

SYK models from 2D gravity

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Outline

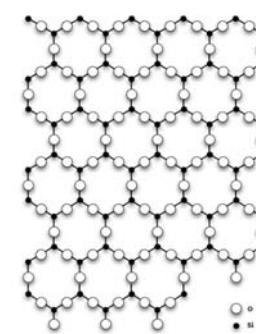
- Spin glass models
- Sachdev-Ye-Kitaev model
- Low energy Schwarzian effective action
- Connection to 2D gravity
- Correlation functions
- Conclusion

Order and disorder

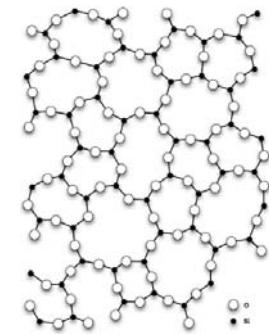
- **Ordered phase**

- Microscopically, perfect periodic arrangement
- Crystal solid, or magnetic (spin) order

Crystalline solid such as [Guartz](#)



Guartz



glass

- **Disordered phase**

- **Annealed** (gradually) one in thermodynamic **equilibrium**
- **Quenched** (rapidly) one in thermal **non-equilibrium**: (some parameters are random variables, which do not evolve)

Amorphous solid such as [glass](#)

Spin glass models

Refer to Micha Berkooz's talk on July 10th

- Spin glass model – disordered magnet ($2s+1$) $SU(2)$ $p = 2$

- Ising model : regular lattice, nearest-neighbor

S. F. Edwards and P. W. Anderson, J. Phys. F 5, 965 (1975).

$$H = - \sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j$$

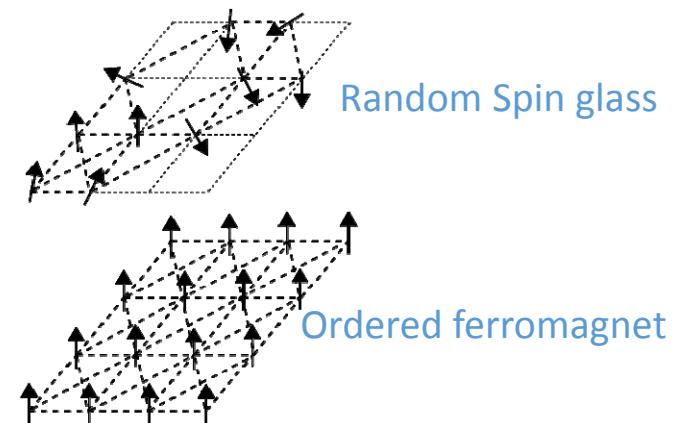
- 2-spin Infinite range random interaction

D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. 35, 1972 (1975).

- p-spin spherical model (pSM) model:
N-particle all to all random p-spin interaction, no spatial structure

$$H = - \sum_{i_1 > \dots > i_p=1}^N J_{i_1 \dots i_p} \sigma_{i_1} \dots \sigma_{i_p} \quad p \geq 3 \quad dp(J) = \exp\left(-\frac{1}{2} J^2 \frac{2N^{p-1}}{p!}\right) dJ \quad \sqrt{\overline{J^2}} \sim \frac{1}{N^{\frac{p-1}{2}}} \Rightarrow H \sim N$$

The factor N^{p-1} is essential in order Hamiltonian $\sim N$



By Leon Balents

SYK model

- SY model $SU(2) \rightarrow SU(M)$ 2SM

A Heisenberg ($p=2$) spin-chain model with $SU(N)$ spin and only $p=2$ -spin with infinite range all-to-random all interactions.

$$\mathcal{H} = \frac{1}{\sqrt{NM}} \sum_{i>j} J_{ij} \hat{\mathcal{S}}_i \cdot \hat{\mathcal{S}}_j$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

- Heisenberg model, no lattice
- Spin glass ($J \ll T$) \leftrightarrow disorderd spin fluid ($J \gg T$) without quasi-particle excitations. In comparison to a ordered (ferromagnetic) spin liquid.

- SYK model $SU(2) \rightarrow 4SM$

A quantum mechanical model composed of N -Majorana fermions with quenched random all-to-all quartic ($p=4$) spin interactions.

Kitaev's talks at KITP, USCB, 2015

$$H = \sum_{1 \leq i < j < k < l \leq N} J_{ijkl} \chi_i \chi_j \chi_k \chi_l \quad \langle J_{i_1 i_2 i_3 i_4}^2 \rangle = J^2 / N^3$$

- Outer-of-time-order correlation (OTOCs) to diagnose quantum chaos: the quantum system is scrambled by an initial perturbation, results in the Loss of phase coherence from electron interactions.
- Quantum chaos and ETH ETH refer to Jan de Boer's talk in July 3rd

The full action of SYK model

- N-Majorana fermion with all to all random quartic interactions at 1-D

$$S = \int d\tau \left(\frac{1}{2} \sum_{i=1}^N \chi_i \partial_\tau \chi_i - \frac{1}{4!} \sum_{i,j,k,l} J_{ijkl} \chi^i \chi^j \chi^k \chi^l \right), \quad \mathbb{E}[J_{ijkl}] = 0 \quad \mathbb{E}[J_{ijkl}^2] = 3!J^2/N^3$$

The action describes dynamics of spin glass, disordered metals without quasiparticles.

