Black holes and vacuum energy

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Introduction

Black hole evaporation by the Hawking radiation

- Black hole is formed by gravitational collapse of matters.
- Hawking radiation appears due to the quantum effects
- Black hole loses energy and finally evaporates.

Information loss problem

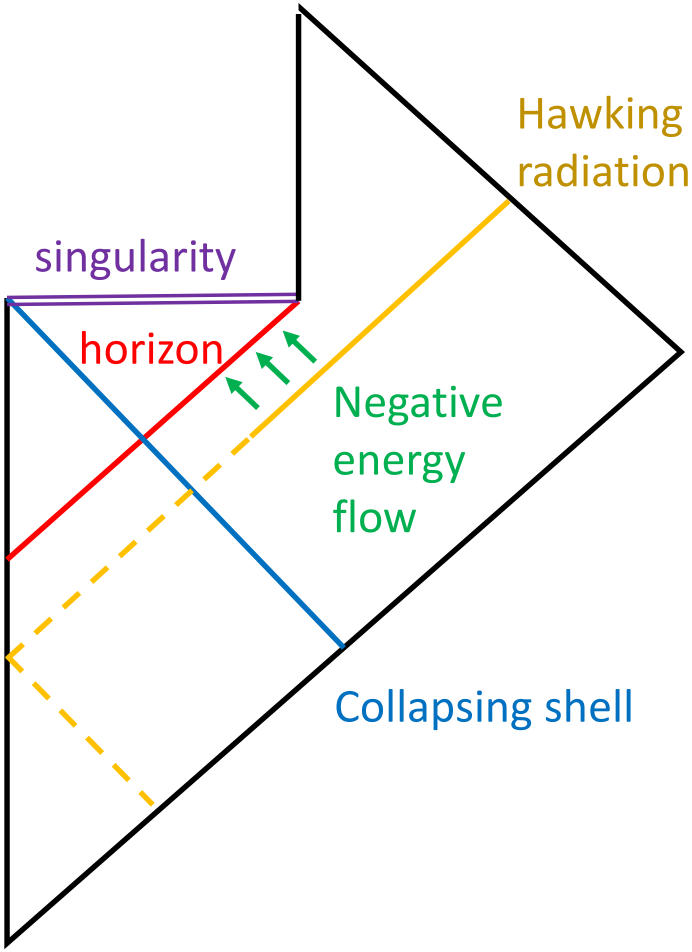
- Information cannot get out from the horizon if once it get inside the horizon
- Information will not be lost for quantum theory

In order to consider this problem, we take into account the back-reaction from the quantum effects.

We consider the Einstein equation in self-consistent manner.

$$R_{\mu\nu} - g_{\mu\nu}R = \alpha\langle T_{\mu\nu}\rangle$$

Hawking radiation from Bulk?



It is sometimes discussed that
Hawking radiation appears in bulk

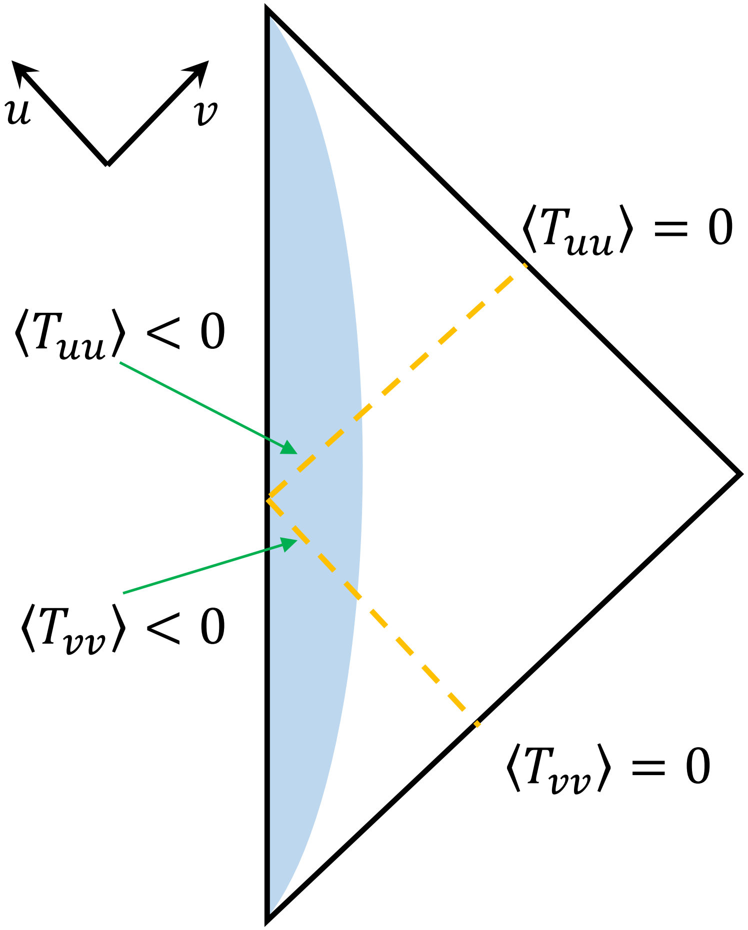
In this picture, Hawking radiation does not take energy from the collapsing matters.

There is also ingoing negative energy flow to satisfy the conservation law.

Because of the ingoing negative energy, black holes lose their energy and eventually evaporates in this picture.

However, the negative energy flow exists even in static backgrounds.

Boulware vacuum



Vacuum state with $\langle T_{\mu\nu} \rangle = 0$ in $r \to \infty$

Vacuum state for static star without horizon

Negative energy at finite r

$\langle T_{uu} \rangle < 0 \qquad \langle T_{vv} \rangle < 0$

"Divergence" of $\langle T_{\mu\nu} \rangle$ at horizon

⇩

We show that no divergence if back reaction from vacuum energy is taken into account

We consider semi-classical Einstein equation with quantum effects in $\langle T_{\mu\nu} \rangle$.

$$G_{\mu\nu}^{(4D)} = \alpha \langle T_{\mu\nu}^{(4D)} \rangle$$

2D model for 4D black hole

Separate 4D metric to angular part and others

$$ds^2 = \sum_{\mu=0,1,2,3} g_{\mu\nu} dx^\mu dx^\nu = \sum_{\mu=0,1} g_{\mu\nu}^{(2D)} dx^\mu dx^\nu + r^2 d\Omega^2$$

We integrate out angular directions

Energy-momentum tensor in 4D and 2D are

$$T_{\mu\nu}^{(4D)} = -\frac{2}{\sqrt{-g_{4D}}} \frac{\delta S}{\delta g^{\mu\nu}} \qquad T_{\mu\nu}^{(2D)} = -\frac{2}{\sqrt{-g_{2D}}} \frac{\delta S}{\delta g_{(2D)}^{\mu\nu}}$$

For $\mu, \nu = 0,1 \ (= t, r)$ $\qquad \langle T_{\mu\nu}^{(4D)} \rangle = \frac{1}{r^2} \langle T_{\mu\nu}^{(2D)} \rangle$

Semi-classical Einstein equation

$$G_{\mu\nu}^{(4D)} = \frac{\alpha}{r^2} \langle T_{\mu\nu}^{(2D)} \rangle$$

2D model for 4D black hole

We focus on s-waves and approximate them by 2D scalar fields.

Static spherically symmetric metric in null coordinates ($(t, r) \to (u, v)$)

$$ds^2 = -C(r)dudv + r^2 d\Omega^2$$

Energy-momentum tensor for 2D scalar fields

$$\langle T^{(4D)}_{\mu\nu} \rangle = \frac{1}{r^2} \langle T^{(2D)}_{\mu\nu} \rangle \qquad \langle T^{(4D)}_{\theta\theta} \rangle = 0$$

2D Weyl anomaly

$$\langle T^{(2D)\,\mu}{}_{\mu} \rangle = \frac{1}{24\pi} R^{(2D)} \Rightarrow \langle T^{(2D)}_{uv} \rangle = -\frac{1}{12\pi C^2} (C\partial_u \partial_v C - \partial_u C \partial_v C)$$

Integrate 2D conservation law

Integration constants

$$\langle T^{(2D)}_{uu} \rangle = -\frac{1}{12\pi} C^{1/2} \partial_u^2 C^{-1/2} + T(u)$$

$$\langle T^{(2D)}_{vv} \rangle = -\frac{1}{12\pi} C^{1/2} \partial_v^2 C^{-1/2} + \bar{T}(v)$$

2D model for 4D black hole

come from Weyl anomaly and does not depend on physical state.

⇨ Negative energy is not physical excitation but simply the vacuum has negative energy, like Casimir effects.

$$\langle T_{uv}^{(2D)} \rangle = -\frac{1}{12\pi C^2}(C\partial_u\partial_v C - \partial_u C \partial_v C)$$

$$\langle T_{uu}^{(2D)} \rangle = -\frac{1}{12\pi} C^{1/2}\partial_u^2 C^{-1/2} + T(u)$$

$$\langle T_{vv}^{(2D)} \rangle = -\frac{1}{12\pi} C^{1/2}\partial_v^2 C^{-1/2} + \bar{T}(v)$$

Integration constants which depend on physical state.

