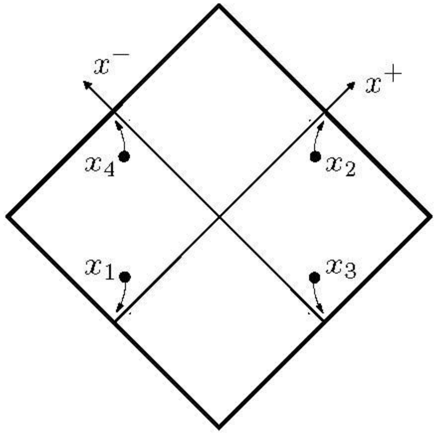
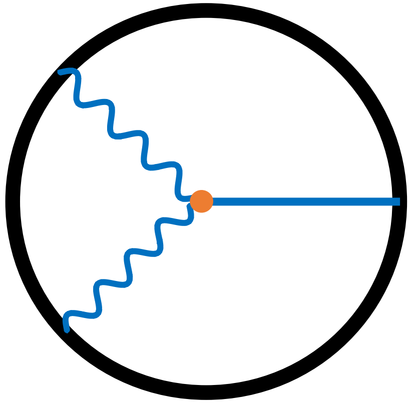


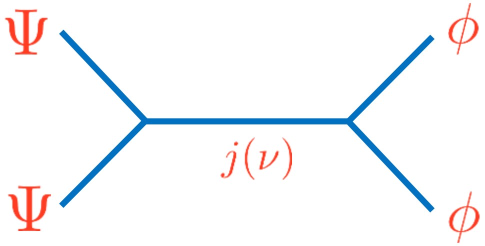


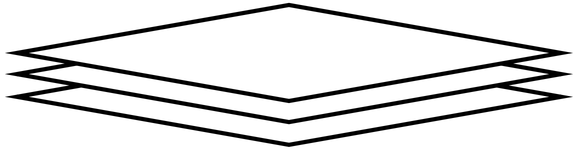
Conformal Bootstrap Constraints on Scattering in Gravity and M-Theory

Eric Perlmutter

Caltech, Simons Collaboration on the Nonperturbative Bootstrap

YITP, 7.3.18





This talk will address the following two questions and their CFT duals:

I. In an AdS effective action, what are the allowed gravitational couplings to matter?

II. What is the effective action of M-theory?

I. Gravitational Couplings to Matter

Low-energy actions are inherently IR objects, but are constrained by UV physics.

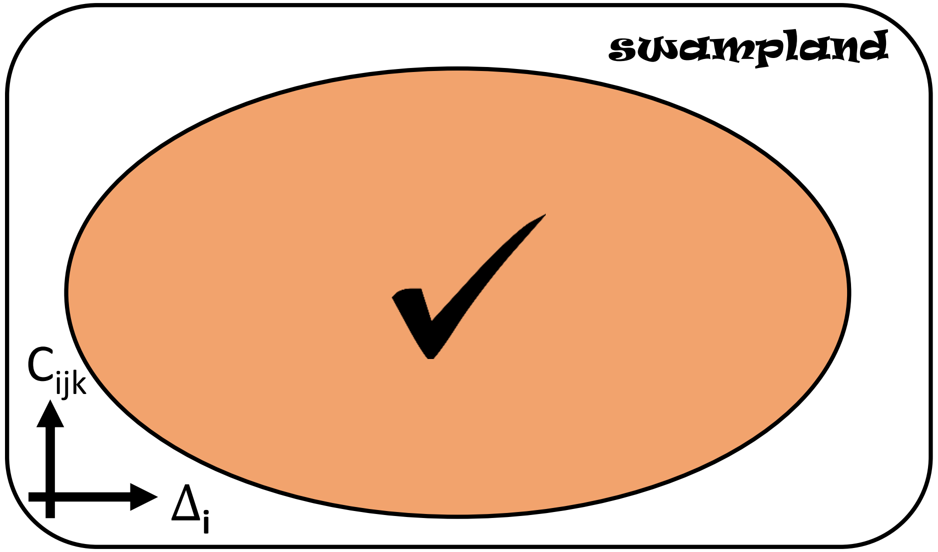
Sometimes these constraints are not immediately obvious from standard effective field theory.

$(\partial\phi)^4$: Sign-definite (causality)

$(\partial^2\phi)^4$: Cannot be part of a **local** action (chaos bound)

The Swampland is populated by examples of illegal effective field theories.

What are all of the laws of The Swampland?



I. Gravitational Couplings to Matter

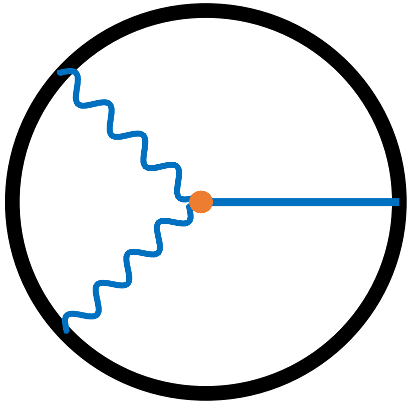
Consider the product of two stress tensors: $T(x)T(0) \sim \sum_{\mathcal{O}} C_{TT\mathcal{O}} \frac{\mathcal{O}(0)}{x^{2d-\Delta_{\mathcal{O}}}}$

This is a central object in any CFT. But we don't understand it well.

What can appear in the TT OPE?

↕

What couples linearly to two gravitons?



We'll focus on the case where O is *not* the stress tensor or a composite thereof.

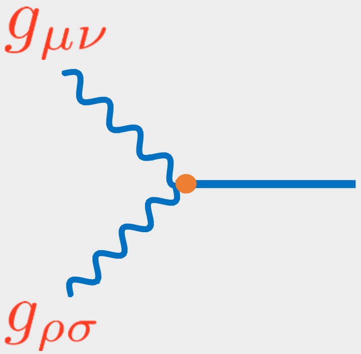
In a large N CFT, such TTO couplings determine whether there exists a consistent truncation to Einstein gravity in AdS.

I. Gravitational Couplings to Matter

In prototypical AdS/CFT, there *is* such a consistent truncation:

$$S_{\text{bulk}} = \int (R + 2\Lambda) + \int \left(\partial_\mu \phi^i \partial^\mu \phi^i + \lambda_{ijk} \phi^i \phi^j \phi^k + \lambda_{ijkl}(\partial) \phi^i \phi^j \phi^k \phi^l + \ldots\right)$$

On the level of cubic couplings,



$$\phi^i = 0 = \langle T_{\mu\nu} T_{\rho\sigma} \mathcal{O}^i \rangle$$

Where does this low-energy constraint come from?

(This question is unique to d>2.)

I. Gravitational Couplings to Matter

In AdS, the first coupling that survives field redefinitions has four derivatives:

$$S_{\rm bulk} \supset \lambda_{TT\phi} \int \phi\, C^2_{\mu\nu\rho\sigma}$$

So the question becomes, what suppresses this term and others like it?

More generally, what is the CFT "dual" of the derivative expansion in the bulk?

Our proposal – based on explicit calculations and previous results on stress tensor three-point couplings – will be: the expansion in the higher spin single-trace gap scale.

Counting AdS derivatives = Counting powers of $\Delta_{\rm gap}$

II. What is the effective action of M-theory?

We utterly lack a non-perturbative description of M-theory...

Perturbatively, M-theory = 11D SUGRA + higher derivative corrections.

Even so, explicit results are scarce:

$$\mathcal{A}^{11}(p_i;\zeta_i) = \mathcal{A}^{11}_{\text{SUGRA,tree}}(p_i;\zeta_i)\Big(1 + \ell_{11}^6 f_{R^4}(s,t) + \ell_{11}^{12} f_{D^6R^4}(s,t) + \dots\Big) + (\text{non-analytic})$$

Known

Unknown

- Results obtained by loop computations in 11D SUGRA + S^1/T^2 compactification + explicit string calculations/S-duality

[Green, Gutperle, Kwon, Russo, Vanhove, Tseytlin, Berkovits, Gomez, Mafra, Schlotterer, Stieberger, D'Hoker, Phong, Gross, Witten,...]

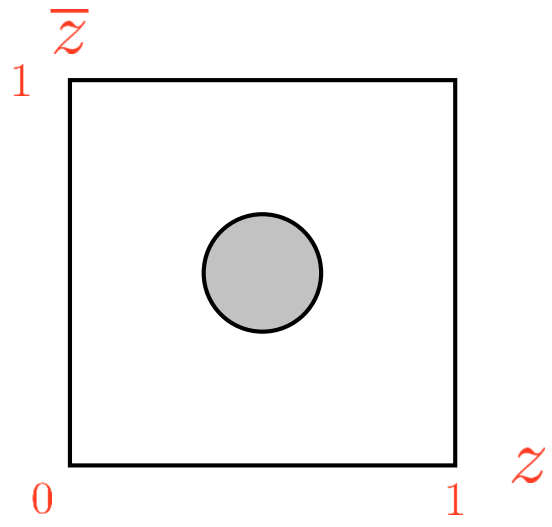
Idea: Use independent calculation of 6d (2,0) CFT data to build M-theory.

- Today, we will recover the M-theory amplitude through 12 derivatives.

[See also Chester, Pufu, Yin]

Tool: Lorentzian CFT correlators

The numerical bootstrap expands in a Lorentzian neighborhood of the crossing-symmetric point.



Recent analytic progress in the bootstrap has mostly come from thinking about other Lorentzian regimes.