- It can be solved by averaging over disorder

- Dynamical mean field (DMF)

$$Z = e^{-\beta F} = \int \mathcal{D}G \mathcal{D}\Sigma \exp(-N\bar{S}), \quad \bar{S} = - \left[\ln \text{Pf}(\partial_\tau - \Sigma) - \frac{1}{2} \int d\tau_1 d\tau_2 \left(\Sigma G - \frac{J^2}{4} G^p \right) \right] \quad G = (\partial_\tau - \Sigma)^{-1}, \quad \Sigma = J^2 G^{p-1}.$$

- Low frequency strong coupling limit, at Large N, exactly solvable

$$1 \ll \beta J, \quad \tau J \ll N$$

$$T = 0 : \quad G(\tau) = \frac{b}{|\tau|^{2\Delta}} \text{sgn}(\tau),$$

$$T \neq 0 : \quad G(\tau) = b \left(\frac{\pi}{\beta \sin \frac{\pi \tau}{\beta}} \right)^{2\Delta} \text{sgn}(\tau),$$

$$G(\tau_1, \tau_2) \rightarrow (\phi'(\tau_1)\phi'(\tau_2))^{1/q} G(\phi(\tau_1), \phi(\tau_2))$$

$$\Sigma(\tau_1, \tau_2) \rightarrow (\phi'(\tau_1)\phi'(\tau_2))^{1-1/q} \Sigma(\phi(\tau_1), \phi(\tau_2))$$

Specialty of SYK model

- (1) Exactly solvable in strongly couple limit or low temperature ($J \gg T$), at large N , in DMF approximation, 2pt, 4pt ...
 $1 \ll \beta J, \tau J \ll N$ [J. Polchinski and V. Rosenhaus, 1601.06768](#)
- (2) Emergent symmetry at low T ($J \gg T$)
 - ($T=0$) Time reparameterization [Diff_1](#) invariant in [Schwinger-Dyson equations](#) and effective (mean field) action, broken [Diff_1](#) from IR.
 - (Finite T) [SL\(2,R\)](#) \sim [SO\(2,1\)](#) isometric to AdS2 in thermal equilibrium Green's function
 - Pseudo-Nambu-Goldstone (pNGBs) modes from symmetry breaking pattern
 - Spontaneous symmetry breaking: low energy vacuum state do not respect [Diff_1](#), but [SL\(2,R\)](#)
 - Explicit symmetry breaking: by the kinetic term, which is relevant at finite frequency
- (3) Exponential growth of [4-pt OTOC](#), [maximally chaotic](#), saturates [MSS maximum bound](#), means fastest possible quantum chaos.
[J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409](#) $\langle \{\psi_a(0), \psi_b(t)\}^2 \rangle \propto \frac{1}{N} e^{\lambda_L t}$
 - The [Lyapunov decay time](#) has a minimum bound (to reach quantum chaos)
$$\lambda_L \leq \frac{2\pi k_B T}{\hbar}$$

SYK model and Quantum gravity

- Motivation: A concrete fully quantum toy model similar to gravity at semi-classical level (QFT in curved spacetime), and identify some universal behaviors in quantum gravity, which involving non-locality at the Planck scale

- SYK model– a simple model of quantum holography
 - Time translation as Lorentz boost near the horizon

$$\tau(t) \equiv t + \epsilon k(t)$$

- Schwarzian action for pNGBs : thermal fluctuating circle in hyperbolic plane (CFT_1).

$$S_{\text{eff}} = -\frac{N\alpha}{J} \int d\tau \text{Sch}(f(\tau), \tau), \quad f(\tau) = \tan(\pi\tau/\beta)$$

- It is conjectured that SYK model is dual to JT model in nearly-AdS2 bulk.

K. Jensen 1605.06098

Jackiw 1985, Teitelboim 1983

J. Maldacena, D. Stanford and Z. Yang 1606.01857

- The SYK models have same low energy dynamics as black holes with AdS2 horizons.

R. A. Davison, W. Fu, A. Georges, Y. Gu, K. Jensen and S. Sachdev, 1612.00849

Refer to Thomas Gerd A Mertens' talk on July 9th

Connection to 2D gravity

- Generally speaking, we expect any theory of QG looks like Einstein gravity at large distance
- **Uniqueness of 2D gravity:** quantization of Einstein-Hilbert action in 2D is special, with **dilaton** degree of freedom

$$L_{HE} = -\frac{1}{2\kappa_D} \sqrt{-g}(R - 2\Lambda) \quad g_{\mu\nu} = \eta_{\mu\nu} + \kappa_2 h_{\mu\nu} \quad \bar{h}^\nu \equiv \partial_\mu h^{\mu\nu} - \partial^\nu h/2! \quad \sqrt{-g} = 1 + \kappa_2 h/2 - \kappa_2^2 (2h_\nu^\mu h_\mu^\nu - \bar{h}^2)/8 + O(\kappa_2^3)$$
$$\mathcal{L}_{gh} = \sqrt{\eta} \bar{h}^\lambda \bar{h}_\lambda / 2 \quad \langle h_{\mu\nu} h_{\rho\sigma} \rangle = \frac{1}{k^2} \left(\eta_{\mu\rho} \eta_{\nu\sigma} + \eta_{\mu\sigma} \eta_{\nu\rho} - \frac{2}{D-2} \eta_{\mu\nu} \eta_{\rho\sigma} \right)$$

- The effective boundary action from **2D gravity** with a **dilaton** field in the bulk, can describe quantum fluctuations of space boundary.