⇩

We consider static black holes without incoming or outgoing energy flow at infinity

$$T(u) = \bar{T}(v) = 0$$

Vacuum energy without back reaction

We consider the fixed background of Schwarzschild BH

$$ds^2 = -\left(1 - \frac{a_0}{r}\right) dt^2 + \frac{1}{1 - \frac{a_0}{r}} dr^2 + r^2 d\Omega^2$$

Quantum effects in energy-momentum tensor

$$\langle T_{uv}^{(2D)} \rangle = \frac{N}{48\pi}\left(\frac{a_0^2}{r^4} - \frac{a_0}{r^3}\right)$$

$$\langle T_{uu}^{(2D)} \rangle = \frac{N}{48\pi}\left(\frac{3a_0^2}{4r^4} - \frac{a_0}{r^3}\right) + \text{Const.}$$

$$\langle T_{vv}^{(2D)} \rangle = \frac{N}{48\pi}\left(\frac{3a_0^2}{4r^4} - \frac{a_0}{r^3}\right) + \text{Const.}$$

$$dv = dt + \frac{dr}{1 - \frac{a_0}{r}}$$

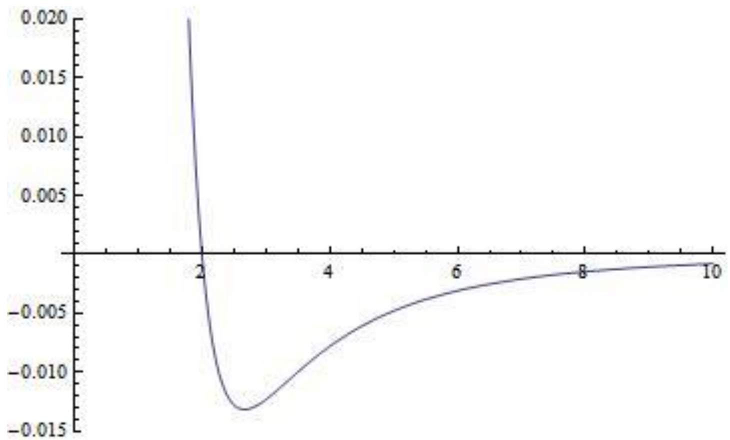
$$du = dt - \frac{dr}{1 - \frac{a_0}{r}}$$

Vacuum energy without back reaction

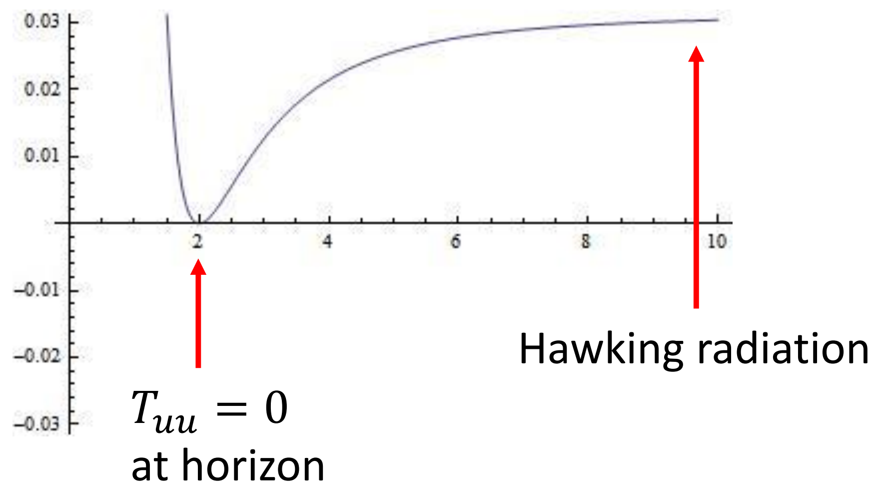
No incoming or outgoing energy at the horizon

Quantum effects give energy flow in $r \to \infty$ (Hawking radiation)

$\langle T_{uv}^{(2D)} \rangle$



$\langle T_{uu}^{(2D)} \rangle$

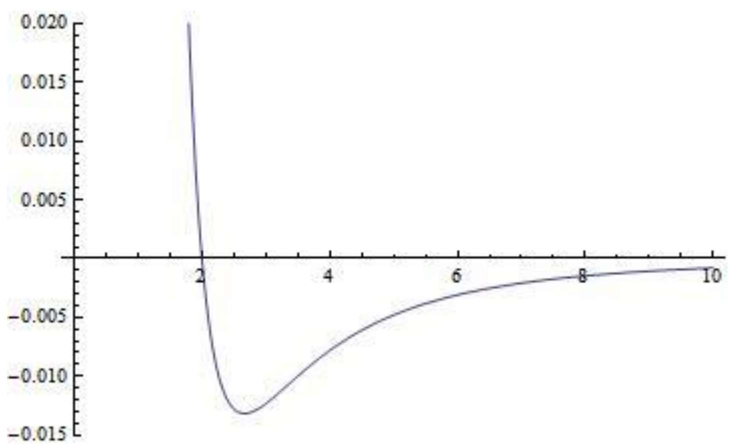


Vacuum energy without back reaction

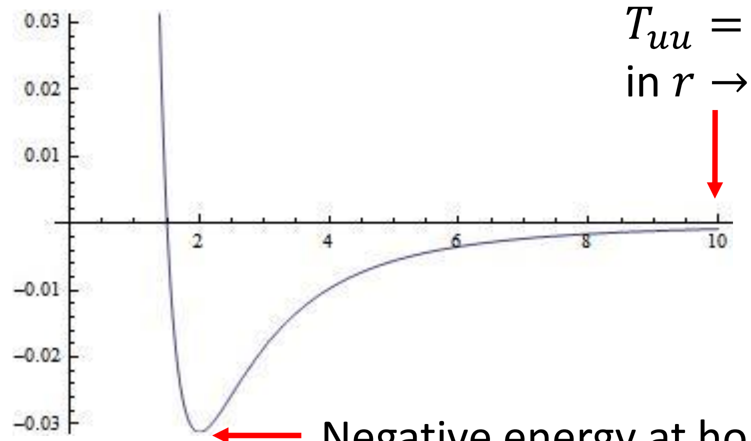
No incoming or outgoing energy in $r \to \infty$

Quantum effects give negative energy outside the horizon

$\langle T_{uv}^{(2D)} \rangle$



$\langle T_{uu}^{(2D)} \rangle$



$T_{uu} = 0$
in $r \to \infty$

Negative energy at horizon

Breakdown of perturbative expansion

Perturbative expansion around classical solution

α: Newton constant
+ numerical factor

$$ds^2 = -C(r)dt^2 + \frac{C(r)}{F^2(r)}dr^2 + r^2 d\Omega^2$$

$$C(r) = C_0(r) + \alpha C_1(r) + \cdots \qquad F(r) = F_0(r) + \alpha F_1(r) + \cdots$$

The leading term $C_0(r)$ is classical solution $\quad C_0 = F_0 = 1 - \frac{a_0}{r}$

By using expansions of Einstein tensor and energy-momentum tensor

$$G_{\mu\nu}^{(4D)} = G_{\mu\nu}^{(0)} + \alpha G_{\mu\nu}^{(1)} + \cdots \qquad \langle T_{\mu\nu}^{(4D)} \rangle = \langle T_{\mu\nu} \rangle_0 + \alpha \langle T_{\mu\nu} \rangle_1 + \cdots$$

semi-classical Einstein equation is expanded as

$$G_{\mu\nu}^{(4D)} = \alpha \langle T_{\mu\nu}^{(4D)} \rangle \quad \Rightarrow \quad \begin{aligned} G_{\mu\nu}^{(0)} &= 0 \\ G_{\mu\nu}^{(1)} &= \alpha \langle T_{\mu\nu} \rangle_0 \\ &\vdots \end{aligned}$$

Breakdown of perturbative expansion

Leading vacuum energy for Boulware vacuum

$$\langle T_{uu}\rangle_0\Big|_{r=a} = \langle T_{vv}\rangle_0\Big|_{r=a} = \frac{1}{8a_0^4}$$

$$g_{uv} = -\frac{r-a_0}{r}$$

First order correction of Ricci tensor

$$R^{(4D)}_{\mu\nu} = R^{(0)}_{\mu\nu} + \alpha R^{(1)}_{\mu\nu} + \cdots$$

$$G^{(1)}_{uu} = R^{(1)}_{uu} \qquad G^{(1)}_{vv} = R^{(1)}_{vv}$$

is calculated as

$$R^{(1)}_{uu} = \alpha\langle T_{uu}\rangle_0 = -\frac{\alpha}{8a_0^4} \qquad R^{(1)}_{vv} = \alpha\langle T_{vv}\rangle_0 = -\frac{\alpha}{8a_0^4}$$

Square of Ricci tensor is

$$R_{\mu\nu}R^{\mu\nu} \sim 2\alpha^2 g^{uv} R^{(1)}_{uu} g^{uv} R^{(1)}_{vv} = \frac{\alpha^2}{32}\frac{1}{a_0^6(r-a_0)^2} + \cdots$$

Perturbative correction diverges at the horizon $r = a_0$.

⇨ We cannot use α-expansion near $r = a_0$.

We solve the Einstein equation without using α-expansion.