- Lightcone
- Double-lightcone
- Regge
- Bulk-point (flat space)

[Fitzpatrick, Kaplan, Poland, Simmons-Duffin; Komargodski, Zhiboedov; Alday, Zhiboedov; Costa, Goncalves, Penedones; Okuda, Penedones; Penedones; Maldacena, Simmons-Duffin, Zhiboedov; Caron-Huot]

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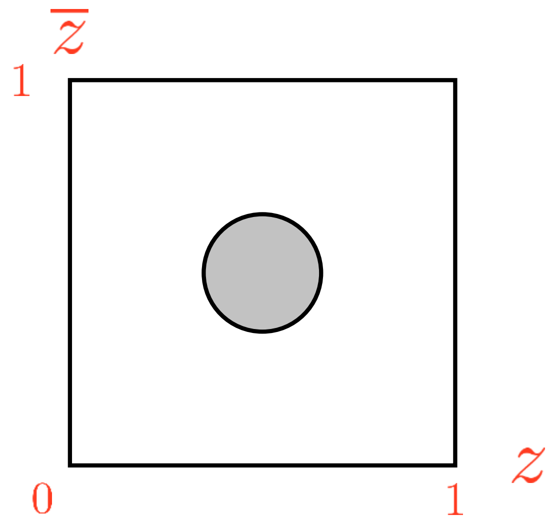

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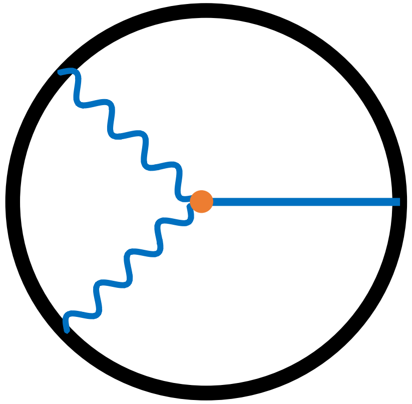
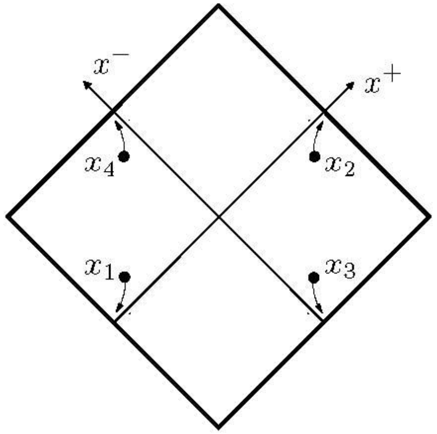
Outline



1. Review: defining holographic CFTs

2. Gravitational couplings to matter from CFT



3. M-theory effective action

4. Toward D^8R^4

Based on 1712.04861 (w/ D. Meltzer), 1805.00892 (w/ S. Chester) and WIP (w/L. Rastelli)

What is a “holographic” CFT?

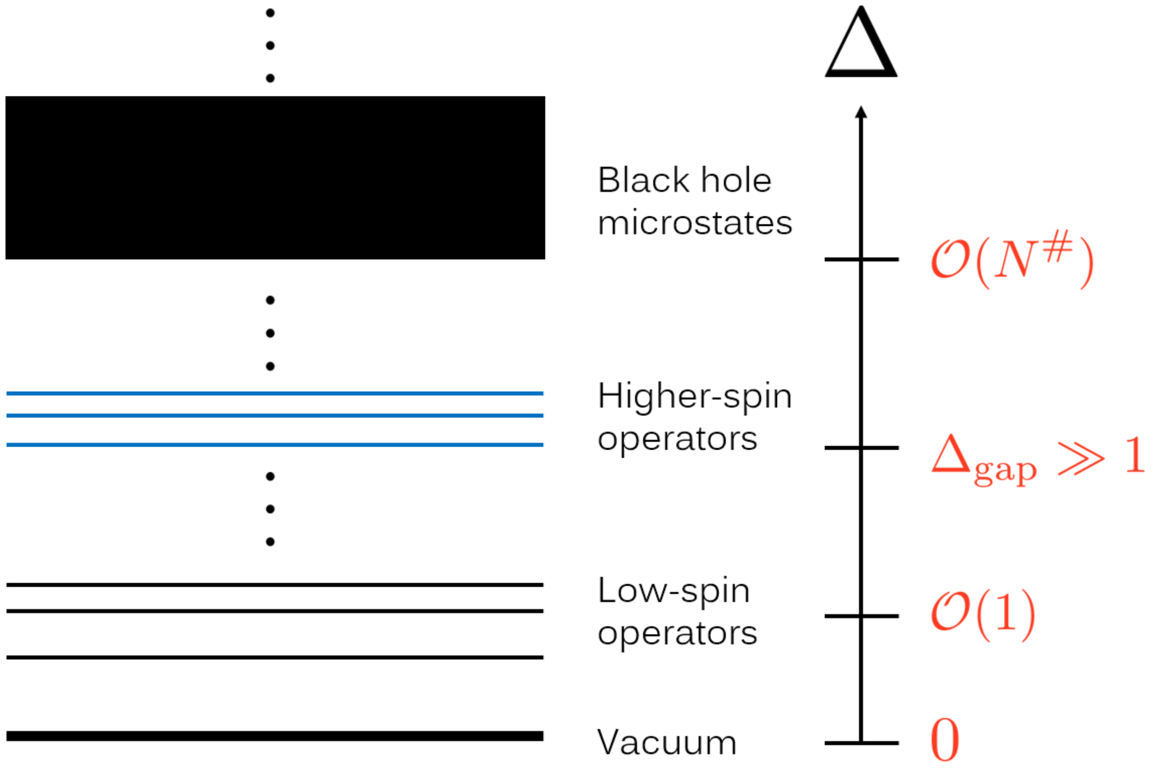
First, a brief review.

Defining Holographic CFTs

What conditions must a large N CFT satisfy such that its bulk dual is “simple”?

(e.g. General relativity + perturbative fields vs. Classical string theory)

Known supergravity examples have a spectrum like this:



Large higher-spin gap = “Strong coupling”

$$\Delta_{\rm gap} \sim M_{HS}$$

Defining Holographic CFTs

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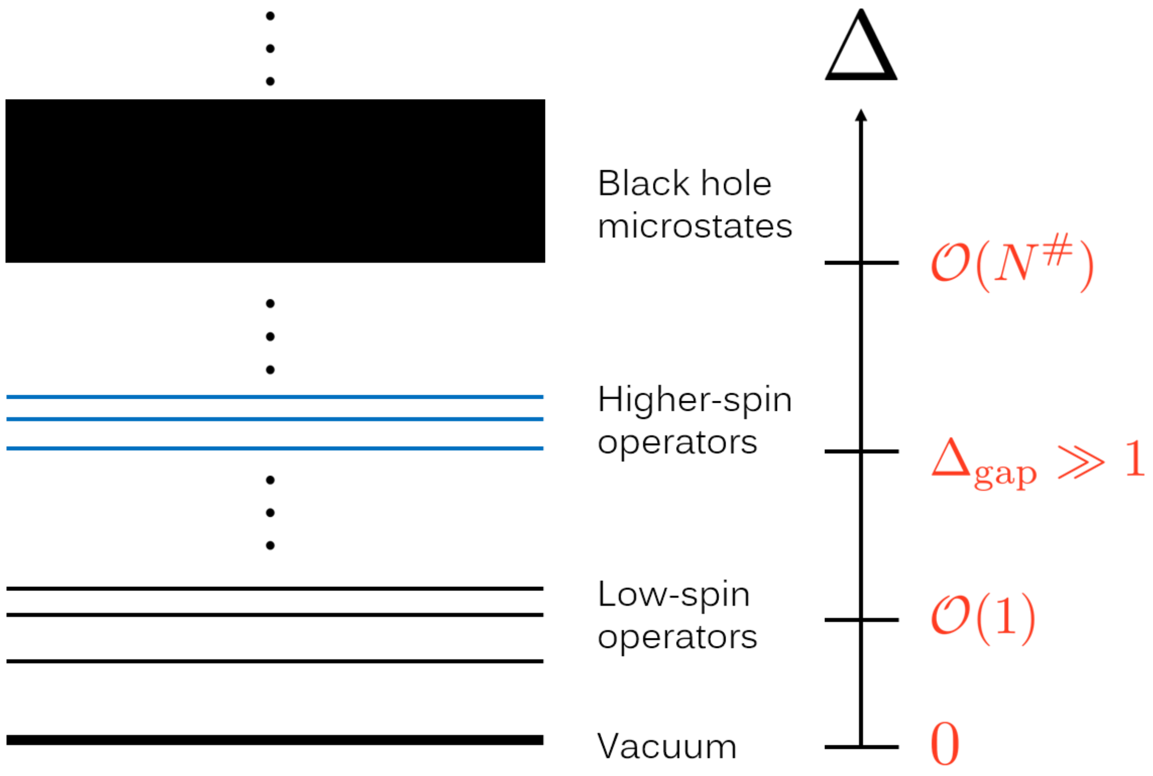


Large N + **Large gap** to higher-spin single-trace operators
=
Weakly coupled, local gravity dual

[Heemskerk, Penedones, Polchinski, Sully]

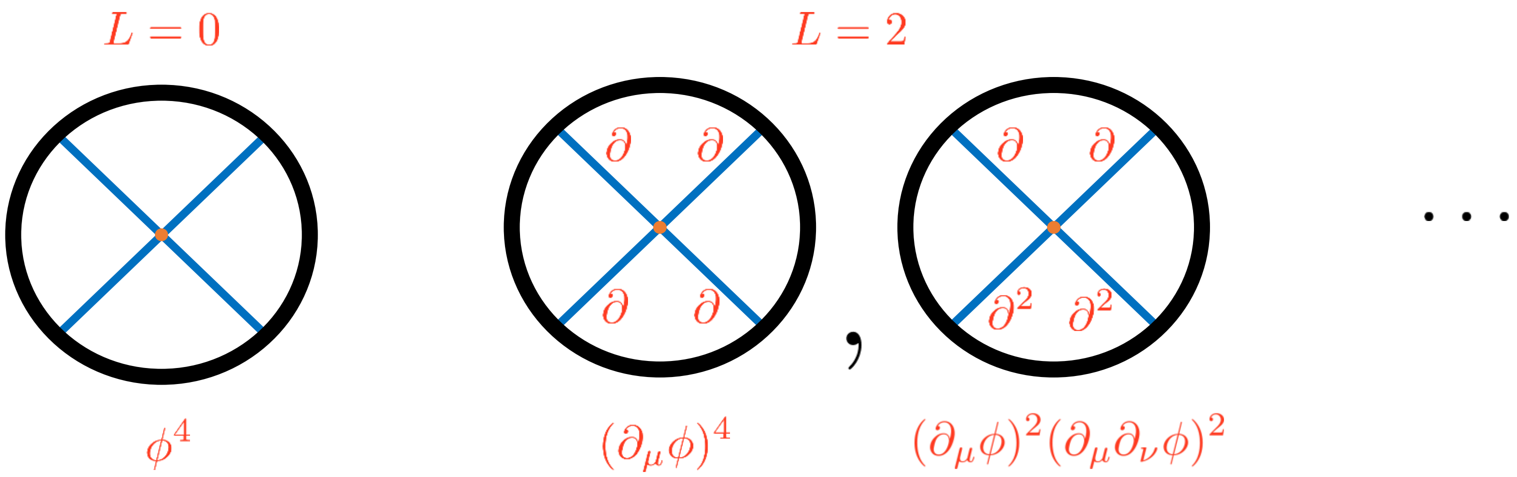
Defining Holographic CFTs

- 1-to-1 map between solutions of four-point crossing and AdS contact interactions.