2D gravity models

- Many models

$$S_{\text{2DG}} = \frac{1}{2} \int d^2x \sqrt{-g} [XR + U(X) (\nabla X)^2 - 2V(X)] \quad \text{Grumiller and Meyer hep-th/0604049}$$

Model (cf. (1.1) or (2.2))	$U(X)$	$V(X)$	$w(X)$ (cf. (2.3))
1. Schwarzschild [5]	$-\frac{1}{2X}$	$-\lambda^2$	$-2\lambda^2\sqrt{X}$
2. Jackiw-Teitelboim [6, 7]	0	$-\Lambda X$	$-\frac{1}{2}\Lambda X^2$
3. Witten BH/CGHS [8, 9]	$-\frac{1}{X}$	$-2b^2X$	$-2b^2X$
4. CT Witten BH [8, 9]	0	$-2b^2$	$-2b^2X$
5. SRG ($D > 3$)	$-\frac{D-3}{(D-2)X}$	$-\lambda^2 X^{(D-4)/(D-2)}$	$-\lambda^2 \frac{D-2}{D-3} X^{(D-3)/(D-2)}$
6. $(A)dS_2$ ground state [10]	$-\frac{a}{X}$	$-\frac{B}{2}X$	$a \neq 2 : -\frac{B}{2(2-a)} X^{2-a}$
7. Rindler ground state [11]	$-\frac{a}{X}$	$-\frac{B}{2}X^a$	$-\frac{B}{2}X$
8. BH attractor [12]	0	$-\frac{B}{2}X^{-1}$	$-\frac{B}{2} \ln X$
9. All above: ab -family [13]	$-\frac{a}{X}$	$-\frac{B}{2}X^{a+b}$	$b \neq -1 : -\frac{B}{2(b+1)} X^{b+1}$
10. Liouville gravity [14]	a	$be^{\alpha X}$	$a \neq -\alpha : \frac{b}{a+\alpha} e^{(a+\alpha)X}$
11. Scattering trivial [15]	generic	0	const.
12. Reissner-Nordström [16]	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$	$-2\lambda^2\sqrt{X} - 2Q^2/\sqrt{X}$
13. Schwarzschild- $(A)dS$ [17]	$-\frac{1}{2X}$	$-\lambda^2 - \ell X$	$-2\lambda^2\sqrt{X} - \frac{2}{3}\ell X^{3/2}$
14. Katanaev-Volovich [18]	α	$\beta X^2 - \Lambda$	$\int^X e^{\alpha y} (\beta y^2 - \Lambda) dy$
15. Achucarro-Ortiz [19]	0	$\frac{Q^2}{X} - \frac{J}{4X^3} - \Lambda X$	$Q^2 \ln X + \frac{J}{8X^2} - \frac{1}{2}\Lambda X^2$
16. KK reduced CS [20, 21]	0	$\frac{1}{2}X(c - X^2)$	$-\frac{1}{8}(c - X^2)^2$
17. Symmetric kink [22]	generic	$-X \prod_{i=1}^n (X^2 - X_i^2)$	cf. [22]
18. 2D type 0A/0B [23, 24]	$-\frac{1}{X}$	$-2b^2X + \frac{b^2q^2}{8\pi}$	$-2b^2X + \frac{b^2q^2}{8\pi} \ln X$
19. exact string BH [25, 26]	(3.11)	(3.11)	(3.13)

$$I(X) := \exp \int^X U(y) dy, \quad w(X) := \int^X I(y)V(y) dy \quad (2.3)$$

2D quantum gravity

- A generic 2D dilaton bulk action with backreaction

$$S_{2D} = \frac{1}{2\kappa_2} \int d^2x \sqrt{-g} \left(\gamma\phi^2 R - V(\phi) + U(\phi)(\nabla\phi)^2 - \frac{1}{2}(\nabla f)^2 + \kappa[\chi R - (\nabla\chi)^2] \right) + \frac{1}{\kappa_2} \int dt \sqrt{-\gamma}(\gamma\phi^2 + \kappa\chi)K + S_{ct}$$

- Dilaton term
- N-conformal massless scalar matter (black hole evaporation, Hawking radiation)
- Back reaction: Polyakov-Liouville action \sim non-local Polyakov action
 - Schwarzian \sim CFT2 anomaly M. Cvetic and I. Papadimitriou 1608.07018
- Gibbons-Hawking boundary term
- Counter term

$$S_P = -\frac{\kappa}{8\kappa_2} \int d^2x \sqrt{-g} R \square^{-1} R$$

Specific 2D gravity models

- JT model : the simplest non-trivial theory of QG [Jackiw 1985, Teitelboim 1983](#)

$$S = \frac{1}{2\kappa_2} \int d^2x \sqrt{-g} [\phi^2 R - V(\phi)] + \frac{1}{\kappa_2} \int dt \sqrt{-\gamma} \phi^2 K, \quad V(\phi) = 2(c_2 - \phi^2), \quad c_2 = 0$$

