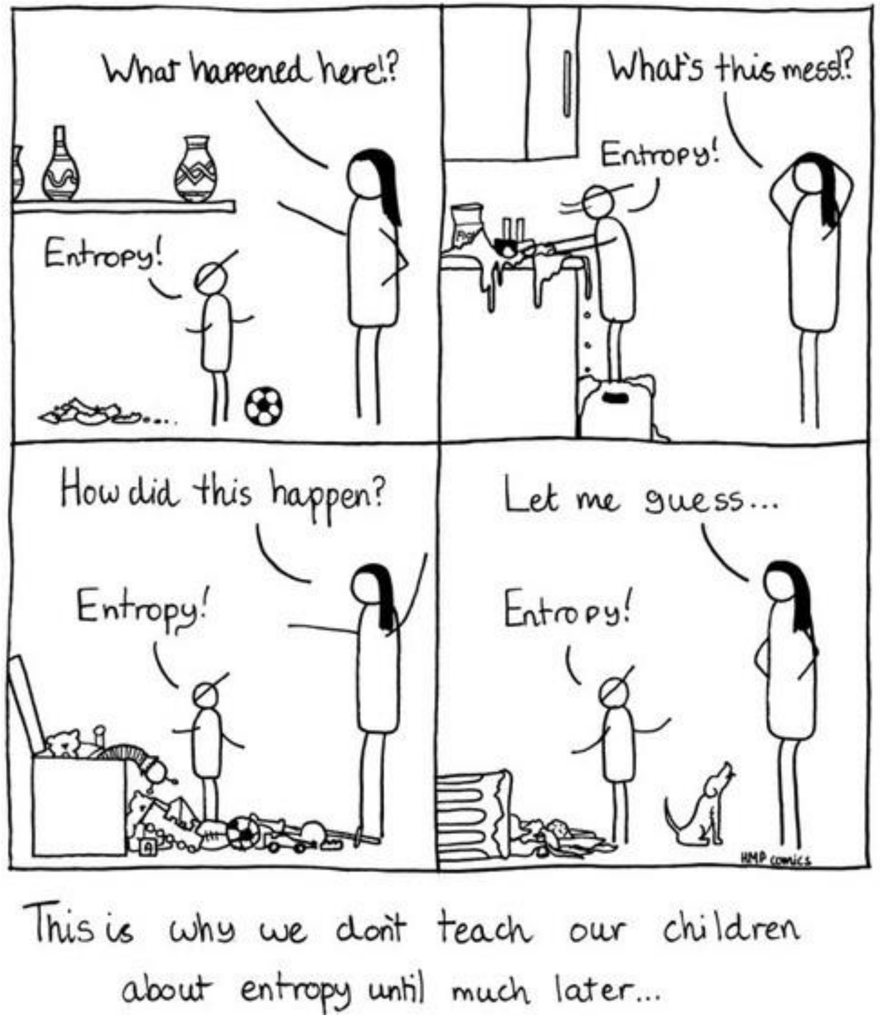
Holographic Second Laws of Black Hole Thermodynamics

Robert Myers
(with Alice Bernamonti, Federico Galli and Jonathan Oppenheim; arXiv:1803.03633)

Second Laws:

- Second law of thermodynamics: entropy of a closed system can never decrease over time



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- Second law of black hole mechanics: horizon area is a nondecreasing function of time in any classical process (assuming the null energy condition)

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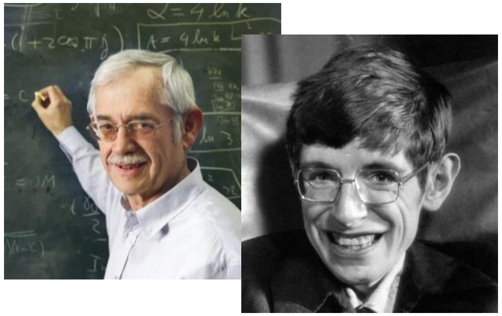
$$S_{BH} = \frac{k_B\, c^3}{\hbar} \frac{A_{horizon}}{4G}$$



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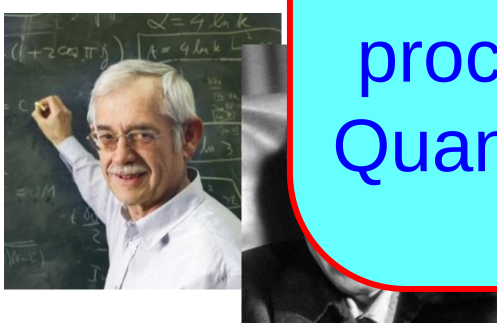
Is there more? Yes!

Quantum thermodyanmics provides additional constraints on thermal processes, which are relevant for Quantum Field Theory and Gravity



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(Brandao, Horodecki, Ng, Oppenheim & Wehner)

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- Distance measure: a measure of how distinguishable two states are; should satisfy

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with $\Lambda(\rho)$, a completely positive trace preserving map

(examples: time evolution or partial trace)

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Where is the thermodynamics??

- $\Lambda_\beta(\rho) =$ couple ρ to thermal bath with temperature $1/\beta$; evolve for some time and trace out thermal bath
- $\sigma = \rho_\beta$, thermal state to which ρ will equilibriate

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- example: relative entropy

$$S(\rho\,|\sigma) = \mathrm{tr}(\rho\,\log\rho) - \mathrm{tr}(\rho\,\log\sigma)$$

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recover standard Second Law!

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Second Laws:

→ $$D(\Lambda_\beta(\rho)\|\rho_\beta) \leq D(\rho\|\rho_\beta)$$

- Renyi divergences:

$$D_\alpha(\rho\|\rho_\beta) \equiv \frac{1}{\alpha - 1} \log \mathrm{tr}\left(\rho^\alpha \rho_\beta^{1-\alpha}\right)$$

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- generalize relative entropy in the same sense that Renyi entropies generalize entanglement entropy

$$\lim_{\alpha\to 1} D_\alpha(\rho\|\rho_\beta) = S(\rho\,|\rho_\beta) = \mathrm{tr}(\rho\,\log\rho) - \mathrm{tr}(\rho\,\log\rho_\beta)$$

- when ρ and ρ_β commute, D_α with $\alpha > 0$ are necessary and sufficient constraints for allowed transitions
- for noncommuting ρ and ρ_β, D_α with $0 \le \alpha \le 2$ provide necessary constraints

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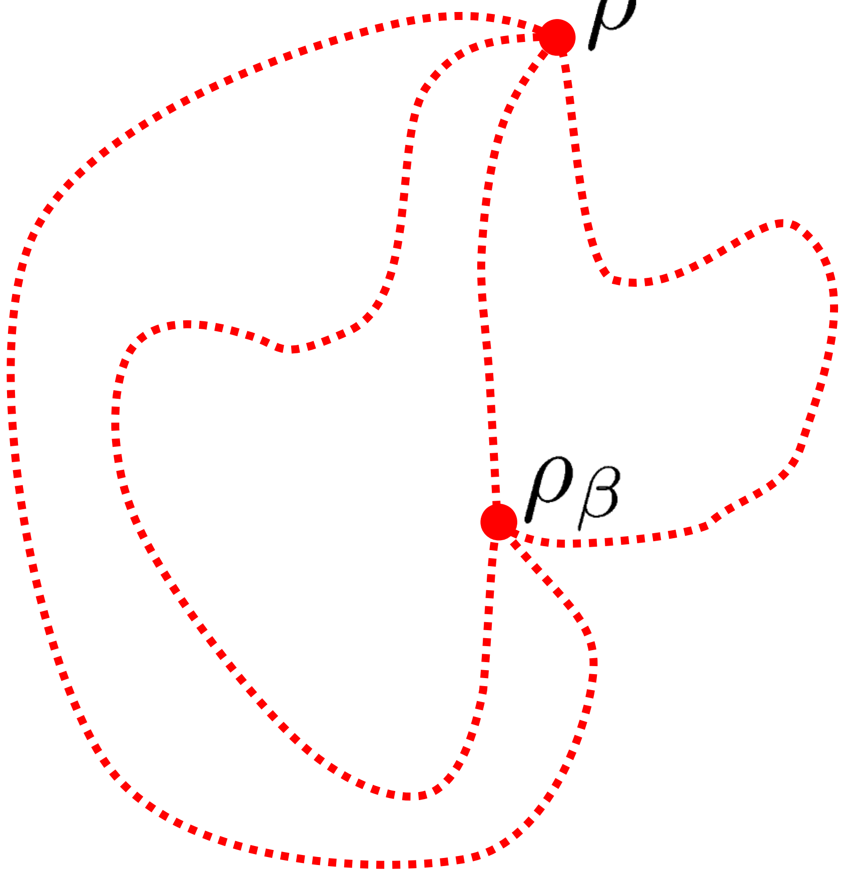
- distance function D orders the states and constrains the allowed transitions, ie, the path towards thermal equilibrium



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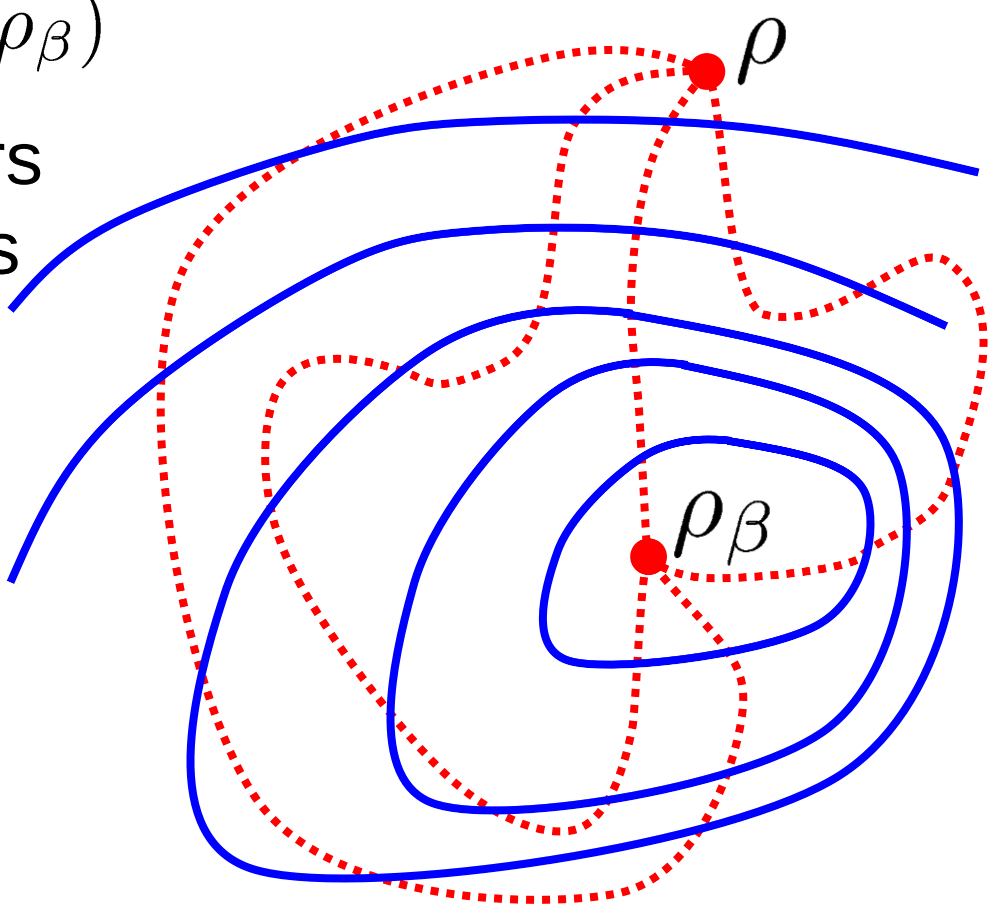

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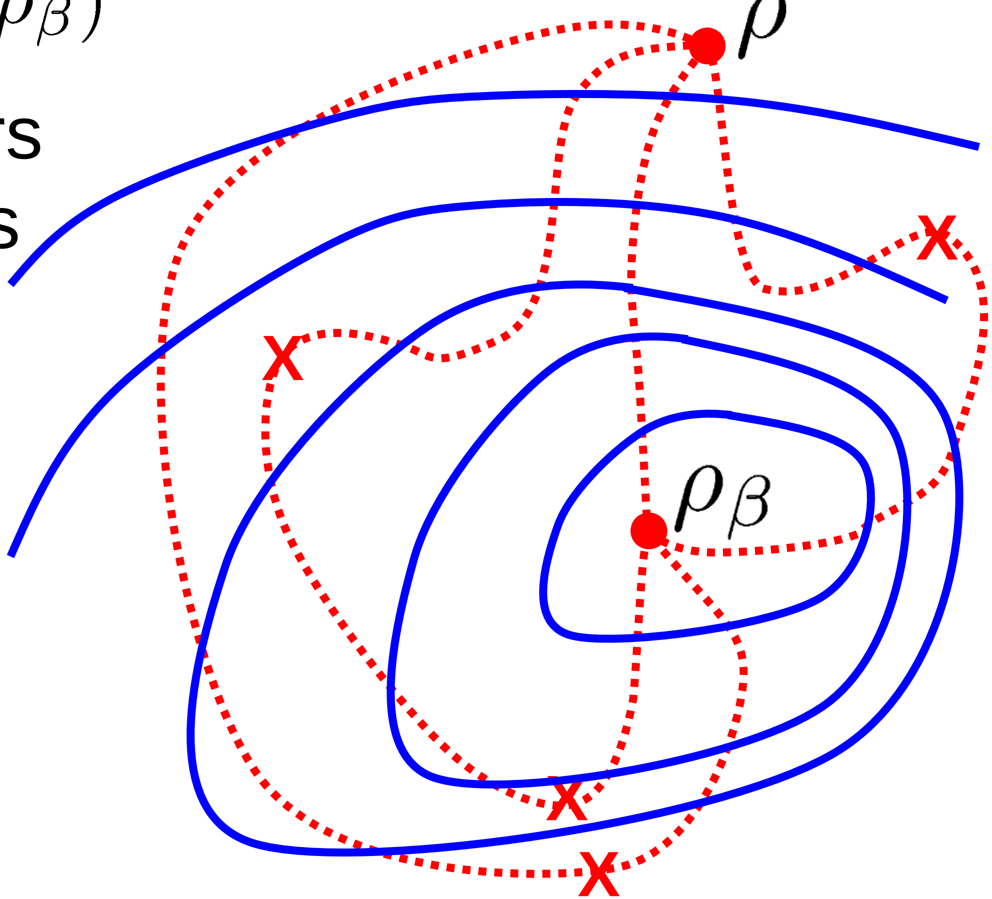

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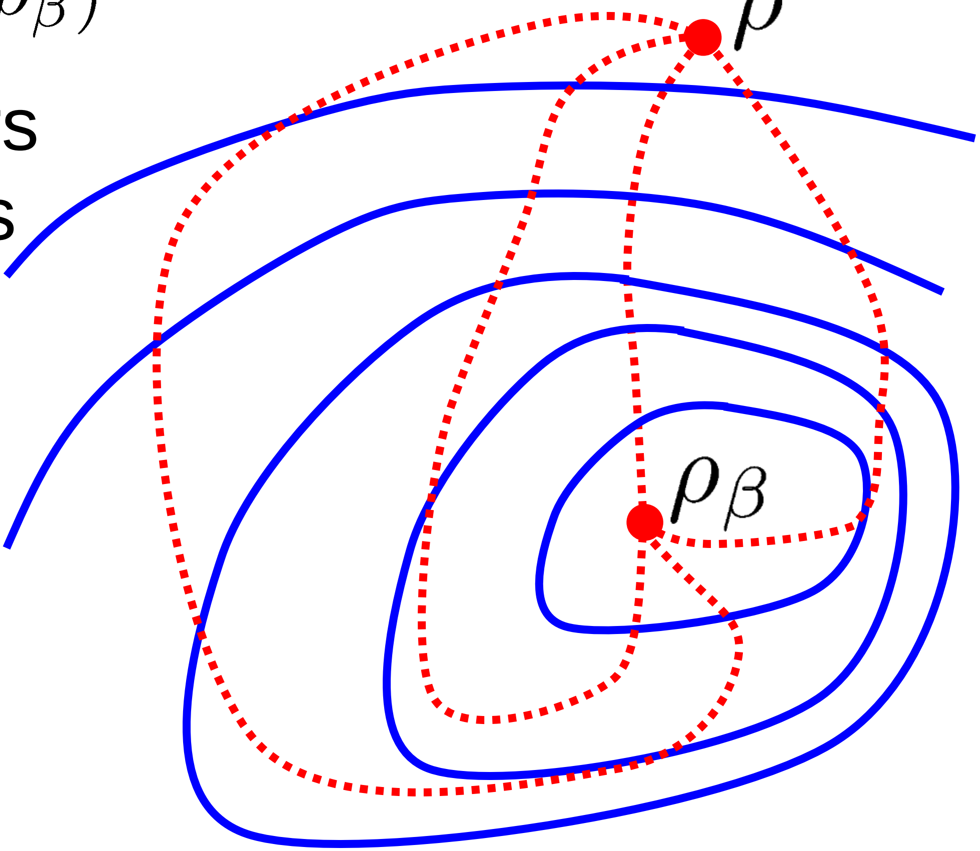

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- Renyi divergences give one-param family of constraints

$$D_\alpha(\Lambda_\beta(\rho)\|\rho_\beta) \leq D_\alpha(\rho\|\rho_\beta)$$

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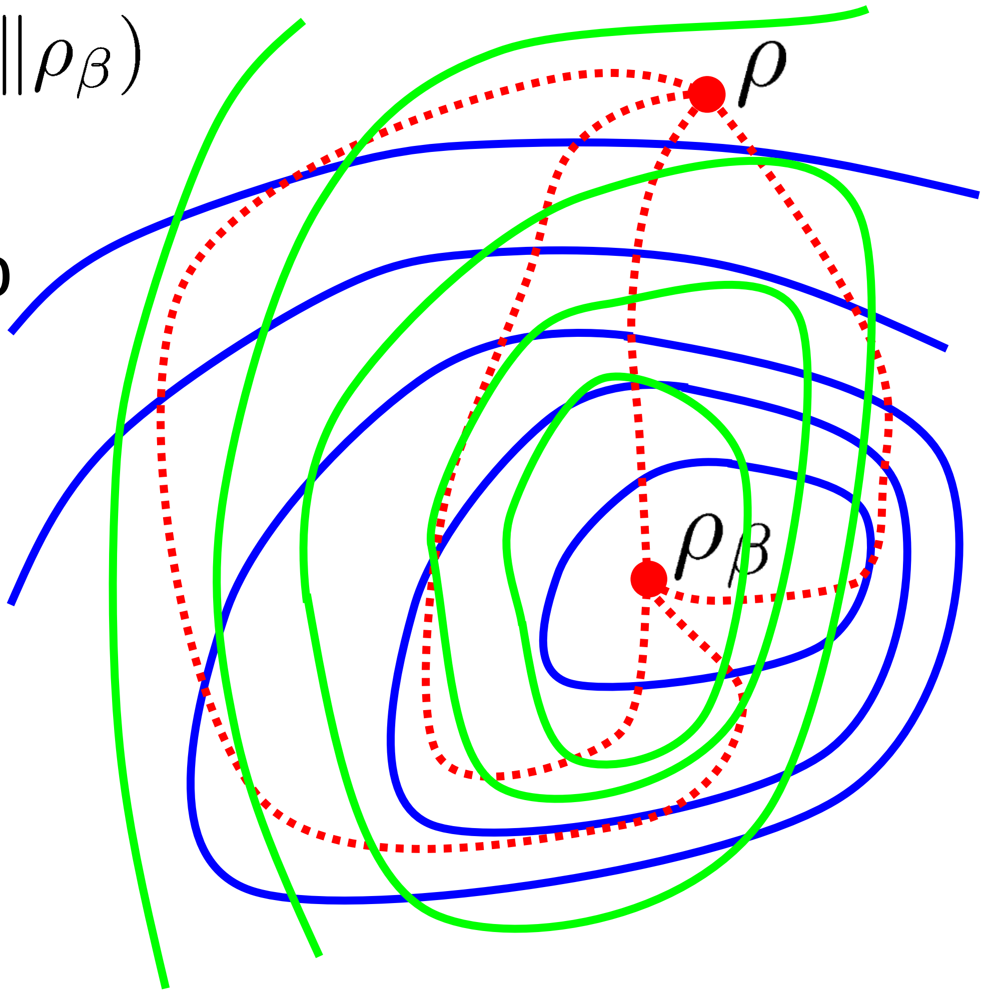

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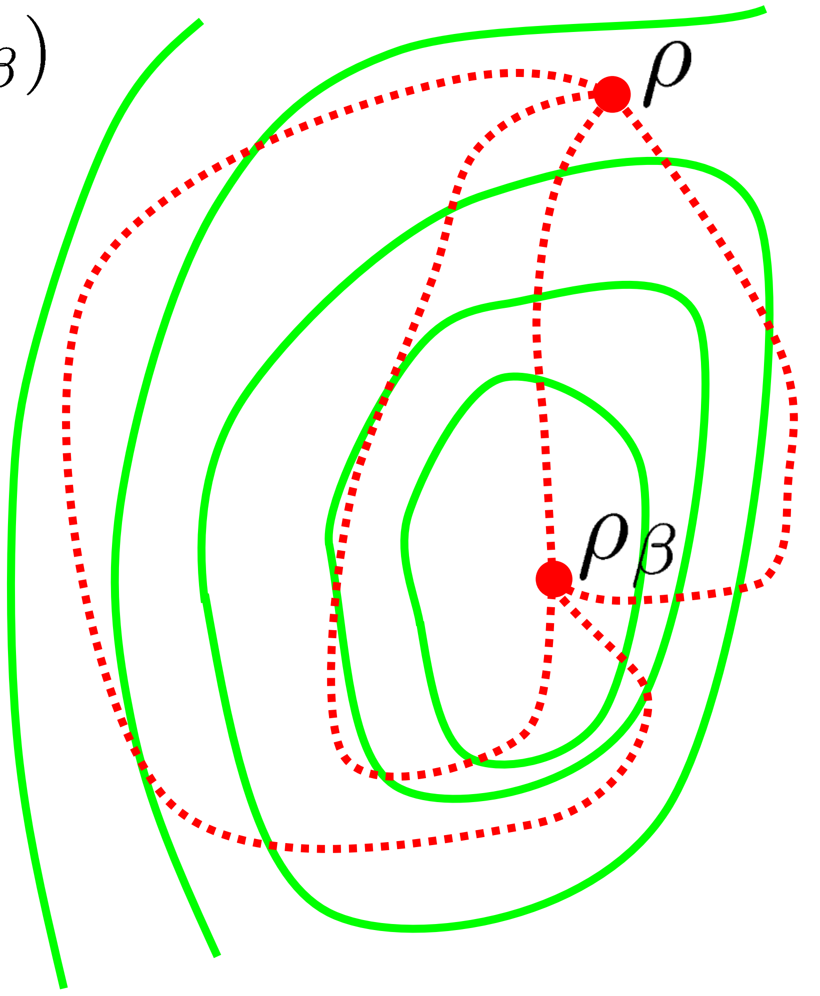