Self-consistent Einstein equation

We solve semi-classical Einstein equation for $g_{\mu\nu}$ and $\langle T_{\mu\nu} \rangle$

$$G^{(4D)}_{\mu\nu} = \frac{8\pi G}{r^2} \langle T^{(2D)}_{\mu\nu} \rangle \qquad \mu, \nu = 0,1 \qquad G_{\theta\theta} = 0$$

where metric and $\langle T^{(2D)}_{\mu\nu} \rangle$ are given by

$$ds^2 = -C(r) du dv + r^2 d\Omega^2$$

$$\langle T^{(2D)}_{uv} \rangle = -\frac{1}{12\pi} (C \partial_u \partial_v C - \partial_u C \partial_v C)$$

$$\langle T^{(2D)}_{uu} \rangle = \langle T^{(2D)}_{vv} \rangle = -\frac{1}{12\pi} C^{1/2} \partial_u^2 C^{-1/2}$$

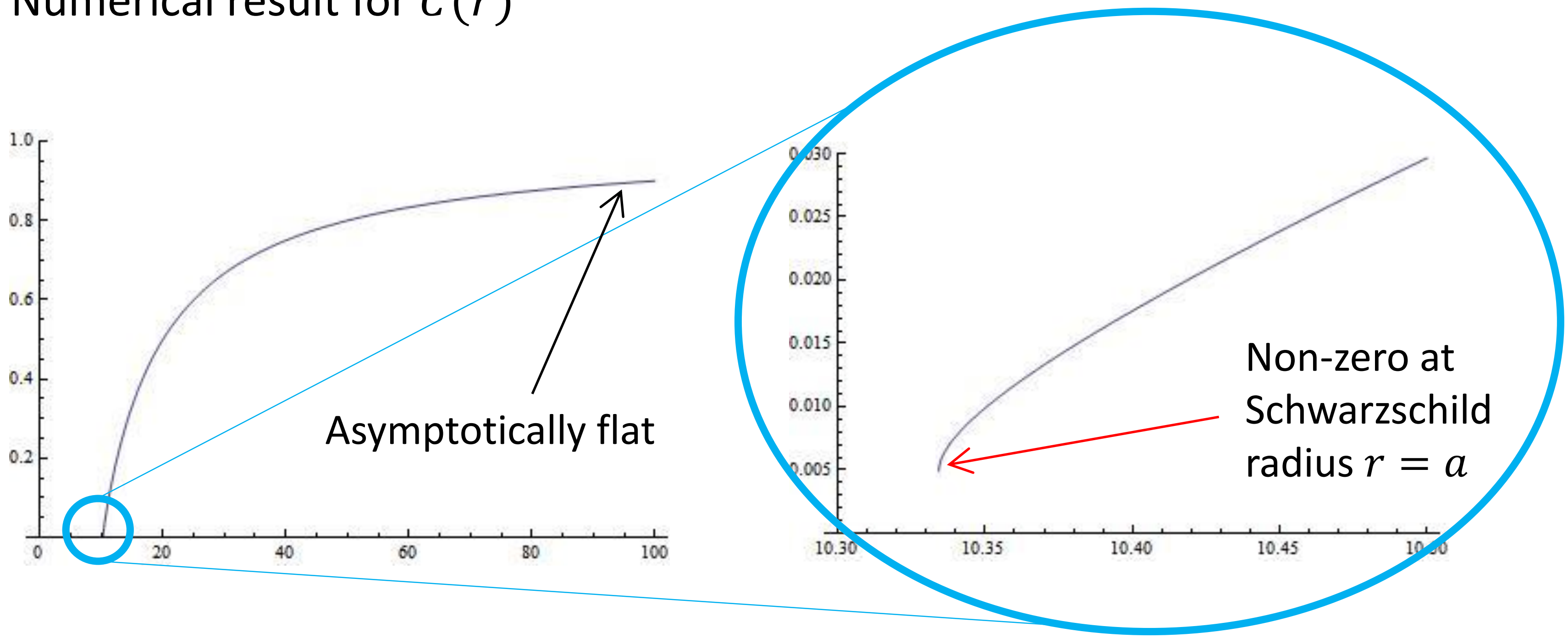
Results

Define ρ by $C(r) = e^{2\rho}$ where $ds^2 = -C(r)dt^2 + \frac{C(r)}{F^2(r)}dr^2 + r^2 d\Omega^2$

ρ satisfies

$$r\rho' + (2r^2 + \alpha)\rho'^2 + \alpha r \rho'^3 + (r^2 - \alpha)\rho'' = 0$$

Numerical result for $C(r)$



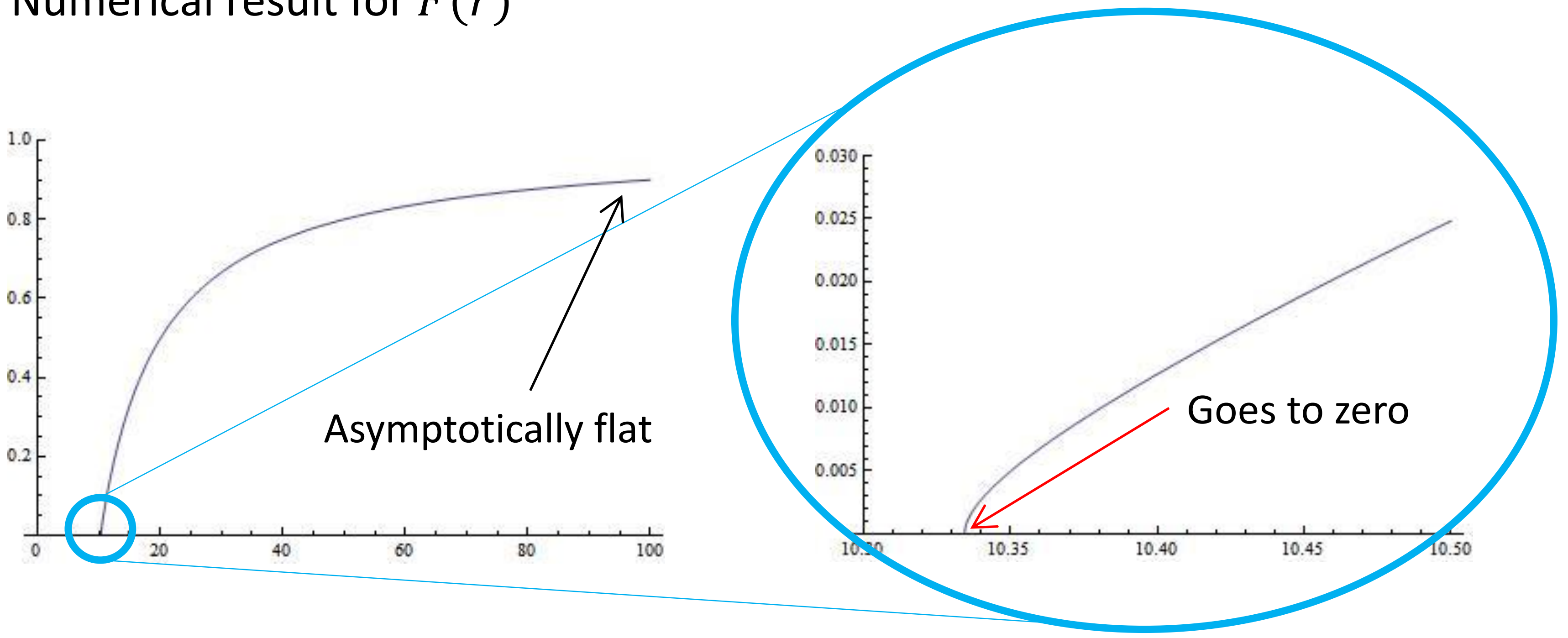
Results

The other component $F(r)$ in $ds^2 = -C(r)dt^2 + \frac{C(r)}{F^2(r)}dr^2 + r^2 d\Omega^2$

By semi-classical Einstein equation, $F(r)$ is related to $C(r)$ as

$$F(r) = \frac{C^{3/2}(r)}{\sqrt{4C^2(r) + 4rC(r)C'(r) + \alpha C'^2}}$$

Numerical result for $F(r)$



Near "horizon" behavior

$(C(r) = e^{2\rho})$

Killing horizon ⇒ $C(r) = 0$ ⇒ $\rho \to -\infty$ ⇒ $\rho' \to \infty$

Assume $\rho' \xrightarrow{r \to a} \infty$, at some point $r = a$,

Differential equation for ρ is approximated as

$$r\rho' + (2r^2 + \alpha)\rho'^2 + \alpha r \rho'^3 + (r^2 - \alpha)\rho'' = 0$$

⇒

$$\alpha a \rho'^3 + (a^2 - \alpha)\rho'' = 0$$

Then, ρ' behaves as $\rho' \sim \frac{k}{\sqrt{r-a}}$ where $k \sim \left(\frac{2a}{\alpha}\right)^2$