- AP model [A. Almheiri and J. Polchinski, 1402.6334](#)

$$c_2 = 1 \quad \bar{V}(\phi) = \phi^{-\frac{\lambda}{2}} V(\phi), \quad \lambda = 0, \quad V(\phi) = 2(c_2 - \phi^2)$$

- CGHS + RST models [Callan, Giddings, Harvey and Strominger hep-th/9111056](#)

$$S_{CGHS} = \frac{1}{2\kappa_2} \int d^2x \sqrt{-g} [\phi^2 (R + 4\lambda'^2) + 4(\nabla\phi)^2] + S_m. \quad S_{RST} = \frac{\kappa}{2\kappa_2} \int d^2x \sqrt{-g} \left[R \left(\chi + \frac{1}{2}\phi \right) - (\nabla\chi)^2 \right] - S_{bd},$$

- Liouville gravity $\bar{V}(\phi) = \phi^{-\frac{\lambda}{2}} V(\phi), \quad \lambda = 4, \quad V(\phi) = -4\lambda'^2 \phi^2$

$$S_L = \frac{1}{2\kappa_2} \int d^2x \sqrt{-g} [\gamma \phi^2 R + 4\lambda \phi^2 (\nabla\phi)^2 + \mu e^{2b\phi^2}]$$

Refer to [Thomas Mertens' talk in July 9th](#)

$$\bar{S} = \int d^2x \sqrt{-\bar{g}} [\phi^2 \bar{R} - \bar{V}(\phi)], \quad \bar{V}(\phi) = \phi^{-\frac{\lambda}{2}} V(\phi).$$

Schwarzian action from 2D gravity

- The 2D dilaton gravity action in **conformal gauge**

$$\begin{aligned} S_{\text{bulk}} &= \frac{1}{\kappa_2} \int d^2x [-4\partial_+(\phi^2)\partial_-\rho + e^{2\rho}(\phi^2 - c_2)], & ds^2 &= -e^{2\rho} dx_+ dx_- \\ S_{\text{bd}} &= \frac{1}{\kappa_2} \int d^2x [\partial_t(\phi^2\partial_t\rho) - \partial_z(\phi^2\partial_z\rho)] - \frac{1}{\kappa_2} \int dt(\phi^2\partial_z\rho), & S_{\text{ct}} &= -\frac{1}{\kappa_2} \int dt \sqrt{-\gamma} c_1 r(\phi) = -\frac{1}{\kappa_2} \int dt e^\rho (\phi^2 - c_2) \end{aligned}$$

- Generic solution to dilaton in JT-AP models [A. Almheiri and J. Polchinski, 1402.6334](#)

$$\phi^2 = c_2 + \frac{c_1 + c_3(f_+ + f_-) + c_4f_+f_-}{f_+ - f_-}.$$

- The backreaction conformal factor and dilatons

$$e^{2\rho} = \frac{4f'_+f'_-}{(f_+ - f_-)^2}, \quad \phi^2 = c_2 + c_1 \frac{1 - \mu f_+ f_-}{f_+ - f_-}.$$

$$\begin{aligned} x_\pm &= t \pm \epsilon & e^{2\rho} &= \frac{1}{\epsilon^2} + \frac{2}{3}\text{Sch} + \frac{4}{15}\text{Sch}^2\epsilon^2 + O(\epsilon^4), \\ \text{Light cones} & \quad \phi^2 = \frac{c_1}{2\epsilon} + c_2 - \frac{c_1}{3}\text{Sch} \epsilon - \frac{2c_1}{45}\text{Sch}^2\epsilon^3 + O(\epsilon^4), \end{aligned}$$

$$\begin{aligned} S_{\text{ren}} &= S_{\text{bulk}} + S_{\text{ct}} \\ &= \frac{c_1}{\kappa_2} \int dt \left(\frac{\text{Sch}}{2} + \frac{\text{Sch}^2\epsilon^2}{4} + \frac{\text{Sch}^3\epsilon^4}{12} + O(\epsilon^6) \right) \end{aligned}$$

Correlation functions of matter

- Effective boundary action

$$S_\chi = \frac{1}{2} \int d^2x \sqrt{-g} [(\nabla\Phi)^2 + m^2\Phi^2]$$

$$= -N \int dt dt' \frac{\Phi_0(t)\Phi_0(t')}{|t-t'|^{2\Delta}} + \dots, \quad \Delta \geq 1$$

$$\Phi(t, z) = z^{1-\Delta}\Phi_0(t) + \dots \quad z \rightarrow 0,$$

$$\Phi_0(t) = [f'(t)]^{1-\Delta} \Phi_0(f(t))$$

$$S_{\text{eff}} = -N \int dt dt' \left(\frac{f'(t)f'(t')}{[f(t)-f(t')]^2} \right)^\Delta \Phi_0(t)\Phi_0(t')$$