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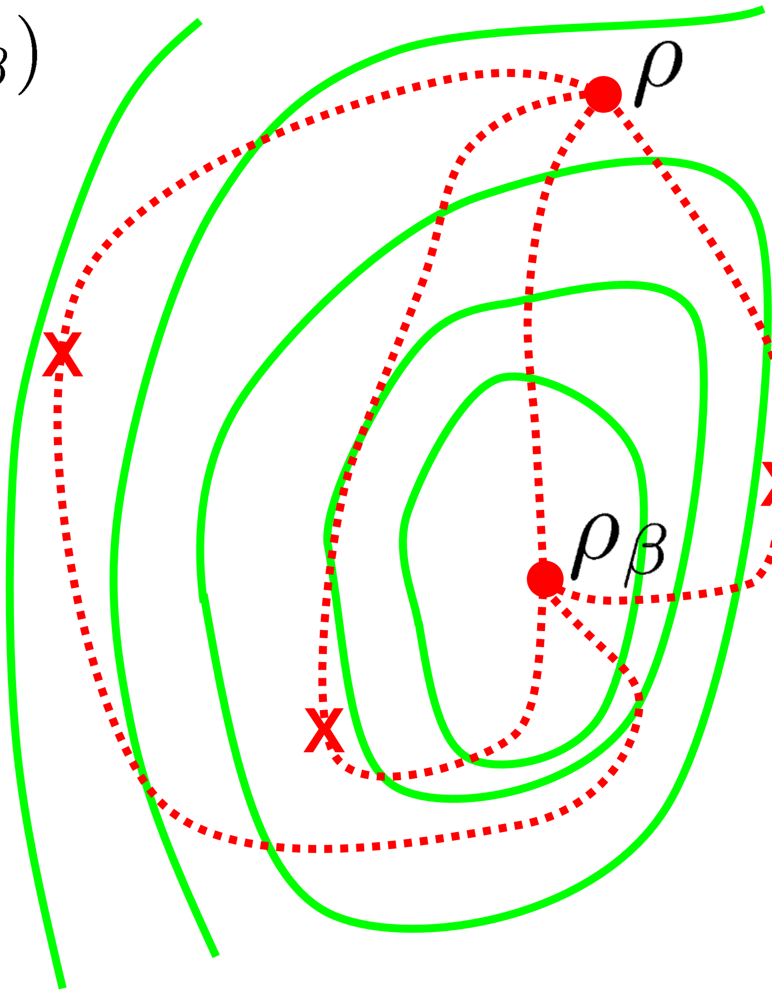

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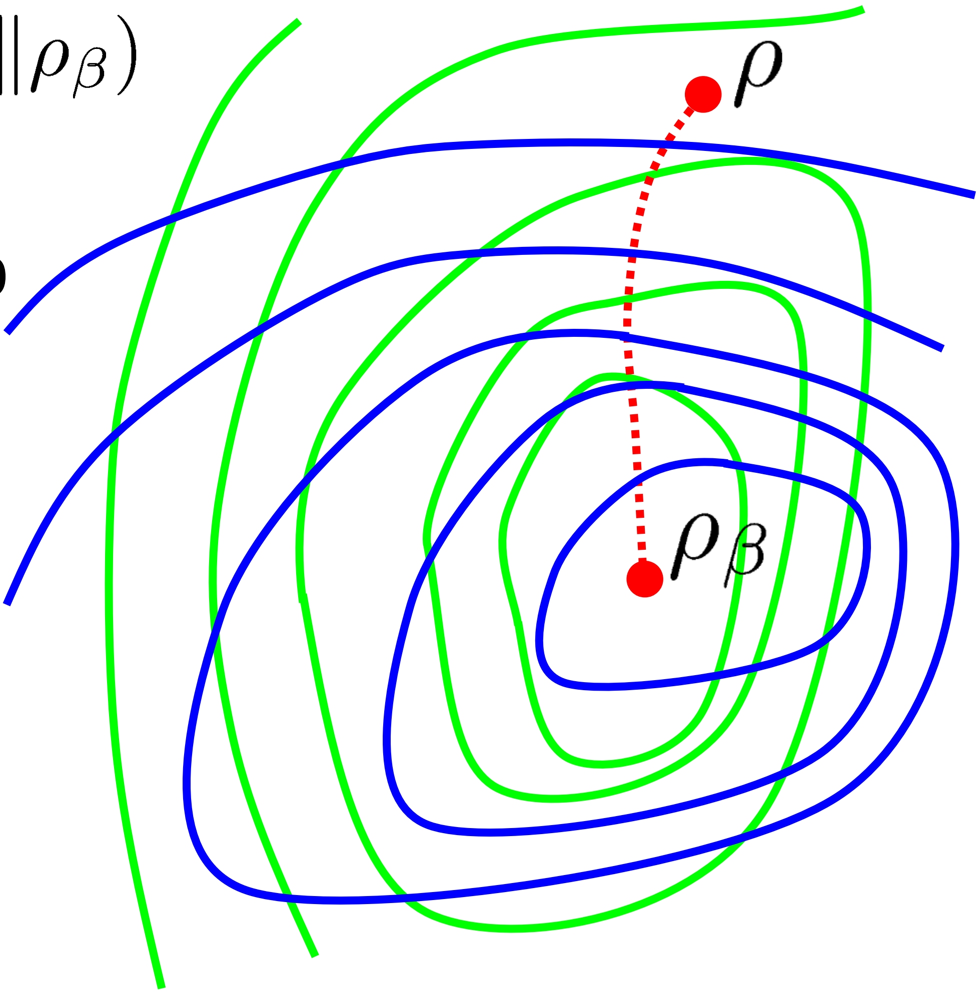

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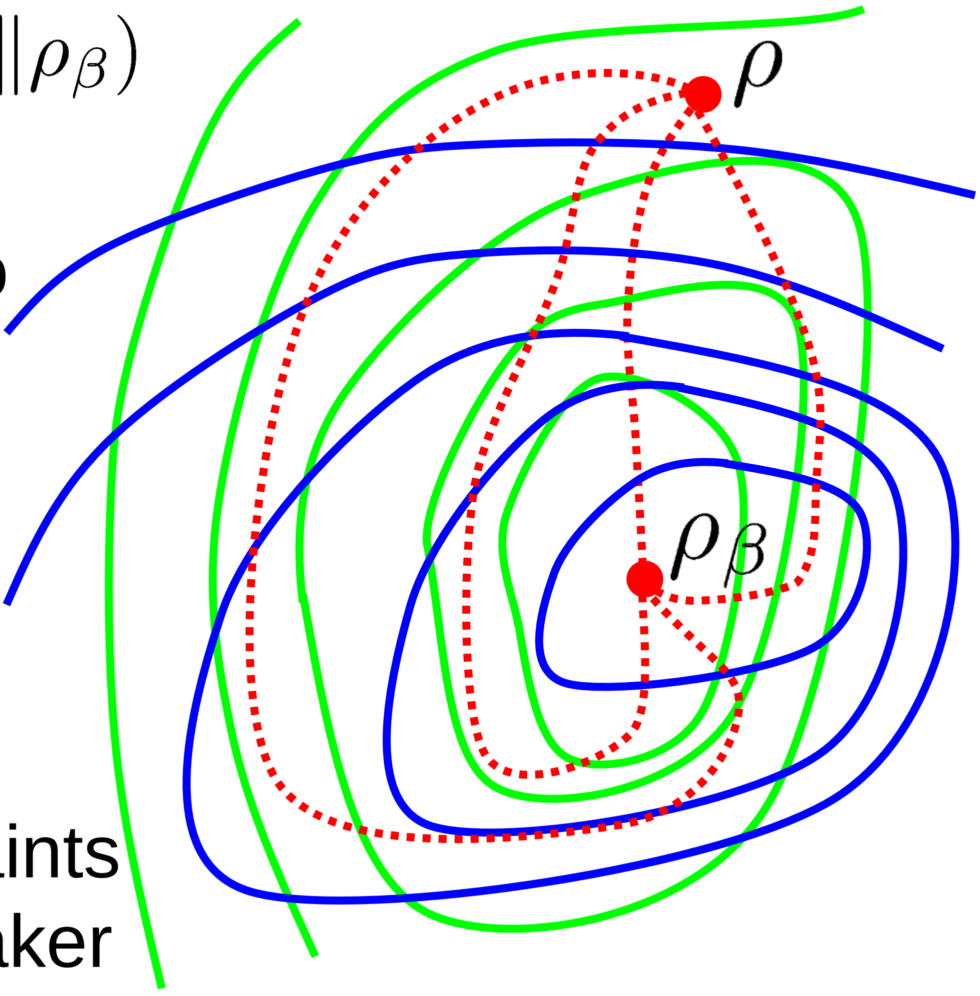

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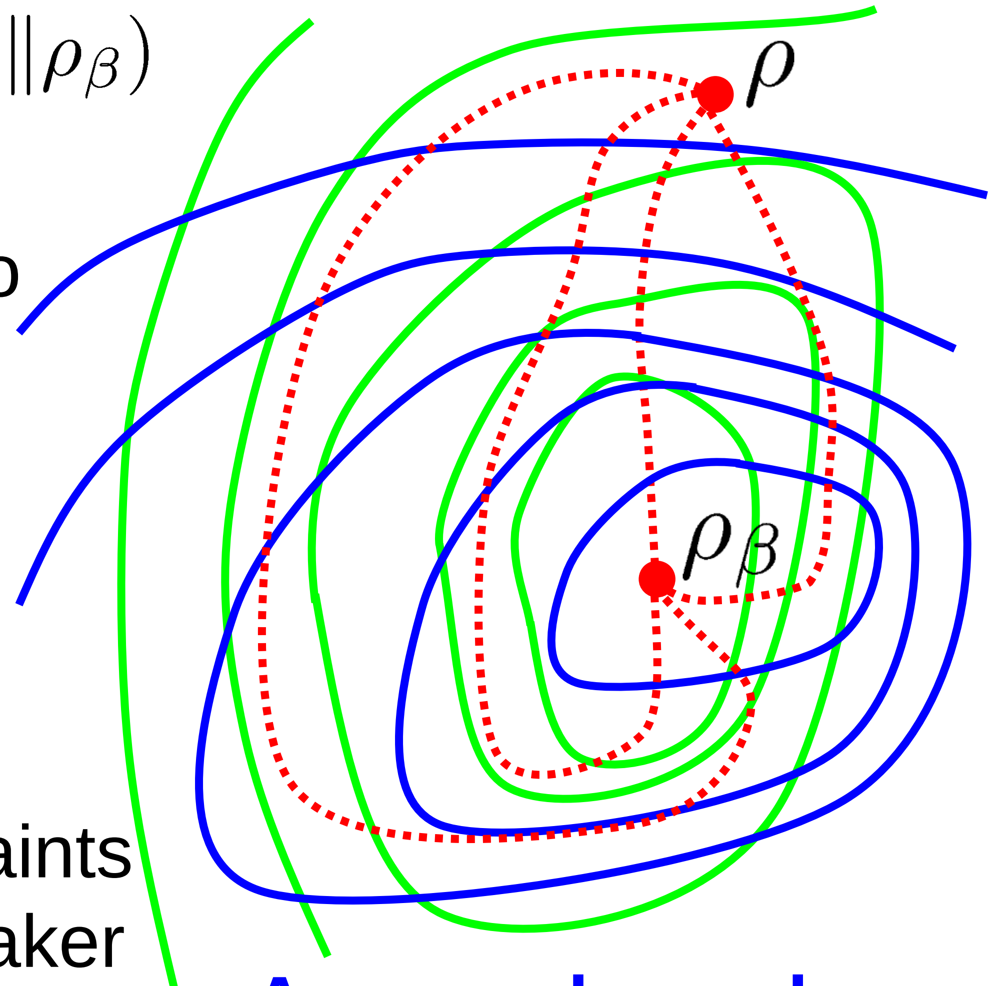

Answer depends on details of system

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$\longrightarrow$ AdS/CFT correspondence

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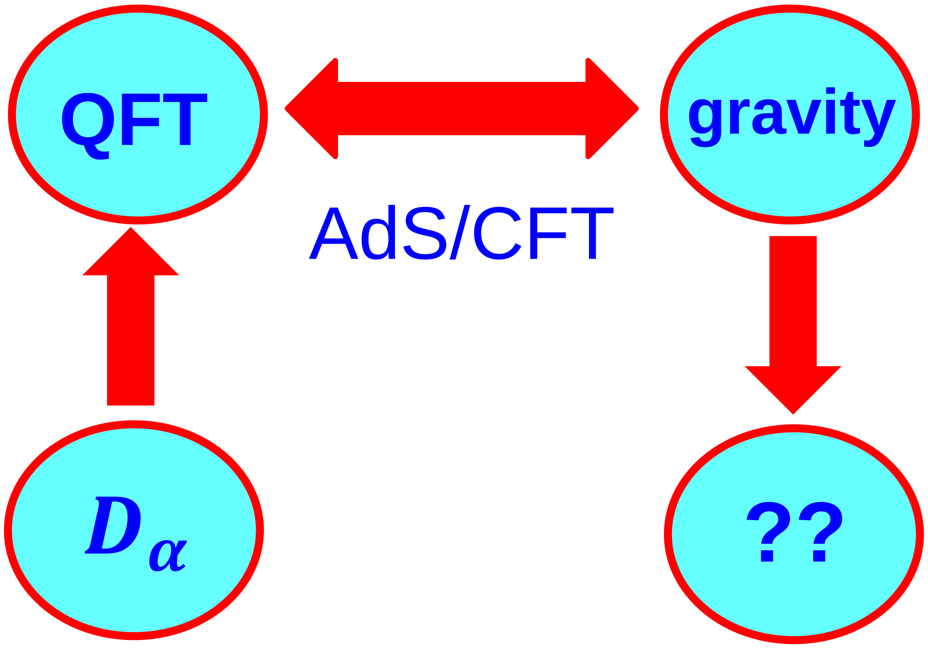

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- recall

$$\rho_\beta = e^{-\beta\, H_{CFT}} =$$

- consider excited state:

$$\rho = e^{-\beta\, \tilde{H}} =$$

where:

$$\tilde{H} = H_{CFT} + \lambda \int d^{d-1}x\, \mathcal{O}_\Delta$$

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$$Z_{\mathrm{CFT}} = \mathrm{tr}\left(\rho^{\alpha}\rho_{\beta}^{1-\alpha}\right) = \mathrm{tr}\left(e^{-\alpha\beta\tilde{H}}\, e^{-(1-\alpha)\beta H_{CFT}}\right)$$

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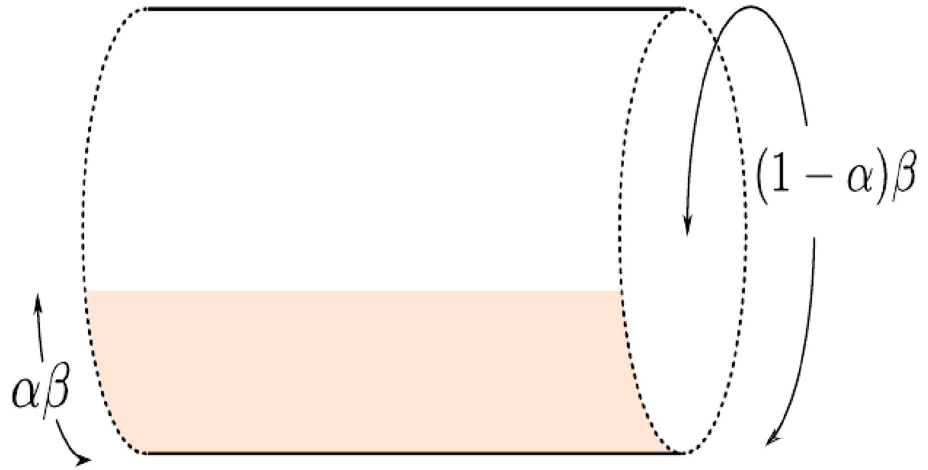

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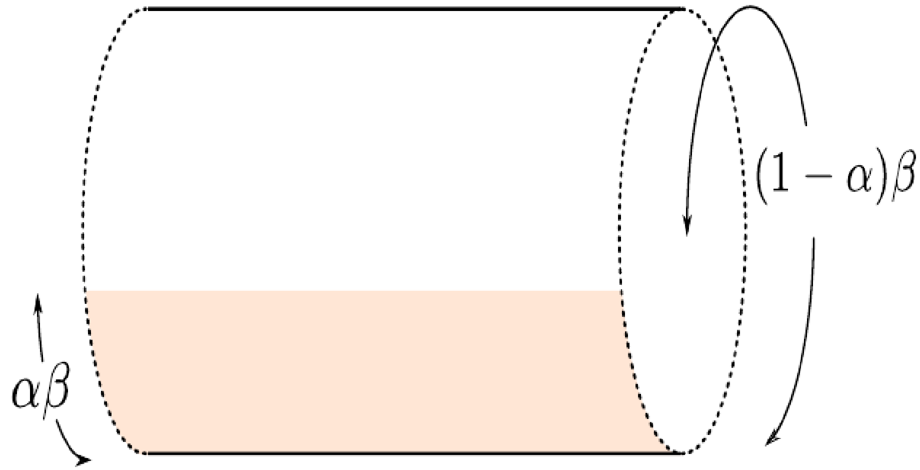

⟶ "Euclidean quantum quench"

$$\lambda(\tau) = [\theta(\tau) - \theta(\tau - \alpha\beta)]\,\lambda$$

- note: $0 \leq \alpha \leq 1$

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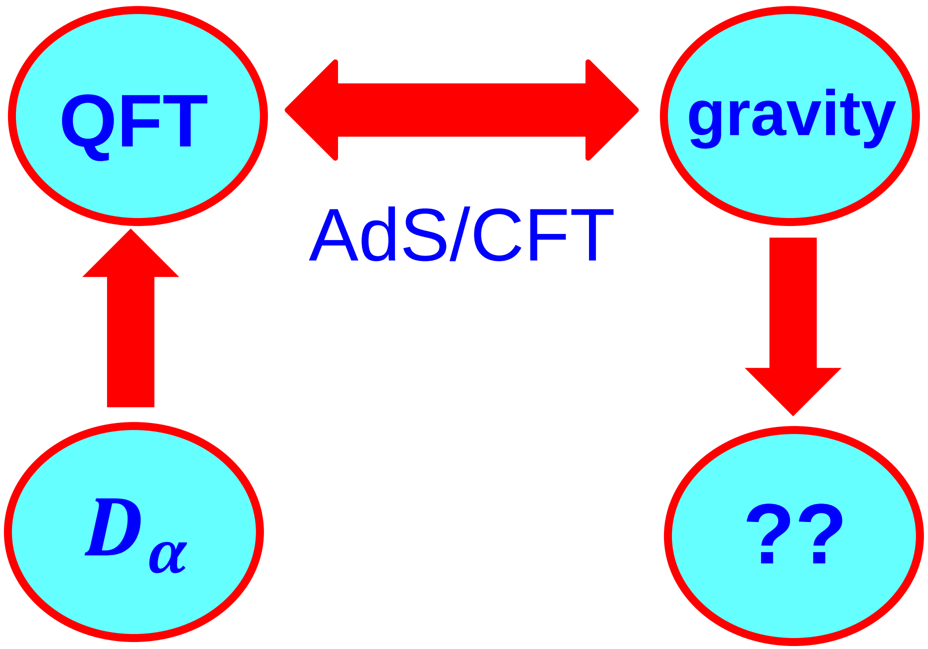
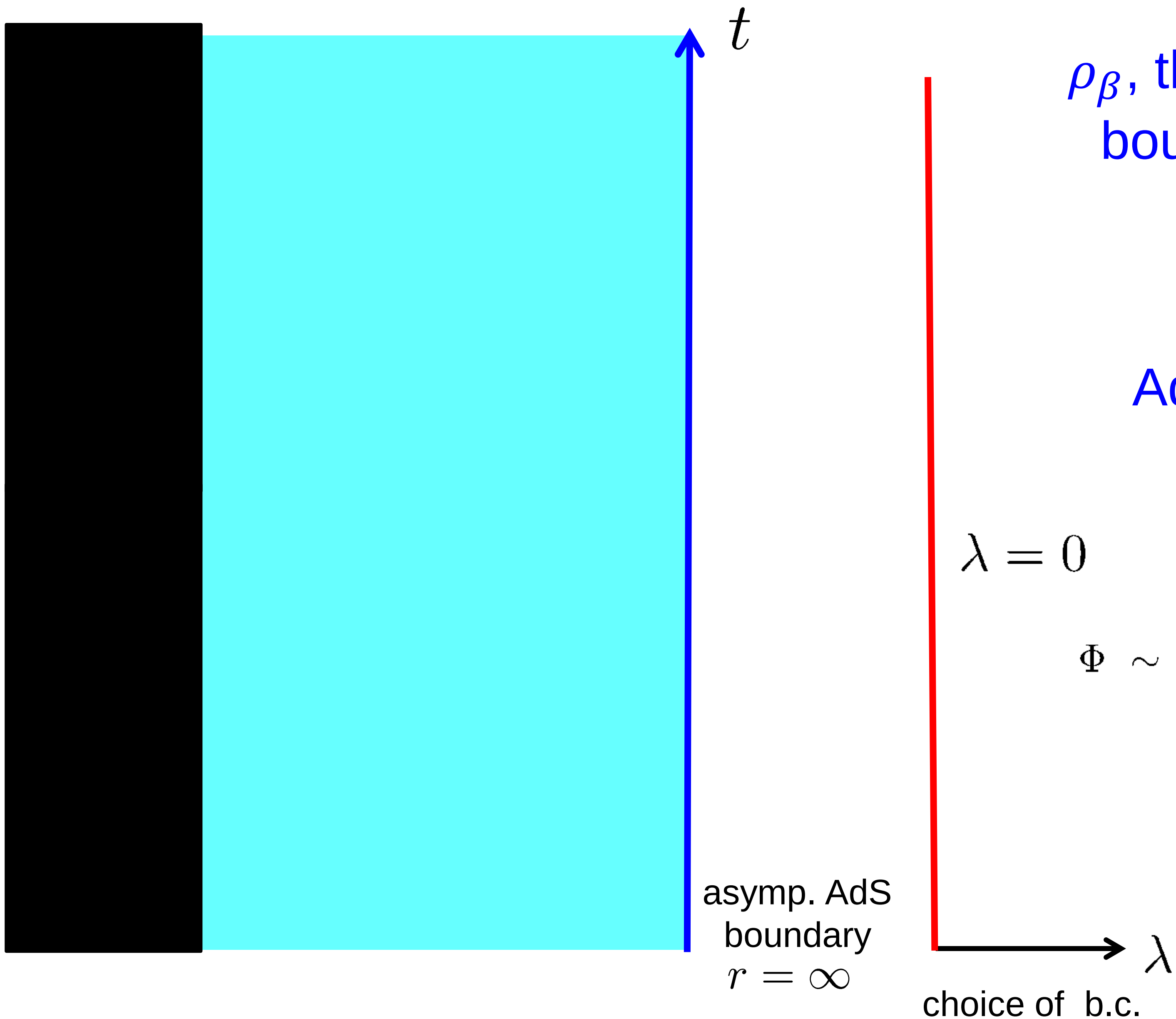