$C(r), F(r)$ behaves near $r = a$ as

$$C(r) = c_0 e^{2k\sqrt{r-a}} \qquad F(r) = \frac{1}{k}\sqrt{4c_0(r-a)}$$

⇓

$C(r = a) \neq 0$ ⇒ No Killing horizon $(C = 0)$ at any finite $r = a$

Near “horizon” geometry

Metric is given by

$$ds^2 = -C(r)dt^2 + \frac{C(r)}{F^2(r)}dr^2 + r^2 d\Omega^2$$

Assuming $\rho' \to \infty$ at $r \to a,\ C(r), F(r)$ behave near $r = a$ as

$$C(r) = c_0 e^{2k\sqrt{r-a}} \qquad F(r) = \frac{1}{k}\sqrt{4c_0(r-a)}$$

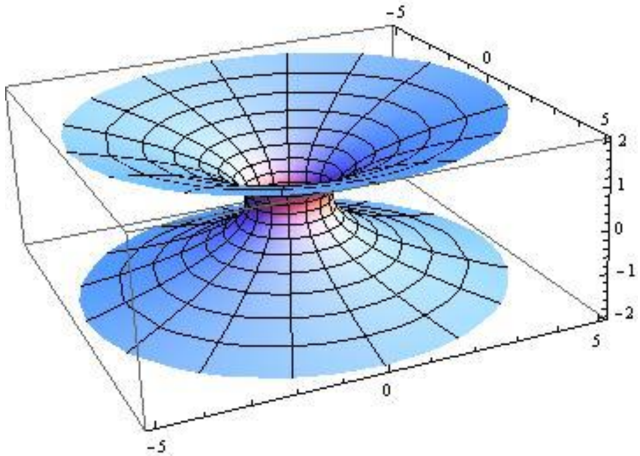
metric near $r = a$

$$ds^2 \sim -c_0 dt^2 + \frac{k\alpha\, dr^2}{4(r-a)} + r^2 d\Omega^2$$

Tortoise coord. x by $r = a + \frac{c_0}{\alpha k} r_*^2$

$$ds^2 \sim -c_0(dt^2 - dr_*^2) + (a^2 + c_1 r_*^2)d\Omega^2$$

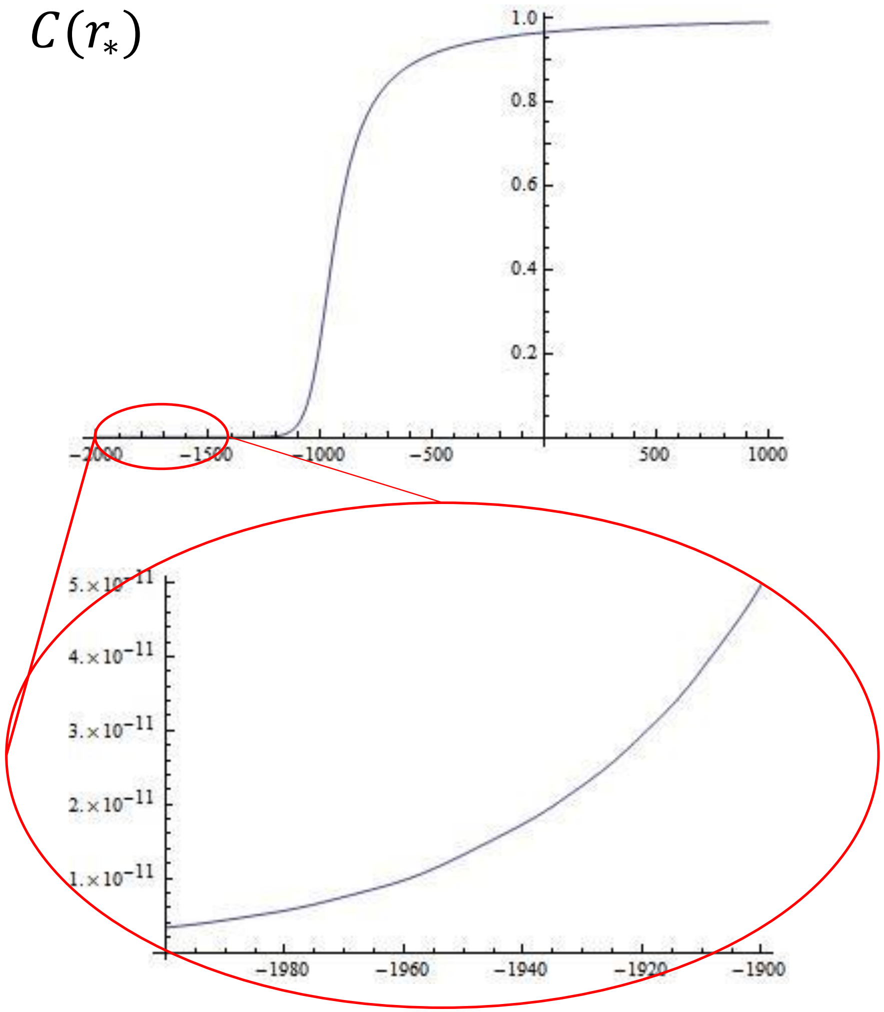
This is wormhole metric



Numerical result

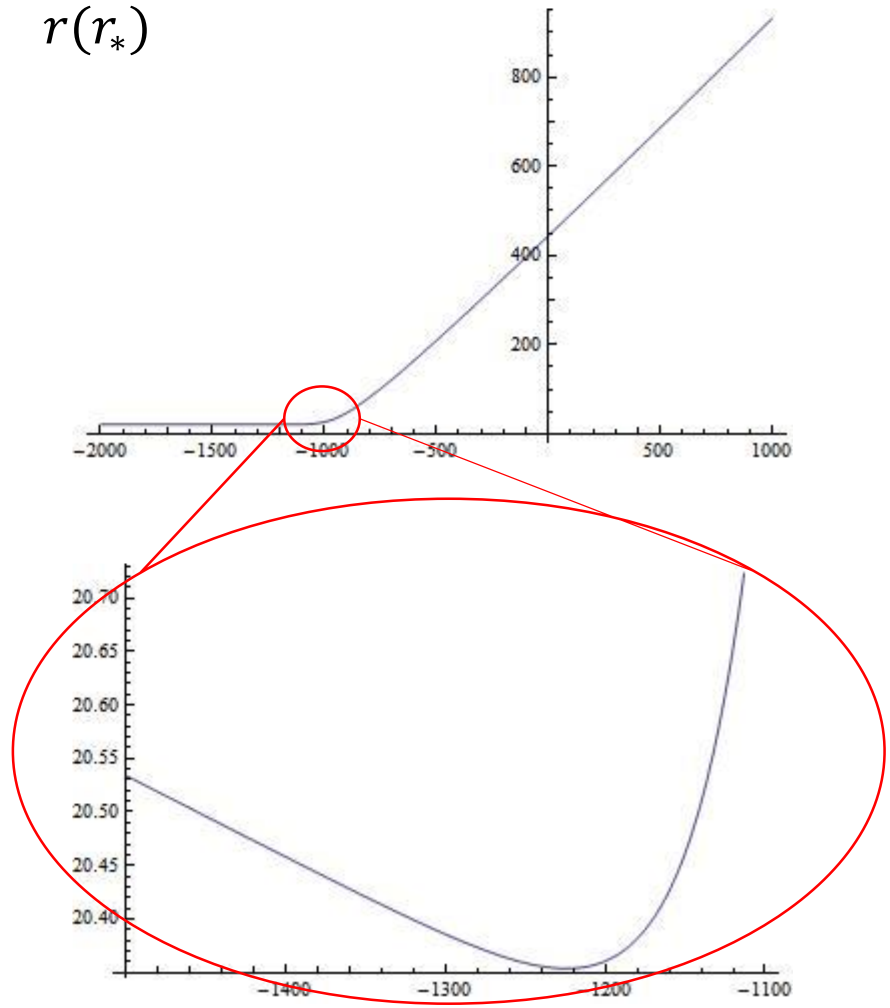
$$ds^2 = -C(r_*)dt^2 + C(r_*)dr_*^2 + r(r_*)d\Omega^2$$

$C(r_*)$



$C(r_*)$ does not go to zero

$r(r_*)$



$r(r_*)$ has local minimum

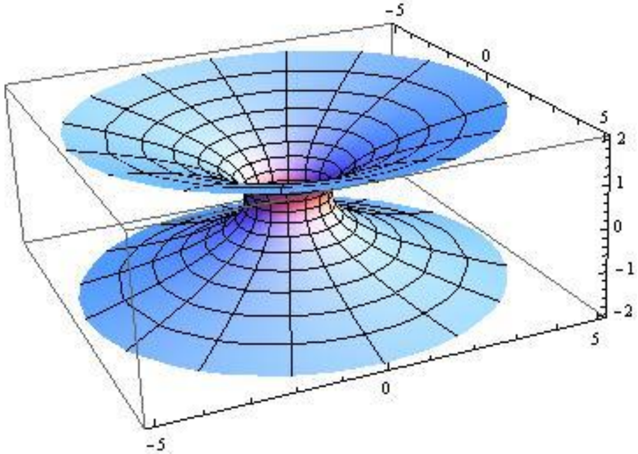
Back reaction from negative vacuum energy

For fixed background of Schwarzschild, negative vacuum energy is finite from the viewpoint of fiducial observer.

For freely falling observer, the negative vacuum energy is infinitely large.

Vacuum energy becomes very large near horizon ⇨ Back reaction from vacuum energy is no longer negligible.

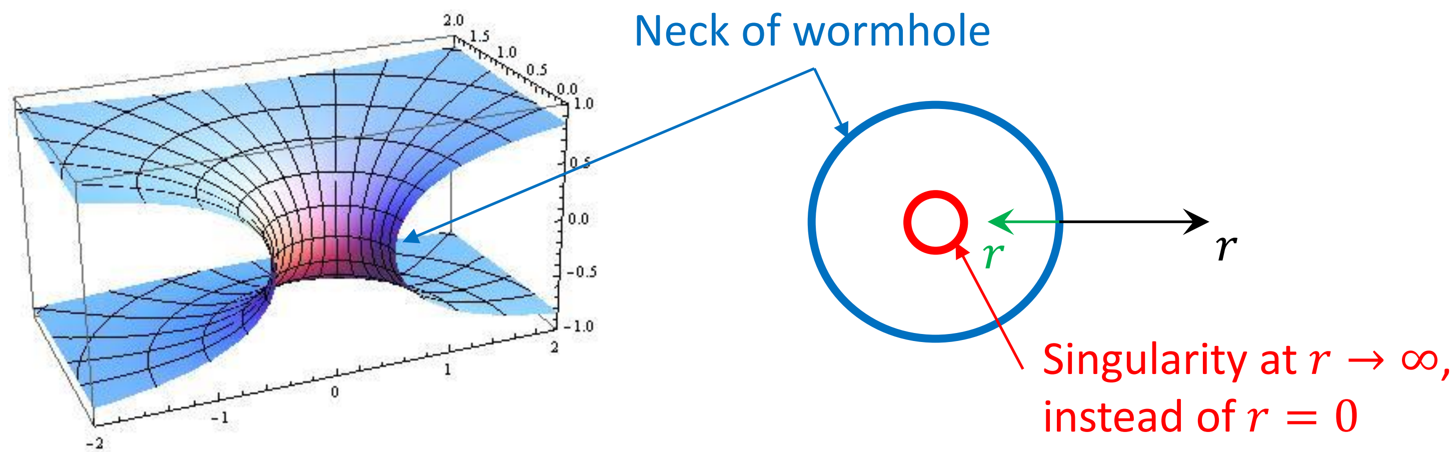
By taking the back reaction into account, the horizon cannot appear if there is negative vacuum energy.