- Generating functions of correlation functions

$$W = \ln \langle Z \rangle = \ln \langle e^{-S_{\text{eff}}} \rangle = \ln \left[\int \mathcal{D}\mu[f] e^{-\bar{S}_0} \left(1 + \epsilon^2 N \int \prod_{i=1}^2 dt_i \Phi_0(t_i) \frac{\mathcal{C}_2(t_{12}) + \epsilon^2 \mathcal{C}_4(t_{12})}{(2 \sin \frac{t_{12}}{2})^{2\Delta}} \right. \right.$$

$$+ \frac{1}{2} \epsilon^2 \int \prod_{i=1}^4 dt_i \Phi_0(t_i) \frac{:\mathcal{C}_1(t_{12})\mathcal{C}_1(t_{34}): + \epsilon^2 (: \mathcal{C}_2(t_{12})\mathcal{C}_2(t_{34}) : + 2 : \mathcal{C}_1(t_{12})\mathcal{C}_3(t_{34}) :)}{(2 \sin \frac{t_{12}}{2})^{2\Delta} (2 \sin \frac{t_{34}}{2})^{2\Delta}}$$

$$+ \frac{1}{6} \epsilon^4 \int \prod_{i=1}^6 dt_i \Phi_0(t_i) \frac{3 : \mathcal{C}_1(t_{12})\mathcal{C}_1(t_{34})\mathcal{C}_2(t_{56}) : + \epsilon^2 (: \mathcal{C}_2(t_{12})\mathcal{C}_2(t_{34})\mathcal{C}_2(t_{56}) : + 6 : \mathcal{C}_1\mathcal{C}_2\mathcal{C}_3 : + 3 : \mathcal{C}_1\mathcal{C}_1\mathcal{C}_4 :)}{(2 \sin \frac{t_{12}}{2})^{2\Delta} (2 \sin \frac{t_{34}}{2})^{2\Delta} (2 \sin \frac{t_{56}}{2})^{2\Delta}}$$

$$\left. \left. + \frac{1}{24} \epsilon^4 \int \prod_{i=1}^8 dt_i \Phi_0(t_i) \frac{:\mathcal{C}_1(t_{12})\mathcal{C}_1(t_{34})\mathcal{C}_1(t_{56})\mathcal{C}_1(t_{78}) : + \epsilon^2 (6 : \mathcal{C}_1(t_{12})\mathcal{C}_1(t_{34})\mathcal{C}_2(t_{56})\mathcal{C}_2(t_{78}) : + 4 : \mathcal{C}_1\mathcal{C}_1\mathcal{C}_1\mathcal{C}_3 :)}{(2 \sin \frac{t_{12}}{2})^{2\Delta} (2 \sin \frac{t_{34}}{2})^{2\Delta} (2 \sin \frac{t_{56}}{2})^{2\Delta} (2 \sin \frac{t_{78}}{2})^{2\Delta}} + \dots \right) \right]$$

$$\left\langle \prod_{i=1}^{2n} \mathcal{O}_{\Phi_0}(t_i) \right\rangle = \frac{1}{Z(\Phi_0)} \prod_{i=1}^{2n} \frac{\delta}{\delta \Phi_0(t_i)} Z(\Phi_0)|_{\Phi_0 \rightarrow 0}$$

- Thermal/Quantum correlation functions

$$f(t) = \tan \frac{t + \epsilon k(t)}{2}$$

thermal fluctuating circle

$$G_2 = N \frac{1 + \epsilon^2 \langle \mathcal{C}_2(t_{12}) \rangle}{(2 \sin \frac{t_{12}}{2})^{2\Delta}}, \quad G_4 = N \frac{\epsilon^2 \langle :\mathcal{C}_1(t_{12})\mathcal{C}_1(t_{34}): \rangle}{(2 \sin \frac{t_{12}}{2})^{2\Delta} (2 \sin \frac{t_{34}}{2})^{2\Delta}},$$

$$G_6 = N \frac{\epsilon^4 \langle :\mathcal{C}_1(t_{12})\mathcal{C}_2(t_{34})\mathcal{C}_1(t_{56}): \rangle}{(2 \sin \frac{t_{12}}{2})^{2\Delta} (2 \sin \frac{t_{34}}{2})^{2\Delta} (2 \sin \frac{t_{56}}{2})^{2\Delta}}, \quad G_8 = N \frac{\epsilon^4 \langle :\mathcal{C}_1(t_{12})\mathcal{C}_1(t_{34})\mathcal{C}_1(t_{56})\mathcal{C}_1(t_{78}): \rangle}{(2 \sin \frac{t_{12}}{2})^{2\Delta} (2 \sin \frac{t_{34}}{2})^{2\Delta} (2 \sin \frac{t_{56}}{2})^{2\Delta} (2 \sin \frac{t_{78}}{2})^{2\Delta}},$$

$$G_{10} = N \frac{\epsilon^6 \langle :\mathcal{C}_1(t_{12})\mathcal{C}_1(t_{34})\mathcal{C}_2(t_{56})\mathcal{C}_1(t_{78})\mathcal{C}_1(t_{9,10}): \rangle}{(2 \sin \frac{t_{12}}{2})^{2\Delta} (2 \sin \frac{t_{34}}{2})^{2\Delta} (2 \sin \frac{t_{56}}{2})^{2\Delta} (2 \sin \frac{t_{78}}{2})^{2\Delta} (2 \sin \frac{t_{9,10}}{2})^{2\Delta}},$$

Two-point thermal correlation function

- Parameterization, $k(t)$ is fluctuation of time

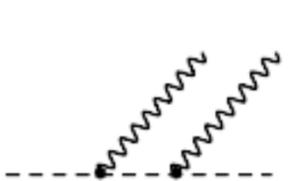
$$f(t) = \tan \frac{t + \epsilon k(t)}{2}.$$

