

The Schwarzian and black hole physics

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Based on [arXiv:1705.08408](#) with G.J. Turiaci and H. Verlinde
[arXiv:1801.09605](#)
[arXiv:1804.09834](#) with H. Lam, G.J. Turiaci and H. Verlinde

Motivation

The Schwarzian path integral

Embedding in 2d Liouville CFT

- Partition function

- 2-point function

- 4-point function

- OTO 4-point function

Conclusion

Definition and Motivation

$S_{Schw} = -C \int dt \{f, \tau\}$ where $\{f, \tau\} = \frac{f'''}{f'} - \frac{3}{2} \left(\frac{f''}{f'} \right)^2$ the Schwarzian derivative

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Appears in:

- ▶ SYK model at low energies

Effective Action (master fields Σ, G) $\sim N \Leftrightarrow$ suppressed at $N \rightarrow \infty$, unless also $\beta J \rightarrow \infty \Rightarrow$ Schwarzian action

- ▶ Jackiw-Teitelboim (JT) 2d dilaton gravity

JT is holographically dual to Schwarzian theory \rightarrow Dynamics of wiggly boundary curve described by S_{Schw}

Compare to CS / WZW, 3d gravity / Liouville topological dualities

The Schwarzian theory: goal

Main goal:

Compute all correlation functions:

$$\langle \mathcal{O}_{\ell_1} \mathcal{O}_{\ell_2} \dots \rangle_{\beta} = \frac{1}{Z} \int_{\mathcal{M}} [\mathcal{D}f] \mathcal{O}_{\ell_1} \mathcal{O}_{\ell_2} \dots e^{C \int_0^{\beta} d\tau \left(\{f, \tau\} + \frac{2\pi^2}{\beta^2} f'^2 \right)}$$

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Bilocal operators: $\left(F \equiv \tan \left(\frac{\pi f(\tau)}{\beta} \right) \right)$

$$\mathcal{O}_{\ell}(\tau_1, \tau_2) \equiv \left(\frac{F'(\tau_1) F'(\tau_2)}{(F(\tau_1) - F(\tau_2))^2} \right)^{\ell} \equiv \left(\frac{f'(\tau_1) f'(\tau_2)}{\frac{\beta}{\pi} \sin^2 \frac{\pi}{\beta} [f(\tau_1) - f(\tau_2)]} \right)^{\ell}$$

Think of this expression as two-point function

$\mathcal{O}_{\ell}(\tau_1, \tau_2) = \langle \mathcal{O}(\tau_1) \mathcal{O}(\tau_2) \rangle_{\text{CFT}}$ of some 1D ‘matter CFT’ at finite temperature coupled to the Schwarzian theory

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- ▶ 1d Liouville [Bagrets-Altland-Kamenev '16, '17](#)
- ▶ 2d BF bulk [Blommaert-TM-Verschelde '18](#) → previous talk
- ▶ 2d Liouville CFT [TM-Turiaci-Verlinde '17, TM '18](#) → this talk
- ▶ Particle in infinite B-field in AdS_2 [Maldacena-Yang \(In progress\)](#), [Kitaev-Suh \(In progress\)](#)
⇒ related to representation theory of universal cover of $\text{SL}(2, \mathbb{R})$ [Iliesiu-Pufu-Wang \(In progress\)](#)

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$$Z(\beta) = \left(\frac{\pi}{\beta}\right)^{3/2} \exp\left(\frac{\pi^2}{\beta}\right) = \int_0^{+\infty} dk^2 \sinh(2\pi k) e^{-\beta k^2} \quad (C = 1/2)$$

Density of states $\rho(E) = \sinh(2\pi\sqrt{E})$

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\Rightarrow Cardy scaling at high energies $\rho(E) \sim e^{2\pi\sqrt{E}}$

