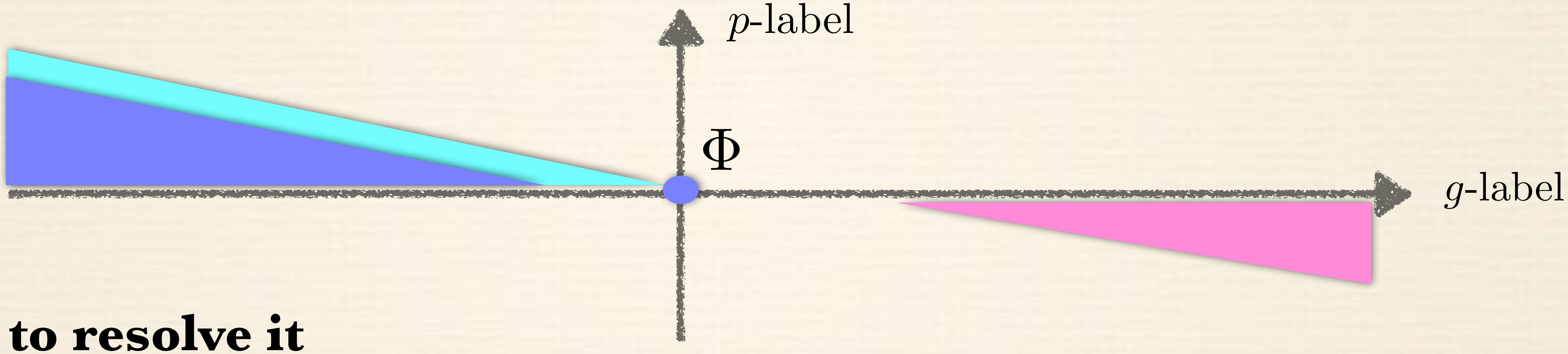
$$(\,S_{\mathsf{bv}}\,,\,S_{\mathsf{bv}}\,)_{\Gamma} = \sum_{k,l}\Big\langle\, \mathbf{M}_k(\eta\,\varphi)^k,\, \boldsymbol{\xi}\,\mathbf{M}_l(\eta\,\varphi)^l\Big\rangle = 0\,.$$

Berkovits' Γ is for partially gauge-fixed theory

BV Master action on Γ :

$$S_{\mathsf{bv}}[\varphi]|_{\Gamma} = \frac{1}{2}\langle \Phi,\, Q\,\eta\,\Phi\rangle + \sum_{g\geq 0}\sum_{p=0}^{g}\langle (\Phi_{-g,p})^{*},\, Q\,\Phi_{-1-g,p}\rangle + \sum_{g\geq 0}\sum_{p=0}^{g}\langle (\Phi_{-g,p})^{*},\, \sum_{n>1}\mathbf{M}_{n}\big(\eta\,(\varphi+\varphi^{*})\,\big)^{n}\rangle\,.$$

Note that $\Phi_{-g,p=g}$ for $g > 0$ behave as auxiliary string fields.



How to resolve it

1. These string fields can have their kinetic terms when we take a bit more complicated constraints. (See JHEP 05 (2018) 020.)
2. One can also remedy it by using unusual assembly of string antifields.
3. You can start with different (unconstrained) master action resolving this problem.

Thank you so much !!