A thermal correlation functions in global AdS2 \sim a zero temperature quantum correlation in Poincare cord.

propagators



$$G_0(t_1, t_2) = \frac{1}{\left(2 \sin \frac{t_{12}}{2}\right)^{2\Delta}} = \left(\frac{\pi}{\beta \sin \frac{\pi t_{12}}{\beta}}\right)^{2\Delta}$$



$$\mathcal{C}_1(t_{12}) = \Delta \left(k'(t_1) + k'(t_2) - \frac{k(t_1) - k(t_2)}{\tan \frac{t_{12}}{2}} \right)$$

$$\begin{aligned} \mathcal{C}_2(t_{12}) &= \frac{1}{2} \Delta^2 \left(k'(t_1) + k'(t_2) + \frac{k(t_2) - k(t_1)}{\tan \frac{t_{12}}{2}} \right)^2 \\ &\quad + \frac{1}{4} \Delta \left(\frac{[k(t_1) - k(t_2)]^2}{(\sin \frac{t_{12}}{2})^2} - 2[k'(t_1)^2 + k'(t_2)^2] \right), \end{aligned}$$

$$G(t_1, t_2) = G_0(t_1, t_2)[1 + \epsilon \mathcal{C}_1(t_{12}) + \epsilon^2 \mathcal{C}_2(t_{12}) + O(\epsilon^3)]$$

$k(t)$ is virtual correction and can only be present as inner lines.

$$\begin{aligned} \epsilon^2 \langle k(t_1)k(t_2) \rangle &= \frac{1}{6\pi C} \sum_{n \neq 0} \frac{e^{int}}{n^4} \\ &= \frac{1}{6\pi C} \left(\frac{1}{24}(|t| - \pi)^4 - \frac{\pi^2}{12}(|t| - \pi)^2 + \frac{7\pi^4}{360} \right), \\ \epsilon^2 \langle k(t_1)k'(t_2) \rangle &= \frac{1}{36\pi C} \text{sgn}(t)(\pi - |t|)(2\pi - |t|)|t|, \\ \epsilon^2 \langle k'(t_1)k'(t_2) \rangle &= \frac{1}{6\pi C} \left(-\frac{\pi^2}{6} + \frac{1}{2}(\pi - |t|)^2 \right) \\ \epsilon^2 \langle k(t_1)k(t_2) \rangle &= \frac{1}{2\pi C} \sum_{n \neq 0, \pm 1} \frac{e^{int}}{n^2(n^2 - 1)} \\ &= \frac{1}{2\pi C} \left(1 + \frac{\pi^2}{6} - \frac{(|t| - \pi)^2}{2} + (|t| - \pi) \sin |t| + \frac{5}{2} \cos |t| \right), \\ \epsilon^2 \langle k(t_1)k'(t_2) \rangle &= \frac{\text{sgn}(t)}{2\pi C} \left((\pi - |t|)(1 - \cos |t|) - \frac{3}{2} \sin |t| \right), \\ \epsilon^2 \langle k'(t_1)k'(t_2) \rangle &= \frac{1}{2\pi C} \left(1 + \frac{1}{2} \cos |t| - (\pi - |t|) \sin |t| \right), \end{aligned}$$

Schwarzian correction to 2-pt correlation

“Slope-dig-rap” shoulder

- 2-pt

$$\beta = 2\pi$$

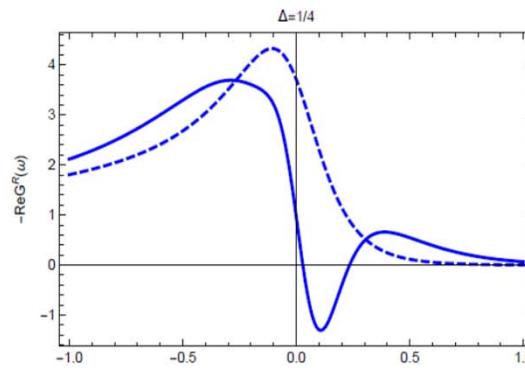
$$C = 1/(2\pi)$$

$$\epsilon^2 \langle \mathcal{C}_2(t_{12}) \rangle$$

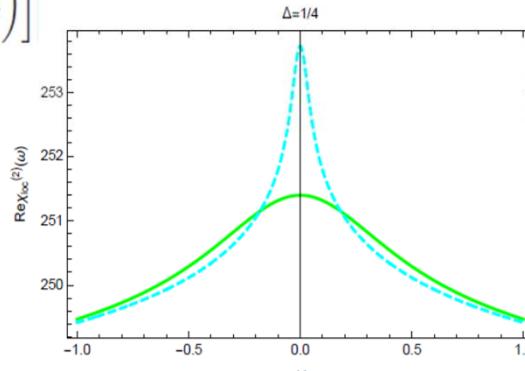
$$= \frac{1}{2\pi C} \left[\frac{\Delta}{4 \sin^2 \frac{t_{12}}{2}} [t_{12}^2 - 2\pi t_{12} + 2(\pi - t_{12}) \sin t_{12} + 4 \sin^2 \frac{t_{12}}{2}] + \frac{\Delta^2}{2} \left(\frac{t_{12} - 2\pi}{\tan \frac{t_{12}}{2}} - 2 \right) \left(\frac{t_{12}}{\tan \frac{t_{12}}{2}} - 2 \right) \right]$$