Holographic translation:



ρ_β, thermal state in boundary theory

AdS black hole

$$\Phi \sim \frac{\lambda}{r^{d-\Delta}} + \frac{\langle \mathcal{O}_\Delta \rangle}{r^\Delta} + \cdots$$

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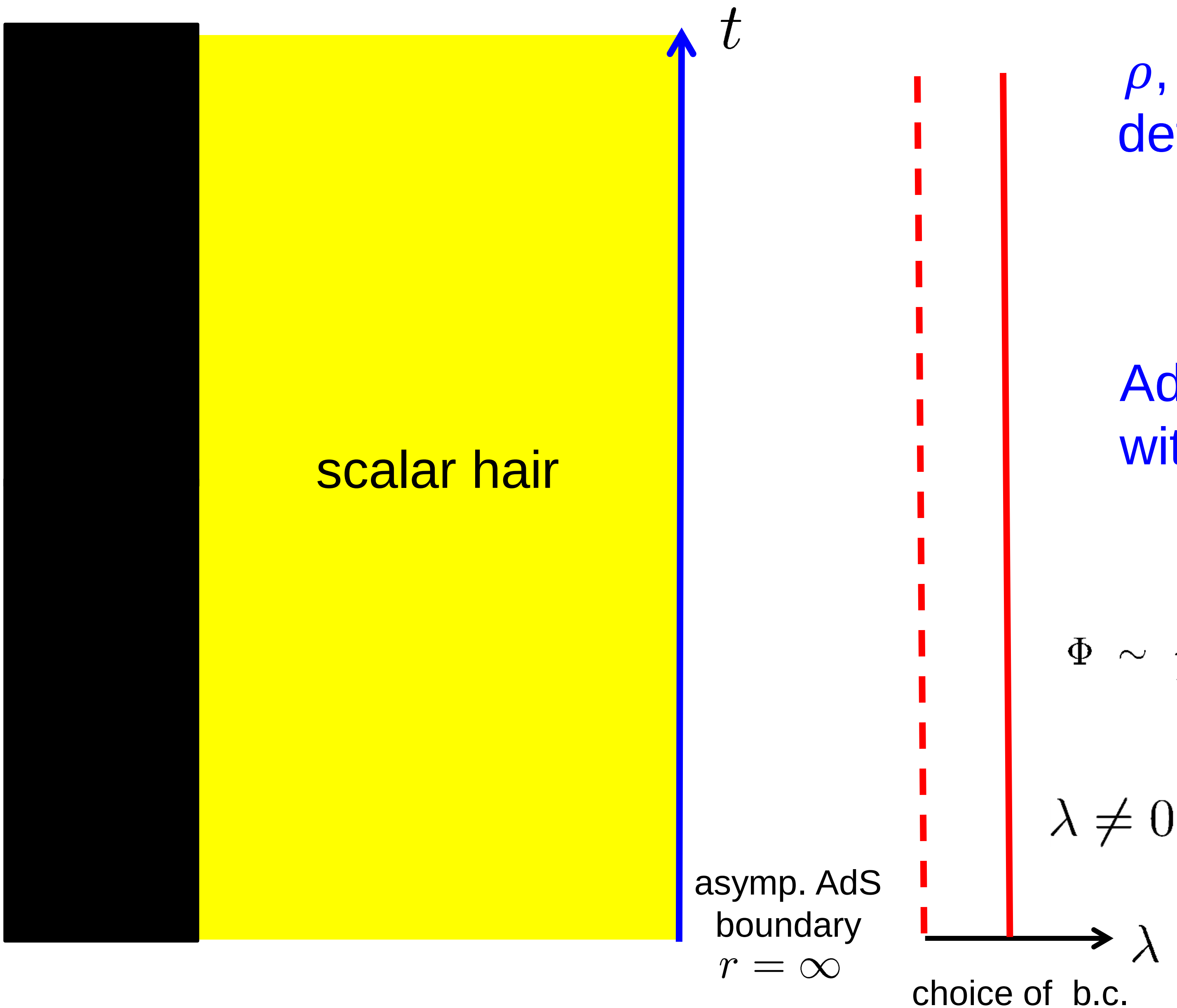

ρ, excited state deformed by O_Δ

AdS black hole with scalar hair

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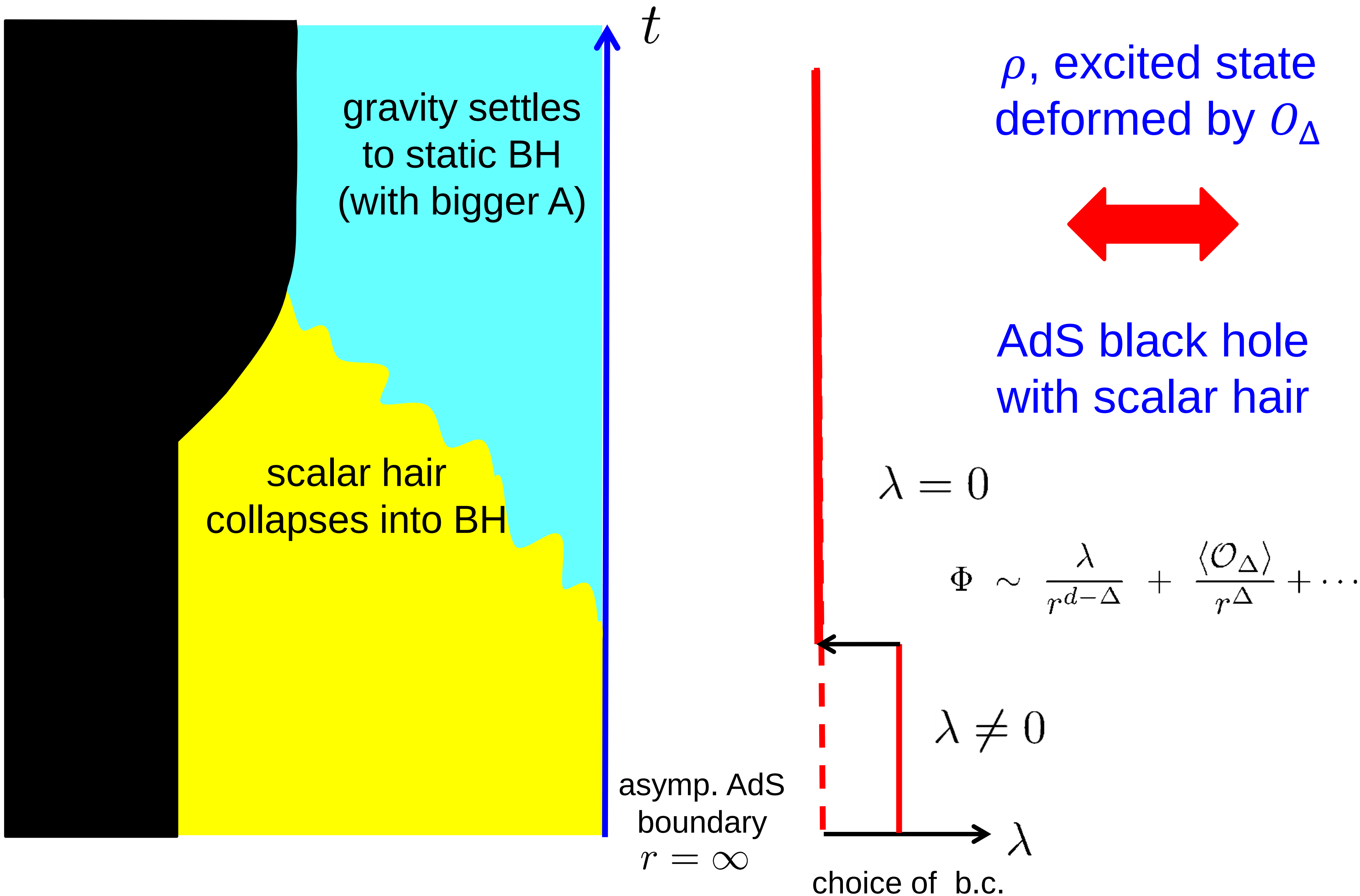

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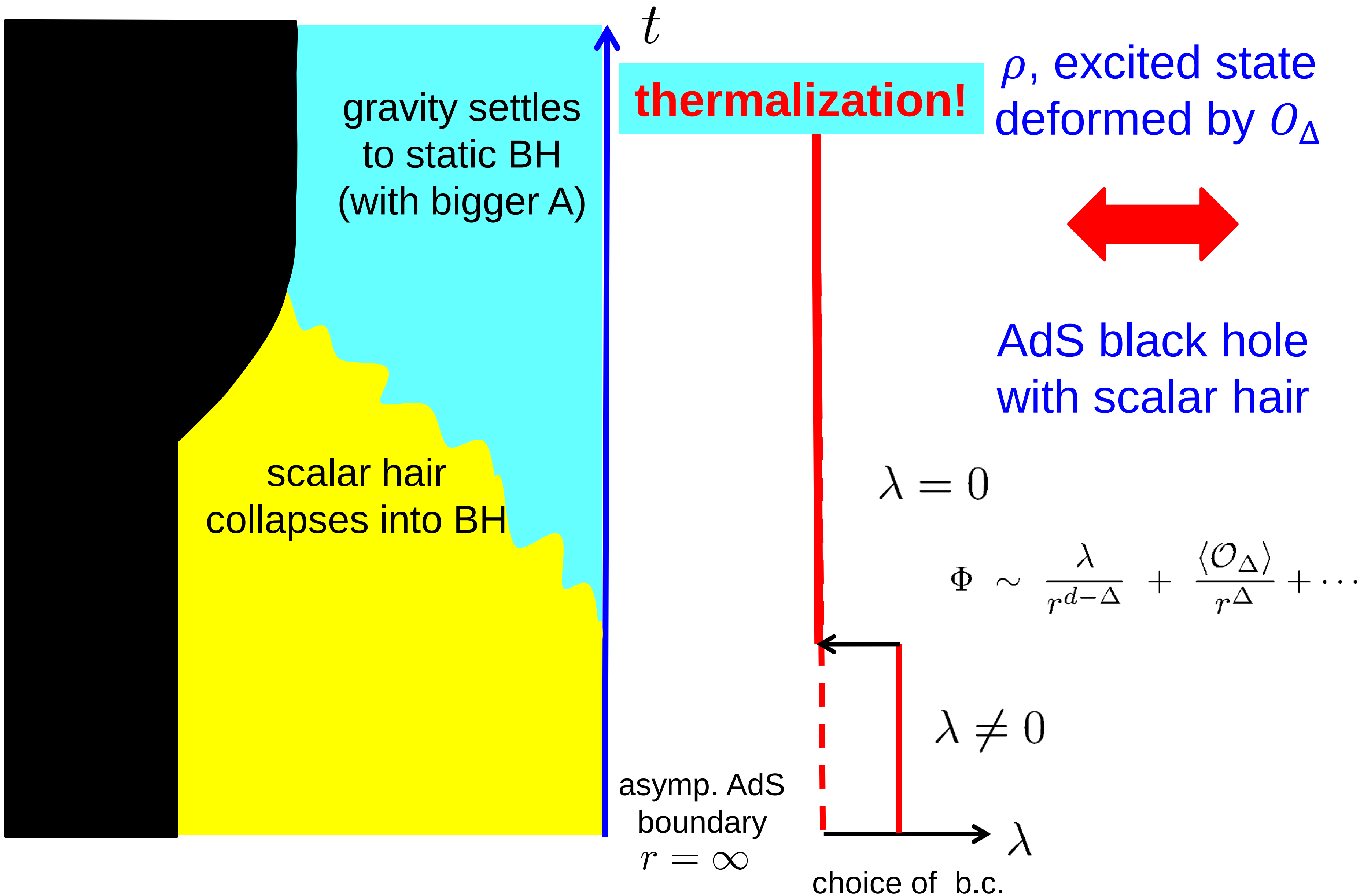

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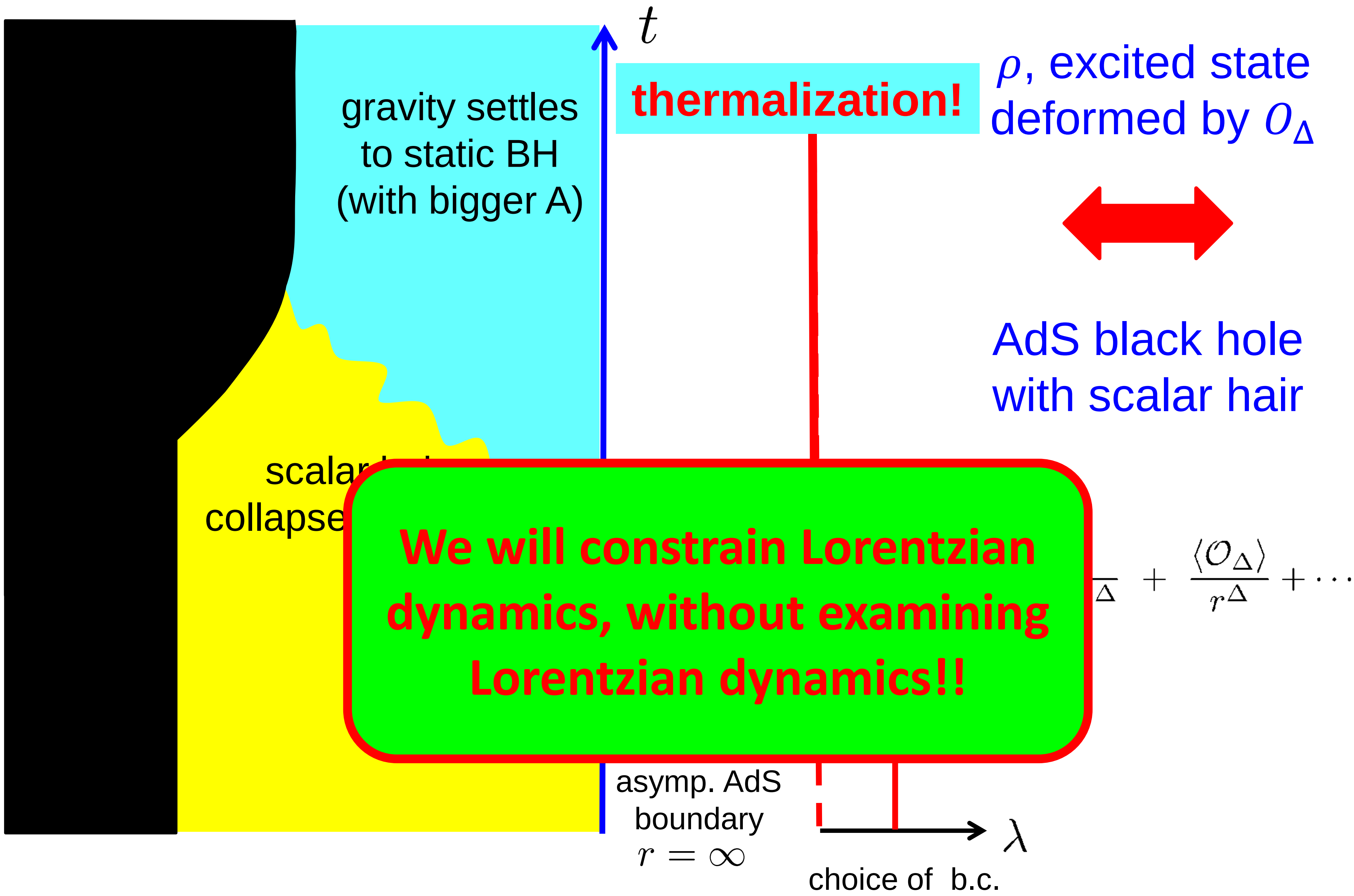
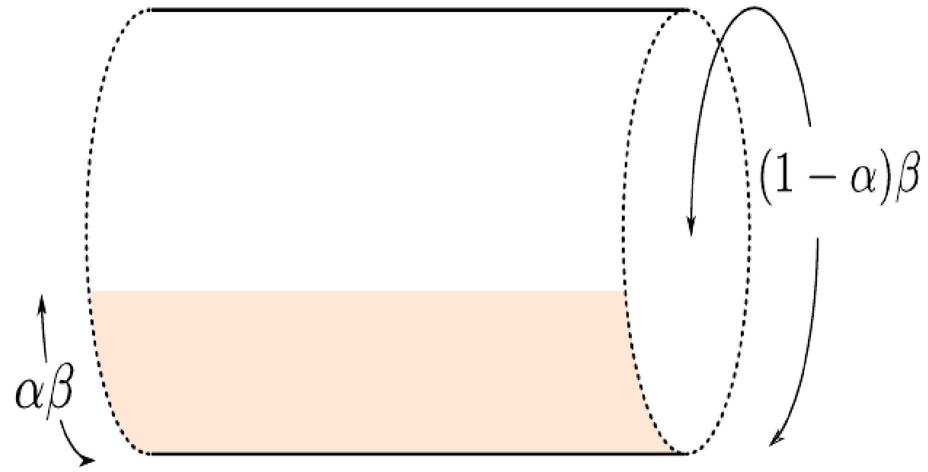


We will constrain Lorentzian dynamics, without examining Lorentzian dynamics!!

Holographic translation of Renyi divergence:

$$Z_{CFT} = \mathrm{tr}\left(\rho^{\alpha}\rho_{\beta}^{1-\alpha}\right) \simeq$$



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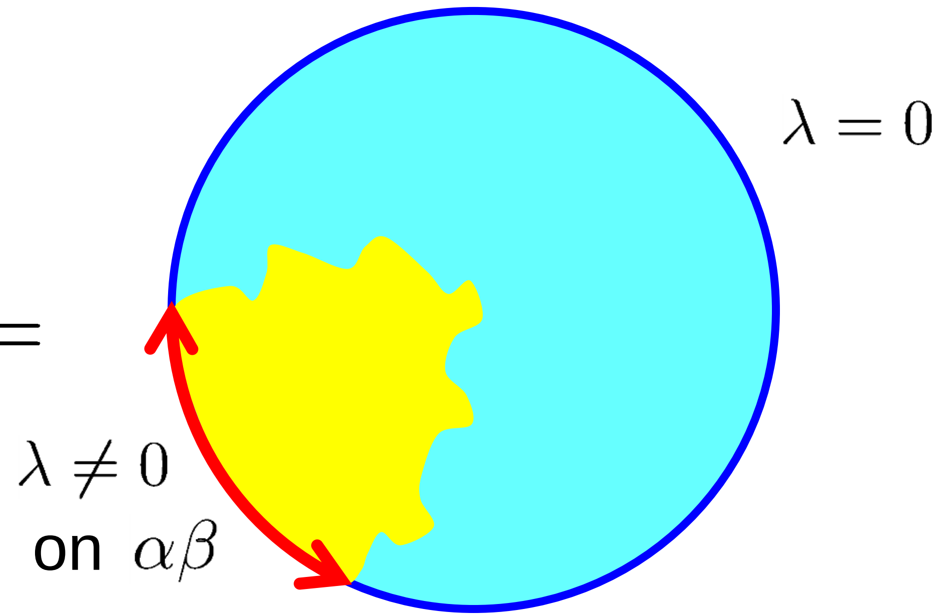
Euclidean (boundary) time: $0 \leq \tau \leq \beta$ $=$



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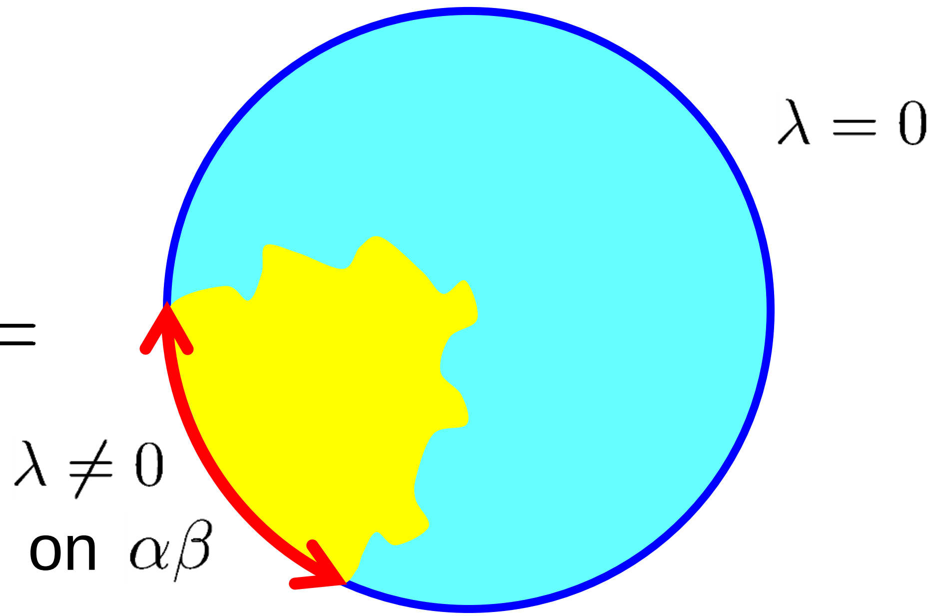

- calculate perturbatively in amplitude of scalar $\lambda/\beta^{\Delta-2}$
  $\longrightarrow$ solve linearized scalar eom in fixed BH bkgd
- only consider $\mathrm{d} = 2$ (ie, Euclidean BTZ black hole)

Final result of holographic calculation:

$$D_\alpha(\rho\|\rho_\beta) = \frac{1}{\alpha-1}\log\frac{tr\left(\rho^\alpha\rho_\beta^{1-\alpha}\right)}{(tr\rho)^\alpha(tr\rho_\beta)^{1-\alpha}}$$

$$\approx \lambda^2\left(\frac{2\pi}{\beta}\right)^{2(\Delta-2)} \frac{c\,L}{6\pi\beta}\,\frac{(\Delta-1)^2}{2^{\Delta+3}}\,\frac{I(\alpha,\Delta)-\alpha I(1,\Delta)}{\alpha-1}$$

where

$$I(\alpha,\Delta) = \frac{2^{2-\Delta}\sqrt{\pi}\Gamma(\Delta)}{\Gamma\left(\Delta+\frac{1}{2}\right)}\int_0^{2\pi\alpha} dp\,(2\pi\alpha-p)\,F\left[\Delta,\Delta,\Delta+\frac{1}{2},\frac{1+\sqrt{1-\tilde{\epsilon}^2}\cos p}{2}\right]$$