$$\mathcal{O} \times \mathcal{O} \sim 1 + \sum_{n,\ell} [\mathcal{O}\mathcal{O}]_{n,\ell} + \cdots$$

[HPPS]

- At O(N^0), this is generalized free field theory. At O($1/N^2$), new solutions: $\gamma_{n,\ell>L} = 0$
  There are $\frac{(L+2)(L+4)}{8}$ solutions: precisely one for every independent, local quartic bulk vertex with ≤ 2L+2 derivatives.



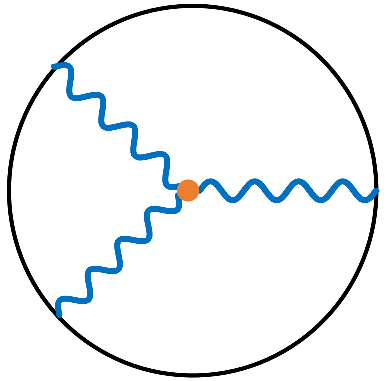
- These vertices are expected to be suppressed by the mass scale of new physics

[Caron-Huot]

Defining Holographic CFTs

The gap condition also suppresses higher-curvature corrections to general relativity.

[Camanho, Edelstein, Maldacena, Zhiboedov]



Large N + **Large gap** to higher-spin single-trace operators
=
Weakly coupled, local gravity dual containing general relativity

$$\langle TTT\rangle \sim \langle TTT\rangle_R + \Delta_{\text{gap}}^{-2}\langle TTT\rangle_{R^2} + \Delta_{\text{gap}}^{-4}\langle TTT\rangle_{R^3}$$

Universality in gravity is an emergent large gap phenomenon in CFT.

Other aspects of higher-d CFTs – e.g. thermodynamics, partition functions, entanglement structure – also signal bulk emergence. All are consistent with the gap hypothesis.

e.g. [Afkhami-Jeddi, Hartman, Kundu, Tajdini; Costa, Hansen, Penedones; Kulaxizi, Parnachev, Zhiboedov; Meltzer, EP; Belin, de Boer, Krufhoff, Michel, Shaghoulian, Shyani; Mefford, Shagloulian, Shyani]

Defining Holographic CFTs

$$\langle TTT \rangle \sim \langle TTT \rangle_R + \Delta_{\text{gap}}^{-2} \langle TTT \rangle_{R^2} + \Delta_{\text{gap}}^{-4} \langle TTT \rangle_{R^3}$$

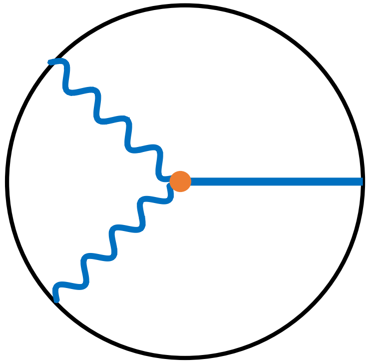
However:

1) No explicit examples where these bounds are even parametrically saturated.

4d CFT: $$\frac{|a-c|}{c} \lesssim \Delta_{\text{gap}}^{-2}$$

In families of supersymmetric CFTs, neither a nor c can depend on Δ_{gap}.

2) What about gravitational couplings to matter?
(Isn't this the hard part of UV completing quantum gravity??)



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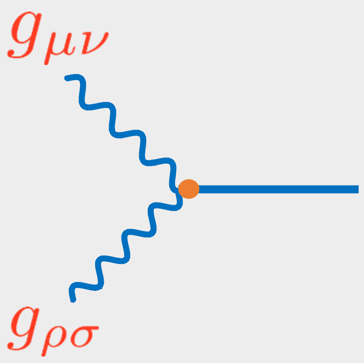
On the level of cubic couplings,



$$\phi^i = 0 = \langle T_{\mu\nu} T_{\rho\sigma} \mathcal{O}^i \rangle$$

Where does this low-energy constraint come from?

(This question is unique to d>2.)

I. Gravitational Couplings to Matter

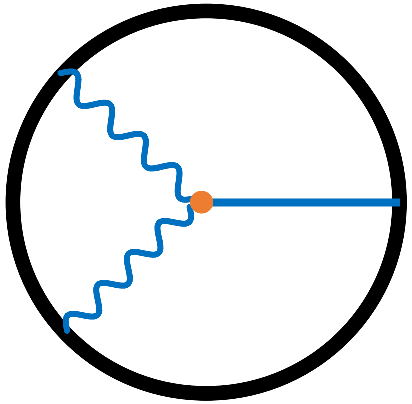
More robust than a-c: <TTO> can vary in 4d N<4 SCFT, even if O is protected.

$$\text{Is } \langle T_{\mu\nu} T_{\rho\sigma} \mathcal{O} \rangle \sim \Delta_{\text{gap}}^{-2}\,?$$

This would imply that consistent truncation to Einstein gravity follows from the absence of light higher spin particles.

In AdS, the first coupling that survives field redefinitions has four derivatives:

$$S_{\text{bulk}} \supset \lambda_{TT\phi} \int \phi\, C_{\mu\nu\rho\sigma}^2$$



What is the CFT dual of the derivative expansion in AdS?

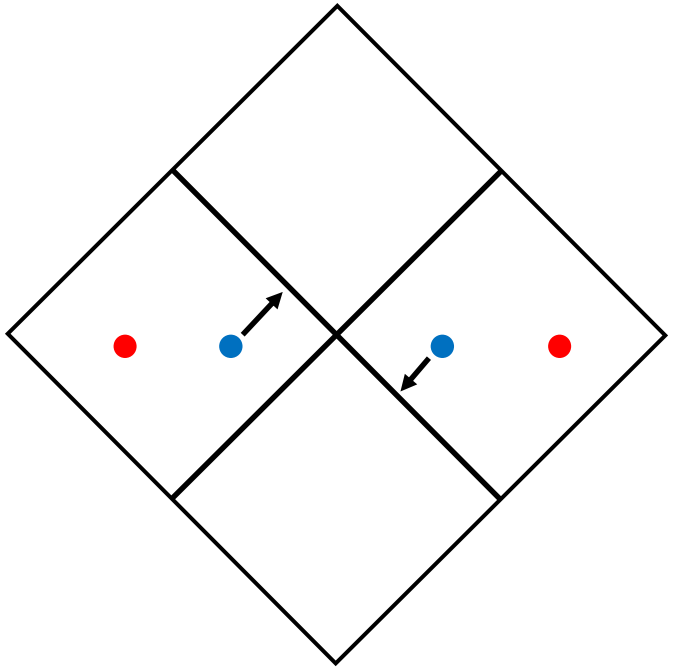
Some notation:

$$\langle \phi(0)\phi(z,\bar{z})\phi(1)\phi(\infty)\rangle =$$

$$= (z\bar{z})^{-\Delta_\phi} G(z,\bar{z})$$

Lightcone limit

The simplest is the lightcone limit: $z \ll 1 \;,\; \bar{z}$ fixed



The "lightcone bootstrap" analytically proved the existence of infinite towers of large spin "double-twist" conformal primaries,

$$\phi(\partial^2)^n \partial_{\mu_1} \dots \partial_{\mu_J} \phi$$

These operators exist to all orders in a 1/J expansion, with anomalous dimensions suppressed by powers of 1/J:

$$\Delta_{n,J\gg1} \approx 2\Delta_\phi + 2n + J - \frac{C^2_{\phi\phi\mathcal{O}_*}}{J^{\Delta_*-\ell_*}} \quad \text{where} \quad \phi \times \phi \sim \mathbf{1} + C_{\phi\phi\mathcal{O}_*}\mathcal{O}_* + \dots$$

Follows from matching singularities in s- and t-channel.