$$\beta = 20\pi$$

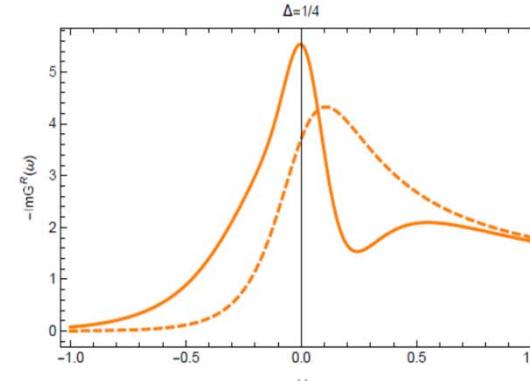
$$\beta = 2\pi$$



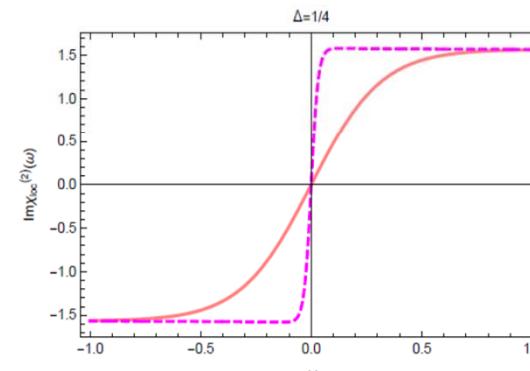
(a)- $\text{Re}G^R(\omega)$



(c) $\text{Re}\chi_{\text{loc}}^{(2)}(\omega)$



(b)- $\text{Im}G^R(\omega) \sim \text{DOS}$



(d) $\text{Im}\chi_{\text{loc}}^{(2)}(\omega)$

$$S_{\text{eff}} = -C_g \int dt \phi_r(t) \text{Sch}(f(t), t)$$

$$C \propto C_g \bar{\phi}_r$$

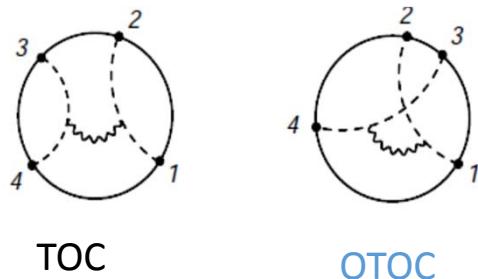
$$\frac{I_S}{N} = \frac{\#}{J} \int d\tau \text{Sch}(f(\tau), \tau) + \dots$$

$$J \sim N/C$$

$$\text{Im } \chi_{\text{loc}}^{(2)}(\omega) \sim \tanh(\omega\beta/2)$$

Four-point correlation function

- 4-pt



$$F_{VWVW}^{(4)} = F_{VvVwW}^{(4)} + \frac{\Delta^2}{C} \left(\frac{t_{23}}{\tan \frac{t_{12}}{2} \tan \frac{t_{34}}{2}} - 2 \frac{\sin \frac{t_{23}}{2} \cos \frac{t_{14}}{2}}{\sin \frac{t_{12}}{2} \sin \frac{t_{34}}{2}} \right)$$

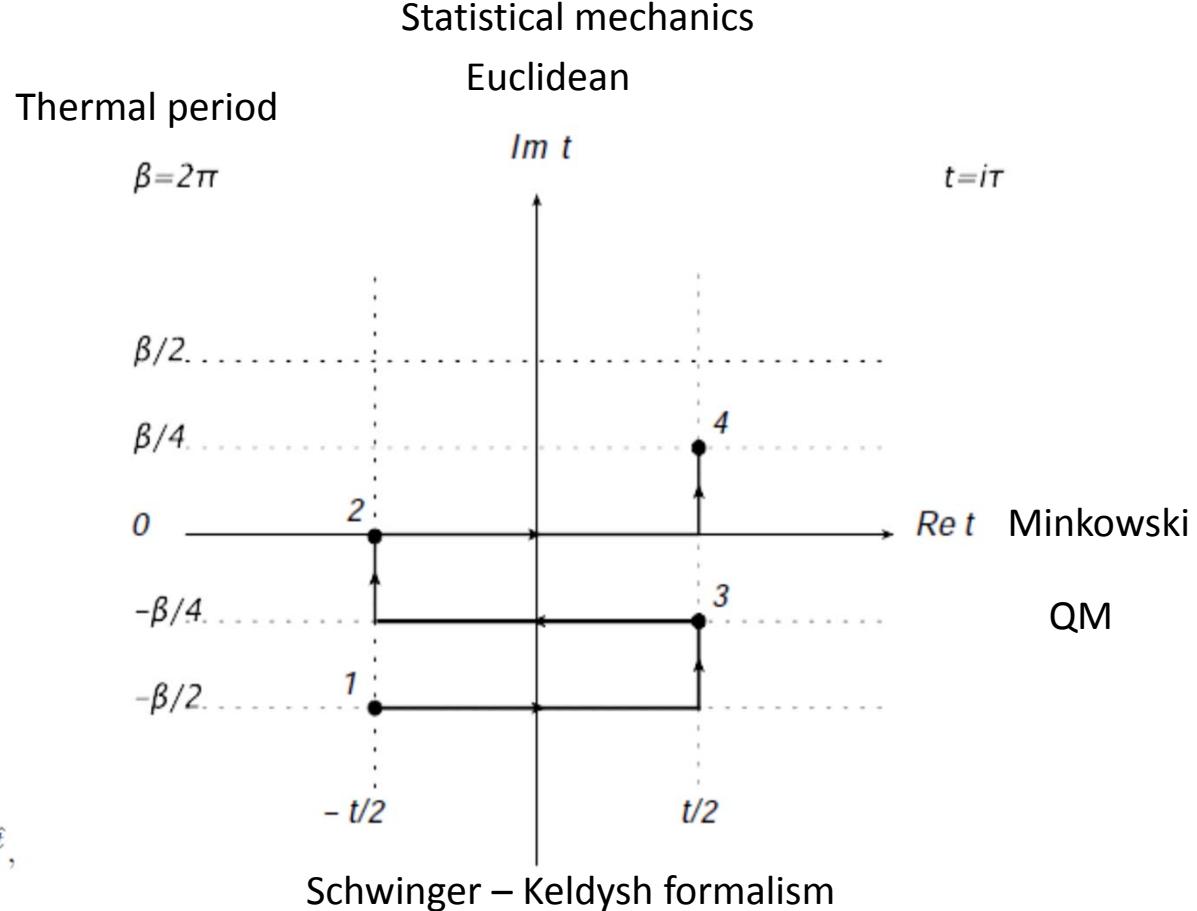
$$F_{VVWW}^{(4)} = \frac{2\Delta^2}{\pi C} \left(1 - \frac{t_{12}}{\tan \frac{t_{12}}{2}} \right) \left(1 - \frac{t_{34}}{\tan \frac{t_{34}}{2}} \right)$$