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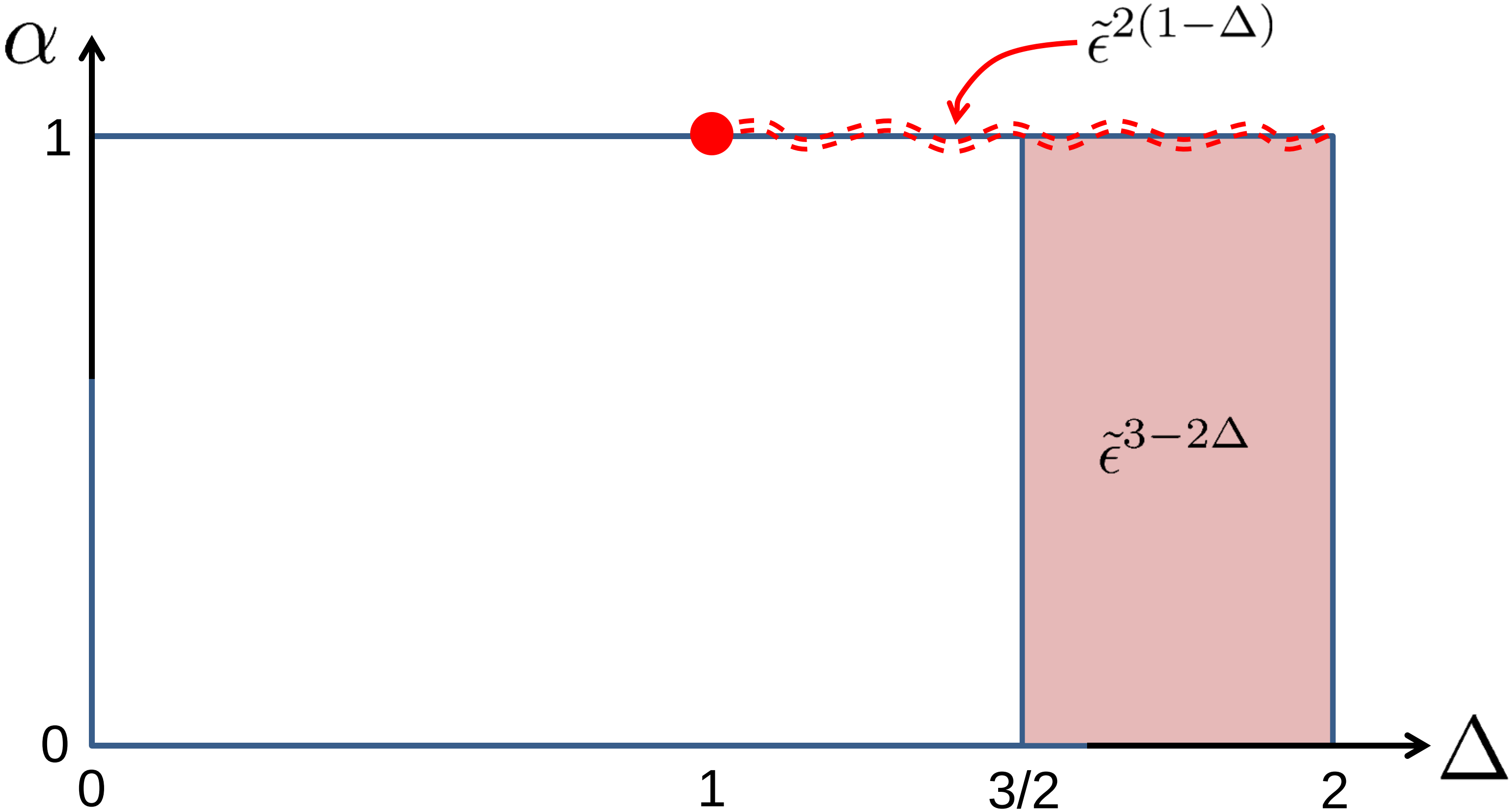

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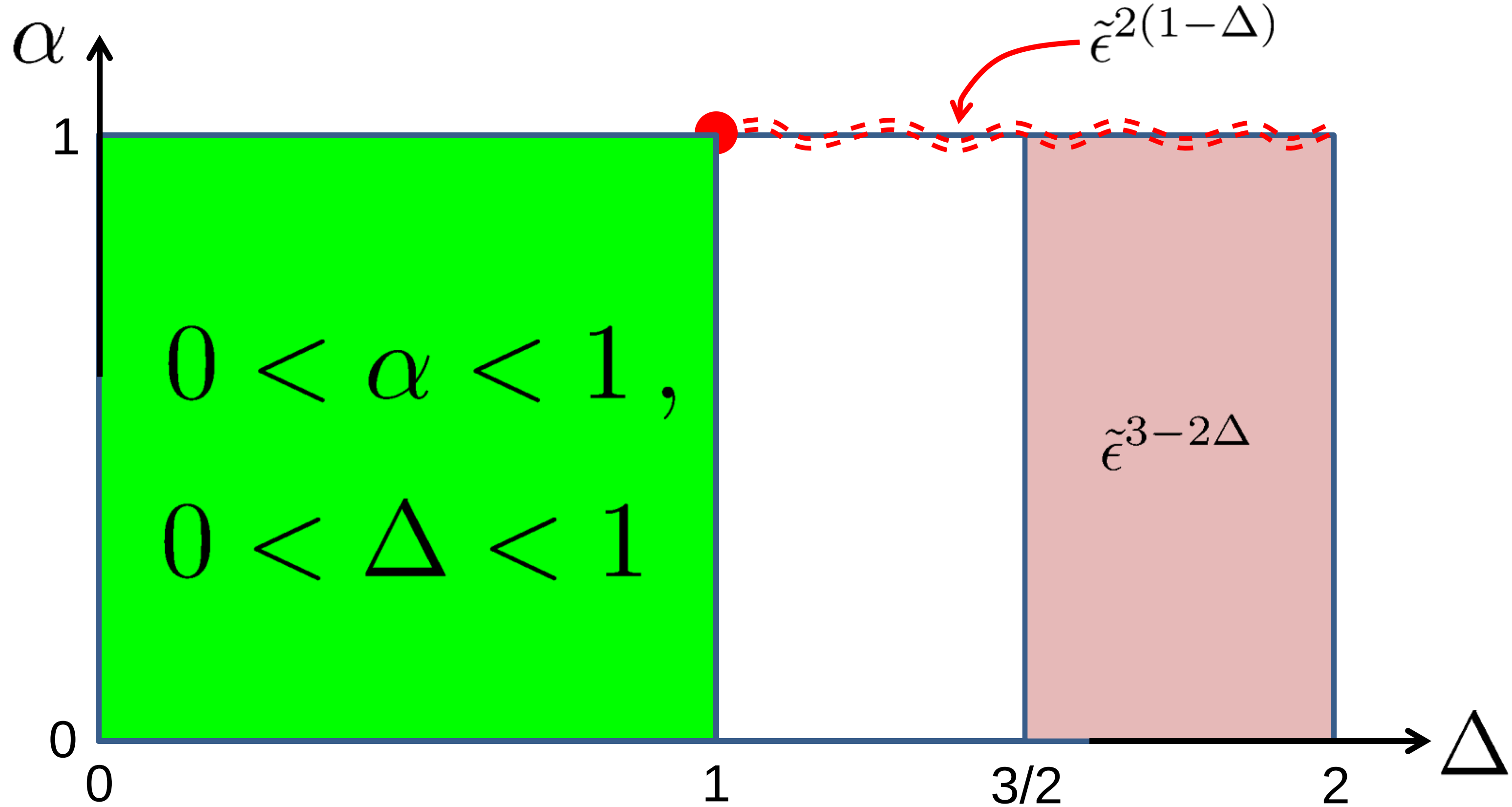
$$D_\alpha(\rho\|\rho_\beta) \;\approx\; \lambda^2 \left(\frac{2\pi}{\beta}\right)^{2(\Delta-2)} \frac{c\,L}{6\pi\beta}\,\frac{(\Delta-1)^2}{2^{\Delta+3}}\,\frac{I(\alpha,\Delta)-\alpha I(1,\Delta)}{\alpha-1}$$



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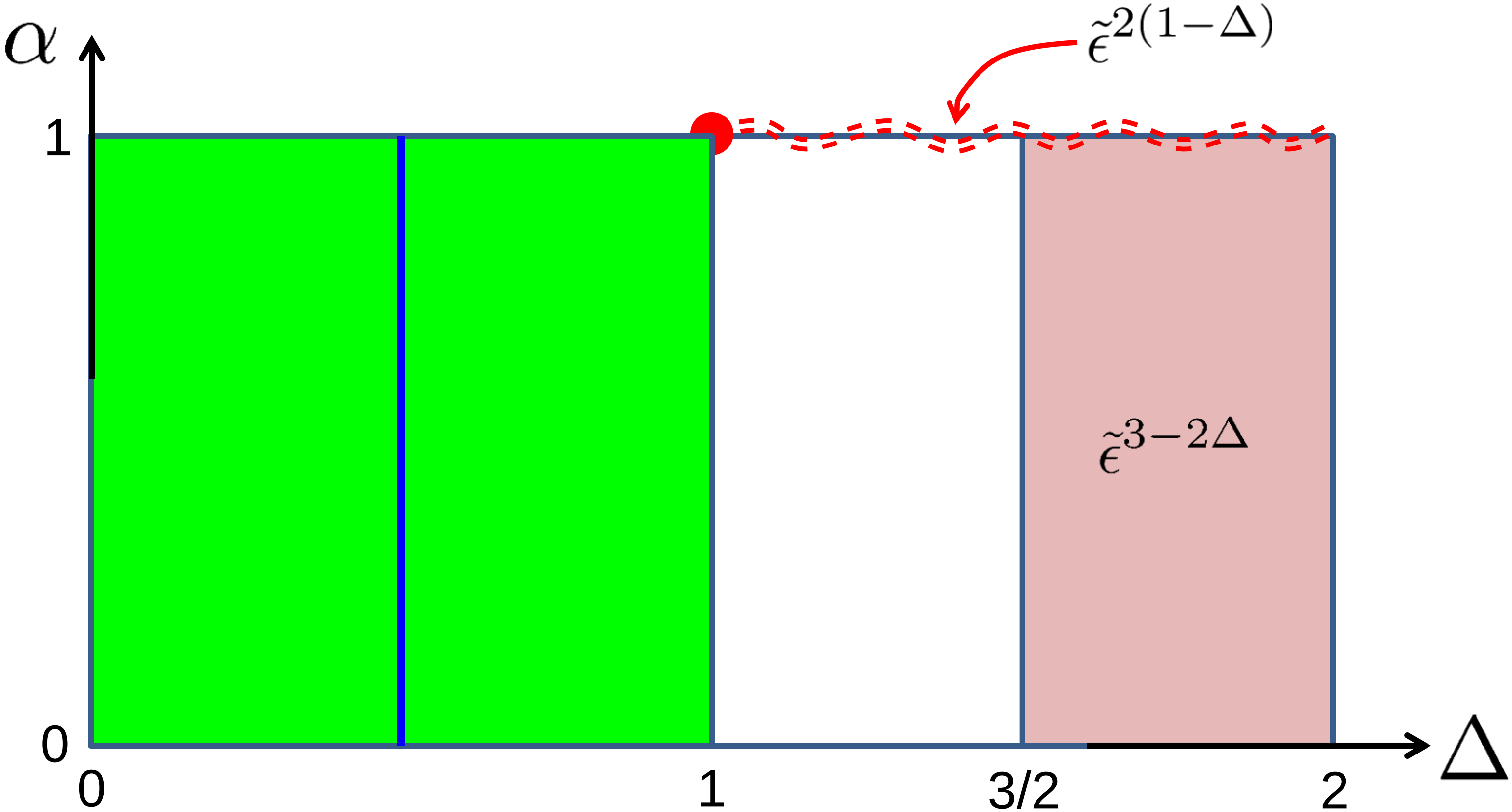

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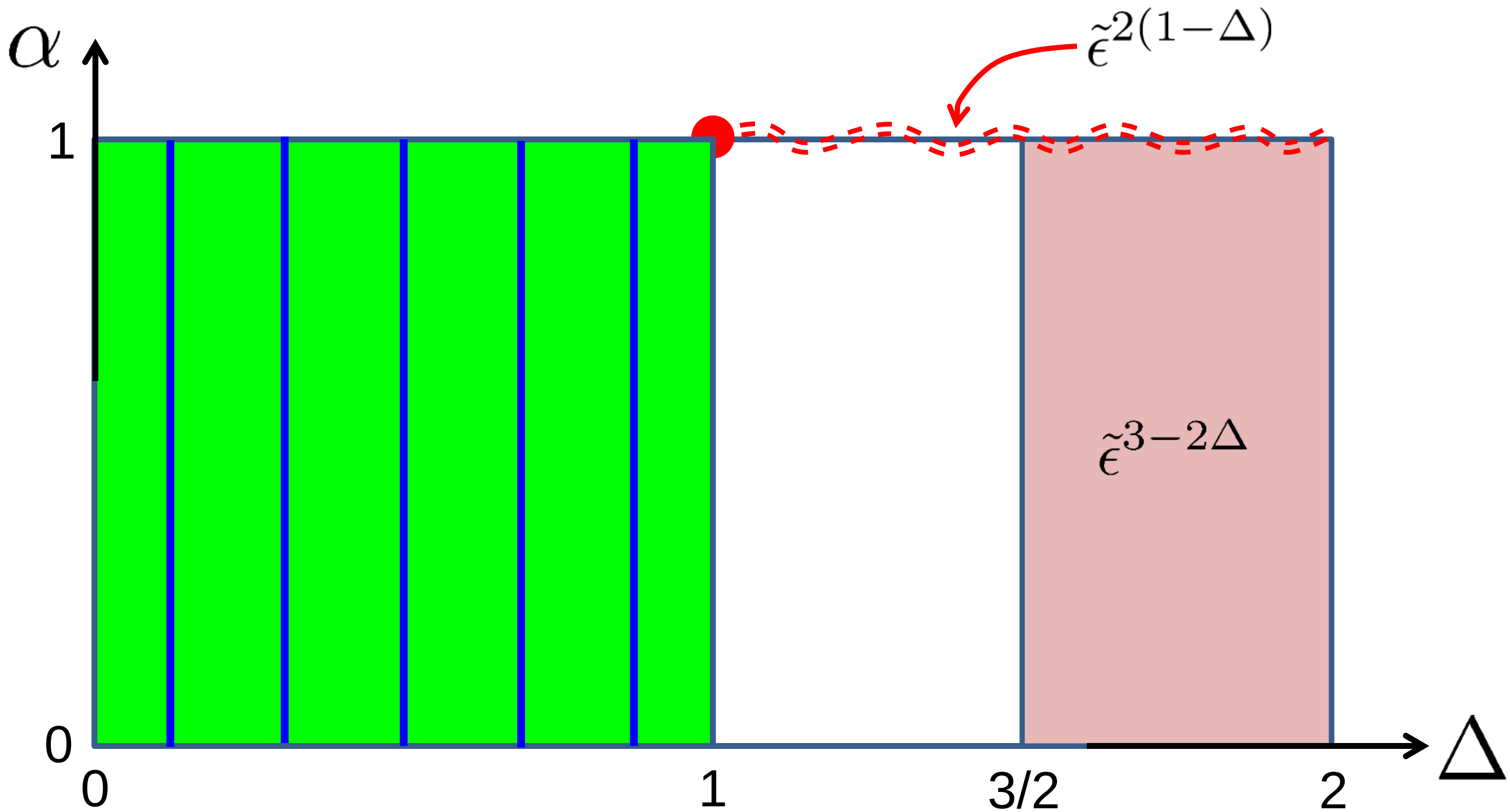

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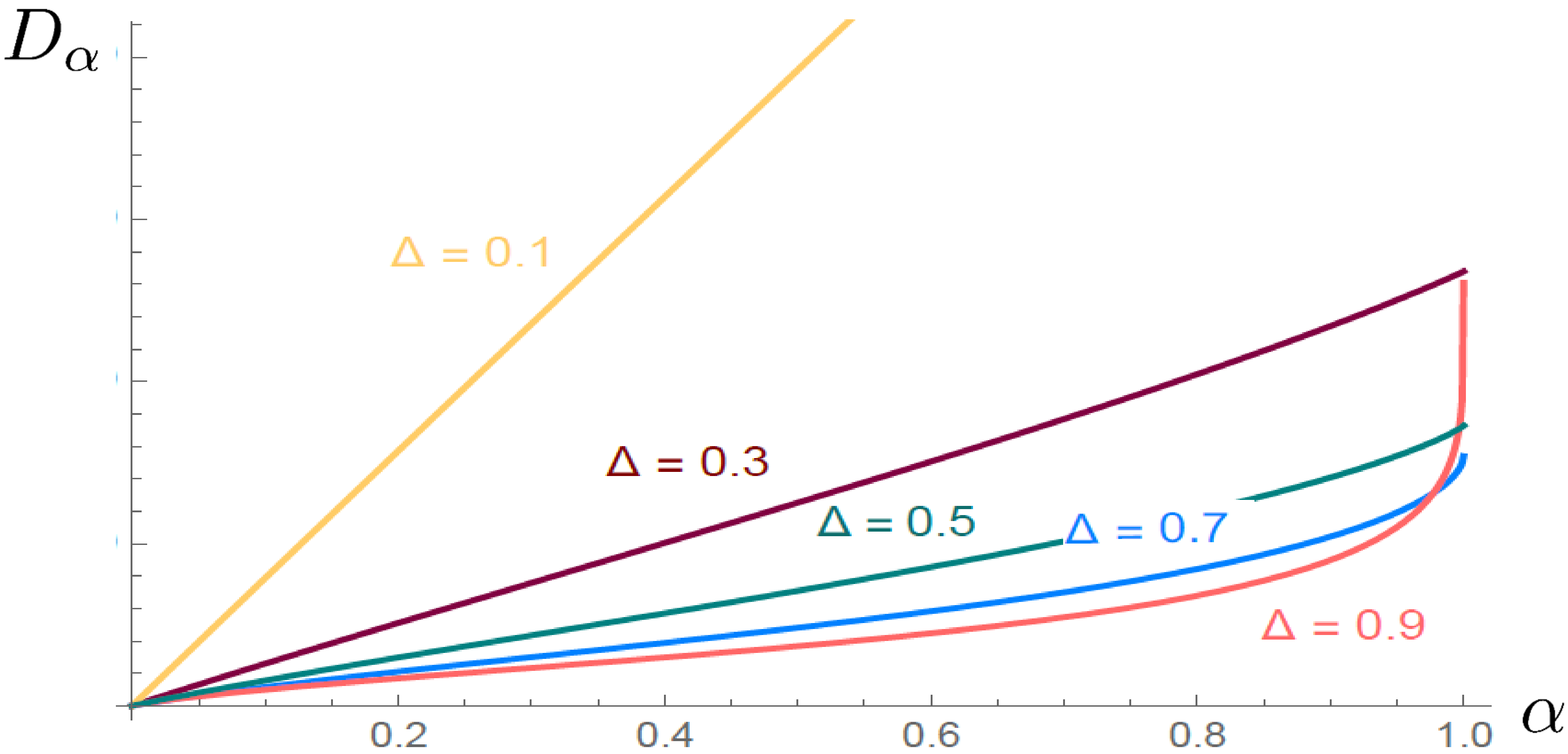

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Final result of holographic calculation:

$$D_\alpha(\rho\|\rho_\beta) = \frac{1}{\alpha-1}\log\frac{tr\left(\rho^\alpha\rho_\beta^{1-\alpha}\right)}{(tr\rho)^\alpha(tr\rho_\beta)^{1-\alpha}}$$

$$\approx \lambda^2\left(\frac{2\pi}{\beta}\right)^{2(\Delta-2)}\frac{c\,L}{6\pi\beta}\,\frac{(\Delta-1)^2}{2^{\Delta+3}}\,\frac{I(\alpha,\Delta)-\alpha I(1,\Delta)}{\alpha-1}$$

- satisfies expected properties of Renyi divergences for $0 \le \alpha \le 1$
  - Positivity: $D_\alpha \ge 0$
  - Continuity in α
  - Monotonicity in α: $\partial_\alpha D_\alpha \ge 0$
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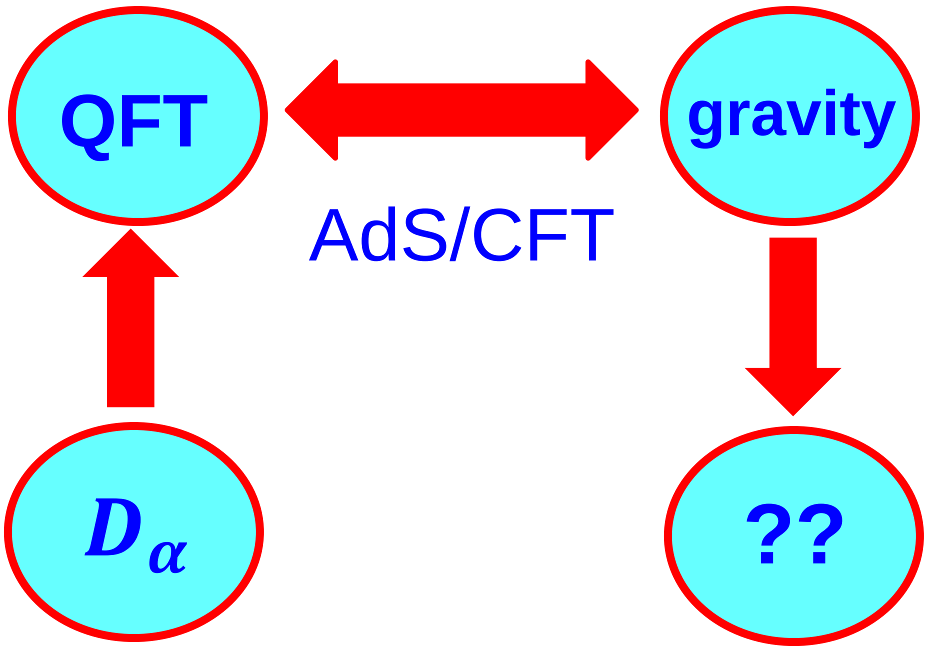
  - Concavity: $(1-\alpha)D_\alpha$ is concave in α
- holographic result matches conformal perturbation theory in boundary theory

Second Laws:

- Renyi divergences give one-param family of constraints

$$D_\alpha(\Lambda_\beta(\rho)\|\rho_\beta) \leq D_\alpha(\rho\|\rho_\beta)$$

- will they provide new constraints for gravitational dynamics that extend beyond the usual second law?



New holographic constraints?