[Fitzpatrick, Kaplan, Poland, Simmons-Duffin; Komargodski, Zhiboedov; Kaviraj, Sen, Sinha; Alday; Simmons-Duffin]

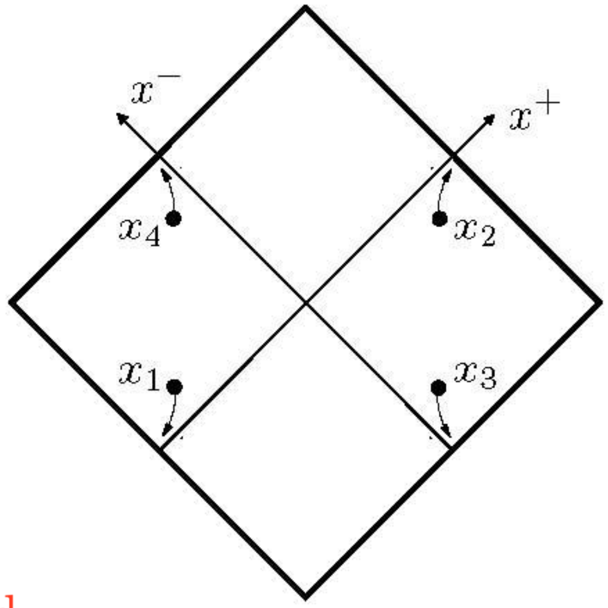
Regge limit

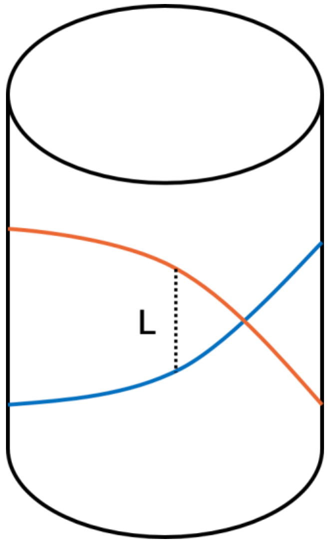
Regge limit = Lorentzian regime of double-null separations

Euclidean four-point functions may be analytically continued:

$$G^{\text{Regge}}(z,\bar{z}) = G(z,\bar{z}) \quad \text{subject to} \quad \begin{cases} (1-\bar{z}) \to e^{-2\pi i}(1-\bar{z}) \\ z \to 0\ , \quad \bar{z} \to 0\ , \quad e^{2\rho} \equiv \frac{z}{\bar{z}} \ \text{fixed} \end{cases}$$

Dual to high-energy, fixed impact parameter scattering in AdS.

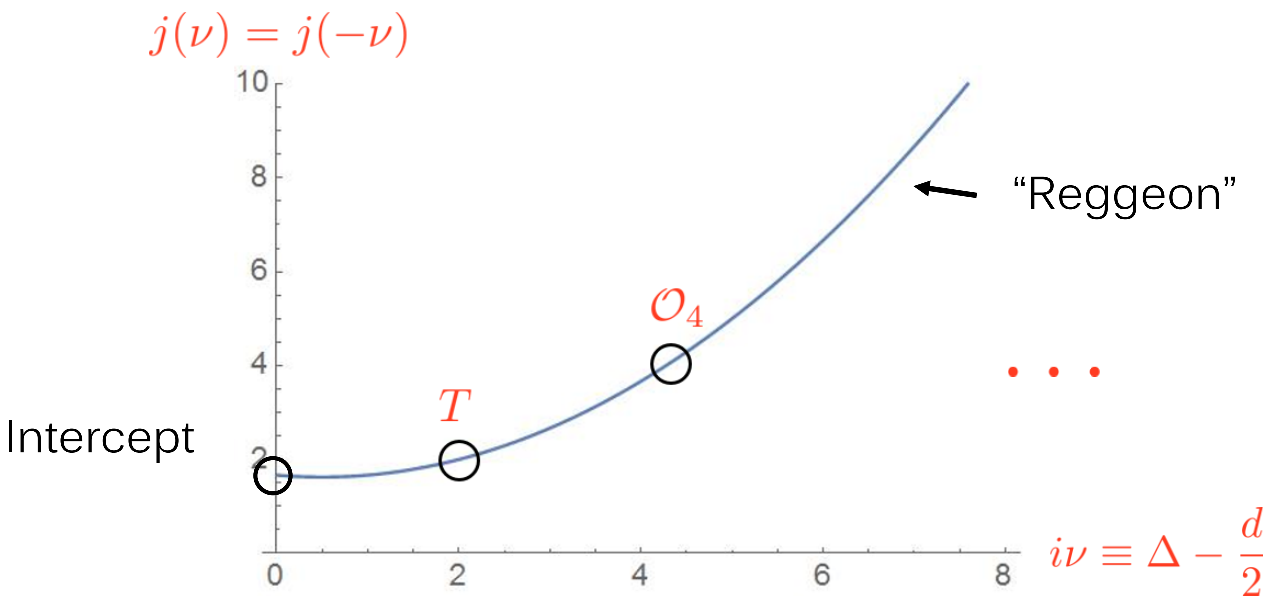




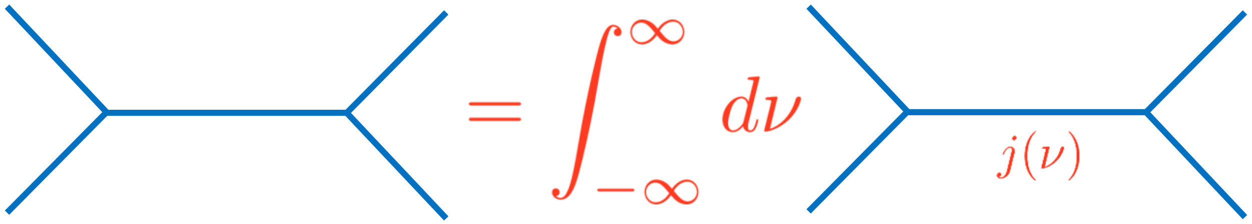
[Cornalba, Costa, Goncalves, Hansen, Penedones; Kulaxizi, Parnachev, Zhiboedov; Li, Meltzer, Poland]

Regge limit

Contributions to CFT correlators in the Regge limit are organized into Regge trajectories:



Leading Regge trajectory = the set of operators of lowest dimension at each spin, analytically continued to continuous spin.

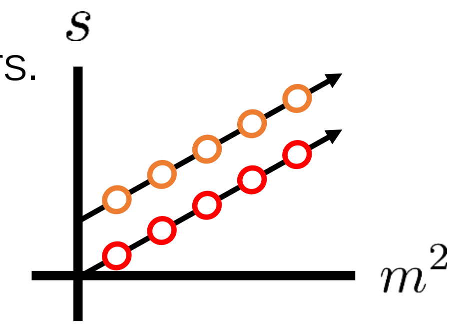


These contributions are computed by "conformal Regge theory"

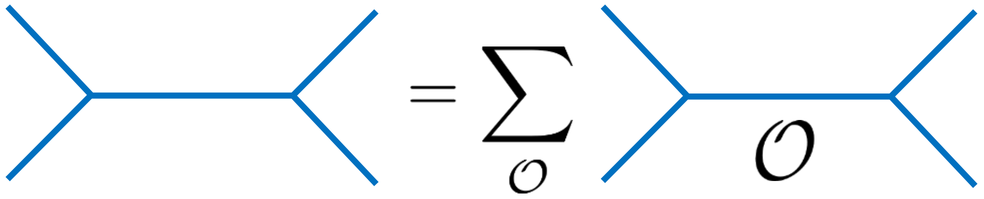
Regge limit as a bootstrap tool

String theory has soft behavior in the Regge limit, despite infinite towers of operators.

This "conspiracy" is intimately tied to the existence of (linear) Regge trajectories.



This has a precise analog in CFT:

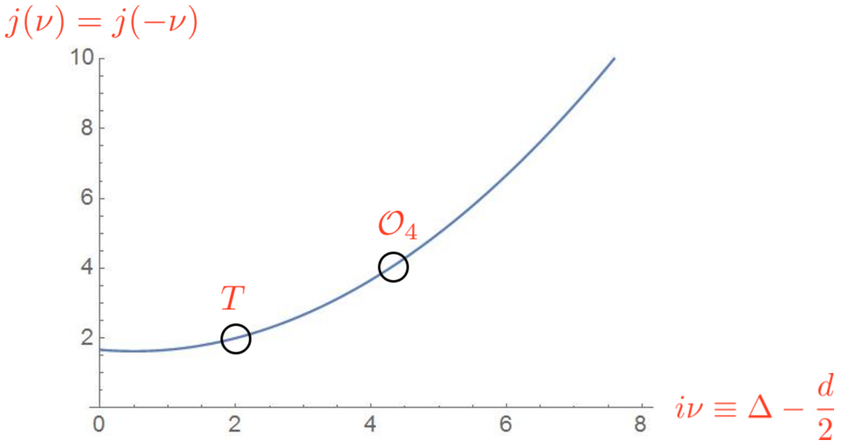


$\sim z^{1-J_*}$ $\sim z^{1-J_{\mathcal{O}}}$

Inversion formula: operators come in families analytic in J, with rigid OPE structure.

Moreover, there is a universal **bound** on the behavior of correlation functions in the Regge limit. At large N, j(0) ≤ 2.

→ "Bootstrap" constraint.



[Maldacena, Shenker, Stanford; EP; Caron-Huot]

TTO Couplings from the Regge Limit

We impose unitarity in the Regge limit of mixed systems of spinning four-point functions.

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External operators carry spin.

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A matrix of four-point functions.

$$\begin{pmatrix} \langle T\phi\phi T\rangle & \langle T\phi\phi\mathcal{O}\rangle \\ \langle \mathcal{O}\phi\phi T\rangle & \langle \mathcal{O}\phi\phi\mathcal{O}\rangle \end{pmatrix}$$