\Rightarrow **2d CFT origin ?**

What about correlators ? Need other techniques

Partition function: Vacuum character

Observation:

$$\mathrm{Tr}_0(q^{L_0}) \equiv \chi_0(q) = \frac{q^{\frac{1-c}{24}}(1-q)}{\eta(\tau)} = \text{[Diagram of a cylinder with a vertical line through the center]}, \quad q = e^{-2\pi t}$$

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with density

$$|\Psi_{ZZ}(P)|^2 = \sinh(2\pi bP) \sinh\left(\frac{2\pi P}{b}\right) \quad c = 1 + 6\left(b + \frac{1}{b}\right)^2$$

In Schwarzian limit: $P = bk$, $b \rightarrow 0$

Path-integral link Liouville and Schwarzian

One can prove via Liouville phase space path integral between ZZ-branes:

$$\int_{\phi(0)=\phi(\tau)} [\mathcal{D}\phi] [\mathcal{D}\pi_\phi] e^{\int_0^\tau d\tau \int d\sigma (i\pi_\phi \dot{\phi} - \mathcal{H}(\phi, \pi_\phi))}$$

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► **Field redefinition** $(\phi, \pi_\phi) \rightarrow (A, B)$ Gervais-Neveu '82:

$$e^\phi = -8 \frac{A_\sigma B_\sigma}{(A - B)^2}, \quad \pi_\phi = \frac{A_{\sigma\sigma}}{A_\sigma} - \frac{B_{\sigma\sigma}}{B_\sigma} - 2 \frac{A_\sigma + B_\sigma}{(A - B)}$$

$$\Rightarrow \mathcal{H} = -\frac{c}{24\pi} \{A(\sigma, \tau), \sigma\} - \frac{c}{24\pi} \{B(\sigma, \tau), \sigma\}$$

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Liouville-Schwarzian Dictionary:

- $T(w) \rightarrow -\frac{c}{24\pi} \{F(\sigma), \sigma\}$
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Liouville cylinder amplitudes with V_ℓ 's between ZZ's

→ Schwarzian bilocal correlators

2-point correlator

Idea: $V_\ell(\tau_1, \tau_2) \equiv e^{2\ell\phi(\tau_1, \tau_2)} \longleftrightarrow \mathcal{O}_\ell(\tau_1, \tau_2)$

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\Rightarrow 3-point function on sphere \Rightarrow large c limit of DOZZ formula

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Result:

$$G_\ell^\beta(\tau_1, \tau_2) = \frac{1}{Z(\beta)} \int d\mu(k_1) d\mu(k_2) e^{-\tau k_1^2 - (\beta - \tau) k_2^2} \frac{\Gamma(\ell \pm i(k_1 \pm k_2))}{2\sqrt{\pi}\Gamma(2\ell)}$$

$$d\mu(k) \equiv dk^2 \sinh(2\pi k)$$

Application: Semiclassics for light operators

Semi-classical regime $C \rightarrow \infty$, $\ell \ll C$: $k_1 \sim k_2 \gg 1$

Redefine $k_1^2 = M + \omega$, $k_2^2 = M$, $M \gg \omega$

$$G_\ell^\pm \sim \int_0^\infty dM e^{2\pi\sqrt{M} - \frac{\beta}{2C}M} \int \frac{d\omega}{2\pi} e^{\pm i\frac{\tau}{2C}\omega + \pi\frac{\omega}{2\sqrt{M}}} \frac{\Gamma(\ell \pm i\frac{\omega}{2\sqrt{M}})}{\Gamma(2\ell)} (2\sqrt{M})^{2\ell-1}$$

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Interpretation:

- M -integral has saddle: $M_0 = 4\pi^2 C^2 / \beta^2$, the JT black hole $E(T)$ -relation

Remaining ω -integral is done explicitly to yield:

$$G_\ell^{\pm, cl}(\tau_1, \tau_2) = \left(\frac{\pi}{\beta \sinh \frac{\pi}{\beta} \tau_{12}} \right)^{2\ell}$$

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- ▶ Quantum black hole M emits and reabsorbs excitation with mass $\sim \ell$ and energy ω