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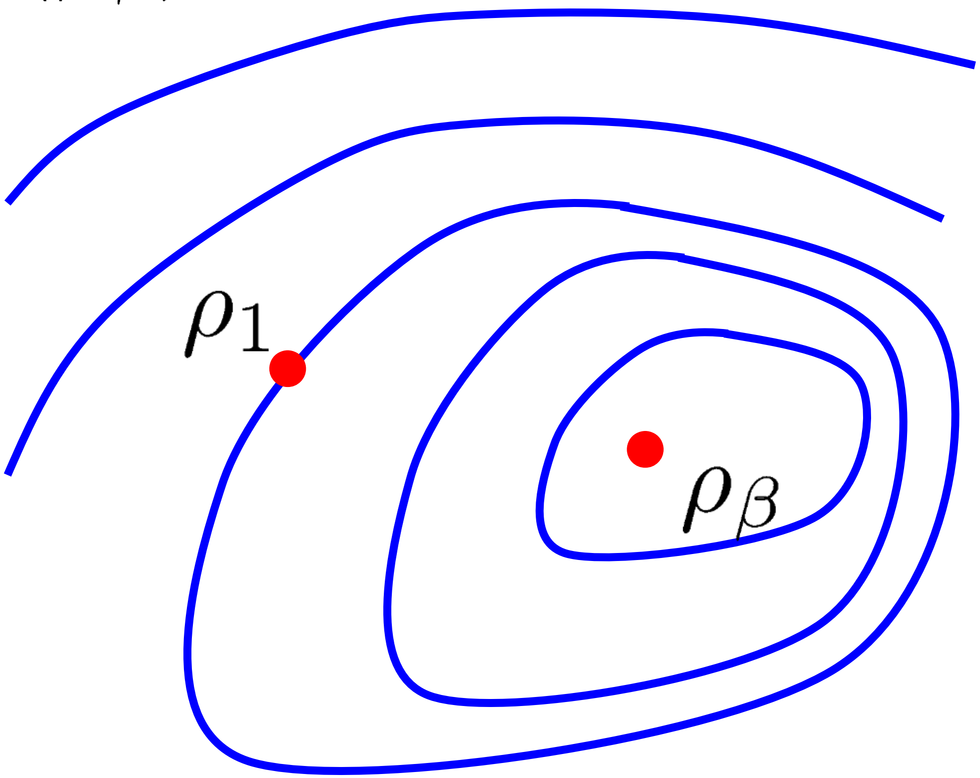
$$D_\alpha(\Lambda_\beta(\rho_1)\|\rho_\beta) \leq D_\alpha(\rho_1\|\rho_\beta)$$



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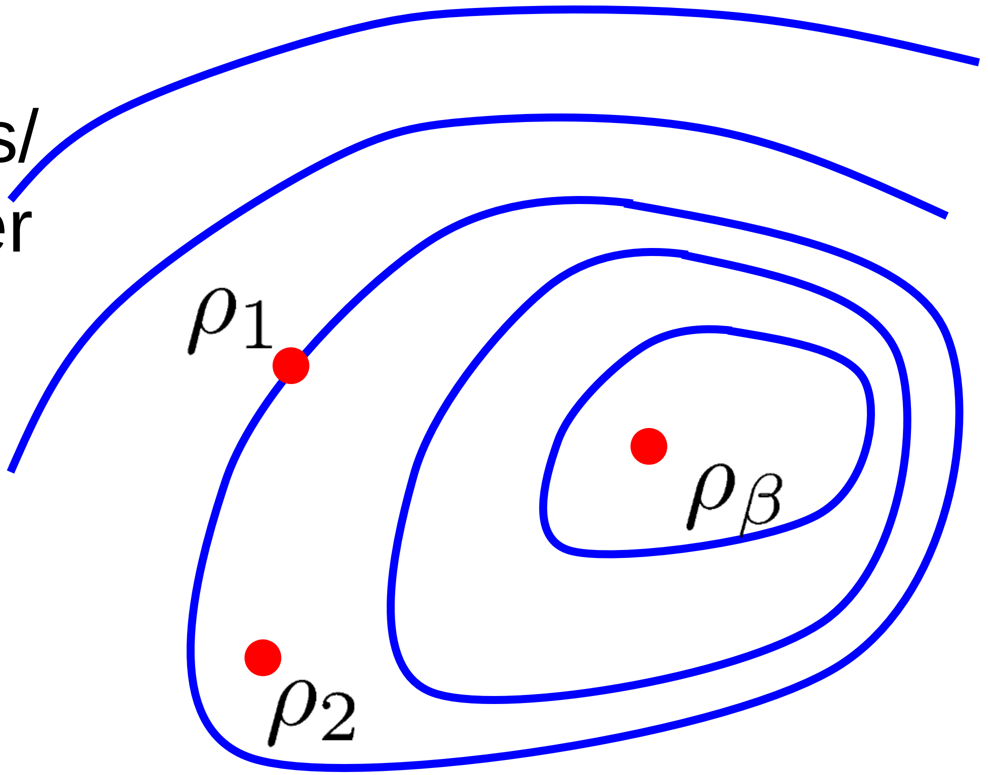

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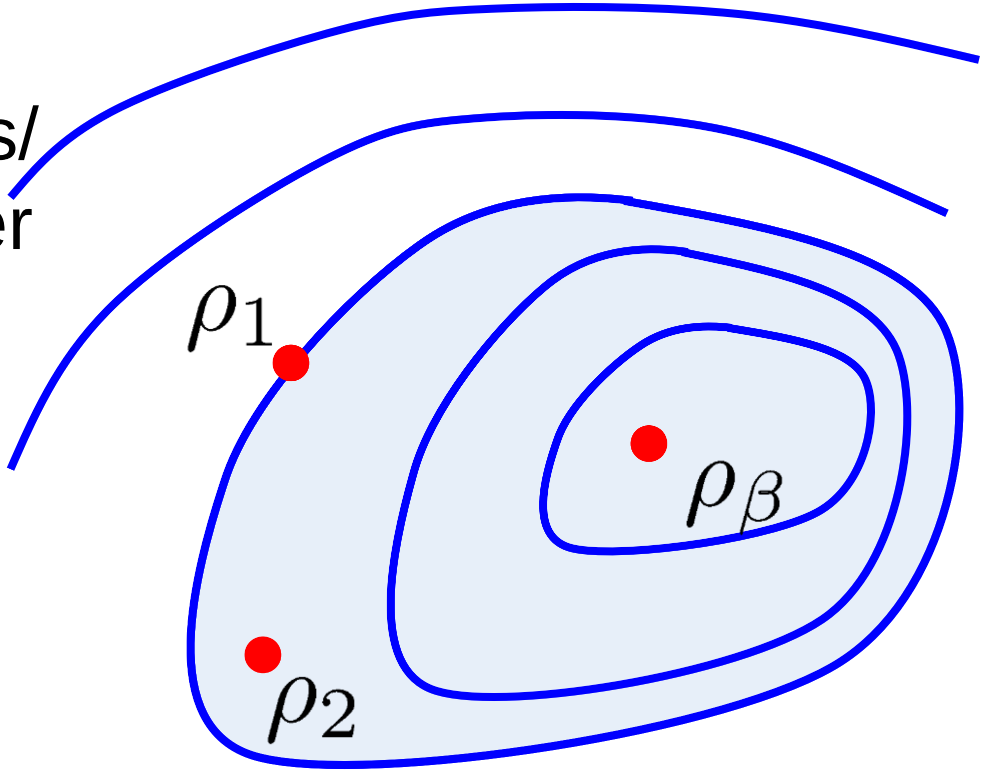

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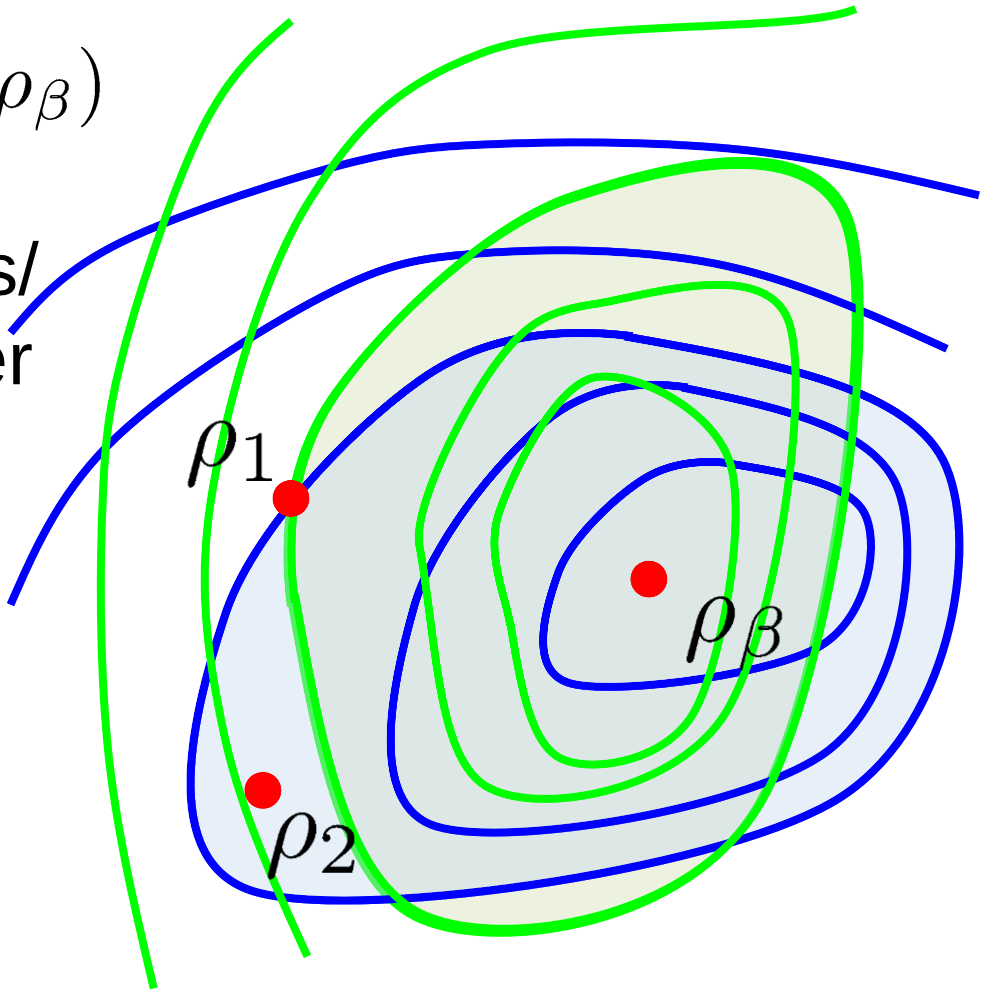
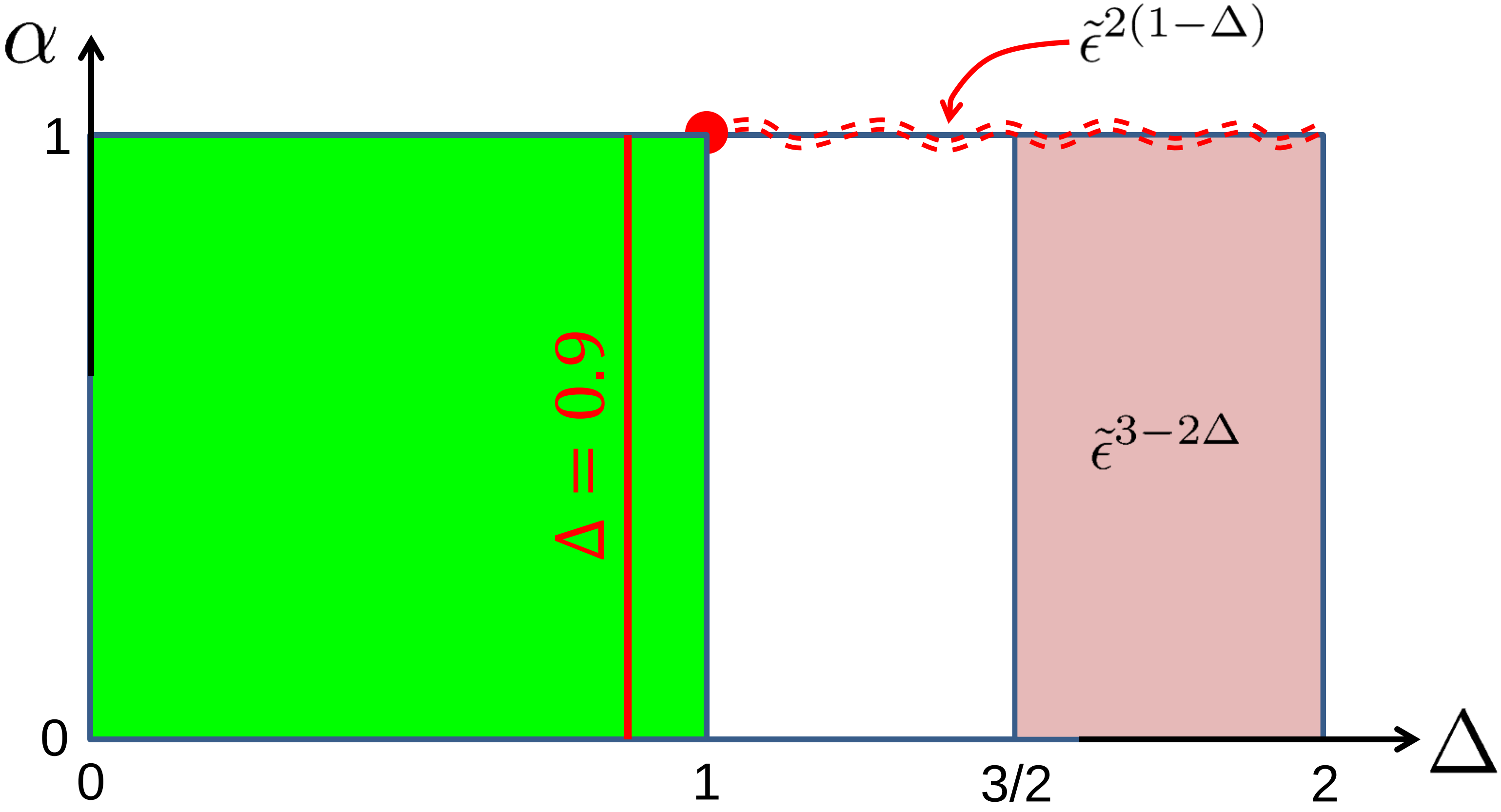
$$D_\alpha(\rho_2 \| \rho_\beta) \leq D_\alpha(\rho_1 \| \rho_\beta)$$

- compare individual states/ families to decide whether a particular transition is ruled out
- ask if new constraints are ever stronger than standard second law (ie, free energy)



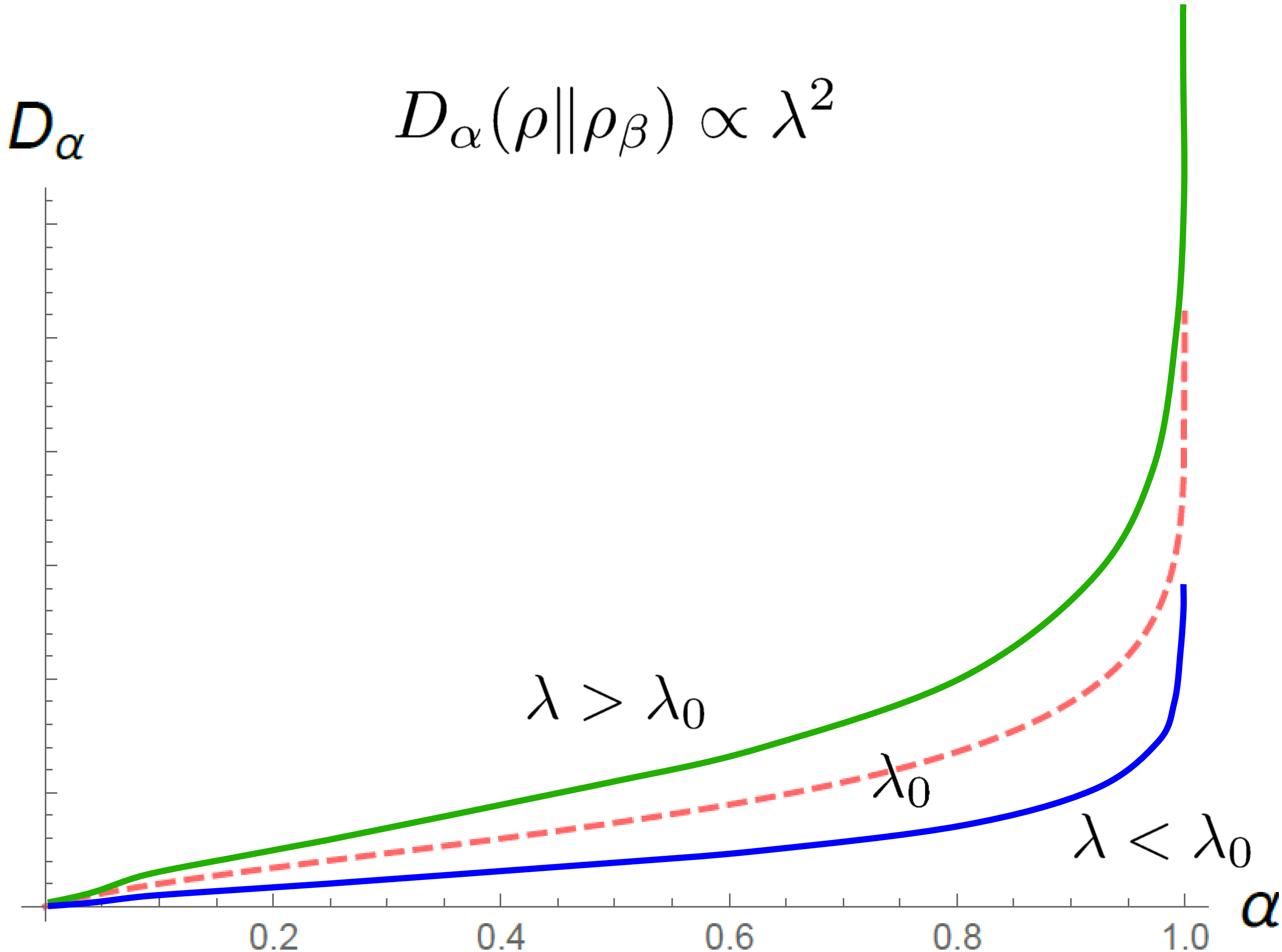
New holographic constraints?

$$D_\alpha(\rho\|\rho_\beta) \approx \lambda^2 \left(\frac{2\pi}{\beta}\right)^{2(\Delta-2)} \frac{c\,L}{6\pi\beta}\,\frac{(\Delta-1)^2}{2^{\Delta+3}}\,\frac{I(\alpha,\Delta)-\alpha I(1,\Delta)}{\alpha-1}$$