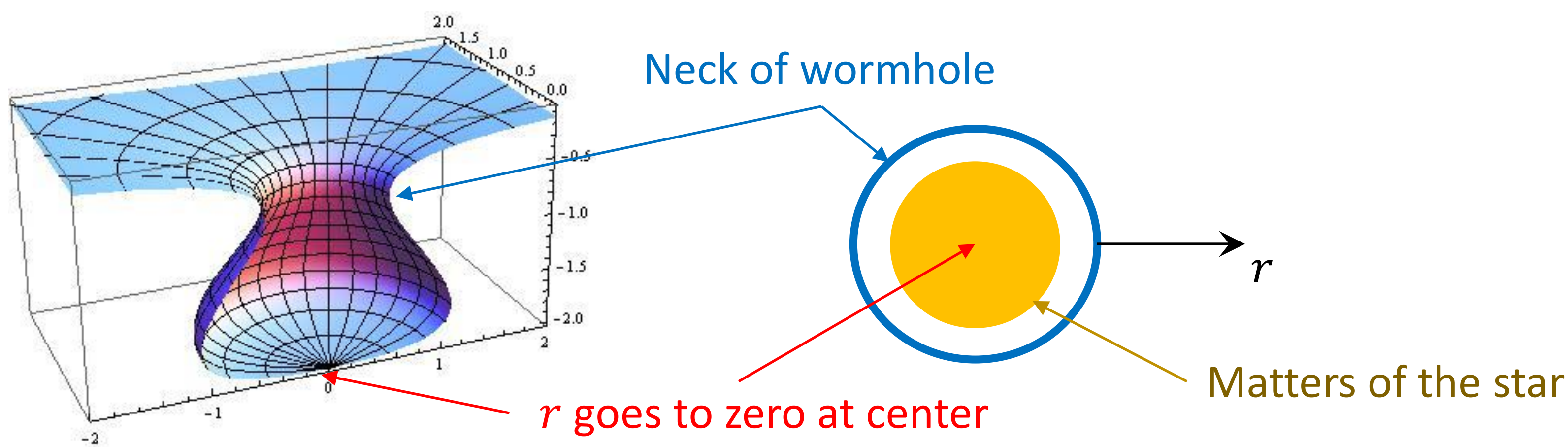
Local minimum of radius r, as wormhole, appear instead of the Killing horizon.

Interior of wormhole

Wormhole like solution is vacuum solution without matters of the star.



For physical situation, there is a star, where matters are distributed

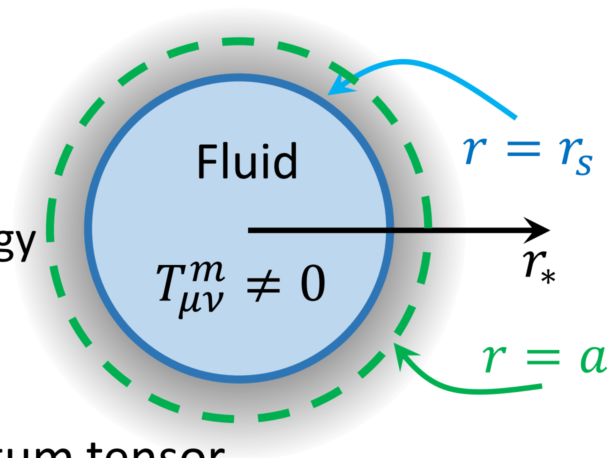


Geometry of interior of black hole

We put the surface of the star at $r = r_s$

Vacuum with negative energy

$T^m_{\mu\nu} = 0$



Fluid fills inside $r = r_s$

Outside: vacuum solution

Inside: solution with $T^m_{\mu\nu} \neq 0$

Energy-momentum tensor

$$\langle T_{\mu\nu} \rangle = T^{\Omega}_{\mu\nu} + T^m_{\mu\nu}$$

Energy-momentum tensor of matters

Energy-momentum tensor of vacuum

$$T^{\Omega}_{\mu\nu} = \frac{1}{r^2} \langle T^{(2D)}_{\mu\nu} \rangle$$

We consider incompressible fluid

$$T^m_{\mu\nu} = (m_0 + P) u_\mu u_\nu + P g_{\mu\nu}$$

m_0: Density (constant)

P: Pressure

Classical star of incompressible fluid

Relation between a and m_0

$$\frac{a_0}{2} = \frac{4\pi}{3} m_0 r_s^3$$

Pressure in classical limit

$$P(r) = 8\pi G \frac{\sqrt{3 - 8\pi G m_0 r^2} - \sqrt{3 - 8\pi G m_0 r_s^2}}{3\sqrt{3 - 8\pi G m_0 r_s^2} - \sqrt{3 - 8\pi G m_0 r^2}}$$

Condition for non-singular pressure

$$m_0 < \frac{1}{3\pi G r_s^2} \quad \Longleftrightarrow \quad r_s > \frac{9}{8} a$$

In classical limit, there is no static star with $r_s^2 > \frac{1}{3\pi G m_0}$

⇩

There are no such condition if quantum effects is taken into account

Semi-classical geometry of interior

Assumption: $T^{\Omega}_{\mu\nu}$ and $T^{m}_{\mu\nu}$ are conserved independently.

Vacuum energy-momentum tensor (approx. by 2D scalar)

$$T^{\Omega}_{uv} = -\frac{N}{12\pi r^2}(C\partial_u\partial_v C - \partial_u C\partial_v C)$$

$$T^{\Omega}_{uu} = T^{\Omega}_{vv} = -\frac{N}{12\pi r^2}C^{1/2}\partial_u^2 C^{-1/2}$$

Energy-momentum tensor for incompressible fluid

r_s: surface of star

Conservation law ⇨ $P = P_0\left[\left(\frac{C(r_s)}{C(r)}\right)^{1/2} - 1\right]$

Tortoise coordinate r_* is convenient to see interior

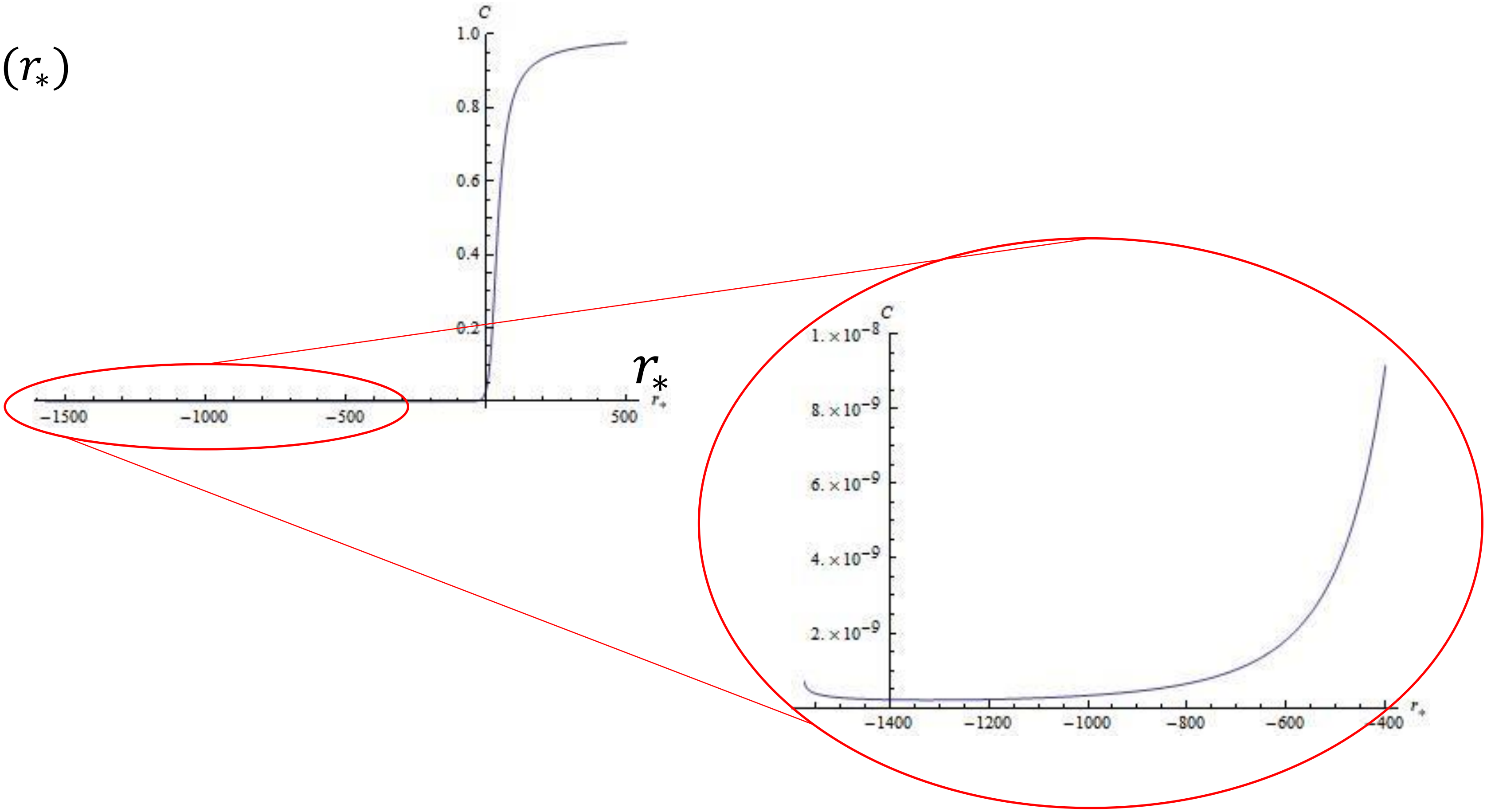
$$ds^2 = C(r_*)(-dt^2 + dr_*^2) + r^2(r_*)d\Omega^2$$