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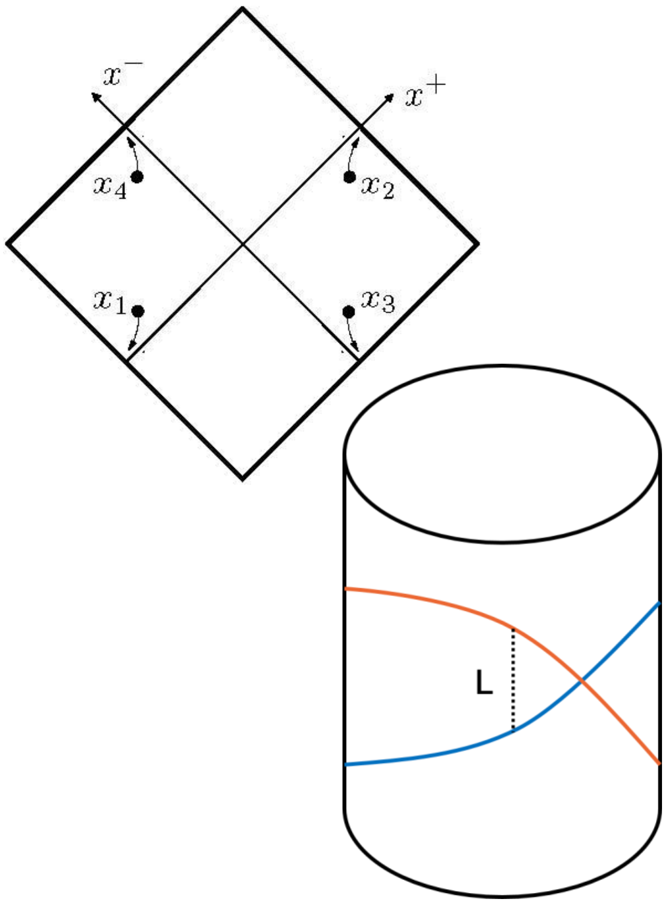
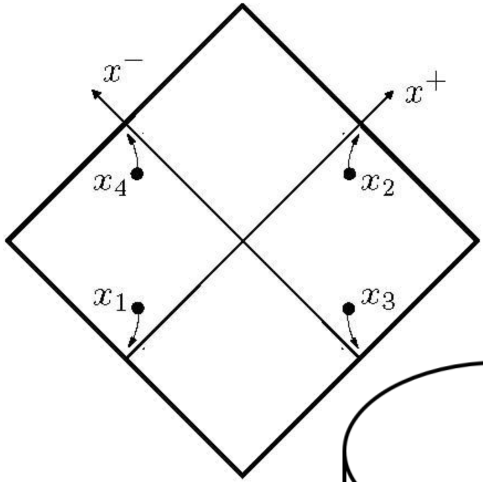
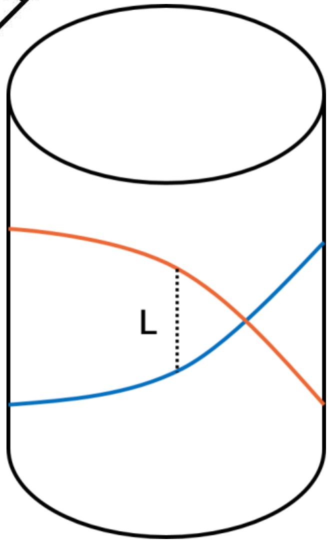


TTO Couplings from the Regge Limit

We impose unitarity in the Regge limit of mixed systems of spinning four-point functions.

Growth of correlations is bounded. (Chaos destroys order.)





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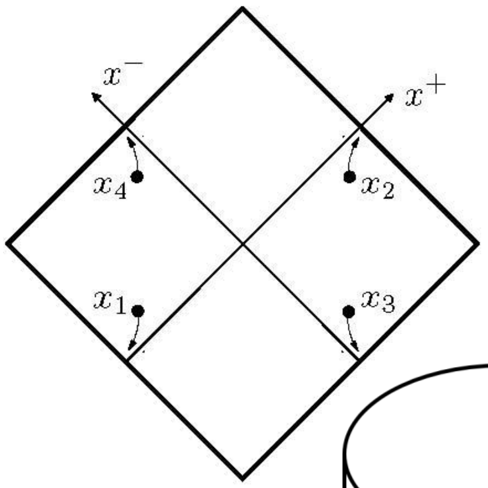

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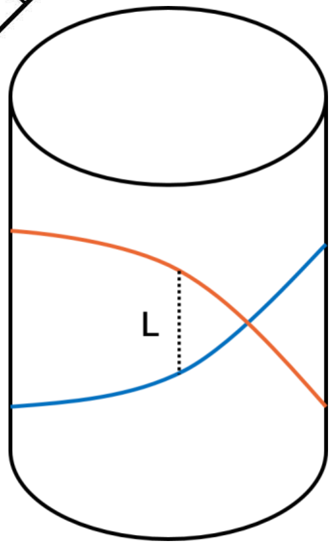

By tuning L, we access different physical regimes:

L>>1: lightcone physics/ANEC

L<<1: probe bulk locality

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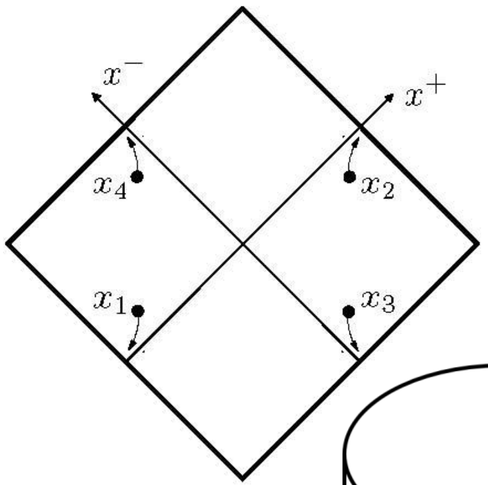

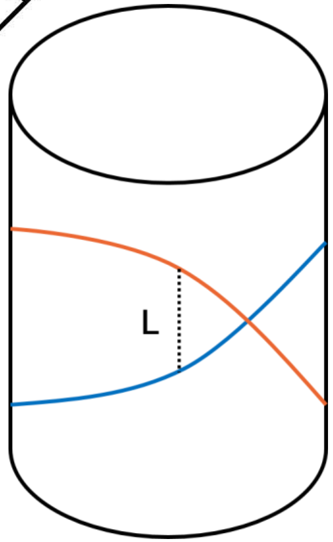

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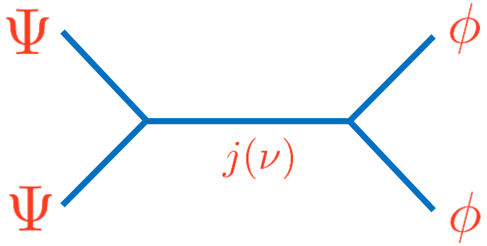
At small impact parameter L ~ $1/\Delta_{\text{gap}}$

$$\begin{pmatrix} \langle TTT\rangle & \langle TT\mathcal{O}\rangle\Delta^2_{\text{gap}} \\ \langle TT\mathcal{O}\rangle\Delta^2_{\text{gap}} & \langle T\mathcal{O}\mathcal{O}\rangle \end{pmatrix} \succeq 0$$

TTO Couplings from the Regge Limit

$$\langle \Psi | \phi\phi | \Psi \rangle$$

- Conformal Regge theory computes the contribution of leading Regge trajectory, parameterized by $j(\nu)$, to this CFT correlator.



- Take "mixed" state: $|\Psi\rangle = |\epsilon \cdot T + c_{\mathcal{O}}\mathcal{O}\rangle$

- In the $\Psi\Psi \to j(\nu) \to \phi\phi$ channel, unitarity upper-bounds the off-diagonal couplings:

$$\det \begin{pmatrix} \langle T j(\nu) T \rangle & \langle T j(\nu) \mathcal{O} \rangle \\ \langle T j(\nu) \mathcal{O} \rangle & \langle \mathcal{O} j(\nu) \mathcal{O} \rangle \end{pmatrix} \geq 0$$

- T lives on this trajectory, at j=2. This bounds <TTO>.

TTO Couplings from the Regge Limit

Some details:

- Solve in saddle-point approximation at high-energy.

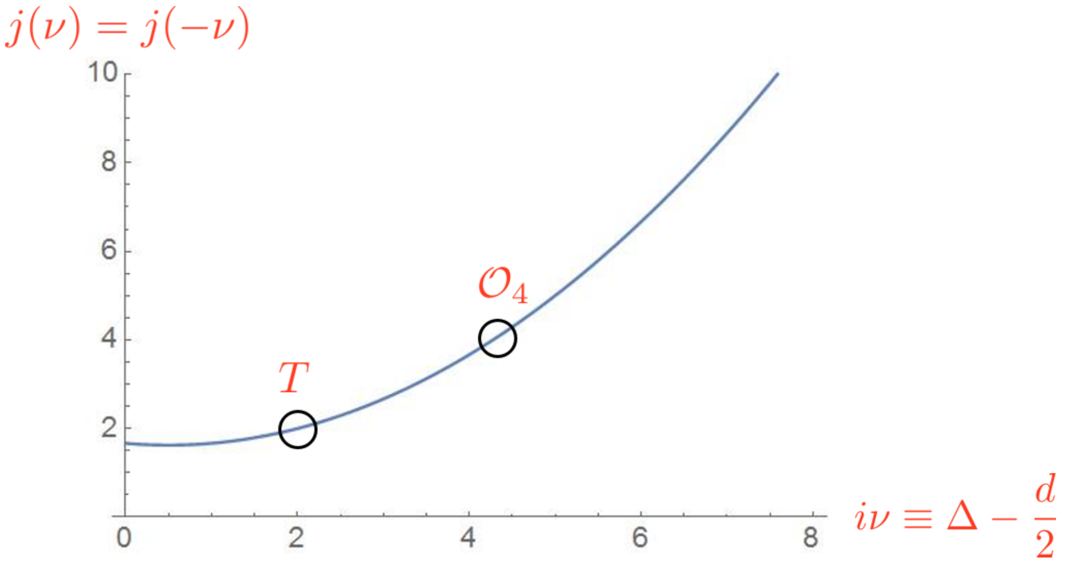
  The solution is a function of L.

  Changing L = sliding along trajectory.

- The Regge limit of spinning three-point structures has extra derivatives for <TTO> versus <TTT>.

  These derivatives act on the Regge limit of the conformal blocks. At small L, the blocks behave like a power law.

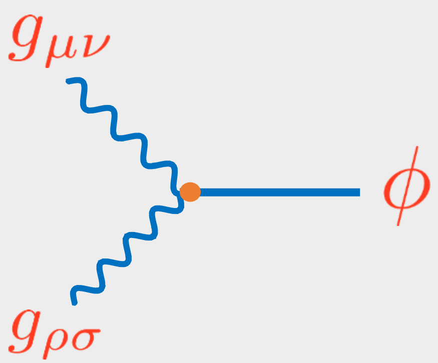
  For scalar O, there are two more derivatives. This leads to suppression by two powers of the gap.



$$\langle T_{\mu\nu} T_{\rho\sigma} \mathcal{O} \rangle \sim \Delta_{\text{gap}}^{-2}$$

The holographic dual of a derivative

Therefore, on the level of cubic couplings, the existence of a consistent truncation to Einstein gravity in a theory of gravity is a consequence of the absence of higher spin matter.



$$\sim \frac{1}{M_{\mathrm{HS}}^2}$$

Similar results for other couplings <TAB> for A,B of spin-0,1,2, conserved and non-conserved.