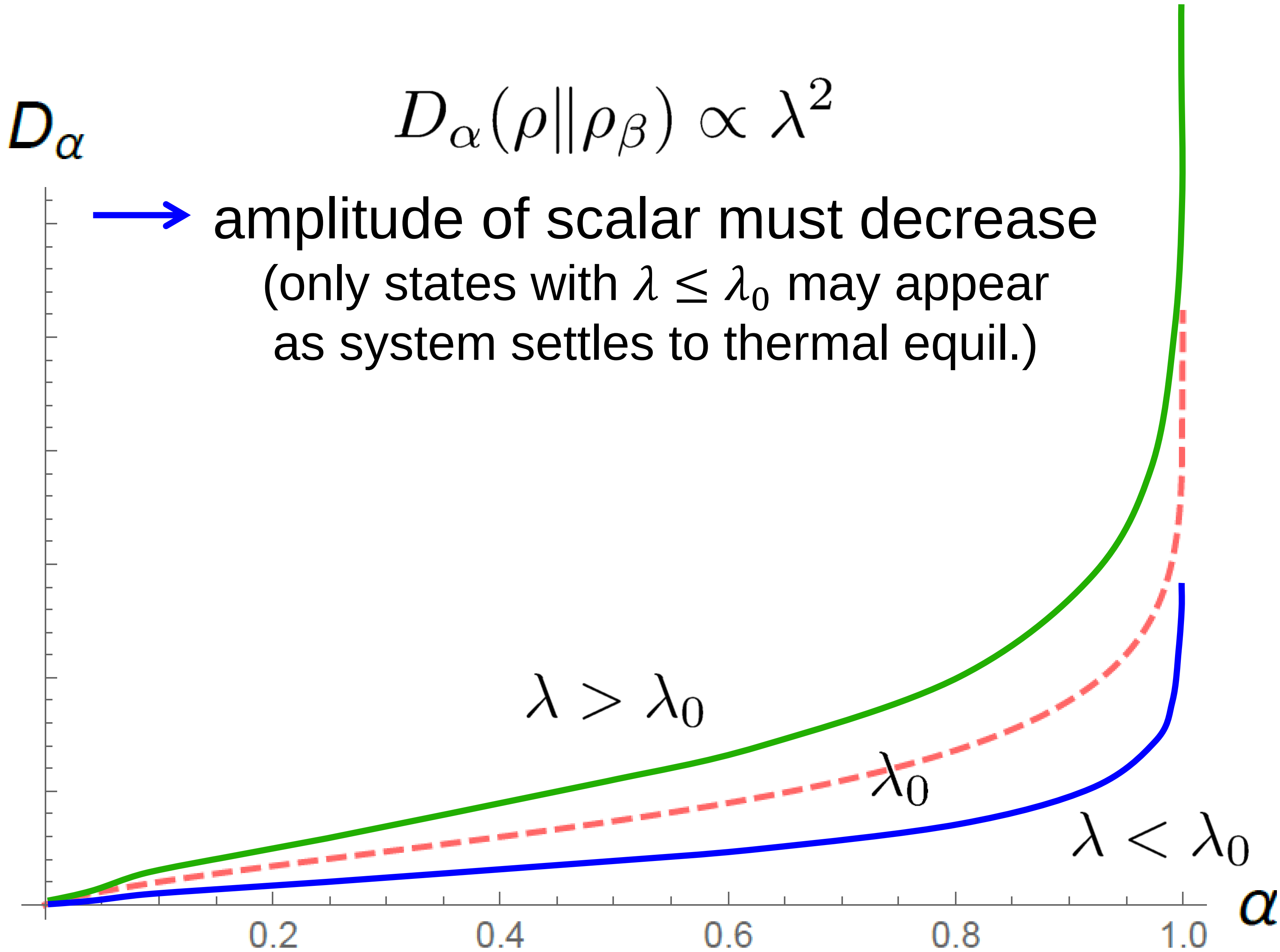


- consider a gravity theory with a single scalar, eg,

$$\Delta = 0.9 \longleftrightarrow m^2 = -0.99$$



D_α
$D_\alpha(\rho\|\rho_\beta) \propto \lambda^2$
$\lambda > \lambda_0$
λ_0
$\lambda < \lambda_0$
0.2
0.4
0.6
0.8
1.0
α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amplitude of scalar must decrease
(only states with $\lambda \leq \lambda_0$ may appear
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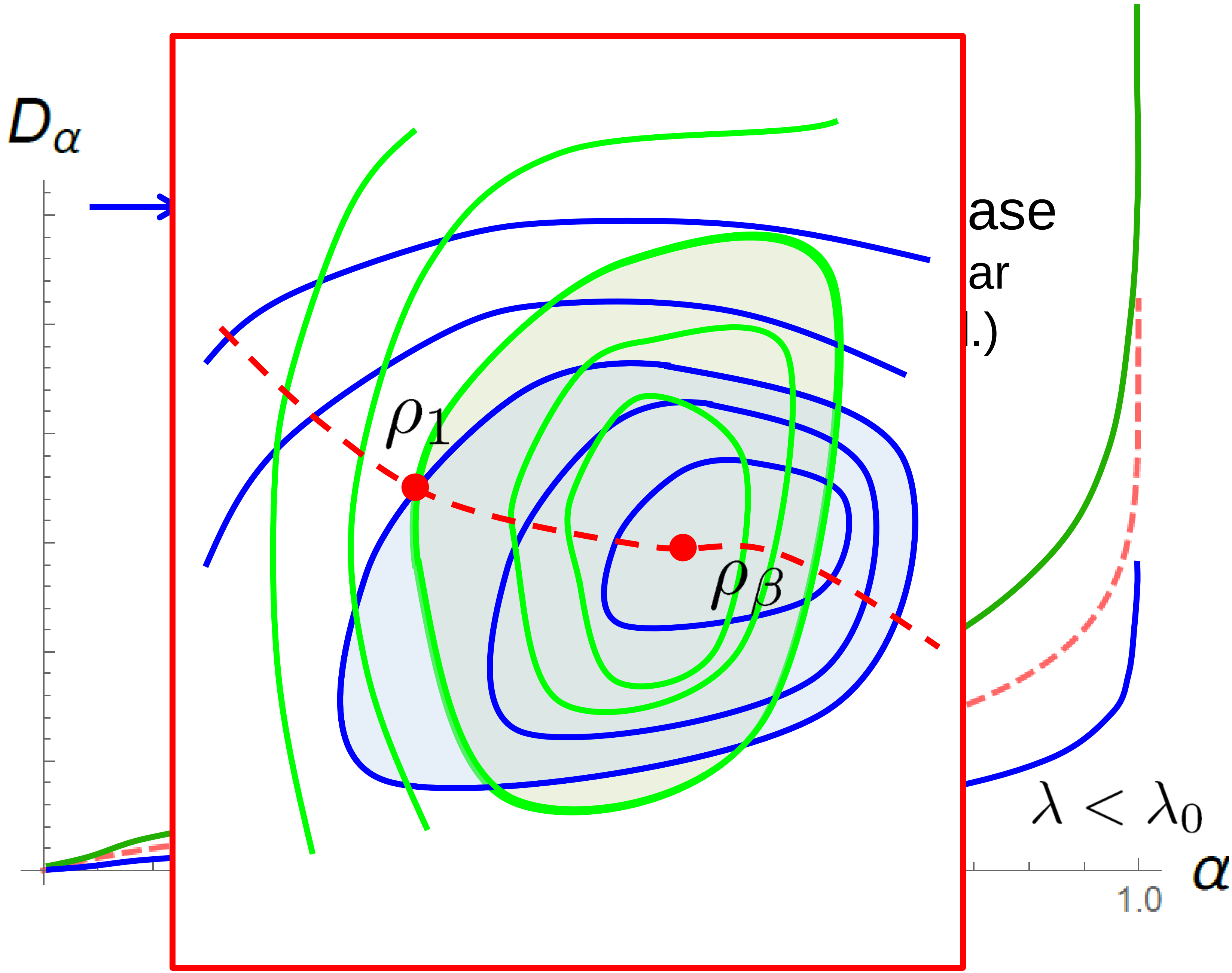

D_α
ase
ar
l.)
ρ_1
ρ_β
$\lambda < \lambda_0$
α
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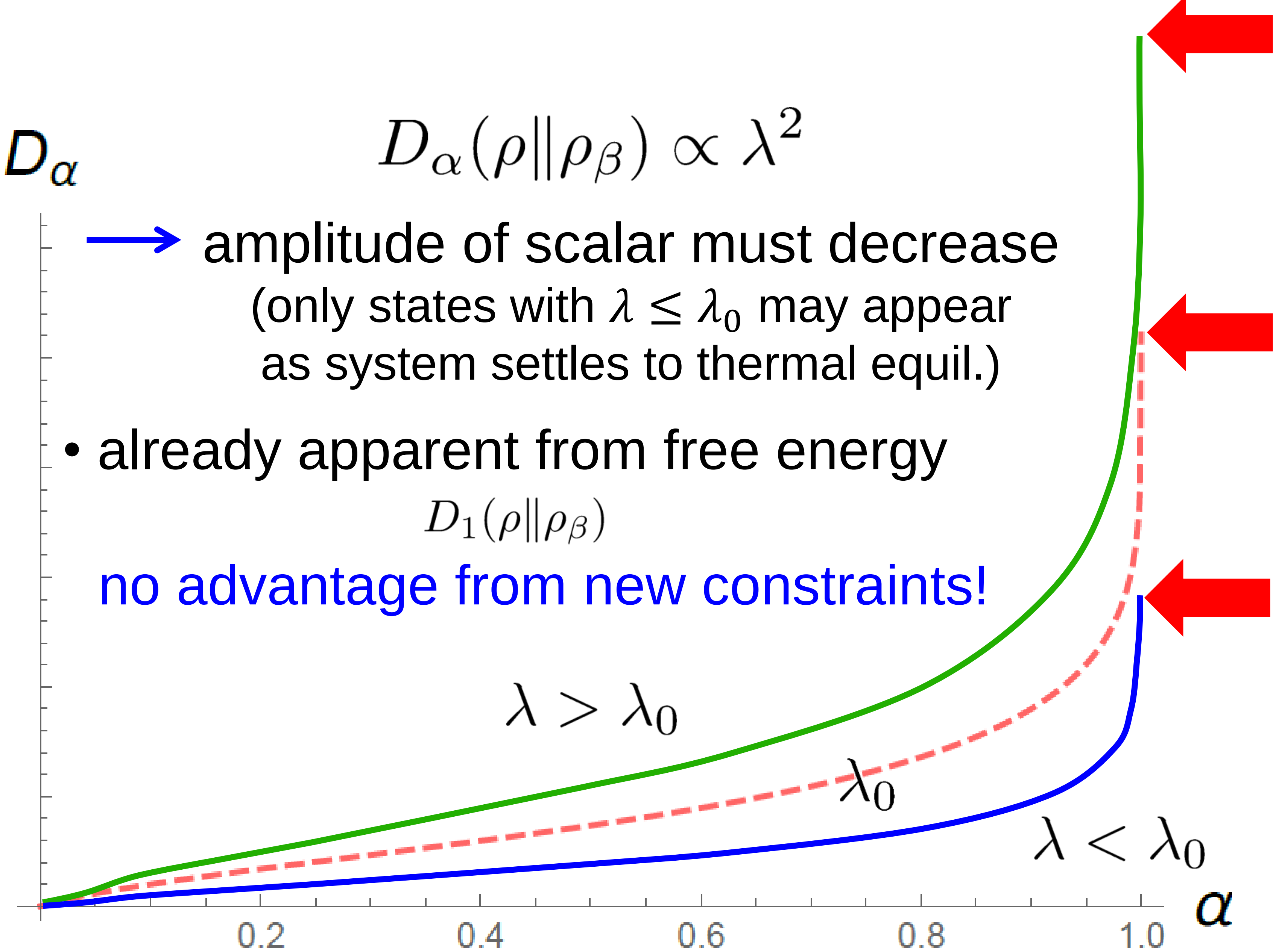
→ amplitude of scalar must decrease
(only states with $\lambda \leq \lambda_0$ may appear
as system settles to thermal equil.)

- already apparent from free energy

$$D_1(\rho\|\rho_\beta)$$

no advantage from new constraints!



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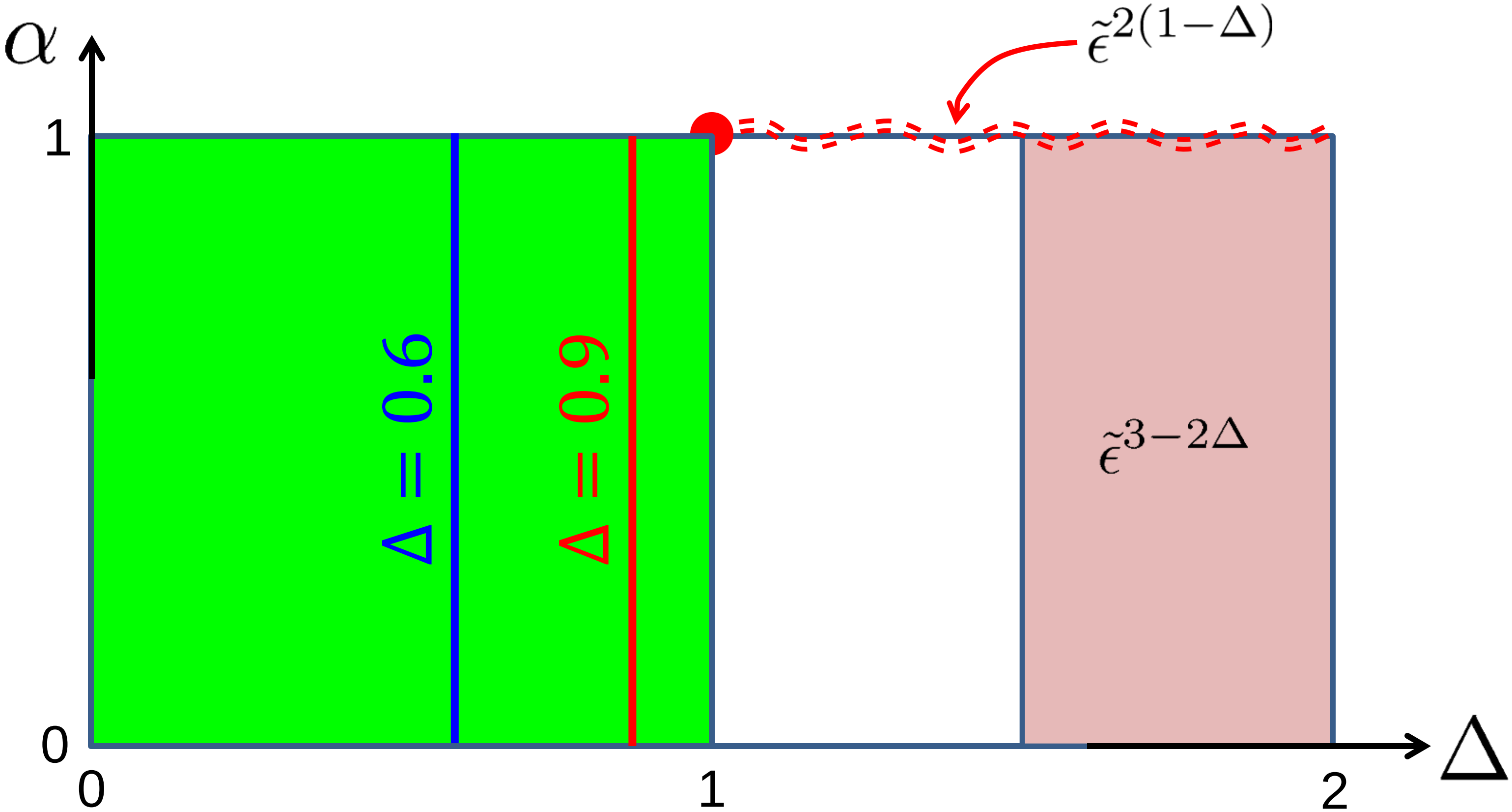


- examine possible transitions from ρ_1 with $\Delta_1 = 0.9$ to ρ_2 with $\Delta_2 = 0.6$

Aside:

- examine possible transitions from ρ_1 with $\Delta_1 = 0.9$ to ρ_2 with $\Delta_2 = 0.6$

two relevant operators in boundary theory
with $\Delta_1 = 0.9$ and $\Delta_2 = 0.6$

two scalars, Φ_1 and Φ_2 , in the bulk gravity theory
with masses ${m_1}^2 = -0.99$ and ${m_2}^2 = -0.84$

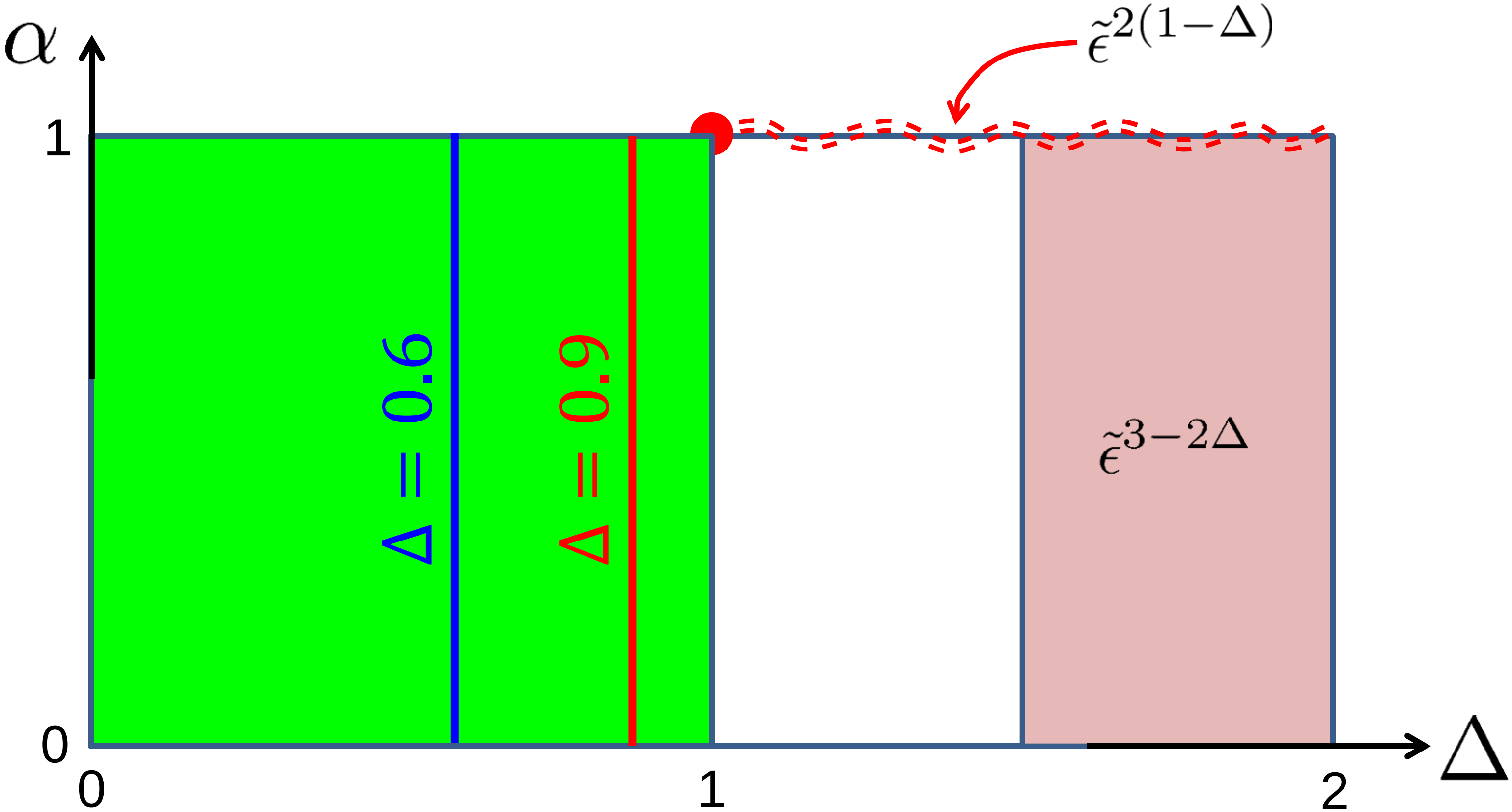
- must include extra bulk interactions, eg,

$$U(\Phi_1, \Phi_2) = \frac{g}{2}\left(\Phi_1\Phi_2^2 + \Phi_2\Phi_1^2\right)$$

to allow transitions between Φ_1 and Φ_2 (ie, ρ_1 and ρ_2)

New holographic constraints?

$$D_\alpha(\rho\|\rho_\beta) \approx \lambda^2 \left(\frac{2\pi}{\beta}\right)^{2(\Delta-2)} \frac{c\,L}{6\pi\beta} \frac{(\Delta-1)^2}{2^{\Delta+3}} \frac{I(\alpha,\Delta)-\alpha I(1,\Delta)}{\alpha-1}$$



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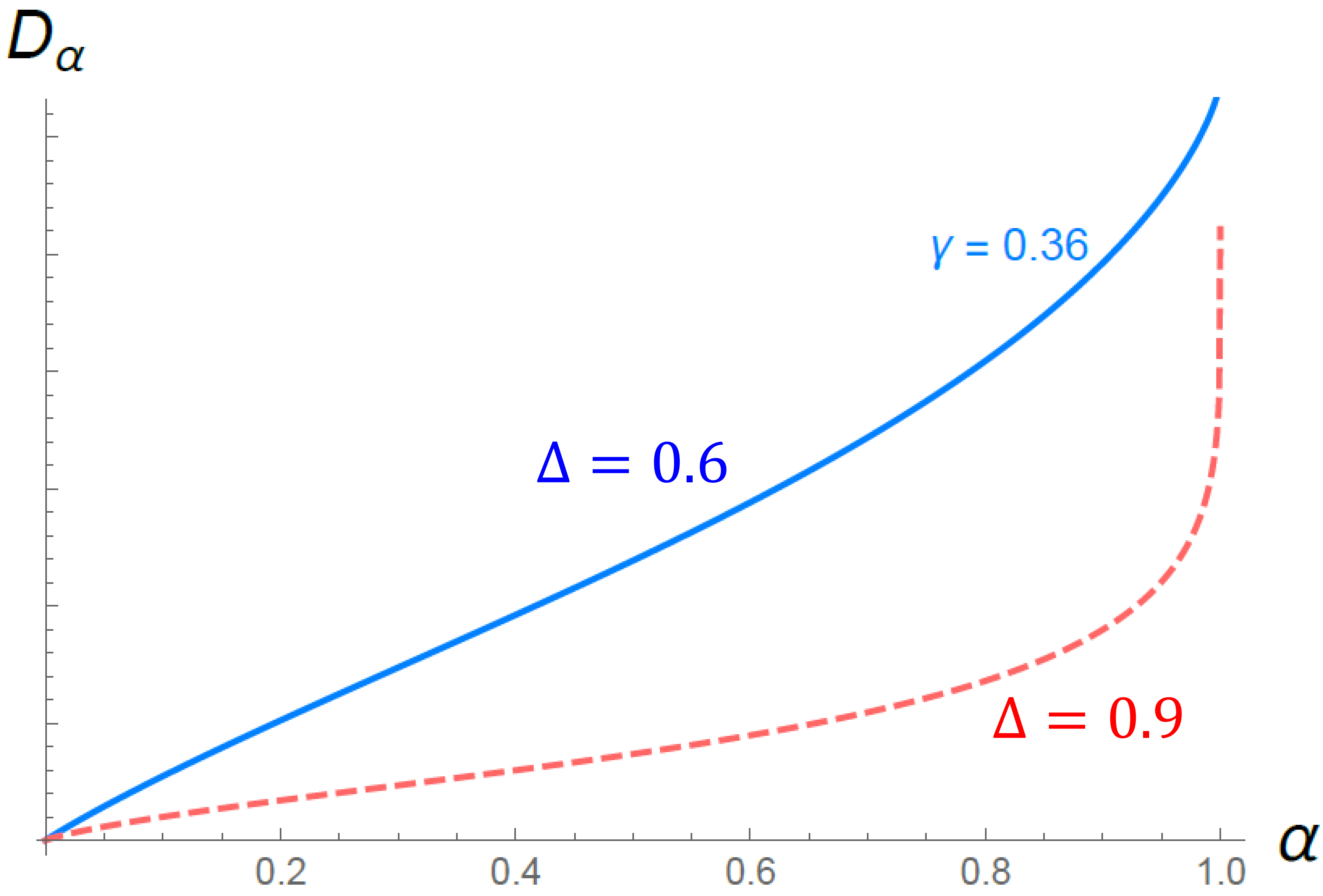

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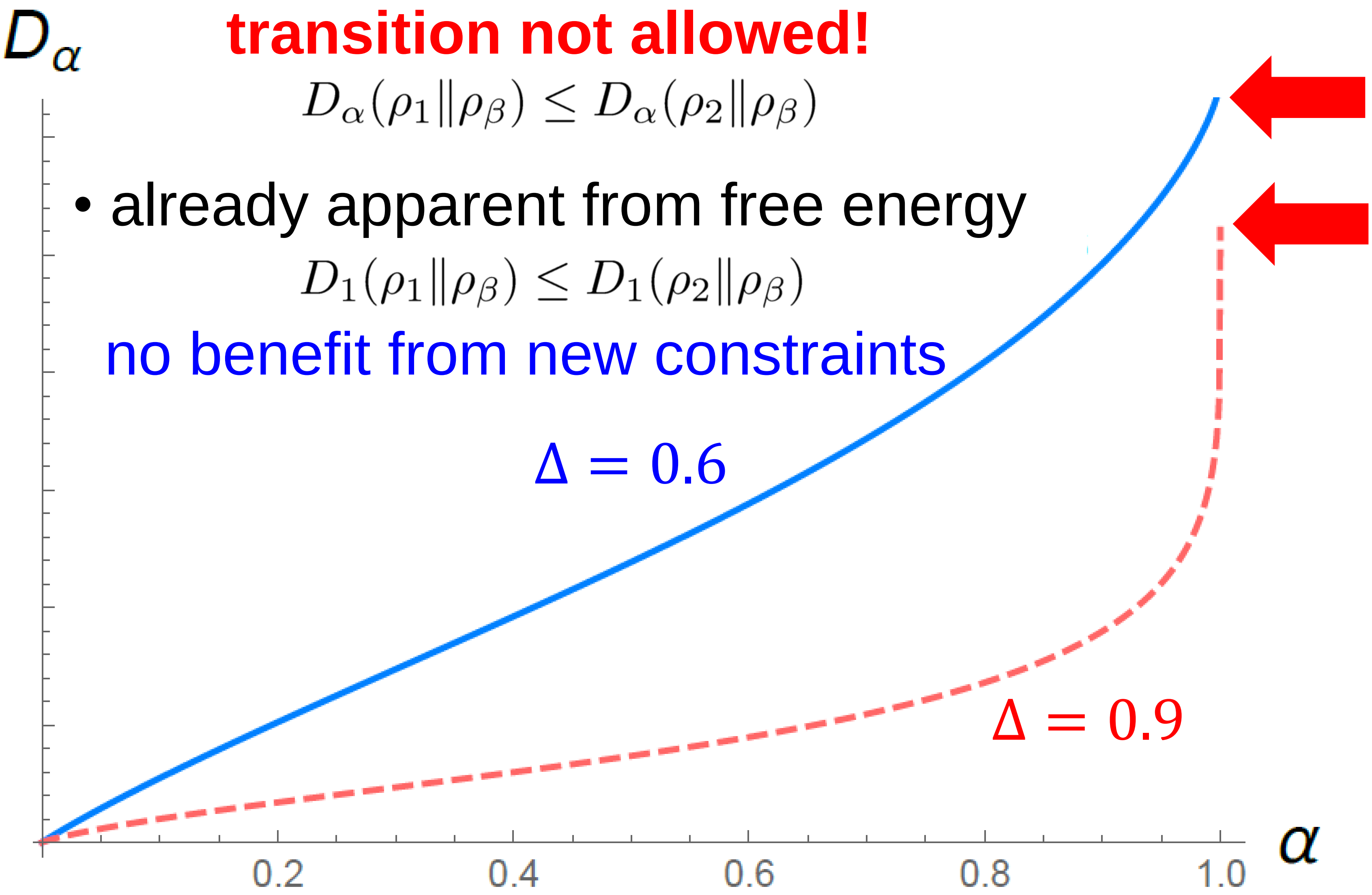
transition not allowed!