Case III: approx. appropriate density ($m_0 \sim \widehat{m}_0$)

Numerical result for $C(r_*)$

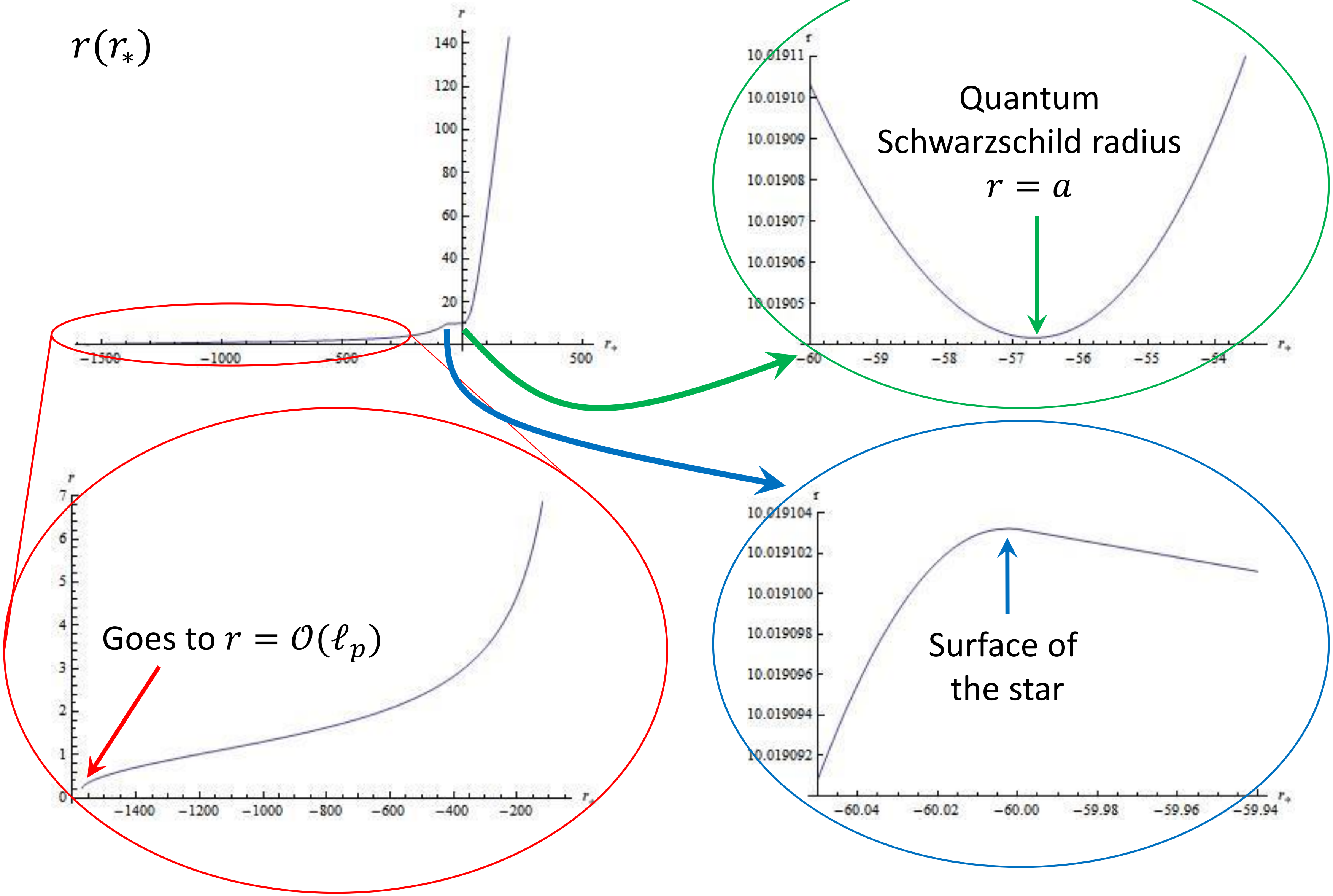
$$ds^2 = C(r_*)(-dt^2 + dr_*^2) + r^2(r_*)d\Omega^2$$

$C(r_*)$

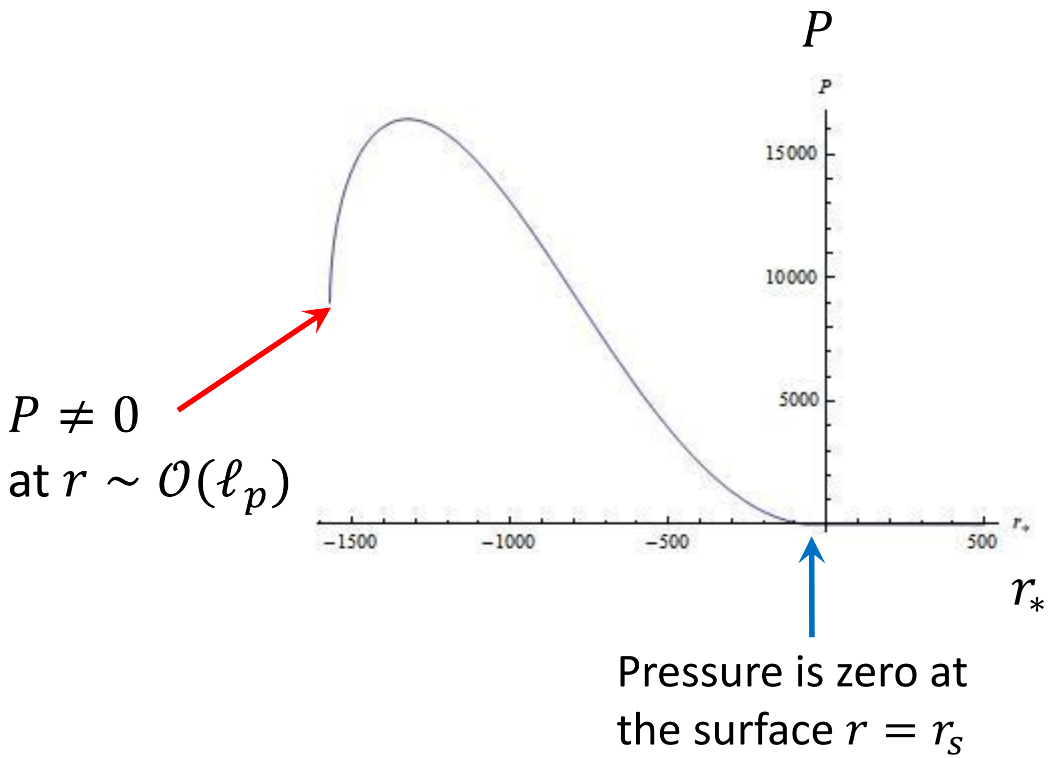


Numerical result for $r(r_*)$

$$ds^2 = C(r_*)(-dt^2 + dr_*^2) + r^2(r_*)d\Omega^2$$



Pressure $P(r_*)$

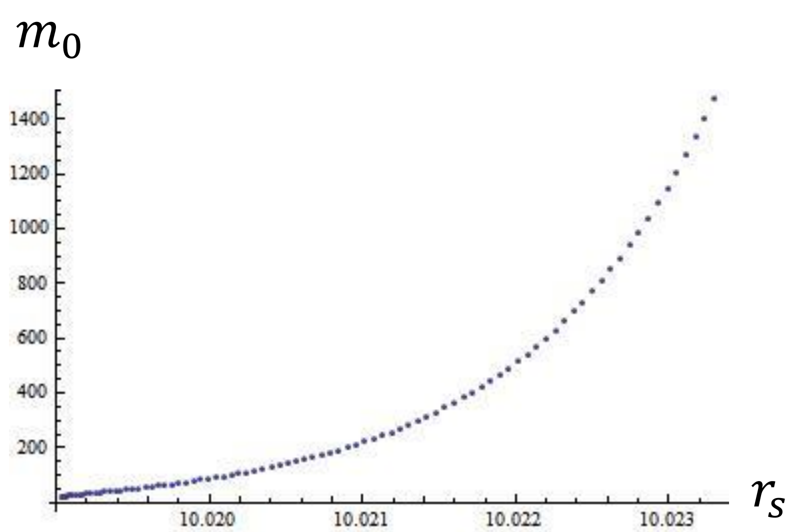


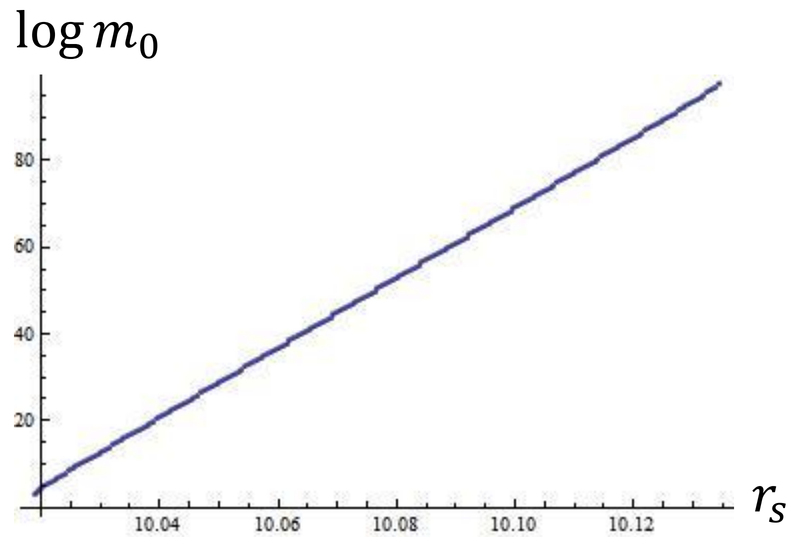
P
P
15000
10000
5000
−1500
−1000
−500
500
r_*
$P \neq 0$
at $r \sim \mathcal{O}(\ell_p)$
Pressure is zero at
the surface $r = r_s$