Time-ordered 4-point function

$$\langle ZZ | V_{\ell_1} V_{\ell_2} | ZZ \rangle$$

Evaluated using Conformal blocks

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$$G_{\ell_1 \ell_2}^\beta = \int dk_1^2 dk_4^2 dk_s^2 \sinh 2\pi k_1 \sinh 2\pi k_4 \sinh 2\pi k_s \\ \times e^{-k_1^2(\tau_2 - \tau_1) - k_4^2(\tau_4 - \tau_3) - k_s^2(\beta - \tau_2 + \tau_3 - \tau_4 + \tau_1)} \frac{\Gamma(\ell_2 \pm ik_4 \pm ik_s) \Gamma(\ell_1 \pm ik_1 \pm ik_s)}{\Gamma(2\ell_1) \Gamma(2\ell_2)}$$

Diagrammatic decomposition

Rules:



$$\begin{array}{c} k \\ \text{---} \text{---} \text{---} \\ \tau_2 \quad \tau_1 \end{array} = e^{-k^2(\tau_2 - \tau_1)}, \quad \begin{array}{c} k_1 \\ \text{---} \text{---} \text{---} \\ \ell \\ \text{---} \text{---} \text{---} \\ k_2 \end{array} = \gamma_\ell(k_1, k_2) = \sqrt{\frac{\Gamma(\ell \pm ik_1 \pm ik_2)}{\Gamma(2\ell)}}$$

- ▶ Integrate over intermediate momenta k_i with measure $d\mu(k) = dk^2 \sinh(2\pi k)$

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Examples:

$$Z(\beta) = \bigcirc$$

$$\langle \mathcal{O}_\ell(\tau_1, \tau_2) \rangle = \tau_2 \overset{\ell}{\text{---}} \tau_1$$

$$\langle \mathcal{O}_{\ell_1}(\tau_1, \tau_2) \mathcal{O}_{\ell_2}(\tau_3, \tau_4) \rangle = \begin{array}{c} \tau_2 \overset{\ell_1}{\curvearrowright} \tau_1 \\ \tau_3 \overset{\ell_2}{\curvearrowright} \tau_4 \end{array}$$

Note: non-perturbative in Schwarzian coupling C

OTO four-point correlator (1)

Swapping two operators in 4-point correlator, means the conformal block is dominated by its primary in a different channel:

$$\mathcal{F}_{P_s} \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} (z') = \int dP_t R_{P_s P_t} \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \mathcal{F}_{P_t} \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} (1/z')$$

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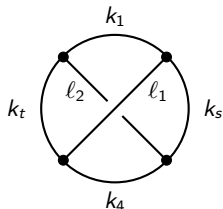
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Explicitly:

$$R_{P_s P_t} \left[\begin{smallmatrix} 2 & 3 \\ 1 & 4 \end{smallmatrix} \right] \sim \left\{ \begin{smallmatrix} \ell_1 & k_2 & k_s \\ \ell_3 & k_4 & k_t \end{smallmatrix} \right\} =$$
$$\sqrt{\frac{\Gamma(\ell_1 + ik_2 \pm ik_s) \Gamma(\ell_3 - ik_2 \pm ik_t) \Gamma(\ell_1 - ik_4 \pm ik_t) \Gamma(\ell_3 + ik_4 \pm ik_s)}{\Gamma(\ell_1 - ik_2 \pm ik_s) \Gamma(\ell_3 + ik_2 \pm ik_t) \Gamma(\ell_1 + ik_4 \pm ik_t) \Gamma(\ell_3 - ik_4 \pm ik_s)}}$$
$$\times \int_{-i\infty}^{i\infty} \frac{du}{2\pi i} \frac{\Gamma(u) \Gamma(u - 2ik_s) \Gamma(u + ik_2 + 4 - s + t) \Gamma(u - ik_s + t - 2 - 4) \Gamma(\ell_1 + ik_s - 2 - u) \Gamma(\ell_3 + ik_s - 4 - u)}{\Gamma(u + \ell_1 - ik_s - 2) \Gamma(u + \ell_3 - ik_s - 4)}$$