$$D_\alpha(\rho_1 \| \rho_\beta) \leq D_\alpha(\rho_2 \| \rho_\beta)$$

- already apparent from free energy

$$D_1(\rho_1 \| \rho_\beta) \leq D_1(\rho_2 \| \rho_\beta)$$

no benefit from new constraints



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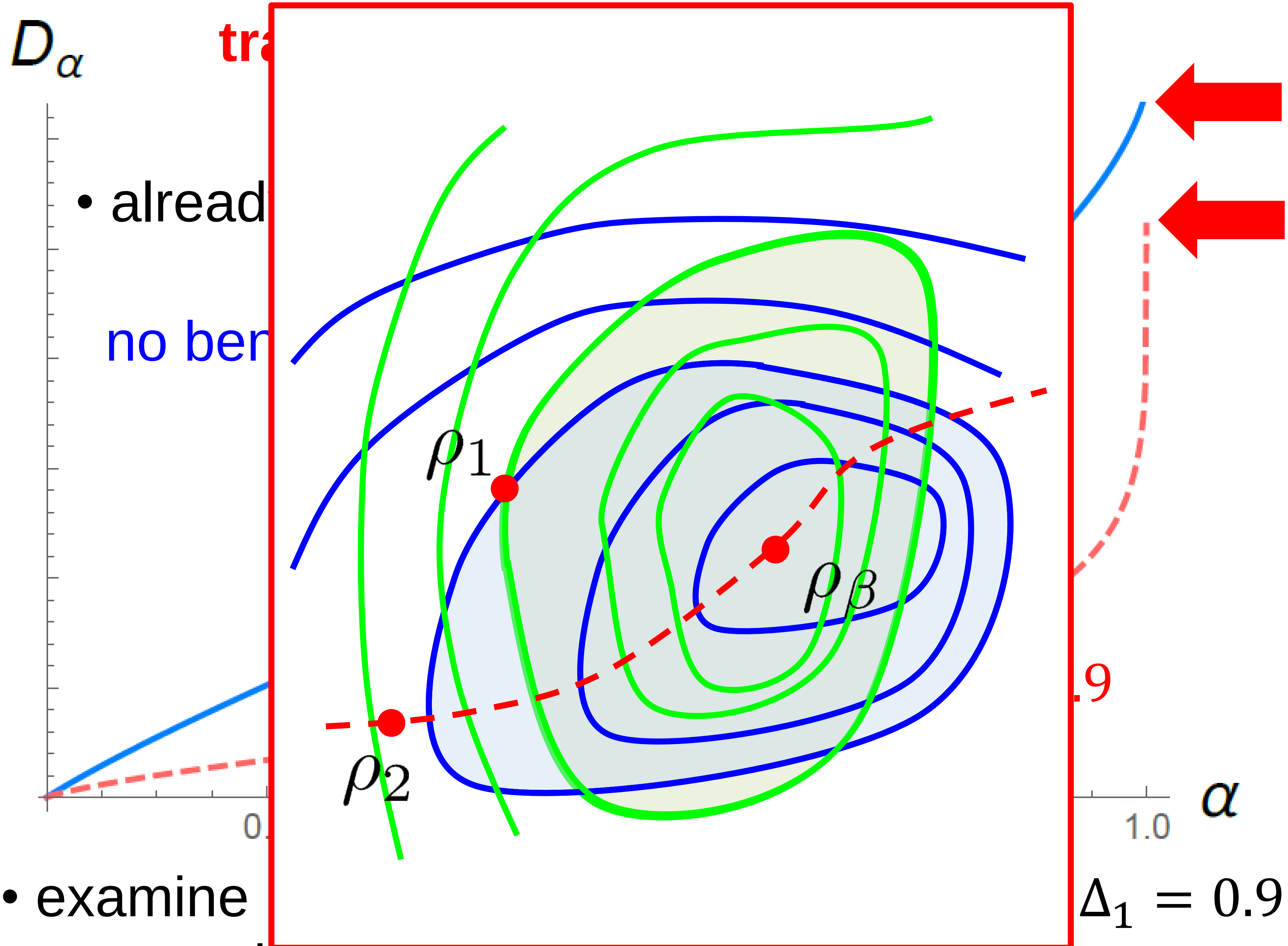
- alread

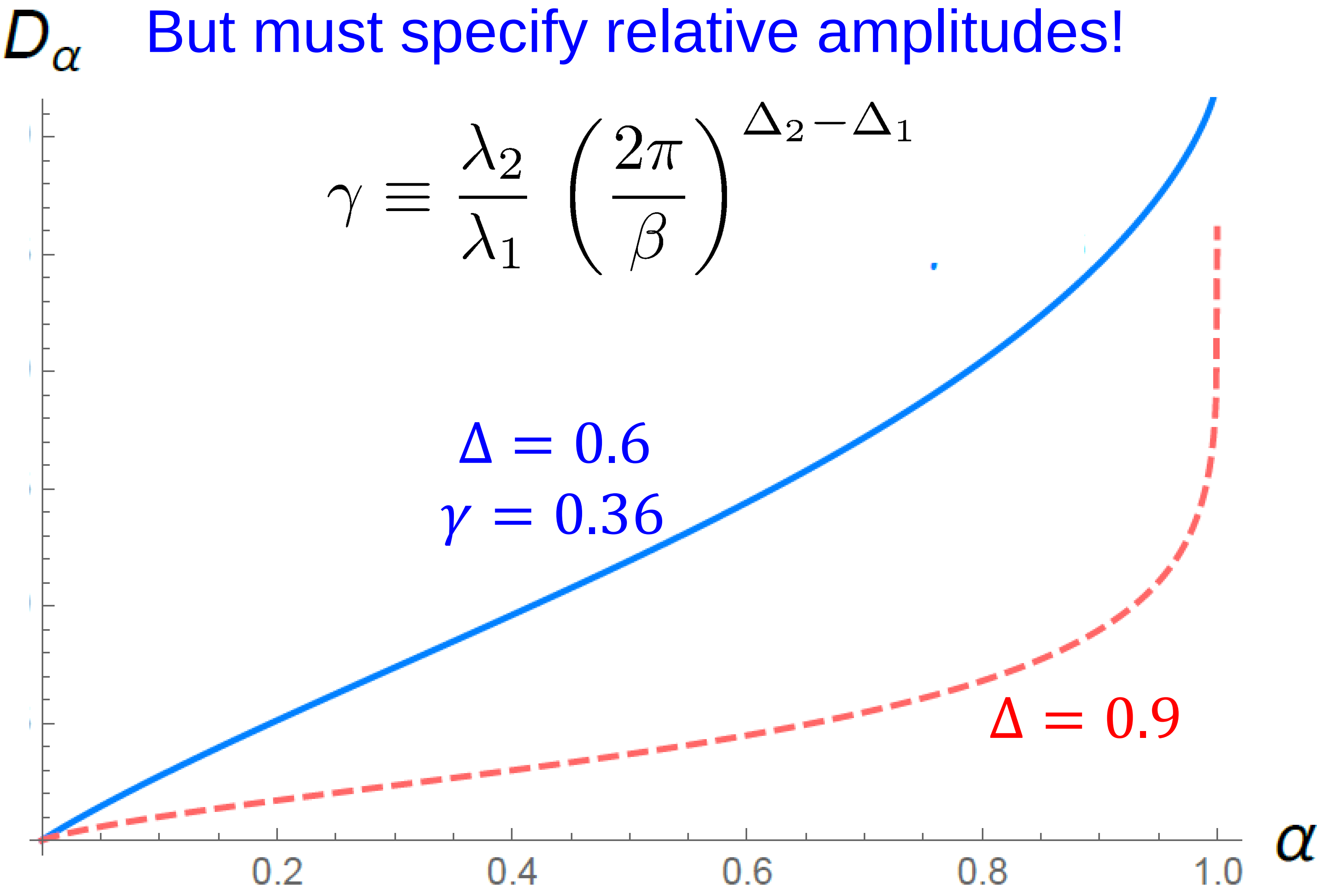
no ben



- examine $\Delta_1 = 0.9$
  to ρ_2 with $\Delta_2 = 0.6$

But must specify relative amplitudes!

$$\gamma \equiv \frac{\lambda_2}{\lambda_1}\left(\frac{2\pi}{\beta}\right)^{\Delta_2-\Delta_1}$$



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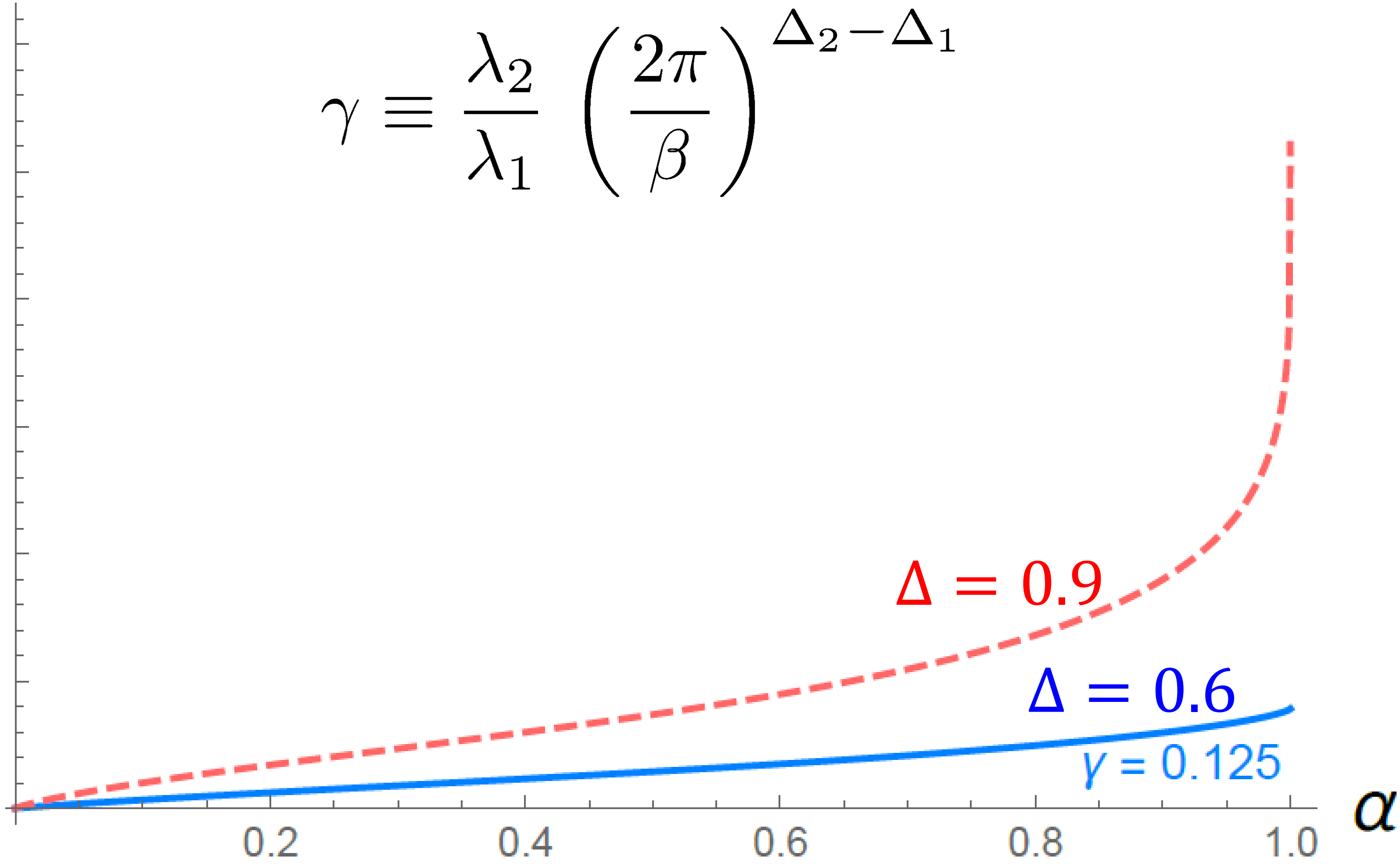

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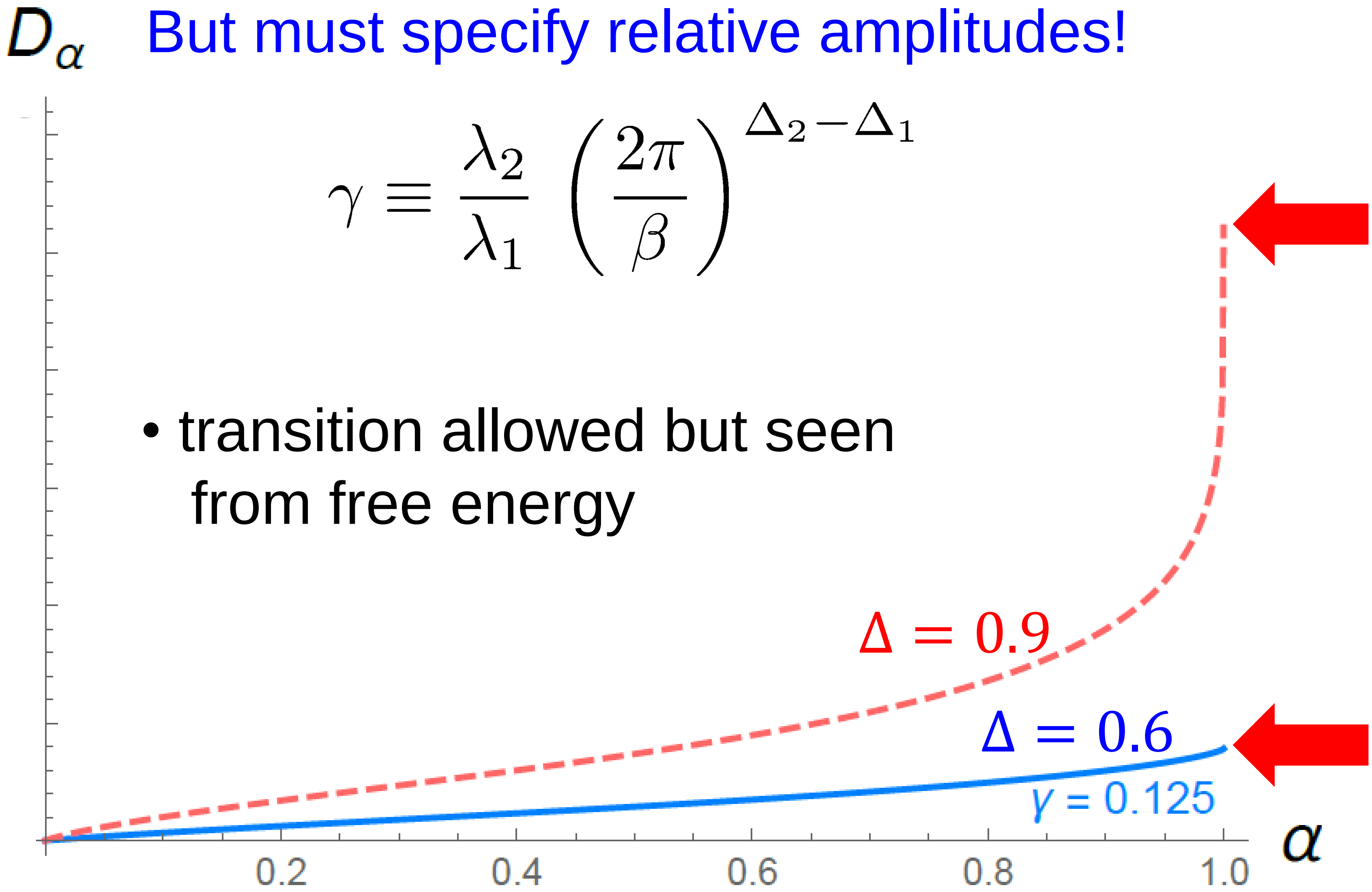
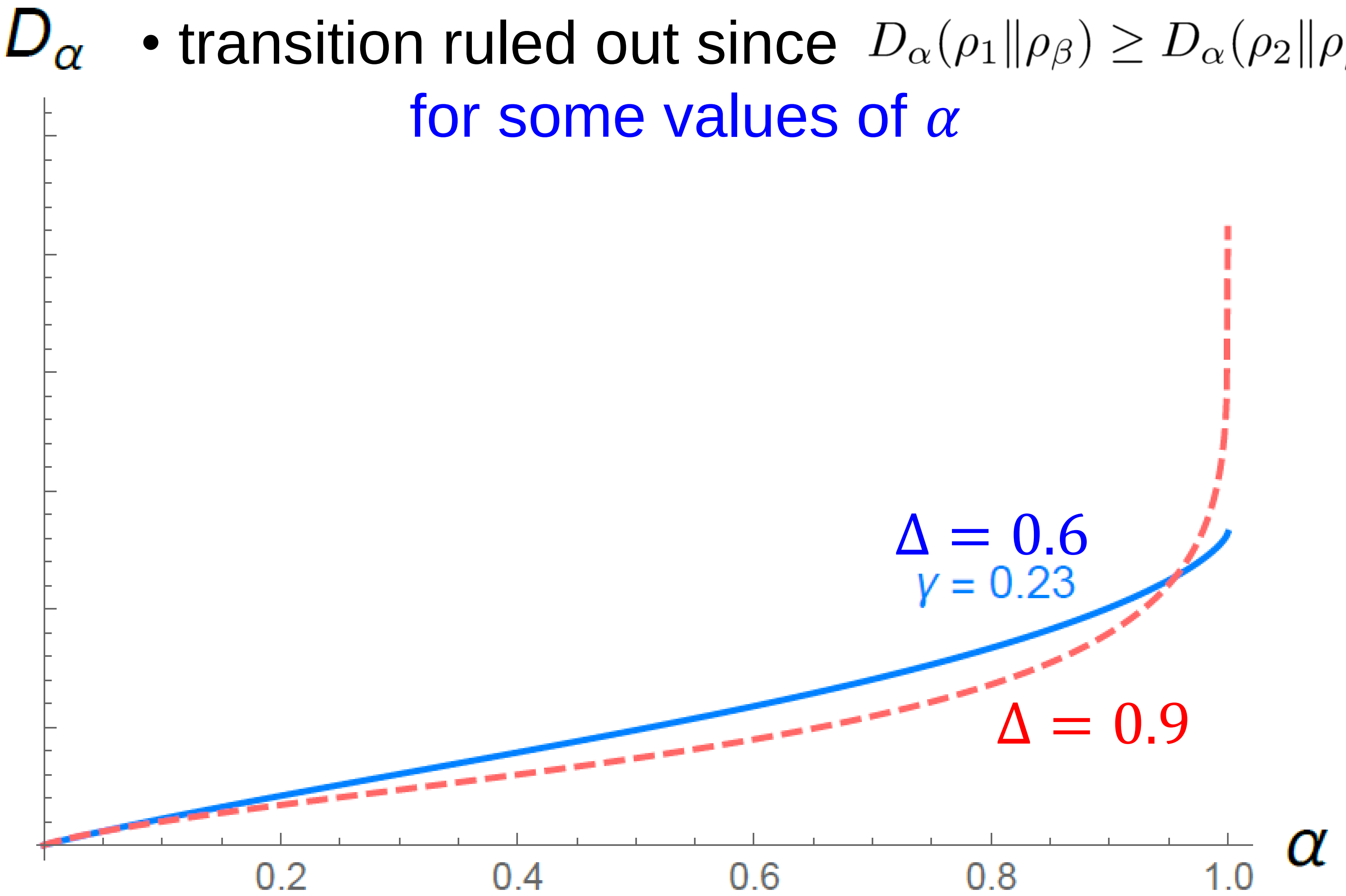


- transition allowed but seen from free energy

- examine possible transitions from ρ_1 with $\Delta_1 = 0.9$ to ρ_2 with $\Delta_2 = 0.6$

• transition ruled out since $D_\alpha(\rho_1 \| \rho_\beta) \geq D_\alpha(\rho_2 \| \rho_\beta)$

for some values of α



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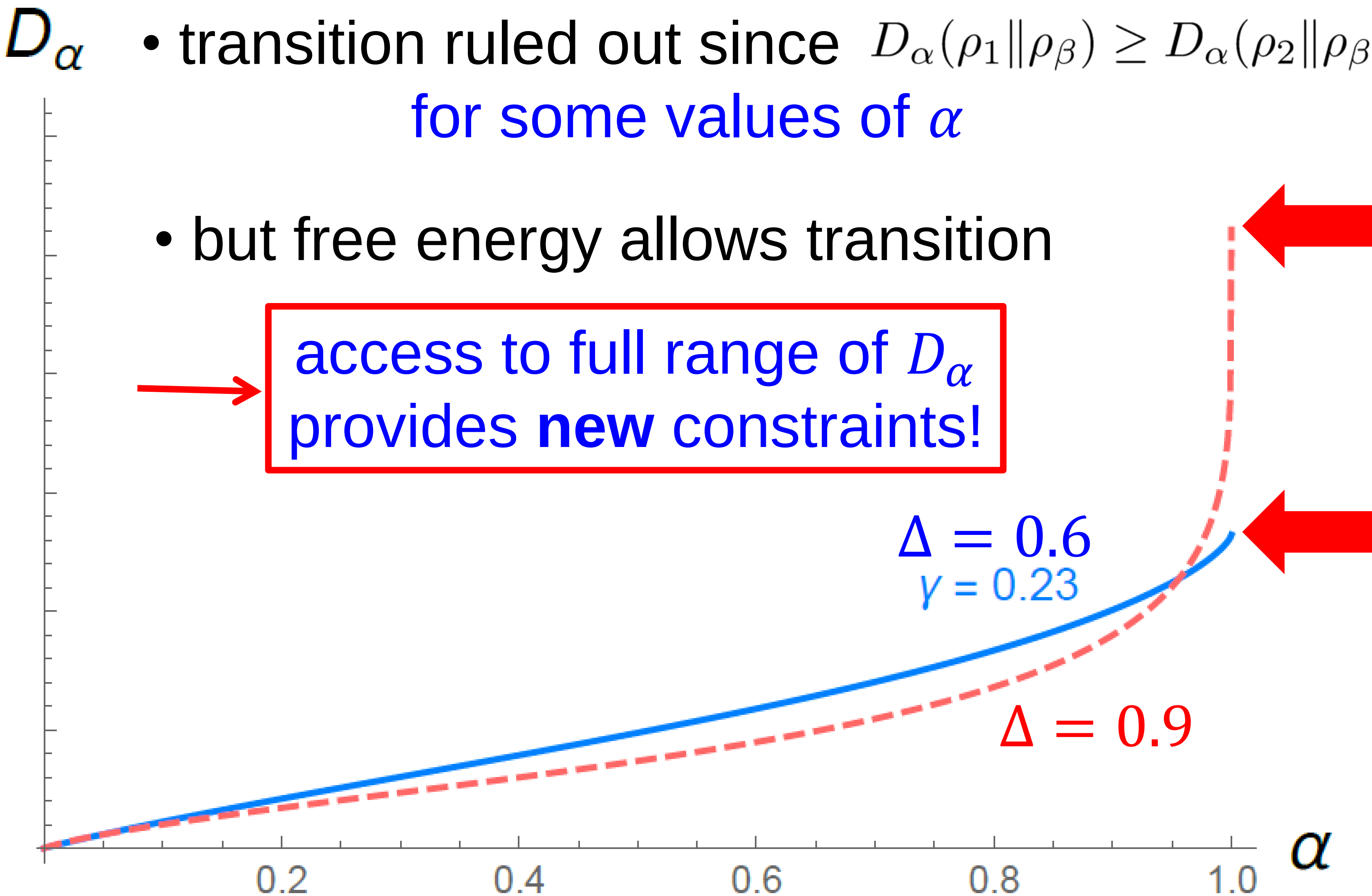
  for some values of α

- but free energy allows transition

access to full range of D_α
provides **new** constraints!