In every case, the interaction is suppressed by the expected power of the gap.

Universal Bounds on the Stress Tensor OPE

At L >> 1, Regge limit = lightcone limit → ANEC bounds. If we don't impose large gap,

$$\boxed{\sum_{\text{Scalar } \mathcal{O}} \langle TT\mathcal{O}\rangle^2 f(\Delta_{\mathcal{O}}) \leq n_B}$$

$$\frac{(d-1)^3 d\pi^{2d}\Gamma\left(\frac{d}{2}\right)\Gamma(d+1)\Gamma(\Delta)\Gamma\left(\Delta-\frac{d-2}{2}\right)}{(d-2)^2\Gamma^4\left(2+\frac{\Delta}{2}\right)\Gamma^2\left(\frac{d+\Delta}{2}\right)\Gamma^2\left(d-\frac{\Delta}{2}\right)}$$

where n_B is the coefficient of the "free boson structure" in TTT: $\langle TTT\rangle = \sum_{i=B,F,V} n_i \langle TTT\rangle_i$

This was first derived via the ANEC [Cordova, Maldacena, Turiaci]

It applies to all CFTs: lightcone physics is universal, not due to large N or large gap.

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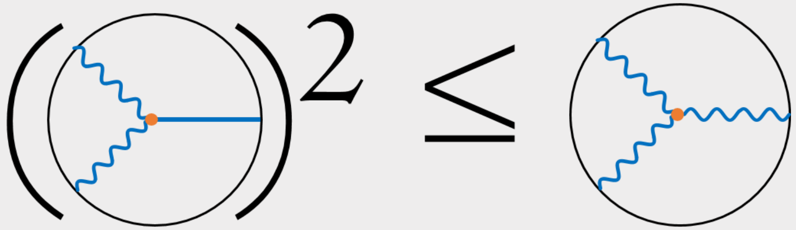
In any consistent theory of AdS quantum gravity, matter couplings to two gravitons are bounded by the graviton self-coupling.



Universal Bounds on the Stress Tensor OPE

We derived similar bounds for O with spin.

For O = non-conserved spin-2, <TTO> has three structures, the same number as <TTT>:

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Using analyticity in spin of CFT data (cf. inversion formula, light-ray operators), all spin-2 operators can be organized into families extending to asymptotically large spin. There is an infinite number of such families, owing to the existence of multi-twist operators.

[Caron-Huot; Kravchuk, Simmons-Duffin]

This implies that, barring the decoupling of an infinite number of spin-2 operators,

$$n_B > 0 \, , \quad n_F > 0 \, , \quad n_V > 0$$

in any interacting CFT. (Free CFT spectra mostly sit at zeroes of f(Δ).) [Zhiboedov]

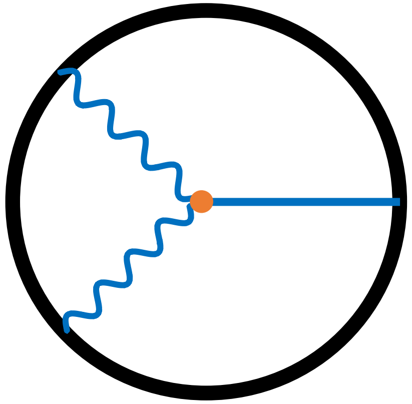
Outline

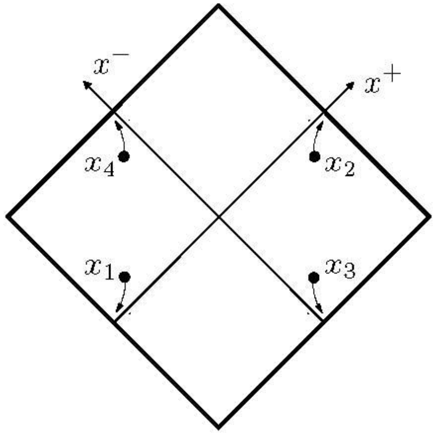
1. Review: defining holographic CFTs

2. Gravitational couplings to matter from CFT

3. M-theory effective action

4. Toward D^8R^4





11D Four-Graviton Scattering

Instead of the effective action, it's less ambiguous to study on-shell amplitudes:

$$\mathcal{A}^{11} = \ell_{11}^{9} \widehat{K}(p_i, \zeta_i) \frac{2^6}{stu} \Big(1 + \sum_{k=0} \ell_{11}^{6+2k} f_{D^{2k}R^4}(s,t) \Big) + (\text{non-analytic})$$

K is an 8-derivative kinematic factor = R^4 in momentum space, linearized in the graviton fluctuation and contracted with polarizations.

From 11D Feynman diagrams + IIA string amplitudes, the following terms are known:

$$f_{R^4}(s,t) = \frac{stu}{3 \cdot 2^7} \qquad \text{1-loop on } S^1_{R_{11}} \times \mathbb{R}^{9,1}$$

$$f_{D^6R^4}(s,t) = \frac{(stu)^2}{15 \cdot 2^{15}} \qquad \text{2-loop on } S^1_{R_{11}} \times \mathbb{R}^{9,1}$$

Finite terms in decompactification limit

11D Four-Graviton Scattering

$$f_{R^4}(s,t) = \frac{stu}{3 \cdot 2^7}$$

$$f_{D^6 R^4}(s,t) = \frac{(stu)^2}{15 \cdot 2^{15}}$$

These terms are special:

1. They are protected by 11D SUSY from higher-loop corrections.

2. They match certain perturbative terms in type IIA, even though M-theory is the strong coupling limit! (The corresponding terms in IIA obey N.R. theorems.)

$$R_{11} = g_s \ell_s = g_s^{2/3} \ell_{11}$$

3. They are finite in the decompactification limit, but are not necessarily the leading terms at their derivative order.

(e.g. in type IIA, D^6R^4 truncates at *three* loops in perturbation theory, not two. The three loop term ~ $(R_{11})^3$ is part of an infinite series of divergent terms of the 1-loop 11D amplitude, which re-sum into a non-analytic contribution due to massless KK modes.)

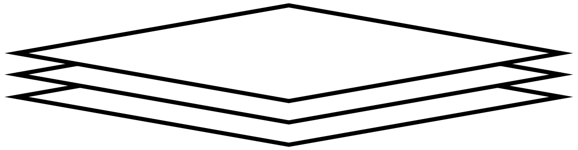
(2,0) CFT basics

- A_{N-1} (2,0) CFT = Worldvolume CFT of N M5 branes in $R^{10,1}$
- Central charge: $c(A_{N-1}) = 4N^3 - 3N - 1$
- ½ -BPS operators dual to scalar KK modes on $AdS_7 \times S^4$:

$$S_k(x,Y) \equiv S_{I_1 \ldots I_k} Y^{I_1} \cdots Y^{I_k} \in \mathcal{D}[k0] \text{ irrep of } \mathfrak{so}(5)_R$$

$$\Delta_k = 2k \ , \ \text{where } k = 2, 3, \ldots, N$$

(Symmetric traceless tensors of so(5))



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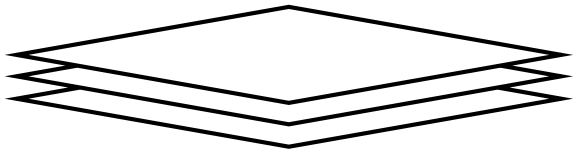

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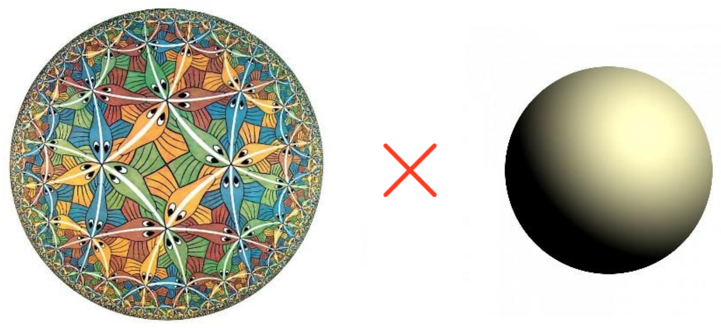


Important basic point:

Because $L_{Sphere} \sim L_{AdS}$, powers of 11D Planck length = powers of 1/c.

$$\mathcal{L}_{11} \approx R + \sum_{k=0}^{\infty} \ell_{11}^{6+2k} D^{2k} R^4 \xrightarrow{\left(\frac{L_{\text{AdS}}}{\ell_{11}}\right)^9 \approx 16c} \mathcal{L}_{11} \approx R + \sum_{k=0}^{\infty} c^{-\frac{2}{3}-\frac{2k}{9}} D^{2k} R^4$$

Many of our results can be understood by following this replacement rule.

(2,0) CFT basics

We will study 4-point functions of the ½-BPS operators:

$$\langle S_k(x_1,Y_1)S_k(x_2,Y_2)S_k(x_3,Y_3)S_k(x_4,Y_4)\rangle = \frac{(Y_1\cdot Y_2)^k(Y_3\cdot Y_4)^k}{|x_{12}|^{4k}|x_{34}|^{4k}}\mathcal{G}_k(z,\bar{z};\sigma,\tau)\,,$$

(R-symmetry cross-ratios: $\sigma \equiv \dfrac{(Y_1\cdot Y_3)(Y_2\cdot Y_4)}{(Y_1\cdot Y_2)(Y_3\cdot Y_4)}\,, \qquad \tau \equiv \dfrac{(Y_1\cdot Y_4)(Y_2\cdot Y_3)}{(Y_1\cdot Y_2)(Y_3\cdot Y_4)}$)

These admit an expansion in superconformal blocks.

$$\mathcal{G}_k(z,\bar{z};\sigma,\tau) = \sum_{\mathcal{M}} \lambda^2_{k,\mathcal{M}}\mathfrak{G}_{\mathcal{M}}(z,\bar{z};\sigma,\tau)$$

All we will need is that the higher-k ½-BPS representations appear in the OPE

$$S_k \times S_k \supset \bigoplus_{n=1}^{k}\mathcal{D}[2n\,0]$$

$$\approx S_{2n} + \text{composites } (:S_nS_n:, \text{etc.})$$

The W_N Chiral Algebra Conjecture

[Beem, Rastelli, van Rees]

Remarkably, the ½ -BPS (and some ¼-BPS) 3-point functions are fixed by the quantum W_N algebra at $c = 4N^3-3N-1$.