Surface at deeper place

Relation between m_0 and r_s for $a = 10$

Surface is inside of $r = a$

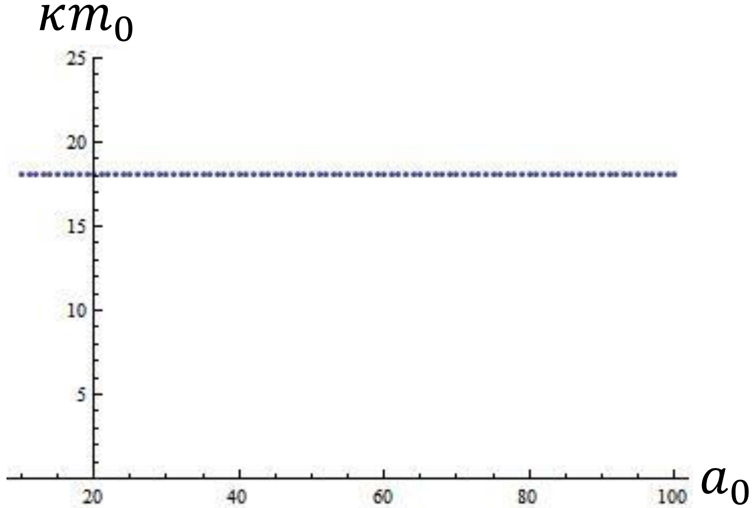


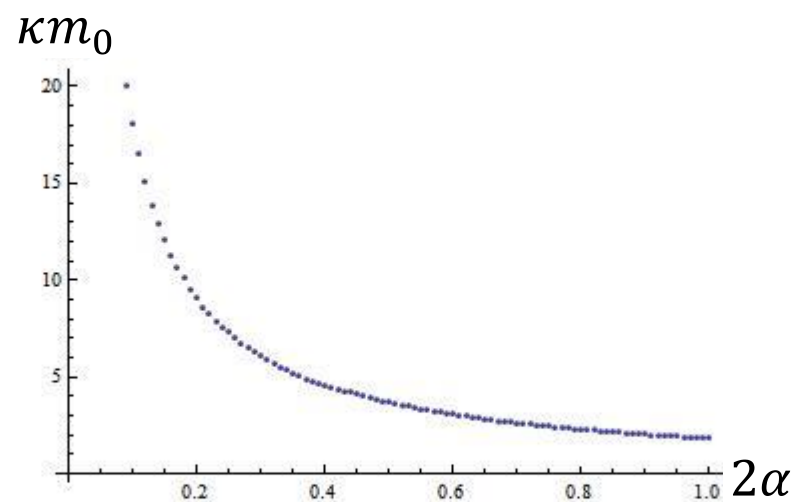


- Density m_0 increases exponentially as surface moves inside
- Difference between local minimum and local maximum of r would be of Planck scale.

Density for $r_s = a$

Density m_0 for the star with surface at neck of “wormhole”





- Density m_0 is independent of mass of black hole a_0
- Density is very large: $m_0 \sim \mathcal{O}(\kappa^{-1}\alpha^{-1}) \sim \mathcal{O}\left(\ell_p^{-4}\right)$

Arbitrarily large star can be non-singular

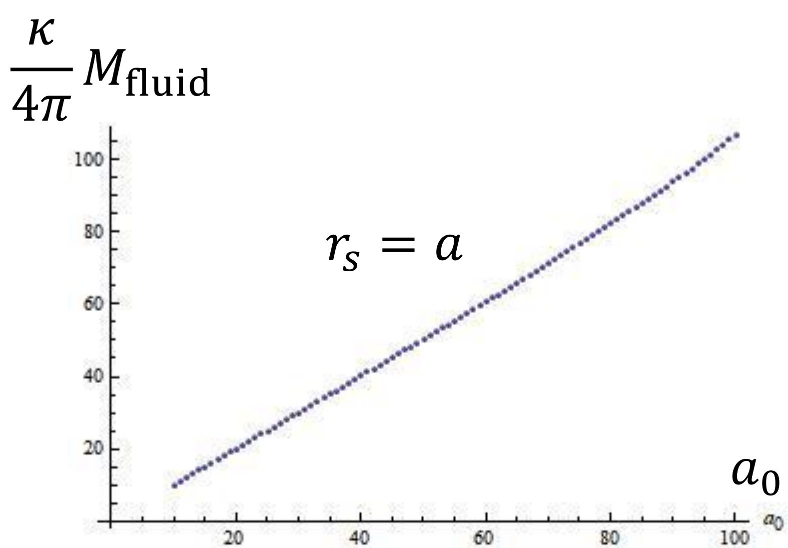
Classical regularity condition for pressure can be violated by arbitrary small m_0

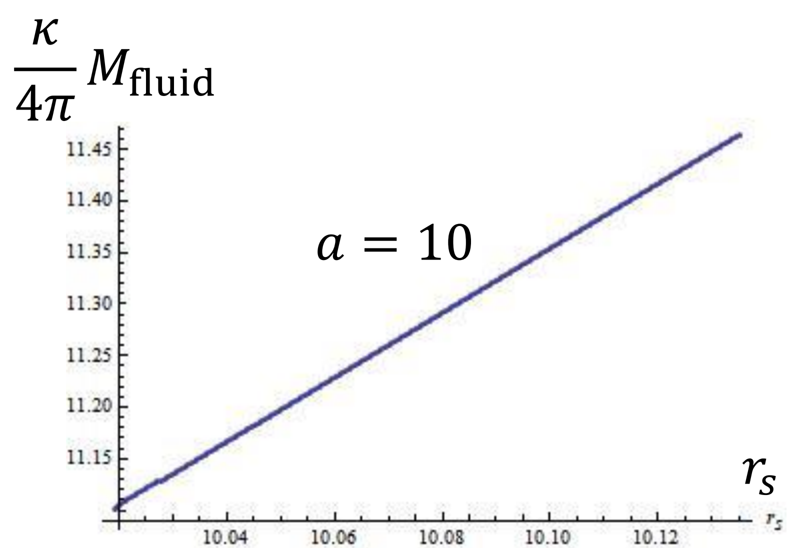
$$m_0 < \frac{1}{3\pi G r_s^2}$$

Mass of fluid and black hole

Komar mass calculated from fluid density and pressure

$$M_{\text{fluid}} = -\int d^3x \sqrt{-g}\left(2T^0_0 - T^\mu_\mu\right) = 4\pi \int dr_*\, r^2 C(m_0 + 3P)$$





- Komar mass of fluid almost reproduce black hole mass
- Fluid mass is slightly larger than BH mass because of negative vacuum energy

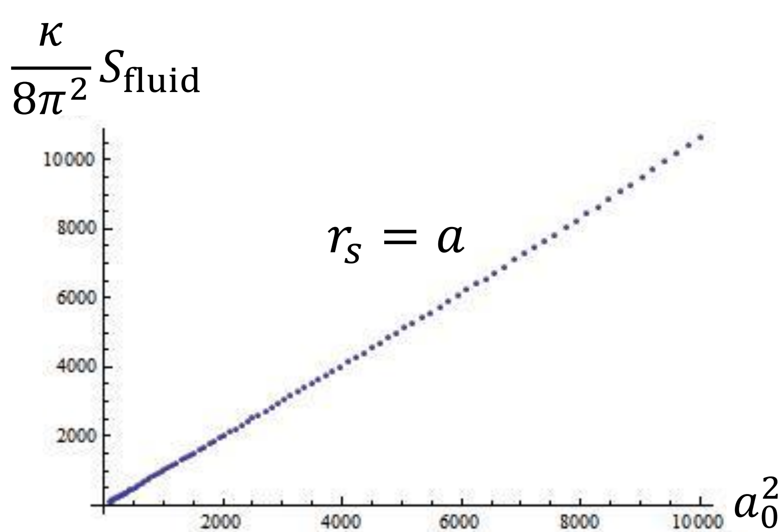
Entropy of fluid and Bekenstein-Hawking

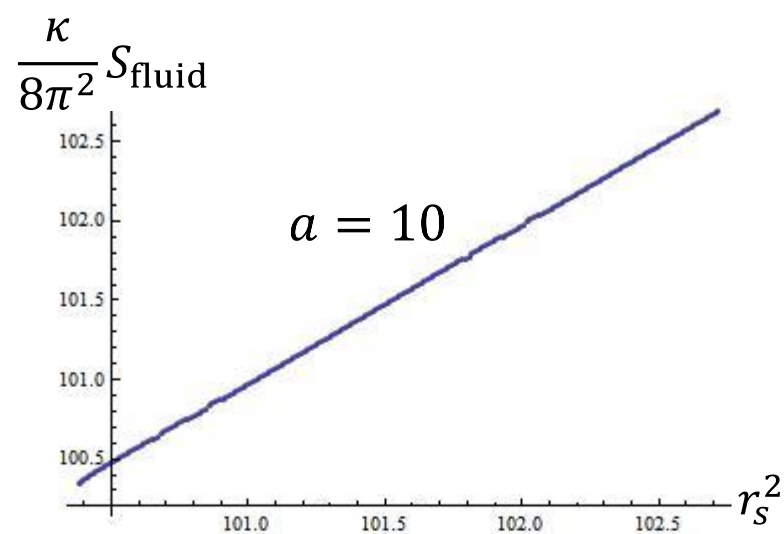
Entropy density from the local thermodynamic relation