J. Maldacena and D. Stanford, 1604.07818

- Maximally chaotic in

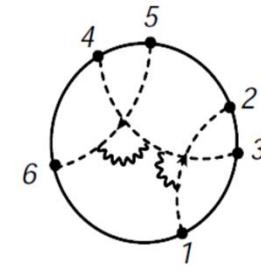
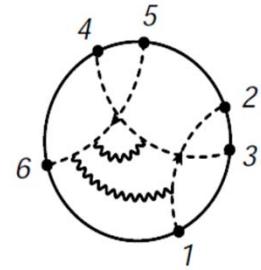
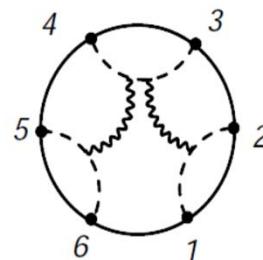
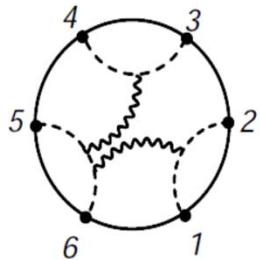
$$G_{VvVwW}^{(4)} = \frac{1}{4^{2\Delta}} \frac{2\Delta^2}{\pi C},$$

$$G_{VWVW}^{(4)} = -\frac{1}{4^{2\Delta}} \frac{\Delta^2}{C} \cosh \hat{t} \sim \frac{1}{4^{2\Delta}} \beta \frac{\Delta^2}{C} e^{\lambda_L \hat{t}},$$



High point correlation functions

- 6-pt



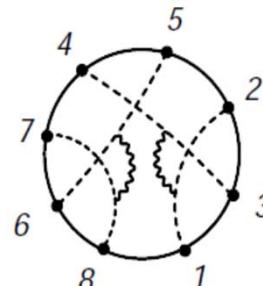
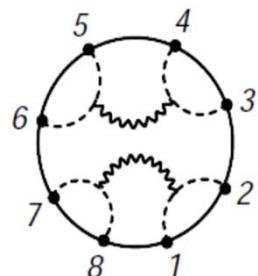
$$F_{VvVwWxx} = \epsilon^3 \langle \mathcal{C}_1(t_{12})\mathcal{C}_1(t_{34})\mathcal{C}_2(t_{56}) \rangle$$

$$\tilde{F}_{VvVwXwx} - \tilde{F}_{VvVwWxx} - \tilde{F}_{VvVwXwx} + \tilde{F}_{VvVwWxx}$$

$$\frac{\Delta^4}{C^2} \cosh^2(\hat{t}) \sim \frac{\Delta^4}{4C^2} e^{2\lambda_L \hat{t}} \quad \lambda_L = 2\pi/\beta$$

F. M. Haehl and M. Rozali, 1712.04963 k-OTCs

- 8-pt



Conclusions

- SYK model is a [microscopic quantum toy model](#) aiming at identifying universal behavior of quantum gravity at semi-classical level.
- It is insufficient to confirm there is [a local bulk dual](#) for the full SYK model.
- In [low energy effective theory](#), SYK model is described by the Schwarzian. The action can be naturally obtained from 2D gravity models.
- We established [a relatively simple reductive method](#) in the framework of QFT [to calculate higher-point correlation functions at large N](#). The 4-pt, 6-pt correlation functions results are consistent with those in SYK model.
- We find low energy Schwarzian correction to retarded Green's function in SYK model, i.e., the [Hubbard band in DOS](#), a signature of strong correlation.

Thank you for your attention!