- examine possible transitions from ρ_1 with $\Delta_1 = 0.9$
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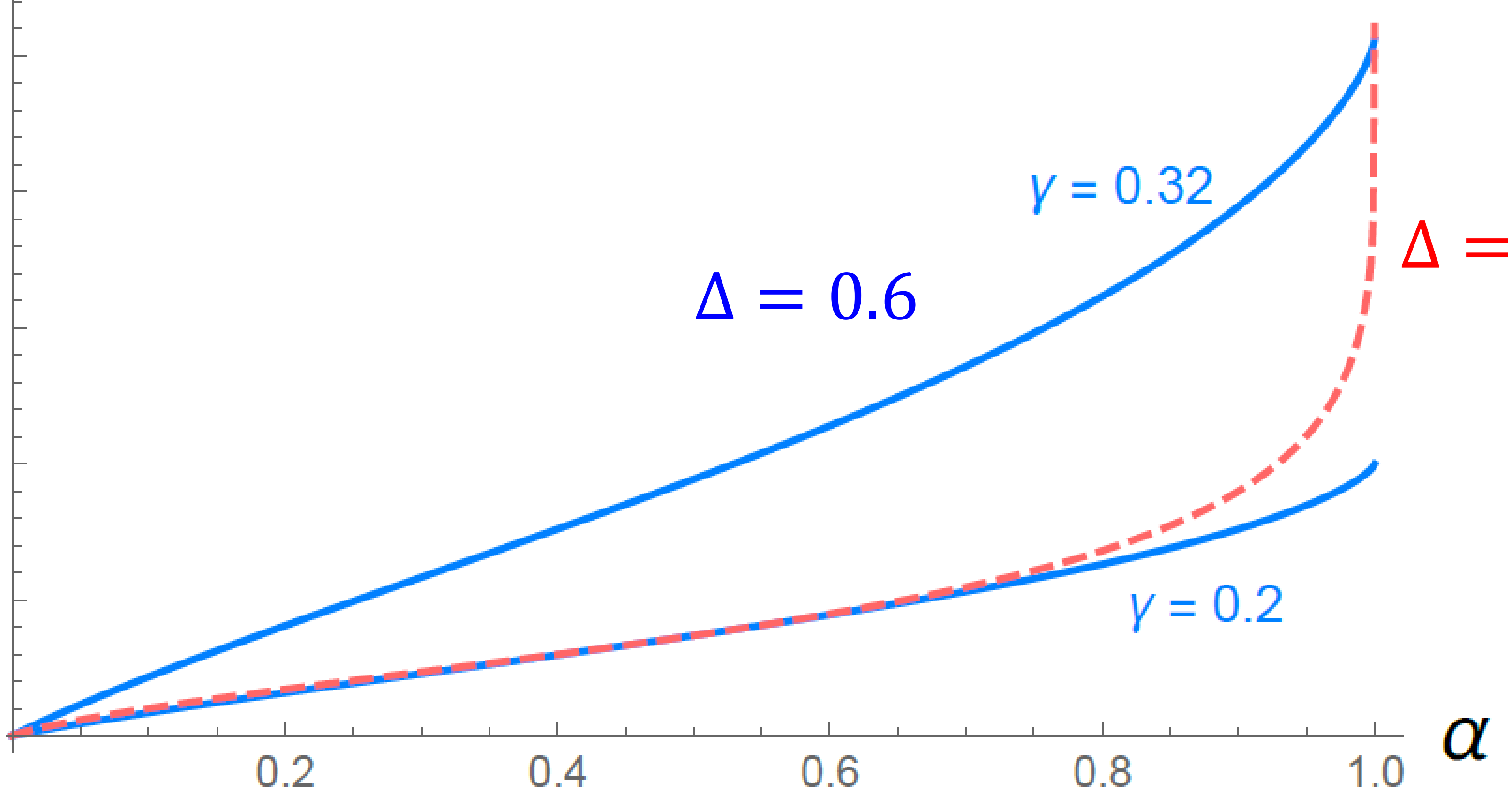
- using only D_1, transition ruled out for $\gamma > 0.32$
- using family of D_α, transition ruled out for $\gamma > 0.2$



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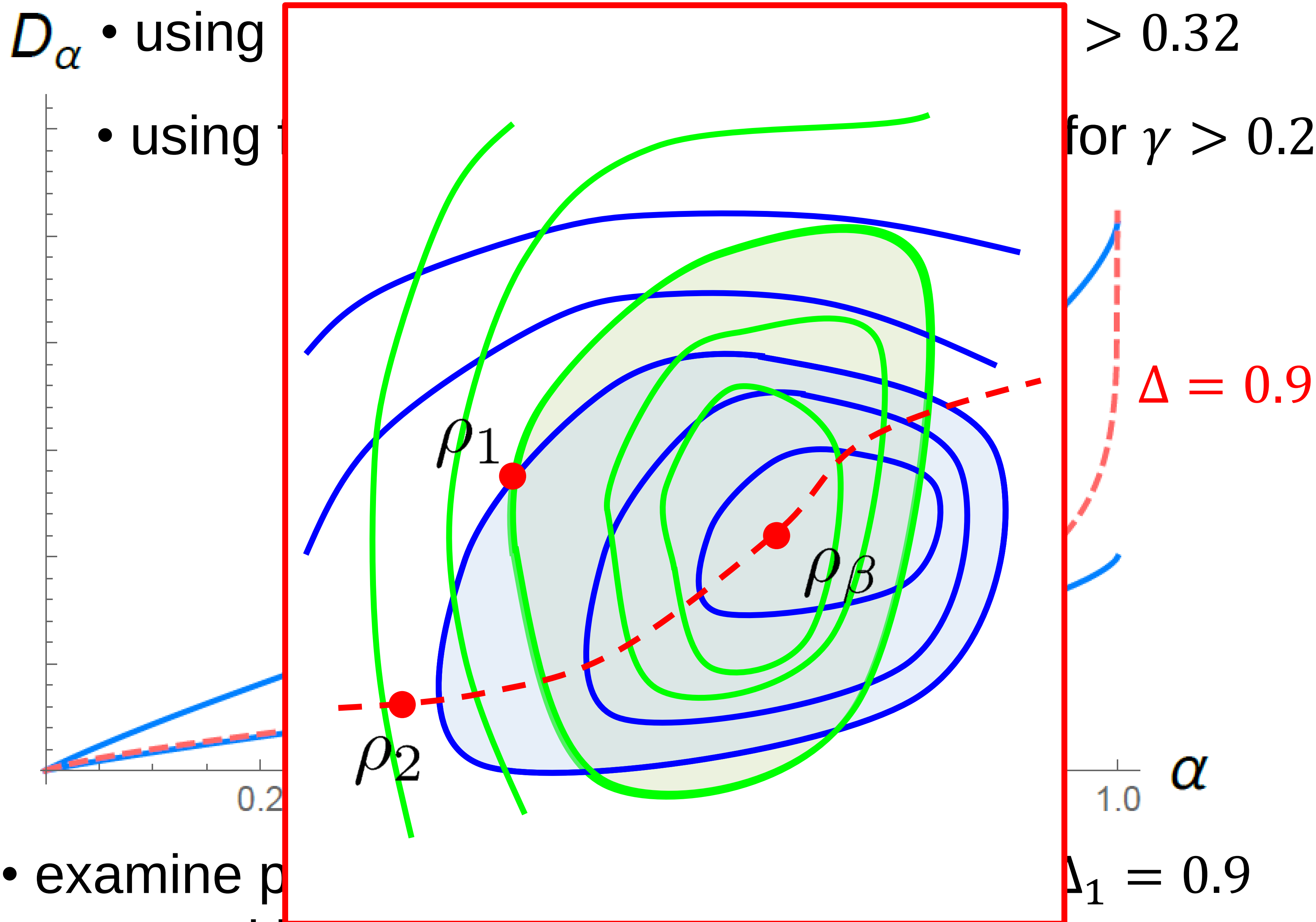
D_α • using [...] > 0.32

• using [...] for $\gamma > 0.2$



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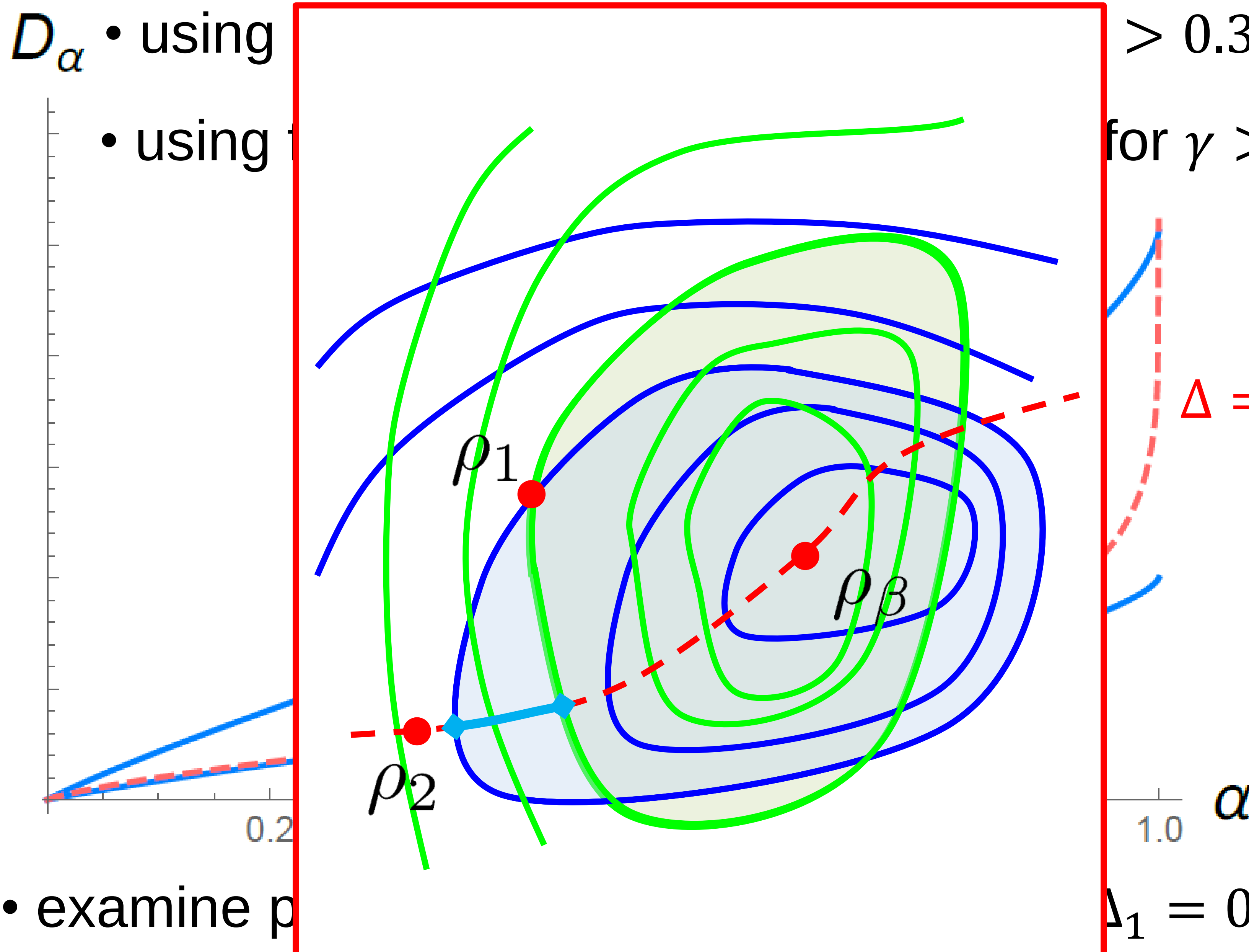

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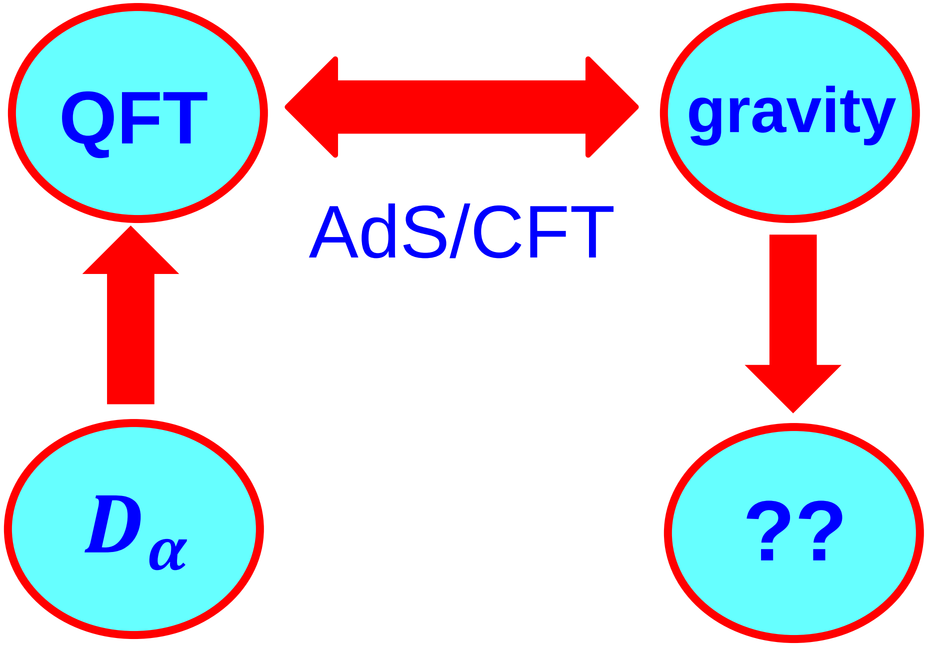
Second Laws:

- Renyi divergences give one-param family of constraints

$$D_\alpha(\Lambda_\beta(\rho)\|\rho_\beta) \leq D_\alpha(\rho\|\rho_\beta)$$

- will they provide new constraints for gravitational dynamics that extend beyond the usual second law?



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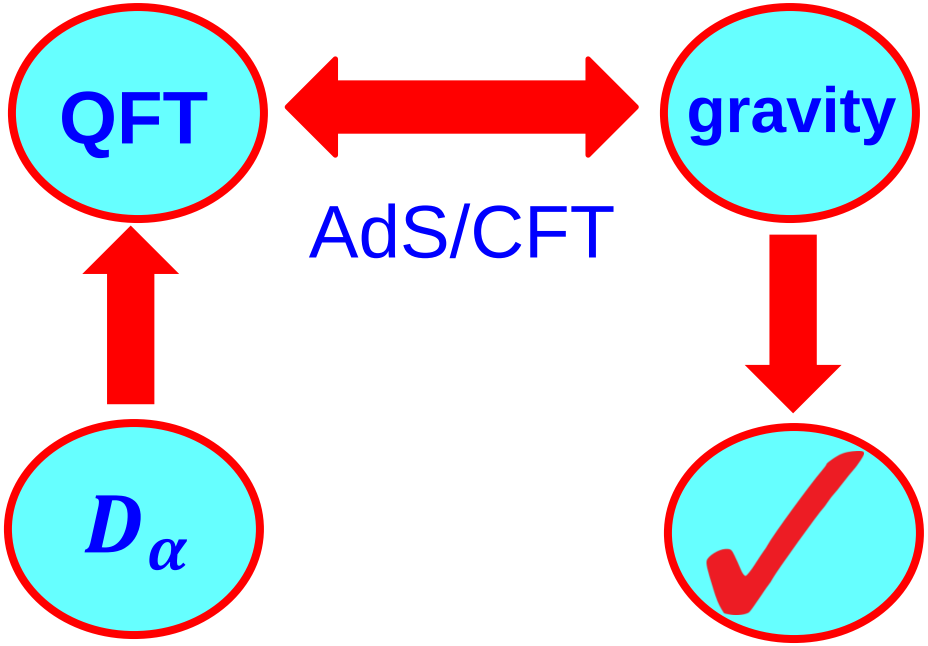

Conclusions & Outlook:

- quantum thermodyanmics provides new constraints on thermal processes, which constrain both thermalization in quantum field theory and gravitational dynamics
- calculated Renyi divergences in CFT only for special class of excited states; need to extend to more general states, more general QFTs and larger range, eg, $\alpha > 1$
- additional constraints from new distance functions
  → general reference states
- gravity constraints indirect through holography

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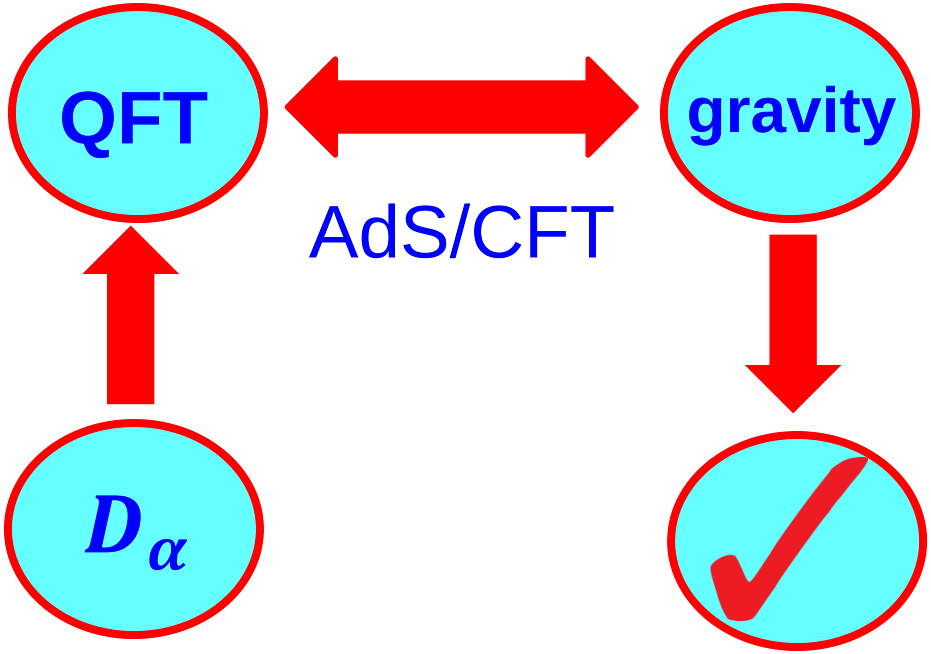

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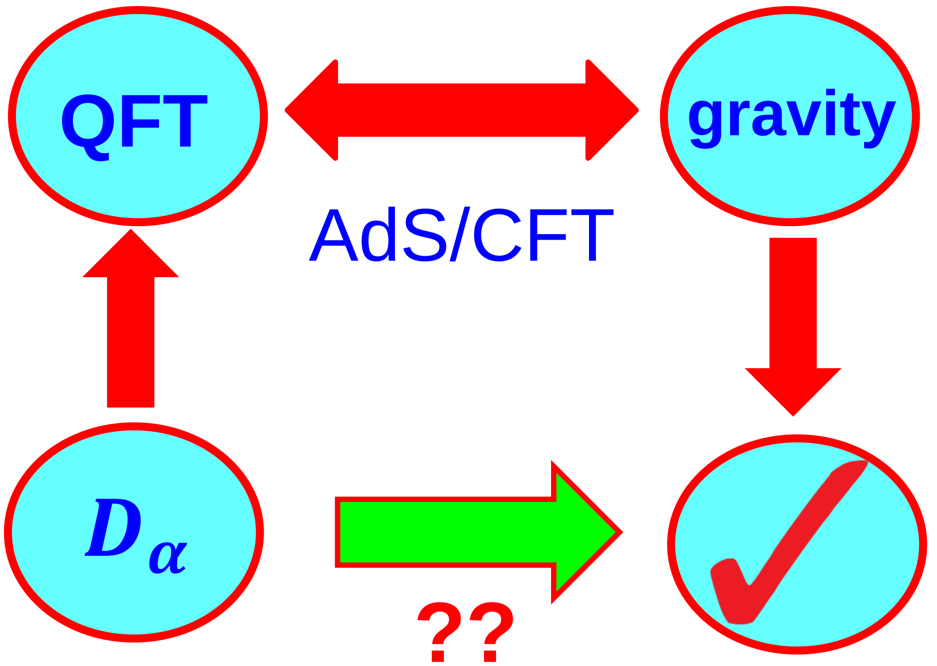

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  $\longrightarrow$ general reference states

- gravity constraints only indirect through holography

  - compute D_α directly in gravity?
  - phrase new constraints "geometrically"?