Generated by holomorphic currents of spins s=2,3,...,N: $W_i(z)W_j(0) \sim \sum_k C_{ijk} \frac{W_k(0)}{z^{i+j-k}}$.

At large c, the operator map is (modulo mixing)

$$S_k \quad \leftrightarrow \quad W_k$$
$$\lambda_{k_1 k_2 k_3} \leftrightarrow C_{k_1 k_2 k_3}$$

This can be derived by correlating R- and x-space cross ratios in the 4-point functions:

$$\begin{aligned} \mathcal{G}_k(z)|_{2d} &\equiv \mathcal{G}_k(z, \bar{z}; \bar{z}^{-2}, (1-\bar{z}^{-1})^2) \, . \\ &= z^{2k} \langle W_k(0) W_k(z) W_k(1) W_k(\infty) \rangle \, , \end{aligned}$$

Crossing + holomorphy imply that the chiral correlator is fully fixed by [2k/3] OPE coefficients.
This is not obvious from the 6d perspective.

[Bouwknegt; Keller; Headrick, Maloney, EP, Zadeh]

The W_N Chiral Algebra Conjecture

This is an extremely powerful claim: quantum M-theory from a familiar chiral algebra!

The lowest non-trivial (i.e. non-Virasoro) structure constant is 3-3-4:

$$\lambda^2_{3,\mathcal{D}[40]} = \frac{1}{6}(C_{334})^2$$

6d 2d

[Gaberdiel, Gopakumar; Prochazka; Linshaw]

$$\lambda^2_{3,\mathcal{D}[40]} = \left.\frac{24(c+2)(N-3)(c(N+3)+2(N-1)(4N+3))}{c(5c+22)(N-2)(c(N+2)+(N-1)(3N+2))}\right|_{c=4N^3-3N-1}$$

The Jacobi identity implies that *all* C_{ijk} are (powers of) rational functions of N and C_{334}.

In the 1/c expansion, this implies the following behavior:

$$\boxed{\begin{gathered}\lambda^2_{k_1k_2k_3} = c^{-1}F_R(c) + c^{-5/3}F_{R^4}(c) + c^{-7/3}F_{D^6R^4}(c)\,, \\ F_i(c) = \sum_{n=0}\alpha_i^{(n)}c^{-n} \text{ for some constants } \alpha_i^{(n)}\end{gathered}}$$

11D Uplift, Part 1

$$\lambda^2_{k_1k_2k_3} = c^{-1}F_R(c) + c^{-5/3}F_{R^4}(c) + c^{-7/3}F_{D^6R^4}(c)\,,$$
$$F_i(c) = \sum_{n=0} \alpha_i^{(n)} c^{-n} \text{ for some constants } \alpha_i^{(n)}$$

$\Longrightarrow$ Minimal explanation: the only protected 11D vertices are R, R^4, D^6R^4.

Argument:
$$\mathcal{L}_{11} \approx R + \sum_{k=0}^{\infty} c^{-\frac{2}{3}-\frac{2k}{9}} D^{2k}R^4$$

$$\Big\downarrow \text{(Legs on } S^4\text{)}$$

$$\mathcal{L}_7 \supset g_{123}(c)\phi_1\phi_2\phi_3 \approx (g_{123}^{(R)} + \sum_{k=0}^{\infty} c^{-\frac{2}{3}-\frac{2k}{9}} g_{123}^{(k)})\phi_1\phi_2\phi_3$$

1. No $c^{-17/9}$, $c^{-19/9}$ → No D^2R^4, D^4R^4 (barring ∞ cancellations)
2. All powers consistent with R, R^4, D^6R^4 + loops thereof

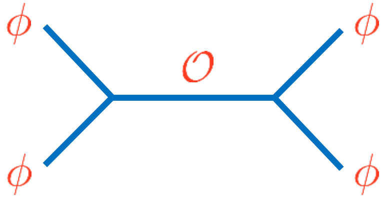
(2,0) 4-point functions

To derive 11D coefficients from CFT, use 4-point functions

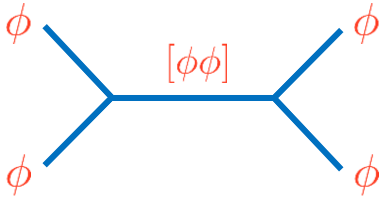
1. Determine via independent CFT computation.
2. Take flat space limit.
   - In Mellin space, take large s,t. (In x-space, this is the “bulk-point limit”.)

$$\mathcal{G}_k(z,\bar{z};\sigma,\tau) = \int_{-i\infty}^{i\infty} ds\, dt\, d\Gamma(z,\bar{z};s,t)\mathcal{M}_k(s,t;\sigma,\tau)$$

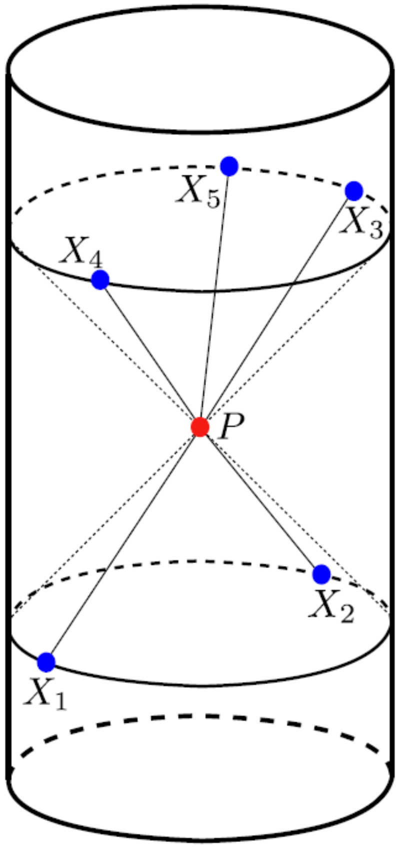
Tree-level Mellin amplitudes are simple:



Simple poles at $s = \tau_{\mathcal{O}} + 2n$



Polynomial

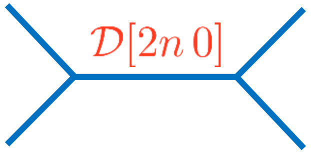


[Polchinski; HPPS; Mack; Penedones; Maldacena, Simmons-Duffin, Zhiboedov]

(2,0) 4-point functions

Accordingly, two types of solutions:

$$\mathcal{M}_{k,\text{mero}} = \sum_{n=1}^{k-1} a_{k,\mathcal{D}[2n\,0]}$$



$$+\,\mathcal{M}_{k,\text{poly}}$$

$$\mathcal{M}_{k,\text{pure}-\text{poly}}$$

In a general (non-supersymmetric) CFT, without using the AdS action we have no control over the polynomial terms.

However, for (2,0), the amplitude must solve the maximal superconformal Ward ID:

$$\mathcal{M}_k(s,t;\sigma,\tau) = \Theta \circ \widetilde{\mathcal{M}}_k(s,t;\sigma,\tau)$$

Difference operator

- Degree-8 in (s,t)
- Degree-2 in (σ, τ)

“Reduced” amplitude

[Rastelli, Zhou]

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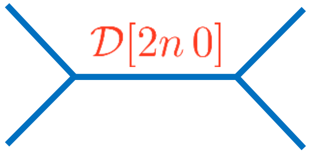
$$\mathcal{M}_{k,\mathrm{mero}} = \sum_{n=1}^{k-1} a_{k,\mathcal{D}[2n\,0]}$$ 

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Correlated!

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- Degree-8 in (s,t)
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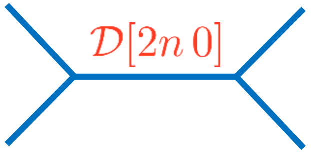
"Reduced" amplitude

[Rastelli, Zhou]

(2,0) 4-point functions

Solve by imposing maximal degree-p in large s,t limit

$$\mathcal{M}^{(p)}_{k,\mathrm{mero}} = \sum_{n=1}^{k-1} a^{(p)}_{k,\mathcal{D}[2n\,0]}$$



$$+ \mathcal{M}^{(p)}_{k,\mathrm{poly}}$$

$$\mathcal{M}^{(p)}_{k,\mathrm{pure-poly}}$$

This is equivalent to solving in the 1/c expansion, due to the correspondence between bulk derivatives (which carry powers of 1/c) and the degree of Mellin amplitudes: (Degree p) = 2p derivatives. Similarly, the cubic coefficients are

$$a^{(p)}_{k,\mathcal{D}[2n\,0]} = \lambda^2_{k,\mathcal{D}[2n\,0]}\Big|_{c^{-\frac{7+2p}{9}}}$$

(An equivalent argument is to use the flat space limit rather than the AdS action.)

11D Uplift Part 2

An algorithm:

1. Find an operator (value of k) whose degree-p Mellin amplitude may be completely fixed using the chiral algebra – that is, for which there is no pure polynomial term.
2. Uplift to 11D.

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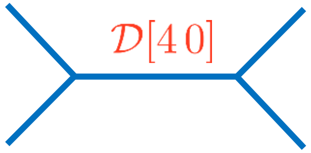

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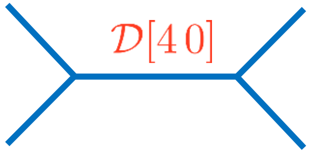

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To uplift to 11D, we need to generalize Penedones' formula to the case of KK modes.