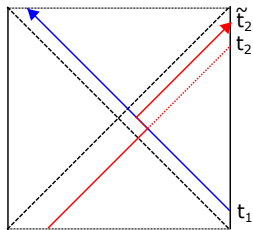
OTO four-point correlator (2)

At the Schwarzian level, this procedure is captured by the diagram:



$$e^{-k_1^2(\tau_3-\tau_1)-k_t^2(\tau_3-\tau_2)-k_4^2(\tau_4-\tau_2)-k_s^2(\beta-\tau_4+\tau_1)} \\ \times \gamma_{\ell_1}(k_1, k_s) \gamma_{\ell_2}(k_s, k_4) \gamma_{\ell_1}(k_4, k_t) \gamma_{\ell_2}(k_t, k_1) \times \left\{ \begin{array}{ccc} \ell_1 & k_1 & k_s \\ \ell_2 & k_4 & k_t \end{array} \right\}$$

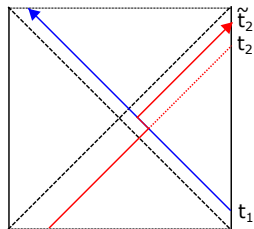
Application: Shockwaves from semiclassics



Time delay:

$$\tilde{t}_2 - t_2 \sim e^{\lambda_M(t_2 - t_1)}, \quad \lambda_M = \frac{2\pi}{\beta_M}$$

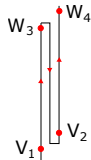
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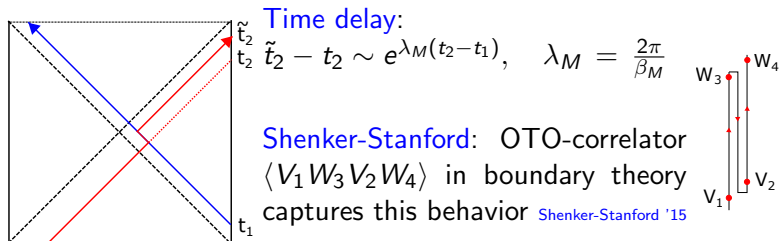
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Shenker-Stanford: OTO-correlator $\langle V_1 W_3 V_2 W_4 \rangle$ in boundary theory captures this behavior [Shenker-Stanford '15](#)



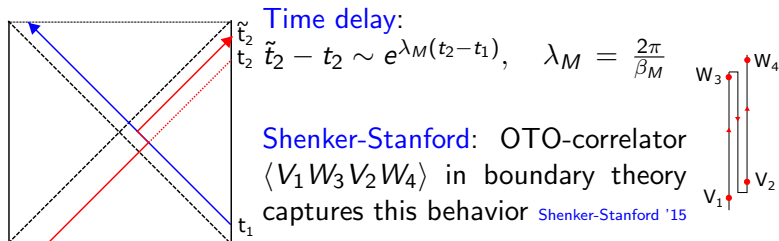
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$\langle V_1 W_3 V_2 W_4 \rangle$, written using shockwaves in the AdS_2 bulk as

$$\int_0^{+\infty} dq_+ \int_0^{+\infty} dp_- \psi_1^*(q_+) \phi_3^*(p_-) \mathcal{S}(p_-, q_+) \psi_2(q_+) \phi_4(p_-)$$

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- ▶ Ψ, Φ = Kruskal wavefunctions = bulk-to-boundary propagators
- ▶ $\mathcal{S} = \exp\left(\frac{i\beta}{4\pi C} p_- q_+\right)$ the Dray-'t Hooft shockwave \mathcal{S} -matrix

Application: Shockwaves from the exact OTO correlator

Large C limit of complete OTO 4-point function, with light ℓ ,
gives full eikonal shockwave expressions

Exact match \Rightarrow Derived full shockwave in semiclassical regime!

Conclusion

Schwarzian QM is relevant as

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Naturally embedded within Liouville theory

Thank you!