$$m_0 + P = Ts$$

Entropy of fluid is calculated by integrating entropy density

$$S_{\mathrm{fluid}} = \int d^3x\sqrt{g_{3D}}\, s = (4\pi)^2 \int dr_*\, a_0 r^2 C(m_0 + P)$$

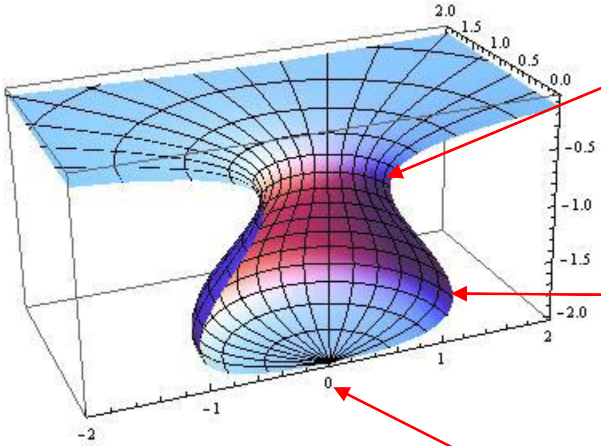




Entropy of fluid agrees with Bekenstein-Hawking entropy

Incompressible fluid

We consider (classical) incompressible fluid + vacuum energy from 2D scalar.



Wormhole-like structure (local minimum of r) instead of the Killing horizon

Local maximum of r is slightly inside the surface of the fluid. r is almost same to that at local minimum.

r goes to zero at the center.
Proper distance from Schwarzschild radius to $r = 0$ is of order of Planck length.

- There is no horizon for arbitrary density and position of the surface.
- Pressure and density are very large but finite.
- The surface is outside the Schwarzschild radius if density is not very large.
- Entropy of the fluid agrees with Bekenstein-Hawking entropy.

Evaporation by Hawking radiation

If we introduce the Hawking radiation, neck of wormhole decreases since mass decreases due to Hawking radiation.

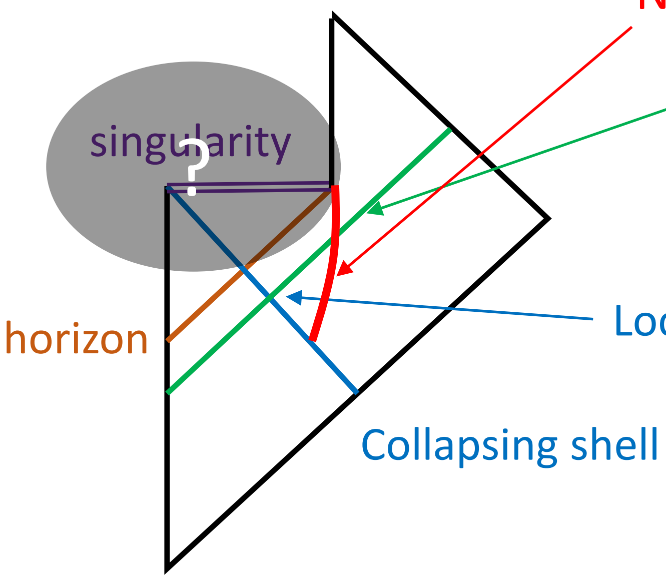
Inside the neck, r decreases along outgoing null line, $\frac{\partial r}{\partial v} < 0$

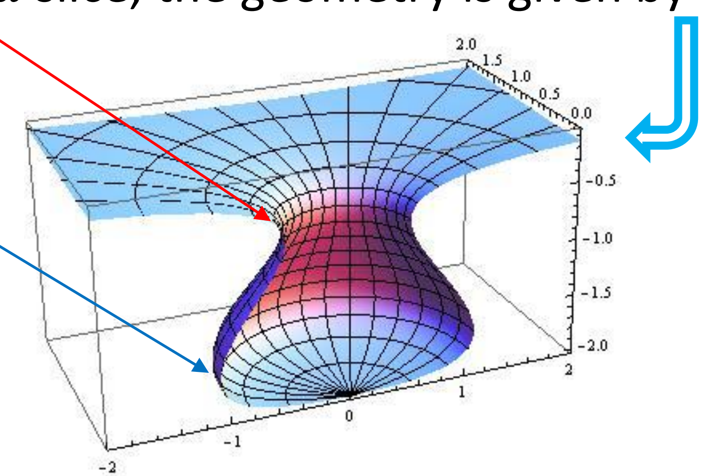
Inside but around the neck, r slightly increases along ingoing null line, $\frac{\partial r}{\partial u} \gtrsim 0$

If the neck reduces with time, r decreases along both outgoing and ingoing lines

Neck is apparent horizon $\frac{\partial r}{\partial v} < 0$ and $\frac{\partial r}{\partial u} < 0$

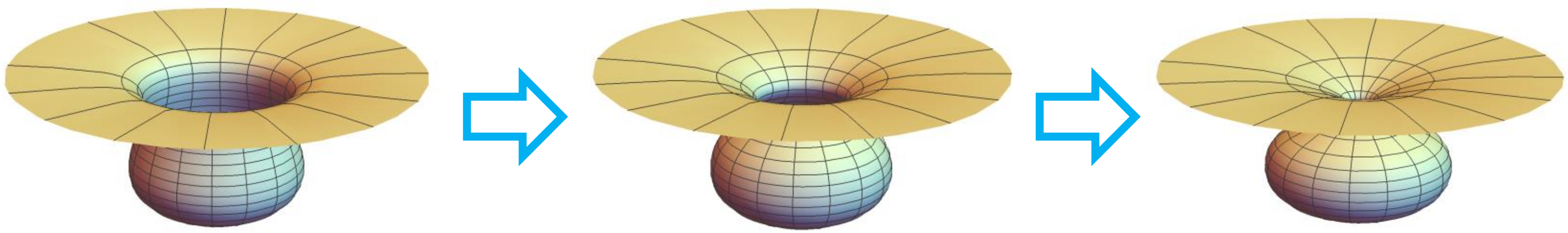
At some constant-u slice, the geometry is given by





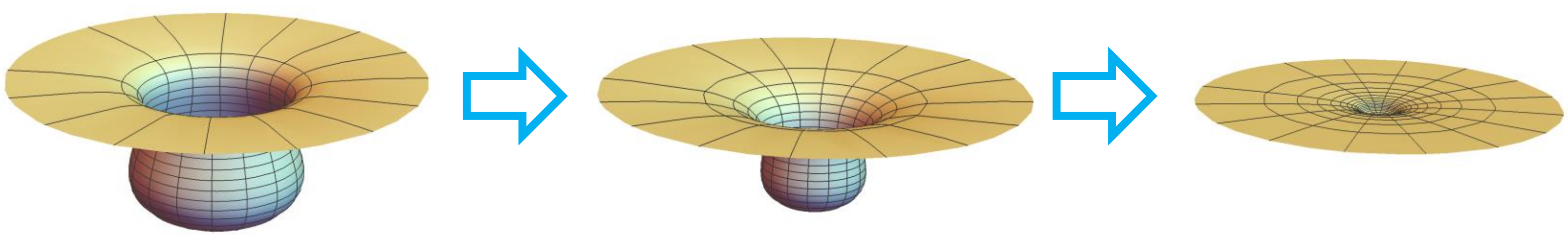
Apparent horizon as shrinking neck of wormhole

Apparent horizon appears simply because the neck is shrinking.



Interior might be disconnected from the outside of black hole after evaporation.

Result for fluid implies that neck and local maximum of r are almost same



In this case, interior will be of Planck scale when the neck is of Planck scale.

Full quantum effects of gravity becomes important and
geometry will simply goes to flat space.

Conclusion

- Quantum energy-momentum tensor for Boulware vacuum diverges at horizon of classical black hole geometry.
- Taking back reaction of quantum effects, there are no divergence even for Boulware vacuum.
- Because of the back reaction from the vacuum energy, static black holes also do not have Killing horizon.
- Geometry has wormhole-like structure.
- Singularity in the other side of wormhole comes from infinitely high density of matter, and will be absent for star with finite size.
- Apparent horizon appears simply because the neck of the wormhole-like structure is shrinking. It is time-like and nothing is trapped.
- The event horizon appears if the interior is disconnected from the outside after the evaporation. This would be no problem for unitarity.
- result for incompressible fluid implies that the interior is also shrinking and everything becomes of Planck scale before interior is disconnected from the outside.

Thank you