The answer must be simple: *all* KK modes uplift to 11D gravitons.

Here is the answer:

$$\lim_{s,t\to\infty} \mathcal{M}_k^{(p)}(s,t;\sigma,\tau) = \mathcal{A}_\perp^{11,(p)}(s,t;\sigma,\tau) P_{k-2}(\sigma,\tau)$$

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Degree-(k-2) polynomial. Carries the "extra" polarizations that the 11D graviton doesn't have.

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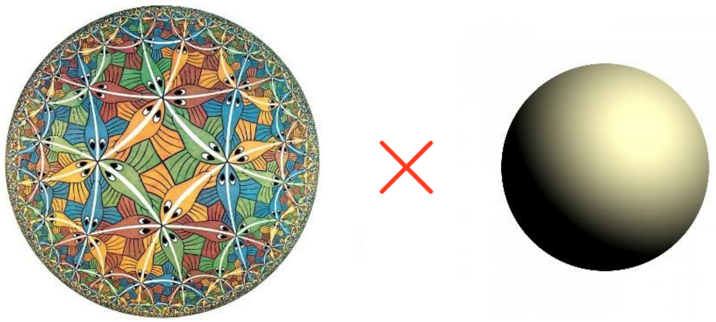
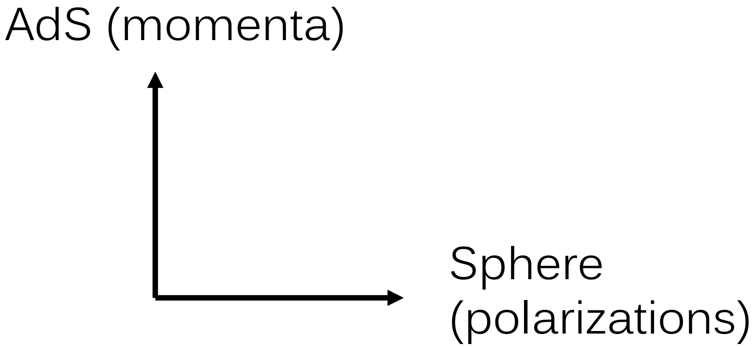
11D graviton amplitude in orthogonal kinematics



$$\mathcal{A}_\perp^{11,(p)}(s,t;\sigma,\tau) \equiv \frac{\mathcal{A}^{11,(p)}(p_i,Y_i)\big|_{p_i\cdot Y_i=0}}{(Y_1\cdot Y_2)^2(Y_3\cdot Y_4)^2}$$



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Take the limit on the level of the SC Ward ID itself:

$$\approx \left(\frac{4(k-1)+p}{128}\right) stu\, \Theta_4^{\text{flat}}(s,t;\sigma,\tau)\widetilde{M}_k^{(p)}(s,t;\sigma,\tau)|_{s,t\to\infty}$$

where

$$\Theta_4^{\text{flat}}(s,t;\sigma,\tau) \equiv \frac{\widehat{K}(p_i,Y_i)\big|_{p_i\cdot Y_i=0}}{4(Y_1\cdot Y_2)^2(Y_3\cdot Y_4)^2}$$

Precisely the kinematic factor appearing in 11D, in orthogonal kinematics.

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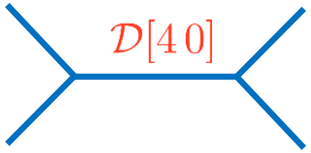
$$\lim_{s,t\to\infty} \mathcal{M}_k^{(p)}(s,t;\sigma,\tau) = \mathcal{A}_{\perp}^{11,(p)}(s,t;\sigma,\tau) P_{k-2}(\sigma,\tau)$$

Using our p=4 solution with OPE coefficient determined by W_N,

$$k=3: \quad \mathcal{M}_{3,\text{mero}}^{(4)} = -36 \cdot 2^{1/3}$$



$$+\mathcal{M}_{3,\text{poly}}^{(4)}$$

we uplift to R^4 in 11D:

$$\boxed{f_{R^4}(s,t) = \frac{stu}{3\cdot 2^7}}$$

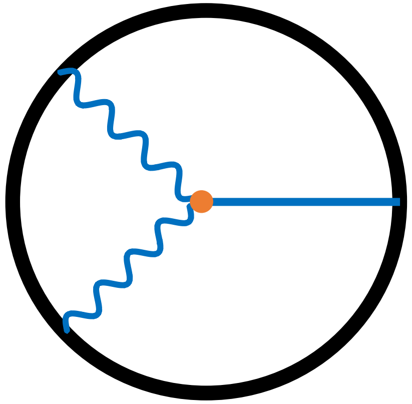
Outline

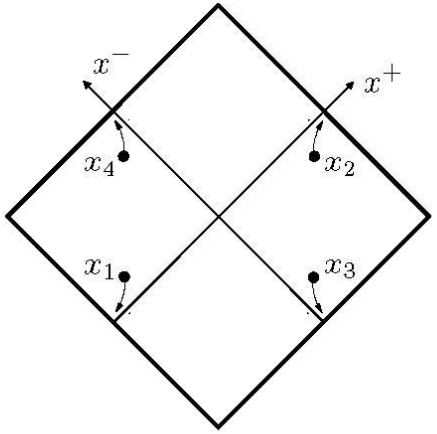
1. Review: defining holographic CFTs

2. Gravitational couplings to matter from CFT

3. M-theory effective action

4. Toward D^8R^4





D^8R^4 and beyond

What is the story with D^8R^4?

1) 11D Feynman diagrams insufficient (no SUSY protection)
2) No relation to IIA string perturbation theory

$$\mathcal{L}_{IIA} \supset (\alpha')^7 f(g_s) D^8 R^4$$

A finite term in 11D requires

$$f(g_s \gg 1) \sim g_s^{14/3}$$

(It is also possible that there are more divergent terms than this, that somehow all contribute to 11D non-analytic terms. Is there a meaningful separation?)

$f(g_s)$ is not known to obey any non-renormalization theorem, despite conjectures!

[See recent work of Bern et al]

D^8R^4 and beyond

The most basic question is:

Is D^8R^4 zero in 11D?

This can be mapped unambiguously to a statement about (2,0) CFT data at $O(c^{-23/9})$.

This data is not ½-BPS. Can the bootstrap calculate it?

e.g. consider O_4, the D[04] (¼-BPS) operator appearing in the OPE of two stress tensor multiplets.

$$\langle S_2 S_4 \mathcal{O}_4 \rangle \big|_{c^{-23/9}} = \# \times (\text{coeff of } D^8 R^4)$$

We derived a precise dictionary between 1/c CFT data and the 11D derivative expansion.

Lorentzian Inversion Formula

1. Write your 4-point function as a contour integral along the principal series

$$\mathcal{G}(z,\bar{z}) = \sum_J \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} d\Delta\, C(\Delta,J) G_{\Delta,J}(z,\bar{z})$$

2. Apply orthogonality condition for conformal partial waves in the principal series
3. Perform contour deformation/analytic continuation in the complex J-plane:

$$C(\Delta,J) = \frac{\kappa_{\Delta+J}}{4} \int_0^1 \int_0^1 \frac{dz}{z^2}\frac{d\bar{z}}{\bar{z}^2} \left|\frac{z-\bar{z}}{z\bar{z}}\right|^{d-2} G_{J+d-1,\Delta-d+1}(z,\bar{z})\, \mathrm{dDisc}[\mathcal{G}(z,\bar{z})]$$

where dDisc may be defined as

$$\mathrm{dDisc}[\mathcal{G}(z,\bar{z})] = \mathrm{Disc}^{\circlearrowleft}[\mathcal{G}(z,\bar{z})] + \mathrm{Disc}^{\circlearrowright}[\mathcal{G}(z,\bar{z})]$$

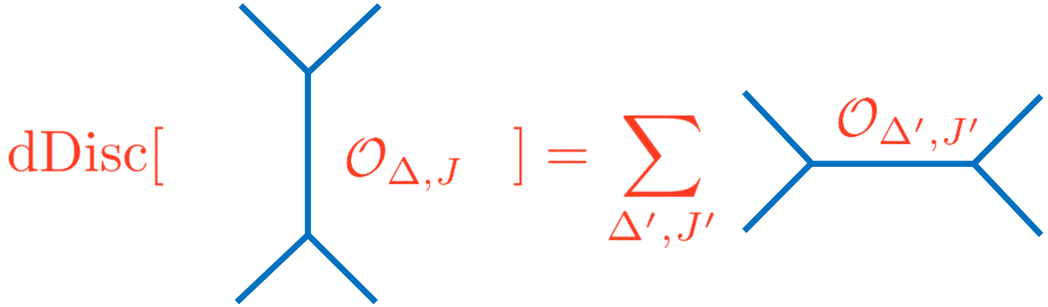
The OPE coefficients are given by the residues:

$$\operatorname*{Res}_{\Delta'=\Delta} C(\Delta',J) = -C_{12\mathcal{O}_{\Delta,J}} C_{34\mathcal{O}_{\Delta,J}}$$

[Caron-Huot]

Lorentzian Inversion Formula

Taking the dDisc of a t-channel conformal partial wave computes the four-point crossing kernel. This is a 6j-symbol of the conformal group.



[Liu, EP, Rosenhaus, Simmons-Duffin, to appear]

When (Δ,J) take double-trace values, dDisc = 0. These are zeroes of the 6j symbol.

In SCFT, many protected operators have precisely these dimensions!

This implies major reduction in determination of OPE data via inversion.

Still, one needs to know something about the long operator data… WIP!

Summary + Future directions

- Derived universal constraints on the CFT stress tensor and its dual gravitational couplings to matter in any theory of AdS quantum gravity,
- Extracted the 11D four-graviton amplitude from (2,0) CFT correlation functions, through R^4.

- D^8R^4 and beyond in 11D… Inversion formula in (2,0) CFT
- Higher derivatives in string theory?
- …